SPATIAL ANALYSIS TECHNIQUES FOR MAPPING THE ANNUAL SURFACE RUNOFF IN SICILY UNDER THE BUDYKO'S FRAMEWORK

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KEY POINTS

- *This work shows a new approach, based on the Budyko framework, for mapping the mean annual surface runoff and deriving the probability distribution of the annual runoff in arid and semi-arid watersheds.*
- *We analytically provide the annual runoff distribution as the derived distribution of annual rainfall and potential evapotranspiration.*
- *The simulated long-term annual runoff and its distribution have been compared with historical records at several gauged stations, obtaining satisfactory matching.*

1 INTRODUCTION

Budyko was one of the pioneers in modeling energy and water balance within a basin, searching, at the same time, links between climate, runoff and evapotranspiration (Budyko, 1961; Budyko, 1974). He empirically derived the homonymous curve which relates evapotranspiration losses to a dryness index, defined as the ratio of potential evapotranspiration to precipitation. This curve describes the partitioning of precipitation into evapotranspiration and runoff, assuming steady-state conditions and that hydrological processes are driven by the macro-climate. These two conditions refer to the temporal and spatial scales of application, namely long-term averages and large catchments. Starting from the semi-empirical equation proposed by Budyko, several formulations have been developed. Two of the most diffused are the Turk-Pike (Turc, 1954; Pike, 1964) and the Fu (Fu, 1981) mono-parametric formulations. Accordingly to the Fu's mono-parametric formulations, the ratio between actual evapotranspiration and rainfall is shaped with only one parameter, commonly named ω, that should be calibrated using measured data and it has been proved to reflect the role of vegetation, soil and topography on precipitation partitioning.

Following the intuition attributable to Yang *et al.* (2007), under the difficulties induced by the seasonality of rainfall in counter-phase with temperature, this work aims to describe a method to estimate annual surface runoff and its interannual variability in Mediterranean arid and semiarid watersheds, under the Budyko framework. A methodology based on the Budyko framework is first used to estimate the long-term water balance components. Two spatial scales are investigated and integrated, namely pixel and basin spatial scale. The first novelty of this work lies in the integration in a GIS framework of the Budyko theory, aimed at deriving the raster layer of long-term runoff for a given region. This will be performed by integrating upstream contributions, which, in turns, can be derived from point information, as rainfall and potential evapotranspiration, and using regional rules for water partitioning.

The second novelty of this work is the analytical derivation of the annual runoff probability distribution. This method is based on a radical simplification of the Budyko equation, under the assumption of negligible water transfer among consecutive hydrological years. Annual rainfall and potential evapotranspiration probability distributions, together with two parameters describing regional simplified water partitioning rules, are the only inputs required for the computation of the annual runoff probability distribution. This novel method is coupled with a similar numerical one, based on Monte Carlo simulations, with the aim to highlight limitations and potentialities of the analytical approach.

The entire island of Sicily (southern Italy) has been used as case study. Regional rules for rainfall partitioning in runoff and evapotranspiration have been derived from observations. Long-term water balance components, recorded in several river basins, have been used in order to fit the parameter ω in Fu's equation. The same curve has been used in order to describe annual water partitioning, after verifying that ω calibrated from the long-term mean water balance is highly correlated with the one optimized from the water balance of individual year. Results point out the possibility to obtain reliable long-term runoff raster layers and probability distributions of annual runoff also at ungauged basins.

2 METHODS

For a given basin the actual evapotranspiration rate E and runoff Q are governed mainly by the amount of available energy and precipitation R. The water balance in a basin for any given time interval can be expressed as the variation of water stored in the basin S, set equal to the difference among precipitation, actual evapotranspiration and runoff. Considering point scale, the water balance has still a physical meaning if we consider Q as the contribution to the downstream runoff and precipitation R as the only water input. Over large time scales, it can be reasonably assumed the steady state condition, which means negligible water accumulation at basin scale. Under this assumption Budyko (1974) suggested a functional link relating the ratio between actual evapotranspiration and precipitation with the aridity index, defined as the ratio between potential evapotranspiration E_p and precipitation. Accordingly to the Fu's (Fu, 1981) mono-parametric formulations, the ratio between actual evapotranspiration and rainfall is shaped with only one parameter, commonly named ω, that should be calibrated using measured data. Under the above mentioned assumptions is thus possible to assess the mean long term runoff $\overline{Q}$ (hereafter the hat indicates long term means) as a function of potential evapotranspiration $\overline{E_p}$ and precipitation $\overline{R}$. While the Fu's equation has been derived for a whole catchment, we tested the hypothesis that this equation is also reasonably valid at point scale and allows a robust estimate of annual runoff when contributions are opportunely averaged at the basin outlet.

Water partitioning rule at the annual time scale may differ from the one depicted by the Fu's equation calibrated at the basin scale for long time scales, because of the water storage presence. In fact, the residual water from a previous hydrological year remains available at the successive year for evapotranspiration in addition to rainfall. Differently, in arid Mediterranean basins, due to low inter-annual water storage, the summer acts as a renewal event which erases the memory of the past dynamics (Viola *et al.*, 2008). As a result, the interannual soil-moisture, evapotranspiration and runoff dynamics can be conceptually represented as a sequence of disconnected years with distinguished wet and dry seasons. Following this consideration, the water partitioning rules given by the Budyko's curve can also be used at the annual time scale, as done by Yang *et al.* (2007) for arid catchments in China.

In this work the Fu's equation, after opportune calibration of the parameter ω, has been used for determining annual runoff Q and evapotranspiration from annual rainfall and potential evapotranspiration. Furthermore, the probability distribution of Q has been obtained as the derived distribution of R and E_p, of course once known the two latter distributions. Assuming that R and E_p are independent random variable, the analytical solution for the derived distribution of Q has been calculated, first, looking for the inverse function of runoff and, then, integrating the joint distribution of rainfall and potential evapotranspiration in the whole domain of physically admissible potential evapotranspiration. Alternatively, the derived distribution has been numerically obtained through Monte Carlo simulations, namely sampling R and E_p from opportune distributions.

3 RESULTS

The considered study area is covered by a considerable number of rainfall, temperature and hydrometric gauging stations managed by the OA–ARRA (*Osservatorio delle Acque–Agenzia Regionale per i Rifiuti e le Acque*) since the 1921. From the original dataset, a total number of 53 hydrometric gauging stations, with more than 10 years of observations within the period 1968-1997, have been selected; such stations collect daily runoff data from basins with areas ranging from 9 to 1086 km^2. Daily rainfall recorded by 248 rainfall stations of the OA–ARRA within the same time period have been aggregated at annual time scale and used to estimate $\overline{R}$ at each gauge location, while daily temperatures, recorded by 75 thermometric stations of the same agency for the same period, have been aggregated at the monthly scale in order to calculate the potential evapotranspiration $\overline{E_p}$ using Thornthwaite equation. Using spatial interpolation procedures (Di

Piazza *et al.*, 2011; Di Piazza *et al.*, 2015), we obtained the raster layer of mean value of annual rainfall $\overline{R}$ and potential evapotranspiration $\overline{E_p}$ at regional scale.

First we created a tool able for deriving and mapping the annual runoff $\overline{Q}$, also at ungauged basins. In order to do that, we assume that the Fu's equation is also valid at pixel scale; under this assumption, it is possible to derive the regional runoff layer using the spatial analysis procedure. The final output of the proposed algorithm is a vector map, shown in Figure 1, in which river is represented with a graduated line (i.e. the line thickness is proportional to the discharge [l/s] above a threshold of 50 l/s). The proposed methodology has been tested against observations for all the gauged Sicilian basins. The R^2 coefficient, calculated considering the whole sample of basins, is equal to 0.91 thus demonstrating that the proposed methodology is effective in reproducing annual runoff. In order to test the robustness of the proposed methodology and prove its performances at ungauged locations, *jackknife* validation has been performed, obtaining good results.

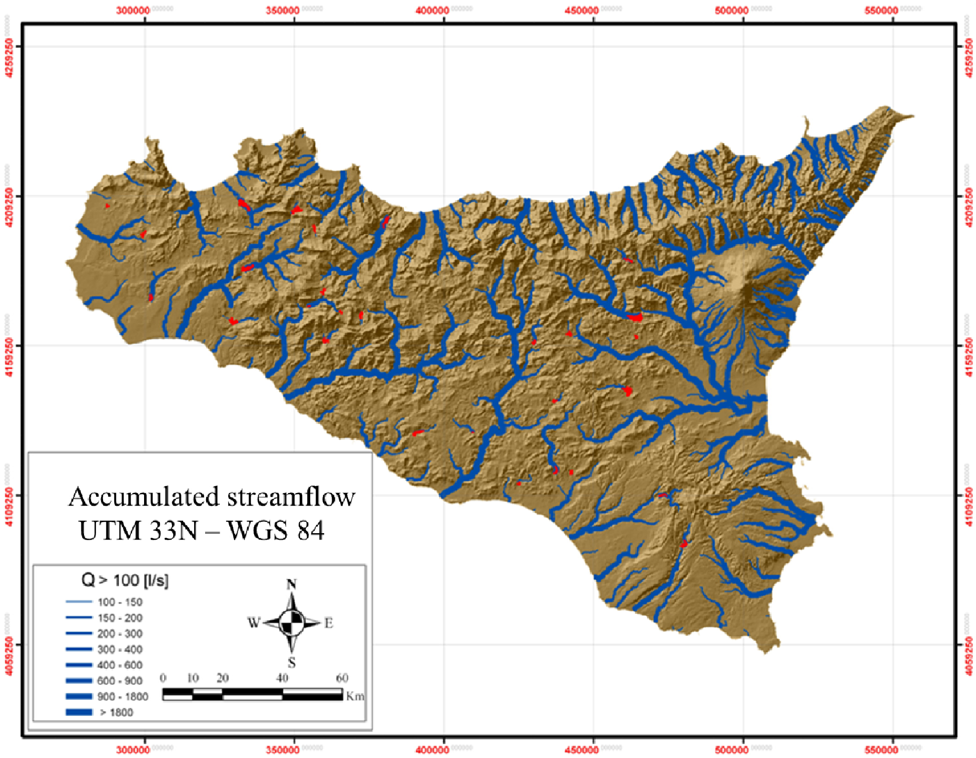

Figure 1. Accumulated streamflow map.

Theoretical cumulative distribution function (*cdf*) of the annual surface runoff has been computed from Monte Carlo simulations. Considering that, for arid catchments, the water balance could be considered as closed within any hydrologic year, the distribution of Q can be easily obtained under the assumption that R and E_p are independent and log-normal distributed. For each of the considered basins, two independent random sampling of R and E_p, have been provided as input to the annual Fu's equation obtaining a large sample of annual values of Q and computing the corresponding theoretical *cdf*. The comparison between observed annual runoff *cdf* and those obtained through the Monte Carlo simulations has been reported, for a representative basin, in Figure 2, together with the *cdf* of annual rainfall and potential evapotranspiration. The graphical comparison between empirical and theoretical *cdfs* of the annual surface runoff shows how the model is able to mimic the observations. A statistical assessment of the distance between the two curves for all the considered basins has been carried out using the Kolmogorov-Smirnov's test. The null hypothesis, that the samples are drawn from the same distribution, cannot be rejected for 87% of the considered basins.

This work has also proposed an analytical methodology that, under some reasonable assumptions, offers the annual runoff probability distribution in a closed form. Under the assumption of simplified rules for water partitioning and assuming that annual rainfall and potential evapotranspiration are independent and normally distributed, it is possible to obtain runoff distribution in all the considered basins. Results of the approach have been reported for the same basin used to show the Monte Carlo method in Figure 2 with the aim of highlighting model potentialities and to offer a comparison with the previous approach (Monte Carlo) and the observed data, showing a really good fit.

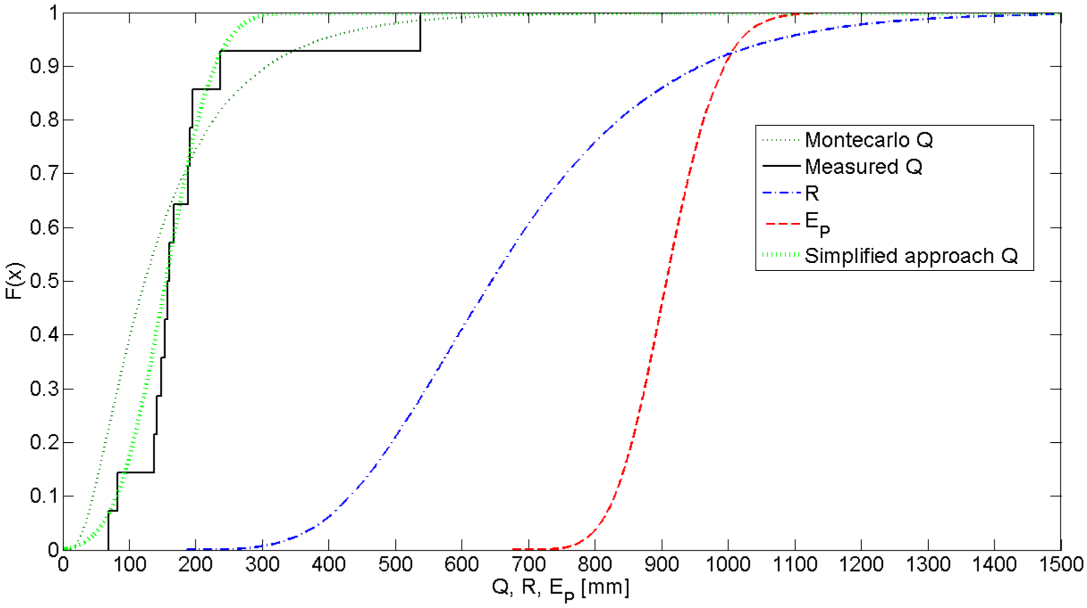

Figure 2. Comparison between observed annual runoff cdf (black line) and these obtained through the Monte Carlo simulations (dark green dots) and the analytical approach (green dashed line) in one of the considered basin. Also climatic forcings are reported: rainfall *R* (blue dashed line) and potential evapotranspiration E_P (red dashed line).

It must be recognized that obtaining the annual runoff probability density function in a closed form as a function of simple climatic statistics and simplified regional partitioning rules represents an important result of this work. Indeed, the annual runoff is fully described using few parameters: the parameter ω of the Fu's equation and the first and second order statistics of annual rainfall and potential evapotranspiration. Apart from the appealing simplicity of the analytical method, it has been verified its ability in reproducing annual runoff distribution in gauged basins. Performances are satisfactory and comparable with those obtained with more complex methods, i.e. Monte Carlo simulations.

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