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VITEBSK STATE MEDICAL UNIVERSITY

MEDICAL AND BIOLOGICAL PHYSICS

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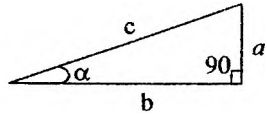
LECTURE №1

DIFFERENTIAL AND INTEGRAL CALCULUS

1. Some data from elementary mathematics and physics, used in lecture

RUSSIAN	*ENGLISH*	*RUSSIAN*	*ENGLISH*
Элементарный	Elementary	Сведения	Information
Математика	Mathematics	Физика	Physics
Уравнение	Equation	Площадь	Area
Круг	Circle	Корень	Root
Свойство	Law, Property	Функция	Function
Скорость	Speed, Velocity	Ускорение	Acceleration
Аргумент	Argument	Знак	Sign, Symbol
Приращение	Increment	График	Graph, Diagram
Закон	Law	Логарифм	Logarithm
lg x	Decimal log: $\log_{10}x$	ln x	Natural log: $\log_e x$
tg x	Tangent: tan x	ctg x	Cotangent: cot x
arcsin x	Arc sine:arcsin x	arctg x	Arc tangent: arc tan x

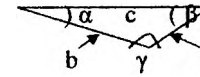
1. 

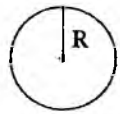
$$\sin\alpha = \frac{a}{c}; \quad \cos\alpha = \frac{b}{c}; \quad tg\alpha = \frac{a}{b}.$$

$$\sin^2\alpha + \cos^2\alpha = 1; \; tg\,\alpha = \frac{\sin\alpha}{\cos\alpha}; \; ctg\,\alpha = \frac{\cos\alpha}{\sin\alpha}$$

Theorem of sine: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

Theorem of cosine: $a^2 = b^2 + c^2 - 2ab\cos\alpha$



2. 

Area of a circle: $S = \pi R^2$, where R is radius, $\pi = 3.141$.

3. $x^2 + px + q = 0$; Roots of a quadratic: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$.

4. Definition of logarithm: $\boxed{\log_b a = c \Rightarrow b^c = a}$; if b = e = 2.718, ln a = c;

Properties of logarithm: $\log ab = \log a + \log b$; $\log \frac{a}{b} = \log a - \log b$;

$$\log a^b = b \log a .$$

5. $(a \pm b)^2 = a^2 \pm 2ab + b^2$; $a^0 = 1$; $a^m \cdot a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$.

6.

Angle	0^0	30^0	45^0	60^0	90^0	180^0	270^0	360^0
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
sin α	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos α	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
tg α	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-	0	-	0
ctg α	-	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	-	0	-

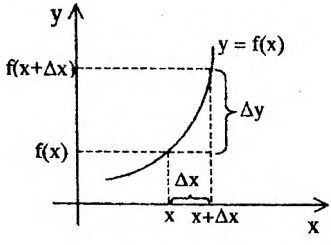


Fig. 1

7. Speed $\vartheta = \frac{\Delta S}{\Delta t}$; acceleration $a = \frac{\Delta \vartheta}{\Delta t}$;

2-nd law of Newton: F = *ma*; Hook`s law: F = kx.

8. y = f (x), y is function, x is argument, Δx is the increment of argument;

Δy = Δf is the increment of function:

$$\Delta f = f(x + \Delta x) - f(x);$$

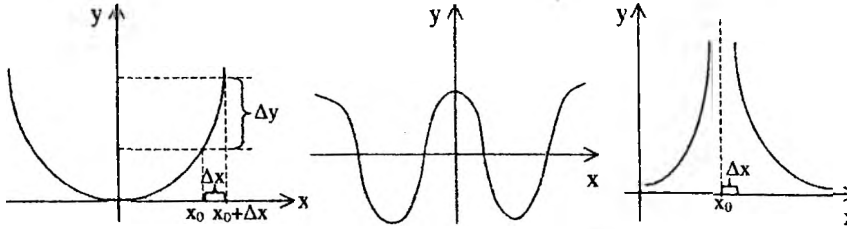
y = kx is linear function, where k = const, the graph of linear function is a straight line.

2. Continuous function and limit of function. Derivative, its mechanical and geometrical sense

RUSSIAN	*ENGLISH*	*RUSSIAN*	*ENGLISH*
Непрерывный	Continuous	Определена	Specified
Можно	Possible	Отрыв	Tearing off
Бумага	Paper	Точка	Point
Бесконечно малая	Infinitesimal	Если	If
Так как	As, since	Тогда	Then

Без	Without	Теорема	Theorem
Производная	Derivative	Пусть	Let
Разрывная	Discontinued	Доказательство	Proof
Окрестность	Neighborhood	Нахождение	Finding
Смысл	Sense, meaning	Средний	Average
Механический	Mechanical	Мгновенный	Instantaneous
Касательная	Tangent	Секущая	Secant straight line

Definition: function y = f (x) is called *continuous* in a point x_0, if it is determined in a neighborhood of this point and $\Delta y \to 0$ at $\Delta x \to 0$. For example:



a) Continuous function: $\Delta x \to 0$, $\Delta y \to 0$.

b) Continuous function

c) Discontinuous function in the point x_0: at $\Delta x \to 0$, $\Delta y \to \infty$

Fig. 2

The graph of continuous function apparently from fig. 2a and fig. 2b is possible to draw not tearing off pencil from a paper.

Definition: number A is called the **limit** of function f (x) at $x \to x_0$, if function $F(x) = \begin{cases} f(x), & x \neq x_0 \\ A, & x = x_0 \end{cases}$ is continuous in the point x_0: $\lim\limits_{x \to x_0} f(x) = A$.

As function F (x) is continuous in the point x_0, then A for example it is possible to take equal $f(x_0)$, i.e. $A = f(x_0)$ and $\boxed{\lim\limits_{x \to x_0} f(x) = A = f(x_0)}$.

Examples:

1. $\lim\limits_{x_0 \to 2} x^2 = f(x_0) = f(2) = 2^2 = 4$.

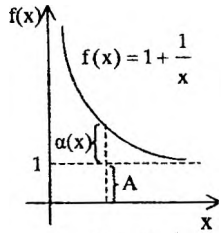


Fig. 3

2. $\lim\limits_{x \to \infty}\left(1 + \frac{1}{x}\right) = \lim\limits_{x \to \infty} 1 + \lim\limits_{x \to \infty} \frac{1}{x} = 1 + 0 = 1$ (see fig. 3.).

Here is used the property of limit:
limit of the sum of function is equal to the sum of limits and the limit of a constant c (at us c=1) is equal to this constant.

Definition: function α (x) is called **infinitesimal,** if $\lim\limits_{x \to x_0} \alpha(x) = 0$ or $\lim\limits_{x \to \infty} \alpha(x) = 0$.

Example: function α (x) = (x – 3) is infinitesimal at $x \to 3$, since,

$\lim\limits_{x\to 3}(x-3)=0$; $\alpha(x)=\dfrac{1}{x}$ is infinitesimal at $x\to\infty$, since $\lim\limits_{x\to\infty}\dfrac{1}{x}=0$.

Theorem 1: let $\lim\limits_{x\to x_0} f(x)=A$, then $f(x)=A+\alpha(x)$, where $\alpha(x)$ is infinitesimal.

Geometrical illustration of this theorem is on fig. 3.

Definition: derivative of function $y=f(x)$ in the point x is the limit

$$f'(x)=\lim_{\Delta x\to 0}\frac{\Delta f}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}.$$

Differentiation is operation of finding of a derivative.

Physical sense of a derivative.

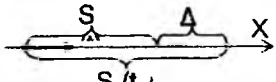


Fig. 4

Let the body moves along axis X with average speed $\vartheta_{AV}=\dfrac{\Delta S}{\Delta t}=\dfrac{S(t+\Delta t)-S(t)}{\Delta t}$. If to reduce an interval of time Δt, i.e. $\Delta t\to 0$ speed of a body at present time is equal to the limit $\vartheta_{INST}=\lim\limits_{\Delta t\to 0}\dfrac{\Delta S}{\Delta t}$.

This speed is called the *instant* and expresses ***mechanical sense*** of a derivative. In general: **derivative = speed of change of a function**.

Through two points on the graph of function $y=f(x)$ (see fig. 5) we shall lead secant straight line M_0M. If $M\to M_0$ then secant straight line will borrow position of the tangent which has been lead in the point M_0. From ΔMNM_0: $\operatorname{tg}\varphi=\dfrac{MN}{M_0N}=\dfrac{\Delta f}{\Delta x}$. Let $M\to M_0$, then $\operatorname{tg}\varphi\to\operatorname{tg}\alpha$ and

$$\operatorname{tg}\alpha=\lim_{\Delta x\to 0}\frac{\Delta f}{\Delta x}.$$

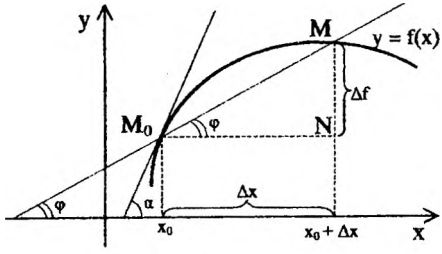


Fig. 5

Last formula expresses **geometrical sense** of a derivative: the derivative from function in the given point is equal to the tangent of the angle of inclination of the tangent this point to axis Ox.

3. General rule of differentiation. Table of the basic derivatives. Derivative of a composite function

RUSSIAN	*ENGLISH*	*RUSSIAN*	*ENGLISH*
Общий	General	Правило	Rule
Дифференцирование	Differentiation	Сложный	Composite
Промежуточный	Intermediate		

Using the general definition of a derivative for finding of derivative it is necessary to make the following steps:

1) to give to x the increment Δx and to find $f(x + \Delta x)$;
2) to find the increment of function $\Delta f = f(x + \Delta x) - f(x)$;
3) to make the ratio: $\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$;
4) to find the limit of the ratio at $\Delta x \to 0$: $f'(x) = \lim\limits_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$.

Example: using the **general rule** to find derivative of function $y = x^2$;

1. $f(x + \Delta x) = (x + \Delta x)^2$;
2. $\Delta f = (x + \Delta x)^2 - x^2 = x^2 + 2x \cdot \Delta x + (\Delta x)^2 - x^2 = 2x \cdot \Delta x + (\Delta x)^2$;
3. $\frac{\Delta f}{\Delta x} = \frac{2x \cdot \Delta x + \Delta x^2}{\Delta x} = 2x + \Delta x$;
4. $\lim\limits_{\Delta x \to 0}(2x + \Delta x) = \lim\limits_{\Delta x \to 0} 2x + \lim\limits_{\Delta x \to 0} \Delta x = 2x + 0 = 2x$.

For simplification of finding of a derivative find derivatives for the basic operations and formulas. Some from them:

1	$(c = const)' = 0$	5	$(U \cdot \vartheta)' = U'\vartheta + \vartheta' U$	9	$(\cos x)' = -\sin x$
2	$x' = 1$	6	$\left(\frac{U}{\vartheta}\right)' = \frac{U'\vartheta - \vartheta' U}{\vartheta^2}$	10	$(tgx)' = \frac{1}{\cos^2 x}$
3	$x^n = n \cdot x^{n-1}$	7	$(cU)' = c \cdot U'$	11	$(\ln x)' = 1/x$
4	$(U \pm \vartheta)' = U' \pm \vartheta'$	8	$(\sin x)' = \cos x$		

Example: $y = 5x^7 - 6\sin x$; $y' = ?$; $y' = (5x^7)' - (6\sin x)' = 5 \cdot 7 \cdot x^6 - 6 \cdot \cos x = 35x^6 - 6\cos x$.

Function is ***composite,*** if it can be submitted as function from function, i.e. $y = f[\varphi(x)]$, where $\varphi(x) = U$ is **intermediate** argument, for example:

Elementary function	Composite function	Intermediate argument
1. $y = x^2$	1. $y = (x + 3)^2$	1. $U = x + 3$; $y_u = U^2$
2. $y = \sin x$	2. $y = \sin 5x$	2. $U = 5x$; $y_u = \sin U$

Theorem 2: let functions $y = f(U)$ and $\varphi(x) = U$ are differentiated in a corresponding points. Then the derivative of composite function $y = f[\varphi(x)]$ is equal:

$$\boxed{y'_x = y'_u \cdot U'_x}\text{.(Chain-rule)}$$

Example: $y = \sin 5x$; $U = 5x$; $y'_x = (\sin U)' \cdot U'_x = \cos 5x \cdot (5x)' = 5\cos 5x$.

4. Derivative of 2nd order and its mechanical sense

Derivative of function y = f (x) is named a derivative of 1-st order. This derivative is function from x and it can be differentiated once again:

$$f''(x) = (f'(x))' = \frac{d^2f}{dx^2} = y''.$$

The derivative from a derivative is known as a **derivative of the second order**. It is similarly possible to receive a derivative of the third order, etc. The derivative of n-th order is designated as $f^{(n)}(x)$.

Example: $y = x^3$; $y' = 3x^2$; $y'' = 6x$; $y^{(3)} = 6$; $y^{(4)} = 0$.

Let's find out **mechanical (physical) sense** of the second derivative. Let the law of motion of a point along axis Ox is expressed by the equation x = f (t). Average acceleration of rectilinear motion in time Δt will be the ratio $a_{AV} = \frac{\Delta\vartheta}{\Delta t}$. The limit of this ratio at $\Delta t \to 0$ is known as **instant acceleration** a_{inst} of the point at this moment of time t:

$$\lim_{\Delta t \to 0} \frac{\Delta\vartheta}{\Delta t} = \vartheta'(t) = a_{INST}; \qquad \boxed{a_{INST} = \vartheta'(t) = x''(t)}.$$

So, the second derivative from a distance X with respect to time t is equal to instant acceleration of rectilinear motion **(mechanical sense of 2-nd derivative).**

Research of functions on extremum.

The maximum and minimum of function is named its extremum. For research of function on extremum with the help of the first derivative it is necessary:

1) to find a derivative of function f '(x);
2) to find roots of the equation (Critical points x_0) f '(x) = 0;
3) if the derivative at transition through a critical point x_0 at increase x changes mark plus (minus) on minus (plus), than in the point x_0 function has maximum (minimum). If at transition through the x_0 the derivative does not change its mark, in this point an extremum is not present;
4) find value of function in the points of extremum.

Example: to research on extremum function $f(x) = 2x^2 + 4x$.

Solution:1) f'(x) =4x+4; 2) f'(x) =0=4x+4=0, from here x = -1;

3) We make the table:

Interval	$-\infty < x < -1$	-1	$-1 < x < +\infty$
x	-2		0
f ' (x)	-4	0	4
f (x)	decreases ↓	minimum	grows ↑

4)$f(-1) = 2(-1)^2 + 4(-1) = -2$.

So, at x = -2 function has minimum.

5. Differential of function, its geometrical sense and application

RUSSIAN	*ENGLISH*	*RUSSIAN*	*ENGLISH*
Дифференциал	Differential	Отсюда	From here

Главная	Main	Часть	Part
Мнимый	Linear	Приближённое значение	Approximate value
Точный	Exact	Кольцо	Ring
Внутренний	Interior	Внешний	External

Proceeding from definition of a derivative $y' = \lim\limits_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$ and by virtue of the theorem 1:

$$\frac{\Delta f}{\Delta x} = f'(x) + \alpha(x), \qquad \text{(i)}$$

where $\alpha(x) \to 0$ at $\Delta x \to 0$. Multiplying both parts of equality (i) on Δx, we shall receive $\Delta f = \underbrace{f'(x)\cdot\Delta x}_{1} + \underbrace{\alpha(x)\cdot\Delta x}_{2}$, i.e. the increment of function Δf can be presented as two parts. The first part depends from Δx linearly since $f'(x)$ from Δx does not depend; in the second part $\alpha(x)$ depends from Δx. Then at $\Delta x \to 0$ the second part can be neglected and $f'(x) \cdot \Delta x$ will be **a body of the increment** of function.

Definition: differential df of function is called the main linear part of the increment (a body of increment) of function:

$$\boxed{df = f'(x)\Delta x}. \qquad \text{(ii)}$$

For $f(x) = x$ the differential is equal $dx = x' \cdot \Delta x = \Delta x$. Therefore (ii) is possible to write down so: $\boxed{df = f'(x)dx}$.

Application of differential for approximate calculations is based on the approached equality

$$\boxed{\Delta f \approx df}.$$

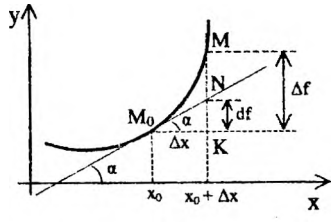


Fig. 6

Let's consider fig. 6. From ΔM_0NK: $NK = M_0K \cdot tg\alpha = \Delta x \cdot f'(x) \Rightarrow NK = df$. So, **the geometrical sense of differential**: it represents an increment of ordinate of the tangent to the graph of this function (fig. 5).

Example: to find differential dy and the increment Δy for function $y = x^2$ at $x = 20$, $\Delta x = 0.1$.

$\Delta f = \Delta y = (x + \Delta x)^2 - x^2 = 2x \cdot \Delta x + (\Delta x)^2 =$
$= 2 \cdot 20 \cdot 0.1 + 0.1^2 = 4.01$.

$df = dy = (x^2)' \cdot \Delta x = 2x \cdot \Delta x = 2 \cdot 20 \cdot 0.1 = 4.00$; i.e. the difference $dy - \Delta y = 0.01$ and it is possible to neglect; $\Delta y \approx dy$.

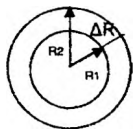


Fig. 7

Example: to find exact ΔS and the approached value of the area of the ring dS (fig. 7), if its external radius is equal 17.02 cm and internal radius is 17 cm.

$\Delta R = R_2 - R_1 = 17{,}02 - 17 = 0.2$ cm; the area of the circle $S = \pi R^2$;

$\Delta S = \pi R_2^2 - \pi R_1^2 = 2.136$ cm^2;

$dS = \left(\pi R_1^2\right)' \cdot \Delta R = 2\pi R_1 \cdot \Delta R = 2.135$ cm^2; $ds \approx \Delta S$.

Example: to find differential of function $y = \sin^3 2x$;

intermediate argument is $U = \sin 2x$; $y = U^3$; $y' = (U^3)' \cdot U'_x$;
$y' = 3 \cdot \sin^2 2x \cdot \cos 2x \cdot 2 = 6 \sin^2 2x \cos 2x$;
$dy = y'dx = 6 \sin^2 2x \cos 2x$.

Physical sense of the differential

As $ds = v\,dt$, the differential of a distance s is equal to that function increment of a way, which will turn out if to assume, that since the given moment of time, the point goes in regular intervals, keeping the got speed.

For example, if the speedometer of the automobile shows 60 km / hour the driver, expecting, that for 1 minute run of the machine will make 1km, actually calculates not increment of a way for 1 minute (which because of non-uniformity of motion can be not equal to 1km), but differential of a way.

Applications of the differential for the estimation of errors

The true value of any physical size can not be taken as result of measurements. So, experimental results obtained due to the measurements usually contain errors. There are absolute errors and relative errors:

1) the absolute error is called the absolute value of the difference between the true and the approximate value of a function: $\Delta y = |f(x+\Delta x) - f(x)|$;

2) the relative error is called the absolute value of ratio of the absolute error to the value of the measured value: $\varepsilon = \left|\frac{\Delta y}{y}\right|$

Example: the radius of a sphere has been found with accuracy till 1%. What accuracy of the given sphere volume?

Given: $\frac{\Delta r}{r} = 1\%$. Solution: we know that $V = \frac{4}{3}\pi r^3$ and

$\Delta V = V'_r \Delta r = \frac{4}{3}\pi\, 3r^2 \Delta r = 4\pi r^2 \Delta r$, then $\frac{\Delta V}{V} = \frac{4\pi r^2 \Delta r}{4/3\pi r^3} = 3\frac{\Delta r}{r} = 3 \cdot 1\% = 3\%$.

6. Functions of many variables. Partial derivatives and differentials. Total differential

At studying of many phenomena in biology, medicine, physics etc. we deal with functions of two, three and more variables, for example, arterial pressure depends on age, time of day, meal, hereditary factors, etc.; the area of rectangular S depends on its length x and width y, i.e. it is function from two arguments: $S = f(x, y)$. The graph of function $z=f(x, y)$ is the surface in space: to each pair values x and y there corresponds some value z (point M).

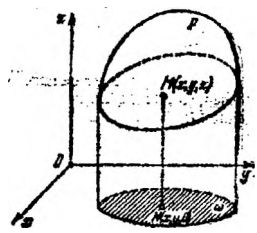


Fig.8

Definition: Partial derivative with respect to x from function of two arguments is called the limit

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x, y)}{\Delta x} = \frac{\partial f}{\partial x} \quad (y = \text{const})$$

Similarly on y:

$$\lim_{\Delta x \to 0} \frac{f(x, y+\Delta y) - f(x, y)}{\Delta y} = \frac{\partial f}{\partial y} \quad (x = \text{const}).$$

In other words, a partial derivative on x from function f (x, y) is called derivative on x, calculated provided that y = const and the partial derivative on y is calculated provided that x is a constant.

Let's find out **geometrical sense** of partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. The geometrical image of function z=f (x, y) is some surface P. Let y = const, then we shall receive a flat curve Γ_x, representing crossing of the surface P by a plane y = const. Then, as well as in case of function of one variable, the derivative $\frac{\partial z}{\partial x}$ is equal to tangent of the angle of inclination of tangent MK to the curve Γ_x in the point M(x,y,z):

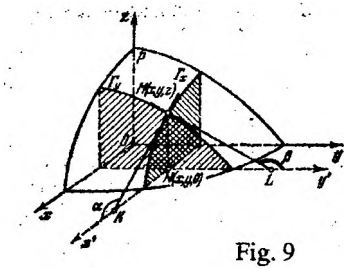


Fig. 9

$$\frac{\partial z}{\partial x} = \text{tg}\,\alpha.$$

Hence, $\frac{\partial z}{\partial x}$ represents **speed of change of function on the direction parallel to axis Ox.** The geometrical sense of the second partial derivative $\frac{\partial z}{\partial y}$ is similarly defined.

Example1: f (x, y) = x^2 siny, to find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$;

$$\frac{\partial f}{\partial x} = 2x \cdot \sin y \quad (y = \text{const}); \qquad \frac{\partial f}{\partial y} = x^2 \cdot \cos y \quad (x = \text{const}).$$

Partial derivatives of any number variables are defined similarly.

Partial differential on x and on y are expressions

$$d_x f = \frac{\partial f}{\partial x} dx \quad \text{and} \quad d_y f = \frac{\partial f}{\partial y} dy.$$

Total differential of function of two variables is equal to the sum of partial differentials:

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy.$$

Total differential is used 1) for the approached calculations and 2) for calculation of errors in measurements.

1) Really, at small increments Δx and Δy: $\Delta f(x, y) = f(x + \Delta x, y + \Delta y) - f(x, y)$ approximately it is possible to replace by increment of differentiated function on differential of this function $df(x, y) = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y$, that is $\Delta f(x, y) \approx df(x, y)$

Equality will be more exact, than it is less $|\Delta x|$ and $|\Delta y|$.

Example 2: the rectangular with the sides x=6m and y=8m is given. How will change the diagonal of this rectangular, if the side x will increase for 5 cm, and the side y will decrease for 10 cm?

Solution: the diagonal of a rectangular u is equal $u=\sqrt{x^2+y^2}$. Therefore, replacing the increment Δu of diagonal by differential du, we shall receive

$\Delta u \approx du = \frac{x}{\sqrt{x^2+y^2}}\Delta x + \frac{y}{\sqrt{x^2+y^2}}\Delta y$. Substituting in the last formula x=6m, Δx=0.05m, y=8m, Δy=-0.1m, let's receive $\Delta u \approx$-0.05m. Thus, the diagonal of a rectangular will decrease approximately for 0.05 m. Exact calculation gives value Δu = - 0.045 m.

2) We shall determine an absolute error Δz of function z=f (x, y), knowing limiting absolute errors Δ_x и Δ_y of arguments x and y: $|\Delta x| \le \Delta_x$, $|\Delta y| \le \Delta_y$. Then replacing the increment of function on its differential, we shall receive

$$|\Delta z| \approx |\frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y|.$$

From here we deduce the approached estimation

$$|\Delta z| \le |\frac{\partial z}{\partial x}|\,|\Delta x| + |\frac{\partial z}{\partial y}|\,|\Delta y|.$$

Hence, for a limiting absolute error z it is possible to accept

$$\Delta_x = |\frac{\partial z}{\partial x}|\,\Delta_x + |\frac{\partial z}{\partial y}|\,\Delta_y. \quad (i)$$

Example 3: hypotenuse of a rectangular triangle is $x=120 \pm 2$ m, sharp angle $y = 30^0 \pm 1^0$. With what accuracy is it possible to find opposite to the given angle cathetus z?

Solution: we have z=x siny (ii); from here $\frac{\partial z}{\partial x}$=sin y, $\frac{\partial z}{\partial y}$=x cosy.

According to (ii) we find z: $z=120 \sin 30^0=60$ m. Then the absolute error z according to (ii) is equal

$$\Delta_z = \sin 30^0 \cdot 2 + 120 \cdot \cos 30^0 \cdot \frac{\pi}{180} = 2.8 \text{ m}$$

Hence $z = 60 \text{ m} \pm 2.8 \text{ m}$

Limiting relative error $\delta = \frac{\Delta_z}{|z|} = \frac{2.8}{60} = 0.046$

Example 4: to find a total differential df of function $f(x,y) = x^2 \sin y$ (see the previous example 1);

as $\frac{\partial f}{\partial x} = 2x \sin y$, $\frac{\partial f}{\partial y} = x^2 \cos y$, than $df = 2x \sin y dx + x^2 \cos y dy$.

Example 5: reaction of an organism (for example, downturn of temperature) on a doze x of a medical product is described by function $f(x, t) = x^2 (a - x) \cdot t \cdot e^{-t}$, where t is time, a is a const. At what value of t at the set doze (x = const) of medicines reaction of the organism will be maximal?

To find max or min functions it is necessary to find its derivative $\frac{\partial y}{\partial t}$ and to equate it to zero $\frac{\partial y}{\partial t} = 0$:

$$\frac{\partial y}{\partial t} = x^2(a-x)\cdot e^{-t} - x^2(a-x)\cdot t\cdot e^{-t} = x^2(a-x)\cdot e^{-t}\cdot(1-t) = 0 \Rightarrow 1-t=0 \Rightarrow t = 1hour\,.$$

7. Indefinite integral and its properties

RUSSIAN	*ENGLISH*	*RUSSIAN*	*ENGLISH*
Обратный	Inverse	Первообразная	Antiderivative
Неопределённый	Indefinite	Множество	Set
Интегральное исчисление – Integral calculus			

The primary problem of differential calculus: on given function F (x) to find its derivative f (x). The integral calculus solves the opposite problem: the derivative f (x) is known, it is necessary to find function F (x), for example, knowing 9 speed, to find displacement S.

Definition: function F (x) is known as antiderivative for function f (x), if $F'(x) = f(x)$.

Example: $F(x) = \frac{x^4}{4}$ is antiderivative for $f(x) = x^3$, as $\left(\frac{x^4}{4}\right)' = x^3$ and, in general,

$F(x) = \frac{x^4}{4} + C$ is also antiderivative for f(x), as (F(x) +C)' = f(x).

Definition: indefinite integral from function f (x) is a set of all antiderivatives of this function, i.e. $\boxed{\int f(x)dx = F(x) + C}$.

Here f (x) is known as subintegral function and f (x) dx is subintegral expression.

Properties of indefinite integral:

1. $\left(\int f(x)dx\right)' = f(x)$;
2. $d\left(\int f(x)dx\right) = f(x)dx$;
3. $\int d\varphi(x) = \varphi(x) + C$;
4. $\int kf(x)dx = k\int f(x)dx$;
5. $\int(f_1(x) + f_2(x) - f_3(x))dx = \int f_1(x)dx + \int f_2(x)dx - \int f_3(x)dx$.

8. Elementary ways of integration

RUSSIAN	*ENGLISH*	*RUSSIAN*	*ENGLISH*
Непосредственное	Immediate, direct	Метод	Method
Замена	Substitution	Переменная	Variable

a) For the basic functions are made tables of integral. To find indefinite integral by the method of **direct** integration, it is necessary to use its properties (1 – 5) and to result in tabulared kind.

Example: $\int(4x^2+\cos x)dx = 4\int x^2dx + \int\cos x dx = 4\cdot\frac{x^3}{3}+\sin x + C.$

b) Method of **substitution** is based on replacement of one variable by another.

Example:

1. $$\int(1+x)^7dx = \left|\begin{matrix}1+x=t\\ d(1+x)=dt\\ dx=dt\end{matrix}\right| = \int t^7dt = \frac{t^8}{8}+C = \frac{(1+x)^8}{8}+C.$$

2. $$\int e^{2x}dx = \left|\begin{matrix}2x=t\\ d(2x)=dt\\ 2dx=dt\\ dx=\frac{dt}{2}\end{matrix}\right| = \int e^t\cdot\frac{dt}{2} = \frac{1}{2}e^t + C = \frac{1}{2}e^{2x}+C.$$

9. Definite integral, its properties and calculation

RUSSIAN	*ENGLISH*	*RUSSIAN*	*ENGLISH*
Отрезок	Segment	Прямоугольник	Rectangle
Внутри	Inside	Нижний	Bottom
Произведение	Product	Верхний	Top
Совокупность	Set, Totality	Работа	Work
Растяжение	Stretch	Пружина	Spring
Разобьём	Divide		
Криволинейная трапеция		Curvilinear trapezium	

Let's find the area of a curvilinear trapeze S_{aABb} (fig. 8). Segment *ab* we shall break on small pieces x_0x_1; x_1x_2; …; $x_{i-1}x_i$. Inside each piece we shall take any point c_i. Product $f(c_i)\cdot\Delta x_i$ is equal the area of a rectangular and sum of such products is equal to the area of all rectangulars: $S_n = \sum_{i=1}^{n} f(c_i)\Delta x_i$. If $\Delta x_i \to 0$, then $S_n \to S_{aABb}$, i. e. $S_{aABb} = \lim_{\Delta x\to 0}\sum_{i=1}^{n} f(c_i)\Delta x_i$.

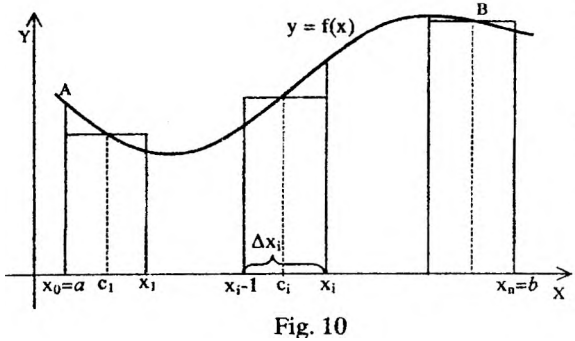


Fig. 10

This limit of the integrated sums is named the **definite integral**:

$$\int_a^b f(x)dx = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i .$$

Numbers **a** and **b** are named bottom and top limits of integration. The area of a curvilinear trapeze expresses **geometrical sense** of the definite integral.

Some **properties of the definite integral:**

1. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$;

2. $\int_a^b (f_1(x) \pm f_2(x))dx = \int_a^b f_1(x)dx \pm \int_a^b f_2(x)dx$;

3. $\int_a^b f(x)dx = -\int_b^a f(x)dx$;

4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$.

The indefinite integral is set of antiderivatives. The certain integral is a number. **Newton – Labnitz's** formula establishes connection between indefinite and definite integrals:

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a),$$

where F (x) is antiderivative for function f (x).

Example: $\int_1^2 x^3 dx = \frac{x^4}{4}\Big|_1^2 = \frac{2^4}{4} - \frac{1^4}{4} = \frac{15}{4}$.

Methods of calculation of a definite integral coincide with methods of finding of indefinite integral since for calculation of $\int_a^b f(x)dx$ it is necessary to find all over again indefinite integral (antiderivative).

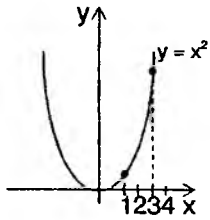


Fig. 11

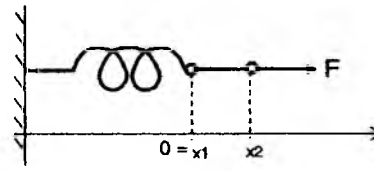


Fig.12

Example: $\int_0^{\pi/3} e^{\cos x} \sin x dx = \left| \begin{array}{l} \cos x = t \\ d(\cos x) = dt \\ -\sin x dx = dt \\ t_{Ver} = \cos 0 = 1 \\ t_{Top} = \cos\frac{\pi}{3} = \frac{1}{2} \end{array} \right| = \int_1^{1/2} e^t(-dt) = \int_{1/2}^1 e^t dt = e^t\Big|_{1/2}^1 = e - \sqrt{e}$.

Example: to find the area of the curvilinear trapeze for the graph of function $y = x^2$ from 1 up to 3 (fig. 11).

$$\int_1^3 x^2 dx = \frac{x^3}{3}\Big|_1^3 = \frac{3^3}{3} - \frac{1^3}{3} = \frac{26}{3} \text{ sq.un.}$$

Example: to calculate the work A at stretching of the spring on $x_2 - x_1 = 0.1$ m, if for its stretching on $\Delta x = 0.01$ m is necessary force $F_1 = 60$ N (fig. 12).

$A = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2} kx dx$; $F_1 = k\,\Delta x \rightarrow$ $k = F_1/\Delta x$; $x_1 = 0$, $x_2 = 0.1$ m; $A = 30$ J.

Differential equations

1. Differential equations. General concepts and definitions

RUSSIAN	*ENGLISH*	*RUSSIAN*	*ENGLISH*
Содержание	Maintenance	Высший	Highest
Решение	Solution	Порядок	Order
Тождество	Identity	Обращать	To transform

Differential is named the equation containing x, y, y', y", …,$y^{(n)}$: $F(x, y, y', y'', \ldots, y^{(n)}) = 0$.

For example: $y' = x^2 + y$ or $y'' + y' + 5 = x$.

The **order** of the differential equation (DE) is named the maximum order of the derivative which is included in DE.

For example: $y'' + y^3 = 0$ is DE of the 2-nd order;

$y' + x^4 = 0$ – DE of the 1-st order.

Function $y = \varphi(x)$ which transform the DE in identity is called **solution** of DE: $F(x, \varphi(x), \varphi'(x), \ldots) \equiv 0$.

For example: $y = \frac{x^2}{2}$ is the solution of DE $y' - x = 0$, as $\left(\frac{x^2}{2}\right)' = x$ and substitution of the derivative in the equation gives $x - x \equiv 0$. The solution will be also function $\frac{x^2}{2} + C$, where C is a constant. This function is called the **general solution**. If C is any number in the general solution we shall receive the **partial** solution.

2. Solution of the differential equations

RUSSIAN	*ENGLISH*
Квадратное уравнение	Square equation
Действительное число	Real number
Комплексное число	Complex number

Мнимая единица	Imaginary unit

a) differential equation **of the first order with divided variables (DE 1)** is called the equation of the kind $f_1(x)\,\varphi_1(y)\,dx + f_2(x)\,\varphi_2(y)\,dy = 0$; $\left[\frac{dy}{dx} = f(x)\varphi(y)\right]$.

For example: $4x^3ydx - \sin x \cdot y^2dy = 0$; $dy - 4x^3dx = 0$ etc.

To decide DE I it is necessary:

1) the members of the equation containing **x** transfer to one part and containing **y** to another (this process is named *division of variables*);
2) to integrate the left and right parts of the equation.

For example: $y' = 5y$; as $y' = \frac{dy}{dx}$, the equation will become $\frac{dy}{dx} = 5y$;

1) let's divide (separate) variables: $\frac{dy}{5y} = dx$;

2) we integrate the left and right parts: $\int\frac{dy}{5y} = \int dx$; $\frac{1}{5}\ln y = x + C$ or $\ln y^{1/5} = x + C$, $\ln\sqrt[5]{y} = \ln e^x + \ln C$, $\sqrt[5]{y} = e^x + C$, $y = C\cdot e^{5x}$ - general solution, at $x = 0$ and $y = 2$ let's receive the partial solution $y = 2\cdot e^{5x}$.

b) the linear homogeneous differential equation of 2-nd order with costant coefficients is called the equation: $y'' + py' + qy = 0$.

For example: $y''- 7y' + 10y = 0$.

Solution:

1. We work out the characteristic (square) equation:
$$k^2 + pk + q = 0;$$
2. We find roots of the characteristic equation:
$$k_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q};$$
3. We write down solution of DE:
   1) if roots $k_1 \neq k_2$ are real numbers, the solution is $y = C_1e^{k_1x} + C_2e^{k_2x}$;
   2) if $k_1 = k_2 = k$, the solution is $y = (C_1x + C_2)e^{kx}$;
   3) if roots $k_{1,2} = \alpha \pm \beta i$ are complex numbers, where $i = \sqrt{-1}$ is imaginary unit, the solution is $y = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x)$.

Example 1: to find the solution of DE: $y'' - 7y' + 10y = 0$;

1. $k^2 - 7k + 10 = 0$;
2. $k_{1,2} = 3{,}5 \pm \sqrt{3{,}5^2 - 10}$; $k_1 = 5$; $k_2 = 2$;
3. General solution of DE: $y = C_1e^{5x} + C_2e^{2x}$.

Example 2: to find the solution of DE $y'' + 4y' + 4 = 0$;

1. $k^2 + 4k + 4 = 0$;
2. $k_{1,2} = -2 \pm \sqrt{4 - 4} = -2$;

3. Solution (general) of DE: $y = (C_1x + C_2)e^{-2x}$.

Example 3: to find the solution of DE $y'' - 4y' + 13y = 0$;

1. $k^2 - 4k + 13 = 0$;
2. $k_{1,2} = 2 \pm \sqrt{4-13} = 2 \pm \sqrt{9} \cdot \sqrt{-1} = 2 \pm 3i$;
3. Solution (general) of DE: $y = e^{2x}(C_1\cos 3x + C_2\sin 3x)$.

LECTURE №2
APPLICATIONS OF DIFFERENTIAL EQUATIONS

1. Differential equations. General concepts and definitions

RUSSIAN	*ENGLISH*	*RUSSIAN*	*ENGLISH*
Уравнение	Equation	Высший	Highest
Решение	Solution	Порядок	Order
Тождество	Identity	Обращать	To transform

Differential is named the equation containing x, y, y', y", …,$y^{(n)}$:

$F(x, y, y', y'', \ldots, y^{(n)}) = 0$.

For example: $y' = x^2 + y$ or $y'' + y' + 5 = x$.

The **order** of the differential equation (DE) is called the maximum order of the derivative that is included in DE.

For example: $y'' + y^3 = 0$ is DE of the 2nd order;

$y' + x^4 = 0$ is DE of the 1st order.

Function $y = \varphi(x)$ which transform the DE to identity is called **solution** of DE: $F(x, \varphi(x), \varphi'(x), \ldots) \equiv 0$.

For example: $y = \frac{x^2}{2}$ is the solution of DE $y' - x = 0$, as $\left(\frac{x^2}{2}\right)' = x$ and substitution of the derivative in the equation gives $x - x \equiv 0$. The solution will be also function $\frac{x^2}{2} + C$, where C is a constant. This function is called the **general solution**. If C is any number in the general solution we shall receive the **partial** solution.

2. Solution of the differential equations

RUSSIAN	*ENGLISH*
Квадратное уравнение	Square equation
Действительное число	Real number
Комплексное число	Complex number
Мнимая единица	Imaginary unit

a) differential equation **of the first order with divided variables (DE I)** is called the equation of the next kind: $f_1(x)\,\varphi_1(y)\,dx + f_2(x)\,\varphi_2(y)\,dy = 0$; or $\left[\frac{dy}{dx} = f(x)\varphi(y)\right]$.

For example: $4x^3ydx - \sin x \cdot y^2dy = 0$; $dy - 4x^3dx = 0$ etc.

To decide DE I is necessary:

3) the members of the equation containing x to transfer to one part, and containing y to another (this process is named *division of variables*);

4) to integrate the left and right parts of the equation.

For example: $y' = 5y$; as $y' = \frac{dy}{dx}$, the equation will become $\frac{dy}{dx} = 5y$;

3) let's divide variables: $\frac{dy}{5y} = dx$;

4) we integrate the left and right parts: $\int\frac{dy}{5y} = \int dx$; $\frac{1}{5}\ln y = x + C$ or $\ln y^{1/5} = x + C$, $\ln\sqrt[5]{y} = \ln e^x + \ln C$, $\sqrt[5]{y} = e^x + C$, $y = C\cdot e^{5x}$, at $x = 0$ and $y = 2$ let's receive the *partial solution* $y = 2\cdot e^{5x}$.

b) the linear homogeneous differential equation of the 2nd order with costant coefficients is called the equation of the next kind: y" + py' + qy = 0.

For example: $y'' - 7y' + 10y = 0$.

The solution:

1. We work out the characteristic (square) equation:
$$k^2 + pk + q = 0;$$
2. We find roots of this characteristic equation:
$$k_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q};$$
3. We write down solution of DE:
   4) if roots $k_1 \neq k_2$ are real numbers, the solution is $y = C_1e^{k_1x} + C_2e^{k_2x}$;
   5) if $k_1 = k_2 = k$, the solution is $y = (C_1x + C_2)e^{kx}$;
   6) if roots $k_{1,2} = \alpha \pm \beta i$ are complex numbers, where $i = \sqrt{-1}$ is imaginary unit, the solution is $y = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x)$.

Example 1: to find the solution of DE: $y'' - 7y' + 10y = 0$;

1. $k^2 - 7k + 10 = 0$;
2. $k_{1,2} = 3{,}5 \pm \sqrt{3{,}5^2 - 10}$; $k_1 = 5$; $k_2 = 2$;
3. general solution of DE: $y = C_1e^{5x} + C_2e^{2x}$.

Example 2: to find the solution of DE $y'' + 4y' + 4 = 0$;

1. $k^2 + 4k + 4 = 0$;
2. $k_{1,2} = -2 \pm \sqrt{4-4} = -2$;
3. general solution of DE: $y = (C_1x + C_2)\,e^{-2x}$.

Example 3: to find the solution of DE $y'' - 4y' + 13y = 0$;

4. $k^2 - 4k + 13 = 0$;
5. $k_{1,2} = 2 \pm \sqrt{4-13} = 2 \pm \sqrt{9}\cdot\sqrt{-1} = 2 \pm 3i$;
6. general solution of DE: $y = e^{2x}(C_1\cos 3x + C_2\sin 3x)$.

3. Using of the differential equations in solution of problems of the medical, biological, physical and chemical contents

Differential equations (DE) are the powerful tool of the explanation of various processes in medicine, biology, physics and chemistry. For example, with help of DE it is possible to solve the following problems and to model processes:

1. Medicine: a) law of dissolution of medicinal forms from tablets; b) modelling of distribution of epidemic etc.
2. Biology: a) law of cell enlargement; b) law of duplication of bacteria; c) ecological models etc.
3. Physics: a) law of radioactive decay; b) law of absorption of light; c) law of free oscillations etc.
4. Chemistry: law of kinetic of chemical processes etc.

Application of differential equations of the 1st order with divided variables

1. **Law of dissolution of medicinal forms from tablets.**

It is necessary to establish time dependence of change of mass of a medical product.

Let m is the mass of substance in a tablet, in time dt was dissolved mass dm. It is possible to assume that dm is proportional to time of dissolution dt and mass of substance m:

$dm = -k \cdot m \cdot dt$.

In the received differential equation we shall divide variables and we shall integrate both parts:

$\frac{dm}{m} = -kdt$; $\int\frac{dm}{m} = -\int kdt$; $\ell nm = -kt + \ell nC$;

$\ell nm = \ell ne^{-kt} + \ell nC$; $m = C \cdot e^{-kt}$.

At t=0 $m=m_0$, whence $C=m_0$ and

$$m = m_0 e^{-kt}$$

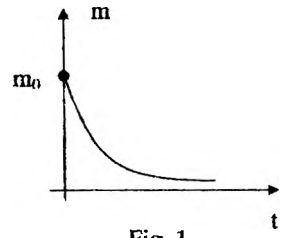


Fig. 1

The formula expresses the law of dissolution of medicinal forms of substance from tablets. According to this formula m decreases on the exponent (fig. 1).

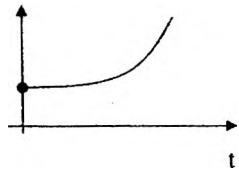


Fig. 2

2. Some other processes, for example, **law of duplication (reproduction) of bacteria** is similarly described.

Let it is necessary to establish time dependence of change of number of bacteria t. At presence of enough of food resources increase of number of bacteria dx in time dt is proportionally to number of bacteria: dx = k x dt. Solving received DE we shall have:

$\frac{dx}{x} = kdt$; $\int\frac{dx}{x} = k\int dt$; $\ell nx = kt + \ell nC$;

$\ell nx = \ell ne^{kt} + \ell nC$; $x = C \cdot e^{kt}$.

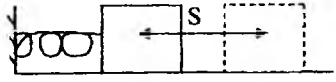
At t=0, $x=x_0$, whence $C=x_0$, hence

$$x = x_0 e^{kt}$$

Thus, under favorable conditions increase of number of bacteria also occurs on exponent law (fig. 2).

Application of the linear homogeneous differential equations of the 2-nd order with constant coefficients

1. **Law of free continuous (harmonious) oscillations.**



$F_{fr}=0$

$F_{el}\neq 0$

Fig.3

Let on a moving body of mass m operates only force of elasticity $F_{el}=-ks$, where k is coefficient of elasticity, s is displacement of the body from position of balance. Force of friction is absent: $F_{fr}=0$ (fig. 3). According to the second law of Newton $ma=F_{el}$, where $a=\frac{d^2s}{dt^2}$ is acceleration of motion of the body. Then

$$m\frac{d^2S}{dt^2}=-kS \text{ or } \frac{d^2S}{dt^2}+\frac{k}{m}\cdot S=0.$$

Having designated $\frac{k}{m}=\omega_0^2$, let's receive DE of harmonious oscillations

$$\frac{d^2S}{dt^2}+\omega_0^2 s=0$$

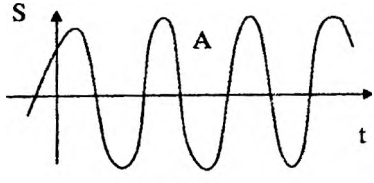
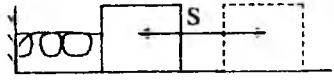


Fig. 4

Let's find its solution. The characteristic equation is $\tau^2+\omega_0^2=0$, which roots $\tau_{1,2}=\pm i\omega_0$ are complex numbers. The general solution is $S=e^{\alpha x}(C_1\cos\omega_0 t+C_2\sin\omega_0 t)$.

Let's designate $C_1=A\sin\varphi_0$, $C_2=A\cos\varphi_0$, then $S=A\sin\varphi_0\cos\omega_0 t+A\cos\varphi_0\sin\omega_0 t$; or

$$S=A\sin(\omega_0 t+\varphi_0)$$



$F_{fr}\neq 0$

$F_{el}\neq 0$

Fig. 5

It is partial solution of DE, where A is amplitude of oscillations, ω_0 is cyclic frequency of oscillations, $(\omega_0 t+\varphi_0)$ is phase of oscillations (fig. 4).

2. **Law of free damped oscillations.**

All real oscillations are at presence of resistance of environment (forces of friction): $F_{fr}=-rv$, where r is coefficient of friction, v is speed of a body (fig.5). Then according to the second law of Newton the equation of motion looks like:

$ma=F_{el}+F_{fr}$ or $m\frac{d^2S}{dt^2}=-kS-r\frac{dS}{dt}$,

whence we shall receive $\frac{d^2S}{dt^2}+\frac{r}{m}\cdot\frac{dS}{dt}+\frac{k}{m}\cdot S=0$.

Having designated $\frac{r}{m}=2\beta$, $\frac{k}{m}=\omega_0^2$, let's receive DE of damped oscillations:

$$\frac{d^2S}{dt^2}+2\beta\frac{dS}{dt}+\omega_0^2 S=0$$

Let's find its solution. The characteristic equation is is $\tau^2+2\beta\tau+\omega_0^2=0$, which roots $\tau_{1,2}=-\beta\pm\sqrt{\beta^2-\omega_0^2}$. Let's designate $\beta^2-\omega_0^2=\omega^2$.

For a case, when a friction is a small value $\beta^2<\omega_0^2$ and roots of the characteristic equation will be complex numbers: $\tau_{1,2}=\beta\pm\omega i$. The general solution:

$$S=e^{-\beta t}(C_1\cos\omega t+\cos\varphi_0\sin\omega t).$$

Let's designate $C_1=A\sin\varphi_0$, $C_2=A\cos\varphi_0$, then $S=e^{-\beta t}A(\sin\varphi_0\cos\omega t+\cos\varphi_0\sin\omega t)$; or

$$S=Ae^{-\beta t}\sin(\omega t+\varphi_0)$$

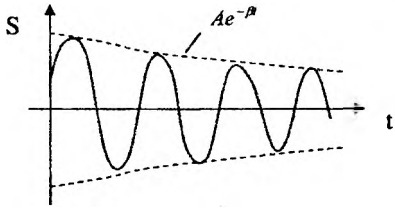


Fig. 6

This formula expresses the law of free damped oscillations with amplitude of $A'=Ae^{-\beta t}$, which decreases on exponent law (fig. 6)

Pharmacokinetic model

Let's find the law of change of concentration of a medical product at various ways and parameters of its introduction and removing. For simplicity we shall present an organism as separate blocks: 1) blood, 2) body - target, 3) the bodies excluding a preparation (kidneys).

1-st case. Single dosing of a medical product (fig. 7). It corresponds to a case, when to the patient have made an injection. After introduction of a medical product of mass m_0, begins its removal from an organism. Speed of removal of the preparation $\frac{dm}{dt}$ is directly proportional to its mass m in the body - target:

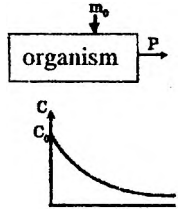


Fig. 7

$$\frac{dm}{dt}=-km,$$

where k is the coefficient describing speed of removal of the given preparation from body into blood. We already solved such equation (look the example 1). The solution of this differential equation is

$$m=m_0e^{-kt}. \quad (1)$$

Concentration of a medical product in a body – target is $c=m/V$, where V is volume of blood:

$$c = \frac{m_0}{V} e^{-kt} = c_0 e^{-kt},$$

where c_0 is initial concentration of a preparation (fig. 7).

2-nd case. Continuous introduction of a preparation with constant speed. It corresponds to a case when to the patient have put a dropper.

In this case change of mass of a medical product $\frac{dm}{dt}$ is defined not only by speed of its removal, but also by speed of its introduction Q (Q is quantity of the medicinal substance entered per unit of time):

$$\frac{dm}{dt} = Q - km.$$

We integrate the left and right parts of the equation with the account, that at t=0 and m=0:

$$\int_0^m \frac{dm}{Q - km} = \int_0^t dt.$$

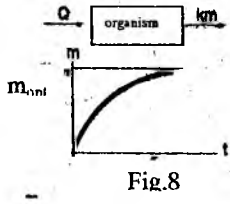


Fig.8

Let's enter the new variable u=Q-km, then du =-kdm, whence dm =-du/k, then

$$\int_0^m \frac{dm}{Q - km} = -\frac{1}{k}\int_Q^{Q-km} \frac{du}{u} = -\frac{1}{k}\ln u\Big|_Q^{Q-km} = -\frac{1}{k}\ln\frac{Q-km}{Q} = t.$$

Whence $\ln\frac{Q-km}{Q} = -kt$ and we shall receive

$$m = \frac{Q}{k}(1 - e^{-kt}) \quad (2)$$

From the received solution is visible, that at $t \to \infty$ item $e^{-kt} \to 0$ and the mass of a preparation tends to a constant value Q/k.

3-d case. A combination of continuous introduction of a medical product (2 case) with introduction of a loading doze (1 case). Adding term by term of right parts of (1) and (2), we shall receive

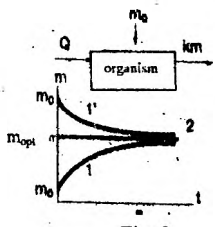


Fig. 9

$$m = \frac{Q}{k} - (\frac{Q}{k} - m_0)\cdot e^{-kt}$$

The graph of this dependence is submitted on fig. 9, curves 1 and 1 '.

If to select corresponding speed of introduction of a medicine $Q = km_{opt}$ and a loading doze $m_0 = Q/k$, then the constant mass of the preparation will be *established instantly* (the direct line 2 on fig. 9).

LECTURE № 3

ELEMENTS OF PROBABILITY THEORY AND MATHEMATICAL STATISTICS

1. Subject of probability theory. Random event. Classical and statistical definition of probabilities. Theorems of addition and multiplication of probabilities

Many phenomena in nature, medicine, technics have casual character. For example, it is impossible to tell beforehand, what side of playing cube 1,2,3,4,5,6 drops out, it is the result of motion of human hand, structure of a surface, motion of air and other random factors. Occurrence in a study of the doctor of the patient with the given illness depends on the patient, a season and from other reasons. Such events are named random. Random events if there are a lot of them submit to the certain laws. For example, if to throw a coin, the arms drop out approximately in half of all cases.

Probability theory is the section of mathematics studying laws inherent in the mass events.

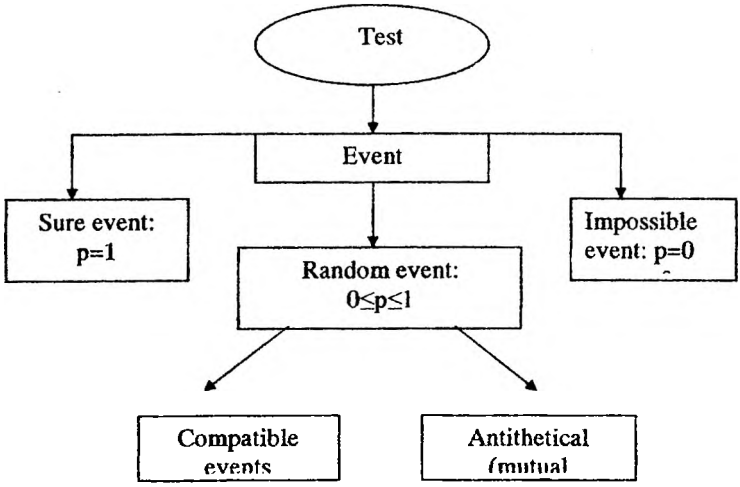
Event is a qualitative result of test (fig. 1). For example, at a shot two events are possible:

1. Hit,
2. Miss.

Events designate by the big letters of the Latin alphabet:

A,B,C,D.....

Classification of events (fig. 1):



Sure is named event which necessarily will take place.

For example: let in a box there are only white spheres;

event A : from the box take out the white sphere; then event A is necessarily happen (sure event).

Impossible are named events which cannot take place at any tests.

For example: let event B: from a box take out the black sphere, in the box there are only white spheres, then event B never happen (impossible).

Random is named event which at test can either take place, or not take place.

For example: in a box are both white and black spheres; event A: from the box the white sphere is taken, B: the black sphere is taken from the box. A and B are random events.

Events A and B are called **compatible,** if occurrence of one of them does not exclude occurrence of another.

For example: let at tossing of a playing cube event A: falling out of 4 points and event B: falling out of even number of points, then events A and B are compatible events.

Mutual exclusive events are called events A and B, if occurrence of one of them excludes occurrence of another.

For example: let in the previous example event B is falling out of odd number of points, then A and B are mutual exclusive events.

Independent are named events, if occurrence of one of them does not influence occurrence of another.

For example: in family two children; let event A: at the first birth will appear the girl; B: at rebirth the boy will appear, A and B are independent events.

Observing some events, we see that one of them are more possible than others, i.e. there is a number expressing a measure of opportunity of occurrence of event. This number is named **probability** of event A and designated by P (A).

The classical probability of event A is the ratio:

$$P(A)=\frac{m}{n} \qquad (1)$$

where m is number of the *equally likely* elementary events *favorable* for A, n is number of *all possible* elementary equally likely events.

Example: at falling out of a cube number of all outcomes is n=6 and all of them equally possible. Let event A is occurrence of even number of points. For A favorable outcomes will be occurrence of numbers 2, 4 and 6. Their number is m=3.

Therefore $P(A)=\frac{m}{n}=\frac{3}{6}=\frac{1}{2}$.

Properties of probability:

1. Probability of sure event is equal to 1.
2. Probability of impossible event is equal to zero.
3. Probability of random event A is $0 \le p(A) \le 1$.

Relative frequency of event A is the ratio

$$W(A)=\frac{m}{n} \qquad (2)$$

where m is number of tests in which event A has appeared, n is number of all lead tests.

Example: to a hospital have arrived 1000 ampoules. 10 ampoules appeared with defects. Find relative frequency of occurrence of ampoules with defects.

Let event A: occurrence of an ampoule with defect, then m=10, n=1000 and $W(A) = \frac{10}{1000} = 0.01$.

Formulas (1) and (2) are similar, but the classical probability is defined before the (or without) tests, it is theoretical value. Relative frequency is empirical value, it is calculated due to tests.

Relative frequency is changed from test to test. But if to pass a lot of tests, values of relative frequency are grouped near certain number which (fig. 2) is known as **statistical probability**.

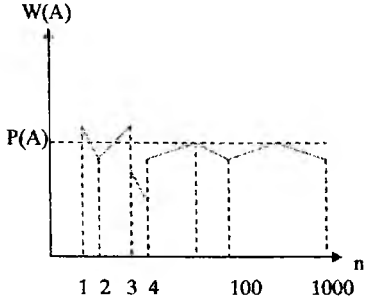
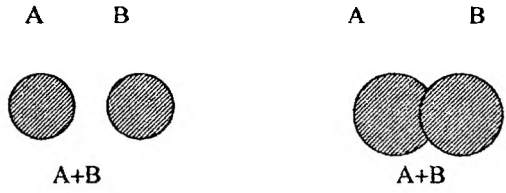


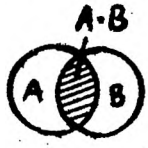
Figure 1

$$P(A) = \lim_{n \to \infty} W(A); \qquad (3)$$

Sum of two events A and B is event A+B consisting in occurrence or A or B or both events simultaneously (fig. 3a, b).



a) Mutual exclusive b) Compatible



c) A·B (multiplication)

Figure 2

Theorem 1: probability of the sum of two mutual exclusive events is equal to the sum of probabilities of these events:

$$P(A+B)=P(A)+P(B)$$

4

If events are compatible then $P(A+B)=P(A)+P(B)-P(A)\cdot P(B)$ (5).

3-white
5-black
2-red

Example 1. In a box there are 10 spheres: 3 white, 5 black and 2 red. To find probability of occurrence of

1) white or black sphere;
2) red or white sphere, if one sphere is taken.

1) Let A is the event consisting in occurrence of white sphere, B is black, C is red. Events A, B, C are mutual exclusive.

$P(A)=\frac{3}{10}, P(B)=\frac{5}{10}; P(C)=\frac{2}{10}$, then P(A+B)=P(A)+P(B)=

$\frac{3}{10}+\frac{5}{10}=\frac{8}{10}; 2. P(A+C)=\frac{3}{10}+\frac{2}{10}=\frac{5}{10}.$

Example 2: two marksmen shoot to targets, probability of hit by the first shot is P (A) =0.7, by the second is P (B) =0.8. Find probability of hit to the target by the first or by the second marksman.

Events A and B are compatible, therefore according to (5)

$P(A+B)=P(A)+P(B)-P(A)\cdot P(B)=0.8+0.7-0.8\cdot 0.7=0.94$.

Multiplication of two events A and B is event $A\cdot B$ consisting in joint occurrence of these events (see fig.3c).

Theorem 2. Probability of joint occurrence of two independent events A and B is equal to multiplication of probabilities of these events:

$$P(AB)=P(A)\cdot P(B) \qquad (6)$$

Example: in a family two children. The probability of birth of a boy is equal to 0.514. Find probability that both children be boys.

Let event A: 1-st child is boy; event B: 2-nd child is boy, events A and B are independent. Then under the formula (6):

$P(AB)=P(A)\cdot P(B)=0.514\cdot 0.514=0.264$.

2. Random variables (variates). Law of distribution and numerical characteristics of discrete random variables

Random is called quantity, which as result of tests can accept any numerical value. **Discrete** is named a random quantity all which possible values are isolated from each other and they can be numbered: $x_1,x_2\ldots x_n$ (fig.4,a). For example: number of patients at a doctor during one day, number of hair on a human head etc.

Continuous (analog) is called quantity, if all its values fill a finite interval or infinite interval (see fig.4b). For example, height, weight, arterial pressure of a patient, temperature of a patient within a day and other physiological and anthropometrical parameters are examples of continuous random variables.

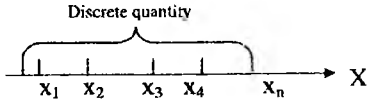


Fig.4a

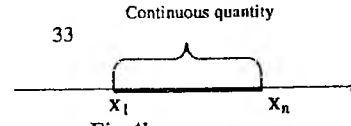


Fig.4b

Law of distribution of a discrete random variable is conformity between its possible values x_i and their probabilities p_i. In simple case it is convenient to set the law of distribution of a discrete random variable by the table (tab.1) or as a polygon (fig.5).

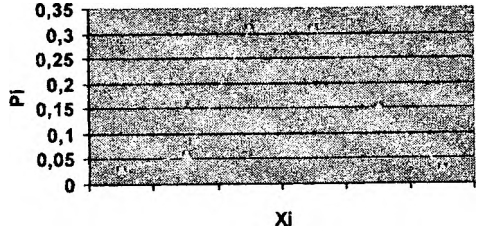
Table 1

X_i	X_1	X_2	...	X_n
P_i	P_1	P_2	.....	P_n

Example (3): in a hospital within a day are born 5 children. Let X_i is number of born boys. P_i is probability of occurrence of 0,1,2,3,4,5 boys. Then the law of distribution X_i can be next (tab. 2):

Number of born boys, X_i	0	1	2	3	4	5
Probability P_i	0.03	0.15	0.31	0.31	0.15	0.03

Fig.5



The important features of distribution of a random variable express not only law of distribution of random variable, but also **numerical characteristics of a random variable**: mean, variance, standard deviation and mode.

Mean M(x) (or **μ**) of a discrete random variable X is determined by the formula:

$$M(x) = \mu = \sum_{i=1}^{n} X_i p_i = X_1 \cdot p_1 + X_2 \cdot p_2 ... + X_n \cdot p_n \qquad (7)$$

Mean has the sense of **average arithmetic value** of a random variable X

$$M(x) \approx \overline{X}$$

In particular, if $P_1=P_2=...=P_n$, then M (x) calculate under the formula of usual average value

$$M(x) = \overline{X} = \frac{X_1 + X + ... X_n}{n}.$$

Example: find a population mean of number of the born boys (see the previous example). Under the formula (7):

$M(X)=0\cdot0.03+1\cdot0.16+2\cdot0.31+3\cdot0.31+4\cdot0.16+5\cdot0.03=2.5$.

For estimation of degree of dispersion of values of a random variable around of its average value the concept **of variance** is entered.

Example: discrete random variables X and Y are set by the laws (table 3 and table 4).

Table 3

X_i	5	10
P_i	0.5	0.5

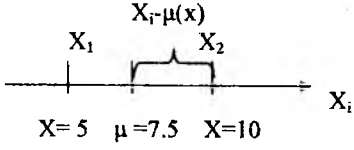
$M(X_i)=5\cdot0.5+10\cdot0.5=7.5$

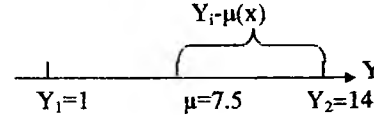
Table 4

Y_i	1	14
P_i	0.5	0.5

$M(Y_i)=1\cdot0.5+14\cdot0.5=7.5$

Fig.6





Means of both quantities are identical M(X)=M(Y)=7.5, but the dispersion of values less at X_i (fig. 6). Difference (x_i - μ) cannot serve a measure of

dispersion, since average arithmetic of these differences tends to zero. It speaks that values of a random variable are located on both sides from μ approximately equally. And if this difference to square and then to find average arithmetic value, the received value already can serve as *measure of dispersion* of random variables around μ.

Variance D(X) of a discrete random variable X is called a mean of square of difference [X - M (X)]

$$D(X) = M[X_i - M(X)]^2 = \sum_{i=1}^{n} [X_i - M(X)]^2 \cdot p_i \qquad (8)$$

For calculation of a variance usually uses more convenient formula which is easy to receive from the formula (8):

$$D(X) = M(X^2) - [M(X)]^2 \qquad (9)$$

Example: determine D (x) from the example (3).

We shall take advantage of the formula (9). In the beginning we shall write down the law of distribution of X^2:

Table 5.

X_i^2	0	1	4	9	16	25
P_i	0.03	0.16	0.31	0.31	0.16	0.03

Then, according to the formula (7) we shall receive:

$M(X^2)=0\cdot0.03+1\cdot0.16+4\cdot0.31+9\cdot0.31+16\cdot0.16+25\cdot0.03=7.5;$

$D(x)=M(X^2)-[M(x)]^2=7.5-2.5^2=1.25.$

The same result can be received under the formula (8).

Standard deviation is

$$\sigma = \sigma(x) = \sqrt{D(x)} \qquad (10)$$

Apparently from definition, this quantity is also an estimation of dispersion, but has the same *dimension*, as well as a random variable, therefore σ use for the characteristic of dispersion more often.

Example: to find σ (X) of example 3. Under the formula (10):

$$\sigma(x) = \sqrt{D(x)} = 1.12$$

Mode Mo is called the value X_m of a random variable having the *greatest* probability. Distributions having one, two and more modes are named accordingly single-mode, two-mode or multimode. In example (3) we deal with two-mode

distribution since to the values X_3=2 and X_4=3 corresponds the greatest probability P (2) =P (3) =0.31. In medicine in many cases knowledge of $\bar{x}$, for example, knowledge of average age of children who were fallen ill with flu less important, than knowledge of age X_m in which disease there is the more often (in particular, at decision of the question: where main preventive actions should be lead: at school or in preschool establishments).

3. Continuous random variables. Normal law of distribution (law of Gauss)

The continuous random variable cannot be characterized by the law of distribution (as a table or polygon of distribution) because two reasons: 1) at a continuous random variable uncountable set of values; 2) probability of its each separate value is equal to zero.

For these reasons for description of distribution of a random variable apply so-called density of distribution.

Density of distribution of probability (or simply density of probability) $f(x)$ is the function which has been picked up so, that the area under the curve corresponding to it in the set limits is probability of event (fig. 7)

$$P(\alpha<X<\beta)=\int_{\alpha}^{\beta} f(x)dx \qquad (11)$$

and

$$\int_{-\infty}^{+\infty} f(x)dx = 1 \qquad (12)$$

Equality (12) is the *normality condition* and expresses the fact, that the probability of hit of a random variable in interval from - ∞ up to + ∞ is equal to 1, as well as the area under the graph of function $f(x)$ will be equal to 1. Therefore, probability of any event connected to a continuous random variable is necessary to understand as some part of all area under the graph of function $f(x)$ - any share of unit. (fig. 7).

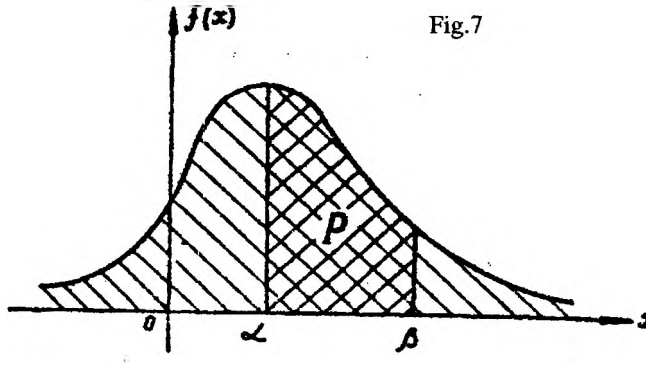


Fig.7

Numerical characteristics of a continuous random variable X with density of distribution $f(x)$ are defined similarly to numerical characteristics of discrete random variables:

$$M(x)= \int_a^b x \cdot f(x)dx \qquad (13)$$

$$D(x) = \sigma^2 = \int_a^b (x-\mu)^2 f(x)dx \qquad (14)$$

Normal law of distribution plays the major role in probability theory. This most frequently law of distribution meeting in practice, for example, distribution of height, mass of peoples, many physiological parameters, errors of measurements, sensitivity of animals of the same kind influence of a medicine, etc.

From here and its name the normal law (Gauss's law).

Normal distribution of a continuous random variable X is called the distribution, if its density of probability f(x) looks like:

$$f(x) = \frac{1}{\sigma\sqrt{2\Pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}} \qquad (15)$$

where μ is mean, σ is standard deviation of x.

The **basic properties** of the curve of normal distribution:

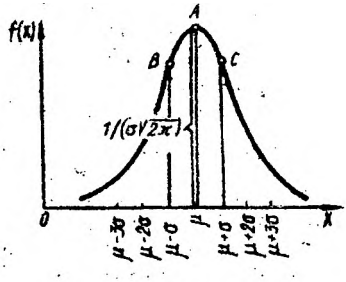


Fig. 8

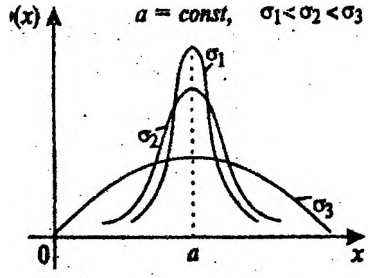


Fig. 9

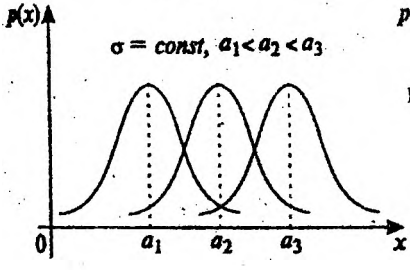


Fig. 10

1. Normal curve is symmetric concerning a straight line x = μ and has a maximum at x= μ,

$f(\mu) = \frac{1}{\sigma\sqrt{2\Pi}}$ (see fig.8)

2. At $x \to \pm\infty$, $f(x) \to 0$. Hence, more value x is deviated from μ, less often is x meet.
3. The curve has two points of excess on distance of ± σ from x = μ (fig. 8)
4. The form of a curve (at μ =const) depends from σ: more σ, the curve is more flat (fig. 9). At σ =const, if changes μ the curve does not change the form and it is shifted to the right or to the left on the axis X (fig. 10).
5. In the interval μ ± σ there are 68.3 % of all values of random variables (fig. 8), **in the interval μ±3·σ there are 99.7 % of all values (it is the rule of « three σ »).**

Normal distribution with parameters μ=0 and σ=1 is called standard normal distribution.

3. Subject of mathematical statistics. Statistical distribution of sample. Histogram

The term "statistics" has taken place from Latin "status": the certain condition and in the beginning was used in a word meaning "political", i.e. description of a state system. Now as **statistics understand**:

1) the science about mathematical methods of ordering and using of the data on the mass phenomena (mathematical statistics). The mathematical statistics is based on concepts of probability theory and puts the purpose to find out to what distribution corresponds the statistical data;

2) special branch of practical activities of people directed on gathering, processing and the analysis of the data on the mass phenomena;

3) statistical data submitted in the reporting of the various organizations, in this sense speak: «statistics of birth rate and death rate in the country».

Each of investigated objects of research in medicine has various characteristics, for example, height (pulse, weight, etc.) of men of the same age changes in a wide range. We shall assume it is necessary to study height of the first year students of medical faculty.

The set consisting of all investigated objects is called **general** or ***population*** (all students of 1-st year) and number of objects of general set is known as its *volume* (n).

If general set contains very big number of objects such studying demands a lot of time and economically is not favorable. Therefore instead of all set study only its some part that named **sample** (for example, students of one group). Sample is used also when studying of objects conducts to their destruction, for example, at carrying out of the analysis of medical products, works with experimental animals, etc. (fig. 11).

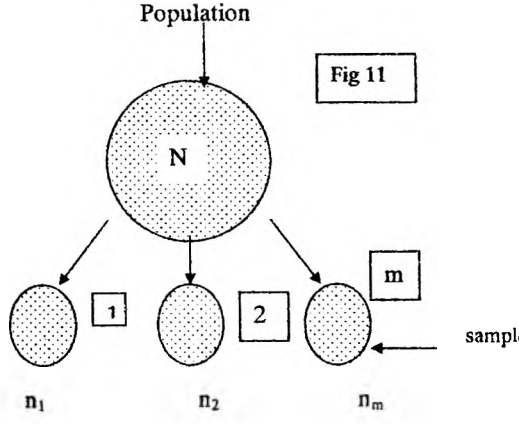


Fig 11

Sample should be **representative,** i.e. correctly to reflect properties of objects of population. For example, the state of health of the people of all city cannot be estimated on sample of one area, since conditions of residing (a gassed condition, humidity, etc.) in the different areas unequal. Therefore sample should represent **randomly** selected objects.

If to write down by way of measurement of value X in a sample we shall receive **a simple statistical series**, for example, height of students in group: 170,184,175,178,175 cm... Such series is inconvenient for the analysis, therefore it is necessary to arrange the data in ascending order with the indication of their repeatability, i.e. to make the **variational series**. If the measured random variable discrete, count up how many time (number m) meets each value and result represent as the table:

Table 6

X	X_1	$<X_2$	$<X_3$	.....	$<X_n$
m	m_1	m_2	m_3	.....	m_n
$P^* = \frac{m}{n}$	$\frac{m_1}{n}$	$\frac{m_2}{n}$	$\frac{m_3}{n}$	.....	$\frac{m_n}{n}$

Received values $x_1,x_2...x_n$ are named **variants**, numbers $m_1,m_2....m_n$ are **frequencies** and their ratio to volume of sample are **relative frequencies** $P_n = \frac{m_n}{n}$.

Sum of all frequencies is equal to volume of sample n: $\sum_{i=1}^{n} m_i = n$.

The table containing values of variants, their frequencies or relative frequencies is named **discrete statistical series of distribution** or **statistical distribution of sample** (tab.6).

In case of continuous quantities (for example, it is studied the weight of newborn) make grouping variant on intervals. Number of intervals K determine under formula of Sterdgess: K=1+3.33·lg n; n is volume of sample; $\Delta X = \frac{X_{max} - X_{min}}{n}$ is width of an interval. Count up frequency m_n and relative frequency $P_n = \frac{m_n}{n}$ for each interval and results will put in the table that is known as statistical interval series (table 7):

Table 7

Interval ΔX	X_0,X_1	X_1,X_2	.....	X_{n-1},X_n
Frequency m	m_1	m_2	.....	m_n
Relative frequency $P^* = \frac{m}{n}$	P^*_1	P^*_2	.....	P^*_n

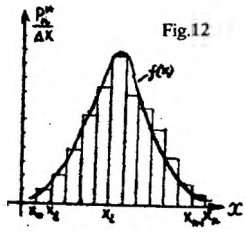


Fig.12

Graphic displaying statistical interval series is the **histogram** (fig.12). For its construction on the axis X lay off the values of random variable broken on intervals ΔX and on the axis Y lay off value of $\frac{P^*_n}{\Delta x}$, it is *density of relative frequency* for the given interval. The area of the i-th rectangular is $S_i = \frac{P^*_i}{\Delta X} \cdot \Delta X = P^*_i$ and proportional to number of the random variables which have got in the given interval. If $\Delta x \to o$ and $n \to \infty$ the middle of the top bases of rectangulars unite in a smooth line, in limit turning to the graph of density of probability $f(x)$, which characterizes distribution of population (fig. 12).

Comparing the received graph with graphs of probability density of typical distributions (normal, exponential etc.) is possible to attribute investigated distribution to some type of distribution.

4. Processing of results of measurements: estimation of random errors of direct measurements

There are direct and indirect measurements. At direct measurements any value is determined with help of any device, for example, measurement of time is made with help of a stop watch, measurement of length by ruler, etc. At indirect measurements a determined value is calculated by results of direct measurements of other values, for example, resistance R is determined under formula R = U / I, and U and I are measured directly by the voltmeter and the ammeter.

Example 1: at seven students of group the pulse rate (number of heart beating per minute X_i) is measured (tab.8)

Table 8

n	1	2	3	4	5	6	7
X_i	60	62	62	64	72	68	65

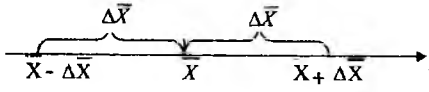


Fig.13

To give the interval estimation of value of pulse rate X (fig. 13) with confidential probability of γ=0.95 (confidential probability is probability with which $\bar{X}$ gets in the interval $\bar{X} - \Delta\bar{X} < \bar{X} < \bar{X} + \Delta\bar{X}$ (or more often $X = \bar{X} \pm \Delta X$); usually in medicine γ=0.95=95%).

Solution: 1. Most probable value of the measured variable is average arithmetic of the received values (mean)

$$\bar{X} = \frac{X_1 + X_2 + ... + X_n}{n} = \frac{60 + 62 + ... + 65}{7} = 65.$$

2. Dispersion of results of measurements is characterized with help of standard deviation S_x

$$S_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{(60-65)^2 + (62-65)^2 + ... + (65-65)^2}{7-1}} = 4.12$$

We check «rule of 3σ»: we reject the measurements (misses) distinguished from $\bar{X}$ more than on 3σ, i.e. 3·4.12=12.36 (of course, if it make a sense). At our example such measurements are not present.

3. S_x is an estimation of variability of data in one series of measurements in the given sample. If we are interested with dispersion of average $\bar{X}$ in several series of measurements of the given quantity, it is necessary to calculate $S_{\bar{x}}$:

$$S_{\bar{X}} = \frac{S_X}{\sqrt{n}} = \frac{4.12}{2.64} = 1.56$$ **(standard deviation for $\bar{X}$).**

4. At small number of measurements (n<30) finding of $\Delta\bar{X}$ with help of normal distribution gives unjustified narrowing of the confidential interval. English mathematician Gosset (his pseudonym is Student) has made the distribution

dependent from n and γ, which gives wider confidential interval. Therefore for finding of $\Delta\overline{X}$ it is necessary $S_{\overline{X}}$ to multiply on coefficient of Student $t_{\gamma;n}$ (number $t_{\gamma;n}$ shows in how many times it is necessary to increase $S_{\overline{X}}$ for cover of $\overline{X}$ with probability of γ). So, under the table is determined $t_{\gamma;n}=t_{0.95;7}=2.447$ and we find the absolute error of $\overline{X}$ or ΔX:

$$\Delta\overline{X} = S_{\overline{x}} \cdot t_{\gamma;n} = 1.56 \cdot 2.447 = 3.82 \approx 4;\ \Delta X = S_X \cdot t_{\gamma;n} = 4.12 \cdot 2.447 = 10.$$

5. **True value of the pulse rate is in the interval:**

$\mathbf{X_{tr}} = \overline{X} \pm \Delta\overline{X} = 65 \pm 4$ **(characterizes variability of data of** $\overline{X}$ **)**

or more often in medicine: $\mathbf{X_{tr}} = \overline{X} \pm \Delta X = 65 \pm 10$ **(characterizes variability of data in the *given* sample, that more often interests the researcher)**

6. Relative error $\varepsilon = \dfrac{\Delta\overline{X}}{\overline{X}} = \dfrac{4}{65} \cdot 100\% = 1.5\% < 5\%$

5. Processing of results of measurements: estimation of random errors of indirect measurements

Example 2: at finding of the volume of cylinder (fig. 14) $V = \pi R^2 H = \dfrac{\pi D^2}{4} H$ six times have measured diameter and height of the cylinder (tab. 9). To find value of the volume of the cylinder (to give the interval estimation of V_{tr}): $V_{tr} = \overline{V} + \Delta\overline{V}$ (or more often in medicine $V = \overline{V} \pm \Delta V$)

Table. 9

n	1	2	3	4	5	6
D, mm	5.8	5.7	5.7	5.9	6.0	5.9
H, mm	8.2	8.2	8.1	8.3	8.3	8.1

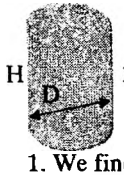
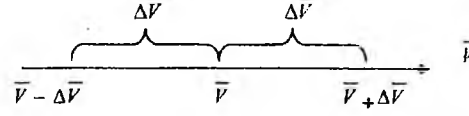


Fig.14



1. We find average values of directly measuring values (means)

$$\overline{D} = \frac{5.8 + 5.7 + \ldots + 5.9}{6} = 5.83; \qquad \overline{H} = \frac{8.2 + 8.2 + \ldots + 8.1}{6} = 8.2\ .$$

2. We find $\overline{V}$: $\overline{V} = \dfrac{\pi \overline{D}^2}{4} \cdot \overline{H} = \dfrac{3.14 \cdot 5.8 \cdot 8.2}{4} = 216.54$.

3. We determine standard deviations of directly measuring variables:

$$S_{\overline{D}} = \frac{S_D}{\sqrt{n}} = \sqrt{\frac{\sum (D_i - \overline{D})^2}{n(n-1)}} = 0.051; \qquad S_{\overline{H}} = \frac{S_H}{\sqrt{n}} = \sqrt{\frac{\sum (H_i - \overline{H})^2}{n(n-1)}} = 0.036.$$

4. Let's determine the standard deviation $S_{\overline{V}}$ for $\overline{V}$. For this purpose in the beginning we shall find partial derivatives

$$\frac{\partial V}{\partial H} = (\frac{\pi D^2}{4} \cdot H)'_H = \frac{\pi D^2}{4}; \quad \frac{\partial V}{\partial D} = \left(\frac{\pi D^2}{4} \cdot H\right)'_D = \frac{\pi HD}{2}, \text{ therefore}$$

$$S_{\overline{V}} = \sqrt{(\frac{\pi \overline{D}^2}{4} \cdot S_{\overline{H}})^2 + (\frac{\pi \overline{H}\,\overline{D}}{2} \cdot S_{\overline{D}})^2} = \sqrt{(\frac{3.14 \cdot 5.8}{4} \cdot 0.036)^2 + \left(\frac{3{,}14 \cdot 8{,}2 \cdot 5{,}8}{2} \cdot 0.051\right)^2} = 3.81$$

5. We determine with help of the table Student coefficient: $t_{0,95;6} = 2.57$.

Then absolute error of $\overline{V}$: $\Delta\overline{V} = S_{\overline{V}} \cdot t_{0,95;6} = 3.81 \cdot 2.57 = 9.8$.

True value (interval estimation of $\overline{V}$) of volume of our cylinder:

$V_{ist} = \overline{V} \pm \Delta\overline{V} = 216.5 \pm 9.8$ (mm^3)

6. Relative error of $\Delta\overline{V}$:

$$\varepsilon = \frac{\Delta\overline{V}}{\overline{V}} = \frac{9.8}{216.5} \cdot 100\% = 4.5\% < 5\%.$$

LECTURE № 4

ELEMENTS OF CORRELATION ANALYSIS

1. Concept of correlation relationship. Method of the least squares

Variables X and Y can be connected by functional and statistical relationships. At **functional** "rigid" relationship between investigated quantities to each value X corresponds certain value Y, for example: in the law of Ohm I=U/R at R=const to each value U corresponds one value of I. Functional relationships are characteristic for laws of physics, chemistry and other natural sciences.

In medical and biologic researches more often meets **statistical** relationships between quantities. For example, at certain change of age of a patient it is not observed strictly certain change of arterial pressure (AP). It speaks that change of AP depends not only on age of the patient, but also from lines of other factors: on sex, state of health, etc.

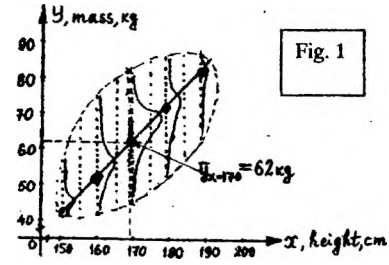


Fig. 1

Statistical relationship is caused by the several reasons: 1) influence on Y not only by X, but also other factors; 2) inevitability of mistakes at measurement of X and Y.

Special case of statistical relationship between X and Y is **correlation relationship, when to each value X the population mean (average arithmetic value) of distribution of other value Y is put in conformity.** For example, relationship between a doze of medical product X and its contents in blood Y. On Y influences the mass of a patient, speed of removing of a preparation and other factors, but at the same patient with growth of a doze of medical product its contents in blood unequivocally increases. Other example of correlation relationship is dependence between height of a person and his mass.

The person with height, for example, 170 cm can have mass both 50 kg, and 90 kg, but majority of people of this height have mass in the interval of 60-80 kg , that is to the given height correspond to the distribution of mass close to normal, with average value of M (Y). In figure 1 all possible values of mass of the person at *the given* height 170 cm are marked by dagger, and average value (62kg) is led round by circle. Clearly, that with increase in height, average value of mass M(Y) of a person will grow also, that is we deal with correlation dependence between height X and mass Y:

$$M(Y_x)=f(x). \qquad (1)$$

All set of values X and Y (points on the graph) forms **the correlation field** which has been led round by a dotted line in figure 1.

The equation (1) is named **equation of regress of Y on X**, and its graph is known as **line of regress**. It is similarly possible to describe and invert correlation dependence $M(X_y) = \varphi(y)$ if it exists. If functions $f(x)$ and $\varphi(y)$ are linear functions that it is possible to estimate on character of arrangement of points of a correlation field, then these functions can be presented as:

$$M(Y_x)=ax + b= (\text{Slope})\cdot(x)+(\text{y-intercept}),$$
$$M(X_y)=cy + d$$

For finding of coefficients **a** (Slope) and **b** (y-intercept), included in the equation of a straight line, we use the method **of the least squares.**

Method of the least squares.

In 1806 the French mathematician Lezhandr has proved, that in the best way relationship between X and Y will be reflected by a direct line $\bar{y}_x = ax + b$ for which the condition (see fig. 2) satisfies:

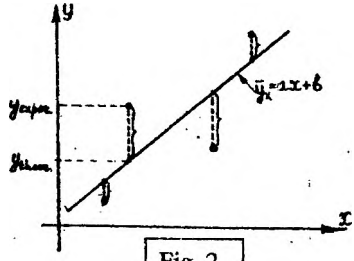


Fig. 2

$$S=\sum_{i=1}^{n}(y_{i\,exp} - y_{i\,theor})^2 = \min, \quad (2)$$

where $y_{i\,exp}$ is value Y received from experiment, $y_{i\,theor}$ is calculate y, laying on a straight line, S is deviation. As $y_{i\,theor}=ax+b$, condition (2) can be written down

$$S(a,b)=\sum_{i=1}^{n}(y_i - ax_i - b)^2 = \min \qquad (3)$$

Expression (3) means, that values of coefficients a and b should be picked up so, that the sum of squares of deviations of ordinates of experimental points from ordinates of points of a smoothing straight line would be **minimal** (fig. 2).

According to rules of research of function of several variables on a minimum the following conditions for partial derivatives of this function of the first and second orders should satisfy:

$$\begin{cases} \dfrac{\partial S}{\partial a}=0 \\ \dfrac{\partial S}{\partial b}=0 \end{cases} \text{ and } \begin{cases} \dfrac{\partial^2 S}{\partial a^2}>0 \\ \dfrac{\partial^2 S}{\partial b^2}>0 \end{cases} \qquad (4)$$

According to (4):

$$\begin{cases} \dfrac{\partial S}{\partial a}=2\sum_{i=1}^{n}(y_i - ax_i - b)(-x_i)=0 \quad (5) \\ \dfrac{\partial S}{\partial b}=2\sum_{i=1}^{n}(y_i - ax_i - b)(-1)=0 \quad (6) \end{cases} \qquad \begin{cases} \dfrac{\partial^2 S}{\partial a^2}=2\sum_{i=1}^{n}x_i^2>0 \\ \dfrac{\partial^2 S}{\partial b^2}=2\sum_{i=1}^{n}1=2n>0 \end{cases} \qquad (7)$$

As the system of inequalities (7) is satisfied at anyone a and b, we shall solve the first system of the equations. We divide both parts of (5) and (6) on 2 and we multiply on (-1):

$$\begin{cases} a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i \\ a\sum_{i=1}^{n} x_i + bn = \sum_{i=1}^{n} y_i \end{cases} \quad (8)$$

The system (8) is known as system of normal equations of Gauss. Solving this system, we shall find a and b, and we shall receive the required equation of a straight line: $\bar{y}_x = ax + b$, where

$$a = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \quad (9), \qquad b = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n\sum x_i^2 - (\sum x_i)^2} \quad (10).$$

Example 1. Concentration of alcohol (Y_i=c) is measured in blood at n=5 volunteers with identical mass after several portions of alcohol (X_i). By method of the least squares determine coefficients a and b of a smoothing straight line $\bar{y}_x = ax + b$. Construct the graph.

Number of portions, X_i	2	2	4	5	8
Concentration, Y_i	0.05	0.06	0.11	0.13	0.22

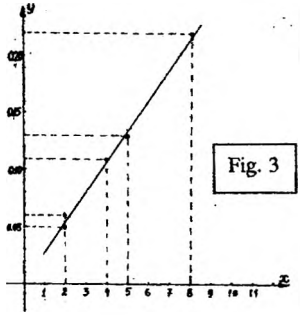


Fig. 3

According to (9) and (10) we shall find preliminary $\sum x_i$, $\sum y_i$, $\sum x_i^2$, $\sum x_i y_i$:

x_i	y_i	x_i^2	$x_i \cdot y_i$
2	0.05	4	0.10
2	0.06	4	0.12
4	0.11	16	0.44
5	0.13	25	0.65
8	0.22	64	1.76
$\sum x_i = 21$	$\sum y_i = 0.57$	$\sum x_i^2 = 113$	$\sum x_i y_i = 307$

$$a = \frac{5 \cdot 3.07 - 21 \cdot 0.57}{5 \cdot 113 - 21^2} = 0.027; \qquad b = \frac{113 \cdot 0.57 - 21 \cdot 3.07}{5 \cdot 113 - 21^2} = -0.00048.$$

The required equation of a smoothing straight line is $\bar{y}_x = ax + b$:

$\bar{y}_x = 0.027x - 0.00048$.

The graph of the required smoothing straight line is resulted on fig. 3.

2. Linear correlation and its characteristics

The establishment **of force and narrowness** of correlation relationship makes a problem *of the correlation analysis*, and *the regression analysis* establishes **the form** of dependence between X and Y (linear, curvilinear) also allows to predict one variable on another.

Historically the theory of correlation in biology began to apply earlier than in other areas of natural sciences. French biologist Z. Kuvie in1800-1805 in «Lectures on

comparative anatomy » has formulated known principle of biological correlation: any part of an organism is by all means coordinated with other parts, hence, on one body it is possible to judge the whole organism. In 1899 Englishman K. Pearson, the founder of the mathematical theory of correlation has deduced the formula connecting height of the modern person with length of his hip. Using this formula, on length of mineral hip Pearson has determined the height of the prehistoric peoples.

For the characteristic **of the form** of the equation of relationship *first of all* it is necessary to take into account theoretical reasons of character of relationship between considered variables. *Second*, character of an arrangement of points of a correlation field also allows doing conclusions about the form of relationship. The extended form of a correlation field and the angle with axes of the graph close to 45° specifies presence of correlation relationship (fig. 5 d, e). If the congestion of points forms a circle or an ellipse, which long axis is parallel to one of axes of coordinates, it is possible to assume, that relationships between variables is absent (fig. 5a).

Force of relationship between X and Y expresses coefficient "a" (slope) found on the formula (9) and is known as **coefficient of regress** (it frequently designate by ρ_{yx}). The coefficient of regress ρ_{yx} shows, on how many units will change on the average Y if change of X will take place equally on unit. The more ρ_{yx} the relationship is stronger. The linear equation of regress can be written down in the standard form

$$y - \bar{y} = \rho_{yx}(x - \bar{x}), \qquad (11)$$

Narrowness of relationship (degree of disorder of points) is estimated with help **of coefficient of correlation r**. We shall receive the working equation for r.

At first sight, for the characteristic of disorder of points it is possible to count up *product* $(x_i - \bar{x})(y_i - \bar{y})$ (on fig. 4 this product is expressed by the shaded rectangular) and then to find *average value* of all products (for elimination of dependence on number of pairs supervision):

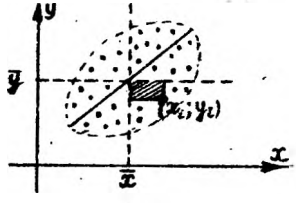


Fig. 4

$$C = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n} \qquad (12)$$

The received variable is called **covariation** that means « the connected variation ». However C depends on the scale chosen on axes. This lack of covariation can be removed if to divide C on product of average quadratic deviations $\sigma_x \cdot \sigma_y$. In result we shall receive the characteristic of narrowness of relationship – **coefficient of correlation r**:

$$r = \frac{C}{\sigma_x \sigma_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n\sigma_x \sigma_y} \qquad (13)$$

Let's open brackets in numerator and we shall take into account, that $\sum_{i=1}^{n} x_i = n\bar{x}$, $\sum_{i=1}^{n} y_i = n\bar{y}$, then

$$C = \frac{\sum_{i=1}^{n} x_i y_i - \bar{x}\sum_{i=1}^{n} y_i - \bar{y}\sum_{i=1}^{n} x_i + n\bar{x}\bar{y}}{n} = \frac{\sum_{i=1}^{n} x_i y_i - \bar{x}n\bar{y} - \bar{y}n\bar{x} + n\bar{x}\bar{y}}{n} = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{n} = \overline{xy} - \bar{x}\bar{y}, \quad (14)$$

where $\frac{\sum_{i=1}^{n} x_i y_i}{n} = \overline{xy}$, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$. (15)

Then the coefficient of correlation r with the account of (14) is equal

$$r = \frac{\overline{xy} - \bar{x}\bar{y}}{\sigma_x \sigma_y} \quad (16)$$

In practice we have the data not about all general population, but only about those variables that are received from experiment (sample). Therefore determine *sample correlation coefficient* r_s, approximately equal to general coefficient of correlation r. Designating average quadratic deviations for sample s_x and s_y, we shall receive

$$r_s = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{s_x s_y} = \frac{\overline{xy} - \bar{x}\bar{y}}{\sqrt{\overline{x^2} - (\bar{x})^2}\sqrt{\overline{y^2} - (\bar{y})^2}}, \quad (17)$$

where $s_x = \sqrt{\overline{x^2} - (\bar{x})^2}$, $s_y = \sqrt{\overline{y^2} - (\bar{y})^2}$, $\overline{x^2} = \frac{\sum_{i=1}^{n} x_i^2}{n}$, $\overline{y^2} = \frac{\sum_{i=1}^{n} y_i^2}{n}$. (18)

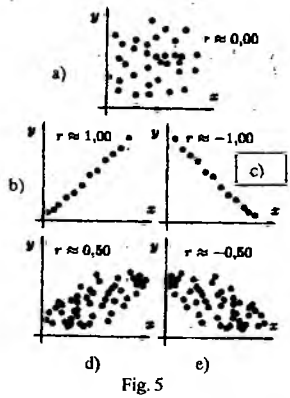


Fig. 5

Properties of correlation coefficient.

1) The value of correlation coefficient changes from-1 up to +1, that is

$$-1 \le r \le 1.$$

2) The closer $|r|$ to unit, the more closely relationship, the closer to a straight line points (fig. 5b, c) are grouped. The following gradation of narrowness of linear correlation relationship is accepted:

Narrowness of relationship	Correlation coefficient r
Relationship is absent	0
Relationship is weak	from 0 up to 0.3
Moderate	From 0.3 up to 0.7
Strong	From 0.7 up to 1
Functional	1

3) The mark of correlation coefficient shows a direction of relationship: a straight line (positive – fig. 5b, d) and inverse (negative, fig. 5 c, e).

Between coefficient of regress ρ_{yx} and correlation coefficient r there is a close relationship: $\rho_{yx}=r\frac{s_y}{s_x}$, therefore $b=\bar{y}-\rho_{yx}\bar{x}$ (19)

Then **predicted value y** at the given value x is equal:

$$y(x)=ax+b=r\frac{s_y}{s_x}x+(\bar{y}-r\frac{s_y}{s_x}\bar{x})$$

3. Testing of significance of a correlation coefficient.

As r_s is determined according to sample, as against correlation coefficient *of all* general population r_s is *random value*. If $r_s\neq 0$, there is a question: whether it speaks really existing linear relationship between X and Y or it is caused by random factors. For the answer to this question the value $t_{experim}$ is calculated:

$$t_{experim}=\frac{r_s\sqrt{n-2}}{\sqrt{1-r_s}}. \qquad (20)$$

Further under the table (page 302 of Lobotzkaya textbook on mathematics), we find value $t_{critical}$, which has Student's distribution at the set significance value p (connected with confidential probability by parity $p=1-\gamma$) and at number of degrees of freedom f=n-2. Then compare $t_{experim}$ and $t_{critical}$: **if $|t_{experim}| > t_{critical}$, that is possible to draw a conclusion, that the correlation coefficient is significant,** otherwise linear relationship can be caused by random factors. If the correlation coefficient appears significant it is possible to predict value of Y at any value X.

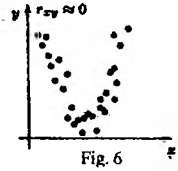


Fig. 6

Remark.

The correlation coefficient characterizes ***relationships*** between variables, ***but does not explain it.*** Presence of correlation between X and Y can be caused by that: variable X influences on Y; variable Y influences on X; on X and Y the third latent factor that creates impression of relationship between X and Y (false correlation) influences. Besides, if r = 0 it not always speaks about absence of statistical relationship between X and Y, relationship can be and nonlinear (fig. 6).

Example 2. In experiment at 13 cats the data about of intrascleral (x) and intraocular pressure (y) are received:

X	19.8	7.8	12.7	13.4	10.3	13.7	16.2	15.4	21.5	8.1	11.7	7.6	6.1
y	32.5	16.1	21.3	26.8	23.4	19.7	22.9	22.2	22.6	17.6	14.3	18.6	21.4

Task:

1) Establish, whether there is a correlation between x and y; determine correlation coefficient r_s.
2) Determine narrowness of correlation relationship.
3) Test the significance of correlation coefficient.
4) Form the equation of regress and find predicated value for y at x=18.

Solution:

1) Let's construct the graph, having postponed along the axis X value of intrascleral pressure x and along the axis of ordinates Y – value of intraocular pressure y. Then to each pair of values x and y on the graph will be correspond the certain point (see fig. 7). On character of an arrangement of points it is possible to assume existence of linear correlation relationship between x and y.

Let's calculate sample coefficient of linear correlation r_s under the formula (17) with the account of (18) and (15):

$$\bar{x} = \frac{1}{13}(19.8 + 7.8 + ... + 6.1) = 12.64 \qquad \bar{y} = \frac{1}{13}(32.5 + 16.1 + ... + 21.4) = 21.49$$

$$\overline{x^2} = \frac{1}{13}(19.8^2 + 7.8^2 + ... + 6.1^2) = 180.5 \qquad \overline{y^2} = \frac{1}{13}(32.5^2 + 16.1^2 + ... + 21.4^2) = 482.2$$

$$\overline{xy} = \frac{1}{13}(19.8 \cdot 32.5 + 7.8 \cdot 16.1 + ... + 6.1 \cdot 21.4) = 283.9$$

$$s_x = \sqrt{180.5 - 12.64^2} = 4.55 \qquad s_y = \sqrt{482.2 - 21.49^2} = 4.51$$

$$r_s = \frac{283.9 - 12.64 \cdot 21.49}{4.55 \cdot 4.51} = 0.595$$

2) Using the table of gradation of an estimation of narrowness of relationship, we judge: relationship of x and y is moderate, positive.

3) For testing of significance of correlation coefficient r_s we shall calculate under the formula (19) value of $t_{experim}$:

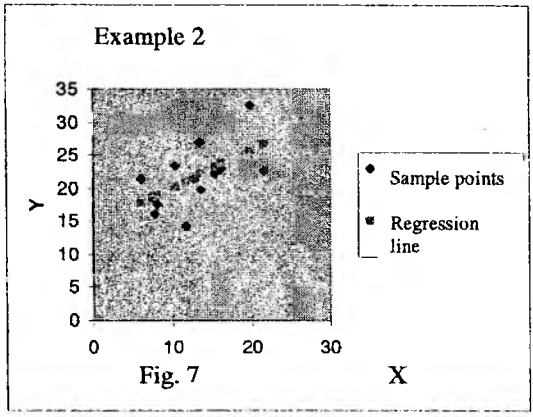


Fig. 7

$$t_{experim} = \frac{0.598 \cdot \sqrt{13-2}}{\sqrt{1-0.598}} = 10.375$$

Under the table we find value of $t_{critical}(11; 0.05)=2.20$. As $|t_{experim}| > t_{critical}$, that is $10.375 > 2.20$, that we judge, that the correlation coefficient is significant.

4) Under the formula (19) we find coefficient of regress:

$$\rho_{yx} = 0.595\frac{4.51}{4.55} = 0.589.$$

Further, substituting ρ_{yx} in the formula (11) and calculating b, we find the equation of regress $y = \rho_{yx}x + b$:

$$y=0.589\,x + 14\,.$$

Coefficients a and b could be found under formulas (9) and (10) also.

And, at last, we calculate predicated value at x=18:

$$y(18)= 0.589 \cdot 18 + 14 = 24.6$$

LECTURE № 5

Testing of statistical hypotheses

1. Concept of a statistical hypothesis

Why statistical methods are used in medicine?

The direct judgement about efficiency of any method of treatment is unreliable because of lines of reasons: biological variability, subjectivity of estimations, psychotherapeutic effect and other reasons. Patients and doctors meet examples of the unreliable, unscientific medical information, for example, in advertising rollers about high efficiency of new medicines or new techniques of treatment. And if it is pardonable to patients to go on occasion at advertising, doctors should approach to such information critically: it is necessary to know, on what numbers of patients, during what type of research results have been received? etc. Skills of a critical estimation are so important and necessary for the modern doctor, as, for example, skill to auscultate the patient.

Development of ideas of a critical estimation of the medical information has resulted in occurrence in 80th years of the last century of the concept of Evidence Based Medicine (EBM). Basic positions of EBM: 1) each decision of the doctor should be based on the scientific data; 2) the weight of each fact is more, than more strictly a technique of research during which this fact is received. As "The gold standard" are considered randomized (i.e. received as result of casual selection) controllable researches. Individual medical experience and opinion of experts or "authorities" are considered as not having a sufficient scientific basis.

One of the major components of EBM is use of the scientifically-grounded statistical methods, one of which is check of statistical hypotheses.

Concept of a statistical hypothesis

The statistical hypothesis (H) is any assumption about *a kind* or *parameters* of population which is checked on the basis of the sample data.

For example: 1) **H:** the weight of newborns is distributed under the normal law (hypothesis about a kind of distribution);

2) **H:** average values ($M_{(Y)} = M_{(X)}$) of arterial pressure in two groups of patients are equal, i.e. both samples are taken from the same population (hypothesis about parameters of distribution).

Not all scientific hypotheses are statistical: so, the hypothesis of De Broglie about wave properties of electrons is not statistical as at it is not present any law of distribution or parameters.

To check of statistical hypotheses are reduced problems of check and estimation of various processes: comparison of medical techniques, characteristics of preparations and medical equipment, efficiency of treatment, duration of illness, profitability, etc.

Zero and alternative hypotheses

Checked hypothesis is called *null* and designated by H_0. The null hypothesis always rejects effect of intervention. Alongside with null hypothesis also is considered one of the *alternative* hypotheses that is designated as H_1.
For example, let a lot of a pharmaceutical medicine supervise on small sample and compare to norm; then null hypothesis H_0: the released lot of pharmaceutical medicine is non-standard (spoilage), and alternative is H_1: the lot corresponds to norm.

Problem of testing of statistical hypotheses

The problem of testing of hypotheses: on the basis of the analysis of the sample data (the incomplete information) to make the decision on validity of one of hypotheses.

2. Type I and type II errors

At check of hypotheses because of presence of the incomplete information can be admitted errors of two kinds (see the table):

1. It is accepted H_1, when actually is true H_0: ***the type I error*** (the erroneous conclusion about existence of distinctions which actually are not present, or "hyperdiagnostics").

2. Is accepted H_0, when is true H_0 ***:the type II error*** (to not find really existing distinctions, or "hypodiagnostics").

The result received at checking ↓	*That is actually*	
	Hypothesis Ho is true	Hypothesis H_1 is true
Hypothesis Ho is accepted	Correct decision of probability 1-α	Wrong decision of probability β, ***type II error***
Hypothesis H_1 is accepted	Wrong decision of probability α, ***the type I error***	Correct decision of probability 1- β **power (sensitivity)**

Probability to make *the type I error* should be small, because must be *the weighty arguments* for recognition, that one method of treatment is better than another, for example. This probability *p* is known *as a significance level α.*

The significance level α is called probability of rejection of a null hypothesis when it is actually true.

More serious consequences of the type I error, it is necessary to take less a significance level. *In medical examinations usually is used $\alpha = 0.05$, or $\alpha = 0.01$ and value $\beta = 0.2$ or 0.1.* It is desirable, that *α and β* be *as small as possible* that can be reached only *incrementing a sample size*. This

deduction is very important because it is directly interlinked to scheduling experiment.

Reasonable parity between α and β find, proceeding from weight of consequences (damage) of each of errors. For example, let it is checked hypothesis H_0 about absence at the patient of the certain disease, and an attribute of disease is size of arterial pressure (AP). Then H_0:AP in norm, i.e. the patient is healthy, H_1: AP differs from norm, i.e. the patient is sick. Then the type I error (rejection H_0 when it is true): confession of the patient as sick, when he actually is healthy. Consequences of this mistake are inconveniences for the patient who, for example, should pass additional inspection or treatment. Other situation in case of the type II error: to recognize the person able-bodied, when he actually is sick. Actually there is a failure from treatment of the patient, it is wasted time and consequences of the type II error can be pitiable. So, in our example with the purpose to reduce probability of the type II error, it is possible to not take the high level of significance α (that is to accept α equal, for example, 0.05, instead of 0.01).

Opposite situations are possible also. Values α smaller than 0.01 are used at statistical detection of toxiferous medical preparations when the major meaning has the guarantee from wrong rejection of a tested hypothesis.

So, at selecting of hypotheses *a null hypothesis* (in comparison with the alternative) should be hypothesis, *which is more dangerously to falsely reject.*

3. Statistical test. Critical areas ("tails")

For checkout of an accepted hypothesis use a random quantity K that is function from the sample data and is called statistical test.

The statistical test is the rule (formula) permitting according to sample to accept or reject a null hypothesis.

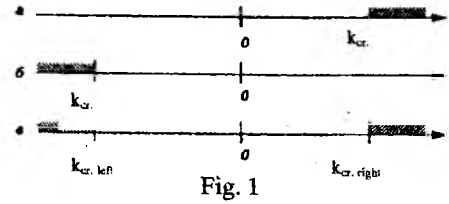
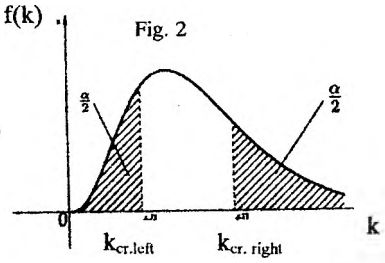


Fig. 1

The statistical test, being a random quantity, has any probability distribution, for example: normal distribution, Student's distribution, Fisher distribution, $\chi 2$-distribution etc.

Depending on the accepted significance level α from all area of allowable values of test K, allocate (see fig. 1) *critical* region. It makes with help of number $\kappa_{cr.}$, that find with help of tables of distribution of each test K. Further works the following rule (**basic principle of testing of statistical**



hypotheses): *if the observed value $k_{obs.}$ of the test K calculated on sample gets to the critical region, zero hypothesis H_0 is rejected for the benefit of alternative H_1 and if $k_{obs.}$ not gets, H_0 is accepted.*

Critical region depending from selection of κ_{cr} can be *unilateral* (right-tailed or left-tailed) or *two-tailed* (fig. 2, 3, 4). For right-tailed (left-tailed) critical region, value K satisfy to the condition: $P(K \geq \kappa_{cr.})=\alpha$ and $P(K \leq \kappa_{cr.})=\alpha$, where $P(\ldots) = \alpha$ is probability, that test K has the value greater (or accordingly smaller) than $\kappa_{cr.}$. $P(\ldots) = \alpha$ is equal to the area of the right or left "tail" on the graph of distribution of probabilities (fig. 3). Similarly, for two-tailed critical region $P(K \leq \kappa_{cr.}) + P(K \geq \kappa_{cr.})=\alpha$, i.e. value α is the area of both "tails" on the graph of distribution of probabilities (fig. 2).

The unilateral critical region should be used, when process interesting us should go only in one direction. For example, there are weighty arguments to assert, that the certain diet will necessarily lower weight of a patient. But even in this case it is necessary to make secure, having chosen two-tailed critical region. In our example it means, that at some people the offered diet can result in increase in weight. Therefore, as practice shows, *in the majority of researches the two-tailed test is applied.*

Selection of statistical test can be compared, for example, to a rule on which the lowest passing score pays off at receipt in high school. Then the rule of calculation of a lowest passing score is a criterion **K**, $\kappa_{obs.}$ is typed quantity of points, a $\kappa_{cr.}$ is a lowest passing score, having overcome which we make any decision (hypothesis).

There are parametrical and nonparametric tests. *Parametrical tests* are used, if samples are taken from population, which submits to known law of distribution, for example, to the normal law of distribution. Normality of distribution of sample should be statistically proved before application of parametrical test.

Nonparametric tests are used, if there is no submission of distribution of sample to the normal law. For example, if the volume of sample is so small, that it is impossible to estimate the law of distribution of the data in sample. Parametrical tests are more powerful, than nonparametric in detection of real effect.

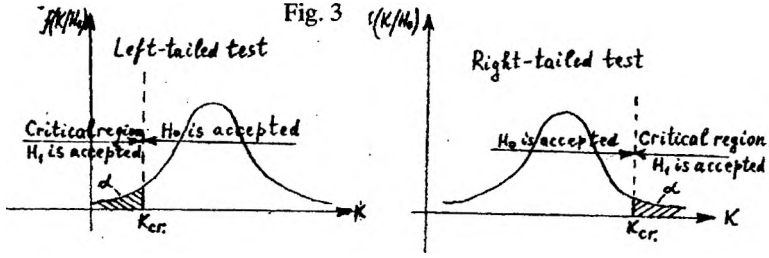
From the researcher using statistical testing of hypotheses in applied problems is necessary to learn to use existing tests.

4. Procedure of testing of hypotheses

Test of hypotheses usually passes the following steps.

1. From one or several populations is collected the initial statistical

Fig. 3



material as two or some samples.

2. A researcher formulates the basic (H_0) and alternative (H_1) hypotheses, and also chooses a significance level α (0.01 or 0.05), corresponding to the purposes of researches.

3. *Must be selected the test K*, which approaches in the given situation and is defined, what test is necessary (right-tailed, left-tailed or two-tailed) and then under corresponding formulas we calculate *value of statistical test* K_{obs}, for available data (samples).

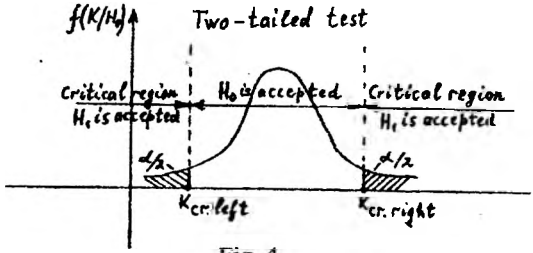


Fig. 4

4. Under the tables, corresponding to the chosen method we find the border *of critical region* $K_{cr.}$ for the accepted significance level (fig. 3 and fig. 4).

5. It is made a decision on validity of the hypothesis Ho or H_1. *If value* $K_{obs.}$ *of the test calculated in the item 3 belongs to critical region (item 4), the basic hypothesis* H_0 *is rejected and accepted alternative hypothesis* H_1*(distinctions between observable values and theoretical are significant, i.e. are caused by error of zero hypothesis). If values of the test do not get to the critical region, the hypothesis* H_0 *is accepted (distinctions are not significant and caused by the casual reasons) (see fig. 3,4).*

5. What means $p < 0.05$?

During of testing of statistical hypotheses except of calculation of statistical test K, in the modern statistical packages is calculated corresponding value *p*, where *p* is a probability that the data corresponds to null hypothesis H_0. Small values of *p* testify about "wonderfulness" of such event and results, that H_0 is rejected. Usually H_0 is rejected, when *p-values* is less *than*

0.05.

Comparing received value p with the accepted significance level α, we make a conclusions about hypotheses:

if $p > \alpha$ (α is the accepted significance level, usually is 0.05), then H_0 is accepted (distinctions are non significant);

if $p < \alpha$, H_0 is rejected (distinctions are statistically significant at $p < 0.05$).

Using of round numbers 0.05; 0.01, etc. as a significance level is a consequence of manual of statistical calculations in precomputer time. Now it is recommended to specify exact value of p (to within three marks), that allows the reader to estimate independently the statistical importance of result, for example, values p=0.049 or p=0.051 should be interpreted practically equally.

Dependent and independent samples

Two samples are **dependent** if the values in one are related to the values in the other in some way. Two samples are **independent** if the values in one are not related to the values in the other. Example of *independent* samples: parameters (for example, arterial pressure) of two groups of patients to which were applied different techniques of treatment.

Examples of *dependent* (connected samples):

1) parameters *of one* group of patients *before and after* influence of any factor, for example, techniques of treatment;

2) parameters of the different parts of the same object, for example, a condition of two finitenesses, one of which was exposed to treatment, and the second is not.

Let's pass to consideration of some most popular statistical hypotheses used in medical researches and examples of their using.

6. Test comparing two means: Student's test

The similar problem arises at comparison of two samples, for example, two groups of the patients, undergone to particular action (for example, treatment on some procedures of two groups of patients, one of which accepts a particular medicinal preparation, and other, control group, accepts placebo(the medicinal form containing neutral matters). Thus comparison of two means allows to judge about degree of action, about significance of a possible effects or its absence.

1. Target setting. From two populations X and Y, distributed under the normal law (testing of normality of both samples will necessarily be spent beforehand), with *equal variances are obtained two* samples of volumes n_X and n_Y accordingly. It is required to compare means of the relevant populations.

Let's consider sequence of the steps at solution of this problem according to the order set above.

2. Tested hypotheses:

$\mathbf{H_0}$: $M(Y_p)=M(X_p)$ (means of two populations are equal);

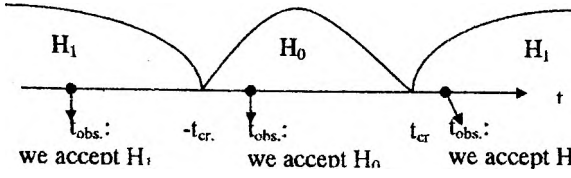
$\mathbf{H_1}$: $M(Y_p) \neq M(X_p)$, i.e. we take two-tailed critical region.

3. For a testing of hypothesis about equality of means of two populations at equal variances is applied Student's test, which value $t_{obs.}$ is calculated under the formula:

$$t_{obs.} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{(n_X - 1)\cdot S_X^2 + (n_Y - 1)\cdot S_Y^2}{n_X + n_Y - 2}}} \cdot \sqrt{\frac{n_Y \cdot n_Y}{n_Y + n_1}},$$

where S_X^2 and S_Y^2 are sample variances, $\bar{x}$ and $\bar{y}$ are means of samples.

Fig. 5



4. From the table of t-distribution we take for a confidence level α =0,05 two-tailed critical region. For this purpose beforehand we determine number of degrees of freedom under the formula:

$$f = n_X + n_Y - 2$$

Critical region for rejection of H_0 (fig. 5):

$$|t_{obs.}| > t_{cr.}$$

7. Test comparing two variances. Fisher F-test

In many clinical examinations it is important testing of a hypothesis about equality of two populations variances of two *normal* samples. This problem can be solved with help *of F-test of Fisher.* The similar problem of comparison of variances arises in case of comparison of precision of measurements, precision of devices, comparison of methods. As the variance characterizes degree of dispersion of values concerning of mean, the best method has minimal variance.

1.*Target setting*. For random quantities X and Y, distributed under the normal law, are obtained two samples of volumes n_X and n_Y accordingly. It is required to compare variances of the relevant populations.

2. Tested hypotheses: $H_0 : D_p(Y) = D_p(X)$ (populations variances are identical).

Alternating hypothesis H_1: $D_p(Y) \neq D_p(X)$ (two tailed critical region).

3. For a testing of hypothesis about equality of variances we use Fisher F-test. We compute concrete values of sample variances S_X^2 and S_Y^2, also we discover the ratio (observed value of test):

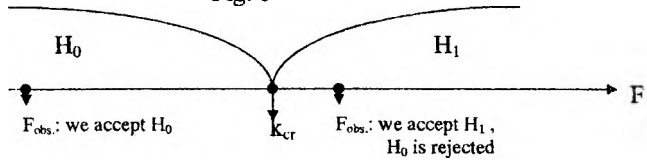
$$F_{obs.} = \frac{S_L^2}{S_S^2},$$

where S_L^2 and S_S^2 are larger and smaller of numbers S_X^2 and S_Y^2.

4. Further under the table of F-distribution for the given confidence level α and for numbers of degrees of freedom $\kappa_1 = n_L - 1$ and $\kappa_2 = n_S - 1$ we find a critical value $\kappa_{cr.} = F_{cr.}(\frac{\alpha}{2}; \kappa_1; \kappa_2)$. It is proved, that in this case the two-tailed critical region can

be exchanged by right-tailed, that is, if $F_{obs.} < \kappa_{cr.}$, that hypothesis H_0 accepts, if $F_{obs.} > \kappa_{cr}$, that difference of variances is significant and H_0 rejected (fig. 6).

Fig. 6



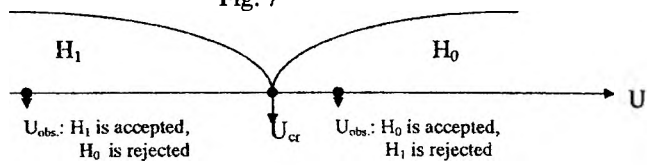
8. Wilcoxon-Mann-Whitney U-test

The given test is nonparametric analog of the t-Student test and is used for testing of a hypothesis about belonging of two *independent* samples to the same population. Here there is no necessity that samples had a normal distribution. The nonparametric tests based on ranks use numbers 1,2,3 ..., featuring their position in a ranked data set.

1. *Target setting.* For random quantities X and Y with unknowns laws of distribution obtain samples of volumes n_X and n_Y. Values of elements are submitted in *a serial scale.* It is required to test a hypothesis about belonging of compared independent samples to the same population. α=0.05 or 0.01.
2. Tested hypotheses: H_0: $M(X)=M(Y)$. H_1: $M(X)\neq M(Y)$.
3. For calculation of U-test it is necessary:

- to arrange numerical values of samples to one general series;
- to enumerate terms of general series from 1 up to $N=n_1+n_2$, where n_1 and n_2 are volumes of the first and second samples. These numbers will be ranks of terms of series. If there are identical values of sample elements, then for these elements give the identical ranks, equal to arithmetic mean of ranks of identical elements;
- for each sample to find the sum of ranks R_1 and R_2;
- to find values U_1 and U_2 (observed values of test):

Fig. 7



$$U_1 = R_1 - \frac{n_1(n_1+1)}{2} \text{ and } U_2 = R_2 - \frac{n_2(n_2+1)}{2},$$

and to pick $U_{obs.}$ as smaller from U_1 and U_2;

4. Under the table of critical values of U-test at the given confidence level to find $U_{cr.}$ If $U_{obs.} > U_{cr.}$, then H_0 is accepted (differences statistically non significant) – fig. 7.

9. Examples of using of statistical tests

Student's t-test. Fisher F-test

Example 1. S.Hejl and co-authors measured diameter of coronary arteries after reception of Nifedipinum (a preparation dilating vessels) and after reception of placebo, and have received two samples of the data of diameter of coronary arteries in mm.
Placebo: 2.5; 2.2;2.6;2.0;2.1;1.8;2.4;2.3;2.7;2.7;1.9;
Nifedipinum:2.5;1.7;1.5;2.5;1.4;1.9;2.3;2.0;2.6;2.3;2.2.
Whether allow indicated data to state, that Nifedipinum influences diameter of coronary arteries?
In other words, it is necessary to test significance of difference of two sample means. Let X is a population from which the first sample (placebo) is extracted; Y is second sample (Nifedipinum). Authors supposed that both populations have the normal distribution (this hypothesis should be tested by statistical methods).
For correct using of a t-Student test, it is necessary to test in the beginning equality of variances of two samples. We shall make it, having taken Fisher F- test.

1) *We set up hypotheses*: H_0: D(X)=D(Y); H_1: D(X)≠D(Y); we take the significance level α=0.05. Critical region is two-tailed.
2) We compute sample variances S_X^2 and S_Y^2:

$$\bar{x} = \frac{1}{11}\sum_{i=1}^{11} x_i = \frac{1}{11}(2.5 + 2.2 + ... + 1.9) \approx 2.29; \qquad \bar{y} = \frac{1}{11}\sum_{j=1}^{11} y_j \approx 2.08;$$

$$S_X^2 = \frac{1}{11-1}\sum_{i=1}^{11}(x_i - \bar{x})^2 = \frac{1}{10}((2.5-2.29)^2 + (2.2-2.29)^2 + ... + (1.9-2.29)^2) = 0.1009;$$

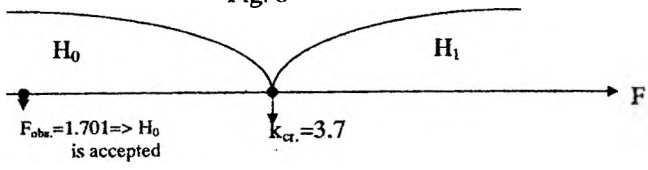
$$S_Y^2 = \frac{1}{11-1}\sum_{j=1}^{11}(y_j - \bar{y})^2 = 0.1716.$$

We compute F_{obs}. As $S_Y^2 > S_X^2$, then $F_{obs.} = \frac{S_Y^2}{S_X^2} \approx 1.701$.

3) Under tables of the table it is calculated $\kappa_{cr.}$:

$K_{cr.}=F_{cr.}(\frac{\alpha}{2}; \kappa_1; \kappa_2)=F_{cr.}(0.025;11\text{-}1;11\text{-}1)=3.72.$

Fig. 8



As $F_{obs.} < \kappa_{cr}$, i. e. 1.701<3.72 and $F_{obs.}$ not belongs to the critical region (fig. 8), we accept the null hypothesis about equality of variances. It allows to compare means with help of the Student's test.

1) *We set up hypotheses*. Null hypothesis H_0: M(Y) = M(X) and the alternative $\mathbf{H_1}$: M(Y)≠ M(X), i.e. a critical region is two-tailed. Significance level is α =0.05.

2) We compute observed value $t_{obs.}$:

$$t_{obs.} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{(n_X - 1)\cdot S_X^2 + (n_Y - 1)\cdot S_Y^2}{n_X + n_Y - 2}}} \cdot \sqrt{\frac{n_X \cdot n_Y}{n_X + n_Y}} =$$

$$\frac{2.29 - 2.08}{\sqrt{\frac{(11-1)1.1009 + (11-1)0.1716}{11+11-2}}} \sqrt{\frac{11\cdot 11}{11+11}} \approx 1.334 .$$

3) Number of degrees of freedom $f = n_X + n_Y - 2 = 11+11-2=20$.

Under the table of t-distribution it is discovered *for α =0.05* critical value $t_{cr.}(\alpha;f)=t_{cr.}(0,05;20)=2.09$. As critical region is two-tailed, it is *the interval* (-∞;-2,09)+(2,09;+∞), and the acceptance region is the interval (-2,09;+2,09).

Comparing $t_{cr.}$ and $t_{obs.}$, we see that $|t_{obs.}| < t_{cr.}$, i. e. 1.334<2.09 and $t_{obs.}$=1.334 hits in region (-2,09;+2,09) of acceptances of the hypothesis H_0 (fig. 9).

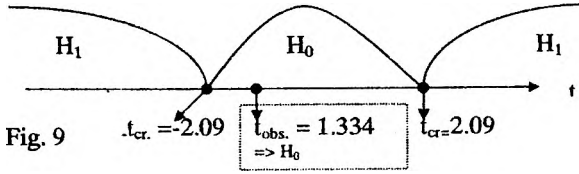


Fig. 9

Conclusion: the Student's test has not revealed essential differences in diameter of coronary arteries under influence of Nifedipinum.

Wilcoxon-Mann-Whitney-test (U-test)

Example 2. Let there are 2 groups of laboratory mice: experienced group (n_1=9) and control group (n_2=11). The mass of mice is measured in grammes.

Experienced n_1=9	64	68	70	72	75	76	79	80	83		
Control n_2=11	60	60	62	66	68	69	70	71	73	78	80

Using U-test to estimate significance of difference of mice mass at α =0.01.

1) We set up hypotheses. H_0: M(X)=M(Y). H_1: M(X)≠M(Y).

2) We arrange numerical values of samples to one general series and we assign ranks in ascending order with count of recurrence:

№	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Rank	1.5	1.5	3	4	5	6,5	6,5	8	9,5	9,5	11	12	13	14	15	16	17	18,5	18,5	20
n_1				64		68			70			72		75	76		79		80	83
n_2	60	60	62		66		68	69		70	71		73			78		80		

We discover the totals ranks of each group:

R_1= 4+6.5+9.5+12+14+15+17+18.5+20=116.5;

R_2= 1.5+1.5+3+5+6.5+8+9.5+11+13+16+18.5=93.5.

We calculate $U_1=116.5 - \frac{9(9+1)}{2}=71.5$; $U_2=93.5 - \frac{11(11+1)}{2}=27.5$.

As $U_{obs.}$ is selected the minimal value 27.5.

3). In the table we find $U_{cr.}(\alpha; n_1; n_2) = U_{cr.}(0.01; 9; 11) = 19$.

We compare: 27.5>19, i.e. $U_{obs.} > U_{cr.}$, hence we accept null hypothesis H_0 (fig. 10).

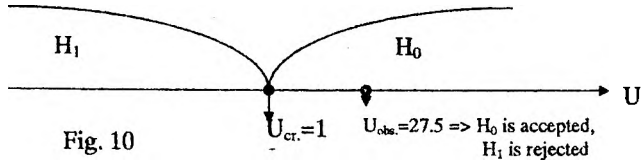


Fig. 10

Conclusion: difference between two means of values of mass of mice statistically non significant.

LECTURE №6

MECHANICAL OSCILLATIONS

1. Concept of oscillatory motion

The processes distinguished by any degree of repeatability are known as **oscillations**. Oscillatory motion and waves caused by it very often meet in nature, medicine and engineering. Make oscillations bridges under action of taking place trains, the eardrum of an ear makes oscillations, parts of buildings vibrate, and the cardiac muscle is rhythmically contracted.

Depending on a physical nature of repeating process can be oscillations: mechanical, electromagnetic etc., we shall consider mechanical oscillations.

2. Not damped harmonious oscillations

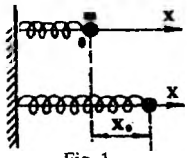


Fig. 1

Let on a body of mass m operates the force aspiring to return it to position of balance (returning force) and proportional to displacement of body from position of balance (fig. 1), i.e. force of elasticity $F_{el} = -kx$. If friction is absent, the equation of the second law of Newton for the body looks like:

$F_{el} = ma$; $\quad m\frac{d^2x}{dt^2} = -kx$ or $\quad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$.

Let's designate $\sqrt{\frac{k}{m}} = \omega_0$, then $\quad \frac{d^2x}{dt^2} + \omega_0^2 x = 0 \qquad (1)$

The equation (1) is the homogeneous differential equation of 2-nd order with constant coefficients. The solution of the equation (1) will be **the law of free or own not damped oscillations:**

$$\boxed{x = A\cos(\omega_0 t + \varphi_0)}, \qquad (2)$$

where A is value of the greatest displacement from position of balance, which is known as **amplitude** (amplitude is a constant, positive value); $(\omega_0 t + \varphi_0)$ is **phase** of oscillations; φ_0 is **initial phase**. Graphically not damped oscillations are submitted on fig. 2:

T is the period of oscillation (time interval of one full oscillation);

$T = \frac{2\pi}{\omega_0}$, where ω_0 is circular or cyclic frequency, $\omega_0=2\pi\nu$, ν is frequency of oscillation.

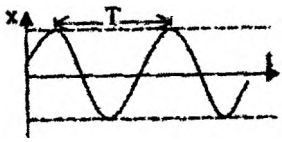


Fig. 2

To find velocity of a material point at harmonious oscillation, it is necessary to find a derivative from (2) with respect to t:

$$\vartheta = \frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t + \varphi_0) = \vartheta_0 \cos(\omega_0 t + \varphi_0 + \pi/2) \ , \ (3)$$

where $\vartheta_0 = A\omega_0$ is the maximal velocity (amplitude of velocity). Taking the derivative of this expression with respect to t we shall find acceleration:

$$a = \frac{d\vartheta}{dt} = -A\omega_0 \sin(\omega_0 t + \varphi_0) = -a_0 \sin(\omega_0 t + \varphi_0 + \pi) = a_0 \cos(\omega_0 t + \varphi_0 + \pi), \ (4)$$

where $a_0 = A\omega_0^2$ is the maximal acceleration.

Comparing (2), (3) and (4) we see, that velocity outstrips displacement on $\pi/2$ and acceleration on π.

3. Damped harmonious oscillations

In real conditions operates force of friction (force of resistance of environment), which at small velocity is proportional to velocity of motion of a body: $F_f = -r\vartheta = -r\frac{dx}{dt}$, where r is coefficient of resistance.

Therefore the equation of motion will become:

$ma = -kx - r\vartheta$ or $m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + kx = 0$, having divided on m let's receive: $\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$, having designated $\frac{r}{m} = 2\beta$, $\sqrt{\frac{k}{m}} = \omega_0$, let's receive:

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0 .$$

This equation is the homogeneous differential equation of second order with constant coefficients. Solution of this equation will be the **law of free damped oscillations**, and will have the following kind (fig. 3):

$$\boxed{x = A_0 e^{-\beta t}\cos(\omega t + \varphi_0)}.$$

From the equation it is visible, that amplitude $A = A_0 e^{-\beta t}$ is not a constant and depends on time and decreases on exponent law. As well as for not damped oscillations, value ω is known as circular frequency: $\omega = \sqrt{\omega_0^2 - \beta^2}$, where $\beta = \frac{r}{2m}$ is **coefficient of attenuation**; φ_0 is initial phase.

Graphically damped oscillations are submitted on fig.3.

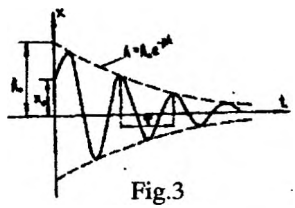


Fig.3

Let's determine period of oscillations $T = \frac{2\pi}{\omega}$ or $T = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}$, whence it is visible, that oscillations in system can arise, if resistance is small: $\beta << \omega_0$; period of oscillations is practically equal to $T = \frac{2\pi}{\omega_0}$.

For characteristic of velocity of attenuation of oscillations the concept of *coefficient of attenuation* is entered: $\beta = \frac{r}{2m}$. Let's find a time τ, for which the amplitude of oscillations will decrease in e times:

$$\frac{A_{(t+\tau)}}{A_t} = \frac{Ae^{-\beta(t+\tau)}}{Ae^{-\beta t}} = \frac{1}{e}, \qquad \text{that is} \qquad e^{-\beta\tau} = e^{-1},$$

whence $\beta\tau=1$, hence $\beta = \frac{1}{\tau}$. *Coefficient of attenuation β is inversely proportional to time interval τ for which the amplitude will decrease in e times.*

The ratio of values of two amplitudes distinguished for the period equal to $\delta = \frac{A_t}{A_{t+T}} = \frac{Ae^{-\beta t}}{Ae^{-\beta(t+T)}} = e^{\beta T}$ and is named **decrement of attenuation**, and its logarithm is called **logarithmic decrement of attenuation**:

$$\lambda = \ln \frac{A_t}{A_{t+T}} = \ln e^{\beta T} = \beta T .$$

4. Energy of oscillatory motion

In general form *kinetic energy* is expressed by the formula: $E_K = \frac{m\vartheta^2}{2}$.

For oscillatory motion it is possible to calculate E_k using the formula:

$$\vartheta = \frac{dx}{dt} = -A_0\omega_0 \sin(\omega_0 t + \varphi_0),$$

$$E_K = \frac{1}{2} m A^2 \omega_0^2 \sin^2(\omega_0 t + \varphi_0) = \frac{1}{2} k A^2 \sin^2(\omega_0 t + \varphi_0).$$

Potential energy of oscillatory motion we shall find from the general formula for potential energy of elastic deformation: $E_P = \frac{kx^2}{2}$. Then

$$E_P = \frac{1}{2} k A^2 \sin^2(\omega_0 t + \varphi_0).$$

Putting kinetic and potential energy, we shall receive *total mechanical energy* of oscillating material point:

$$E_{TOTAL} = E_K + E_P = \frac{1}{2} k A^2 \sin^2(\omega_0 t + \varphi_0) + \frac{1}{2} k A^2 \cos^2(\omega_0 t + \varphi_0) =$$

$$= \frac{1}{2} k A^2 \left[\sin^2(\omega_0 t + \varphi_0) + \cos^2(\omega_0 t + \varphi_0)\right] = \frac{1}{2} m \omega_0^2 A^2; E_{TOTAL} = E_k + E_p = const$$

i.e. at absence of forces of friction total mechanical energy of system *does not change* (values m, ω_0, A are constants). Graphic dependence of kinetic, potential and total energy of oscillating system on time is shown on fig.4.

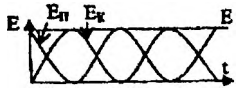


Fig.4

5. Forced oscillations

Free oscillatory motion of mechanical system always is damped due to presence of friction. For not damped oscillations it is necessary to fill from outside losses of energy on friction. For this purpose it is necessary to influence on system with external periodically varied force: $F = F_0 \cos\omega_{EXTER} t$.

The external force providing not damped oscillations is known as *constraining force*. These oscillations which arise in system at participation of the external force varied under the periodic law are known as the **forced oscillations**. The differential equation of oscillation will have the following kind: $m\frac{d^2x}{dt^2} = -kx - r\vartheta + F_0\cos\omega_{EXTER}t$, or $\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = f_0\cos\omega_{EXTER}t$,

where $f_o = \frac{F_0}{m}$; x is displacement of a material point in the established forced oscillations: $\boxed{x = A\cos(\omega_1 t + \varphi_0)}$, where $A = \frac{f_0}{\sqrt{(\omega_0^2 - \omega_{EXTER}^2)^2 + 4\beta\omega_{EXTER}^2}}$.

From the formula of displacement is visible:

1) The established forced oscillations occurring under influence of harmoniously varied constraining force also are harmonious.

2) Frequency of the forced oscillation coincides with frequency of constraining force.

3) The forced oscillations are shifted on a phase concerning of constraining force.

From expression for amplitude follows, that it is directly proportional to amplitude of constraining force and has complex dependence on β, ω_o, ω_{EXTER}. If ω_o and β are certain values, amplitude of the forced oscillations has the maximal value at some certain frequency of constraining force.

Phenomenon of sharp increase of amplitude of the forced oscillations at tend of frequency of constraining force to own frequency of oscillating body is known as *resonance*. Oscillations occurring at it are called resonant, and their frequency ω_{res} is **resonant frequency** of oscillations.

Resonant circular frequency can be found if to find minimum of the denominator in the expression for amplitude: $\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2}$. Then, having

substituted this expression to the formula for amplitude instead of ω_{EXTER}, let's receive: $A_{res} = \frac{f_0}{2\beta\sqrt{\omega_0^2 - \beta^2}}$.

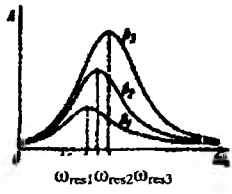


Fig. 5

Graphically dependence of amplitude A of forced oscillations on frequency of constraining force at different values of coefficient of attenuation is shown on fig.5 ($\beta_1>\beta_2>\beta_3$). Degree of increase of amplitude or acuteness of a curve of resonance depends on coefficient β, than β is less than a resonance is sharper. **At $\beta=0$: $\omega_{res} \approx \omega_0$, i.e. the resonance in system without attenuation comes, when frequency of constraining force coincides with frequency of own oscillations.**

Resonance in one case happens **useful** since thus action of insignificant stimulating force it is possible to cause oscillations with rather big amplitude. Such action of resonance is shown, for example, at percussion when each cavity of a body at tapping resounds on the certain frequency. The device of a capsule of a phonendoscope is based on the phenomenon of a resonance. Harmful action of a resonance is connected with destructions which it can cause.

Separate organs of a person have own frequency. If the coefficient of attenuation of internal organs would be small the resonance in these organs under action of external sound fluctuations would result in damage of these bodies. However, such phenomena at moderate external influences practically are not observed, since β in biological systems is considerably great. And nevertheless the resonant phenomena at action of external mechanical oscillations take place in biological systems. In it consists one of the reasons of negative influence of infrasonic oscillations and vibrations on human organism. For example, resonant oscillations of a human head at frequencies of 8-27 Hz can become the reason of reduction of visual acuity.

6. Addition of harmonious oscillations of identical direction

Cases when the body participates simultaneously in several oscillations are possible. For example, the various sound waves simultaneously perceived by human ear force eardrum to take part at once in several harmonious oscillations (to hear voices of many people).

Let's consider addition of two harmonious oscillations of identical direction and identical frequency. Displacement X of oscillating body will be the sum of displacement X_1 и X_2, which will be written down as follows:

$$X_1 = A_1\cos(\omega_0 t + \varphi_1), X_2 = A_2\cos(\omega_0 t + \varphi_2).$$

Displacement of resulting oscillation can be received having combined these expressions and having made the appropriate trigonometrical transformations. But we shall take advantage of the **method of vector diagrams,** which differs the greater simplicity and presentation. The essence of the method:

1) Sine wave variable is represented by a rotating vector, which length in the chosen scale expresses amplitude of a sinusoid.

2) The angle, formed by the vector with positive direction of axis x at the initial moment of time is equal to an initial phase.

3) Speed of rotation of the vector is equal to angular frequency.

4) Instant values of sine wave variable are expressed by projections of the rotating vector to axes of coordinates (fig.6).

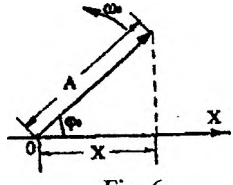


Fig. 6

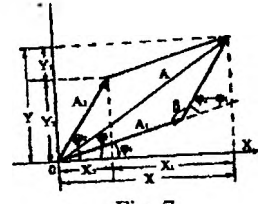


Fig. 7

Let's present both oscillations with help of vectors A_1 and A_2. Let's construct by rules of addition of vectors the resulting vector A. It is easy to see, that a projection of this vector to the axis X is equal to the sum of projections of composed vectors $X = X_1+X_2$, hence, the vector A represents resulting

oscillation (fig.7). This vector rotates with the same angular velocity ω_o as well as the vector A_1 and A_2, so resulting motion will be harmonious oscillation with frequency ω_o, amplitude A and the initial phase φ. From construction it is visible, that $\angle B = [\pi - (\varphi_2 - \varphi_1)]$, then

$$A^2 = A_1^2 + A_2^2 - 2A_1A_2\cos[\pi - (\varphi_2 - \varphi_1)] = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1);$$

$$tg\varphi = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}.$$

Analyzing the first expression we come to **conclusions**:

a) if difference of phases of both oscillations $\varphi_2 - \varphi_1$ is equal to zero, the amplitude of resulting oscillation is equal to the sum to amplitude A_1+A_2;

b) if a difference of phases $\varphi_2 - \varphi_1 = \pm\pi$, i.e. both oscillations are in antiphase, the amplitude of resulting oscillation is equal to $|A_1 - A_2|$.

If frequencies of oscillations X_1 and X_2 are not equal, then A_1 and A_2 will rotate with different velocities. In this case the resulting vector A pulses on size and rotates with changeable velocity. Hence, resulting motion will be not harmonious oscillation and represent some complex oscillatory process.

7. Addition of mutually perpendicular oscillations

Let's consider result of addition of two harmonious oscillations of identical frequency ω occurring in mutually perpendicular directions along axes x and y. For simplicity reference mark we shall choose so that the initial phase of the first oscillation was equal to zero:

$$\begin{cases} x = A\cos\omega t \\ y = B\cos(\omega t + \alpha) \end{cases}$$

where α is a phase difference of both oscillations, A and B are amplitudes of added oscillations. Then the equation of a trajectory of resulting oscillation is resulted by exception from system of t. From here

$$x/A = \cos\omega t;$$

$$y/B = \cos(\omega t + \alpha) = \cos\omega t\cos\alpha - \sin\omega t\sin\alpha.$$

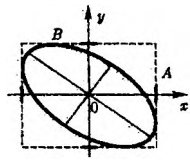


Fig. 8

Replacing in the second equation cos ωt on x/A and sin ωt on $\sqrt{1-(x/A)^2}$, let's receive after simple transformations the equation of an **ellipse,** which axes are focused concerning axes of coordinates any way (fig. 8):

$$\frac{x^2}{A^2}-\frac{2xy}{AB}\cos\alpha+\frac{y^2}{B^2}=\sin^2\alpha\,.$$

As the trajectory of resulting oscillation has the form of ellipse such oscillations are named *elliptically polarized.*

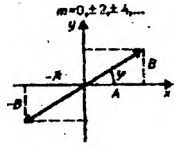


Fig.9

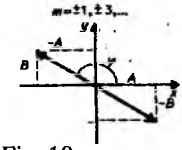


Fig. 10

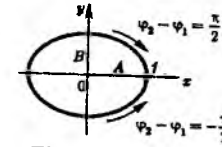


Fig. 11

Orientation of an ellipse and the sizes of its axes depend on amplitudes of added oscillations and a difference of phases. We shall consider some special cases:

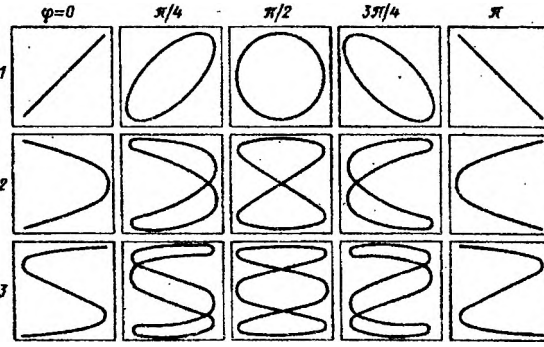


Fig.12

1) α=π·m (m=0,±1,±2, ...). In this case the ellipse turns into a piece of straight line: y = (B/A) ·x, where mark «plus» corresponds to zero and even m (fig. 9), and mark «minus» to odd values of m (fig. 10). Resulting oscillation is harmonious oscillation of frequency ω along a straight line. Such oscillation are named *linearly polarized.*

2) α =(2m+1)(π/2) (m=0,±1,±2, ...). In this case the equation will become:

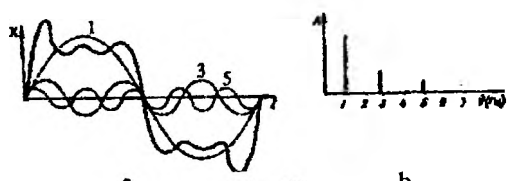
$$\frac{x^2}{A^2}+\frac{y^2}{B^2}=1.$$

This equation of an ellipse, which axis coincide with axes of coordinates, and it semiaxis are equal to amplitudes A and B (fig. 11). If A=B the ellipse turns to a circle. Such oscillations are named *polarized on a circle.*

If frequencies of added mutually perpendicular oscillations are different, the kind of the received curves depends on a parity of amplitudes, frequencies and difference of phases of added oscillations. The received curves are named **figures of Lissajous** (fig. 12). The ratio of frequencies of added oscillations is equal to the ratio of number of crossings Lissajous's figure with the straight lines parallel to axes of coordinates. By the form of figures it is possible to *determine unknown frequency or to determine the ratio of frequencies of added oscillations.* Therefore the analysis of Lissajous's figures is widely used method of research of ratio of frequencies and differences of phases of added oscillations and also forms of oscillations.

8. Composite oscillation and its harmonious spectrum

Oscillatory motion at which displacement changes in time under any law (except for harmonious) is known as composite oscillation. Any composite oscillation can be submitted as the sum of simple oscillations, that considerably simplifies its analysis. Decomposition of composite oscillation is frequently dictated by necessity of practice. This question in general view was solved by mathematics **Fourie** which has shown, that **periodic function of any complexity can be submitted as the sum of simple harmonious oscillations, which frequencies are multiple to frequency of complex periodic function.** Set of simple oscillations on which it is possible to spread out the given composite oscillation is named *harmonious spectrum.* Such decomposition of nonharmonic function on harmonious oscillations is known as *harmonious frequency analysis.* Decomposition of composite oscillation on simple harmonious oscillations making it, is carried out more often on the



a Fig. 13 b

basis of its diagram. Special devices named harmonious analyzers are applied to the analysis. Similar devices are applied at special researches of oscillatory processes in medicine, for example, at researching of oscillations of biopotentials of brain, written down on a tape. Such decomposition of biopotentials of brain can be used with the diagnostic purpose.

In the spectrum of composite oscillation are specified frequencies and amplitudes of all simple oscillations making it. Usually the spectrum is represented as the diagram: (fig. 13 b), on which horizontal axis frequencies are postponed, and for everyone frequency (harmonic) of simple oscillation corresponds ordinate, appropriated to amplitude of this oscillation (linear spectrum). In figure 13a is given the diagram of composite oscillation 3 and its harmonics of frequencies 1 Hz, 3 Hz and 5 Hz.

LECTURE №7

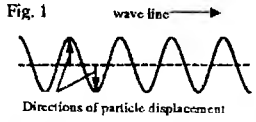
MECHANICAL WAVES. ACOUSTICS

1. Mechanical waves. Equation of wave. Wave equation

If any body makes oscillations in the elastic medium it cooperates with particles of environment and forces them to make *forced oscillations*. Gradually more and more removed particles are involved in oscillatory motion.

Process of propagation of oscillations or special disturbances of a condition of substance or field in space eventually is named a **wave.** The wave is called **cross wave,** if particles of the medium oscillate in directions perpendicular to a direction of distribution of the wave (fig. 1), and **longitudinal** (fig. 2), if oscillations of particles of environment occur in the direction of propagation of the wave.

Fig. 1



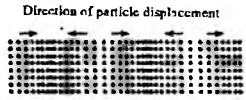


Fig. 2

Elastic waves arise due to the connection existing between particles of environment: moving of one particle from position of balance results in moving the next particles. This process is distributed in space with the certain velocity.

Equation of wave expresses dependence of displacement of the oscillating point, participating in wave process, on coordinate of its equilibrium position and time. For a wave extending along axis OX in a general view this dependence looks like: $S = f(x, t)$.

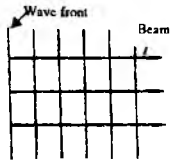


Fig. 3

Let's deduce the equation of a *flat wave* (fig. 3). If a source of waves is in a point with coordinate X = 0 (point A), fig. 4, the equation of oscillations is defined by the formula: $S = A\cos\omega t$.

To the point B with some coordinate X disturbance will come in time τ, therefore oscillations in this point are late:

$S = A\cos[\omega(t-\tau)]$ or

$S = A\cos\left[\omega\left(t-\frac{x}{\vartheta}\right)\right]$, where

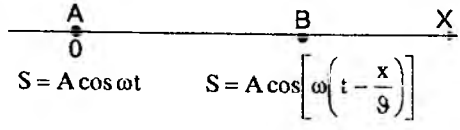


Fig. 4

ϑ is velocity of propagation of a wave. Thus it is supposed, that during propagation of a wave there is no its attenuation. Time of delay can be expressed: $\tau = \frac{x}{\vartheta} = \frac{x}{\lambda\nu} = \frac{xT}{\lambda}$, where λ is length of a wave, then:

$$S = A\cos\left[\omega\left(t - \frac{x}{\vartheta}\right)\right] = A\cos\left(\omega t - \omega\frac{x}{\vartheta}\right);$$

or: $$\boxed{S = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\vartheta T}\right) = A\cos 2\pi\left(t\nu - \frac{x}{\lambda}\right)}, \quad \lambda = \vartheta T.$$

The received expression is the **equation of a flat wave** extending along axis X.

The equation of any wave is the solution of some differential equation, named *wave*. To establish a kind of the **wave equation**, we shall take the second partial derivatives with respect to x and time t from the equation of a flat wave $S = A\cos\left(\omega t - \omega\frac{x}{\vartheta}\right)$:

$$\frac{\partial S}{\partial t} = -A\sin\left(\omega t - \omega\frac{x}{\vartheta}\right)\cdot\omega; \qquad \frac{\partial^2 S}{\partial t^2} = -A\omega^2\cos\left(\omega t - \omega\frac{x}{\vartheta}\right); \tag{1}$$

$$\frac{\partial S}{\partial x} = +A\sin\left(\omega t - \omega\frac{x}{\vartheta}\right)\cdot\frac{\omega}{\vartheta}; \qquad \frac{\partial^2 S}{\partial x^2} = -A\frac{\omega^2}{\vartheta^2}\cos\left(\omega t - \omega\frac{x}{\vartheta}\right). \tag{2}$$

Comparing with the second derivatives, we find, that at multiplication of both parts of the equation (1) on $\frac{1}{\vartheta^2}$ the right parts of the equation (1) and (2) will be equal and means are equal also left parts:

$$\boxed{\frac{\partial^2 S}{\partial x^2} = \frac{1}{\vartheta^2}\cdot\frac{\partial^2 S}{\partial t^2}} \tag{3}$$

The equation (3) also is the required **wave equation** since it is received from the equation of the flat wave extending along axis X and represents a special case more the general equation:

$$\frac{\partial^2 S}{\partial Z^2} + \frac{\partial^2 S}{\partial Y^2} + \frac{\partial^2 S}{\partial X^2} = \frac{1}{\vartheta^2}\frac{\partial^2 S}{\partial t^2}.$$

2. Energy of a wave. Umov`s vector

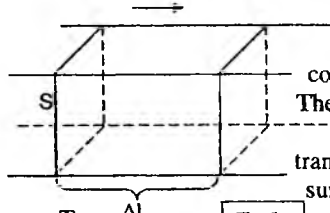


Fig. 5

Wave process in the environment is connected with propagation of energy of oscillations. The quantitative characteristic of the transferred energy is a **stream of energy Φ**: energy E, transferable by a wave per unit of time through some surface **Φ=E/t.**

To count up energy, transferable by a wave, we shall allocate mentally some platform S, located perpendicularly to direction of propagation of the wave (fig. 5). Let during the initial moment of time (t=0), the front of a flat wave coincides with this platform. In time t>> T (where T – the period) the front of a wave will move on distance Δl then the weight of substance Δm will be involved in oscillatory process. Full energy the weight

Δm, participating in oscillatory movement, it is defined under the formula: $E = \frac{\Delta m \omega_0^2 A^2}{2}$. Energy, transferable by the wave in a medium for a time unit through the unit platform is known as **intensity of wave** I. Then:

$$I = \frac{\Phi}{S} = \frac{E}{St} = \frac{\Delta m \omega_0^2 A^2}{2St} = \frac{\Delta l S \rho \omega_0^2 A^2}{2St} = \frac{1}{2}\rho\omega_0^2 A^2 \vartheta,$$

where ϑ is velocity of propagation of a wave.

So: $I = \frac{1}{2}\rho\omega_0^2 A^2 \vartheta = w\vartheta$, where $w = \frac{1}{2}\rho\omega_0^2 A^2$ is volumetric density of energy. Intensity of a wave is measured in Wt/m^2.

Vector $\vec{I}$ showing direction of propagation of waves and equal to stream of energy of waves, taking place through the unit platform, perpendicular to this direction is named **Umov's vector** : $\vec{I} = w\vec{\vartheta}$.

3. Doppler effect

Dopple effect is changing of frequency of the waves perceived by an observer (the receiver of a wave) owing to relative motion of the source of waves and observer.

By theoretical consideration of the phenomenon it is necessary to note, that all velocities (velocity of propagation of oscillations, velocity of the observer and the source) are counted concerning of air. We shall consider the elementary cases when a source of waves and the observer move concerning a medium *along one straight line*. Velocity of propagation of waves in the medium we shall count equal to ϑ, velocity of a source is ϑ_s; velocity of an observer is ϑ_{obs}.

Let the observer is

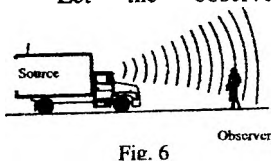


Fig. 6

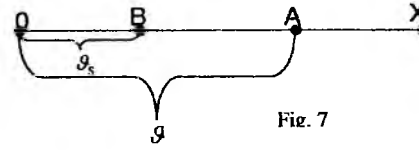


Fig. 7

motionless, and the source of waves moves with constant velocity ϑ_s in the direction of the observer (fig. 6). We shall accept, that frequency of oscillations of the source is equal to ν, and $\vartheta_s < \vartheta$. We shall consider, that the source is in the beginning of coordinates (X=0, fig. 7). Oscillations arise in the point X=0 at the moment of time t=0. For one second the source will emit in the medium ν waves. The waves, which are emited by the source per 1 second will pass distance ϑ and front of a wave there is in the point A. When the source emits last from ν waves, it will pass a way ϑ_s and appears in the point B. Hence, ν waves are placed on distance $\vartheta - \vartheta_s$, therefore the length of the wave perceived by the observer will be equal: $\lambda' = \frac{\vartheta - \vartheta_s}{\nu}$, and frequency $\nu' = \frac{\vartheta}{\lambda'} = \frac{\vartheta}{\vartheta - \vartheta_s}\nu$. Having

divided numerator and the denominator on ϑ, we shall receive: $\nu' = \frac{\nu}{1 - \frac{\vartheta_s}{\vartheta}}$, i.e. the observer will perceive a sound with frequency **greater**, than frequency of a stationary source. If the source moves *from* the observer than ϑ_s will be with minus and ν' will be less, i.e $\nu' = \frac{\nu}{1 + \frac{\vartheta_s}{\vartheta}}$ (see the table 1).

Table 1

Source	Observer	Frequency,perce-ived by observer	Source	Observer	Frequency,perce-ived by observer
□	☺	$\nu = \nu_{OBS}$	□→	←☺	$\nu_{OBS} = \frac{\upsilon + \upsilon_{OBS}}{\upsilon - \upsilon_S}\nu$
□	←☺	$\nu_{OBS} = \frac{\upsilon + \upsilon_{OBS}}{\upsilon}\nu$	←□	☹→	$\nu_{OBS} = \frac{\upsilon - \upsilon_{OBS}}{\upsilon + \upsilon_S}\nu$
□	☹→	$\nu_{OBS} = \frac{\upsilon - \upsilon_{OBS}}{\upsilon}\nu$	←□	←☹	$\nu_{OBS} = \frac{\upsilon + \upsilon_{OBS}}{\upsilon + \upsilon_S}\nu$
□→	☺	$\nu_{OBS} = \frac{\upsilon}{\upsilon - \upsilon_S}\nu$	□→	☺→	$\nu_{OBS} = \frac{\upsilon - \upsilon_{OBS}}{\upsilon - \upsilon_S}\nu$
←□	☺	$\nu_{OBS} = \frac{\upsilon}{\upsilon + \upsilon_S}\nu$	erythrocyte ⊙→ ϑ_{OBJ}	Generator	$\nu_{SHIFT} = \frac{2\vartheta_{OBJECT}}{\vartheta_{US}}\nu_G$

Frequencies, perceived by the observer are specified in the table depending on relative motion of the source and the observer.

Second case: observer moves with a velocity ϑ_{OBS} towards a stationary source. Thus he meets on the way for the same interval of time more waves, than at absence of motion. It means, that frequency ν' perceived by him is more, than frequency of the source, i.e $\nu' = \frac{\vartheta + \vartheta_{OBS}}{\lambda}$. Velocity of propagation of the wave concerning the observer becomes equal to $\vartheta + \vartheta_{obs}$, and the length of wave thus will not change. Taking into account, that $\lambda = \frac{\vartheta}{\nu}$, we shall receive $\nu' = \frac{\vartheta + \vartheta_{OBS}}{\vartheta}\nu = \nu\left(1 + \frac{\vartheta_{OBS}}{\vartheta}\right)$, i.e. the observer perceives the **greater frequency** of oscillations, than frequency of the source. If the observer leaves from a source $(-\vartheta_{OBS})$the frequency perceived by him will be smaller, i.e $\nu' = \nu\left(1 - \frac{\vartheta_{OBS}}{\vartheta}\right)$.

Doppler effect is observed in a waves of any type: in waves on water, sound, radio, light waves.

For example, the doppler locator is specially used by road police for definition of automobiles, which drivers exceed allowable velocity of motion.

Using of Doppler effect for determination of velocity of moving bodies has found application in medicine. We shall consider the following system: the generator of US waves is combined with the receiver, frequency of the generator is ν_G. The object moves in the medium with a velocity ϑ_0. Velocity of propagation of ultrasound (US) is ϑ. The ultrasonic wave, reflected from moving object will have frequency ν'. The receiver, owing to Doppler's effect, perceives already another frequency ν_{REC}. The difference of frequencies will be equal $\nu_{SHIFT} = \nu_{REC} - \nu_G$ and is known as **Doppler's shift**. In medical appendices velocity of US is more than velocity of moving object $(\vartheta > \vartheta_0)$. For this case

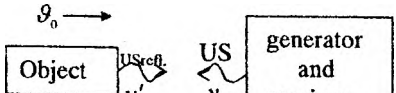


Fig. 8

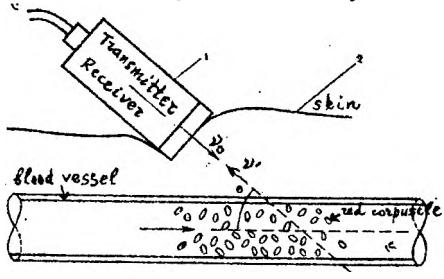


Fig. 9

$$\nu_{SHIFT} = \frac{2\vartheta_0}{\vartheta}\nu_G$$

Doppler effect is used for determination of a velocity of a blood-groove (fig. 9), velocity of motion of valves and walls of heart (Doppler echocardiography) and for other bodies.

4. Nature of sound. Physical characteristics of sound waves

In the narrow sense of the word as "acoustics" understand the doctrine about sound, i.e. about elastic oscillations and waves in the various mediums perceived by human ear.

Sound oscillations are a special case of the mechanical oscillations extending in space in the form of longitudinal waves.

Source of a sound are bodies (firm, liquid, gaseous), making oscillations as result of any mechanical influences. The varying body radiates in the medium, for example, to air elastic longitudinal wave which achieving the ear causes acoustical sensations.

Sounds are subdivided: 1) tones or musical sounds; 2) noise; 3) sound impacts.

Tone is the sound being periodic process with frequency constant or naturally changing in time. If it is harmonious process tone is called *simple or pure*. To nonharmonic fluctuation there corresponds complex tone. Simple tone can be received with the help of a tuning fork, a sound generator. Complex tone is created by musical instruments, the device of speech. Complex tone can be spread out on simple. The *least frequency* ν_0 of such decomposition corresponds to *the basic*

tone. Other harmonic (overtone) have frequencies $2\nu_0$ $3\nu_0$, ... Set of frequencies with the indication of their relative intensity (amplitude) is named **acoustic spectrum**, it is the important physical characteristic of complex tone.

Noise is named a sound distinguished by complex dependence not repeating in time (rustle, scratch, applause, sound from vibration of machines).

Sound impact is a short-term sound influence (clap, explosion).

Sound tone is characterized by frequency (period), amplitude, harmonious spectrum, and also intensity or force of a sound and sound pressure.

Intensity or force of a sound is called density of stream of energy of sound wave: $I = \frac{\Delta E}{\Delta S \Delta t}$ (Wt/m^2). Sound pressure is an additional pressure Δp, which arises in the medium at passage of sound waves. Sound pressure is measured in N/m^2.

For a flat harmonious wave pressure is connected with intensity of sound by the ratio: $I = \frac{p_0^2}{2\rho\vartheta} = \frac{p_{eff}^2}{\rho\vartheta}$, p_0 is peak value; p_{eff} is effective value of pressure.

Human ear perceives the wide range of intensity. For created of sensation of a sound it is necessary, that intensity exceeded some minimal value I_0, named **a threshold of audibility**. For example, on frequency of **v=1kHz** threshold of audibility is $\mathbf{I_0=10^{-12}\ Wt/m^2}$ **or** $\mathbf{p_0=2\cdot10^{-5}\ Pa}$. On the other hand, sounds of very big intensity are not perceived as a sound, causing only sensation of pain in ear. The maximal value of intensity at which excess there is a sensation of pain is known as **threshold of painful sensation:** $\mathbf{I_{max}=10\ Wt/m^2}$ **or** $\mathbf{p_{max}=60\ Pa}$ **for frequency v=1 kHz**. The ratio $I_{max} / I_0 = 10^{13}$.

As the range of intensity perceived sounds is rather great, it appears convenient to *compare intensity of sound in logarithmic scale*. The scale of levels of intensity is created as follows: value I_0 is accepted for the initial level and level of intensity of any sound with intensity I express through the decimal logarithm of its ratio to I_0: $\boxed{L_B = \lg\frac{I}{I_0}}$. In this scale a level of intensity express in **Bells (B)**. Bell is unit which has received the name in honour of the inventor of phone Alexander Bell (1847-1922).

If level of intensity of some sound is L=1B, ratio of its intensity to I_0 will be equal to 10 (lg10=1), if L=2B, $I/I_0=10^2$ ($\lg10^2=2$). Bell is rather big unit, therefore usually a level of intensity express in **decibels**: **1B=10dB,** then $\mathbf{L_{dB}=10\ lg\ I/I_0}$ or $L_{dB}=20\lg p/p_0$. The threshold of audibility I_0 has the level of intensity **L=0dB** and painful threshold is **130dB.**

5. Propagation of sound waves in a medium. Wave resistance

Sound is propagated in any medium, velocity of its propagation does not depend on frequency of oscillations, but depends on elasticity and density of medium, and also from its temperature. In air at t=0 ^{0}C $\vartheta = 331.5$ m/s. With growth of temperature, velocity increases. In firm and liquid mediums velocity of sound is

more. For water it is equal to 1500 m/s. This velocity corresponds approximately to average velocity in **soft tissues** of a person. Sound wave, meeting on the way a body raises in them oscillations. At hit on the body the part of energy is reflected and refracted (laws of reflection and refraction of sound waves are similar to laws for light), the part of energy can be absorbed by a body (absorption can be total). The certain part of energy can leave from a body. Thus, energy of a wave can be divided on reflected, absorbed and also past through the medium.

As it has already been told, intensity of a wave $I = p_0^2/2\rho\vartheta$ is defined by sound peak pressure and product $\rho\vartheta = \omega$ which is known as **wave resistance**. Wave resistance is the major characteristic of a medium, determining condition of reflection and refraction of waves on its border. We admit, that the flat wave falls normally to border of two mediums, its intensity in the first medium is I_1, intensity of a past wave in the second medium is I_2. The ratio $\beta = \frac{I_2}{I_1}$ (1) is called **coefficient of penetration** of a sound wave. This ratio (β) depends on lines of factors, including from frequency of oscillations of a sound wave.

Let's result examples of the average values of this ratio for frequency of 512 Hz: open window – 1; the wall brick – 0.032; felt of thickness 2.5 cm – 0/55.

Soft tissues have the big absorption, therefore them apply when it is desirable to reduce reflection of sound from walls.

Reley has shown, that the coefficient of penetration of a sound is defined by the formula: $\beta = 4\frac{\rho_1\vartheta_1/\rho_2\vartheta_2}{[\rho_1\vartheta_1/\rho_2\vartheta_2 + 1]^2}$. (4)

The greatest value which can have coefficient β is equal to 1. From (4) it is visible, that $\beta = 1$ if $\rho_1\vartheta_1 = \rho_2\vartheta_2$. So, at equality $\omega_1 = \omega_2$ of two mediums the sound wave will pass (at normal falling) border of the unit without reflection. If wave resistance of the second medium considerably exceeds wave resistance of the first, then $\beta \approx 4\cdot\rho_1\vartheta_1/\rho_2\vartheta_2$ (as $\rho_1\vartheta_1/\rho_2\vartheta_2 << 1$).

Examples of wave resistance of some substances at 20°C:
air – 440 kg·м$^{-2}$c^{-1}; concrete – 4800000 кг·m^{-2}c^{-1}; water–1440000 кг·м$^{-2}$c^{-1}.

For concrete $\beta_{con} = \frac{4\cdot 440}{4\,800\,000}100\% = 0.037\%$.

Conclusion: *only very small part of sound energy passes from air in concrete. For water:* $\beta_{H_2O} = 0.122\%$.

Such calculations can be used for estimation of levels of intensity of loud noise and an opportunity of sound insulation.

Elimination of sources of harmful sounds or easing of their action with help of sound-proof materials is under the control of public health service, as **noise renders harmful influence on human health**. Normally allowable level of intensity of noise is 40 – 50 dB. Maximum permissible level (for high frequencies) is 75 – 80 dB and 90 – 100 dB for low-frequency sounds.

For measurement of level of intensity of noise is used special device (*audio noise- meter*). Sound oscillations will be transformed in to electric at it.

Noise is a complex disturbance, having many component frequencies and producing varying effects on different individuals. Table 2 illustrates some common sources of noise and their average intensity levels, sometimes reffered to as noise levels.

Table 2. Common sources of noise

Source	Intensity level (dB)	Sound intensity Wt/m^2	Sound pressure Pa
"Silence"(threshold of hearing)	0	10^{-12}	$2\ 10^{-5}$
Library	20	10^{-10}	$64 \cdot 10^{-6}$
Average home	30	10^{-9}	$2 \cdot 10^{-4}$
Backgrond music	40	10^{-8}	$2 \cdot 10^{-3}$
Speech at 0.6 m	60-80	10^{-6}-10^{-4}	0.2-0.02
Heavy traffic	80	10^{-4}	0.2
Pneumatic drill	90	10^{-3}	0.64
Factory	80-130	10^{-4}-10	0.2
Jet overhead	100	10^{-2}	2
Thunder overhead	110	10^{-1}	6.4
Threshold of feeling	120	1	20

High intensity noise is generally accepted as greater than 85 dB and as the level increases there are a number of harmful results:

a) change in a hearing acuity and possible damage to the cochlea;
b) stimulation of receptors in the scin;
c) significant changes in the pulse rate;
d) vibrations of muscles and incoordination;
e) feeling of fear, annoyance, dissatisfaction;
f) inability to perform skilled and unskilled tasks;
g) nausea, vomiting, dizziness (>130 dB);
h) pain in middle ear (≈140 dB);
i) temporary blindness (>140 dB);
j) mild warming of body surfaces (>150 dB);
k) minor permanent damage if prolonged (≈160 dB);
l) major permanent damage in a short time (≈ 190 dB).

LECTURE №8

CHARACTERISTICS OF ACOUSTICAL SENSATION. ULTRASOUND. INFRASOUND

1. Physics of hearing

Let's consider some questions of physics of hearing, concerning to a structure of an ear and perception of a sound.

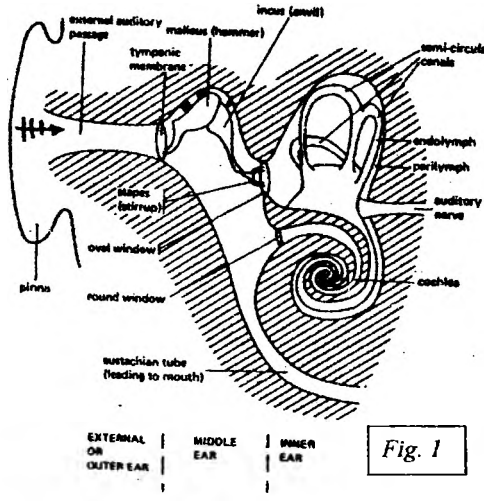


Fig. 1

The ear will consist of three parts: external, middle and inner (fig. 1).

The external ear of the person will consist of an auricle and the external auditory passage having length of 2.7 cm and reminding a cavity pipe, closed from one end by eardrum. As is known, the resonance is observed, if the length of the resonator makes 1/4 of a wavelength. External auditory passage has therefore *resonant frequency* of

$$\nu_{res.ear} = \frac{\upsilon_{air}}{4\lambda} = \frac{300 m/s}{4 \cdot 2.7 \cdot 10^{-2} m} \approx 3000 Hz$$

At coincidence of frequency of a falling sound to own frequency of fluctuations of the external ear (that is at resonance) sensitivity of ear is maximal. Besides, the auricle promotes maintenance of the directed reception and concentration of a sound.

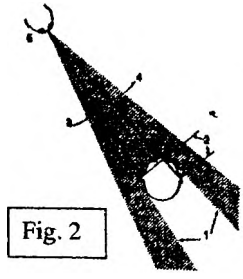


Fig. 2

Localization of sources of a sound (*binaural effect*) is based on two mechanisms. First, for low frequencies the ear is capable sensitively locate delay of arrival of a sound on time to the left and right ear that is to catch the difference of phases of arrival sound wave (on fig. 2: 3 and 4 is way of sound). More there will be the value of this delay is more the angle on the source of sound. At the person distance between ears on the average is 0.17 m (on fig.2: 2 is additional way for the right ear) Therefore the maximal value of delay at propagation of a sound at right angle to the average plane of a head is equal to $\Delta t = 0.17/330 = 5 \cdot 10^{-4}$s (fig. 2).

Second, for high frequencies the ear mainly reacts to a difference of the sound intencities which has achieved of the right and left ear. For a sound with frequency, for example, 10000 Hz the wavelength makes 3.3 cm therefore the sound is

reflected by a head with formation of a sound shadow 1 (fig. 2) for one of ears. As result, intensity of the waves falling on the right and left ear, will be various. At the person the differential threshold on frequency makes 1 dB for 1000 Hz, therefore, if the difference of intensities reaches of 1 dB it is already enough of it for approximate localization of a source of sound.

Middle ear is the device, transforming sound fluctuations of air in sound fluctuations of the liquid environment of an inner ear. Wave resistance of the liquid environment of an inner ear is approximately equal to wave resistance of water. As it has been shown earlier, at direct transition of a sound from air in water it is transferred only 0.123 % of falling energy and it is not enough. Therefore the *basic purpose of the middle ear* - to reduce to the minimum of loss of energy of a sound at reflection, to receive the win in pressure (speak, that the middle ear will coordinate wave resistance of air and a liquid of an inner ear). It is reached due to two factors.

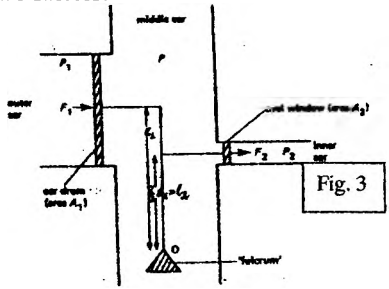


Fig. 3

First, on the an ear there are three jointed among themselves acoustical bonelets (malleus (hammer 4 fig. 4), incus 5 (anvil) and stapes 6 (stirrup)), forming system of levers. The system of levers gives a win in force at the person in $l_1/l_2 \approx 1.3$ times (fig. 3). Second, the area of an eardrum of a person is S_1=64 mm^2, that there is more than area of oval window of an inner ear S_2=3 mm^2, therefore both membranes and connecting them bonelets carry out function of the transformer of pressure. We shall count up the win in pressure (p_2/p_1).

On an eardrum sound pressure p_1 operates with force

$$F_1 = p_1 S_1.$$

On oval window of inner ear force F_2 operates, creating sound pressure of p_2 in the liquid environment

$$F_2 = p_2 S_2.$$

From here

$$\frac{F_1}{F_2} = \frac{p_1 S_1}{p_2 S_2} = \frac{l_2}{l_1}$$, whence the win in pressure

$$\frac{p_2}{p_1} = \frac{S_1}{S_2} \cdot \frac{l_1}{l_2} = 20 \cdot 1.3 = 26,$$

or in logarithmic units

$$L_{dB} = 20 \cdot \lg \frac{p_2}{p_1} = 20 \cdot \lg 26 = 20 \cdot 1.415 \approx 28 dB.$$

Other important function of the middle ear is *protection of inner ear against the big mechanical loadings* at influence of very loud sounds (more than 90 dB). It occurs due to the reflex relaxation of muscles of bonelets of an middle ear.

Let's estimate amplitude of displacement of eardrum for threshold pressure of intensity of a sound: very small ($I=10^{-12}$ Wt/m2) and very big (I=10 Wt/m2 and more).

As thickness of an eardrum is very small in comparison with length of a wave velocity of its moving coincides with velocity of particles in a flat wave in air υ.

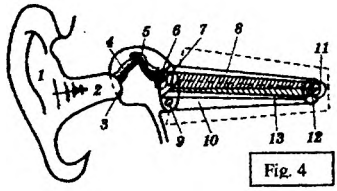


Fig. 4

Earlier we have found expression for intensity of a wave

$$I = \frac{\rho A^2 \omega^2}{2} \upsilon ,$$

whence the amplitude of fluctuations A is equal

$$A = \sqrt{\frac{2I}{\rho (2\pi \nu)^2 \upsilon}} ,$$

where $I=10^{-12}$Wt/m^2 is the minimal intensity of wave perceived by an ear, ρ=1.3 kg/m3 is density of air, ν=1000 Hz is frequency of fluctuations of particles of a wave, υ = 330 m/s. Substituting numerical values of the specified values in the formula for A, we shall receive $A \approx 10^{-11}$m. This size is less than radius of atom. It is no wonder therefore that Corti's organ, containing nervous cells of inner ear, has no blood vessels that pulsations of a blood pressure did not actuate hair cells and did not cause acoustical sensations. Similar calculation for intensity of a sound, at which the eardrum collapses (160 dB) gives result: A=1 mm.

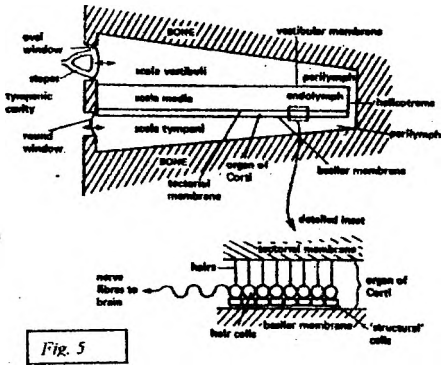
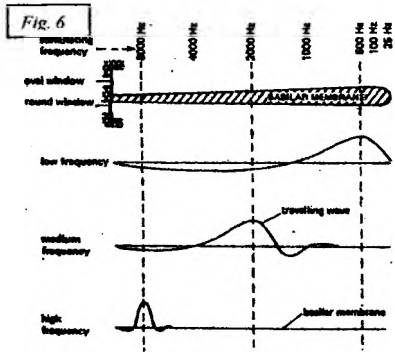


Fig. 5

The middle ear unites to an atmosphere through eustachian tube.

Body of the inner ear is the cochlea (dotted line on fig. 4), transforming mechanical fluctuations in electric signal. Except for cochlea the vestibular apparatus concerns to an inner ear, which to acoustical function has no attitude. On fig. 4 are shown:7-oval window, 8-vestibular ladder (scala vestibuli), 9-round window, 10- tympanic ladder (tympanic cavity), 11-helicoterma, 12-cochlea cavity, 13-basilar (basic) membrane.

The cochlea of the person has the spiral form and forms 2.5 coils with the length about 35 mm. In a cochlea three parallel channels (cavities) filled by a liquid settle down (fig. 4,5). Vestibular and tympanic channels are filled by perylimph and connected in top of cochlea through a small opening –helicoterm. These two channels are separated from each other by the cochlea cavity. It is filled by endolymph and separated from the vestibular cavity by Reissner's membrane, and



from the tympanic cavity – by basilar (basic) membrane. On the basic membrane is organ of Corty containing receptor(hair) cells and transforming mechanical oscillations to the electric signal.

Nobel prize winner Bekesy has established, that the basic membrane has non-uniform mechanical properties. At influence by acoustic fluctuations on the basic membrane the wave is propagated. Depending on frequency this wave fades on miscellaneous: low frequencies (less than 100 Hz) cause fluctuations of the remote sites of the basic membrane close of helicoterm, and high frecuencies actuate a site near to the window (fig. 5 and fig. 6). Perception of sound frequencies is defined by localization of the maximal fluctuations of the basic membrane (fig. 6). Fluctuations of the basic membrane cause deformation of hair cells which are settling down in the organ of Corti inside of tympanic cavity (fig. 5). Deformation of hair cells causes generation of an electric signal.

2. Characteristics of acoustical sensation, their connection with physical characteristics of sound. Dependence of loudness on frequency. Weber-Fehner's law

Sound tone is characterized by ***frequency (period), harmonious spectrum, intensity or force of sound and sound pressure***. All these characteristics of a sound are **physical or objective** characteristics. However the sound is estimated by the person subjectively, i.e. the sound has also physiological characteristics which are reflection of its physical characteristics. A problem of system of sound measurements is to establish this connection and thus to enable at research of hearing at various people to compare value judgment of acoustical sensation to the data of objective measurements.

Frequency of oscillations of sound wave is estimated as **height** of sound (height of tone). More frequency of oscillation is more height of perceived sound.

Other physiological characteristic is a **timbre** which is determined by **spectral structure** of a complex sound. Complex tones of identical basic frequencies can will be distinguished under the form of oscillations and accordingly on a harmonious spectrum. This distinction is perceived as a timbre (colouring of sound). For example, ear distinguishes the same note reproduced on different musical instruments.

Loudness is one more value judgment of a sound which characterizes a *level of acoustical sensation*. It depends, first of all, on **intensity and frequency** of a sound.

Human ear is unequally sensitive to various frequencies at the same intensity. **The range of frequencies perceived by human ear is 16 Hz-20 kHz.** Let's consider in the beginning dependence of sensitivity of ear on frequency. Ability of the person to perceive high-frequency sounds worsens with the years. The young man can hear sounds with frequency up to 20 000 Hz, but already in the middle age the same person is not capable to perceive sounds with frequency above 12-14 kHz. Within the limits of frequency of **1000-3000 Hz sensitivity is the greatest.** Sensitivity is reduced to frequency of 16 Hz and 20 kHz. It is obvious, that character of change of threshold of audibility is opposite to change of sensitivity of an ear, i.e. at increase in frequency from 16 Hz, it in the beginning is reduced, in the field of frequencies of 1000-3000 Hz remains almost constant, then again raises. It is reflected on the graph of dependence of change of a threshold of audibility from frequency (see fig. 7).

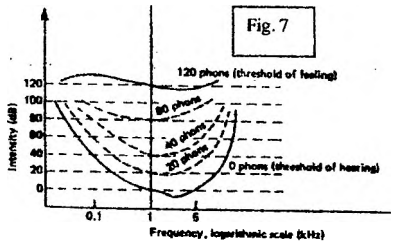


Fig. 7

The schedule is constructed in logarithmic scale. The top curve on the schedule corresponds to a **painful threshold (or threshold of feeling).** The bottom curve is named curve of **threshold of audibility (or threshold of hearing)**, i.e. $I_0 = f(\nu)$ at the level of loudness equal to "0".

Loudness of a sound depends on its intensity. It is the **subjective** characteristic of sound. These two concepts are inadequate. Dependence of loudness on intensity of sound has the complex character caused by sensitivity of an ear to action of sound waves. The person can approximately estimate absolute intensity of sensation only. However he precisely enough establishes the difference at comparison of two sensations of different intensity. It has caused occurrence of **comparative method** of measurement of loudness. Thus measure not absolute value of loudness and its parity with some other value which is accepted for an initial or zero level of loudness.

Except for it has agreed at comparison of intensity and loudness of sound to use tone with frequency of 1000 Hz, i.e. the comparative method is applied to count loudness of tone frequency of 1000 Hz (standard for the scale of loudness). Therefore there are two scales: 1-st for measurement of levels of intensity; the second - for measurement of levels of loudness.

On the basis of creation of the scale of levels of loudness lays important **psychophysical law of Weber-Fechner**.

*According to this law, **if to increase irritation in geometrical progression (i.e. in identical number of times) the sensation of this irritation grows in arithmetic progression (on identical value).*** For example, if intensity of a sound accepts a number of consecutive values: $a\,I_0$, $a^2\,I_0$, $a^3\,I_0$ ($a > 1$ - some constant) changes of loudness of sound corresponding to them will be equal to $E_0, 2E_0, 3E_0$. It means that loudness of a sound is **directly proportional to the logarithm of sound intensity.**

If operates sound irritant with intensity I on the basis of Weber-Fehner law the level of loudness E is connected with the level of intensity as follows:

$$E = KL = K\lg\frac{I}{I_0}, \qquad (1)$$

where I/I_0 is relative force of irritation, K is coefficient of proportionality dependent on frequency and intensity, equal to 1 for ν =1000 Hz. Hence, if to accept K=1 on all frequencies according to the formula (1) we shall receive the scale of levels of intensity; at K≠1 - a scale of loudness, where unit of measurement will be any more decibel and **phon**. Taking into account, that on frequency 1 kHz scales of loudness and intensity coincides, means

$$E_{ph}=10\cdot\lg\frac{I}{I_0}.$$

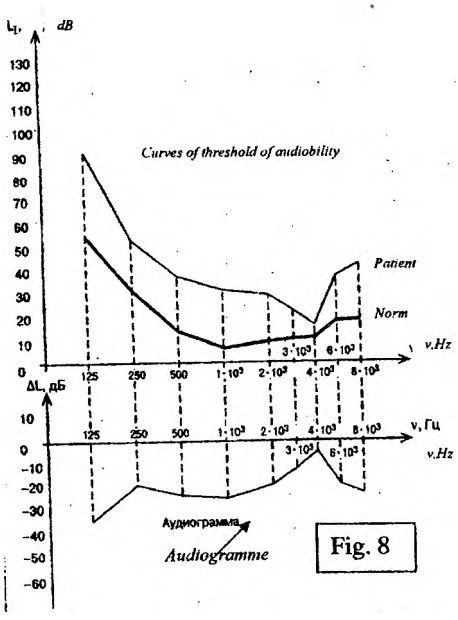


Fig. 8

Dependence of loudness on intensity and frequency of oscillations in system of sound measurements is defined on the basis of experimental data by means of schedules which refer to as curves of equal loudness, i.e.

I=f (ν) at E = const. We had been constructed a curve of zero level of loudness or threshold of audibility [I_0 = f (ν)]. This curve is the basic (zero level of loudness E_{ph} =0).

If to construct similar curves for various levels of loudness, for example, steps through 20 phons will turn out the system of graphs (fig. 7) which enables to find dependence of a level of intensity on frequency at any level of loudness. These curves are constructed on the basis of the average data which have been received from people with normal hearing. The bottom curve corresponds to a threshold of audibility, i.e. for all frequencies $E_{ph=0}$ (for frequency ν =kHz intensity $I_0=10^{-12}$ Wt/m^2). Research of sensory acuity is called **audiometry**. At audiometry on the special device **(audiometer)** define at the patient the threshold of audiobility on different frequencies. The received graph is named **audiogramme**. Loss of hearing is defined by its comparison with the normal curve of threshold of audibility (fig. 8):

$\Delta L=L_{norm}-L_{patient}$

3. Sound methods of research in clinic

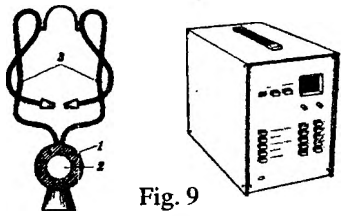
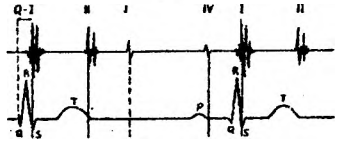


Fig. 9

Fig.10



The sound phenomena accompany with a number of processes occurring in an organism, for example, work of heart, breath, etc. Direct auscultation of the sounds arising inside an organism make one of the major receptions of clinical research and is known as **auscultation.** This method is known since 2-nd century b.c.. For this purpose use a **stethoscope:** the device as direct wooden or plastic tube with a small bell on one end and the flat basis on opposite for applying of ear. The sound from surface of a body to ear is carried out both a column of air and walls of the tube.

For auscultation is used a **phonendoscope** (fig. 9) consisting from hollow capsule 1 with membrane 2 putting to a body of the patient. From a capsule there are two rubber tubes, which are inserted into ears of the doctor. The resonance of column of air in the capsule strengthens a sound.

The method, applied for diagnostic of cardiovascular system is called **phonocardiography (PCG)**: graphic registration of tones and noise of heart (fig. 10) with the purpose of their diagnostic interpretation. Record is made with the help of phonocardiograph (fig. 9), consisting of microphone, amplifier, system of frequency filters and the recording device.

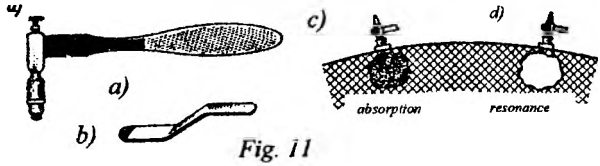


Fig. 11

Distinct from two specified methods is **percussion**; it is method of research of internal bodies by means of knock on surface of body and the analysis of sounds arising at it. Character of these sounds depends on a way of knock and properties (elasticity, density) of tissues which are taking place near to the place on which it is made the knock. Knock can is made by special **hammer** (fig.11,a) with the rubber head, a plate from the elastic material named **plessimeter** (fig. 11,b), or by knock bent finger of one hand on phalanx of a finger of another, imposed on the body. At impact on the surface of body there are oscillations which frequencies have a wide range. One oscillations will quickly fade, others owing to resonance will amplify and will be audible (fig. 11c,d). On tone of sounds define condition and topography of internal bodies.

Fig. 12

Research of hearing in clinic we can do not only with the help of audiometer, but also with help of **tuning forks**. In clinic are used tuning forks with low frequency (128 Hz) and high frequency (2048 Hz). For example, in method of Weber (fig. 12) is used the low-

frequency tuning fork for definition of audibility of sound at bone conductivity. In norm both ears perceive sound of tuning fork equally. At defect of the sound-conducting apparatus lateralization of sound occurs to the sick ear and at defect of sound-perceiving apparatus to the healthy ear.

3. Ultrasound (US), sources of US. Features of distribution of ultrasonic waves

Ultrasound is called sound oscillations which frequency borrows the range from 20 kHz up to 10^{10} Hz. The top limit is accepted completely conditionally from such reasons, that the length of wave in substance and tissues for such frequency appears is commensurable with intermolecular distances in view of that speed of distribution of US in water and tissues is identical.

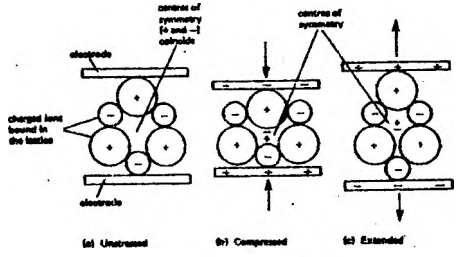


Fig. 13

The greatest distribution, both in technics and in medical practice piezoelectric radiators of US have received. As piezoelectric radiators use crystals of quartz or the synthetic ceramic, lead zirconate titanate etc. **Piezoeffect** (*direct*) is called phenomenon of occurrence on surfaces of the mentioned crystal plates of opposite on mark charges under action of mechanical deformations (fig. 13). After removal of deformation charges disappear.

There is also **inverse piezoeffect** which has found application and in medical practice for reception of high-frequency US. If on silvered sides of surface of plate of the piezoelement to submit alternating voltage from generator, the quartz plate will come in fluctuation in a step of alternating voltage of the generator. The amplitude of oscillations will be *maximal*

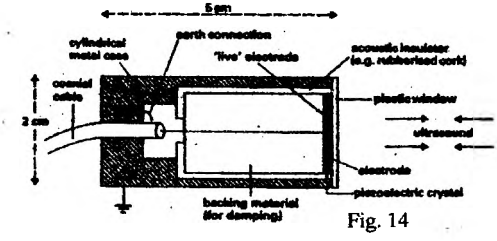


Fig. 14

when own frequency of quartz plate ν_0 coincides with frequency of generator ν_g, i.e. there will be *resonance* ($\nu_0 = \nu_g$). The *detector* of US can be created on the basis of direct piezoelectric effect. Thus under influence of US-waves there is a deformation of a crystal that results in occurrence of alternating voltage which can be measured or fixed on the screen of an electronic oscillograph after its preliminary amplification. A piezoelectric transducer can operate as US generator (radiator) and US detector. A typical transducer used in medical applications is shown in fig. 14.

The ultrasound we can get with help of the devices based on the phenomenon of

magnitostrictia (for reception of low frequencies). Magnitostrictia is the phenomenon, which will consist in change of length of the ferromagnetic core placed in a high-frequency magnetic field. End faces of this core will radiate *low-frequency* US. Except for the specified sources of US there are *mechanical sources* in which mechanical energy will be transformed to energy of US oscillations.

By the nature US as well as sound is the mechanical wave extending in the elastic medium. Speeds of propagation of sound and ultrasonic waves are approximately identical. However the length of wave of US is much less, than for sound. It allows focusing easily US oscillations.

Ultrasonic wave has the much greater intensity than sound, owing to the big frequency it can achieve several Watt on square centimeter (Wt/cm^2), and at focusing it is possible to receive US with intensity of 50 Wt/cm^2 and more.

Propagation of US in the medium differs (due to small length of a wavc) other feature: liquids and firm bodies represent good conductors of US, air and gas are bad conductors. So, in water US fades in 1000 times more poorly than in air. At propagation of US in the non-uniform environment arises its reflection and refraction. Reflection of US on border of two mediums depends on parity of their wave resistance. If US in the medium with wave resistance $\omega_1 = \rho_1 \vartheta_1$ falls perpendicularly on a flat surface of the second medium with $\omega_2 = \rho_2 \vartheta_2$, part of energy will pass through the boundary surface and part will be reflected. The coefficient of reflection will be equal to zero, if $\rho_1 \vartheta_1 = \rho_2 \vartheta_2$, i.e. US energy will not be reflected from border of the unit of surfaces and will pass from one medium to another lost-free. For the borders of «air – liquid», «liquid – air», «firm body – air» and on the contrary the coefficient of reflection will be equal almost to 100 %. It speaks that air has very small acoustic (wave) resistance.

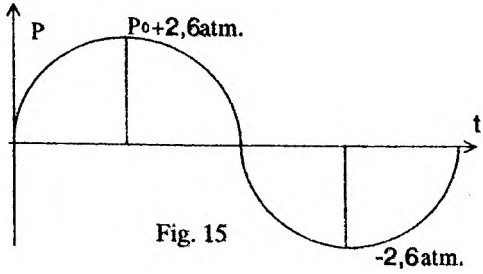


Fig. 15

Therefore in all cases of connection of a radiator of US with the irradiated medium, for example, with a body of the person, it is necessary to watch that between radiator and a tissue there *was no even a minimal air layer* (wave resistance of biological mediums in 3000 times is more of wave resistance of air). To exclude air layer, a surface of US-radiator covered by a layer of oil or it is rendered by a thin layer on a surface of a body.

At propagation of US in the medium arises sound pressure which changes, accepting positive value in the field of compression and negative in the area following it. So, for example, at intensity of ultrasound of 2 Wt/cm^2 in tissues of the person pressure is created in the field of compression of + 2,6 atm., which in the following area passes in value of - 2,6 atm. (fig. 15). Compression and low preassure, created by ultrasound result in formation of breaks of continuous liquid with formation of microscopic cavities (**cavitation**). If this process occurs in a

liquid bubbles are filled by pairs of liquid or the gases dissolved of it. Then on a place of a cavity the site of compression of substance is formed, the cavity slams quickly, allocated a significant amount of energy in small volume that results in *destruction of microstructures of substance.*

4. Medical and biologic application of ultrasound

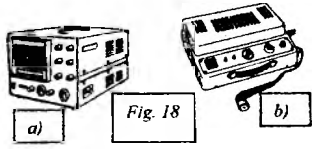
Medical and biologic actions of US are wide.

THERMAL ACTION of US has the important value since to processes of metabolism in biological objects significant temperature dependence is peculiar. The thermal effect is defined by the absorbed energy. Thus is used small intensity of US (about 1 Wt/cm^2). The thermal effect causes expansion of tissues and blood vessels therefore the metabolism amplifies, amplification of blood-groove is observed. Due to thermal action of the focused ultrasound it is possible to use US as the scalpel for cutting not only soft tissues, but also a bone tissue. Now the method of "welding" damaged bone tissues is developed.

Fig. 16

MECHANICAL ACTION. Mechanical oscillations of particles of substance in the ultrasonic field can cause positive biological effect (micromassage of tissue structures, fig. 16). Microvibration concerns to this kind of influence at cellular and subcellular level, destruction of biomolecules, destruction of microorganisms, viruses, destruction of malignant tumors, stones in bladder and kidneys also. The ultrasound is used for crushing substances, for example, at manufacturing of colloid solutions, medicinal emulsions, aerosols. By destruction of vegetative and animal cells from them allocate biologically active substances (enzymes, toxins). US cause damages and reorganization of cellular membranes, change of their permeability.

Fig. 17



a) Fig. 18 b)

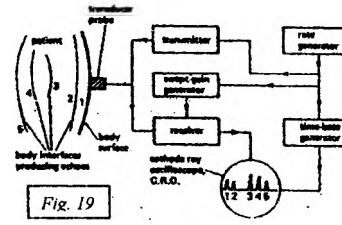


Fig. 19

PHYSICAL AND CHEMICAL ACTION of ULTRASOUND. Action of ultrasound is possible to speed up some chemical reactions. Consider that it is connected with activation of molecules of water, which then break up, forming active radicals of H^+ and OH^+.

The medical and biologic application of US can be divided on two directions: **diagnostics and therapy**. To the first concerns echo-ranging methods with using mainly pulse radiation. It is echoencephalography: definition of tumors and hypostases of brain (on fig. 18,a is shown echoencephalograph "Echo - 12").

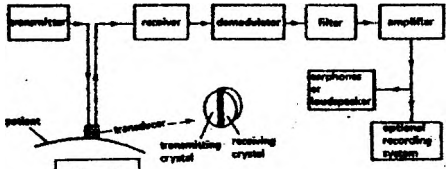


Fig. 20

Echo-ranging methods are based on reflection of US from border of the unit of mediums with various densities. The A-scan system (fig. 19) is basically range-measuring system. It operates by recording the time taken for an US pulse to travel to interface in the body and then be reflected back. The time-measuring instrument is the cathode ray tube which must therefore be synchronized with the transmitter/receiver system. To this method concerns also ultrasonic cardiography – measurement of the sizes of heart in dynamics. US location is used and in ophthalmology (fig. 17) for determination of the sizes of eye mediums. Ultrasonic Doppler`s effect is used for studying character of motion of mitral valves and speed of blood-groove (the principle of the continuous wave Doppler system is shown on fig. 20).

Rather big future of ultrasonic holographic methods of reception of the image of such bodies as kidneys, heart, stomach, etc.

Ultrasonic therapy concerns to the second direction. Usually are applied US with frequency of 800 kHz and intensity of 1 Wt/cm^2. On fig. 18, b the device used for these purposes is shown. And initial mechanisms of action are mechanical and thermal action on a tissue.

5. Infrasound, features of its propagation. Action of infrasound on biological objects

INFRASOUND (IS) is called sound oscillations which top range does not exceed of 16 – 20 Hz. The bottom range is 10^{-3} Hz. The big interest is represented IS by frequency of 0.1 and even 0.01 Hz. IS are part of noise. Sources of IS are movement (storm) of sea or river water, noise of a wood, wind, lightning discharges, earthquake and collapses(landslides), oscillations of the bases of buildings, machine tools, roads from moving transport. IS arises during vibrations of mechanisms, at blowing by a wind of buildings, trees, columns, at movement of the person and animals.

Characteristic property of IS is its **small absorbability** by mediums. Therefore it is propagated to the big distances. IS is well propagated in a tissues of organism, especial in a bone tissue. Speed of IS in air is 1200 km/h, in water is 6000 km/h.

IS oscillations have biological activity which speaks concurrence of their frequency to the alpha - rhythm of human brain.

IS of frequency 1-7 Hz with intensity of 70 db within 8-10 minutes of an irradiation is caused: dizziness, nausea, difficulty of breath, feeling of oppression, headache and asthma. All these factors amplify at repeated influence of IS. IS of the certain frequency can result to **fatal** outcome.

Vibrations of mechanisms are source of US. In connection with adverse action of vibration and of IS on an organism of the person there is vibrating illness.

It results in the beginning to atrophy of muscles of hands and other bodies, to downturn of sensitivity to mechanical vibrations, to occurrence of spasms of fingers of hands, foots and other bodies.

Assume, that the initial mechanism of action of IS on human organism has the **resonant nature**. Internal bodies have own frequency of oscillations. At influence by IS with the frequency equal own there is a resonance which causes the specified unpleasant sensations, and in some cases can result in heavy consequences: to cardiac arrest or break of blood vessels.

Frequency of own oscillations of a human body: in a prone position (3 – 4 Hz), standing – (5 – 12 Hz), of thorax – (5-8 Hz), of abdominal cavity – (3 – 4 Hz) and other bodies correspond to frequency of IS. Decrease of level of intensity of infrasound in inhabited, industrial rooms and on transport is one of problems of hygiene.

LECTURE №9

HYDRODYNAMICS OF VISCOUS LIQUID

1. Stationary current of a liquid. Continuity equation of flow filament

Hydrodynamics is the section of physics that study questions of motion of liquids (incompressible) and its interaction with environmental solid bodies.

Liquid medium makes the greatest part of human organism, its motion provides metabolism and supply of cells by oxygen and therefore mechanical properties and current of liquids represent interest for physicians and biologists.

Consider the established or **stationary current** of liquid, i.e. such current at which speeds of particles of liquid in each point of a stream do not changes.

Stationary current is characterized by **lines of a current,** i. e. imagined lines, conterminous to trajectories of particles. Tangents to lines of a current shows direction of speed vector of particles of a liquid, density of these graphically represented lines is proportional to speed. A part of a stream of liquid limited from different directions by lines of a current forms **a tube of a current** or **jet** (fig. 1).

Let's allocate a tube of the current so narrow, that speeds of particles υ in its any section S perpendicular to axis of the tube is possible to count identical on all section. As speed of particles is directed along of line of a current, then particles of a liquid can not fall outside the limits of this tube. Then volume ΔV of incompressible liquid, proceeding through any section of a tube per unit of time, remains the constant:

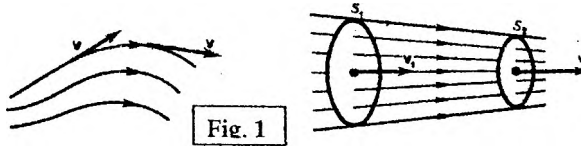


Fig. 1

$$\boxed{\Delta V = \vartheta \cdot S = const} \qquad (1)$$

The parity (1) expresses **continuity equation of flow filament**, since only at continuous current through any section for same time pass identical volumes of incompressible liquid.

Then: $\vartheta_1 S_1 = \vartheta_2 S_2$ (see fig.1), whence $\frac{\vartheta_1}{\vartheta_2} = \frac{S_1}{S_2}$ i.e. average speeds of current in various sections of a pipe are inversely proportional to the areas of these sections.

From the continuity equation of flow follows, that also for a real liquid at the established current on a pipe of variable section, quantity of the liquid, proceeding for same time through any section of the pipe remains constant: Q = const. In particular, with big accuracy it is carried out for blood current in large blood vessels in time equal to several intimate cycles, directly following one after another.

2. Equation of Bernoulli and its consequences

One of the most important equations used for the description of moving liquids, for the first time was received by Swiss mathematician and physicist Daniel Bernoulli (1700-1782). For deduction of the equation Bernoulli has assumed, that we deal with **ideal liquid**. It means that we neglect any forces of viscous resistance and friction.

Let's consider stationary current of an incompressible ideal liquid. We shall allocate the volume of liquid limited by walls of a narrow tube of current and perpendicular to lines of current by sections S_1 and S_2 (fig. 2). In time Δt this volume will be displaced along the tube of current and the border of volume S_1 will receive displacement Δl_1, and border S_2 displacement Δl_1. The work made at it by forces of pressure is equal to the increment of total energy (E_K+E_P) in considered volume of liquid: $A = E_K + E_P$.

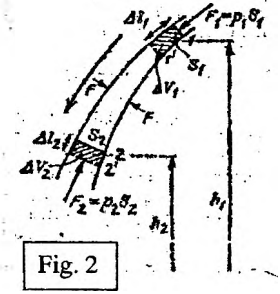


Fig. 2

Forces of pressure upon walls of a tube of current are perpendicular in each point to the direction of moving of liquid owing to what works do not made. Work of forces of the pressure enclosed to sections S_1 and S_2 is equal to zero. This work is equal to:

$$A = p_1 S_1 \Delta l_1 - p_2 S_2 \Delta l_2 = (p_1 - p_2)\Delta V \qquad (2)$$

Total energy of considered volume of liquid is

composed from kinetic energy and potential energy in the field of forces of terrestrial gravitation. Owing to stationary current total energy of that part liquids, which is limited by sections 1' and *2* (the internal not shaded part of the tube of current on fig. 2) in time Δt does not change. Therefore the increment of total energy is equal to the difference of values of total energy of the shaded volumes ΔV_2 and ΔV_1, which mass is $\Delta m = \rho \Delta V$ (ρ is density of liquid).

Let's take section S of a tube of current and displacement Δt so small that all points of each of the shaded volumes could attribute the same speed υ, pressure p and heights h. Then for the increment of total energy:

$$\Delta E = \left(\frac{\rho \Delta V \upsilon_2^2}{2} + \rho \Delta V g h_2 \right) - \left(\frac{\rho \Delta V \upsilon_1^2}{2} + \rho \Delta V g h_1 \right) \qquad (3)$$

Having equated expressions, (2) and (3) having reduced on ΔV and having transferred members with identical indexes in one part of equality, we shall come to the equation

$$\boxed{\frac{\rho \upsilon_1^2}{2} + \rho g h_1 + p_1 = \frac{\rho \upsilon_2^2}{2} + \rho g h_2 + p_2}$$

$$\boxed{p + \rho g h + \frac{1}{2} \rho \upsilon^2 = const.} \qquad (4)$$

It is **equation of Bernoulli** which can be referred not only to sections of a tube, but also to the points, located along some of lines of current. Summands, included in equation of Bernoulli, have dimension and sense of pressure. Pressure p is called **static pressure** (external): it is not connected to motion of a liquid and can be measured, for example, by a manometer moving together with liquid. Pressure $\frac{1}{2} \rho \vartheta^2$ is named **dynamic:** it is caused by motion of liquid and shown at its braking. Pressure $\rho g h$ is **hydrostatic** (weight) pressure. In a condition of weightlessness hydrostatic pressure is absent, with increase of overloads it grows. In these terms Bernoulli equation can be formulated as the law: **at stationary current of ideal liquid the sum of statical, dynamic and hydrostatic pressure is the constant in various points of a line of current, in any cross section of stream.**

For a *horizontal tube* of current hydrostatic pressure remains the constant (since h = *const)* and can be transferred in the right part of the equation (4) which in this case can be written down so:

$$p_{st}+p_{dyn}=\text{const}$$

Statical pressure is caused by potential energy of a liquid (energy of pressure), dynamic pressure by kinetic. From last equation follows conclusion named **rule of Bernoulli**: *statical pressure of a nonviscous liquid at current on a horizontal tube grows in the places, where speed of liquid decreases and on the contrary.*

Equation of Bernoulli received for the ideal case is possible to use and for explanation of real examples.

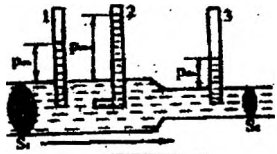
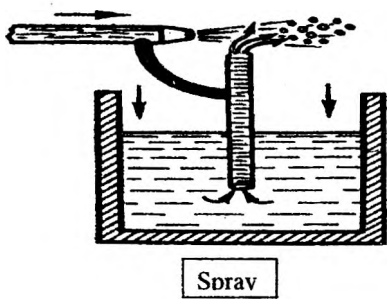


Fig.3

On fig. 3 monometric tubes 1 and 3 show size of statical pressure in the different sections of a horizontal tube. As the bottom section of these tubes is parallel to lines of current, it do not show dynamic pressure. Dynamic pressure is determined on difference between total pressure p_{total} and static:



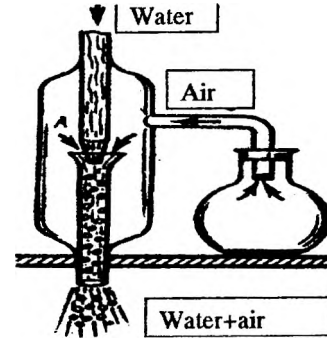
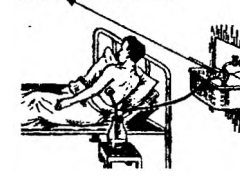


Fig. 4 Water-jet pump

$p_{dyn}=p_{total}-p_{st}$. For measurement of total pressure is used the tube bent under the right angle to stream (fig. 3). Here $S_2<S_1$, $\vartheta_2 > \vartheta_1$ and $p_2<p_1$. The section of tube S_2 can be made so narrow, that owing to small static pressure (below



atmospheric) in this section will be sucked air or a liquid (soaking up action of jet). It is used in water-jet pumps, medical inhalers, sprays (fig. 4)

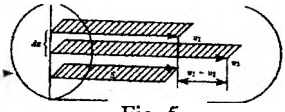


Fig. 5

3. Viscosity of liquid. Newton's equation

In a real liquid between molecules operates the force of a mutual attraction causing **internal friction** or **viscosity.** Internal friction, for example, causes force of resistance at stirring of liquid, slows down speeds of falling body thrown in it.

Newton has established, that force F_{fr} of internal friction between two layers of liquid moving with various speeds is directly proportional to area S of adjoining layers and to gradient of speed $\frac{d\vartheta}{dx}$ between them (fig. 5) (the gradient of speed between layers is the change of speed dv, divided to length dx in the direction, perpendicular of speed):

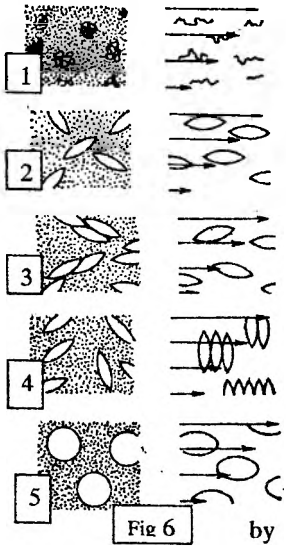


Fig 6

$$F_{fr} = \eta \cdot S \frac{d\vartheta}{dx}, \qquad (6)$$

where η is the coefficient of proportionality, it is known as **coefficient of internal friction or dynamic viscosity** (or simply **viscosity**). Viscosity depends from condition and molecular properties of a liquid. The equation (6) is the **equation of Newton.**

Unit of viscosity in SI is $N\frac{s}{M^2} = Pa \cdot s$, in system SGS is $din \cdot \frac{s}{CM^2}$, This unit is known as puas (P).

Connection between them: 1Pa s=10 P.

In practice viscosity of a liquid is characterized by **relative viscosity** at which understand the ratio of viscosity η of the given liquid to viscosity of water η_w at the same temperature:

$$\eta_{rel} = \frac{\eta}{\eta_{water}}$$

From majority of liquids (water, low-molecular organic connections, true solutions, fused metals and their salts) coefficient of viscosity depends only by nature liquids and temperature (with rise of temperature η goes down). Such liquids are known as **Newtonians.**

At some liquids, mainly high-molecular (solutions of polymers) or representing dispersive systems (suspensions, emulsions) coefficient of viscosity depends also from the mode of current (from the pressure and gradient of speed). At their increase viscosity of a liquid decreases owing to infringement of internal structure of stream of a liquid. Such liquids are named structural - viscous or **non – Newtonians** (on fig. 6 some reasons of non- Newtonian behaviour of solutions and suspensions are shown: 1-unreelings of balls, 2-modification of orientation, 3-disaggregation, 4-aggregation, 5-deformation). Their viscosity characterize in so-called conditional coefficient of viscosity, which concerns to the certain conditions of current of a liquid (pressure, speed).

Blood represents suspension of uniform elements in albuminous solution - plasma. Therefore it should be referred to **non - newtonians** liquids. Besides at current of blood on vessels is observed concentration of uniform elements in the central part of a stream, where viscosity is accordingly increased. But as viscosity of blood is not so great, by these phenomena frequently neglect and count its coefficient of viscosity as the constant. **Relative viscosity of blood** in norm is 4.2-6. At diseases it can be reduced up to 2-3 or raise till 15-20.

Viscosity of a liquid is measured with the help of viscosimeters. With help of capillary viscosimeter of Ostwald (fig. 7) it is possible to measure viscosity of gases (from 10^{-5} Pa·s) and liquids (up to 10^{4} Pa·s). In the U-shaped tube one of which knees has the capillary 2 pour a researched liquid. With help of a rubber pear the liquid from the big tank 3 is sucked in the tank 1 above of mark A. Then, having removed the pear, is measured time of flow of liquid between labels A and

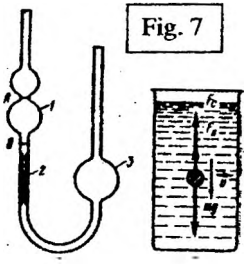


Fig. 7

B. Comparing this time with time of flow of reference liquid (distilled water) can be determined viscosity of a researched liquid.

Method of incident blob (fig. 7) is used in the viscometers, which has been set up on the Stokes law. Measuring the velocity of the blob it is possible to find viscosity of a liquid.

In rotational viscosimeters (fig. 8) the fluid is between two coaxial cylinders. One of cylinders (rotor) is twirled, and second is immobile. The turning moment operating on the fixed cylinder will be proportional to viscosity.

Now for determination of viscosity of blood in clinic is used Gess capillary viscometer (fig. 9). Distances passed by distilled water and blood in tubes 1 and 2 on capillary a_1b_1 and a_2b_2 will be proportional to their viscosities.

Fig. 8

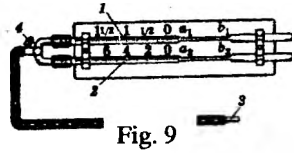


Fig. 9

4. Laminar and turbulent current. Reynolds number

The elementary type of motion of a liquid is **laminar current** in which layers of stream gradually replace each other (fig. 10, on the left, fig. 11,a). At low speeds current of stream through a pipe be laminar. But at the big speed laminar current can not be kept. Current becomes non-uniform, in the stream develop casual whirlwinds and resistance sharply grows. This type of stream is called **turbulent** (fig.10, on the right; fig. 12,b). Turbulent current is characterized by local changes of pressure in the liquids causing oscillatory motion of its particles, which is accompanied by the sound phenomena (noise, murmur), due to which turbulent current is easily found out. Experiment shows, that turbulence arises, when the certain combination of variables more some critical value. This combination is known as **number of Reynolds**:

Fig.10

$$\mathrm{Re} = \frac{2\rho r \vartheta}{\eta},$$

where ρ and η accordingly density and viscosity of a liquid, r is radius of a pipe, ϑ is average speed of a liquid. It is easy to check up that Re is dimensionless variable. The stream through the pipe is laminar, if $\mathrm{Re} \leq 2000$. For Re>3000 stream is turbulent. A mode with Re between 2000 and 3000 is intermediate in which current not stably and can change casual between laminar and turbulent.

The stream of blood in aorta in a condition of rest is laminar and can be

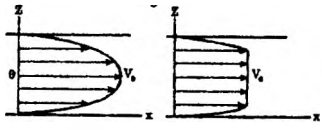


Fig 11,a Fig. 11, b

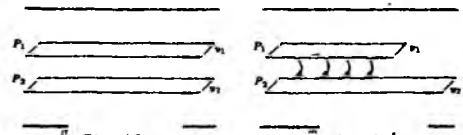


Fig. 12, a Fig.12, b

expressed as product of the area of cross section of aorta S and average speed of stream, i.e.: $Q = S \cdot \bar{\vartheta}$.

However during the intense activity speed $\bar{\vartheta}$ of the blood consumption grows and it results that Re number achieves 2000 and current becomes turbulent. Thus the velocity profile of current essentially varies: the parabolic profile of a laminar flow (fig. 11, a) is substituted by more flat profile (fig. 11,b) for a turbulent flow. In fine blood vessels speed remains enough small and a stream still laminar.

5. Current of a viscous liquid on pipes. Formula of Poiseuille. Hydraulic resistance

Current of a viscous liquid on pipes represents special interest for physicians since the blood system consists basically of cylindrical vessels of different diameter. Owing to symmetry it is clear, that in a pipe the particles of the current liquid equidistant from its axis have identical speed. The greatest speed has the particles moving along axis of pipe and the layer of a liquid closest to the pipe is motionless. Provisional distribution of speed of particles of liquid in section of a pipe is shown on fig. 13.

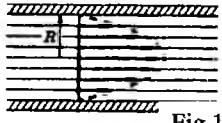
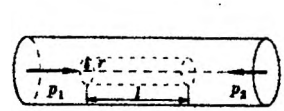


Fig.13

Let's allocate mentally cylindrical volume of a liquid of some radius r and length l (see fig. 14, a) and we shall determine dependence $\vartheta = f(r)$. At the end faces of the cylinder allocated with us are supported pressure p_1 and p_2 accordingly, that causes resulting force:



a

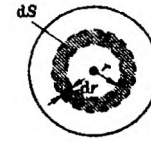


Fig. 14 b

$$F = p_1\pi r^2 - p_2\pi r^2 = (p_1 - p_2)\pi r^2$$

On the lateral surface of the cylinder from the part of environmental layer of the liquid operates the force of internal friction equal in this case to:

$$F_{fr} = \eta\frac{d\vartheta}{dr}S = \eta\frac{d\vartheta}{dr}2\pi rl$$

where $S = 2\pi rl$ is the area of lateral surface of the cylinder. As the liquid goes in regular intervals, the forces working on the allocated cylinder are counterbalanced: $F=F_{fr}$. Substituting in this equality expression for F and F_{fr} and taking into account, that speed decreases with increase of r, i.e. $\frac{d\vartheta}{dr} < 0$ (means F_{fr} it is necessary to take with "minus"), we receive:

$$(p_1 - p_2)\pi r^2 = -\eta\frac{d\vartheta}{dr}2\pi rl$$

Hence, we have:

$$d\vartheta = -\frac{p_1 - p_2}{2\eta l}rdr$$

Let's integrate this equation:

$$\int_0^{\vartheta} d\vartheta = -\frac{p_1 - p_2}{2l\mu}\int_R^r rdr$$

Here the bottom limits correspond to the layer, adjoining to internal surface of pipe ($\vartheta = 0$ at r = R), and the top limits are variables. As a result of integration is received **parabolic dependence of speed** of layers of a liquid on their distance up to axis of pipe (see bending around the ends of vectors of speed on fig. 13):

$$\vartheta = \frac{p_1 - p_2}{4l\eta}(R^2 - r^2)$$

From this expression is visible, that the layer of current along the axis of pipe has the greatest speed (r = 0, v=max):

$$\vartheta_{max} = \frac{(p_1 - p_2)R^2}{4l\eta},$$

and for the layer, adjoining to the internal surface of pipe (r = R) speed is equal to «0».

Let's establish from what factors depends volume Q of the liquid proceeding through a horizontal pipe per 1 second. For this purpose we shall allocate a cylindrical layer of radius r and thickness of dr (see fig. 14, a). The area of section of this layer is $dS = 2\pi r dr$, since the layer thin, it is possible to count that it moves with identical speed ϑ. For 1 s the volume of liquid dQ is transferred through the layer:

$$dQ = \vartheta dS = \vartheta 2\pi r dr.$$

Substituting (8) in (10), we receive: $dQ = \pi \frac{p_1 - p_2}{2 \cdot l \cdot \eta}(R^2 - r^2) r dr$, whence integration on all section we find:

$$Q = \pi \frac{p_1 - p_2}{2 \cdot l \cdot \eta} \cdot \int_0^R (R^2 - r^2) r \cdot dr = \pi \frac{p_1 - p_2}{2 \cdot l \cdot \eta} \left(R^2 \cdot \frac{r^2}{2}\Big|_0^R - \frac{r^4}{4}\Big|_0^R \right) = \pi \frac{p_1 - p_2}{2 \cdot l \cdot \eta} \cdot \frac{R^4}{4},$$

or

$$\boxed{Q = \frac{\pi \cdot R^4}{8 \cdot \eta} \cdot \frac{p_1 - p_2}{l}}; \quad \left(Q = \frac{\pi R^4}{8\eta} \cdot \frac{dp}{dl} \right)$$

This dependence is known as **Poiseuille's formula**. This formula can betray and such kind:

$$Q = \frac{p_1 - p_2}{\omega}, \text{ where } \omega = \frac{8l\eta}{\pi R^4}$$

Value ω is named **hydraulic resistance**. It is inversely proportional to the fourth degree of radius and considerably grows with reduction of radius of a pipe. We shall notice, that if R is doubled, Q grows in 16 times. Similarly, at reduction of R the stream considerably will decrease. So, if something will result in exhaustion of arterial walls (that causes reduction R), the weakened stream of blood can cause

stenocardia, which is characterized by pains in a breast, accompanying with the general indisposition.

Improvement of condition can be achieved entering some substances, for example, nitroglycerine, which weakens muscles of arterial walls, increases R and accordingly, increases the stream of blood and reduces loading on heart.

The analogy between electric and hydraulic resistance allows to use in some cases the rule of finding of electric resistance of series and parallel connections of conductors for determination of hydraulic resistance of system in series and in parallel connected pipes. For example, the general hydraulic resistance of three pipes connected in series and in parallel (fig. 15) is calculated accordingly the formulas:

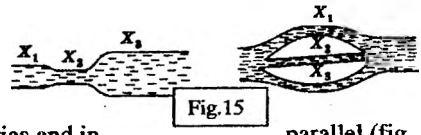


Fig.15

$$X = X_1+X_2+X_3,$$

$$X = \left(\frac{1}{X_1}+\frac{1}{X_2}+\frac{1}{X_3}\right)^{-1}.$$

To give to Poisseuille equation more general expression fair and for pipes of variable section, we shall replace $(p_1-p_2)/l$ by gradient of pressure dp/dl, and then

$$Q = \frac{\pi r^4}{8\eta}\cdot\frac{dp}{dl}.$$

Let's establish in different places of a horizontal cylindrical pipe of different section on which the viscous liquid flows manometrical tubes (fig. 16,a). They show, that static pressure along the pipe of variable section decreases proportionally to l: dp/dl=const. As value of Q is identical (incompressible liquid) according to the last formula,

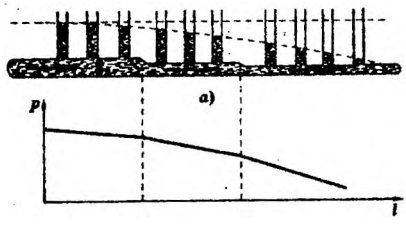


Fig. 16 b)

gradient of pressure is more in pipes of smaller radius. The graph of dependence of pressure from distance along pipes of different radius is approximately shown in figure (fig. 16, b).

LECTURE №10

PHYSICAL BASES OF HEMODYNAMICS

1. Motion of liquid and blood on pipes with elastic walls. Blood system as branching of pipes

Blood circulation is one of the most important processes occuring in human organisms. The section of biophysics studying laws of motion of blood on vascular system is named **hemodynamics** (Greek "haima" means blood). General laws of current of liquids investigated by hydrodynamics are established within the framework of classical physics and are the basis for description of complex hemodynamics processes in a human organism. However properties of blood in many respects are different from properties of liquids used in engineering and having elastic walls and repeatedly branching blood vessels considerably differ, for example, from system of water pipes. Therefore the biophysics considers only simplified model of blood circulation.

One of **features** of physical model of cardiovascular system is **elasticity** of its walls. Elasticity is ability of material to test more or less significant elastic convertible deformations at rather small efforts.

Walls of blood vessels differ on its structure. Aorta and large arteries have the walls consisting from muscular fibres, elastin and collagen. Elastin supposes deformations up to 200-300 %, collagen up to 10 %. Arterioles consist completely only of muscular tissues which extensibility is much less. Walls of capillaries are not covered by elastic or muscular tissue.

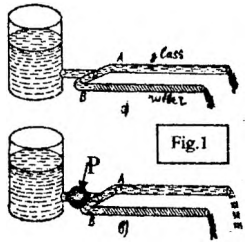


Fig.1

Current of liquid on pipes (vessels) with elastic walls has the certain specificity (fig. 1). At constant pressure elasticity of walls of tube has no essential value. For example, it is possible to observe identical continuous stationary stream of liquids from glass (rigid) and rubber (elastic) pipes (p=const).

If through tubes to pass the pulsing stream, using for this purpose periodically working pump P, that character of the outflow of liquid be various:

from the rigid tube will be pulsating flow, from elastic be continuous. When such pump pushes liquid into a pipe with the elastic walls already filled by a liquid,

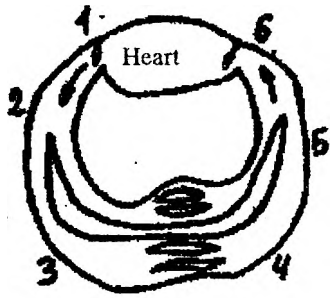


Fig. 2

pressure in the tube raises, the wall of it is stretched and contains surplus of liquid. Then, when pressure of the pump falls, the wall of tube is reduced and potential energy of walls passes to kinetic energy of liquid, therefore surplus of liquid from the initial site of pipe passes in its following site.

Second feature of cardiovascular system is that it represents the **system closed, repeatedly ramified and filled by a liquid pipes**, motion of liquid in which occurs under action of rhythmically working delivery pump (heart). From fig. 2 is visible connection in series of aorta (1-2), arteries and arterioles (2-3), capillaries (3-4), venules (4-5) and veins (5-6), and also parallel connection of arteries and arterioles, capillaries and venules. The general hydraulic resistance of these connections can be determined by analogy to laws of connection of resistors: for connection in series: $Z=Z_1+Z_2+...+Z_n$; for parallel: $\frac{1}{Z}=\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\cdots+\frac{1}{Z_n}\right)$.

Let's consider gemodynamic parameters in the different sites of vascular system. Hydraulic resistance Z substantially depends from radius of vessel $Z \propto \frac{1}{R^4}$. Ratio of radiuses for various sites of vascular system: R_{aor}: R_{art}: $R_{cap} \approx 3000:500:1$, therefore it is possible to write down the parity $Z_{cap}>Z_{art}>Z_{aor}$. The area of total section of all capillaries in 500-600 times is more then cross section of aorta. Under the law of indissolubility of stream it means that $\vartheta_{cap} \approx \frac{1}{500}\vartheta_{aor}$. If in aorta average speed is equal approximately to 0.5 m/s, then in capillaries is 0.3-0.5 mm/s. In capillary network at slow speed of motion there is metabolism between blood and tissues. On fig 3 the curve of distribution of linear speeds along vascular system is given.

Reasons of blood motion: 1. ΔP is a primary factor of blood motion.

2. Muscles of a sceleton is the muscular pump (at presence of veins valves).

3. Negative pressure in cava (p in cava is equal to -4,-6 mm Hg).

4. Due to elastic properties the potential energy of walls is transformed to kinetic energy of blood motion.

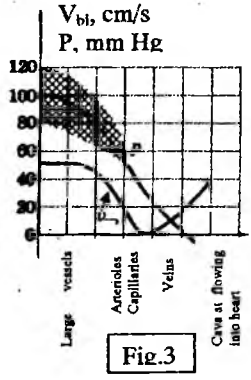


Fig.3

At reduction of heart pressure of blood in aorta has fluctuations. We shall consider average pressure for period. Change of average pressure along vessels can be described by law of Poiseuille ($\Delta p = QZ$). Heart throws out blood under average pressure p_{av}. Average pressure at process of promotion on vessels decreases. As Q=const and $Z_{cap}>Z_{art}>Z_{aor}$, then for average values of pressure: $\Delta p_{cap}>\Delta p_{art}>\Delta p_{aor}$. In large vessels average pressure falls on 15 % and in fine vessels on 85 %. It is meant that the most part of the energy spent by left ventricle of the heart on exile of blood is spent for its current on small vessels. Distribution of pressure (excess above atmospheric) in various departments of vascular channel is submitted on fig 3. Negative value of pressure means, that it below atmospheric. The shaded area corresponds to fluctuation of pressure: p_s is systolic pressure ≈120 mm Hg; p_d is diastolic pressure ≈80 mm Hg.

2. Propagation of pulse waves

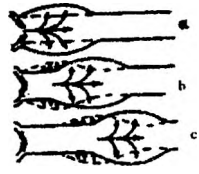


Fig. 4

After each reduction of heart along aorta in direction from heart to periphery runs the wave of deformation (as waves on surface of water from a stone thrown in it). And if on the artery which is taking place near to surface of body (for example, at wrist) to put a finger it will feel these waves as pushes (pulse). Wave of pressure arising at it is named the **pulse wave**. On fig 4 formation of pulse waves is shown. Amplitude of pressure in pulse wave decreases on exponential law.

Speed of pulse wave in large vessels as follows depends on their parameters (formula of Moens and Corteveg):

$$\vartheta_{PULSEWAVE} = \sqrt{\frac{Eh}{\rho d}},$$

where E is module of elasticity of a vessel; h is thickness of its wall; d is diameter of a vessel; ρ is density of blood.

Analytical researches show, that value $\frac{h}{d}$ changes insignificant at different people and practically does not depend on type of an artery. Therefore, it is possible to count that speed of pulse wave changes only from elasticity of walls of vessel and its module of elasticity. With age and also at the diseases accompanying with increase of E (hypertension, atherosclerosis) speed ϑ can be increased almost in 2-4 times in comparison with norm. It allows to use change of ϑ at statement of diagnosis.

Speed of pulse waves is easy for measuring. So, from the moment of intimate reduction before occurrence of pulse in the beam artery of the person there is about 0.1 s and the distance from heart up to the place of measurement of pulse about 70 cm that gives value of speed is $\vartheta_{PULSEWAVE} = 7\frac{M}{s}$. For this time blood passes in artery distance only 5 cm. Hence, speed of propagation of pulse waves is much more than linear speed of blood flow.

Alongside with pulse wave in system "vessel - blood" can be propagated also sound waves, which speed is very great $\left(\vartheta_{SOUND} \approx 1500\frac{m}{s}\right)$.

3. Infringements of hemodynamics parameters of vascular system

Basic function of cardiovascular system is maintenance of continuous motion of blood on capillaries where there is metabolism between blood and tissues.

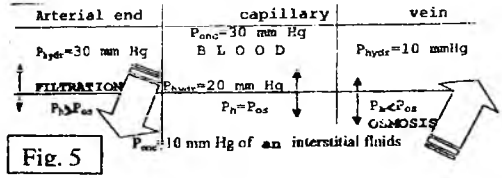


Fig. 5

Distinguish **two basic mechanisms of transportation of**

substances: *diffusion exchange* of molecules caused by distinction of concentration of these molecules on the different sides of wall of vessels and *filtrationreabsorption mechanism*. Filtration-reabsorption mechanism is motion of liquid through the pores in capillary wall under action of gradient of pressure.

Under action of pressure in capillary the liquid aspires to leave from capillary in tissues (filtration) and under action of pressure in tissue liquid comes back again in a capillary (reabsorption).

Under normal physiological conditions usually the filtration occurs in the arterial end and reabsorption in the venous end of capillary and between them there is a balance (fig. 5).

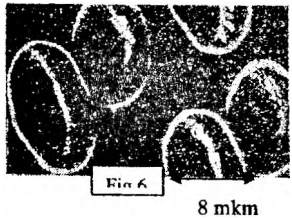


Fig.6

8 mkm

Infringements of hemodynamics parameters of vascular system and structure of capillary wall inevitably result to infringement of metabolism. The reason of infringement of hemodynamics can be changes of apertures of vessels and reological properties of blood. Under **rheology of blood (hemoreology)** understand study of biophysical features of blood as viscous liquid. **Blood is non-newtonian liquid**. It represents suspension of uniform elements in solution - plasma. 93 % of uniform elements make erythrocytes, which representing flexible biconcave disks (fig. 6) of diameter $d_{er} \approx 8$ mkm. The important parameter is the ratio of volume of erythrocytes (V_{er}) to volume of plasma (V_{pl}), which is known as **hematocrit**. In norm $\frac{V_{er}}{V_{pl}} \approx 0.4$.


Fig. 7

With increase of hematocrit viscosity of blood grows.

Erythrocytes can "stick together" with each other forming units which are named **monetary columns** (fig.7). In large vessels (aorta, arteries) erythrocytes gather in units and in this case viscosity of blood η=0,005 Pa·s. In small vessels (fine arteries, arterioles) units break up on separate

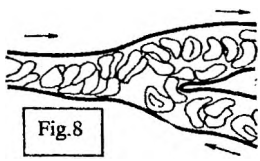


Fig.8

erythrocytes, thus reducing viscosity of blood. In microvessels (capillaries) erythrocytes are easily deformed, becoming similar to a dome (fig. 8) and pass not collapsing through capillaries even of diameter 3 microns. At illness named *spherocytosis* erythrocytes have the *spherical form*. At motion of such erythrocytes through capillaries their membranes are stretched and the certain numbers of erythrocytes collapses. Reduction of their quantity in blood conducts to *anemia*. At anemia viscosity of blood decreases and makes of 0,002-0,003 Pa·s. In connection with reduction or increase of viscosity of blood the gradient of pressure in a vessel changes that causes change of capillary pressure and appearance of *edema*. It arise if too many liquid is filtered from capillaries in tissues in comparison with it reabsorption.

As rule, movement of blood on vessels is **laminar**. However occurrence of turbulence in some cases is possible. Whirlwinds of stream already initially exist, when blood is pushed out from ventricle to aorta. At places of branching of vessels and at increase of speed of blood stream (for example, at muscular work), flow can become turbulent and in arteries. Turbulent current can arise also in the field of narrowing of a vessel, for example, at formation of blood clot. It is connected with additional expense of energy and in blood system it can result to additional loading on heart. The noise arising at turbulent current of blood can be used for diagnostics of diseases. At defeat of valves of heart there is so-called intimate noise.

On capillar exchange can affect narrowing of diameters of vessels or local expansion. During ageing of organism and at the certain diseases (excessive feed, smoking, abusing alcohol) there is atherosclerosis, i. e. thickening of walls of an artery due to sedimentation on their surface of cholesteric plaques, that conducts to narrowing of diameter of artery.

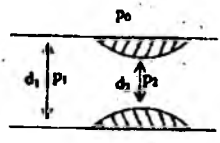


Fig.9

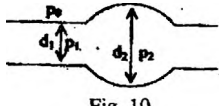


Fig. 10

We admit, that on some site of large artery in diameter d_1 there was a narrowing in diameter d_2 (fig.9). Current of blood on artery will occur until static pressure p_2 in the place of formation of narrowing will exceed external pressure p_o (let's count

it approximately equal to atmospheric). At reduction of narrowing up to d_{min} under action of external pressure p_o the aperture in the place of narrowing is closed. Blood from left ventricle should be thrown out under the big pressure than p_1 otherwise pressure at the end of vessel becomes lower than norm, it will result in downturn of capillary pressure. Heart will work in conditions of the raised loading.

Some pathological processes can result to local reduction of durability and elasticity of blood vessels. As result on this site there is a **swelling of vessel (aneurysm)** (fig 10). Speed of blood stream ϑ_2 in a place of development of aneurysm will be less then speed ϑ_1 in the not deformed part. On the basis of equation of Bernoulli pressure p_2 will be more of pressure p_1 and more than external pressure p_o. Arisen aneurysm under action of the increased pressure will tend to expansion. In result it is possible having dug aneurysms.

Fig.11

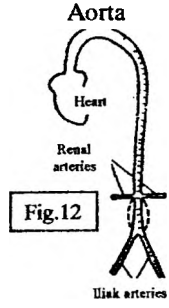


Fig.12

Sometimes meets the aneurysm of aorta *in the place of its branching* in which there is reflection of puls waves (fig. 11, fig. 12). Under action of reflected puls wave, wall of aorta extends above norm. The reasons of aneurysms in this case are not only in increase of pressure in the field of the reflected wave, but also in change of mechanical properties of vessels with age.

4. Modelling representations of process of blood circulation

Quantitative calculation of hemodynamic phenomena becomes simpler at using of models and analogies.

Let's consider the hydrodynamical model of blood system offered by **Franc**.

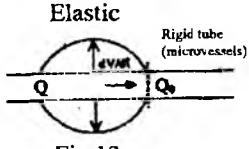


Fig.13

The arterial part of system of blood circulation is modeled by the **elastic tank** (ET) (see fig 13). As blood is in the elastic tank, its volume at any moment depends on pressure p :

$$V=V_0+k\cdot p, \qquad (1)$$

where V_0 is the volume of the tank, when p=0, k is coefficient of proportionality between pressure and volume. Differentiate (1), we shall receive:

$$\frac{dV}{dt} = k\frac{dp}{dt}. \qquad (2)$$

From fig 13 obviously that $Q = \frac{dV}{dt} + Q_0$, (3)

where Q is volumetric speed of blood, which acts to ET from heart, Q_0 is volumetric speed of blood, which flows in peripheral system.

On the basis of equation it is possible to write down for peripheral part of system: $Q_0 = \frac{p - p_v}{Z}$, where p is pressure in ET; p_v is venous pressure, which can be accepted equal to zero. Then:

$$Q_0 = \frac{p}{Z} \qquad (4)$$

Having united (3), (2) and (4) we shall receive:

$$Q = k\frac{dp}{dt} + \frac{p}{Z} \text{ or } Qdt = kdp + \frac{p}{Z}dt. \qquad (5)$$

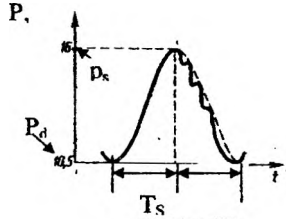


Fig.14

Let's consider this differential equation for time of **diastole,** when Q=0, we shall receive:

$$0 = kdp + \frac{p}{Z}dt \text{ or } \frac{dp}{p} = -\frac{dt}{kZ}. \qquad (6)$$

Having integrated (6), we shall receive expression of *dependence of pressure from time in the tank after a systole* (fig. 14):

$$\boxed{p = p_s \cdot e^{-\frac{t}{kZ}}}. \qquad (7)$$

On the basis of (4) we shall receive: $\boxed{Q = Q_s \cdot e^{-\frac{t}{kZ}}}$ (8)

The formula (8) expresses dependence of speed of blood flow from time, where Q_s is volumetric speed of outflow of blood from ET at the end of systole.

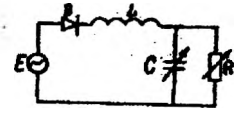


Fig. 15

Curves of dependences (7) and (8) are represented the exponents. The given model roughly describes the real phenomenon however it is simple and truly reflects process to the end of diastole.

By analogy to mechanical model it is possible to construct **electric model** (fig 15). The vascular system is compared to analog electric circuit in which E is **generator** of alternating voltage modelling **heart**; B is the **rectifier**, analogue of **valves of heart**; L is the **coil of inductance** modelling **inertial properties of blood**; C is **condenser** modelling **elasticity of vessels**; R is **resistor** modelling **hydraulic resistance**. R and C are represented by variables since both hydraulic resistance and elasticity of vessels can change. Force of current in a circuit will be analogue of the volumetric charge of liquid Q, and potential φ will be analog of pressure p.

5. Work and capacity of heart

Causing motion of blood in vascular system, heart makes work, which turns to energy of stream of blood and spent for overcoming of viscosity in vascular system.

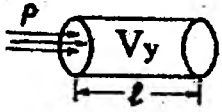


Fig. 16

Let's calculate the work made at unitary reduction of the left ventricle. We shall represent V_s (shock volume of blood) as the cylinder (fig. 16). It is possible to count, that heart pushes this volume to aorta by section S on distance l at average pressure p. Work made at it A_1 is equal: $A_1=F\cdot l=p\cdot S\cdot l=p\cdot V_s$.

Work is spent for the impart of kinetic energy to this volume of blood A_2:

$A_2 = \frac{m\vartheta^2}{2} = \frac{\rho \cdot V_y \cdot \vartheta^2}{2}$, where ρ is density of blood, ϑ is speed of blood in aorta.

Thus, work of the left ventricle A_{left} is equal:

$$A_l = A_1 + A_2 = pV_s + \frac{\rho \cdot V_s \cdot \vartheta^2}{2}.$$

Pressure in the right ventricle approximately in 5 times less than in the left, therefore work of the right ventricle is equal $A_r = \frac{1}{5}A_l = 0{,}2A_l$.

Work of all heart then is equal:

$$A = A_l + A_r = A_l + 0{,}2A_l = 1{,}2\left(pV_s + \frac{\rho \cdot V_s \cdot \vartheta^2}{2}\right)$$

Having substituted in this formula value of p_{av}=13-10^3 Pa; V_s=60 ml =6-10^{-5} m^3; ρ=1.05·10^3 kg/m^3; ϑ=0.5 m/s let's get the job of single reduction of heart in condition of rest (A≈1J). If to take into account that duration of a systole about of t≈0.3s, that average capacity of heart during one reduction is equal: $\overline{N} = \frac{A}{t} = 3.3Wt$.

At physical loading grows systolic volume of blood, speed of current of blood in aorta is increased also. Work of heart is sharply increased, the kinetic component grows.

6. Physical bases of clinical method of measurement of blood pressure

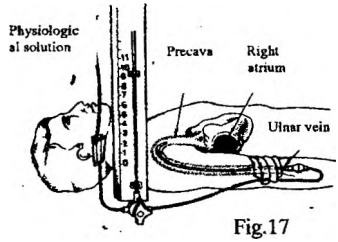


Fig.17

Pressure of blood plays the big role in diagnostics of many diseases. There is **direct** measurement of blood pressure. It is carried out by introduction of catheter directly into a blood vessel or cavity of heart (fig. 17). Catheter is filled by isotonical solution and transfers pressure of blood from the entered end to the external measuring device. Direct measuring is practically unique method of measurement of pressure in a cavity of heart and the central vessels. Such measurement is connected with loss of blood and painful sensations.

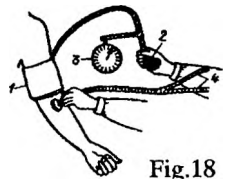
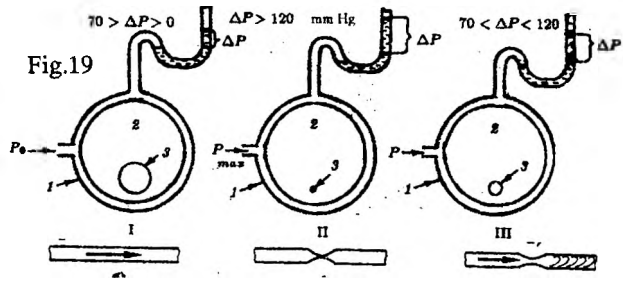


Fig.18

More perfect bloodless way of measurement of blood pressure was offered in 1896 year by Italian doctor **Riva-Rochi** and advanced in 1905 year by Russian doctor **Korotkov** (fig.18). The method is based on listening of the *noise* created by pulse waves.



At measurement of

pressure at a person between the shoulder and elbow impose the cuff C (fig 20) and pump up in it air until in humeral artery the current of blood does not stop and pulse does not disappear (II). Then air from the cuff with help of rubber pear gradually let out and pressure upon artery weakens. When pressure upon artery *becomes equal to systolic* blood starts to push through the squeezed artery and in it creates *turbulent stream* accompanying by **noise (tones of Korotkov)** (fig.19, III). These noises are listened through phonendoscope and on manometer P registered appropriate to this moment systolic pressure (fig. 20). At further decrease of pressure in the cuff, aperture of artery is gradually restored up to normal, current of blood becomes laminar and noises are disappear. Indications of manometer at the moment of **disappearance of noise** are corresponds to **diastolic** to pressure.

If the muscles of hand are weakened, pressure of air inside cuff having elastic walls is approximately equal to pressure in the soft tissues adjoining with cuff. It is the **basic physical idea** of bloodless method of measurement of pressure.

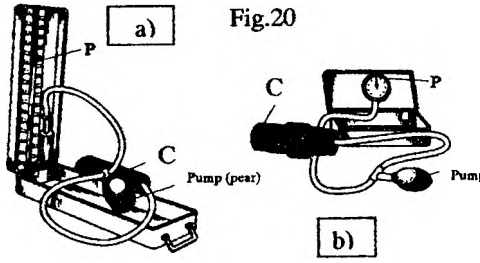


Fig.20

To measurement of pressure apply the devices shown in figure 20: a) sphygmomanometer with mercury manometer; b) tonometer with metal мmembrane manometer. Here: C is cuff, P is manometer.

LECTURE №11

ELECTRIC DIPOLE

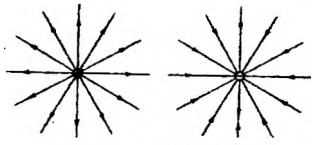
PHYSICAL BASES OF ELECTROCARDIOGRAPHY

1. Basic characteristics of electric field

Electric field is the version of matter by means of which force influence on electric charges is carried out which are taking place in this field.

Characteristics of electric field which is generated by biological structures are source of information on condition of an organism.

Unit of charge is Coulomb (C), 1C=1 A·s.



a) Fig. 1

The **force characteristic** of electric field is **intensity E** equal to the ratio of force working in the given point of field on a trial charge to value of this charge: $\vec{E} = \frac{\vec{F}}{q}$. (1)

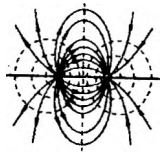


Fig. 2

Intensity is vector which direction coincides with direction of the force working in the given point of field on positive dot charge. Its dimension is [E]=N/C or V/m.

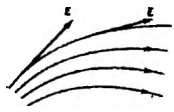


Fig. 3

Graphically electrostatic field is represented with help of lines of intensity (force lines): these are lines tangents to which in each point coincide with direction of vector $\vec{E}$ (fig. 3). On fig. 1 a) and 1 b) lines of intensity accordingly for the positive and negative charges are shown, and on fig. 2: lines of intensity for the system of the positive and negative charges.

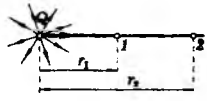


Fig. 4

Intensity of field of a dot charge in vacuum in scalar form is determined under the formula: $E = \frac{|q|}{4\pi r^2 \varepsilon_0}$ (2)

Power characteristic of electric field is the **potential (U or φ).** In practice often is used concept of **potential difference** between points 1 and 2, that is known as **electric voltage** (fig. 4): $U = \varphi_1 - \varphi_2$.

The potential difference of two points in the field is equal to the ratio of work of forces of field on moving of dot positive charge from one point of a field to another to value of this charge (fig. 4): $U = \varphi_1 - \varphi_2 = \frac{A}{q}$.

Potential of the given point of field is equal to work, which is made with forces of field at moving of the unit positive charge from the given point to infinity (or in the point where the potential of the field is accepted equal to zero).

The potential in any point of field in vacuum for the **dot charge** is determined under the formula: $$\varphi = \pm \frac{q}{4\pi\varepsilon_0 r}, \qquad (3)$$

where sign (+) concerns to the case of positive charge, and sign (-) to the case of negative charge.

Unit of potential is called **volt (V)**, $1V = \frac{1J}{1C}$.

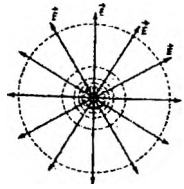


Fig. 5

The surface for which all points have the identical potential is called **equipotential** (dotted lines on fig. 5 and fig. 2). Force lines and equipotential surfaces are mutually perpendicular. Between potential and intensity in the given point of field there is the dependence:

$$E = -\frac{d\varphi}{dl}, \qquad (4)$$

where dφ is change of potential: $d\varphi = \varphi_2 - \varphi_1$, dl is small displacement from the given point along a line of intensity. Sign "minus" is caused by that intensity of a field is directed aside decrease of potential.

For homogeneous field change of potential on unit of length of line of intensity (along the line of intensity) is called **potential gradient** and is designated as **grad φ**. **Potential gradient is the vector directed aside the greatest increase of potential. Vectors of intensity and potential gradient are equals in each point of field on magnitude and are guided to opposite directions: E= - grad φ.**

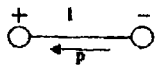


Fig. 6

2. Electric dipole. Dipole in electric field

Electric dipole is the system consisting from equal dot charges opposite on sign (fig. 6), which centers are on distance *l*. The main characteristic of a dipole is **dipole moment P=q·l,** it is equal to product of value of charge on distance *l* **(arm).** Unit of the moment of a dipole is Coulomb · meter: **P=q·l**. Dipole moment is the vector directed *from negative charge to positive*. Dipoles are polar molecules of proteins, if the centers of positive and negative charges are on some distance *l* (fig. 7). Dipoles are molecules of amino acids and also water (fig.8) and other substances included in tissues of organism.

In complex albuminous molecules each connection can have the dipole moment. Therefore the dipole moment $P_м$ of complex molecule will be equal to the sum of dipole the moments of separate connections:

$$P_м = \sum_{i=1}^{n} P_i$$

At placing of a dipole in the constant electric field of intensity E (fig. 9), on the dipole will operate pair of forces: +F=q·E and -F=-q·E, aspiring to establish dipole along the field. Phenomenon of orientation of dipoles in electric field has

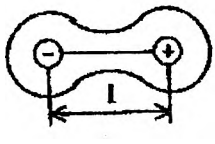


Fig. 7

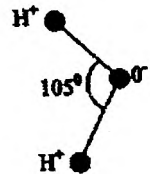


Fig. 8

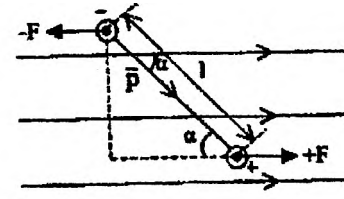


Fig. 9

received the name of **polarization**. Moment of pair forces will be equal (apparently from fig. 9) to M=q·E·1·sin α and in the vector form: $\overline{M} = \overline{P} \cdot \overline{E}$.

If dipoles to place in the variable electric field, they will turn about position of balance in step to change of field. On such turns of dipoles some energy or capacity of variable electric field will be spent, which will be allocated as *heat* inside the medium consisting of dipoles.

Dipole molecules of tissues of organism also make turns in alternating electric field with allocation some heat. This phenomenon has found application at physio-therapy by field of UHF.

3. Electric field of a dipole

A dipole represents system of two opposite charges and creates electric field in medium surrounding it. We shall find expression for intensity and potential of this field in the beginning on axis of a dipole and then in any point of space around of dipole:

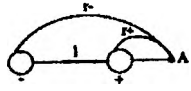


Fig.10

a) let it is necessary for us to find value of intensity in the point A on axis of the dipole (fig. 10). As l is small, $r_+ \gg l$, $r_- \gg l$, hence, $r_+ \approx r_- = r$. The field in the point A will be created by charges q_+ and q_-. Therefore

$$E_A = E_+ + E_- = k\left(\frac{q_+}{r_+^2} - \frac{q_-}{r_-^2}\right) = \frac{kq(r_-^2 - r_+^2)}{r_+^2 \cdot r_-^2} = \frac{kq(r_- + r_+)(r_- - r_+)}{r^4} = \frac{k2qrl}{r^4} = \frac{k2P}{r^3}, \quad (5)$$

where k is a constant depending from choice of sistem. Or $E_A = \frac{2P}{4\pi\varepsilon\varepsilon_0 r^3}$ in SI.

For value of potential of field we shall receive expression:

$$U_A = E_A \cdot r = \frac{2P}{4\pi\varepsilon\varepsilon_0 r^2}, \quad (6)$$

where $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m is electric constant.

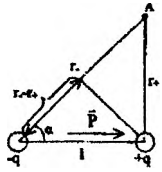


Fig. 11

b) Let it is necessary for us to find potential in the point A removed from charges accordingly on distance r_+ and r_- (fig. 11). In this case, as well as intensity, potential is created by both charges and it can be found under the formula:

$$U_A = kq\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = k\frac{q(r_- - r_+)}{r_+ \cdot r_-} = k\frac{q \cdot l \cdot \cos\alpha}{r^2} = k\frac{P \cdot \cos\alpha}{r^2},$$

$(r_+ >> l; r_- >> l; r_- - r_+ \approx l \cdot \cos\alpha)$, where k is constant depending on choice of system.

Or in SI: $$U_A = \frac{P \cdot \cos\alpha}{4\pi\varepsilon\varepsilon_0 r^2}. \quad (7)$$

Using the formula (7) it is possible to find the potential difference of two equidistant points A and B: where k is a constant depending from choice of system.

$$U_A - U_B = \frac{P}{4\pi\varepsilon\varepsilon_0 r^2}(\cos\alpha_A - \cos\alpha_B). \qquad (8)$$

So, under the formula (7) we can find potential in any point of the medium surrounding the dipole, and under the formula (8) we shall find the potential difference in two points. It is necessary to note that human heart also represents the dipole, creating in space surrounding it electric field and accordingly some potential difference.

4. Concept of dipole electric generator (current dipole)

The equivalent electric generator is the model physical system. Almost in all existing physical models electric activity of bodies and tissues reduce to the certain *set of current electric generators*, which are taking place in the volumetric conducting medium. The equivalent circuit of current generator in the conducting medium is submitted on fig. 12. In this circuit E is EMF of source; resistor R is equivalent resistance of the conducting medium. Resistance r is internal resistance of current generator. Resistance r many times over exceeds resistance R of the conducting medium (r>>R). Under the law of Ohm: $I = I_0 = \frac{E}{(R+r)}$, where I and I_0 are force of current in the generator and total current in the medium. But as r>>R and $I = I_0 = \frac{E}{r}$. From the last formula it is possible to draw the conclusion: force of current in the medium will remain constant since the current does not depend from R (resistance of medium).

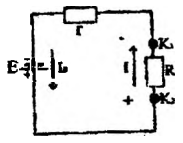


Fig. 12

In the circuit resulted on fig 11 TERMINALS K_1 and K_2 of source of voltage it is possible to present as dipole, i.e. **equivalent electric generator is current electric dipole** or the system consisting of positive pole (source of electric current or *outflow*) and the negative pole (*inflow*) located on small distance from each

other. The spatial structure of electric field created by such generator in the medium is defined by position of its poles.

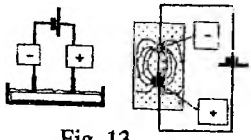


Fig. 13

The current dipole is not pure abstraction: it is possible to make its actual physical analog, for example, if to join two wires to cell (battery, generator) and to sink wires ends in to saline solution (fig. 13). If in a bath with solution to sink two electrodes to which to join the voltmeter, indications of the voltmeter will equal to the potential difference between points of the field where electrodes are dropped.

Main parameter of the current electric dipole is the **current dipole moment (or dipolar moment)**: $\bar{D} = \bar{I} \cdot l$, which is vector variable. In resulted formula I is the current in dipole (equal to the total current in the medium), l is distance between poles. Direction of vector of dipole moment is accepted from the negative pole to positive (coincides with direction of current inside a dipole).

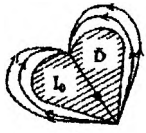


Fig. 14

Human heart is possible to assimilate to a current dipole (fig. 14). A current I_0 inside "dipole – heart" flows from the minus to plus and in the medium from plus to minus. Current I_0 is equal to the total current proceeding through the medium surrounding a dipole. *In the heart - dipole the negative pole will be area of muscle, where there is an excitation. This area is* ***sinoauricular*** *node* (fig. 14). A current dipole as the charging dipole creates in the conducting medium electric field. Lines of intensity of the electric field of charging dipole are identical with lines of intensity of the electric field of the current dipole (they coincide with lines of current).

Dipoles depending on their size are shared on two types: dot and finite. Dot is called a dipole which occupies infinitesimal volume of space with $l \to 0$.

Potential of the field of a finite current dipole in the point A can be expressed by the formula similar to the formula (7), if in last q (p=q·l) to replace on I and $\varepsilon\varepsilon_0$ on $\frac{1}{\rho}$:

$$U_A = \frac{D \cdot \rho}{4\pi r^2} \cdot \cos\alpha_A \quad , \qquad (9)$$

where α is the angle between direction of registration of potential and direction of the vector of dipole moment (D). Potential difference between two points A and B:

$$U_A = U_A - U_B = \frac{D \cdot \rho}{4\pi r^2} \cdot (\cos\alpha_A - \cos\alpha_B) \qquad (10)$$

From formulas (9) and (10) follows that, as well as for a charging dipole **the potential difference for the current dipole is directly proportional to the dipole moment D and inversely proportional to the square of distance r from negative pole of the dipole.** Such potential differences created by a heart - dipole are projected on surface of skin of the person, removed with the help of electrodes and registered with help of electrocardiograph.

5. Electrography. Physical bases of electrocardiography

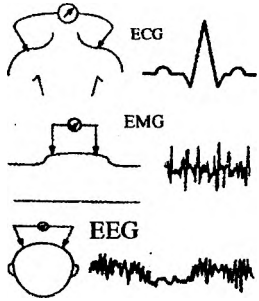


Fig. 15

Living cells of human tissues and plants are sources of electric potentials which have received the name of **bioelectric potentials** or **biopotentials.** Registration of biopotentials of tissues and bodies with the diagnostic (research) purpose has received the name of **electrography** (fig. 15).

Electrocardiography (electrocardiogram) ECG (EKG) is registration of the biopotentials arising in cardiac muscle at its excitation;

Electromyography (EMG, fig. 16) is method of registration of bioelectric activity of muscles. EMG is used for diagnostic of diseases or clearing up of physiological state of muscles. It allows to find out pathogeny (the mechanism of development) of disease, and also to inspect productivity of therapy. At EMG electrodes of small square are superimposed in the relevant points along nervous trunk or muscle. In norm the arbitrary traction is accompanied by

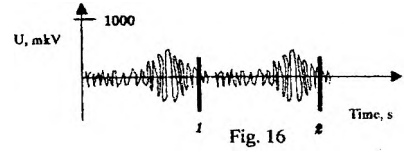


Fig. 16

the expressed myogram of frequency 100 Hz and more and amplitude up to 2000 mkV. On fig.16 is shown EMG of common extensor of fingers of dextral hand (slow rhythm of inflection - extension).

Electroencelography (EEG, fig. 17) is registration of bioelectric activity of brain. **Electroencephalogram** is the curve reflecting integrated activity of large number of pyramidal neurons of cortex of cerebrum and propagation of waves of excitation on neurons. The rest potential of pyramidal cells is from - 50 up to-80 mV, and amplitude of action potential is 60 – 100 mV at duration of 0.5-2 ms. For filing of oscillations of electrical potential in time electrodes are superimposed in various points of integument of head or immediately (during surgical operation) on particular departments of brain. Number of electrodes depending on the purpose is varied from 2 up to several tens. Electroencephalograms look like the composite oscillations with various frequencies (1-100 Hz) and amplitudes as in time dependence and from position of electrodes on surface of head. For examination of electrical activity of brain at various functional states consider prime sine wave oscillations on which under the theorem of Fourier is possible to analyze the composite oscillation (EMG). So, at wake person dominates α-rhythm: oscillations over frequency of 8-13 Hz. It is observed also β- rhythm with frequency of 14-35 Hz, γ-rhythm – 35-70 Hz, δ-rhythm (0.5-3 Hz), θ-rhythm (4-7 Hz), etc (fig. 18). On occurrence or vanishing of particular rhythm it is possible to judge character of functional state of a brain, to estimate pathologies. The basic rhythms are miss or less exhibited at epilepsy, tumors of cortex of cerebrum, etc.

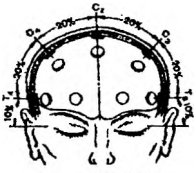


Fig.17

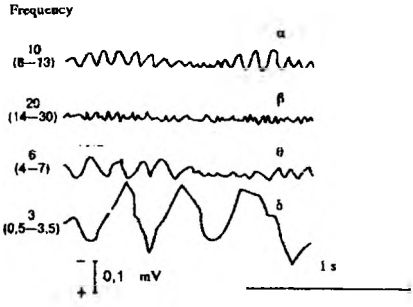


Fig. 18

Electroretinography (ERG) is registration of biopotentials of retina of an eye.

It is used as additional method of diagnostic at diseases of retina. After impulse illumination of retina filing of potentials is yielded through the electrode solder in contact lens which superimpose approximately. Then yield transient flashout of potent flashlight valve.

Comparison of various aspects of electrography on amplitude of signals in mV and on frequency band in Hz is shown in the table.

Parameters	ECG	EEG	EMG	ERG
Amplitude, mV	0.1-5.0	0.01-0.5	0.01-50	0.05-0.2
Range of frequencies, Hz	0.5-400	1-1000	1-10000	0.5-15

In most cases biopotentials are removed by electrodes not directly from body (heart, a brain) and from other "next" tissues in which electric fields are created by these bodies.

Let's stop in more detail on physical bases of ECG. The **physical approach** to electrocardiogram consists in creation (choice) of model of the electric generator which corresponds to the picture of removed potentials. It is experimentally established that each cell which it is possible to assimilate to a current dipole at excitation generates action potential. In the excited myocardium always there are many dipoles (we shall name these elementary). We shall note, that dipole potential of anyone elementary dipole is proportional to $D_i \cdot \cos\alpha_i$ (D_i is the module of vector $\overline{D_i}$), i.e. the projection of vector of dipole moment on straight line, connecting beginning of the dipole and the point of measurement of potential:

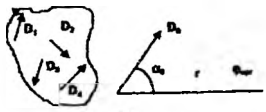


Fig. 19

$$Ui = \frac{\rho \cdot D_i \cdot \cos\alpha_i}{4\pi r^2}. \qquad (11)$$

Potential U of the electric field of heart develops from dipole potentials of elementary dipoles. As during each moment of cardiocycle rather small site of a myocardium is raised, distance from all dipoles up to the point of measurement of

potential are approximately equal to each other and U can be described by expression:

$$Ui = \frac{\rho}{4\pi r^2} \sum_{i=1}^{n} D_i \cdot \cos\alpha_i , \qquad (12)$$

where r is identical to all dipoles distance up to the point of measurement of potentials; n is number of dipoles. Proceeding from the specified representations, human heart is possible to consider as **the multipole generator (dipole)** giving total action potential.

In the formula (12) sum of projections can be considered as the projection of vector of dipole moment $\bar{D}_0$ of one current dipole for which $\bar{D}_0 = \sum_{i=1}^{n} \bar{D}_i$. This dipole is called **equivalent dipole of heart.** Thus, the potential of external electric field of heart can be presented as dupole potential of one equivalent dipole:

$$U = \frac{\rho D_0 \cos\alpha}{4\pi r^2}, \qquad (13)$$

where α is angle between $\bar{D}_0$ and direction of registration of potential; D_0 is the module of a vector $\bar{D}_0$. Model in which electric activity of myocardium is replaced by action of one dot equivalent current dipole and potentials of external field are described by expressions (12) and (13) is called **the dipole equivalent electric generator of heart or integral electric vector of heart** $\vec{P}_H = \vec{D}_0$.

Validity of the equation (13) proves to be true that potentials measured on a surface of body during the fixed moment of cardiocycle are approximately directly proportional to cosα and inversely proportional to r^2, i.e. **the physical approach to ECG consists in a choice (creation) of model of the electric generator, which corresponds to the picture of potentials removed from the surface of body.**

6. Theory of leads of Einthoven. Vectorelectrocardiography

It is experimentally established that heart - dipole during excitation generates action potentials, which on a surface of human skin give the lines of equal

potentials (equivalent lines). On fig 20 position of the dipole moment of current dipole of heart and equipotential lines is shown.

If through a heart - dipole to lead the straight line "ab" and through the center of arm of dipole perpendicular "00``",then on the part of positive pole of dipole – heart are settle down positive and on the part of negative - negative lines of identical potential.

The line "00``" have zero potential. From the drawing is visible that if to apply electrodes on various points of surface of body of the person it is possible to determine the potential difference ΔU of these points.

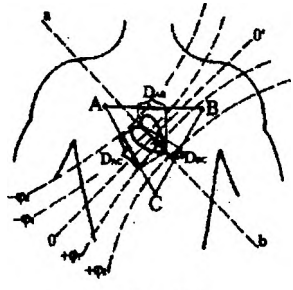


Fig. 20

For the first time theoretically proved points on a surface of human body from which it is possible to take potentials of heart have been offered by Einthoven. Dipole representation about heart underlies the theory of leads of Einthoven. Einhoven is the Dutch scientist-father of electrocardiography. **The main postulates of Einthoven`s theory are:**

1) **the electric field of the heart is represented as electric field of current dipole $\vec{P}_H$ which is called the integral electric vector of heart;**

2) **$\vec{P}_H$ is in homogeneous isotropic conducting medium that is tissues of organism;**

3) **$\vec{P}_H$ permanently changes its direction and value.**

Its beginning is in the auriculoventricular node and may be considered as its constant position, the end of the vector for one cycle of the heart work makes the compound spatial curve and its projections on the frontal plane forms three loops which are marked as P, QRS and T (fig. 22 and 23).

On the surface of thorax Einthoven has determined three points A, B, C (fig. 20), having connected their it is possible to receive equipotential triangle (triangle of Einthoven) in which center there is a dipole - heart, generating action potentials.

The potential difference between any tops of triangle will be directly proportional to the projection of the moment of current dipole on any of sides of triangle: $\Delta U_I \sim D_{AB}$, $\Delta U_{II} \sim D_{AC}$, $\Delta U_{III} \sim D_{BC}$ (or $U_I:U_{II}:U_{III}=P_I:P_{II}:P_{III}$ on fig. 21). At taking of electrocardiogram electrodes settle down in the points, which can be counted electrically equivalent to points A, B, C of Einthoven`s triangle. Einthoven has suggested to place electrodes not in tops A, B, C and on right arm (RA), left arm (LA) and left leg (LL) (fig. 21). On terminology of physologists **the difference of biopotentials registered between two points of a body is known as « lead».**

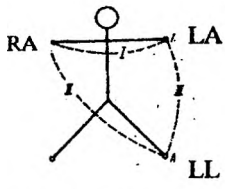
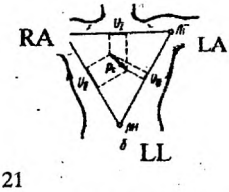


Fig. 21



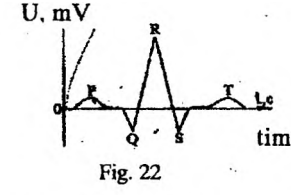


Fig. 22

Distinguish the first lead - (RA-LA); II-nd - (RA-LL) and III-d -(LA-LL), corresponding potential differences: ΔU_I; ΔU_{II}; ΔU_{III}. As the electric moment of current dipole - heart changes in time in leads will be received **time dependences of voltage (potential difference)** that is called **electrocardiogram** (fig. 22).

To maximal values of potentials in various time intervals of reduction of heart had been gave names P, Q, R, S, T (**waves of ECG**). On fig. 22 is shown the normal human electrocardiogram. At pathology the form of the waves, their size is changed, that will allow to use electrocardiogram for the purposes of diagnostics. Features of ECG (fig.22, fig.23):

the P wave occurs during the depolarization of the atria, which causes atrial contraction (fig. 26);

the QRS wave corresponds to the depolarization (and subsequent contraction) of the ventricles (fig. 26);

the T wave occurs during the ventricular repolarization which corresponds to the relaxation of the ventricles.

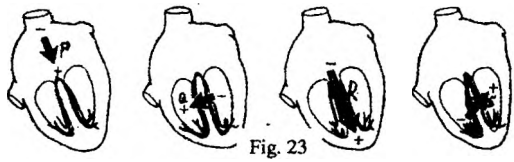


Fig. 23

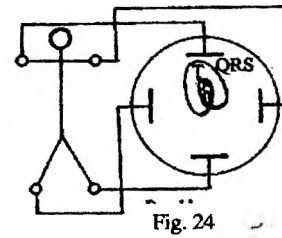


Fig. 24

The scheme of recording of the QRS wave of the ECG in three standard leads is shown in fig. 25. The sings (+) and (-) correspond to the sings on the axes of the ECG in the corresponding leads. Electrocardiogram does not give representation about spatial arrangement of vector D_0. However, for the diagnostic purposes such information is important.

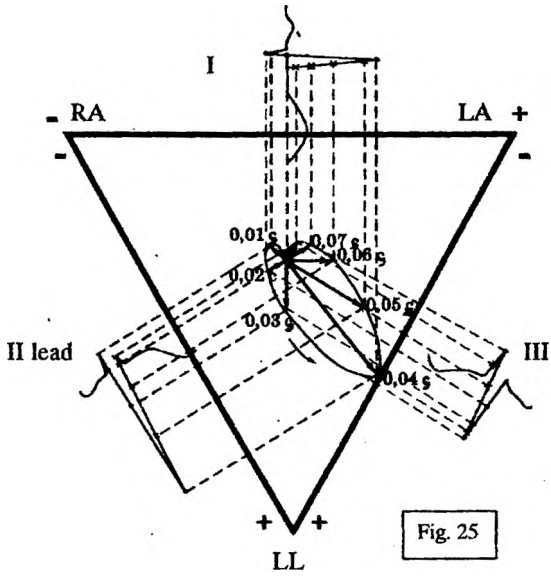


Fig. 25

In this connection the method of spatial research of electric field of heart named **vector - electrocardiography** is applied.

Vector – electrocardiogram (fig. 24) (flat) is geometrical place of the points corresponding to the end of the vector $\bar{D}_0$ (which position changes during an intimate cycle). On projection to any plane change of position of $\bar{D}_0$ is possible to write down with help of electronic beam. On the screen of the cathode ray oscilloscope of vectorcardioscope are observed separate loops: P, complex QRS and T, which give to the doctor more information at setting of diagnosis than waves on electrocardiogram.

The projection of vectorcardiogramm on the plane can be received by addition of voltages of two mutually perpendicular leads (fig. 25). At many illnesses of heart the form of flat VECG is sharply transformed and it is used in the diagnostic purposes. So, for example, in loop QRS in projection to the horizontal plane there is no bottom part at heart attack of some sites of heart.

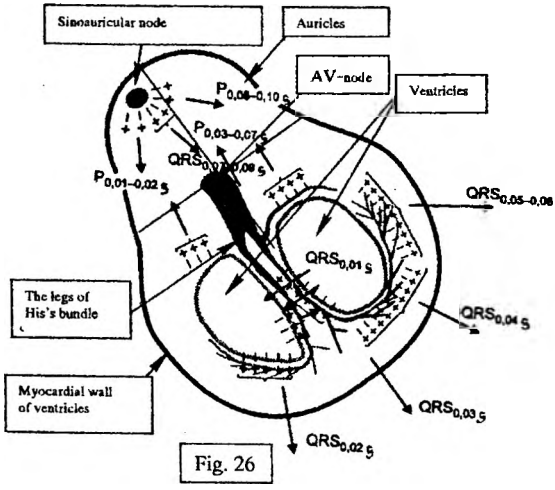


Fig. 26

The initial initiating pulse originates in sinus node (rhythmmaker cells) and propagates in the dextral auricle. After activation of auricles impulse propagates on conductive system of heart (atrial conducting system, ventricular conductive system: Boundle of His, boundle branches, Purcinje fibers) and attains myocardium of ventricles with spread of activation from endocardium to epicardium (fig. 26).

LECTURE №12

DIRECT CURRENT. ACTION OF A DIRECT CURRENT ON ORGANISM

1. Electroconductivity of biological tissues and liquids for direct current. Phenomenon of polarization

Passive electric properties are inherent to biological objects: *resistance* (R), *electroconductivity* $\left(K=\frac{1}{R}\right)$, *specific resistance* (ρ), *specific conductivity* $\left(\gamma=\frac{1}{\rho}\right)$, *capacity* (C), *dielectric permeability* (ε). Study of their passive electric properties is importance for understanding of structure and physical and chemical condition of biological substance.

Biological objects have properties of conductors and dielectrics. Presence of free ions in cells and tissues causes conductivity of these objects. Dielectric properties of biological objects and value of dielectric permeability is defined by their structural elements (membranes) and the phenomenon of polarization.

Biological objects are various formations with different electric resistances, which can change at action of electric current that causes difficulties of measurement of these resistances.

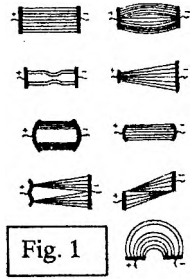


Fig. 1

Any part of human organism has certain conductivity, which is defined by presence of free carriers of charges, i.e. the certain quantity of positive and negative ions. Electroconductivity of separate sites of organism, on which electrodes are imposed, depends *from resistance of skin and hypodermics*. Resistance of skin is defined by its condition, thickness, age, humidity, impurity, etc. **Inside of organism a current is propagated basically on blood vessels, muscles, environments of nervous trunks and intercellular liquid and the path of the current essentially differs from theoretical distribution of force lines between electrodes** (fig. 1). It is experimentally established, that *conductivity of tissues and bodies depends from their functional condition and hence can be used as a diagnostic parameter*. So, at inflammatory processes cells swell, intercellular intervals will decrease, therefore electric resistance will increase and conductivity decreases. Physiological phenomena connected with allocation of sweat are accompanied by increase of electroconductivity.

Electroconductivity for direct current determine *by the bridge method* and also by the **method of ammeter and voltmeter**. We shall consider this method.

Let there is some conductor representing a living tissue by section S and length l. Then its resistance will be equal:

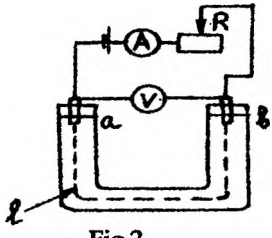


Fig.2

$$R = \rho\frac{l}{S}, \qquad (1)$$

where ρ is specific resistance of the conductor (substance) expressed in Ohm·m. From here:

$$\rho = \frac{RS}{l}. \qquad (2)$$

The value inversely proportional to specific resistance is known as *specific electroconductivity* $\gamma = \frac{1}{\rho}$. It is measured in $Ohm^{-1}\cdot m^{-1}$. From the formula (2) is visible, that if we know the area of electrodes S and distance between electrodes l, we shall find γ (R = U/I).

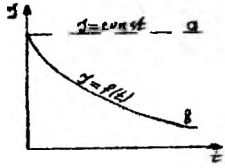


Fig.3

Value of R we can find by the method of voltmeter and ammeter for a direct current. For this purpose U-shaped tube (fig. 2) fills by blood or other biological liquid. Platinum electrodes *a,в*, which do not cooperate with a solution are placed in the tube. Specific electroconductivity γ determine under the formula:

$$\gamma = \frac{l}{S}\cdot\frac{I}{U}. \qquad (3)$$

Determination of specific electroconductivity is connected with the certain complexities. At passage of direct current through living tissues some features are observed: force of current does not remain the constant in time, though the voltage does not changes. Force of current after switching on of source starts to decrease continuously, and after a while is established at a constant level (fig. 3). Thus it decreases in many times in comparison with previous values. Reduction of a current in time is caused by the **phenomenon of polarization** in tissues (on fig. 4 some mechanisms of polarization of a dielectric are shown: bias of electronic cloud of atom and orientation polarization). At passage of direct current through biological system, in it arises increasing up to some limit EMF of polarization (E_p), directed opposite to the enclosed voltage that results to reduction of current. EMF of polarization is a function of time $E_p(t)$. Then the law of Ohm for biological object should be written down so:

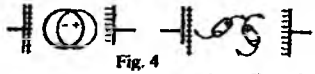


Fig. 4

$$I = \frac{U - E_p(t)}{R}.$$

Building blocks of a tissue are cells, which are flowed by intercellular lymph. Such block represents two mediums: electrolytes, conducting current (intercellular lymph and cytoplasm) are divided by a layer of dielectric (cytoplasmic membrane, possessing capacity properties). Under the action of external electric field, opposite charged ions concentrate at opposite sections of

interior surface of cellular membrane. In outcome, the polarization field is directed opposite to external field. Motion of ions will stop. From lateral side of membrane ions of opposite sign are drawn up. In outcome is formed a condenser, in which the lipide layer of membrane serves as dielectric (fig. 5).

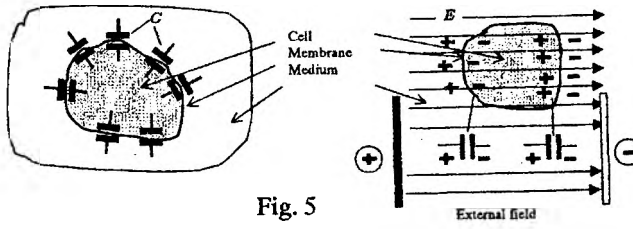


Fig. 5

Process of motion of charges under action of electric field and formation there of EMF directed against an external field is called polarization (fig. 5).

If on the direct current polarizing effects at electrodes are significant, measurements carry out on alternating current at which polarizing effects on frequencies more 500 кHz are small.

We shall result values of specific electroconductivity of some biological tissues and liquids. A spinal liquid: 1.81 $Ohm^{-1} \cdot m^{-1}$; muscles: 0.5 $Ohm^{-1} \cdot m^{-1}$; bone tissue: 10^{-7} $Ohm^{-1} \cdot м^{-1}$.

Now method of measurement of electroconductivity is widely applied in biological and medical researches. Convenience of application of the given method, that is used the voltage (less than 50 mV), not bringing essential changes to the physical and chemical processes occurring in biological object. The method has found application at studying of the processes occurring in cells and tissues at change of a physiological condition, at pathological conditions, at action of damaging factors: temperature, radioactive radiation, ultrasound, etc.

2. Mechanisms of action of direct current on an organism.

Action of a direct current on human organism depends from force of the current, therefore essential factor is electric resistance of tissues. As it has already been told, electric properties of various tissues are different. Good electroconductivity in relation to a direct current have liquid mediums of organism: spinal liquid, blood, plasma of blood, an intercellular liquid, etc. The big resistances have the bone tissue, skin. Specific resistance of a dry skin is approximately equal to 10^7 Ohm·m. The damp skin has smaller specific resistance $\approx$ 2000 Ohm·m, that even at small voltage can cause significant current through a body. Meeting the big resistance of skin, energy of a direct current in part turns to heat and it causes activization of blood circulation and amplification of biochemical processes. But the thermal effect is not the only thing.

The basic component of action of a direct current is its influence on parity of various ions in tissues. For normality of various tissues (as well as for their excitation) crucial importance has not the ion concentration, but their

quantitative relation, in particular, between ions of sodium and kalium on the one hand, and divalent ions of calcium and magnesium with another:

$$\frac{[Na]^+ + [K]^+}{[Ca]^+ + [Mg]^+}.$$

At increase of number of ions of kalium and sodium there is an excitation, at increase of ion concentration of magnesium and calcium there is a drop of intensity of zoetic processes.

The human body substantially will consist of the biological liquids containing a plenty of ions, which participate in various exchange processes. Under influence of the enclosed potential difference in electrolyte there is the counter moving of opposite charged ions.

Moving with different speed, ions accumulate at cellular membranes, at connecting tissue environments on their both sides, on border «soft tissues – skin». This phenomenon has received the name of *interstitial polarization* (fig. 6). In figure interstitial polarization is shown: on border «soft tissues - skin» and at cellular and other environments.

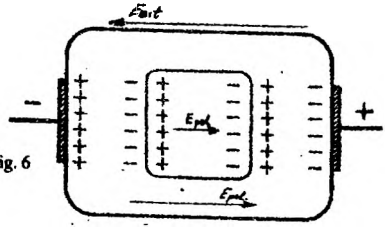


Fig. 6

The counter electric field named **polarizing** is formed and there is an interstitial polarizing current of inverce direction. On the one hand, it creates additional resistance to a working current, on the other hand, such sites inside tissues are places of the most active action of current.

Thus, initial action of a direct current is connected with motion of ions, change of usual concentration in various elements of tissues that can cause excitation or braking of activity of cells, change of acid-alkaline balance, the water-maintenance and other properties of tissues. It causes change of functional condition of the cell and reaction of all organisms to a direct current.

3. Galvanizing. Device of galvanizing

Application of electricity with the medical purpose began in extreme antiquity, when still people did not reflect on essence of the phenomena occuring at it.

Fig. 7

Scientific study of action on human organism of electric current began in end of XYIII century, after the discoveries made by Italian scientist L. Galvani and A. Volta on the basis of which have been received new sources of current.

However, only in XX century development of physics, electronics, physiology promoted scientifically proved perfection existing and development of new effective methods of electrotherapy. Many of them have been created in the Soviet Union. One of methods of electrotherapy

is galvanizing.

The medical method at which is used action on a human tissue of direct current up to 50 mA, density up to 0.1 mA/cm^2, voltage 60 – 80 V is known ***as galvanizing*** (on fig. 7 is shown general galvanization on Vermel).

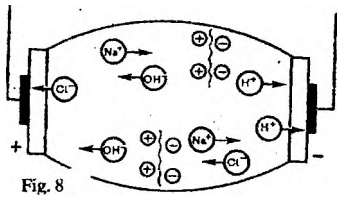


Fig. 8

A current from a source is brought to a body with the help of wires and metal electrodes (fig. 8). Electrodes make usually of sheet of lead by thickness of 0.3 – 0.5 mm depending on the values of electrodes. Galvanizing is carried out with the help of liquid electrodes also, as the little baths filled with water. In them place corresponding hand or foot of the patient. As tissues of organism contain electrolytes and hence the opposite charged ions, for example: $NaCl \leftrightarrows Na^+ + Cl^-$, that in the place of contact of the electrode with a body occurs electrolysis: neutral atoms, for example, sodium and chlorine are allocated. At the anode, incorporating with water, chlorine forms the acid, and at the cathode sodium, incorporating to water, forms alkali which causes burns or irritation. Therefore imposing of metal electrodes directly on a skin is not allowable.

For prevention of it between the skin and electrodes place moistened in the physiological solution or in water cloth padding (fig. 8).

Direct current for galvanizing receives by transformation of alternating current of city network. For this purpose is used the lamp or semi-conductor rectifier with the electric filter. *The device for galvanizing is the two-period rectifier.*

The basic circuit of the device for galvanizing is submitted on fig. 9.

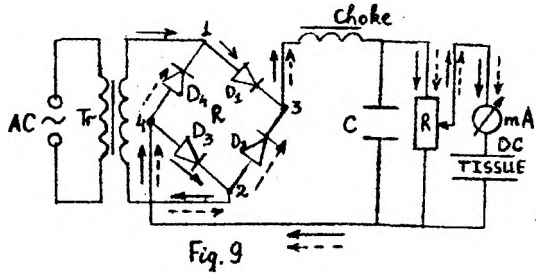


Fig. 9

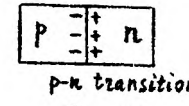


Fig. 10

The device contains transformer **Tr**. It is intended for downturn of a voltage and safety of the patient.

The rectifier **R** will consist of 4 semi-conductor diodes **D** connected under the bridge circuit. One diagonal of the bridge in the points 1 and 2 is connected with the secondary winding of the transformer, the second diagonal in the points 3 and 4 is connected to resistor **R**.

Work of the rectifier is based on property of electronic-hole transition of the semi-conductor diode (fig. 10). At contact of two semiconductors with electronic

(n) and hole (p) conductivity arise the potential barrier (p-n-transition), which interferes with transition between semiconductors of the basic carriers of charge.

For formation of a current in the circuit with p-n transition it is necessary to apply the external voltage: on the part of the p-semiconductor must be (+) and on the part of the n-semiconductor must be (-). At change of polarity the current will not be. If to p-n transition to apply alternating voltage the current in the circuit will pass only in one direction from "p" to "n" during the one half-cycle, the following half-cycle of current will not be (fig. 11,b). This property of p-n – transition (unilateral conductivity) is used for rectification of alternating current. At switch on of initial winding of the transformer to the network, in secondary there is a altarnating voltage and potentials of points 1 and 2 alternately become positive and negative.

When the potential of the point 1 is positive and in the circuit there is no filter the current passes through the diode D_1, the resistor R, the diode D_3 to the point 2 (continuous lines). When the potential of the point 2 is positive the current goes through the diode D_2, resistor R, diode D_4 to the point 1 (shaped line).

These processes will repeat in the step of change of a voltage, but through resistor R the current will always proceed in one direction. It is possible to present all processes by graphs of current or voltage (fig. 6): a – current in the secondary winding, b – if instead of 4 diodes one diode would be switched on, c – current after the bridge of 4 diodes, d,e – current, smoothed only by choke or only by the condenser, f - combined effect of all devices of the scheme – a direct current.

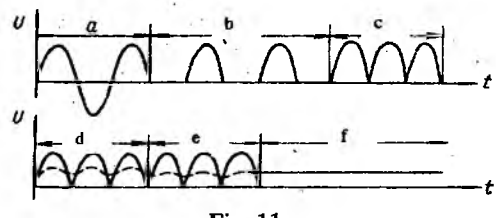


Fig. 11

If there was no filter, through the resistor the pulsing current (constant on direction, but variable on value) would proceed, such current is not applied to galvanizing since makes strong irritating action. The filter consisting of the choke, connected in series with resistor, and one or two condensers C is applied to smoothing a pulsation. The choke represents the coil of inductance with the iron core. In it at pulsation of current arise EMF of self-induction, interfering changes of the current. At the moment of increase of current, EMF of self-induction is opposite to direction of a current and limits its increase. At the moment of reduction of pulsing current, EMF of self-induction coincides with direction of current and, hence, supports it. As a result of work of choke pulsation of current will be a little smoothed.

Condensers, being gradually charged during increase of a pulse and being gradually unloaded at its reduction also promote smoothing of pulsations of current. As result of joint action of the choke and the condenser through the resistor R the current not varying almost on value, i.e. constant will proceed.

According to the Ohm`s law this current creates the constant voltage which is used with the consumer connected to plugs 5 on the resistor.

4. Medicinal electrophoresis

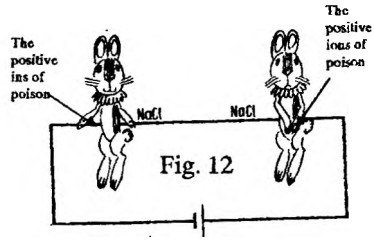


Fig. 12

Direct current is used in medical practice for introduction of medicinal substances through skin or mucous membranes. This method has received the name of **electrophoresis of medicinal substances**, i.e. galvanizing is combined with introduction into a tissue of medicinal substances.

Introduction of medicinal substances shows the following imagined experience. To two rabbits shave sites of skin on both sides and to these sites attach the napkins moistened with a solution of strychnine and solution of NaCl (fig. 12). On napkins impose electrodes and pass the current of 50 mA. After some time the rabbit at which strychnine on the anode perish at the typical phenomena of poisoning with this substance. The second rabbit, for which strychnine on the cathode does not perish, but if to change the direction of current he will die also.

Human skin in usual conditions has very small permeability for ions. It is caused by that pores of skin are filled with air. Large organic ions cannot penetrate through skin at all. As walls of skin pores have an electric charge, that at imposing of external electric field there is electro osmotic motion of liquid from within tissues or outside, air thus is superseded from pores, it replaced with a liquid, permeability of skin considerably increases. Quantity of entered medicinal substance at electrophoresis will depend on quantity of electricity past through the electrodes and from concentration of entered substance.

Direct current for electrophoresis receives from the device of galvanizing. For carrying out of electrophoresis the cloth padding placed under electrodes moisten with a solution of medicinal substance. *From a padding under the positive electrode enter into a tissue of an organism positive ions of metals (from solutions of their salts), vitamins B_1, B_{12}, E, K, Mn, Mg; under the negative electrode are entered acid radicals, negative ions, ions of some organic connections, for example, penicillin, cocaine, bromine, iodine etc.*

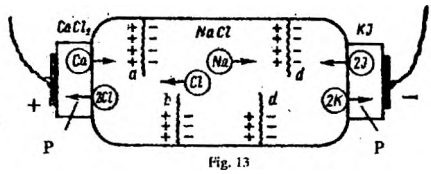


Fig. 13

On fig. 13 some object including conductivity tissues of an organism containing solution of NaCl is shown, the paddings (napkins) moistened with solution of $CaCl_2$ and KJ and electrodes. Motion of ions (arrows) and accumulation of ions on tissue partitions (polarizing phenomena) is shown.

At negative electrode will have neutralization of K ions, then reaction with water and formation of H_2 and KOH and also transition of iodine from the padding into a tissue and motion to the positive electrode. At positive electrode are formed Cl_2, HCl and ions of calcium (Ca) will leave into tissue.

Time of carrying out of procedure depends on speed of ions. Speed of ions is established under action of intensity of electric field E and force of resistance of environment, which will be increase with growth of speed. When the force working on the part of electric field will be equal to force of resistance, the ion will move with constant speed ϑ. This dependence is expressed by the formula $\vartheta=uE$, where the coefficient of proportionality u is known as *mobility of ions*. It has dimension $[\frac{m^2}{V \cdot s}]$. Mobility of an ion depends from resistance of medium to movement in it of ion (from viscosity, from temperature of medium, etc.) and from an ion (from the form of ion, its charge). This implies that medical electrophoresis proceeds variously at different patients and also at use of various medicinal solutions.

Medicinal electrophoresis is the joint action of constant electric current and medicinal substance. Electric current resulting tissues including receptors to the condition of hyperactivity raising them makes more sensitive to action of medicines.

LECTURE №13

ALTERNATING CURRENT. NATURE OF CAPACITIVE PROPERTIES OF TISSUES OF A HUMAN ORGANISM

1. Reception of alternating current. Its basic characteristics

Alternating current (AC) is a current periodically changing on value and direction. The most widespread is the sine wave alternating current which instant value varies in time under the law of sine (or cos). Let's consider physical bases of reception of an alternating current and the principle of device of a generator. Let in the field of a constant magnet (B=const) rotates the conducting framework with constant angular speed ω (fig.1). Then instant value of the stream connected with the contour will be equal to:

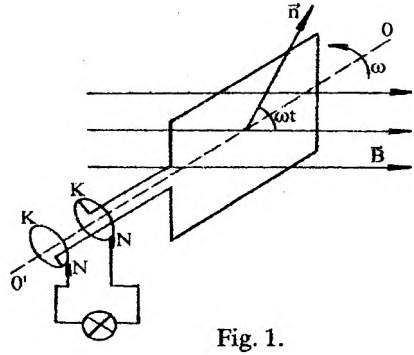


Fig. 1.

$$\Phi = BS\cos\varphi = BS\cos\omega t,$$

where S is the area limited by the contour; B is an induction of the magnetic field; $\varphi = \omega t$ is the angle of rotation of the contour, counted from its initial position, when $S \perp B$. According to the law of Faraday in the framework arises EMF of induction:

$$E = -\frac{d\Phi}{dt} = BS\omega\sin\omega t,$$

where $BS\omega = E_m$ is maximal (peak) value of EMF of induction, i.e. $E = E_m \sin\omega t$. Hence, if in the uniform magnetic field rotates the conducting contour, in it there is variable EMF changing under the law of sine in regular intervals. This EMF creates a sine wave alternating current in the contour:

$$I = \frac{E}{R} = \frac{E_m}{R}\sin\omega t = I_m \sin\omega t,$$

where R is resistance of the contour and of the electric circuit in which the electric current (by means of brushes N sliding on rings K) is allocated; I_m is peak value of an alternating current; ω is circular frequency; $\varphi = \omega t$ is a phase of a current.

The alternating current also is characterized by period T and frequency ν and $\omega = \frac{2\pi}{T} = 2\pi\nu$. Graphically value EMF and an alternating current will be represented by two sinusoids (values change in identical phases). The considered way of reception of alternating current underlies at the heart of device of the industrial generator of alternating current. In industrial generators the magnetic field is created by a powerful electromagnet. The rotating contour consists from n

coils (connected in series)of the wire which has been reeled up on the ferromagnetic core (rotor of the generator). Therefore EMF, excited in such generator will be equal: $E = BSn\omega \sin \omega t$.

For the characteristic of an alternating current the concept of **working (effective)** or root-mean-square value of **current** is entered. **The effective (virtual) value of AC current is defined as that steady (constant) current which would develop the same quantity of heat in the same time in the same resistance as is done by the DC:**

$$I_{ef.} = I_{w.} = \frac{I_m}{\sqrt{2}}, \quad U_{ef.} = U_{w.} = \frac{U_m}{\sqrt{2}}.$$

The devices included in a circuit of alternating current (ammeter, voltmeter) show effective values of a current and a voltage. When it is stated that the AC potential difference between the supply mains is 220 Volts, this means the effective potential difference is the 220 Volts and the peak value $U_0 = \sqrt{2} \times 220 = 311$ Volts.

2. Various kinds of electric resistance in a circuit of alternating current

a) *Active resistance* in a circuit of alternating current.

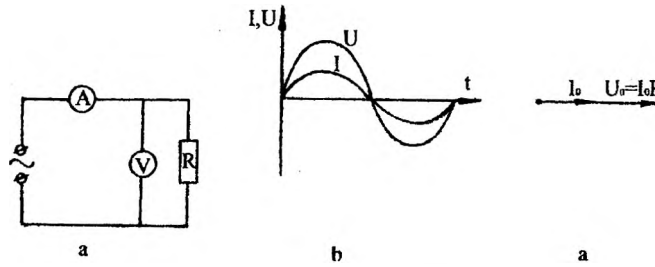


Fig. 2

Resistance R in a circuit of alternating current (fig. 2a) is called the active since at passage of a current in it there is an irreversible loss of energy. At presence in a circuit only R a voltage is $U = U_0 \sin \omega t$ and $I = \frac{U}{R} = \frac{U_0}{R} \sin \omega t = I_0 \sin \omega t$, i.e. the current and the voltage coincide on a phase. The graph of a current and a voltage, and also the vector diagram of amplitudes of a current and a voltage are shown on fig. 2 b, c.

b) *Inductive resistance* (*inductance*) in a circuit of alternating current.

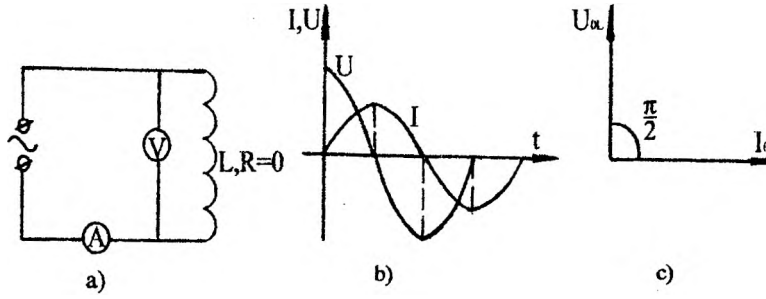


Fig. 3

Let's consider a case, when the circuit contains only the coil of inductance with the small resistance R (R=0) and significant inductance L (fig. 3a). Let in the circuit there is alternating current: $I = I_0 \sin \omega t$. It causes in the coil EMF of self-induction E_L, which at any moment is opposite to enclosed voltage U_L and counterbalances it: $U_L = -E_L$, but $E_L = -L\frac{dI}{dt}$, then:

$$U_L = I_0 \omega L \cos \omega t = U_{0L} \sin\left(\omega t + 90^0\right),$$

where $U_{0L} = I_0 \omega L$ is peak value of the voltage. From this formula follows, that $I_0 = \frac{U_{0L}}{\omega L} = \frac{U_{0L}}{R_L}$,

where $R_L = \omega L$ is inductive resistance of the coil. At only induced resistance in a circuit calorification is not presence, as R=0. The role of inductance is reduced to accumulation of magnetic field energy and returning of this energy back to a cell. There is a periodic energy transfer from a current source to a circuit and from the circuit to the current source, in ideal case loss-free of energy.

R_L increases with growth of frequency of alternating current. Dimension of inductive resistance is Ohm.

For a circuit with inductance in which *the voltage outstrips a current on* 90^0, wave and vector diagrams are submitted on fig. 3 b, c.

c) *A capacitance* in a circuit of alternating current.

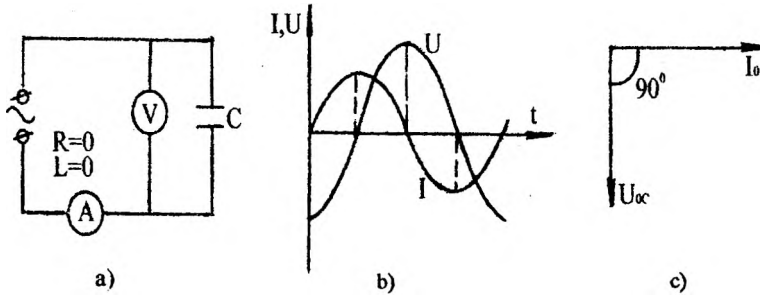


Fig. 4

Let's consider a case, when the condenser C is included in a circuit (fig. 4a) (by resistance and inductance of bringing wires is possible to neglect). Let the current in the circuit changes under the law: $I = I_0 \sin\omega t$. The voltage on plates of the condenser can be presented by the following formula: $U_c = \frac{q}{C}$. Current in the circuit: $I = \frac{dq}{dt}$, $dq = Idt$, $q = \int I_0 \sin\omega t dt = -\frac{I_0}{\omega}\cos\omega t$, then

$$U_C = \frac{q}{C} = -\frac{I_0}{\omega C}\cos\omega t = \frac{I_0}{\omega C}\sin(\omega t - 90^0) = U_{0C}\sin(\omega t - 90^0),$$

where $U_{0C} = \frac{I_0}{\omega C}$ is the peak value of the voltage enclosed to the condenser. Peak value of the current is $I_0 = \frac{U_{0C}}{\frac{1}{\omega C}} = \frac{U_{0C}}{R_C}$, where $R_C = \frac{1}{\omega C}$ is capacitance. It decreases with growth of frequency. R_C has dimension of Ohm.

In a circuit with only capacitance the voltage enclosed to plates of the condenser lags behind on phase of current on 90^0. It is reflected on the wave and vector diagrams on fig. 4 c, b. In the circuit with the condenser calorification is not presence, as ohmic resistance of conductors equal to null (warming of a dielectric in the alternating electric field is not taken into account, it will be surveyed later). The role of capacity is reduced to accumulation of electrical field energy of the condenser and returning of this energy back to a current source. There is a periodic energy transfer from a current source to the circuit and from the circuit to the current source, in ideal case loss-free of energy.

3. Total resistance (impedance) in a circuit of alternating current Resonance of a voltage

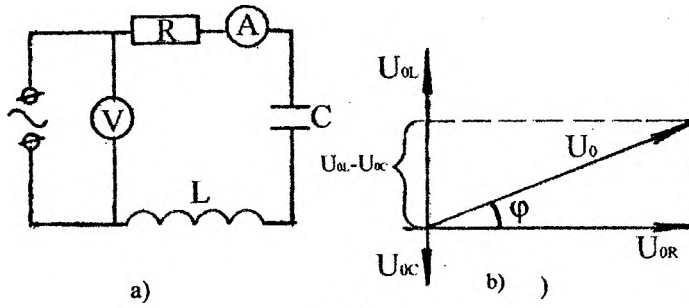


Fig. 5

Let's consider a circuit consisting from R, L and C joined in series (fig. 5a). The current is equal $I = I_0 \sin \omega t$. We shall define how the voltage will change. The sum of falling of voltages on R, L and C is equal to the enclosed voltage:

$$U_0 = U_{0R} + U_{0L} + U_{0C} = I_0 R + I_0 R_L + I_0 R_C = I_0 R + I_0 \omega L + I_0 \frac{1}{\omega C}.$$

Owing to presence of a phase difference between U_L, U_C and the current I (U_R is in identical phase with the current) these voltages can be put vectorially and under theorem of Piphagor enclosed voltage U_0 (fig. 5 b) is equal:

$$U_0 = I_0 \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = I_0 Z,$$

where $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ is known as total resistance, or *impedance of a circuit*. The law of Ohm for the given circuit will be written down so:

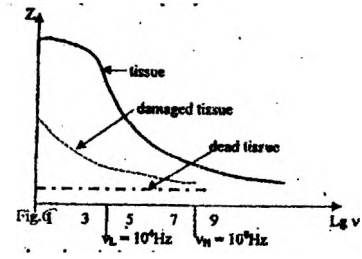
$$I_0 = \frac{U_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

The difference of phases between the current I and the voltage U is defined by the angle φ between vectors U_0 and U_R. Then: $U = U_0 \sin(\omega t + \varphi)$. From diagram on fig. 5b follows that $\operatorname{tg}\varphi = \frac{I_0 \omega L - I_0 \frac{1}{\omega C}}{I_0 R} = \frac{\omega L - \frac{1}{\omega C}}{R}$.

From the formula for Z follows that than closer on value ωL and $\frac{1}{\omega C}$, the less impedance Z and the is more current in this circuit. At $R_L=R_C$ or $\omega L=\frac{1}{\omega C}$ total resistance is Z=R and current achieves of the greatest value caused only by active resistance of a circuit: $I_{0RES}=\frac{U_0}{R_0}$. This phenomenon is known as *electric resonance,* which is provided by selection corresponding L and C. Resonance in the series circuit is called *resonance of voltage,* as thus occurs mutual indemnification of voltage U_L and U_C (they are directed opposite), each of U_L and U_C can significantly exceed voltage U enclosed to the circuit.

4. Total resistance (impedance) of tissues of human organism. Using of the method of electroconductivity in medicine

At work with biological objects found that on high frequencies (10^7 Hz) electroconductivity is much higher than for low frequencies. At increase of frequency, electrical conductivity increases up to some maximal value. On fig. 6 is resulted a curve of dependence of resistance of a muscle from frequency (curve of dispersion).



The zone of dispersion of electroconductivity usually varies in the interval 10^2 – 10^8 Hz. The dispersion of electroconductivity of living tissues on low frequencies is connected with polarization, but with increase of frequency the polarizing phenomena will decrease.

For a damaged tissue the steepness of dispersion decreases and for a dead tissue the graph is represented by a line parallel to axis X (fig. 6). Steepness of dispersion K express by the ratio of value of the resistance measured on low frequency to the value of the resistance measured on high frequency. If two resistances are measured on different frequency under the same conditions, the ratio between them appears to constants for a normal condition of the given tissue. Usually choose for measurement of frequencies 10^4 Hz and 10^6 Hz, as at frequency of 10^6 Hz in many cases is observed maximal electroconductivity and for frequency of 10^4 Hz is observed change of a curve dispersion: $K=\frac{Z_{10^4}}{Z_{10^6}}$. For a tissue the specified coefficient K aspires to 1.

Explaining passage of an alternating electric current through biological objects recognize that **resistance of living cells consists from *ohmic and capacitor* resistance.** Inductive elements in biological objects are absent.

Tissues of human organism will consist of the cells washed by a tissue liquid. Such element represents two mediums with good conductivity of current (a tissue liquid and cytoplasm of a cell) divided by badly conducting layer of a cellular membrane. Such system has electric capacity. In tissues there are the macroscopical formations consisting of various connecting mediums and partitions (badly conduct a current), on which both parties there are tissues well conducting electric current. It gives to tissues capacitor properties also.

Ohmic resistance does not depend from frequency, and capacitor resistance considerably decreases with increase of frequency and it results to increase of conductivity of all capasitance-resistance system. For connection in series of R and C total resistance is defined under the formula: $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$.

Presence at biological systems of capacities proves by the presence of *shift of phases* between a current and a voltage. The angle of shift of phases is defined by a ratio between capacitor and ohmic resistance, and for their series connection is equal: $tg\varphi = \frac{1/\omega C}{R}$. For biological systems the big value of this angle is characteristic. It shows that the share of a capacitance in tissues is great.

Let's result examples of value of angles of shift of the phases received on frequency of 10^3 Hz:

Human skin: 55^0;
muscle of rabbit: 65^0;
nerve of frog: 64^0.

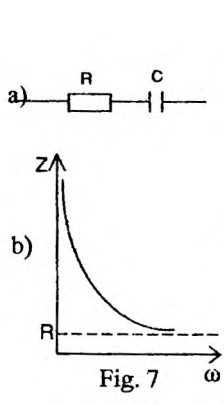


Fig. 7

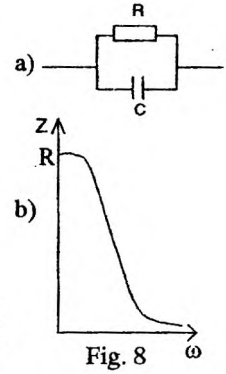


Fig. 8

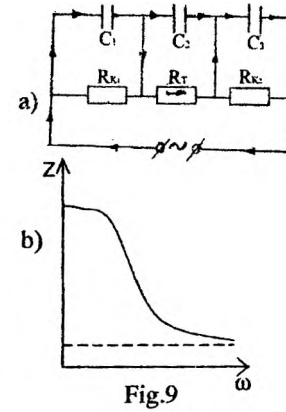


Fig.9

Taking into account that the total value of resistance (impedance) of living objects is submitted only by the geometrical sum of ohmic and capacitor resistance, for characteristic of conductivity of a current by living tissue use the *equivalent circuits*, i.e. to such combinations of ohmic resistance and capacity, which in some approximation can model electric parameters of tissues. Elementary

of them are circuits with connection of R and C in series (fig. 7a) and with parallel connection of these elements (fig. 8a).

But these elementary circuits cannot be completely applicable for living cells. As follows from the graph of dependence of Z from ω for the first circuit (fig. 7b): if $\omega \to 0$, then resistance $Z \to \infty$, that contradicts to experience.

From the graph of dependence of Z from ω for the second circuit (fig. 8 b) is visible, that at $\omega \to \infty$ $Z \to 0$, that on experience does not prove to be true.

The circuit combining first two circuits is most successful. One of them is represented on fig. 9a. On this circuit R_{K_1} и R_{K_2} are resistance of a skin; R_T is resistance of a tissue; C_1, C_2 and C_3 are capacity shunting these resistance. Arrows show the direction of alternating current in one of half-cycles. Resistances R_{K_1} and R_{K_2} are very great and alternating current through them does not pass. On fig. 9b the graph of dependence of Z from ω for this circuit is given that corresponds to the skilled data. There are other complex equivalent circuits, however any of them in accuracy cannot reproduce the laws inherent to complex biological systems.

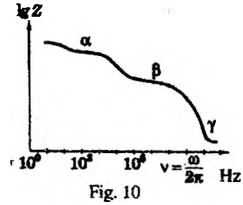


Fig. 10

On fig. 10 is presented the graph of the frequency dependence of impedance of a muscular tissue. For compactness the curve builds in logarithmic coordinates. From the graph two singularities of this association are visible: 1) the smoothly varying decrease of impedance with increase of frequency; 2) presence of three areas of frequencies in which the deviation from a common course of dependence occurs: Z is not varies. They have been called accordingly as fields of α-, β - and γ - dispersions of impedance. Presence of these fields of impedance speaks, that with increase of frequency of alternating electric field in appearance of polarization participate different structures of biological tissues: at low frequencies all structures react to change of a field (α - dispersion), with increase of frequency large molecules - dipoles of organic junctions and molecules of water (β - dispersion) react, and at the most major frequencies molecules of water (γ -dispersion) react only. With increase of frequency of an electric field of ever less structures will react to a change of this field, the capacity of tissues decreases that leads to increase of impedance Z. Hence, at a common trend to decreasing of Z there are fields with smaller decreasing of Z.

The method of electroconductivity on an alternating current in living tissues and cells is used in biological researches and medicine for estimation of pathological processes. For example, at measurements in the field of low frequencies is observed increase of resistance of a tissue at an inflammation at the first stages. Current of low frequency goes mainly through intercellular spaces. At inflammation as result of swelling of cells the section of intercellular intervals decreases, that attracts increase of ohmic resistance, while the capacity of cells at early stages of inflammation remains constant.

In diagnostics it is used as the method of measurement of the angle of shift of phases. At some diseases (thyroid gland) or at physiological changes (ageing of an organism) appreciable change of the angle of shift of phases is found out.

For characteristic of physiological condition of tissue is used value of a steepness of dispersion (K). This criterion applies for example at estimation of viability of the tissues intended for transplantation.

From physiological condition depends also impedance of tissues. So, at filling vessels by blood impedance of tissues is changed. Blood has smaller resistance than walls of vessels or cells and at filling of tissues by blood during a systole total resistance of tissue decreases and at diastole increases. **The diagnostic method based on registration of an impedance of tissues during cardiovascular activity is called rheography**. With help of this method receive *rheograms* of a brain (rheoencephalography), heart (rheocardiography), lungs, liver, vessels and finitenesses. Measurements usually carry out on frequency of 30 kHz at current no more 10 mA.

Rheography allows to give notion of arterial blood filling, states of tone of arterial vessels, venous reflux, microcirculation to spot magnitudes of shock and minute volume of circulation.

The complete impedance will consist of stationary value and variable component. The direct component is stipulated by common blood filling, it is a pedestal, or base impedance; the variable component or the pulse impedance is caused by oscillations of blood filling during a cardiac cycle. This magnitude is very small and makes of 0.5-1 % from the complete impedance, but for the sake of its study and it is offered reography. Hence, the *rheogram is a curve of pulse oscillations of the variable part of impedance, reflecting volumetric modifications of blood supply of organs at transiting on them of pulse wave.*

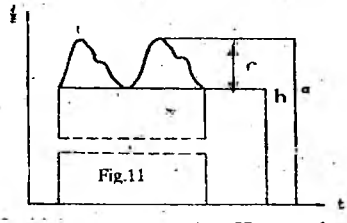


Fig.11

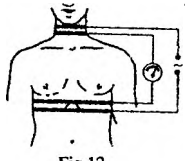


Fig.12

At conducting of rheocardiography the active electrode put above the apex of heart, and passive is plased in the Botkin point, that allows to find a modification of blood filling of chambers of heart. At tetrapolar rheography (the stroke output of heart, minute volume of a blood-groove is determined, etc.) are used two pairs of electrodes (fig. 12): I is the generating circuit (superposition of current electrodes), U is the metering circuit (measured voltage is determined only by a change of impedance Z).


Fig.13

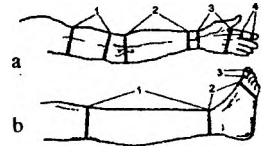


Fig.14

At rheoencephalography electrodes are fastened on the head that reflects processes in frontal and occipital departments of a blood supply of the brain (fig. 13).

Rheovasogram file from various sections of upper (shoulder 1, forearm 2, hand 3, dactyls 4, fig. 14) and lower (hips, shanks 1, autopodiums 2, fingers 3, fig. 14, b) extremities for determination of intensity of peripheric circulation, etc.

LECTURE №14

HIGH-FREQUENCY ELECTROTHERAPY AND ELECTROSURGERY

1. Influence of radiowaves on biological structures

Every possible electromagnetic (EM) waves are possible to present as a uniform scale. All scale is conditionally subdivided on **6 ranges**: radiowaves, infra-red radiation, visible light, ultra-violet radiation, X-ray radiation and γ-radiation (fig. 1). The most long-wave site of a scale is radiowaves. Their length of a wave makes 10^3-10^{-3} m and frequency $3 \cdot 10^5$-$3 \cdot 10^{11}$ Hz. It is necessary to mean, that these borders are accepted conditionally. Radiowaves share on long, middle, short, UHF and the microwave ranges.

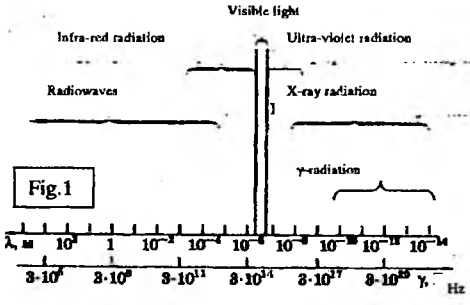


Fig.1

Interest of physicians to EM waves of high frequency (HF: 200 kHz - 30 of MHz), ultrahigh frequency (UHF: 30-300 MHz) and microwave frequency (MWF – over 300 MHz – *such division is accepted in medicine-see the table*) began to be shown in connection with development of broadcasting on these frequencies, when their influence on the attendants has been noticed: *rise of temperature, ache in joints, hyperhidrosis, drowsiness etc.*

Low (LF)	Up to 20 Hz
Sound (SF)	20 Hz - 20 кHz
Ultrasonic (USF)	20 kHz-200 kHz
High (HF)	200 kHz-300 MHz
Ultrahigh (UHF)	30 MHz-300 MHz
Microwave (MWF)	300 MHz-300 GHz
Extremely-high (EHF)	From above 300 GHz

Artificial sources of radiowaves are broadcasting and television stations, radars and satellite systems of communication. For creation of EM waves there are special generators, which basic part is the oscillatory contour consisting of the condenser of capacity C and the coil of inductance with inductance L. Frequency ν of oscillatory contour is defined by Thomson's formula $\nu = \frac{1}{2\pi\sqrt{LC}}$ and depends only from C and from L.

On frequency about 10^{10} Hz these stations can give capacity up to 30-10^9 Wt in a pulse. Intensity of radiowaves of *0.1 Wt/m²* for a human organism is

considered as *safe*. However, in zones where intensity reaches of $100\ Wt/m^2$ stay of a person is *forbidden* by standards established by the World organization of public health services (WHO).

At passage of EM waves through a layer of substance by thickness of x intensity of a wave I decreases according to the law of Buger $I=I_0e^{-\mu x}$, where μ *is propagation coefficient*. The value μ defines by nature of substance and lengths of a wave.

This wave attenuation is caused by that the part of energy of radiowaves at their interaction with biological structures turns to heat. Allocation of heat occurs due to generation of alternating currents of conductivity in electrolits (blood, lymph, cytoplasm of cells) and due to turns of dipole molecules of dielectrics of tissues, i.e. due to polarization.

Feature of propagation of radiowaves in living organisms is strong dependence of electric properties (dielectric permeability ε and specific conductivity γ) from frequency. At action on a tissue of UHF and MWF waves is marked their fast attenuation and fast transformation of their energy into heat.

2. Heating of conductors by high-frequency current. Diathermy. Electrosurgery. Darsonvalization

At heating by a high-frequency current a biological tissue with specific resistance ρ settles down between two electrodes with area S, which are directly imposed on tissue (fig. 2). Distance between electrodes is *l*.

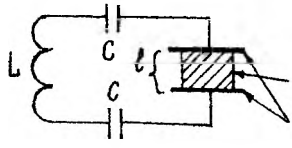


Fig.2

According to the law of Joule - Lents, quantity of heat Q, which is allocated at passage through a conductor of resistance R at current of I in time t will be equal:

$$Q = I^2Rt = I^2\frac{\rho l}{S}t = \left(\frac{I}{S}\right)^2 \cdot \rho \cdot l \cdot S \cdot t = j^2\rho Vt,$$

where $j = \frac{I}{S}$ is density of a current; $V=S\cdot l$ is volume of a tissue.

Having divided Q on volume and time, we shall receive quantity of heat q allocated per unit of time per unit of volume of substance:

$$\boxed{q = \frac{Q}{Vt} = j^2\rho}.$$

It is necessary to mean, that for a sine wave alternating current value j represents effective value of density of current, i.e. $j_{EFF} = j = \frac{j_0}{\sqrt{2}}$, where j_0 is peak value of density of current.

a) Diathermy

Passing of currents of high frequency through a tissue is used in physiotherapeutic procedures and is known as **diathermy** (greek. – dia – through + term – heat). At diathermy apply a current of frequency from 1 up to 1,5 MHz, a voltage 100-250 V (!), and a current from 1 up to 3 A (!!). Diathermy allows to increase local temperature of tissues on 2-5^{0}C, that results to expansion of blood vessels, increase of blood circulation and results to activization of some biological processes. As blood, muscles, liver, lung have small specific resistance, they are heated up poorly. Skin and hypodermic cellular tissue have the big specific resistance therefore they are heated up more strongly. Such unproductive allocation of heat in a skin and hypodermic cellular tissue is **weakness of diathermy.** Besides, method of diathermy assumes very dense contact of a site of a body with electrodes. At bad contact there can be burns. For these reasons now diathermy *replace with another more effective methods* of high-frequency influence.

b) Electrosurgery

Now currents of high frequency *(ν =1-2 MHz)* are used for the surgical purposes (electrosurgery). They allow to dissect a tissue (**diathermotomy**) or to weld tissues (**diathermocoagulation**). The electric circuit the same, as well as at diathermy (fig. 3).

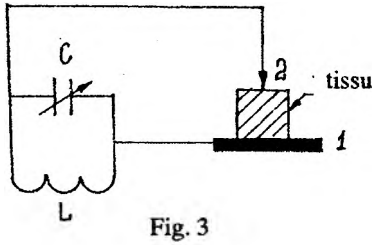


Fig. 3

The bottom electrode 1 has the big area and it is called *inactive electrode* and the top electrode 2 has very small area, it is made as a scalpel or a needle and is known as active electrode.

At electrotomy the section of tissues is carried out as result of intensive steam formation of a tissue liquid in area adjoining to the electrode 2. The density of a current at electrotomy reaches up to 40 kA/m^2. The electrosurgery has a number of advantages in comparison with usual surgery: **small loss of blood owing to coagulation of walls of blood vessels; small postoperative pains, bactericidal action.**

Diathermocoagulation is used for welding of blood vessels, alveoluses, for a burning out of malignant tumours and in other cases. Density of current at coagulation from 5 up to 10 kA/m^2.

c) Darsonvalzation

The method of treatment with frequency from 200 up to 500 kHz at the voltage up to 30 kV (!) and a current 15-20 mA is called **local darsonvalization.** The method has received the name in honour of offered it French physics and biologist G.A.D'Arsonval. The form of pulses is shown on fig. 4.

The *electric circuit* of darsonvalization is submitted on fig. 5. The current to patient P acts from a source of high-frequency fluctuations S through the vacuum glass electrode (fig. 6) or filled by graphite.

The second electrode is not present as the circuit is closed through a body of the patient and the medium by

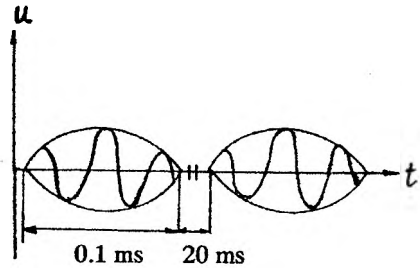


Fig. 4

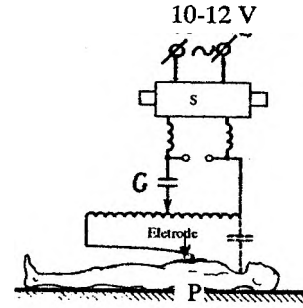


Fig. 5

so-called currents of displacement (the dotted image of the condenser). Heating of tissues at darsonvalization is practically imperceptible, as force of a current is very small. At local darsonvalization there is *irritation of skin receptors by fine jump sparks between a body of the patient and an electrode*. It causes expansion of capillaries and arterioles in the operative range of electrode, strengthens of blood circulation, stimulates healing of wounds and ulcers, improves metabolism and takes effect sedative analgesic effect. Essential action is rendered with polarizing effects on cellular membranes.

Fig. 6

3. Heating of a conductor in a variable magnetic field. Inductothermy

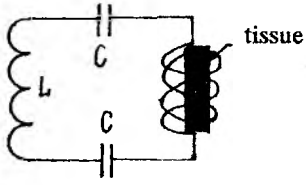


Fig. 7

The electric circuit of heating of tissues in a variable magnetic field is submitted on fig. 7. A tissue is placed in the coil inside which there is a variable magnetic field of frequency *10-15 MHz*. This field creates (induces) vortical currents in conducting tissues. These currents can be used for warming up of tissues and bodies. Such medical method is known as **inductothermy.**

Let's calculate quantity of heat allocated in this case. EMF of induction ε_i is equal:

$$\varepsilon_i = -\Phi_t' = -(BS)_t' = -(B_0 S\cos\omega_0 t)' = B_0 S\omega_0 \sin\omega_0 t,$$

where B_0 is the maximal value of induction of a magnetic field.
Under the law of Ohm:

$$I = \frac{\varepsilon_i}{R} = \frac{B_0 S\omega_0 \cdot \sin\omega_0 t}{\rho\frac{l}{S}} = \frac{B_0\omega_0}{\rho}\cdot\frac{S^2}{l}\cdot\sin\omega_0 t,$$

where $I_0 = \frac{B_0\omega_0}{\rho}\cdot\frac{S^2}{l}$; $I_{ef} = \frac{B_{ef}\cdot\omega_0}{\rho}\cdot\frac{S^2}{l}$.

Then $j_{ef} = \frac{I_{ef}}{S} = \frac{B_{ef}\cdot\omega_0}{\rho}\cdot\frac{S}{l}$.

Earlier we have shown that the quantity of heat allocated in 1 m^3 tissues for 1 second is equal: $q = j_{ef}^2\cdot\rho = \frac{B_{ef}^2\cdot\omega_0^2}{\rho}\cdot\frac{S^2}{l^2} = k\frac{B_{ef}^2\cdot\omega_0^2}{\rho}$,

Fig. 8

where $\kappa = \frac{S^2}{l^2}$ is the factor dependent on the sizes of a sample.

Thus *at imductothermy the quantity of heat allocated in tissues is proportional to squares of frequency and induction of a magnetic field and is inversely proportional to specific resistance*. Hence, at inductothermy tissues with *smaller specific resistance (blood, liver) are more heated up*. In comparison with diathermy inductothermy gives deeper warming up, as it is carried out on higher frequencies. Inductothermy gives good results at treatment of chronic inflammatory processes in deeply laying tissues: bronchitis, pneumonia, cholecystitis, nephritis, etc (fig. 8).

4. Heating of conductors and dielectirs in ultrahigh-frequency electric field. UHF-THERAPY

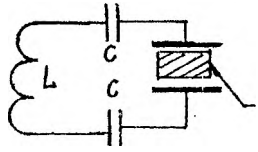


Fig. 9

The electric circuit of heating of a tissue by UHF waves is submitted on fig. 9. **Electrodes do not concern a tissue**. The tissue is located between plates of the condenser (*therapeutic electrodes*) in which there is a variable electric field of frequency 40-50 MHz, that on the order is higher than at diathermy. These fields concern to the ultrahigh frequencies, therefore the corresponding physiotherapeutic method has received the name of UHF-THERAPY.

a) Heating of *conductors* in the ultrahigh-frequency electric field

Capacity of a current in a conductor is $P = \frac{U^2}{R}$, taking into account that U=E·l, and $R = \frac{\rho\cdot l}{S}$, let's receive $P = \frac{E^2\cdot l^2\cdot S}{\rho\cdot l} = \frac{E^2}{\rho}\cdot S\cdot l = \frac{E^2}{\rho}V$, where V is volume of a tissue. Having divided P on V, we shall receive the *quantity of heat allocated per unit of time per unit of volume of the conducting tissue:*

$$\boxed{q = \frac{P}{V} = \frac{E^2}{\rho} = E^2\gamma},$$

where E is effective value of intensity, i.e. $E = E_{ef} = \frac{E_0}{\sqrt{2}}$; E_0 is peak value of intensity of the electric field.

b) Heating of dielectrics by ultrahigh-frequency electric field

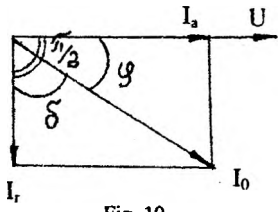


Fig. 10

Let's consider dielectric with relative dielectric permeability ε, taking place in the alternating electric field. In real dielectric there are free ions and electrons, which move under action of the electric field and heat up dielectric, i.e. there is *active current of conductivity* I_{con}.

Except for it, in dielectric there is rotation of dipole molecules under action of field and their orientation along power lines of a field, i.e. is *oriental current* I_{or}.

As there is the resistance of medium interfering such orientation of dipoles, part of energy I_{or} goes on heating of dielectric, i.e. there is an *active part* $I_{a.or}$ of oriental current, other part $I_{r.or}$ *be reactive* (it does not result in heating). Hence, in dielectric there is active current $I_A=I_{con}+I_{a,or}$ (created by currents of conductivity and the active part of oriental current) and the reactive current $I_r=I_{r,or}+I_{el}$, where I_{el} is the current caused by electronic polarization. On the vector diagram the active current coincides at direction with a voltage, and reactive lags behind on $\pi/2$. The general current I_0 is equal to the vector sum of I_a and I_r (fig. 10). The angle between general current I_0 and reactive I_r is called the *angle of dielectric losses* δ. Let's find a tangent of the angle of dielectric losses. From fig.10: $tg\delta = \frac{I_A}{I_r}$ and it characterizes a share of energy of the electric field spent in dielectric on heating.

From triangles we shall write down:

$$I_{A.ef} = I_{R.ef} \cdot tg\delta; \qquad I_{A.ef} = I_{0.ef} \cdot \cos\varphi; \qquad I_{R.ef} \cdot tg\delta = I_{0.ef} \cdot \cos\varphi;$$

$$P = U_{ef} \cdot I_{0.ef} \cdot \cos\varphi = U_{ef} \cdot I_{R.ef} \cdot tg\delta; \qquad I_{R.ef} = U_{ef} \cdot \omega c;$$

$$U_{ef} = E_{ef} \cdot l; \qquad c = \frac{\varepsilon \cdot \varepsilon_0 \cdot S}{l}; \qquad V = S \cdot l;$$

$$P = U_{ef}^2 \cdot c \cdot \omega \cdot tg\delta = E_{ef}^2 \cdot l^2 \frac{\varepsilon\varepsilon_0 S}{l} \cdot \omega \cdot tg\delta = E_{ef}^2 \varepsilon\varepsilon_0 \cdot V \cdot \omega \cdot tg\delta;$$

$$\boxed{q = \frac{P}{V} = E_{ef}^2 \varepsilon\varepsilon_0 \cdot \omega \cdot tg\delta}.$$


Fig.11

Comparing with formulas for conductors and dielectrics in the field of UHF, it is possible to note that quantity of heat allocated in both cases are directly proportional to the square of effective intensity of the electric field and for dielectrics depends from frequency of electric field. In UHF devices use frequency of 40.68 MHz (fig. 11). *For such frequency*

dielectric tissues of human organism are heated up more strongly than conducting tissues.

At UHF – therapy heating of bone, muscular and fatty tissues occurs more intensively, than heating of blood vessels, lymph nodes (fig. 15). UHF-therapy takes effect of number of physical and chemical influences: amplification of activization of enzymes, change of PH of cytoplasm.

5. Microwave therapy

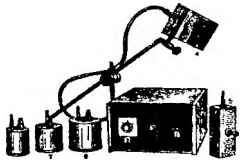


Fig. 12

The physiotherapeutic methods based on application of electromagnetic waves of the microwave band have received the name of **microwave therapy.** For this kind of high-frequency therapy are allocated waves of 6.5 dm and v=460 MHz (decimeter or DMW-THERAPY) and 12.6 cm and v=2375 MHz (centimetric or CMW-THERAPY). Electromagnetic oscillations of the MICROWAVE creates *magnetron generator* (the magnetron is device combining functions of electronic lamp and oscillatory contour: fig.12, 13). The electromagnetic wave directs on a corresponding site of body by the special radiators, which are looking like hollow cylinders (fig. 12, 13). Depth of penetration of electromagnetic waves is defined by a structure of tissue. Centimetric waves

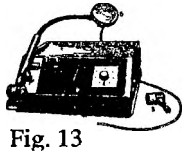


Fig. 13

Fig. 14

will penetrate into an organism on depth of 2-6 cm, and decimeter waves on depth of 7-9 cm (fig. 15). The mechanism of physiological action of microwave therapy, as well as the mechanism of any other method consists of initial and secondary actions. Initial action is the direct influence of microwaves on tissues, and secondary action is arising reply on initial action. *Initial action takes place directly in an irradiated site of a body and consists from thermal and not thermal components.* Now the theory about thermal action of MICROWAVES on biological objects is most developed. The electromagnetic wave polarizes molecules of substance and periodically changes their orientation as they are electric dipoles. It influences on ions of biological objects and causes alternating current of conductivity. Thus, *in substance arise both a current of displacement and a current of conductivity.* It causes heating of substance. The big role in heating of substance has currents of displacement caused by reorientation of dipole molecules of water. Therefore **the greatest absorption of energy occurs in tissues rich by water: in muscles and blood.**

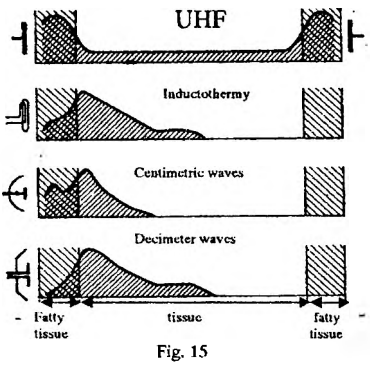


Fig. 15

Bones and fatty tissues are heated up less. On the border of the unit of two mediums with different coefficients of absorption of electromagnetic waves, for example, tissues with the various maintenance of water, there can be *standing waves* creating **local overheating.** Such phenomena arise in tissues with insufficient blood supply (crystalline lens of eye and vitreous body).

The quantity of heat allocated per unit of time per unit of volume of a tissue is defined under the formula:

$$q = kI^2 \varepsilon v^2.$$

Not thermal action is reduced to various electrochemical changes and structural reorganizations in complex bicolloidal systems (change of osmotic pressure in cells, change of permeability of biomembranes, colloidal conditions of cytoplasm). These changes influence exchange processes in cells. It is necessary to note, that this action is investigated less of thermal action.

As to the secondary mechanism, that it is reduced basically to influence of the absorbed energy on receptors. The irritation from receptors acts through nervous channels to the central nervous system. Thus, the local irradiation results in the general physiological effect (fig. 14).

LECTURE №15

Characteristics of pulse currents. Physical bases of electrostimulation of tissues and bodies

1. Pulse currents and its characteristics

Last some tens years were conducted purposeful studying of opportunity *of pulse modes* in treatment, the equipment was developed for reception of pulse currents, new techniques of electrotreatment were created. It speaks that leading of energy of the physical factor to an organism in the separate portions divided by pauses allows: 1) to reduce heat buildup in tissues and electric loading by cardiovascular and nervous systems, 2) to carry out *selective* influence on the certain bodies and systems by selection of corresponding parameters of influence.

Pulse modes have received the widest application in electrotherapy for the following purposes:

1. Reception of soothing effect at action on peripheral nervous system, on structures of brain (short-impulse electroanalgesia, diadynamic therapy or Bernard's currents, electric sleep, transcranial analgesia, etc.).
2. Normalization of condition of the central nervous system (transcranial analgesia etc.).
3. Amplification of blood circulation, normalization of functions of many bodies and systems (diadynamic therapy, electropuncture, etc.).
4. Electrostimulation.

Electric pulse this short-term change electric voltage or current on a background of some constant value (fig. 1).

Pulses share on two groups: *videopulses* or electric pulses of a direct current or voltage (these are understood in physiology as the term «electric pulse»), and *radio impulses* or the modulated electromagnetic fluctuations (see fig. 7).

Pulses under the form are:

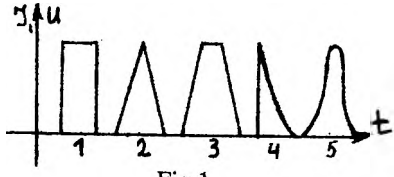


Fig.1

1) rectangular;
2) sawtooth;
3) trapezoidal;
4) exponential;
5) campaniform, etc. (see fig. 1).

Let's consider parameters of these pulses.

Characteristic sites of a pulse are (fig. 2):
1-2 -wavefront; 2-3 - top;
3-4 - back front;
4-5 - wavetail.

In a real pulse the moments of the beginning t_1, transition from front to top t_2 and to the end t_5 precisely are not determined (see fig. 3).

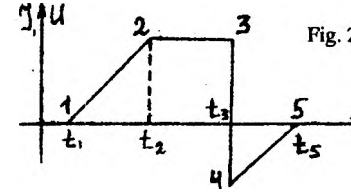


Fig. 2

For reduction of a possible error allocate the moments of time at which U or I have values 0.1 U_{max} and 0.9U_{max}, where U_{max} is amplitude, i.e. the greatest value of a

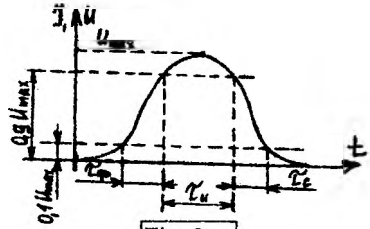


Fig. 3

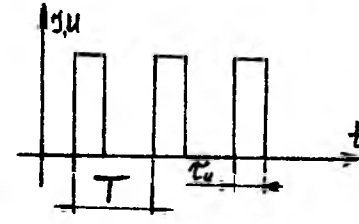


Fig. 4

pulse. On fig. 3 are shown:

τ_Φ- duration of front;

τ_C - duration of back front;

τ_U - duration of pulse.

The relation $\frac{0.9U_{max} - 0.1U_{max}}{\tau_\Phi} = \frac{0.8U_{max}}{\tau_\Phi}$ is called the **rate of pulse rise** of front. If pulses repeat **(pulse current)** is used one more characteristic - **period of recurrence T**, it is average time between the beginnings of the next pulses (fig. 4). **Frequency of recurrence** is equal $f = \frac{1}{T}$. **Relative pulse duration** is called relation: $Q = \frac{T}{\tau_U} = \frac{1}{f \cdot \tau_U}$. Quantity $K = \frac{1}{Q} = f \cdot \tau_U$ is known as **duty factor.**

2. Electroexcitability of tissues, rheobase, chronaxia. Weiss - Lapik equation, Dubois – Reymond's law

The irritation of a muscle is influenced with rate of pulse rise of front, ratio of amplitude and duration of a pulse, and also frequency of recurrence of pulses. We shall stop more in detail on each of these factors.

1). Influence of pulse rise of front.

Still Volta tried " to bring in a measure " to studying irritating action of a current, but at him it has turned out nothing, because the paw of a frog was much more sensitive, than his best electroscope. Dubois - Reymond (Berlin professor, father of electrophysiology) already had new sources of irritation - galvanic cells and sensitive galvanometers. First of all, Dubois tries to irritate a muscle, switching on direct irritating current of different force. He managed to measure that minimal force of a current which forces a muscle to be reduced. This minimal force of a current of Dubois has named *threshold force of irritation (or threshold of current)*, and it was found out, that the threshold is not an absolute constant: different muscles have different thresholds.

Then he finds out, how not the direct current, and a current gradually increasing on time operates on a muscle. As appeared, that if force of current to

increase gradually, at the certain value of the current there is reduction of muscle (traction), which proceeds until the current passes through the muscle. Such condition comes at that greater force of current, than more slowly current increases. In other words, the more the rate of pulse rise of front, then at smaller force of current will come reduction of muscles (*traction is proportional to speed of change of force of current*: **Dubois – Reymond's law**).

It testifies that muscles adapt to change of force of a current, there are ionic compensatory processes. Moreover, at slow increase of force of a current the muscle can not be excited at all.

In figure 5 point with arrow is the moment of excitation, the more slowly increases the current, then later and at the greater force of current there is excitation. For slowly increasing current (a straight line 3) excitation does not come at all. This phenomenon is known as *accommodation* and speaks by partial inactivation of Na channels and activation of K channels. The rate of pulse rise of front *of a rectangular pulse* is very big (theoretically it is infinite), therefore for such pulses threshold force of a current is less, than for others (a point 4 on an axis of currents I).

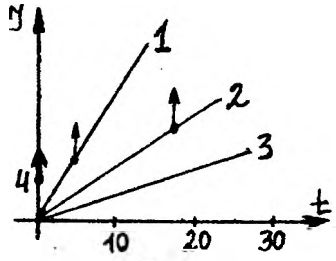


Fig. 5

2). Influence of amplitude and duration of a rectangular pulse on irritation of a muscle.

The reason of irritation of cells is reduced to *polarizing effects* since at passage of a current on the opposite ends of a cell collect heteronymic ions. Excitation of a cell occurs when concentration of ions on opposite surfaces of a membrane reaches of breakdown voltage value. At small currents the "breakdown" of membrane will not be at all, since ions collecting on its surfaces will be scattered in the parties by thermal movement. Thus, the irritation of a tissue has a threshold, the current is lower of threshold is not felt.

At passage through a tissue of a short-term pulse, ions because of the inertia have not time to come in motion, their congestion at a membrane insignificantly and irritation of tissue is insignificant.

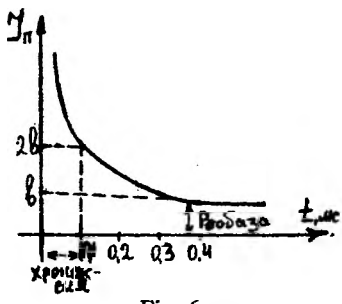


Fig. 6

Dependence of value of threshold current $I_п$ on time is expressed by formula of **Weiss** (1901), received by him empirically:

$$I_п = \frac{a}{t} + в, \qquad (1)$$

where "a" и "$в$" are constants (see fig. 6). Constant "$в$", determining minimal force of threshold current necessary for irritation at long influence of current ($t \to \infty$) is named **rheobase**.

If both parts of the equation to multiply on t, we''ll receive $I_п = a + вt$ or $a = I_п t - вt$, i. e. "a"

is a charge, which is necessary for passing to cause excitation at very short time of influence.

Time t necessary for irritation at the force of current equal of two rheobases is named **chronaxia** (greek: time + measure). The value of chronaxia is parameter of speed of occurrence of excitation and speaks about a level of excitability of a tissue. For skeletal muscles and nerves of mammal chronaxia is approximately equal to ten-thousand shares of second. Lapik has checked up and has confirmed equation of Weiss and has constructed around this equation a number of theories and has entered some new terms (rheobase, chronaxia), therefore *the formula (1) carries the name* of the equation **of Weiss - Lapik**.

To each point of the curve «force – duration» (fig. 6) and to the points laying above of the curve correspond pulses causing reduction of muscles. The pulses corresponding to the points located below the curve do not cause irritation.

3). Influence of frequency of pulses on irritation of a muscle.

If to change frequency f a pulse current, keeping condition of excitation (the area is higher than curve on fig. 6) separate reductions everyone will be closer and more close to each other. At some frequency there will come long reduction or ***tetanus***. The effect which causes a pulse current is similar to effect from alternating current of corresponding frequency.

3. Electrostimulation. Kinds of electrostimulation of heart

Electrostimulation is application of a pulse current with the purpose of excitation or amplification of activity of the certain bodies, muscles and nerves.

It is possible to stimulate by pulse currents many organs and systems by means of application of the relevant procedures. In practical work the widest application has electrical stimulation of heart (special section of medicine) and electrical stimulation of motor nerves and muscles.

Bases of the modern method of electrical stimulation have been included by French neuropathologist Duchene (1855). Having initiated to study of medical application of electricity in particular electropuncture, he has found that stimulate traction it is possible without piercing of skin, and only through the metal electrode which has been wrapped up by the wet tissue i.e. how will carry out electrical stimulation today.

Main principle of an electrical stimulation is deriving of optimum physiological effect at the least by-effects (including unpleasant sensations). Therefore at electrical stimulation various currents with various parameters are applied depending on excitability of tissues and their functional state. Activity of any physical factor is founded on uptake of energy and its transformation inside a cell to energy of biological processes.

Electrostimulators can be sectioned on *stationary, wearables and implanted.* To *stationary* it is possible to refer the universal electrostimulator УЭИ-1, devices АСМ-2 и АСМ-3 (devices for a stimulation of muscles), device "Neuropulse", the device for stimulation by sine modulated currents of muscles - "Amplipulse", etc.

So, device УЭИ-1 is the generator of impulse current of the rectangular and exponential shape. Parameters of impulses can be governed over a wide range.

For *implanted* stimulators, for example, for cardiostimulators a serious problem are power supplies which should is long-term and economically function.

To special kind of electrostimulators concern devices, that in the coded shape transmitted information usually accepted by organs of sense. A similar stimulator is wearable cochlear prosthesis conversing the sound information to electrical signal, i.e. in essence, exchanging a cochlea of inner ear.

Observations for electrical stimulation are extensive: prophylaxis of atrophy of muscles at a hypokinesia, infringement of innervation, for restitution of the broken motive functions at paresis and paralyses, for improving respiration, for a stimulation of muscles with the purpose of improving circulation, normalization of the broken lipometabolism and diminution of redundant mass of a body, etc.

Before electrical stimulation it is necessary to lead *electrodiagnostics* to find those parameters of a current and modes of exposure at which there is optimum effect. For example, for excitation of muscles in electrogymnastics are used currents of the exponential shape of ν = 8 - 80 Hz and the triangular shape of ν = 100 Hz.

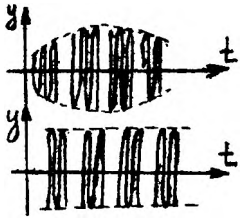


Fig. 7

Last time currents of acoustical frequency 2-10 kHz more often apply for electrical stimulation of intact motor nerves and innervated them of muscles, modulated in the series of oscillations effective similarly to impulses of current (see fig. 7). The groups of such series alternated to pauses ensure change of traction by their rest.

Square-wave pulses are used in the following methods. Electric sleep is the method of medical action on structures of brain. To this procedure apply square-wave pulses of frequency 5-160 imp/s and by duration of 0.2-0.5 ms. Force of an impulse current is 1-8 mA.

Transcranial electroanalgesia is the method of medical action on integuments of head by the impulse currents calling anesthesia or downstroke of intensity of pain sensations. For this method the following modes of exposure (fig. 8) are used:

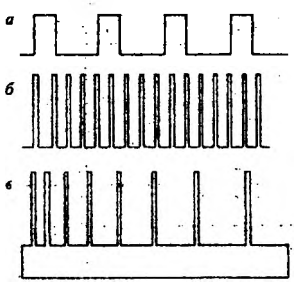


Fig. 8

The basic views of the impulse currents used at transcranial electroanalgesia

а) square-wave pulses of voltage up to 10 V and with frequency of 60-100 imp/s of duration 3.5-4 ms, following by bundles till 20-50 impulses;

б), в) square-wave pulses of a stationary and variable off duty factor of duration 0.15-05 ms and voltage up to 20 V the following with frequency of 150-2000 imp/s. Force of impulse current thus does not exceed of 1 mA. Select of parameters (frequency, duration, off duty factor, amplitude) is carried out individual *for* each patient.

Hulf-sine pulse currents are used in *diadynamic therapy.* The basic views of such currents are submitted on fig. 9:

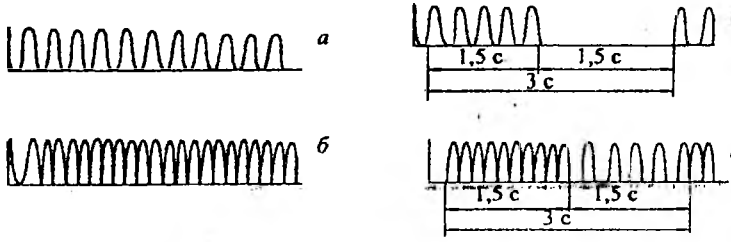


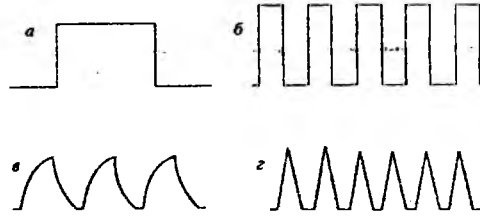
Fig. 9

Basic kinds of diadynamic currents:

а) hulf-wave continuous of frequency 50 Hz;
б) full-wave continuous of frequency 100 Hz;
в) hulf-wave rhythmic is intermittent hulf-wave current which premises with pauses of duration (1-1.5 s);
г) modulated by different periods on duration.

Bernard's currents represent diadynamic currents, i. e. impulses with the back front having the shape of exponential curve, frequency of these currents is 50-100 Hz. In a method offered for the first time by Bernard lays *the failure from rhythmicity of stimulation.* Now this principle is used more widely: random change of parameters of impulses hinders with various adaptable processes in tissues, thus raising efficiency of physiotherapeutic procedure. The basic views of the impulse currents used in electrotherapy are shown on fig. 10:

Fig. 10



а) direct current with interruption,
б) impulse current of the rectangular shape,
в) impulse current of the exponential shape,
г) impulse current of the triangular acuminate shape.

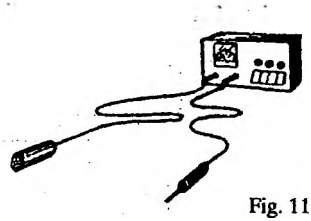


Fig. 11

Electropuncture is medical action by pulse and alternating currents on biologically active points (BAP). On the modern representations such points are morpho-functionally isolated fields of tissues posed in hypodermic fatty tissue. They have increased electrical conductivity in relation to fields of skin environmental them. On this property devices for searching BAP and actions on them (fig. 11) are built.

The operating voltage of measuring apparatuses does not exceed of 2 V. Measurings will be carried out as follows: the patient holds a neutral electrode in the hand and the doctor affixs to explored BAP a measuring electrode - probe of the small area (dot electrode).

It is observationally shown: current intensity flowing past in a metering circuit depends on pressure of the dot electrode on the surface of skin. Therefore always there is dispersion in a measurand. Besides elasticity, depth, damp of skin in various fields of a body and at various people are different, therefore it is impossible to enter uniform norm.

Electrical stimulation (ES) of heart for the first time has been applied in the USA (Zoll, 1952) and now has come in wide cardiological practice.

In the basis of ES of heart lays that fact, that by electrical impulses of a definite form, amplitude and duration it is possible to exchange natural signals of centers of automatism of heart, driving by its rhythm.

Distinguish *time (temporary) and stationary value ES of heart* and to destination: *preventive, diagnostic and medical.*

For *time* ES of heart are used skin, hypodermic and cardial electrodes. At patients after operation on heart thin electrodes in heart which free ends output outside for a time cardiac activation or taking out of ECG can be abandoned. Advantage of time ES of heart is in opportunity of momentary stimulation of ventricles at menacing states.

Stationary ES of heart is carried out by implantation of electro-cardiostimulator (ECS) with electrodes. At implantation of the cardiostimulator consisting of the generator and electrodes, the generator of impulses is disposed in a pouch shaped by the surgeon below clavicle. The modern ECS have small dimensions and mass of 29-40 g and represent a metal box (for shielding from outside electromagnetic noises), in which basic volume is completed by the power supply, and the electronic part on a chip borrows some tens cubic millimetres.

Depending on needs, the electronic part can ensure and such functions as programming ща parameters with help of outside programming device, accumulation of the data in storage, dialogue (telemetering) communication with the doctor, carrying out of electrophysiological testing of heart, etc.

Energy source are batteries from Li or radionuclide devices; prognostic reserve of energy is designed for 8-12 years.

Some firms release ECS, capable to increase frequency of ES under activity of mechanical vibrations of a body of the patient during walking, run, etc. exercise stresses.

Impulses from a cardiostimulator given immediately on heart have usually rectangular shape of duration 0.15 - 3 ms, frequency of 1-1.2 Hz, with amplitude of 5 - 15 mA and if it is necessary to execute a cardiac activation at accident through thorax, than amplitudes are necessary to increase in 10 times.

Heart is capable to fulfill the important function on pumping-over of blood only at strictly synchronized activity of fibrils of cardiac muscle. If the current from an exterior source transits through heart, it can give muscular fibrils in excitation and the action potential will be spread on heart on all directions that will cause not coordinated contraction of ventricles. This phenomenon is known *as ventricular fibrillation.* It can arise also under activity of other reasons: poisoning, sudden refrigerating at a jump into water, etc.). Once having arisen the fibrillation spontaneously does not stop, even if the reasons caused it are eliminated. Within 1-2 minutes the cardiac muscle which is not gaining coronary blood weakens, normal contractions will stop, that can give lethal outcome. However the fibrillation can be stopped, if to irregularly contracting muscles to guide the exterior, transient impulse with great strength of a current. Under activity of this impulse muscles of heart are reduced simultaneously and after extinction of the impulse there is a simultaneous weakening of muscles and rhythmic activity of heart is recommenced. The current for fibrillation should be not less than 1 A, at smaller current not all fibrils will be contracted rhythmically. For this reason the electrical shock by voltage of 220 V more often gives lethal outcome, since through a cardiac muscle transits the current 0.1-0.2 A, which calls fibrillation.

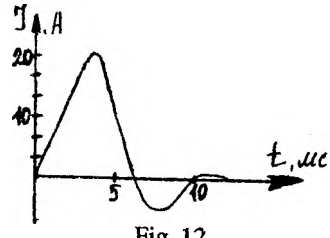


Fig. 12

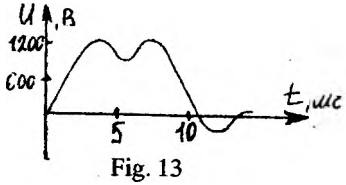


Fig. 13

The Soviet scientists led by N.L.Gurvich (1939) posess a merit of making of the first condenser defibrillator of direct current and development of theoretical bases of cardiac defibrillation (Юньев, 1939). In defibrillators of condenser type the direct current is used: impulses sine, regarding cases, the rectangular or trapezoidal shape are made. The matter is that the undesirable peak of current (up to 20 A, 6000 V) which appears in the beginning of the discharge can damage a myocardium (see

fig. 12). To reduce such hazard, in a defibrillator the inductive coil is used which allows to generate the impulse of necessary shape, for example, double-peak, trapezoidal, etc. of greater duration and with significant smaller peak voltage (up 1200 V) (see fig. 13 and fig. 14).

The condenser is charged up to a high voltage, electrodes are affixed to the thorax.

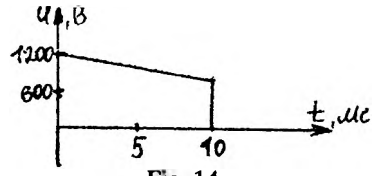


Fig. 14

Then the condenser is promptly discharged, and for making a current of 1 A through heart, it is necessary to pass through a body current approximately of 10A at voltage up to 6 kV. In some defibrillators (for treatment of arrhythmia) is used sync of a discharge impulse with ECG of the patient in order the discharge happened in necessary fields of the ECG curve.

Since 1970 designed and in 1980 are applied completely implanted defibrillators of heart. Origin of a fibrillation of ventricles will be recognized automatically and later 15s the device generates 1-st discharge (E = 25 J, duration t = 3 - 8 ms). At absence of effect some more discharges are superimposed.

LECTURE №16

PHYSICAL BASES OF STRUCTURE AND FUNCTIONING OF BIOLOGICAL MEMBRANES. TRANSPORT PHENOMENON

1. Basic functions and structure of biological membranes

Structural unit of a living organism is the cell which is carrying out all basic vital signs. With the help of modern methods of research of structure of biological objects is established that in creation of structure of a cell and its functioning biological membranes have the important value. Membranes, being universal structures, carry out diverse and extremely important functions for a cell: they surround all cytoplasm and limit it from environment, provide durability and autonomy of cells *(mechanical function)*. Besides a membrane forming an external boundary layer of a cell, similar membrane structures penetrate cytoplasm and form environments of all cellular organelles: nucleus, mitochondria, lysosome, complex Goldgi. Different sort of membrane structures in organisms of animals and person make great surface – tens thousand square meters. Such extensive structural system specifies its important functional value. *Membranes adjust the metabolism of a cell and serve as its osmotic barrier (cytoplasmatic membranes); membranes are a regulator of cell division, play the big role in generation and conduction of potentials, in cellular breath, membranes are the place of localization (form a basis, a matrix) for membrane enzymes, macropower connections, receptors and other molecules built in membranes; membranes are sensitive receivers and converters of light, sound, mechanical and chemical signals of external world.* A number of vital processes proceed on biological membranes.

Many illnesses are connected with infringement of normal functioning of membranes: cancerogenesis, atherosclerosis, poisoning, virus and infectious diseases, damage of organism by UV and radioactive radiation. Therefore treatment is frequently connected with influence on membranes with the purpose of normalization of their functions.

On the basis of the analysis of numerous researches of structure and properties of membranes, *Danielli and Dowson in 1935* have offered model of the structure of a biological membrane which basically has not undergone essential changes till our time. According to this model the membrane has structure of a *sandwich: double lipid layer squeeze between layers of proteins* (fig. 1). Lipid molecules in layers are located perpendicularly to the surface of a membrane; hydrophilic ends of molecules of lipids are directed outside and

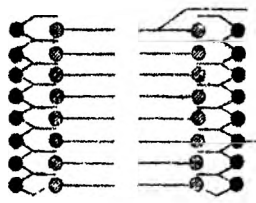


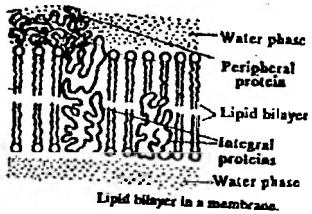
Fig. 1

hydrophobic to the center of a membrane. Presence of lipids in structure of biological membranes is supported by results received at measurement of electric parameters of a cell, which testify to high resistance of a cellular membrane (≈ 1000 Ohm/cm^2) characteristic for lipids and significant capacity (0.05 мсF/m^2). Lipids are connected with each other by waterproof interactions, lipids and proteins are connected by electrostatic forces. Albuminous molecules in the globular form cover the double layer of phospholipids from both sides giving to it thus the certain elasticity, stability to mechanical damages and low surface tension (0.1 din/cm). Polar groups of molecules of proteins are directed aside of water phase and not polar groups aside of lipids.

In 1956 the model has been advanced: are entered into consideration "pores" to explain free penetration through a membrane of water by diffusion and such hydrophilic connections, as urea. Diameter of a pore is 0.35-0.8 nm. Due to presence of polar groups in pores, they usually have an electric charge, that renders the big influence on process of penetration of soluble particles through pores, in particular, of ions.

In 1972 *Singer and Nicholson* on the basis of the results received by physical and chemical methods of research, have offered the generally accepted *fluid-mosaic* model, according to which phospholipids are form the double layer, but not necessarily continuous (the double layer of phospholipids, inlaid by proteins).

Not all surface of a biological membrane is covered with proteins. Distinguish *superficial (or peripheral) and integrated* proteins (fig.2). Due to these proteins the important functions of a membrane are carried out: permeability, active transport through a membrane, generation of electric potential. Lipids are in the liquid phase. Proteins are included in the layer of lipids, but their polar groups keep contact with water phase; there are proteins which penetrate a membrane through, they form membrane channels, some proteins are shipped in it half. One proteins are connected with each other, others are surrounded by lipids. Distribution of proteins is non-uniformly. According to the electronic-microscopic data, their concentration on the internal surface is higher than on external.



Lipid bilayer in a membrane.

Fig.2

Proteins are rather mobile, i.e. the membrane is not motionless structure. Lipids and proteins exchange by places, moving as along its plane and across, so-called "flip-flop". The structure of membranes includes also other chemical compounds: cholesterol, glycolipids and glycoproteins.

2. Lipids model membranes

Natural membrane is very complex system therefore scientists try to create the various models reproducing its structure and properties.

Models as *artificial monomolecular films* (monolayer lipid membrane - MLM) give representation about the organization of molecules of lipids in

membranes. The technics of reception of artificial flat membranes has been developed by *Lengmur* in 1917.

Fig. 3

If on a surface of water to put a drop of dissolved in any flying solvent phospholipids, then after distribution of their molecules on a water table and evaporation of solvent, the monomolecular film is formed (fig. 3). At full saturation of the superficial layer adsorbed molecules of lipids settle down perpendicularly to the surface of water in that way, that hydrophilic (polar) head is immersed in water and not polar hydrocarbonic circuit is directed vertically upwards. Such focused layer of molecules is called "*paling of Lengmur*". Thickness of such monomolecular film in case of the stearin acid is 2.5 nm. Molecules of phospholipids in such monolayer are settle down as densely, as in one of bilayers of a membrane. If a glass plate to lower in water on which surface there is a monomolecular film, this film can be transferred on the surface of the plate. At repeated immersions on the plate are arise bimolecular films.

The second model of membranes is *flat bilayers lipids membranes* (BLM). For the first time such membrane was created by *P.Muller* with the colleagues in 1962. For this purpose on the aperture in a teflon plate rendered a solution of phospholipids solvent (geptane). Then the plate placed in the solution of KCl. On a measure of diffusion of the solvent from the drop in a water phase, the aperture appeared closed by bilayer of phospholipid membrane by thickness of 5-7 nm and with diameter a little more of 1 mm. Such membrane can to exist long time in a water solution of salts. Models of such membranes have played the big role in finding-out of the mechanism of action of substances - ionophors (for example, antibiotics of type of valinomicyne), and also of some other connections modifying permeability of a membrane, including medical products, toxins, etc. On similar models researchers studied electroconductivity, transport through membranes of ions, their permeability for various substances and also mechanical and optical properties.

The third model system which is widely used now is *liposomes*. Liposomes are phospholipids bubbles (vesicules). They can be received by stirring of dry phospholipids in the water-salt buffer. Suspension of liposomes is very convenient object for study of structure of lipid bilayer (fig. 4), as the structure of this layer in liposomes close to a structure in biological membranes, membranes of mitohondria, erythrosites.

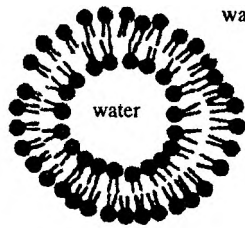


Fig. 4

Liposomes have found application in medicine. For example, it is possible to include a medical product inside of a liposome and to use it as *phospholipid microcapsule* for delivery of a medicine into a certain body and tissues. Liposomes are not toxic (at correct selection of lipids) and are completely acquired by an organism.

The basic value of the method of artificial membranes: it allows to study complex biophysical processes in membranes, as researches are carried out on simple systems.

3. Some physical properties of membranes and methods of their researching

The basis of a biological membrane is the double layer of molecules of phospholipids. *Lipid component* of membranes defines their following properties: *mechanical, optical, electric* (R, C) and osmotic (tightness for ions and permeability for water). Membrane contains different phospholipids. For example, in the membrane of erythrocyte there are 20 phospholipids.

From the *electric point* of view the membrane represents *dielectric* with relative dielectric permeability ε from 2 up to 6. It is experimentally shown, that between two sides of a membrane there is the potential difference of 50-80 mV. Taking into account thickness of a membrane this potential difference results to occurrence of high intensity of the order of 10^4-10^5 V/cm^{-1}.

Lipids are make 20-30 % of dry weight of a membrane, thus is considered, that on one molecule of a protein are 75-90 molecules of lipids necessary approximately. The proteins included in membranes are various.

The molecule of phospholipids consists from two functionally various parts. Not polar hydrophobic "tail" (the rests of fat acids) is a long direct hydrocarbonic circuit (CH_2; CH_3), to which cannot join molecules of water. This hydrocarbonic circuit joins to more complex structure ("head"), containing atoms of carbon, hydrogen, oxygen, phosphorus and nitrogen. This part of the molecule is polar and draws to itself molecules of water, which also are polar. Thus, one end of phospholipid molecule has hydrophilic and the second has hydrophobic properties. In the head of phospholipid are available opposite charged groups located on some distance from each other, i.e. representing a *dipole*.

Biological membranes, which thickness is approximately equal to 7-8 nm, represents as though two monolayers on the surface of water - lipid. The force compressing the monolayer in the membrane is a surface tension on the border of water - lipid phase. This *superficial tension has value of* 0.03-1 *mN/m* and results to compression of bilayer. *Viscosity* of the lipid layer of membranes on two orders is higher than viscosity of water and is equal to *30-100 mPa·s* (it is comparable with viscosity of sunflower oil). Many illnesses are connected with deviation of microviscosity of lipid phase from norm. For example, cancerogenesis is connected with decrease of microviscosity, and at ageing of organism viscosity increases.

Change of a condition of lipid molecule connected with change of temperature, chemical updating of a "tail" or with change of a charge of the head is accompanied by change of the area borrowed with molecules. To the same result is result interaction of membranes with medical products, for example, with anesthetic.

In biological membranes lipids are mainly in the *liquid crystal condition.* At change of a surface tension of a liquid on the border the membrane - environmental solution or superficial charge or temperature, in lipid phase of membranes can occur local or general *phase transitions* from liquid crystal to the gel - condition. Transitions are caused by complex physical properties of phospholipids, which belong to connections with long hydrocarbonic chains. They are capable to form some crystal forms. The liquid crystal condition of bilayer has smaller viscosity, smaller orderliness of molecules, the bigger ionic conductivity, the bigger solubility of substances than the firm condition. Thickness of liquid crystal of bilayer is less than of gel-condition. Structure of molecules in liquid and solid condition is different, that confirms X-rays analysis. In the liquid phase, molecules of phospholipids can form cavities ("kinks"), into which molecules of penetrating substance are capable to enter, and which are capable to move.

Method of the *fluorescent analysis* is used for research of some physical properties of biological membranes (with using of fluorescent probes and labels). In the normal condition the membrane does not fluoresce. For researches it is necessary to enter into membrane molecules or molecular groups capable to fluorescence. The fluorescent analysis enables to investigate mobility of phospholipids molecules in a membrane, to estimate viscosity of lipid phase (microviscosity of membranes) on displacement of fluorescence in the shorter area of the spectrum at increase in viscosity.

Microviscosity can be estimated on a degree of *polarization P* of fluorescent radiation at illumination of the membrane by the polarized light: $P = \frac{I_{pol}}{I_{tot}} < 1$, where I_{pol} is intensity of polarized light; I_{tot} is total intensity of fluorescence.

Radiation of fluorescence appears only in part polarized (the membrane is shined by completely polarized light). More mobility of a molecule (fluorescing), then is less a viscosity and less degree of polarization.

The fullest representation about aggregate of lipid bilayer gives the method of *radiospectroscopy*: *electronic paramagnetic resonance* (EPR) with use of the method of spin probes and the *nuclear magnetic resonance (NMR).*

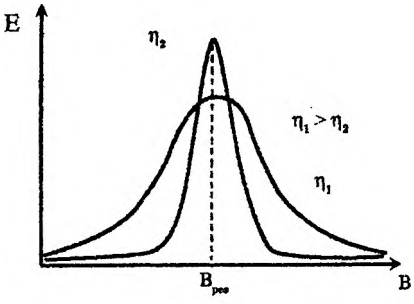


Fig. 5

EPR is phenomenon of sharp increase of absorption of energy of electromagnetic radiation (microvawe range) by the system of paramagnetic particles (have not coupled electrons and not compensated magnetic moment), placed in a constant magnetic field at resonant frequency of the wave ν_{res}. Spectrum of EPR is dependence of energy (E) of absorption of the electromagnetic wave from value of magnetic induction B (fig. 5). With help of EPR it is possible to study only the objects having not coupled electrons

(*free radicals*). Therefore into investigated system making corresponding chemical synthesis enter spin labels (spin-probes). It is connections representing various radicals ($\dot{N}O$), which can be attached to any atom of carbon of the hydrocarbonic chain of a molecule of lipid.

Application of EPR is based on dependence of the form of the curve of absorption on properties of environment of a free radical and first of all from microviscosity of the medium (fig. 5). In the liquid crystal phase circuits of phospholipids have more significant mobility, than in the solid phase.

By this method had been marked decrease of mobility of phospholipids at increase of the maintenance of cholesterol, at action of some medicinal substances. The increase of mobility is marked at thyrotoxicosis and a number of other pathologies.

Study of mobility of various sites of the unmodified molecules of phospholipids allows to carry out the method of a nuclear magnetic resonance (NMR). The NMR is the phenomenon of sharp increase of absorption of energy of an electromagnetic wave by system of the nuclear paramagnetic nucleus placed into the constant magnetic field at resonant frequency of waves ν_{res}. The biological object contains many paramagnetic nucleus $^{1}_{1}H$ of protons, that enables to apply them for researching by method of NMR. At NMR frequency of the variable electromagnetic field is less than at ENR.

4. General equation of transport phenomenon. Diffusion. Fick's equation

The important element of functioning of membranes is their ability to pass or not pass molecules (atoms) and ions.

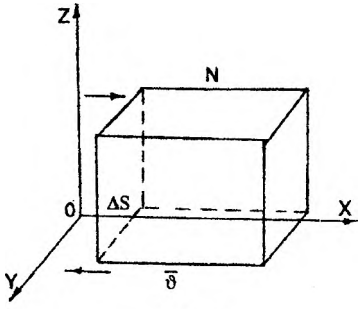


Fig. 6

Permeability is caused by a lot of the important physical phenomena and, in particular, by transfer by molecules of substances (during movement) of the next physical characteristics: mass, energy, impulse, etc. The mechanism of transfer of all these characteristics is identical and all phenomena connected with their transfer are incorporated by the general name *of the transport phenomenon:*

$$\Delta(N\varphi) = -\frac{1}{3}\lambda\bar{\vartheta}\frac{\Delta(n_0\varphi)}{\Delta X}\Delta S\cdot\Delta t \qquad (5)$$

where φ is movable physical value (mass, impulse, energy, charge) in time Δt through the platform ΔS, v is speed of the molecule, λ is free length, $\Delta(N\varphi)=(N\varphi)_1-(N\varphi)_2$ is quantity of physical value, which transport N molecules, $\frac{\Delta(n_0\varphi)}{\Delta X}$ is gradient of $(n_0\varphi)$, directed from the right to the left, and transfer of φ occurs in the opposite direction: from area, where concentration of molecules is more n_{01}, to the area, where n_{02} is less.

At diffusion as transferable value will be the mass of the molecule φ=m, then $n_0\varphi=n_0m=\rho$ is density and $\Delta(N\varphi)= \Delta(Nm)= \Delta M$, where ΔM is mass of gas at diffusion in time Δt through the platform ΔS (fig. 7):

$$\Delta M = -\frac{1}{3}\lambda\bar{\vartheta}\frac{\Delta\rho}{\Delta x}\Delta S\cdot\Delta t = -D\frac{\Delta\rho}{\Delta x}\cdot\Delta S\cdot\Delta t .$$

Last expression is *the equation of diffusion or Fick's law*, where $-\frac{1}{3}\lambda\bar{\vartheta} = D$ is coefficient of diffusion, $\frac{\Delta\rho}{\Delta x}$ is gradient of density of gas.

The equation of diffusion can be written down and through the density of stream of substance: $\Phi = \frac{\Delta M}{\Delta t \Delta S}$, then $\Phi = -D\frac{dc}{dx}$;

where $\frac{dc}{dx}$ is gradient of concentration. The mark "-" specifies that transfer occurs aside decrease of concentration.

LECTURE №17

MASS TRANAFER THROUGH BIOLOGICAL MEMBRANES

1. Passive transport of molecules (atoms) through biological membranes. Versions of passive transport

In an alive organism constantly occur processes of metabolism and interchange of energy between cells and the extracellular environment through membranes. Distinguish *passive and active transport* of substances through biological membranes. *Passive transport is not connected with an expense of chemical energy:* there is the diffusion of molecules and ions to the direction from places with the greater concentration to the area with smaller concentration and moving of ions to the direction of forces of the electric field.

Versions of passive transport are:1) simple diffusion, 2) facilitated diffusion , 3) filtration and 4) osmosis (fig. 1).

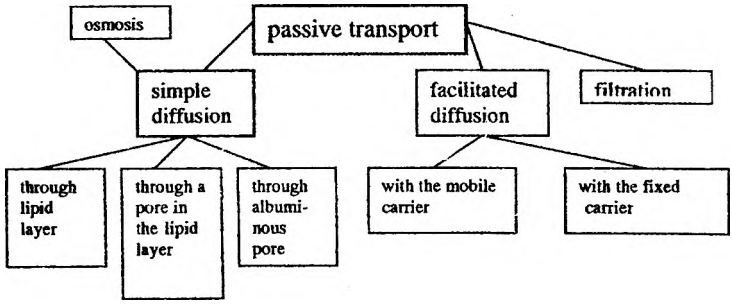


Fig. 1

1) We shall consider as the example of **simple diffusion** (fig 3a) a stream of not charged particles through a biological membrane by thickness l. We shall transform Fick's equation $\Phi = -D\frac{dc}{dx}$ with reference to a biological membrane.

If concentration C_i of particles on the left side of the membrane (internal) is higher than on the right side (external) C_e (see fig.2) inside a membrane the gradient of concentration is created. In Fick equation the gradient of concentration can be counted a constant, then:

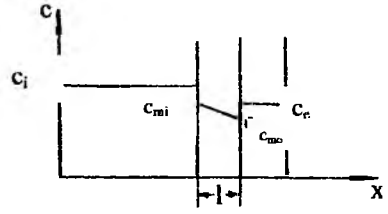


Fig.2

$$\Phi = -D\frac{C_{me} - C_{mi}}{l},$$

where C_{me} and C_{mi} are concentrations of substance on membrane at its borders,

determined by concentration C_i and C_e in water phase and by coefficient of distribution K of substance between the membrane and environmental water phase; l is thickness of the membrane, $K = C_{mi}/C_i = C_{me}/C_e$. Let's finally receive:

$$\Phi = -\frac{Dk}{l}(C_e - C_i) = -p(C_e - C_i),$$

where $P = Dk/l$ is *coefficient of permeability*. This equation is known under the name of the ***law of Fick for passive transport of substances (diffusion) through a membrane*** (the equation for density of stream at diffusion through a membrane). In an alive cell such diffusion provides passage of oxygen and carbonic gas and also of some medicinal substances and poisons.

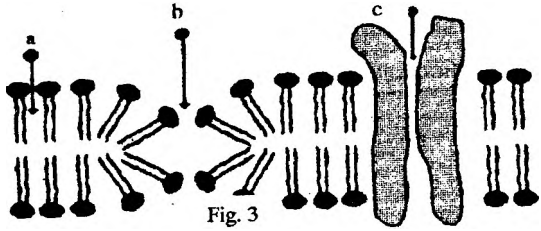


Fig. 3

Diffusion can pass also through *lipid and albuminous pores or channels* (fig. 3b,c) that form a passage in the membrane. This kind of transport supposes penetration through a membrane not only small molecules, for example, molecules of water, but also larger ions. Permeability thus is defined by sizes of molecules.

Diffusion through a pore is described by diffuse equation also, however, presence of pores increases coefficient of permeability. Channels can show selectivity in relation to different ions, it is shown in different permeability for them.

2) ***The facilitated diffusion*** occurs at participation of molecules - carriers. It was revealed that speed of penetration into a cell of glucose, glycerin and amino acids has no linear dependence on the difference of concentration (fig. 5). At the certain concentration of substances speed of their penetration is much more, than it is necessary to expect for simple diffusion. At increase of difference of concentration speed grows to a lesser degree, than it follows from the equation of simple diffusion.

In this case is observed the facilitated diffusion. The mechanism of it: given substance A independently badly penetrates through a membrane, but speed of diffusion considerably grows, if molecules A form a complex with molecule X of auxiliary substance (fig. 4), which as believe is dissolved in lipid membranes. On the surface of the membrane molecule A forms with molecule X complex AX, which diffuse into the cell. At the internal surface molecule A is released and molecule X diffuse outside and contacts with new molecules A. If concentration of

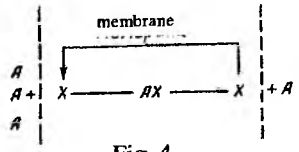


Fig. 4.

substance in the medium to raise up to such degree, that all molecules of substance X will be used, speed of diffusion at increase of concentration of substance A will not grow.

Ability of the molecule of valinomicyne (antibiotic) is most known to transfer through model bilayer membranes ions of potassium. The molecule of the antibiotic grasps ion of K^+, forming complex soluble in lipids, and passes through the membrane. For ability to transfer ions through a membrane valinomicyne and others related to it connections have received the name of *ionophores.*

The facilitated diffusion can is carried out also by means of *fixed carriers.* Carriers X can form a time chain across membrane (fig. 5) or cover from within a pore and the molecule of transferable substance A is capable to move from its one link to another (in variant of go-ahead transfer).

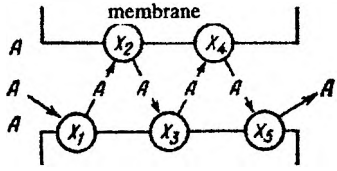


Fig. 5

Differences of the facilitated diffusion from a simple:

a) transport of substance occurs with the help of a carrier much faster;

b) the facilitated diffusion possesses property of saturation: at increase in concentration on the one hand of membrane density of stream of substance grows only up to some limit when all molecules of a carrier are already borrowed;

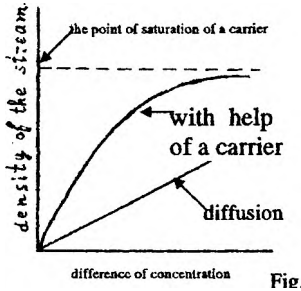


Fig. 6

c) the competition of transferable substances is observed, some substances are transferred thus better than others: so, glucose is transferred better, than fructose, fructose is better, than ksilose, etc.

d) There are the substances blocking facilitated diffusion, forming a complex with molecules of carrier.

3) Moving of a liquid through the pores in a membrane under action of gradient of hydrostatic pressure is known as *filtration.*. This phenomenon is observed at transport of water through walls of blood vessels (capillaries). The phenomenon of filtration plays the important role in many physiological processes. For example, formation of initial urine occurs as the result of a filtration of blood plasma under action of pressure of blood. At some pathology filtration amplifies that results to hypostases.

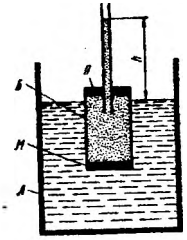


Fig.7

4) *Osmosis* is primary motion of molecules of water through semipermeable membranes from places with smaller concentration of the dissolved substance to the places with the greater concentration. Osmosis is simple diffusion of water from the places with its greater concentration to the places with smaller concentration. For example, we shall consider the big vessel (fig. 7) with water in which the

cylinder with a manometrical tube is placed. The cylinder is filled with a water solution of sugar. The bottom of the cylinder is the semipermeable membrane passing molecules of water and not passing larger molecules of sugar. As concentration of molecules of water outside of the cylinder is more than in the cylinder, diffusion of molecules of water through the membrane inside of the cylinder will be observed until hydrostatic pressure of a column of liquid of height h will not interfere with water inflow. This pressure designated by π and equal to ρgh is known as *osmotic pressure:*

$$\pi = \rho gh = i\, c\, R\, T,$$

where I is Vant-Hoff's coefficient showing for electrolytes in how many times osmotic pressure of a solution of electrolyte is more or less than osmotic pressure of a solution of nonelectrlyte at the same conditions; at dissotiation electrolits on ions $i > 1$.

c is the molar concentration of a solution, T is temperature.

Solutions with identical osmotic pressure are called *isotonic*, for example, osmotic pressure of human blood is 0.77 MPa; the same pressure has *0.86 % solution of NaCl*. Osmosis plays the big role in many biological phenomena, for example, causes gemolysis of erythrotytes in hypotonic solutions.

Graham made use of the semipermiable properties of pig's bladder to separate crystalloids from colloids. This process is called dialysis and is used for purification of blood when the patient suffers from a defective kidney.

2. Transport of ions through a membrane. Nernst-Planck's equation

On a biological membrane there is a potential difference, hence, in a membrane there is an electric field which influences diffusion of the charged particles as ions and electrons. Between intensity of the field and potential there is the known parity: $E = -d\varphi/dx$. If the charge q of an ion is equal $Z{\cdot}e$, on it force $f = Eq = -Z \cdot e\dfrac{d\varphi}{dx}$ will operate and the force working on one mole of ions will be equal: $f_1 = -Z \cdot e \cdot N_A \dfrac{d\varphi}{dx} = -ZF\dfrac{d\varphi}{dx}$, where N_A is number of Avogadro, $F=eN_A$ is number of Faraday. Speed of the directed motion of ions is proportional to driving force: $\vartheta = U_m f_1 = -U_m ZF\dfrac{d\varphi}{dx}$, where U_m is the mobility of ions expressed for mole.

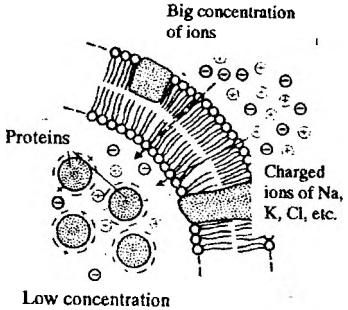


Fig.8

The stream of ions past through platform S per 1 second will be equal to volume of a parallelepiped (9S) multiplicated on molar concentration of ions C (kmole/m^3), then density of the stream: $\Phi = 9C = -U_m ZFC\frac{d\varphi}{dx}$.

Generally transport of ions is defined by two factors: by gradient of concentration and action of an electric field

$$\Phi = -D\frac{dc}{dx} - U_m ZFC\frac{d\varphi}{dx}.$$

A.Enshtein has proved that the coefficient of diffusion is proportional to temperature $D=U_mRT$, then:

$$\boxed{\Phi = -U_m RT\frac{dc}{dx} - U_m ZFC\frac{d\varphi}{dx}} \quad (1)$$

(1) is ***the Nernst-Planck's equation***.

3. Active transport of substances. Molecular organization of systems of active transport. Sodium-potassium pump

Transport of substances from a cell to environment and from environment into a cell can be carried out not only with help of various kinds of passive transport owing to which gradients of a cell tend to reduction. Passive transport always aspires to equalize the non-uniformity in distribution of substances between the cell and environment.

Alongside with passive transport in membranes of a cell there is a transport of molecules and ions aside the greater electrochemical potential (molecules are transferred to the area of their greater concentration, and ions against the force working on them from the part of an electric field).

At this transport the cell should make the certain work and to spend on it free energy. Such transport of substances is known as **active transport.** The majority of scientists hold the opinion that in cells there are the pumps working due to free energy of hydrolysis of ATP carried out by special transport proteins, that are known as **ATPase** (or enzymes – carriers); **ATPase** is denoted by letter E. There are three basic systems of active transport of ions in the living cell, providing transport of ions Na^+ and K^+; Ca^+ and H^+ through biological membranes.

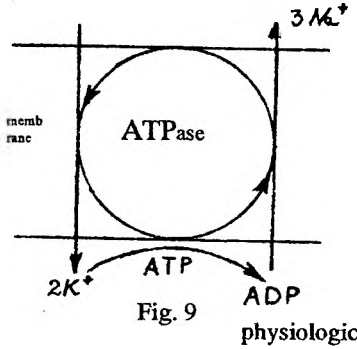


Fig. 9

A plenty of works is devoted to research of active *transport of ions of potassium and sodium* through cellular membranes. It speaks their big role in such important phenomena as generating of bioelectric potentials and transporting out of excitation. In opinion of lines of scientists in a membrane there is a mechanism named connected «sodium - potassium pump», which provides presence of two counter streams of these ions during physiological rest of the cell, i.e. in membranes there is

one general carrier of ions of potassium and sodium. Three ions of Na^+ transferred from a cell separates on its external surface; then to the carrier join some ions of potassium, which are transferred on the internal surface of the membrane.

Let's consider the basic stages (on Vladimirov Yu.A.) of the transport of K^+ and Na^+ ions connected with hydrolysis of ATP through biological membranes. Process of transport of ions K^+ and Na + proceeds in some stages:

1. The first stage of work of carrier E: linkage on the internal surface of the membrane of substrat: three ions of Na^+ and ATP in the complex with Mg^{2+}. At this stage which is made active by ions of Na^+ occurs phosphorylation of E inside the cell and from the complex [Mg^{2+} -ATP] there is the complex [Mg^{2+} -ADF]:

$$3Na^+ + [Mg^{2+} - ATP] + E \rightarrow [Mg^{2+} - ADF] + (3Na^+)E \sim P.$$

Ions of sodium join the certain center of linkage on the surface of carrier E.

2. Transport of the center of linkage on the external surface of the membrane (translocate №1). Such transport is accompanied by change of spatial structure of the ion - transport complex:

$$(3Na^+)E \sim P \rightarrow E \sim P(3Na^+).$$

3. Detachment on the external surface of $3Na^+$ and their replacement by $2K^+$ from environment (exchange of ions):

$$E \sim P(3Na^+) + 2K \rightarrow E \sim P(2K^+) + 3Na^+.$$

4. Disconnection of rest of the phosphoric acid P (designed by Φ):

$$E \sim P(2K^+) \rightarrow E(2K^+) + H_3PO_4.$$

5. Transport of ions of the center of linkage with ions of potassium on the internal surface of the membrane (translocate №2):

$$E(2K^+) \rightarrow (2K^+)E.$$

6. Disconnection of $2K^+$ and connection of $3Na^+$ and phosphorylation of E:

$$3Na^+ + [Mg^{2+} - ATP] + (2K^+)E \rightarrow 2K^+ + [Mg^{2+} - ADP] + (3Na^+)E \sim P.$$

Sodium-potassium pump works due to energy of hydrolysis of ATP with formation of molecules of ADP and inorganic phosphate:

$$ATP=ADP+P.$$

Work of the pump is convertible. The gradient of concentration of ions promotes synthesis of molecules ATP from molecules of ADF and phosphate P:

$$ADP+P=ATP.$$

It has experimentally been proved that for lines of tissues at expenditure of one ion of ATP 3-4 gramme - equivalent of ions of sodium are transferred.

Till now it was not possible to find out one of the major questions at work of the pump: why on the internal surface of a membrane the carrier has affinity to sodium, and on external to potassium.

Transport of $2K^+$ inside of a cell and emission of $3Na^+$ outside results to transport of one positive charge from cytoplasm to environment.

It promotes to occurrence of membrane potential (with "minus" inside of a cell). Thus, Na^+-K^+ - pump is electrogenic.

The value of work which is necessary for making for transport of ions at functioning of a carrier depends as on gradients of concentration of K^+ and Na^+ and from membrane potential $\varphi_м$:

$$\Delta G = 2RT \ln \frac{|K^+|_{in}}{|K^+|_o} + 3RT \ln \frac{|Na^+|_o}{|Na^+|_{in}} + ZF\varphi_M,$$

where Z=1, as in the cycle of work of E one positive charge is transferred to area of higher concentration; $[K+]_{In}$ and $[Na+]_{In}$ are concentrations of ions in the internal medium; $[K+]_o$ and $[Na+]_o$ are concentrations of ions outside; F is number of Faraday; T is absolute temperature of a cell; R is universal gas constant.

As shows calculations, in a nervous fibre of squid this energy is 41.2 кJ/mole. About the same work makes in each cycle carrier E in cytomembrane of muscular cells.

LECTURE №18

BIOELECTRIC POTENTIALS

1. Membrane potentials and their ionic nature

All processes in organism are accompanied by occurrence in a cells and tissues of electric potentials which have received the name of bioelectric.

Distinguish the *oxidation-reduction* potentials arising owing to transportation of electrons from one molecule to anothers. There are *membrane* potentials which arise owing to a gradient of concentration of ions and their transport through a membrane. The biopotentials registered in an organism are basically membrane. They are accessible for measurement and are used in the diagnostic purposes.

The ***membrane potential*** is called potential difference between *internal and external* surface of a membrane:

$$\varphi_м = \varphi_{in} - \varphi_{ex}. \qquad (1)$$

Let's consider the example of occurrence of potential on a membrane (semipermeable). Let the vessel represented on fig. 1 is divided by the membrane not passing the organic connections; both compartments are filled by solution of KCl and then in the compartment 2 we add potassium salt of hemoglobin (KHb). Potassium ions will pass from the right half of vessel to the left owing to a difference of their concentration ($C_1 > C_2$). Negative ions of Hb^- concentrates on the right party of the membrane and keep positive ions of potassium at the left. The membrane is polarized, on it arises membrane potential determined on the basis of the following equation (**Nernst equation**):

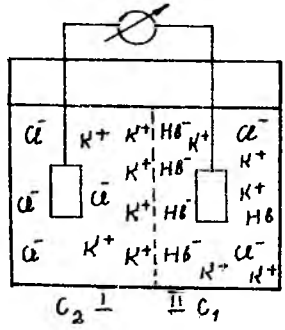


Fig. 1

$$\varphi_м = \frac{RT}{ZF}\ln\frac{C_1}{C_2} \qquad (2)$$

where C_1 is concentration of ions in the area from there is a diffusion; C_2 is concentration of ions in the area where there is a diffusion; T is absolute temperature; F is number of Faraday (96500 C/mole); Z is a charge of an ion (in units of elementary charge); R is a universal gas constant (8,3 J/C mole). As follows from the equation, the *membrane potential depends on temperature and from value of a concentration gradient of ions diffusing through a membrane.*

Something similar to the specified example takes place in a living cell in which concentration of potassium ions is more than in environmental intercellular liquid.

The membrane theory of biopotentials is well theoretically developed and confirmed by brilliant experiments. It has been put forward in 1902 by Bernstein. However only in 50th years this theory has been advanced and experimentally proved by Hodikin to which posess the basic ideas and theories about a role of

ionic gradients in occurrence of potentials and about the mechanism of distribution of ions between a cell and environment.

2. Resting potential

The ***resting potential*** *is the stationary difference of the electric potentials registered between external and internal surfaces of a membrane in not excited condition.*

The resting potential is defined by different concentration of ions on the different sides of a membrane and diffusion of ions through a membrane.

In a resting normally functioning cell always there is a potential difference between cytoplasm and environment. In 1838 Matteuchi for the first time has established, that the external surface of muscles is charged positively and internal is negative. This potential difference peculiar to a condition of rest also has received the name *of the resting potential* or *membrane potential.*

Certain time counted, that the resting potential arises owing to damage of a cell or a tissue between damaged and undamaged sites. Damaged part has negative charge in relation to intact. However it was revealed, that the potential difference can arise and for the undamaged cells. Experimentally value of the resting potential is measured with the help of microelectrodes one of which was entered inside of the cell (fig. 2) and the second was located in extracellular liquid (1). If both electrodes to place into extracellular liquid, the device will not record a potential difference (2).

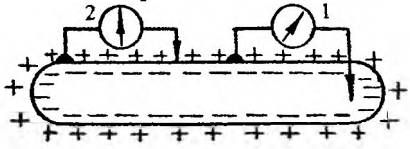


Fig. 2

According to the modern views, the reason of occurrence the resting potential of cells is non-uniform distribution of potassium ions between contents of a cell and environment. Concentration of ions of potassium in cells at 20-40 times exceeds their contents in environment. This point of view is based that ions of potassium inside a cell are in a free condition and can easily to diffuse.

Surplus of positive charges of ions of potassium inside of cells is compensated basically by organic anions of acetic acid and others, which maintenance in cells rather small. It agrees of Hodgkin, Huxli, Katz a cellular membrane permeable in a condition of rest only for ions of potassium. Ions of potassium diffuse due to a gradient of concentration through a cellular membrane in environmental liquid, anions cannot penetrate through a cellular membrane and they are remain on its internal side. The external surface of the membrane thus is charged positively and internal is negative. If to accept, that the the resting potential is defined only by diffusion of ions of potassium from cytoplasm outside, its value φ_M can be found from Nernst equation for equilibrium of membrane potential:

$$\varphi_M = -\frac{RT}{F} \ln \frac{[K^+]_{in}}{[K^-]_{ex}}. \qquad (3)$$

Potential difference between the internal and external surface of a membrane of the various cells designed on Nernst equation is very close to measured in experiment with the help of endocellular microelectrodes.

At measurement of the resting potential it was found out that in one cases its measured value corresponds to the value theoretically calculated on Nernst equation, in other cases between the measured and calculated values there are *significant distinctions*. It speaks that on value of the resting potential alongside with ions of K^+ other ions penetrating through a membrane influence and shows that «potassium theory» of the resting potential is imperfect, incomplete and not completely explains the observable facts.

For the quantitative description in conditions of permeability of membranes for several ions Hodgkin and Katz used representation that the resting potential not equilibrium, but stationary by the nature, i.e. it reflects a condition of system, when through a membrane continuously there are counter streams of K^+, Na^+, Cl^-. Total density of the stream is equal to the sum of density of positively charged ions (cations) and minus the sum of density of streams of univalent anions: $\Phi=\Phi_{K}^{+}+\Phi_{Na}^{+}-\Phi_{Cl}^{-}$.

Before Φ_{Cl} there is the mark "minus", which is taking into account the negative charge of the ion of chlorine.

The total density of the stream at stationary condition is equal to zero ($\Phi=0$), i.e. number of the different ions which are taking place per unit of time through a membrane inside of a cell is equal to number ions leaving the cell through the membrane.

It is possible to receive equation for the resting potential on a membrane for 3 ions:

$$\varphi_M = -\frac{RT}{F}\ln\frac{P_K[K^+]_i + P_{Na}[Na^+]_i + P_{Cl^-}[Cl^-]_e}{P_K[K^+]_e + P_{Na}[Na^+]_e + P_{Cl^-}[Cl^-]_i}.$$

This equation is known as the equation of stationary potential of ***Goldman – Hodgkin - Katz*** or ***Goldman equation***. The last transforms to the equation of Nernst, if permeability of one of ions is much higher than for others. For example, in huge axon of squid $P_K:P_{Na}:P_{Cl}=1:0.04:0.45$, i.e. P_K appreciably is higher than for other ions, therefore the numerator and the denominator of the equation are approximately equal accordingly $P_{K^+}\cdot[K^+]_i$ and $P_{K^+}\cdot[K^+]_e$ and the equation transforms to the equation of Nernst for K^+, i.e. the Nernst equation is the special case of the equation of Goldman. Calculated on the Goldman equation the potential of a huge axon of a squid coincides with found in experiment.

3. Action potential

The action potential *is the electric impulse caused by change of ionic conductivity of a membrane in connection with propagation on nerves and muscles of a wave of excitation.*

Experiments on research of action potential are leaded (basically Hodgkin and its employees) on huge axons of a squid by the method of microelectrodes and by the method of labelled atoms.

Cells of various tissues at action of various irritants (mechanical, thermal, electric) are capable to pass to the condition of excitation. Excitability is an ability of cells to a prompt reply to the irritation, exhibited in aggregate of physical, physical and chemical processes and functional changes.

Experiments shows, that the excited site of a cell becomes electronegative in relation to not excited, that shows on redistribution of ions in the excited site. This redistribution of ions have temporary character and after the termination of excitation again is restored the resting potential.

It has been shown, that occurrence of action potential is connected with increase of permeability of membranes for ions of sodium and the subsequent amplification of diffusion of these ions on a concentration gradient inside of the cell that results to change (reduction) of membrane potential or depolarization of membrane.

Reduction of membrane potential of below a critical level results to the even greater increase in permeability of the membrane for sodium and increase in permeability is accompanied by amplification of diffusion of Na^+ in cytoplasm, that causes still big depolarization of membrane. On data of Hodgkin the ratio of the coefficient of permeability of the membrane of axon of squid for the phase of depolarization: $P_K:P_{Na}:P_{Cl}=1:20:0.45$. If to compare it with the similar parity in the rest condition : $P_K:P_{Na}:P_{Cl}=1:0.04:0.45$, it is visible, that for K^+ and Cl^- in the first phase of excitation permeability does not change and for Na^+ it increases in 500 times. The amplified stream of positively charged ions of Na^+ inside of a cell causes in the beginning disappearance of a superfluous negative charge on the internal surface of the membrane (***the phase of depolarization***), and then results to overcharding of membranes, i.e. the external surface of the membrane on the site of excitation becomes negative in relation to not excited. This phase of development of potential of action is called ***the phase of reversion***.

Coming in a cell of ions of sodium proceeds until the internal surface of membrane will not get the positive charge sufficient for the equilibration of the gradient of concentration of Na^+ and the terminations of its transition inside of the cell.

The raised permeability of the membrane for ions of sodium proceeds very short interval of time (0.5-1ms). Then the stream of K^+ ions from the cell amplifies, that results to restoration of the resting potential on the membrane, i.e. its external surface due to left outside of K^+ ions again gets the positive charge and internal negative.

This phase during which polarization of a membrane comes back to the initial level is called the phase of ***repolarization***.

The phase of repolization is always more long than the phase of depolarization. Thus, restoration of the resting potential occurs not as result of return moving ions of Na^+, but owing to output of equivalent number of ions of K^+.

In the dormant (rest) period the normal condition of concentration is restored, that is caused by work of Na – K pump which provides active transport of these ions.

Action potentials represent short-term quickly changing potentials. Duration of action potentials in nervous fibres makes 1 ms, in the cardiac musle ≈ 300 ms. The value of action potential of a nerve is equal 100-110 mV, for a skeletal muscle is 120 mV. Change of potential on a membrane can be presented graphically on the example of huge axon of a squid (fig. 3). So in axon of the squid the the resting potential is equal (– 45mV) and the internal surface of the membrane is electronegative. At excitation the potential of the internal surface of the membrane becomes equal to +40mV and there is inversion of the mark of potential. Hence, total value of action potential makes 85mV that considerably exceeds the resting potential.

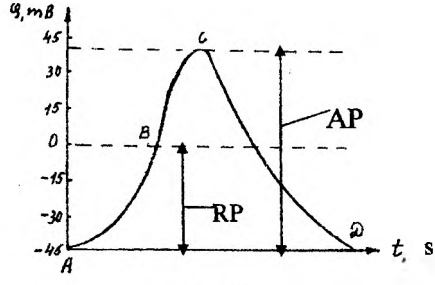


Fig. 3

On fig. 3: AB is the phase of depolarization; BC is the phase of reversion; CD is the phase of repolarization, AP is action potential, RP is resting potential.

Thus, on the basis of generalization of the big experimental material it has been established, that the action potential arises as result of adittional in comparison with rest diffusion of sodium ions from the environment into the cell. Actually, formation of action potential is caused by two ionic streams through a membrane.

Streams of ions of Na^+ and K^+ are approximately equal on value, but are shifted on time. Due to this shift in time occurrence of potential of action is possible. If streams of sodium and of potassium through a membrane synchronized, they would compensate each other and any change of membrane potential could not occur. Prominent feature of potential of action is the period of refractory during its development and the residual phenomena during of 1-3 ms after removal of excitation. It testifies that permeability of a membrane is finally restored not right after the termination of action potential.

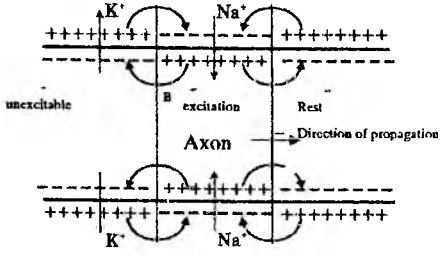


Fig. 4

4. Propagation of action potential on a nervous fibre

The action potential having arisen in one excited site of a nervous fibre is quickly distributed on its surface. Propagation of action potential (AP) is caused occurrence of the so-called ***local currents*** circulating between excited and not excited sites. We shall consider propagation of excitation on

example of propagation of a nervous impulse on axon. In a condition of rest external surface of membrane has the positive charge and internal is negative. At the moment of excitation polarity of the membrane (on the excited site) changes on the negative (fig. 4). As a result of it, between excited and not excited sites arise a potential difference that results to occurrence of electric currents between these sites (local currents or currents of action). On the surface of the membrane the current flows from not excited site to excited, and inside to the opposite direction.

The local current causes increase of permeability of the next not excited sites of a membrane that reduces their resting potential on absolute value, i.e. the potential raises. When depolarization will reach of critical (threshold) value, in this site there is the action potential. On the site earlier excited at this time there are regenerative processes of repolarization. This process repeatedly repeats and provides propagation of the impulse of excitation further on based sites.

Under influence of local currents the wave of excitation is propagated without attenuation in one direction. There can be a question, why excitation is distributed on axon not in both sides from a zone of excitation (in fact local currents has reached flow in both sides from the excited site). The matter is that excitation can be propagated in the field of a membrane which are taking place in the rest condition, i.e. in one side from the excited site of axon. In other side the nervous pulse cannot be propagated, as areas through which has passed excitation some time remain unexcitable (refractory).

In nervous fibres character of propagation of excitation depends on presence or absence in them of *myelin* sheaths (neurilemma), which increase specific resistance of a membrane and its thickness. In non-myelin nervous fibres excitation is propagated continuously along all membrane. All sites of membrane thus become excited. In myelin nervous fibres having thick aheaths, which through 1-3 mm interrupt with formation of nodes of Ranvier (sites free from myelin) (fig. 5), excitation is propagated a little on other. Myelin is dielectric, therefore local currents through myelin sheath cannot proceed. They circulate between nodes of Ranvier. At excitation of one node of Ranvier between it and the next node there are local currents and the pulse as though jumps on the second interception, from the second on the third, etc. Such way of carrying out of a nervous impulse is called *saltatory*. At blocking of one from nodes of Ranvier by any anesthetizing substance, the impulse at once is transferred to the third node. At blocking at once of two interceptions the pulse cannot be propagated further (resistance between 1-st and 4-th node is great). In myelin fibres speed of propagation of the nervous pulse in 10 times is higher than in non-myelin at identical diameter of fibres. Calculations shows, that speed of propagation of excitation on smooth non-myelin nervous fibres is approximately proportional to the square root of radius ($\vartheta \approx \sqrt{r}$). These speeds of proragation at non-vertebrates

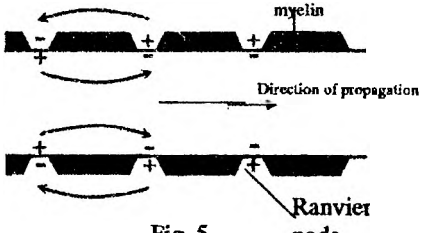


Fig. 5

reach of 20-30 m/s and are provided with the big diameter (up to 1 mm) of their fibres. Defect of myelin sheath results to infringement of propagation of action potential on the nervous fibre and to heavy nervous diseases.

LECTURE №19

INTERFERENCE AND DIFFRACTION OF LIGHT. PRINCIPLE OF X-RAYS CRYSTAL ANALYSIS

1. Interference of light waves. Coherence

Interference of light is used in medicine in interferometers, interference microscopes, at determination of sensitivity of retina and in many other cases.

Superposition of two or several waves resulting to steady in time amplification of oscillations in some points of space and attenuation in others, is known as **interference.** In usual conditions often there is superposition of light waves from various sources, but interference of light is not observed. Each such source (a lamp, a flame, the Sun, etc.) represents set of big number of radiating atoms. Difference of phases of oscillations which radiate such sources *will not be constant* and quickly and randomly changes in time. The interference of light can arise only from the coordinated ***coherent sources,*** which provide constant in time difference of phases $\Delta\varphi$ of composed waves in various points. The waves adequate to this condition are known as *coherent*, i.e. ***at identical frequency (length of wave) have the constant phase difference.*** The interference could be carried out from two sine wave waves of identical frequency. It is possible to receive coherent waves having divided a wave from one source on two parts (by reflection or refraction) and then to reduce these two waves together (fig. 1). Pinholes are equidistant from S and are close to each other. Spherical waves spread out from S. Spherical waves also spread out from pinholes.

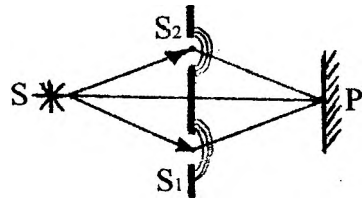


Fig. 1

Two waves received by such way will be coherent and at superposition can interfere. In practice division of one light wave on two waves can be carried out by means of the opaque screen with two small apertures. According to principle of Huygens-Fresnel, source S (fig. 1) creates in apertures of the screen sources of secondary waves S_1 and S_2 , which will coherent. The first supervision of interference was carried out by T.Young in 1802, having passed solar beams through very small aperture in the opaque screen. The second way of reception of coherent sources is based on reflection of light from two flat mirrors established under the angle α, close to 180^0. This optical system is called ***mirrors of Fresnel.*** Imaginary images S_1 and S_2 serve as coherent sources of basic source S (fig. 2). The picture of interference arises in the point P.

Next way consists in reception of imaginary image S' of the source S by the special single-layered mirror ***(Lloyd's mirror).*** Sources S and S' (fig. 3) can be

considered as coherent. They create the interference picture in the point A of screen Э.

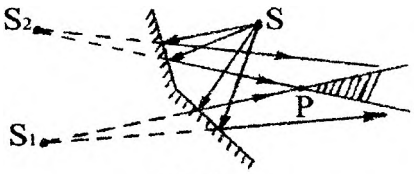


Fig. 2

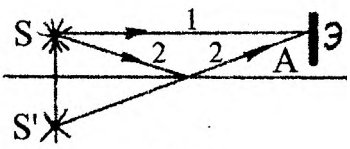


Fig. 3

For monochromatic light the interference picture represents a number of alternating dark and light strips (maxima and minima). In case of white (not monochromatic) light maxima for different λ settle down in the different places, because of what interference strips extend and get iridescent colouring.

Formation of coherent waves and their interference occurs also at hit of light on thin transparent plate or film. Due to reflection of light from both surfaces of the film there is a splitting of falling light beam, and there are conditions for interference of light. It speaks occurrence of iridescent colouring of soap bubbles, coloring of film of oily substances on surface of water or of wings of butterflies and other insects, coloring of internal surface of bowls, feathers of some birds (humming-bird, peacocks).

Let's execute calculation of interference picture, when two coherent waves from sources S_1 and S_2 pass different ways r_1 and r_2 and interfere in the point M

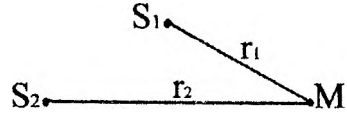


Fig. 4

(fig. 4), i.e. between them arises a **path difference** $\Delta=r_2-r_1$(geometrical difference of ways). If waves are propagated in the medium with refractive index of n, we can speak about **optical path difference** $\delta=n\cdot\Delta$. Oscillations of vectors of electric intensity E in the point M removed from sources on distance r_1 and r_2 accordingly from each source, occurs under the harmonious law (amplitudes of both oscillations we shall accept identical and we shall designate by E_m). Then $E_1 = E_m \cdot \cos 2\pi(\nu t - \frac{r_1}{\lambda})$,

$E_2 = E_m \cos 2\pi(\nu t - \frac{r_2}{\lambda})$.

Addition of harmonious oscillations of one direction and identical frequency with the phase difference $\Delta\varphi = \varphi_1 - \varphi_2$ as it has been shown earlier, gives resulting oscillations of the same frequency. The amplitude of resulting oscillation of light vector is expressed as:

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\Delta\varphi, \qquad (1)$$

and for the case considered by us:

$$E = \sqrt{E_m^2 + E_m^2 + 2E_m \cdot E_m \cdot \cos\Delta\varphi} = E_m\sqrt{2(1+\cos\Delta\varphi)} = 2E_m\cos\frac{\Delta\varphi}{2}. \qquad (2)$$

From the formula $\left[\cos^2\varphi = \frac{1+\cos 2\varphi}{2}\right]$ follows $\frac{1+\cos\Delta\varphi}{2} = \cos^2\frac{\Delta\varphi}{2}$.

Let's determine:

$$\Delta\varphi = \varphi_1 - \varphi_2 = 2\pi(\nu t - \frac{r_1}{\lambda}) - 2\pi(\nu t - \frac{r_2}{\lambda}) = \frac{2\pi}{\lambda}(r_2 - r_1) = \frac{2\pi}{\lambda}\Delta.$$

If light is propagated in the medium with the refractive index n:

$$\Delta\varphi = \frac{2\pi}{\lambda}\delta.$$

Substituting in the formula (2) value of $\Delta\varphi$, we shall receive:

$$E = 2E_m \cos(2\pi\frac{r_2 - r_1}{2\lambda}) = 2E_m \cos\frac{\pi\cdot\Delta}{\lambda},$$

i.e. E depends on value Δ.

In the points where Δ is equal to ***odd number of lengths of half waves****, i.e.:* $\boxed{\Delta = (2k+1)\frac{\lambda}{2}}$ *k=0,1,2,3,..., value is* $\cos\frac{\Delta\varphi}{2} = 0$ *and the amplitude of resulting oscillation is equal to zero. In these points are formed* ***interference minima****. If the difference of pathes is equal to* ***even number lengths of half waves*** *(or to the integer of lengths of waves)* $\boxed{\Delta = 2k\frac{\lambda}{2} = k\lambda}$ *that* $\cos\frac{\Delta\varphi}{2} = 1$ *and* $E=2E_m$ ***(interference maximum)****.*

2. Diffraction of light. Diffraction of light on a slit in parallel beams

At propagation of waves in the medium containing heterogeneity the phenomenon of *diffraction* is observed. ***Diffraction*** *is bending by waves of the obstacles meeting on their paths, or in more comprehensive sense diffraction is any deviation of advance of waves near to hindrances from laws of geometrical optics*. The opportunity of supervision of diffraction depends on *a parity of wavelength of light and the sizes of obstacle.* The phenomenon of diffraction is shown more strongly, if the ***sizes of an obstacle (slit) are comparable to length of a wave*** λ. The phenomenon of diffraction of light naturally is not observed almost, because the sizes of the most part of bodies environmental us are *incommensurable to the wavelength of light*. Owing to diffraction the shadow image of object ceases to be similar to the subject.

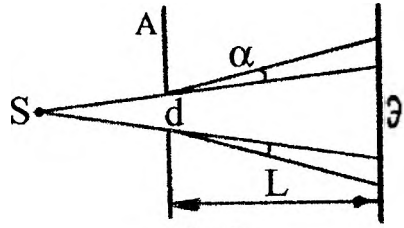


Fig. 5

In experience diffraction can be observed at formation of shadow from an obstacle as thin wire or hair and also at passage of light through an aperture of small size (share of millimeter) (fig. 5).

If between the screen E and a source of monochromatic light S to place other

opaque screen A with the small aperture d, the border of the geometrical shadow will not be sharp. It is especially appreciable, when the size d of aperture is very small in comparison with distance L from the screen up to the aperture (d <<L). Then the stain on the screen will be submitted as system of the alternating light and dark rings gradually passing each other, grasping the area of a geometrical shadow and also leaving for its limits. It speaks about not rectilinear propagation of light from the source S, about bending of light waves at edges of aperture in the screen A. At use of white light diffraction picture gets iridescent colouring. Diffraction of light speaks occurrence of iridescent rings around of light source, when air is sated with a fog or dust, colouring of pearls (diffraction of white light on the alien smallest contained in it particles).

Diffraction is defined by wave properties of light and to explain this phenomenon is possible by *principle of Huygens-Fresnel according to which: **points of medium of which were reached with front of a wave are sources of elementary secondary waves, which are coherent.*** Let light from the source S falls on the screen through the round aperture "ab" in the screen B (fig. 6). Each point of site "ab" of front of light wave S_1 is the secondary light source. Secondary sources are coherent, therefore beams (waves) proceeding from them 1,2; 3,4, etc. will be interfere among themselves.

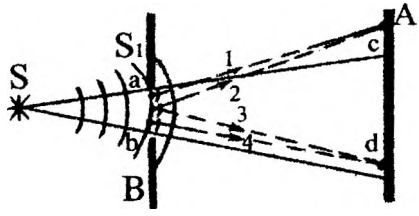


Fig. 6

Depending on path difference of beams on the screen A in the points c; d, etc. will arise interference maxima and minima, i.e. ring-shaped diffraction picture.

To define result of diffraction in some point of space, it is necessary to calculate according to principle of Huygens-Fresnel the interference of the secondary waves, which have got in this point from a wave surface. For a wave surface of the any form such calculation is difficult. But on occassion (the spherical or flat wave surface, a symmetric arrangement of a point concerning wave surface S_1 and opaque barrier A) calculations are simple. The wave surface thus is breaking on separate sites (*zones of Fresnel)* that simplifies mathematical calculation.

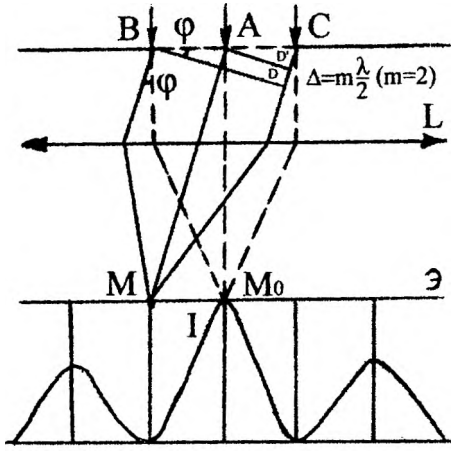


Fig.7

Let's consider result of diffraction of flat monochromatic waves on the slit which have been cut out in the opaque barrier (fig. 7) and

having constant width BC=a. Wave falls normally to slit as plane-parallel bunch of light. All points of the wave surface open by the slit are the centers of the secondary waves extending behind the slit on all directions. To represent all these secondary waves is impossible, therefore on fig. 7 are shown only secondary waves extending under the angle φ. Beams, diffract under the angle φ will be collected in the point M of the screen and interfere. At $\varphi = 0$ all waves will come in the point M_0 in the identical phase and will strengthen each other, on the screen will appear the most light strip (the central maximum). To define result of interference at $\varphi \neq 0$, from the point B we shall lead perpendicular BD to the direction of bunch of secondary waves. Optical ways of beams from BD up to point M of the screen are identical (lens L of additional path difference does not bring), therefore the path difference DC of extreme beams is equal: $\Delta = a \sin \varphi$. We break DC into the pieces equal to $\lambda/2$ (in figure 2 pieces are shown). Generally this path difference contains m half waves: $a \sin \varphi = m \frac{\lambda}{2}$. Let's lead from the point D' straight line D'A parallel of BD and we shall divide BC into two equal *zones of Fresnel* BA=AC. To any secondary wave going from any point of one zone of Fresnel it is possible to find in the next zones such secondary waves, that the path difference between them will be $\lambda/2$. For example, the secondary wave going from the point C in the chosen direction passes up to the point M distance on $\lambda/2$ more, than the wave going from the point A, etc. Hence, the secondary waves going from two next zones will extinguish each other, since differ on phase on π. The number of zones stacked in a slit, depends on length of wave λ and angle φ. *If slit BC is* ***broken at construction on odd number of Fresnel zones*** *$m = (2k+1)$, and DC on odd number of the pieces equal to $\lambda/2$, in the point M is observed* ***diffraction maximum***, *i.e.:* $\boxed{a \sin \varphi = (2k+1)\frac{\lambda}{2}}$ *$k=0,\pm1, \pm2,\ldots$ is order of the maximum.*

Condition of ***diffraction minima****:*

$$\boxed{a \sin \varphi = 2k\frac{\lambda}{2} = k\lambda} \quad (m=2k).$$

On fig. 7 is shown the case, when m=2, that corresponds in the point M to the diffraction minimum. Thus, on the screen E the system of light (maxima) and dark (minima) strips symmetrically located to the left and to the right from central ($\varphi = 0$) the brightest strip will turn out. Intensity I of other maxima decreases on measure of removal from the central maximum (see fig. 7). If the slit is shined with white light, on the screen E is formed the system of color strips; only central maximum will keep color of falling light, as at $\varphi = 0$ amplify all length of waves of light.

3. Diffraction grating. Diffraction spectrum

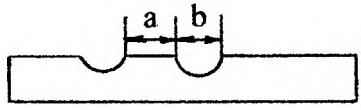


Fig. 8

Diffraction grating is the optical device representing a glass plate on which by the diamond edge renders a plenty of parallel lines with intervals between them. The intact

glass between lines serves as slits of the grating. Total width of the slit "a" (fig. 8) and interval "b" between slits is named *the constant or the period of diffraction grating: d=a+b.* The best diffraction gratings have up to 1200-1500 slits per millimeter (n), the period of diffraction grating is $d = \frac{1}{n}$, n is general number of lines of a diffraction grating.

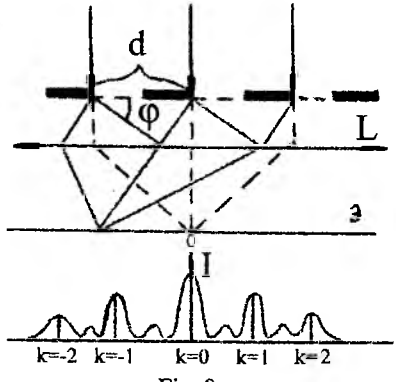


Fig. 9

Let's consider diffraction (fig. 9) of the flat monochromatic wave falling normally on the diffraction grating. If on the grating falls the beam of monochromatic beams, the secondary waves going from slits on all directions are coherent and will interfere, forming a diffraction picture. If between the screen and the diffraction grating to place a collecting lens L (the screen is located in the focal plane of the lens), there is diffraction picture which is growing out of two processes: diffraction of light from each separate slit and interference of light from slits. The basic features of this process are defined by the second phenomenon.

Let's consider the beams falling on the left edges of slits (N=3). Due to diffraction, light from slits will propagated in every possible direction. The path difference of beams from extreme points of two next slits and diffracting under the angle φ, is defined: $\Delta = d\sin\varphi$. If this path difference will be equal to zero or an integer number of wavelengths, at interference there are main maxima for which the condition satisfies: $d\sin\varphi = k\lambda$, where k is the order of the main maxima (k=0; ±1; ±2; ±3; ...). At performance of the condition $d\sin\varphi = (2k+1)\frac{\lambda}{2}$ will arise interference minima.

Maxima will symmetrize concerning central (k=0; φ =0). ***Expression*** $d\sin\varphi = k\lambda$ ***is named the basic formula of a diffraction grating.*** Possible number a maximum is limited, it cannot be more than d / λ. Between the main maxima are formed minima (additional), which number depends on number of slits of a grating (N). Between the main next maxima is settled down (N-1) additional minima. On fig. 9 distribution of intensity of maxima on the screen for the grating with N=3 is shown. At plenty of slits separate additional minima practically do not differ and all space between the main maxima looks dark. As the amplitude of light oscillations in maxima is proportional to number of slits, intensity of maxima is proportional to the square of number of slits (N^2), i.e. than is more number of slits of a diffraction grating, than the main maxima are especially sharp: $E_m \sim N$; $I \sim E_m^2$; $I \sim N^2$.

If on a grating falls white light for all values of wavelengths position of maxima of zero order coincide (k=0 falls; φ =0), position of maxima of higher order will be various (more λ than is more φ) for the given value k. Therefore the central maximum will look like a narrow white strip (fig. 10) and each of lateral maxima represents multi-coloured strip of certain width. All spectra are symmetric concerning zero maximum and are inverted to it by short-wave Ф (violet) edge. The most intensive is the spectrum of 1-st order (k=1).

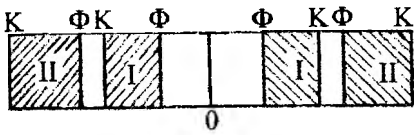


Fig. 10

Thus, diffraction grating decomposes complex light into a spectrum and consequently with success is applied in spectral devices, for example, in diffraction spectroscope; it is a device, employee to measurement of length of light waves, i.e. for carrying out of the spectral analysis.

4. Diffraction of electromagnetic waves on spatial structures. Bases of X-ray crystal analysis

Diffraction of waves can occur on small heterogeneities and particles. Most simple case is when heterogeneity forms periodic structure. Diffraction gratings are examples of the periodic structure. 3 measured structure crystals are natural, in which as the scattering centers serve units (atoms, ions) of a crystal grating. In a crystal it is possible to allocate directions along which diffract waves reinforce each other. The beam of monochromatic radiation, passing through such structure forms on the screen plane two-dimensional diffraction picture, i. e. system of the light spots (maxima) located in the certain order. On the location of these maxima, their relative intensity and the wavelength it is possible on the basis of corresponding calculations to define spatial 3 measured structure of the object, which has caused diffraction. As such objects there can be large molecules.

However precise diffraction picture can be received only, if the period of structure **d** will be a little bit more of the wavelength λ (d> λ). This restriction does not allow carring out diffraction of light on crystals, as the period of a crystal grating (distance between planes of crystal) in thousand times (~10^{-10} m) is less than length of light wave. However for X-rays the condition d> λ is carried out. In 1912 M. Laue having passed the narrow beam of X-rays allocated by lead diaphragms through the monocrystal (the crystal plays a role of a spatial grating) has received on photographic plate the diffraction picture (lauegram) as

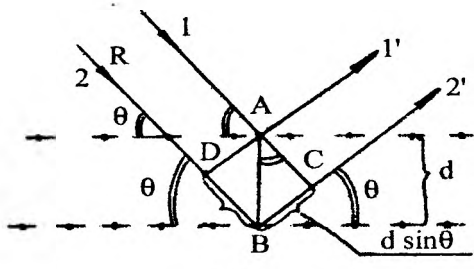


Fig. 11

dark spots (diffraction maxima). *The order of arrangement of particles in a grating of the crystal defines the order and symmetry in arrangement of diffraction maxima.*

Diffraction of X-rays can be observed at reflection from a crystal (fig. 11). Beams 1 and 2 interfere having reflected from two next layers (d is distance between the next nuclear layers). Path difference Δ =DB+BC of beams is equal to $2d\sin\theta$, where θ is the angle of sliding. These waves reinforce each other, i.e. form maxima in directions for which the path difference is multiple of λ. Therefore is possible to write down:

$$\boxed{2d\sin\theta = k\lambda}, \qquad (3)$$

where $k = \pm1; \pm2; \pm3; \ldots$ is the order of maxima. *It is **Woolf – Bragg's formula.*** At falling of monochromatic X-ray radiation on a crystal under various angles, the maximum will take place for the angles adequate to the formula of Woolf - Bragg. The angle θ is measured in photo of diffraction pictures (on position of diffraction maxima).

X-ray crystal analysis is the method which on diffraction picture received on unknown crystal structure by means of X-rays of known length allows to find the arrangement of particles making this structure (to determine d). It has the big practical value for biology, as it is the most effective method of definition of spatial structure of crystal connections. For monocrystal the method of "rotating crystal" is usually used (fig.12). At rotation of the crystal the different systems of planes get under the falling beam of X-rays in position at which is carried out the condition (3). It means that at the certain angles θ on the photographic plate will appear diffraction maxima. Knowing λ and having determined from experience the θ and order of spectrum k, under the formula (3) calculate distance d between corresponding layers of structural particles.

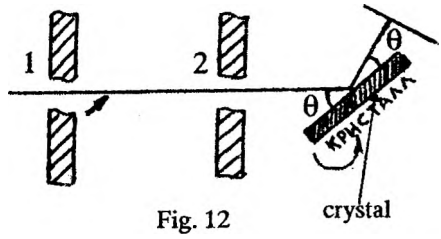


Fig. 12

For polycrystalline bodies use the method of powders (method of Debuy and Sherer) (fig. 13). Among the big number of fine crystals always will be such crystals, for which the condition (3) satisfies. As the condition (3) is identical for many crystals on miscellaneous oriented, diffract beams form the cone, which top lays in researched object in space and the angle is equal to 4θ. ***The X-rays crystal analysis is widely applied at research of structure of biological molecules (DNA, proteins).*** By described method deciphers spatial structures of hemoglobin, ribonuclease, etc. Being based on the analysis of roentgenograms, F.Kric and J. Watson have reproduced spatial structure of DNA and have been awarded with the Nobel Prize. With help of X-rays crystal analysis

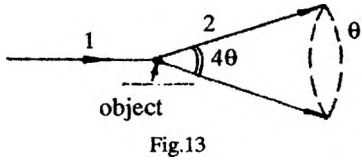


Fig.13

it was possible to understand functioning of molecules of enzymes, to find out structural basis of many hereditary diseases, structure of viruses, etc.

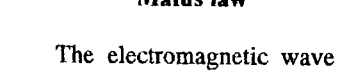
LECTURE №20

POLARIZATION AND DISPERSION OF LIGHT

1. Polarization of light. Light natural and polarized. Malus law

The electromagnetic wave emitted by separate atom can be presented as oscillation of 2 mutual perpendicular vectors of intensities of electrical (E) and magnetic (H) fields. Both vectors change in the plane, perpendicular to velocity vector. Such electromagnetic wave is called *plane-polarized.* By reviewing phenomenon of polarization we further shall conduct all reasonings concerning of vector E, as experience and the theory shows, that chemical, physiological, etc. actions of light on material are stipulated mainly by vector E. Light from the Sun, from filament of bulb, etc. is *unpolarized,* natural. In such light vectors of E from the different elementary radiators have various orientations of oscillations. Projections of vectors E of natural light on the plane, perpendicular to velocity, will look like figured on fig. 1a. All orientations are equiprobable and amplitudes of values E are equal in all directions.

If there is a preferable direction of oscillations, such light is called *part polarized* (fig. 1b). Natural light is possible to turn into polarized, i.e. to polarize with the help of the devices called as **polarizers**. The diagram of polarized light is presented on fig. 1c. Polarizers are capable to pass only component of vector E laying in the certain plane PP′, called as the *principal plane* of a polarizer (fig. 2). Thus through the polarizer passes polarized light, which intensity is equal to half of intensity of incident light: $I = I_{NAT}/2$



a b Fig. 1 c

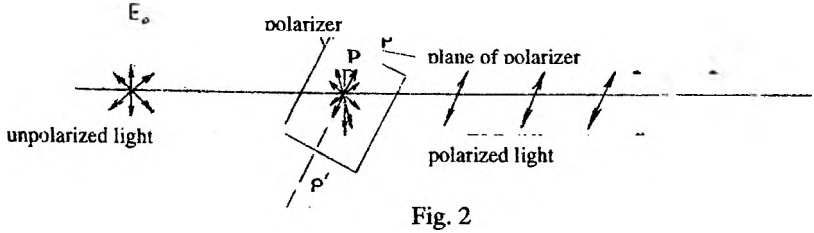


Fig. 2

At rotation of polarizer concerning of the ray of natural light, the vibration plane of plane-polarized light rotates also, but intensity of it does not change.

The polarizer can be used for analysis of polarized light, then it is called as the *analyzer.*

If the plane-polarized light falls on the analyzer of intensity E_0, it passes only component $E = E_0 \cos\varphi$, where φ is angle between principal planes of the

analyzer and polarizer. As light intensity is proportional to square of amplitude of oscillations, then

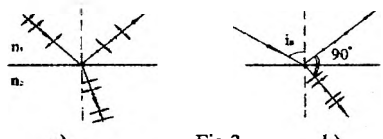
$$\boxed{I = I_0 \cos^2 \varphi}. \qquad (1)$$

The equation (1) is **Malus' law**, where I_0 is intensity of the plane-polarized light, which has left polarizer, I is the light intensity, which has left the analyzer.

Properly from the equation (1), that at rotation of the analyzer concerning the beam, intensity of light which has left the analyzer is changed from 0 up to I_0. If at rotation of analyzer concerning the ray intensity of light, which has left it does not change, the incident light is natural; if light changes under the law of Malus, then light is plane-polarized.

Eye of a person is not capable to distinguish polarized light from natural, but approximately 25-30 % of people have this ability, though almost never suspect about it.

2. Polarization of light at reflection and refraction on border of two dielectrics



a) Fig.3 b)

If the angle of incidence of light on the border of 2 dielectrics is not equal to zero, the reflected and refracted rays appear in part polarized. In the reflected ray oscillations, perpendicular to the plane of incidence prevail (on fig. 3a they are marked out by points); in the refracted ray prevail oscillations parallel to the plane of incidence (on fig. 3a they are marked out by two-sided arrows). The natural ray is conditionally designated by alternating arrows and points.

Degree of polarization depends on the angle of incidence of rays and on the refractive index of reflecting medium. At angle of incidence, satisfying to the requirement

$$\boxed{tg\, i_B = n}, \qquad (2)$$

where n is refractive index of the second medium concerning the first, the reflected ray is completely polarized, refracted is in part polarized, but degree of its polarization the greatest. The **equation (2) is Brewster's law**, angle i_B is called *Brewster's angle*. It is easy to test, that at falling of light under the Brewster's angle, the *reflected and refracted rays are mutually perpendicular* (fig. 3 b).

Thus, the border of two dielectrics or a dielectric and vacuum is a polarizer. The polarity effect of a reflected light is used, for example, at detection from air of a film of petroleum on water plane.

The Brewster's law is inapplicable in case of reflectance of light from a surface of conductors (metals).

It is possible to achieve, that the refracted ray will be completely polarized. For this purpose as polarizer is used the pile of the glass plates located one after another. At realization of the Brewster's law the degree of polarization of refracted ray will increase in accordance with transit of plates.

3. Polarization at birefringence (double refraction)

At transit of light through some crystals the light ray is parted on two rays. This appearance has received the name of **birefringence** (fig. 4).

One of rays on fig. 4 (**o**) fulfils to the Snell refraction law, it is called ***ordinary***; for the second ray which has been marked out by **e** (which is known as ***extraordinary***) the ratio $\frac{\sin i_{INC}}{\sin i_{REFR}}$ does not remain the constant and depends on the direction of ray incidence.

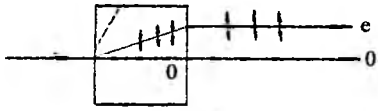


Fig. 4

The ordinary ray at normal falling of light on a surface of crystal passes not refracting as it follows from the Snell refraction law, extraordinary ray (fig. 4) refracts.

Crystals have directions for which ordinary and extraordinary rays are propagated not being parted and with equal velocity. These directions are termed as ***optic axises*** of a crystal. If such direction is one, crystals are termed ***uniaxial***, if two are termed *diaxonic*. To uniaxial crystals concerns the Iceland spar (type of calcium carbonate $CaCO_3$), quartz, tourmaline (the composite aluminosilicate). On fig.4 this direction is shown by the shaped line. Plane which is taking place through optic axis and light ray is called the **main plane**. Both rays, which have left a crystal are completely polarized in the mutually perpendicular planes. Oscillations of an ordinary ray are perpendicular to the main plane and oscillations of extraordinary are in the main plane. Birefringent crystals immediately are not used as polarizers, however from them produce special polarization prisms.

In some crystals one of rays is absorbed more strongly another. This appearance is known as ***dichroism***. The strong dichroism has crystal of the *tourmaline*. In it the ordinary ray is practically completely absorbed on length of 1mm and light which has left is plane-polarized. The same property has the **polaroid**, it is celluloid film on whom is lined the quantity of equally oriented crystals of herapathite (iodine sulphate – quinine). The layer of polaroid depth of 0.1 mm completely absorbs the o-ray. Hence, the polaroid can be used as polarizer.

High-quality polarizer is the polarization prism of the Nicol (or primely the **Nicol**). Action of the Nicol sets up on birefringence of the Iceland spar $CaCO_3$. The Nicol is the prism intagliated by the special mode from iceland crystal, cut almost on diagonal and pasted together by Canada balsam (pitch of spruce fir), which *refractive index (n = 1,550) lays between values of refractive indexes of iceland crystal for ordinary ray (n_0 = 1.658) and extraordinary ray (n_e = 1.486).* It allows having puck up in appropriate way angles of the prism to provide *total reflection* of **o**-ray on the border with Canada balsam. This ray is absorbed by black lower plane of prism. The extraordinary ray goes out

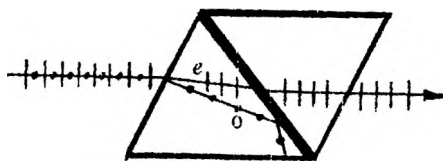


Fig. 5

from prism parallel to lower plane (fig. 5).

Defect of the tourmaline and polaroids in comparison with the Nicol is their bad wavelength characteristics. The white light after their transit becomes painted, while the Nicol is transparent in the visual part of spectrum.

Double refraction is explained by *anisotropy* of crystals (it is dependence of optical properties from direction). The majority of diaphanous crystals are optically anisotropic. In them speed of light and, hence, refractive index are various on different directions.

4. Optical rotation. Polarimetry

At transit of the plane-polarized light through some materials is observed rotation of the plane of oscillation of E. Such materials are termed *optically active.* To them belong crystals (quartz, film-pitch), pure liquids (turpentine, nicotine) and solutions (water solutions of Saccharum, tartaric acid, etc.). Light after escaping of material is plane-polarized, but the vibration plane of its vector E appears rotated on the angle φ. Optical rotation for the first time was revealed on crystals of *quartz.*

For crystals the rotation angle of polarization plane is proportional to the path *l*, traversed by the ray in the crystal:

$$\boxed{\varphi = \alpha l},$$

where α is specific rotation, it is equal to rotation angle of polarization plane of material by the layer of unit length. It is accepted to express it in degree/ mm.

Specific rotation depends on wave length λ of light in which observation is conducted. This dependence is called the ***dispersion of rotatory power***; it is individual for each material.

In solutions the rotation angle of polarization plane is proportional to the path of ray in solution *l* and concentration of active material C:

$$\boxed{\varphi = \alpha \cdot C \cdot l},$$

where α is specific rotation. *l* is accepted to express in dm., C in g / sm^3, φ in degree.

If concentration presented in grammes in 100 sm^3 of solution, then

$$\boxed{\varphi = \frac{\alpha \cdot C \cdot l}{100}}.$$

In dependence on direction of rotation of polarization plane, optically active materials are sectioned into the right and laevorotatory. There is left and dextrorotary quartz (if to look towards to a ray), saccharum, etc. materials. Numerical values of rotational constant for both types are equal.

Optical activity of materials is stipulated by *asymmetry* of their molecules, which are not having neither planes, nor the center of symmetry. Optical activity of many biopolmers is stipulated, in particular, by casual frame of their moleculas. All proteins built only from laevorotatory aminoacidic oddments. Apparently because of it organic isomers can strongly discriminate on physiological action. For example, the dextrorotary nicotine is more toxicant, than the laevorotatory

nicotine, laevorotatory epinephrine renders more the strong hormonal action, than dextrorotary epinephrine.

Except of natural optical activity, the material can have synthetic optical activity, which originates in it under effect of external actions, for example, at addition of material into a magnetic field (the phenomenon of Faraday).

If between the crossed polarizers to locate optically active material the visual field will brighten up. Again to receive the dark field is necessary to rotate the second polarizer on the angle φ. Knowing specific rotation α of the given material, length *l*, having measured φ, it is possible to find c. Such method of determination of concentration of material is known as **polarimetry** or **sacharimetry**. The devices used for this purpose are known as **polarimeters**. Schema of polarimeter is presented on fig. 6.

Light from light source S in succession passes collecting lens L_1, through light filter Φ, which has choose homogeneous rays, polarizer П, conversing homogeneous rays in polarized. The rays located to the center, pass through quartz plate K in the tube T, and extreme only through the tube. Then rays transit through the analyzer A and collect by lens L_2; in it focus there is the virtual image of the field of vision divided on three parts. Optical rotation estimates by the angle φ, on which is necessary to rotate the analyzer to reduce initial illuminating intensity of the field of vision. The analyzer paired to the circular scale, permitting to measure value of the angle.

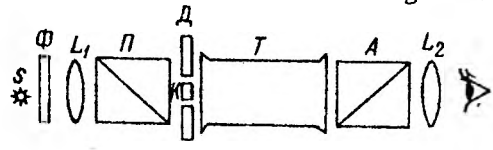


Fig. 6

In medicine the polarimeter is used for determination of saccharum in urine, for biophysical researches. By this method is possible to distinguish dextrorotary paravariations of materials from laevorotatory. For example, laevorotatory Chloromycetin is the fissile antibiotic, while dextrorotary chloromycetin has no medical properties. Use of dispersion of rotation gives good outcomes at research of biopolimers. It is very sensitive to any changes in molecular composition.

5. Polarizing microscope

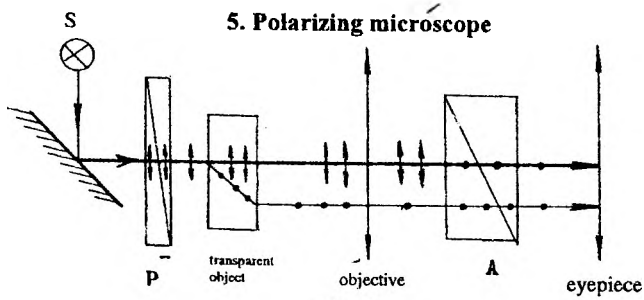


Fig. 7

Viewing transparent biological objects in a microscope, it is difficult to reveal different structures, therefore sometimes use polarization microscopy.

The polarizing microscope (fig. 7) is analogous to a routine biological microscope, but has analyzer A between objective and eyepiece and polarizer П before condenser. Thus, the object is illuminated by polarized light and is looked through the analyzer. If to cross polarizer and analyzer the visual field be dark. *Series of tissues (muscle, osteal, nervous, optical mediums of eye) have* ***optical anisotropy*** *(difference of optical properties on different directions).* If between polarizer and analyzer to locate any tissue with the anisotropic structures then light, past polarizerб be have double refraction in tissue also. In this connection polarized light is not extinguished completely by the analyzer, and the relevant structures appear light on a blanket dark background. For example, for estimation of the mechanical strain incipient in osteal tissues, from the transparent isotropic material, for example, from the Perspex frame flat model of bone (fig. 8). Loading, we can produce anisotropy of the Perspex that becomes noticeable on the reference pattern of strips and spots. On this pattern it is possible to make conclusions about mechanical strains incipient in models, so, and in a nature. The polarizing microscope makes available observation of objects, which are difficult for observing by other methods (chromosomes, process of division, etc.).

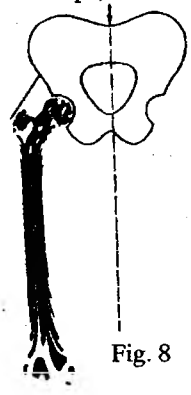


Fig. 8

6. Dispersion of light

Dispersion of light is called the phenomenon caused by dependence of refractive index of material **n** from frequency (or lengths) of light wave **λ**: **n=f (λ)**.

In most cases on the border of two different transparent mediums the short-wave radiation refracts more strongly, than the long-wave radiation.

Distribution of any radiation on lengths of waves (or on frequencies) is known ***as spectrum*** of this radiation.

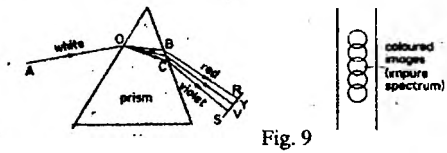


Fig. 9

By corollary of light dispersion is decomposition of white light to spectrum at passage through a prism, for the first time explored by Newton (fig. 9).

To explain dispersion is possible as follows. Under action of transiting electromagnetic wave electrons of medium start to make the harmonic forced oscillations with frequency equal to frequency of the transmitted wave. Oscillating electrons radiate secondary waves of the same frequency. Between the primary wave and secondary waves the phase shift is formed, caused by retardation of oscillations of electrons. The resultant wave (from initial and secondary waves) is

dephased also in comparison with the initial and consequently has other speed of propagation.

This phase shift depends on the oscillation frequency of electromagnetic field, i.e. *light of various lengths of waves will have different speeds of propagation ϑ and so, different refractive indexes n, as* $n = \frac{c}{\vartheta}$, where c is speed of light in vacuum. The least refractive index will have red colour and the greatest is violet.

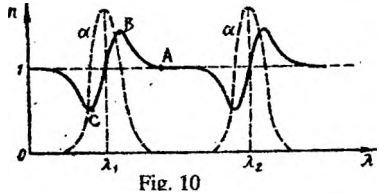


Fig. 10

For all transparent colourless materials function $n = f(\lambda)$ is shown on fig. 10.

With decrease of wave length the refractive index in the beginning is incremented. The quantity $\frac{dn}{d\lambda}$ is known as **dispersion of material**, it is $\frac{dn}{d\lambda} < 0$ on part AB. Such dispersion is called the ***normal***. At very small lengths of waves the part, where $\frac{dn}{d\lambda} > 0$ can appear. On fig. 10 it is part BC. Such dependence n from λ is termed ***as abnormal dispersion***.

The abnormal dispersion is observed on those parts of lengths of waves, where there is light absorption (the absorption coefficient on fig. 10 is shown by the dot line) that corresponds to resonant requirement $\omega = \omega_0$, where ω is frequency of transmitted wave, ω_0 is natural frequency of oscillations of electrons of medium. The abnormal dispersion enables to find frequencies of eigentones of electrons in atoms and molecules and on this basis to judge their structure.

The phenomenon of dispersion in various optical systems plays positive and negative role. In lenses of cameras, microscopes dispersion of light produces chromatic aberration that worsens the image, as at chromatic aberration the luminous point emitting the white light looks like the iridescent stain.

7. Spectroscopic devices

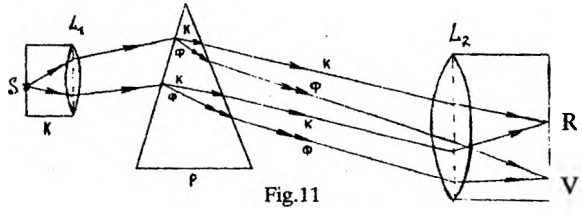


Fig.11

Dispersion of light finds practical application in spectral prismatic devices.

The elementary scheme of the spectral device with a prism is figured on fig. 11.

Light from slit S illuminated by a light source falls on the lens L_1 and, transiting through it forms

parallel bundle, since the slit is posed in the focal plane of the lens. The slit and lens L_1 are posed in a tube termed as *collimator* K. After refraction in prism P, cones of light of different lengths of waves are focalized by lens L_2 on the screen, where a series of monochromatic images of the slit is gained.

Depending on the way of recording of a spectrum, spectroscopic instrumentations are sectioned on the following types:

a) SPECTROGRAPHS in which obtain *photos* of a spectrum;

b) SPECTROSCOPES in which the spectrum is considered by human eye through eyepiece;

c) MONOCHROMATORS intended for separation of radiation in a particular narrow part of spectrum; it has the second slit in the screen, on which the smoothly rotating the prism output various fields of spectrum. Variable width of exit slit allows to separate the convenient interval of wavelengths $\Delta\lambda$ for examination. Monochromators usually are amounting part of the more composite devices.

d) SPECTROPHOTOMETERS intended for deriving and simultaneous photometric measurement of spectral lines (measuring of their relative intensity). With this purpose explored radiation with help of a monochromator sequentially is output on a photoelectric cell or the photomultiplier, conversing light signal into electrical.

One of the basic characteristics of the spectral device is its **resolving ability**. Resolving ability R of the prism as well as a diffraction grating, characterizes property of the device to divide the radiations distinguished on lengths of waves on quantity $\Delta\lambda$. The less this interval is more resolving ability of the device. It expresses through a dimensionless quantity, equal to $R = \lambda / \Delta\lambda$.

There are three types of spectrums: continuous, striped (band) and line.

Incandescent solid and fluid bodies and gases at major pressure give ***continuous spectrum*** in which one colour gradually transfers to another. An example of continuous spectrum is the spectrum of white light. In it is conditionally accepted to discriminate seven primary colours.

Line spectrums will consist of the separate narrow lines of various colour parted by dark gaps. Such spectrums receive from atoms of luminous gases or the steams, which are taking place in the unloaded state. They appear as a result of electronic transitions inside atoms and ions of some elements. For observation of ruled spectrums sometimes use, that gases shine, when through them transits electric current. To receive a line spectrum of materials which in routine requirements are in solidity, it is possible to inject their grains into a torch flame. The study of line spectrums has shown that *each chemical element gives the line spectrum, which is not conterminous to spectrums of other elements.*

Band spectrums are look like the separate light lines parted by dark gaps. Many of strips, by viewing through a spectroscope with major resolving ability, disintegrate on a series of separate lines. Band spectrums are characteristic *for molecules* of heated gases and steams and grow out changes of electronic, oscillatory and rotary energies of molecules.

All these three views of spectrums are ***emission spectra***. Besides them there are absorption spectrums, which gain as follows. White light from a radiant pass

through investigated material (gas, pairs, solution) and guide on a spectroscope. In this case on backgrownd of continuous spectrum will be visible the dark lines posed in the particular order. Their number and the order allow to judge about composition of investigated material. Experiments confirm that *lines of absorption always precisely correspond to lines of emission in the spectrum of gas or the steam, immersing light.* This dependence expresses ***Kirchhoff's law**: any materia absorb those beams, which itself can emit.* This law originating the dark lines apparent in a spectrum of sunlight speaks. They always borrow the same place and in basic represent lines of absorption of steams and the gases environmental the Sun, which temperature is much lower, than on its surface.

8. Spectrum analysis

On a line spectrum of steams of any material it is possible to establish, what chemical elements enter its composition. Such method of definition of chemical composition of material is termed *as **qualitative spectrum analysis**.* Precise location of lines in the radiation spectrum of each element is given in express tables of spectral lines. The spectrum analysis of gases can be carried out and on absorption spectrums.

The methods *of the **quantitative** spectrum analysis* permitting on ***intensity*** of light emission of lines of chemical element to determine its percentage in assay designed. It allows to find presence of very much mass of element ***(up to 10^{-6}–10^{-9} g)***.

For definition of elements with low excitation energy (2 – 4eV) is used the photometry of flame. The abbreviated plan is on fig. 12.

Light from a flame into which assay is injected, pass through the light filter C having narrow transmission band $\Delta\lambda$ in spectral region, relevant to the required element. Further light guide on a photoelectric cell, connected with a sensing galvanometer. Having a gang of replaceable light filters, it is possible to study various fields of spectrum and to establish, whether there is at assay any element. In a flame with major precision are defined all alkaline and alkali-earth elements, manganese, chromium, etc. Photometry of flame is applied to the analysis of connatural waters, biological fluids, chemical fertilizings and medical specimens.

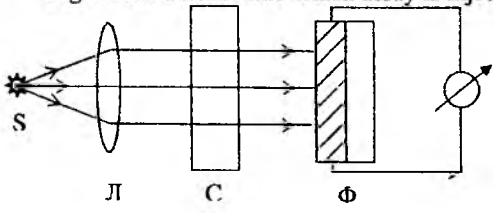


Fig. 12

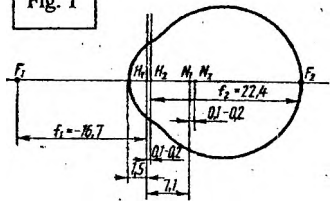
LECTURE №21

OPTICAL SYSTEM OF EYE. BIOPHYSICAL BASES OF VISUAL RECEPTION

1. Optical system of eye. Accommodation

Human eye is the original optical device borrowing in section of geometrical optics special place. It speaks that many optical tools are designed for visual perception of their indications. On the other hand human eye (and animals) as the biological system advanced during evolution, gives some ideas on designing and improvement of optical systems.

Fig. 1



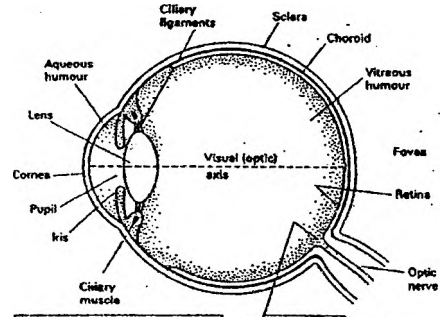


Fig. 2

Let's consider the structure of an eye. The eyeball has almost spherical form of diameter in axial direction 24 – 25 mm (fig. 1). Eye contains *photoconductive* and *photoperceptible* device. Walls of eye consist of three concentric located mediums: external, average and internal. External medium **(sclera)** in the forward part of eye (cornea) passes to the transparent convex **cornea** (fig. 2). Separated from sclera, cornea has the form of a spherical cup of diameter about 12 mm; thickness of cornea is about 1 mm. Radius of it curvature is 7-8 mm, the refractive index is 1.38. To sclera adjoins **vascular medium,** which internal surface is covered by a layer of the pigmentary cells, interfering internal dispersion of light in eye. To vascular medium in the back part, named bottom of the eye, adjoins **retina**, containing photoperceptible device of an eye. It consists of the smallest cells - rods and cones, providing twilight and color vision. In the forward part the vascular medium passes to iridescent, painted at various people differently and having in the center small round aperture that is known as *pupil.*

The *iris* of an eye is the original diaphragm regulating diameter of the pupil (from 2-3 mm at bright up to 6-8 mm at weak illumination) and by that the light stream getting to the eye. The space between the iris of eye and the cornea **(the forward chamber)** is filled by the transparent liquid close on optical properties to water. Directly behind the pupil is located the **crystalline lens,** it is the elastic transparent body having the form of the biconvex lens (n=1.4). Diameter of crystalline lens is 8-10 mm, radius of curvature of the forward surface is 10 mm

and back is 6 mm. Curvature of crystalline lens can change with help of the *circular ciliary muscle*. All internal cavity of eye is filled by transparent jellylike liquid – the **vitreous body** (n=1.33).

Retina serves as photoperceptible screen on which turns out the valid and reduced image of a subject considered by eye. *Refracting system of an eye: the cornea, the moisture of the forward chamber, the crystalline lens, the vitreous body* represent the centre optical system with the optical axis, which is taking place through the geometrical centers of crystalline lens, pupil and cornea. For optical system of an eye, as well as for any optical system is possible to specify six cardinal points with which help determine direction of rays of light: it are two main points (H_1 and H_2), two central points (N_1 and N_2) and two focuses (F_1 and F_2) (fig. 3). Through two main points perpendicularly to optical axis pass two main planes (I and II). On identical distance from them there are focuses. At construction of images use the following rules. The beam going from a subject through focus and crossing the first main plane on height h from the optical axis leaves the plane II on the same distance h from axis and in parallel to it. The beam going from a subject to one central point N_1, leaves other central point N_2 in parallel the initial direction (see fig.3). Average position of cardinal points in the human eye is defined by results of research of set of people with normal sight. Both main and both central points in the average eye are located close from each other. In an eye still distinguish the ***visual axis,*** which is taking place through the center of crystalline lens and the *yellow spot* (site on retina of the greatest light sensitivity) and determining *direction on which the eye has the best sensitivity. The angle between the main optical and visual axes is* 5°. This angle has

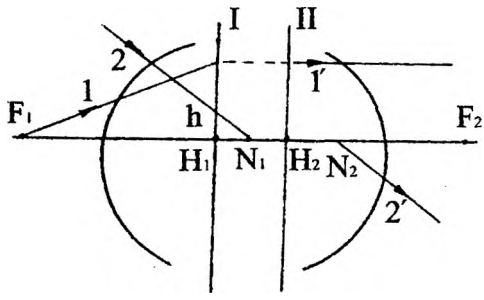


Fig. 3

Fig. 4

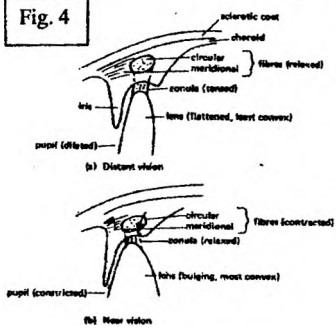
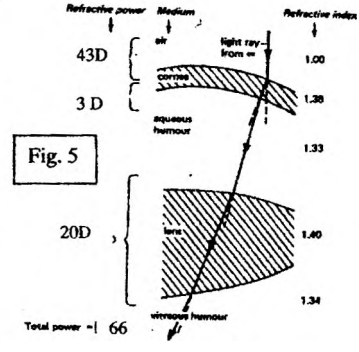


Fig. 5



practical value. If it is great, there is impression of squint. It should be taken into account at determination of distance of centre to centre of spectacles lenses.

The healthy eye adapts to vision of the subjects located from it on distance of 10-15 cm and up to infinity. This ability of an eye is known as *accommodation.*

Accommodation *is the ability of eye to change optical force due to change of curvature (shape) of crystalline lens, that allows to receive precise images of subjects on retina.* When the eye is accommodated for distant vision, both the circular and meridional fibres of the ciliary muscle are relaxed, thus stretching the zonula which squeezes the elastic lens to a flattened shape (fig. 4). In order to increase the refractive power of the lens when viewing a close object, both sets of fibres of the siliary muscle contract. Each has the effect of releasing the tension in the zonula, which then allows the lens to bulge. Thus there is increase of optical force (D): $D = (n-1)(\frac{1}{R_1}+\frac{1}{R_2})$, where R_1 and R_2 are radiuses of curvature of crystalline lens. Proceeding from the formula of lens $D = \frac{1}{d}+\frac{1}{f}$ it allows at increase of D to reduce the distance up to object (d), not changing distance up to the image (f). As whole the optical system of an eye operates as the *converging lens with variable focal length.* All system of eye in not intense condition (rest of accommodation) has ***optical force (refractive power) about 65 diopters***. *The basic refraction of light occurs on external surface of cornea on border with air* (fig. 5). ***Cornea has optical force about 43 diopters***. *Optical force of crystalline lens is* ***≈20 diopters*** (at vision of the removed subjects). At vision of close subjects curvature of crystalline lens increases and optical force of eye can reach of 70-75 diopters (*limit of accommodation*).

At the adult person (healthy) at approach of subject to eye up to distance of 25 cm accommodation is made without a pressure and due to a habit to descry the subjects which are taking place in hands, the eye more often accommodates on this distance named as **distance of the best vision** (or **least distance of distinct vision**). The minimal distance up to a subject, corresponding to the maximal accommodation, defines position of a so-called **near point of clear vision** (near point of the eye). Position of near point depends on age of a person. With the years this distance increases and accommodation decreases.

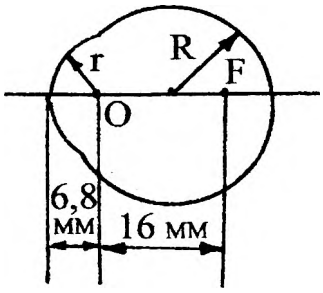


Fig. 6

For construction of the image of subjects on retina of an eye and the analysis of the phenomena connected to it is used the *reduced or the resulted eye,* which is considered as *homogeneous spherical lens*. Some circuits of the resulted eye rather close among themselves are offered. On fig. 6 (the eye on Verbitsky) such circuit is resulted: radius r of the forward refracting surface is 6.8 mm, radius R of sphere is 10.2 mm, length on the axis is 23.4 mm; n=1.4. The optical center O of

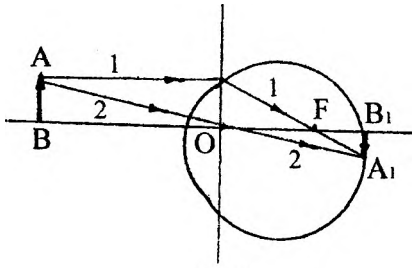


Fig. 7

lens is on the distance of 6.8 mm from top of the forward refracting surface, and main focus F on distance of 16 mm from the optical center, i.e. inside the lens. In the resulted eye on Verbitsky the main and central points for simplification of constructions, as well as for a thin lens are combined and are in the point O (the center of the lens) (fig. 5).

Construction of the image of subjects in the resulted eye is done by rules for a single thin lens. The subject usually settles down behind the double focal length, and the image A_1B_1 results on the back surface of the reduced eye by *valid, inverted and reduced* (fig. 7).

2. Resolution of eye. Defects of optical system of eye

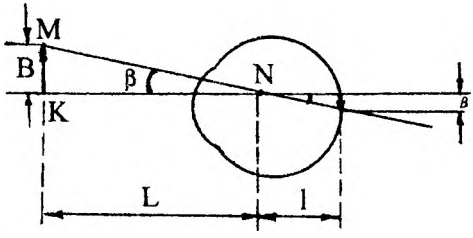


Fig. 8

The size of the image on the retina depends not only from the size of a subject, but also from distance to the eye, i.e. from the angle under which the subject is seen (fig. 8).

For the characteristic of size of the image on the retina enter concept of the *visual angle*. It is the angle β between the beams going from extreme points of the subject through conterminous central points (fig. 8). The size of the image on retina is $s = l \cdot \beta$, where l is distance between the uniform central point and the retina (l≈17 mm). The formula is fair, if the visual angle is small. From the resulted figure is easy to establish connection of size B of the subject with the distance L of subject from the central point of the eye and visual angle of sight: $B = L\beta$, then

$$s = \frac{lB}{L}.$$

Resolution of eye can be estimated by the ***minimal*** *visual angle* β_{min} *under which two next points of a subject are visible separately*. This angle finally defines the size of the image on the retina. Within the limits of the yellow spot at good illumination, eye of the person starts to perceive two points if $\beta \geq 1'$. *The value* $\beta_{min} = 1'$ *characterizes resolved ability of an eye (the maximal visual acuity) and is defined by structure of the retina.* Two next points are visible separately, if their images get on *different* receptors. This condition also defines the limit of resolution of eye. At normal vision the person can see separately from distance of 25 cm two

points, taking place from each other on distance of 70 microns. The size of the image on the retina thus is equal to 5 microns that corresponds to average distance between cones. Therefore, if the image of two points on the retina will borrow smaller distance of 5 microns, these points are not resolved, i.e. eye of them does not distinguish separately. In medicine resolution of an eye estimate **visual acuity** $V=1/\beta_{min}$. ***For norm of visual acuity is accepted 1 and in this case the least visual angle is equal to*** 1′. At infringements visual acuity in so much time is less than norm, in how many times the least angle of sight at deviation from norm more than one minute. If for the patient the minimal angle of sight is equal 4′, visual acuity is equal 1:4=0.25. Visual acuity is the basic function of eye by which is guided at selection of glasses.

Main optical characteristic of eye is represented with position of back focus concerning of retina. It is known as clinical refraction of eye (fig. 9). If the point of

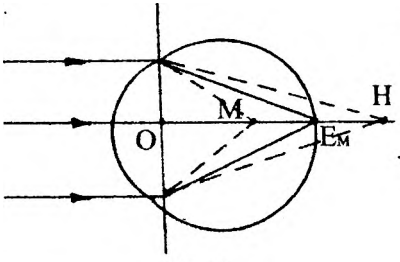


Fig. 9

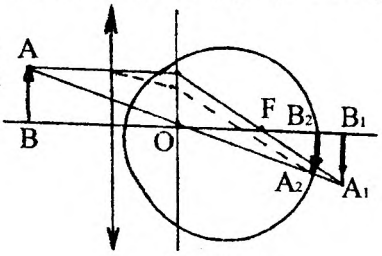


Fig.10

focus lays on the retina, the refraction is called **emmetropic** (Em), if behind the retina the eye is **hypermetropic** (H), if before the retina the eye is **myopic** (M). Only the first refraction provides (at rest of accommodation) the precise image of far subjects on the retina and, hence, normal vision.

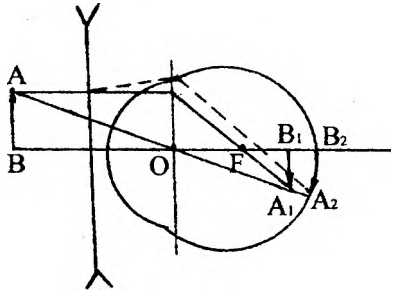


Fig. 11

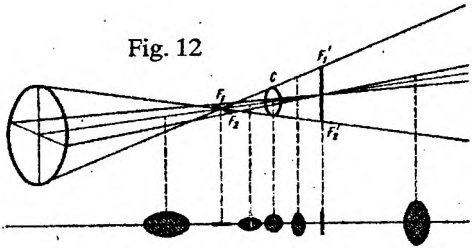
The most widespread defect of vision is **short-sightedness (myopia)**, connected with 1) *the lengthened form of the eyeball* and less often with 2) *excessive optical force of refracting mediums* of the eye at his normal form. Thus sharp image A_1B_1 of far subjects is formed in the plane laying a little bit ahead of retina (fig. 6). Such eye does not see distinctly far subjects. Accommodation thus is useless, as it increase excessive for the given form of eyeball optical force of an eye even more. For correction of short-sightedness is necessary to reduce optical force of the eye by application of glasses with **disseminating** (negative) lenses. Thus image A_2B_2 of the distant subjects turns out on the retina (fig. 11). Continuous lines the

path of beams without the lens is shown, by a combination of shaped and continuous lines are shown with the lens.

The **far-sightedness (hypermetropia)** is connected with insufficient refracting ability of the eye or with the short form of the eyeball. Image A_1B_1 of the far subjects turns out behind of retina (fig. 10). With help of accommodation of eye in part eliminates this defect, however limits of accommodation are limited and such eye does not see distinctly close located subjects. Glasses are applied to elimination of far-sightedness with **collecting** (positive) lenses, which strengthen refracting ability of the eye and provide sharp image A_2B_2 on the retina (fig. 10).

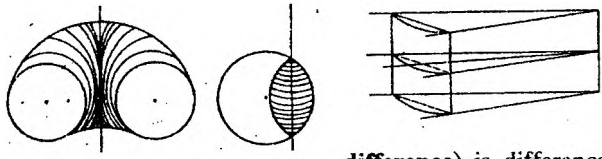
With the years there is change of elastic properties of a crystalline lens and there is the *age far-sightedness.*

Fig. 12



More seldom there is defect of vision **astigmatism,** which is connected with non-uniform refraction in various meridian planes of the eye (fig. 12). This phenomenon is caused by *defect of the correct spherical form* of external surface of the cornea and the crystalline lens. If on such eye falls the bandle of parallel rays, instead of one focal point rays are agglomerated in two segments.

a) Fig. 13 b)



Segments lays in the focal planes of principal cuts. Measure of astigmatism (so-called **astigmatic difference**) is difference of refractive force of two principal cuts (in diopters). The more astigmatic difference is more distance between horizontal and vertical segments. Significant astigmatism causes distortion of the form of a subject on the retina, stretching them at length or width. Astigmatism demands correction of vision also, though it is dependent kind of clinical refraction, and accompanies to emmetropia, hypermetropia or myopia more often. If to look on an astigmatic eye in front and mentally to dissect his by planes, which are taking place through the pole of cornea, appears that the refractive index of this eye smoothly changes from the biggest value in one of sections up to the smallest in the other section, which will be perpendicular to the first. If in each section the refraction remains the constant it is the *correct astigmatism.* If for various beams within the limits of the certain section the refraction appears various, this kind is named *wrong astigmatism.* Glasses can correct only correct astigmatism. Correction of astigmatism is carried out by cylindrical (fig. 13,b) and spherocylindrical lenses (fig. 13,a).

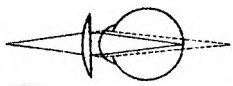


Fig.14

To defects of sight also concerns **presbyopy** or age weakening of accommodation (because of loss of elasticity by crystalline lens). For its correction are used positive lenses (fig.14).

Special case is the **aphakia**; it is condition after extraction of the grown turbid crystalline lens (*cataract*). Thus there is the far-sightedness of very high degree demanding correction by strong positive lenses (of 8-13 diopters).

Both eyes work for the person as the coordinated system forming a uniform image of the seen subject. Ability to form such image from images of two eyes is known as **binocular sight**. Simultaneously, binocular sight allows to estimate remoteness of mediumal subjects. This ability of an eye is called three-dimensional sight. The place in space where visual lines of two eyes are crossed is known as a point of fixing.

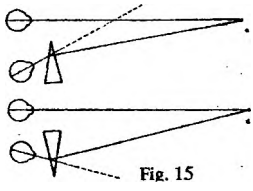


Fig. 15

Infringement of binocular sight is shown as the **squint** more often. The squint is the deviation of the visual line of one of eyes from the joint point of fixing. On the direction of deviation of visual line of the eye distinguish the squint converging, missing and vertical. For correction of the strabismus use prismatic lenses (fig. 15).

3. Bases of photometry

Discussion of questions of biophysical processes of visual perception demands of knowledge of the basic concepts concerning measurement of light values. This circle of questions studies *photometry* (photo=light, metry=measurement)

One of photometric quantities is the **light flux (illuminating power)**(Φ), which represents energy of (dW) the light wave, taking place per unit of time through the given surface: $\Phi = \frac{dW}{dt}$. The light flux is measured in **lumens (lm).**

Light exposure (intensity of illumination) is a ratio of the light flux to the area of this surface: $E = \frac{d\Phi}{dS}$. Light exposure is measured in **lux**.

Physiological action of light on the person substantially depends on light exposure. At small light exposure the eye hardly distinguishes fine subjects and quickly gets tired. At big light exposure light renders harmful action on the retina and excite of nervous system. Therefore hygienic norms of light exposure of inhabited and industrial rooms are established. For example, in an auditorium (at the level of surface of a table) light exposure should be **150 lux**, in rooms in a hostel is **50 lux**, etc.

Light intensity of a source (I) is the ratio of the light flux to value of the solid angle inside which this flux is propagated:

$I = \frac{d\Phi}{d\Omega}$, where Ω is the solid angle, measured in **steradians**. Light intensity is measured by **candela (cd)**. It is basic unit in system of SI.

Brightness (L) is the value numerically equal to the ratio of the light flux inside the solid angle to value of this angle and to the area of the radiating platform: $L = \frac{d\Phi}{d\Omega dS \cos\alpha}$, where α is the angle between the normal to the surface and axis of the solid angle. As $\frac{d\Phi}{d\Omega} = I$, then $L = \frac{I}{dS\cos\alpha}$.

Unit of measurements of brightness is 1 Nit =1 cd/ m^2. Brightness of a sheet of the white paper at reading should be not less than 10 cd / m^2.

4. Sensitivity of eye for light and color. Adaptation

Light getting to the eye is focused with the help of crystalline lens on the layer of photosensitive cells of retina (fig. 16). Photosensitive elements of the retina **cones** are located in the yellow spot. **Rods** settle down on edges of yellow spot and on other surface of retina. The number of cones is equal approximately to 7 million and number rods are approximately 130 million.

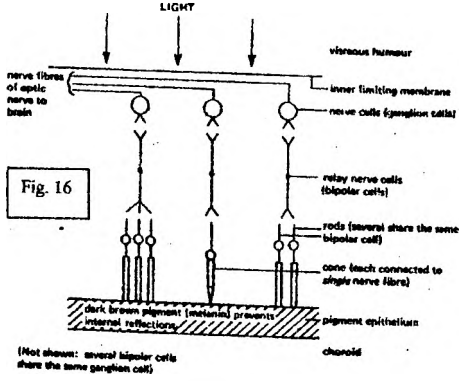


Fig. 16

Rods do not distinguish color and responsible for *black-white vision or twilight vision.* Only the light irritation of cones causes sensation of color and due to presence of cones is carried out day time sight. Cones are concentrated in the center of the retina because color vision is carried out in conditions of bright illumination at the narrowed pupil, passing light basically on the central part of the retina.

Sensitivity of rods is much higher, than cones that function only at light exposure more than 10^{-2} lux, whereas rods react to light even at light exposure up to 10^{-6} lux. It is easy to be convinced in it on twilight, when it seems to us, that all subjects lose the colouring.

Perception of light as well as the perception of sound, submits to law of Weber-Fechner according to which the change of force of light sensation is proportional to the logarithm of the ratio of intensity of two compared light streams.

The photosensitivity of the eye changes over wide range due to visual adaptation. It is known, that at input in poorly covered room, in the beginning the person does not distinguish subjects and for their distinction needs certain time, i.e. transition from day time vision to twilight vision demands certain time. This process is known as adaptation. **Adaptation** is the ability of eye to adapt to various brightnesses. Before full adaptation time of 30-40 minutes sometimes is necessary.

Adaptation allows to the eye to function normally in the range of brightness 10^{-7}-10^5 cd/m^2.

Researches have shown that minimum quantity of light, which should fall on the surface of an eye for creation of light sensation, makes from 60 up to 150 photons of yellow - green light. To retina reaches even less photons. About 50 % is absorbed by crystalline lens, about 4 % is reflected from the cornea. Thus, on share of photoreceptors there are some photons from number of the photons, falling on the cornea. Recent researches have shown that threshold of sensitivity of the eye adapted to darkness, for yellow - green light makes only **2-3 photons**. Photoreceptors will transform light energy into electric with coefficient of amplification of 10^5-10^6. Such big amplification allows even to individual photons to create the nervous impulse and accordingly light sensation.

Thus, the eye is one of the most sensitive devices.

Human eye reacts to electromagnetic waves with length of wave approximately from 400 up to 780 nanometers. And even in the specified interval sensitivity of eye for different lengths of waves is difference. Sensitivity of the eye for longer and shorter waves is sharply reduced. The greatest sensitivity human eye has to wavelength of λ_{max} =555nanometer, i.e. to green color. If to take some sources of the different colors of identical capacity, they will be submitted to the eye not equally brightly. For example, that red light seem so bright, as well as green, is necessary, that its capacity exceeded capacity green in some times. Therefore, for the characteristic of spectral sensitivity of an eye enter value, which is equal to the ratio of capacity of radiation with length of wave λ_{max} =555 nanometer to capacity of radiation with wavelength of λ, causing brightness of the same sensation, as well as radiation of wavelength λ_{max}. This value is called *relative spectral light efficiency* V_λ (sometimes is used the old name: relative luminosity): $V_\lambda = \frac{P_{\lambda=555\,нм}}{P_{\lambda\,нм}}$. Value of function V_λ for various wavelengths have been determined by averaging results of numerous measurements. For green color of λ_{max} =555nm value $V_\lambda = 1$.

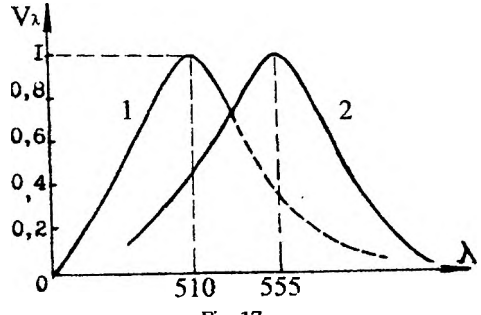


Fig. 17

The graph of dependence of V_λ from wavelength λ is called *spectral sensitivity* of the eye.

Curves (fig. 17) of spectral sensitivity in conditions of day time (2) vision are received and compared with curve of sensitivity in conditions of twilight vision (1). At day time vision the maximum of sensitivity corresponds to wavelength of λ_{max}=555nm, at twilight to wavelength of 510 nanometers, i.e. the curve (2) is displaced concerning the curve (2) of day time vision to the direction of short

wavelengths and the maximum is in the field of dark blue color. Therefore, at consideration of a subject at first at strong illumination, and then at weak illumination, notice displacement of colouring of the subject in the dark blue part of spectrum.

It is not necessary to think, however, that the eye is not sensitive to the radiations laying outside of range of 400-780 nanometers. The person can perceive radiation in ultra-violet area up to 300 nanometers and in infra-red area up to 950 nanometers, but sensitivity of the eye to these waves in billion times is less, than for λ_{max} =555 nanometers.

Crystalline lens and vitreous body almost completely absorb ultraviolet. Therefore at ablation of a crystalline lens (concerning of cataract), sensitivity of the eye to the ultraviolet considerably grows.

The maximum of curve luminosity of day time vision corresponds to the maximum of the sunlight, which past through atmosphere and has got on the surface of Earth. In it the expediency of the organization of eye as photosensitive device is shown.

5. Biophysical bases of visual reception

Light, getting to the eye is fixed by optical system of the eye on the retina, which represents multilayered cellular system (fig. 16). Photoreceptor cells are in the back layer of the retina, basing by the photosensitive segments on the layer absorbing photons of epithelial cells painted by dark pigment. To get in photoreceptors for light is necessary to pass preliminary through the layer of nervous cells that however does not reduce sensitivity of the eye, as these cells are transparent for seen light. *Such position of photoreceptors protects these cells from external influences better and prevents hit on them of the photons, reflected and absent-minded by other sites of the eye; it improves visual acuity.* Each rod and cone will consist of the external and internal segment containing the nucleus and mitochondrias, providing power processes (fig. 18: 1) cross section of eye; 2) cone; 3) rod; 4) disk of the outer segment of the rod; 5) a fragment of the membrane of the disk with the molecula of rhodopsin built-in it; 6) retinal in two states; the M is accumulation of mitochondrions). At the end of the internal segment inverted to light, there is the synoptic contact with the nervous fibre.

Let's consider the structure and functions of rods. The external segment of rods consists from the pile of photosensitive disks, in which is built the visual pigment **rhodopsin** (protein) of red color (fig. 18).

Each disk by thickness about of 20 nanometers, the flattened out balloon reminding by self, will consist from bilayers lipids membranes with molecules of proteins penetrating it. A plenty of disks in the pile increases the general photosensitive surface sensory of the cell that raises probability of absorption by cell of a photon.

Visual disks are formed during all human life. They gradually move along the segment and on the end are separated, then are absorbed by cells of pigmentary epithelium and then disks are blasted.

The visual pigment **rhodopsin** is the complex protein. It will consist of protein **opsin** and chromophor group **retinal** (aldegid group of vitamin A). If with food in an organism acts insufficient quantity of vitamin A, process of synthesis of visual pigments is broken, that results to deterioration of the twilight vision named *"night blindness"*.

Retinal can be in two isomer configurations: cis- or trance - configuration. The molecule of trance - retinal has the straightened form. Cis-retinal has the bent form, turn of group of atoms begins from the eleventh atom of carbon therefore isomer is called 11-cis-retinal. The bent molecule of 11-cis-retinal in darkness forms the complex with opsin and densely enters into the corresponding deepening in the molecule of opsin. At illumination cis-retinal passes to the steadier transform and the straightened molecule of the trance - retinal is not located in the deepening, leaves it and chip off from opsin. **Disintegration of rhodopsin on retinal and opsin results to excitation of receptor cell and to occurrence of generating potential.** Disconnection between opsin and retinal results to decolouration of rhodopsin. Return process of transformation the trance - retinal into cis-retinal occurs under action of enzyme retinalisomerase, then cis-retinal joins to opsin. In the retina at constant illumination takes place stable equilibrium at which rate of decay of rhodopsin is equal to speed of its restoration takes. In darkness speed of regeneration of rhodopsin reaches of maximum and the eye gets the maximal sensitivity.

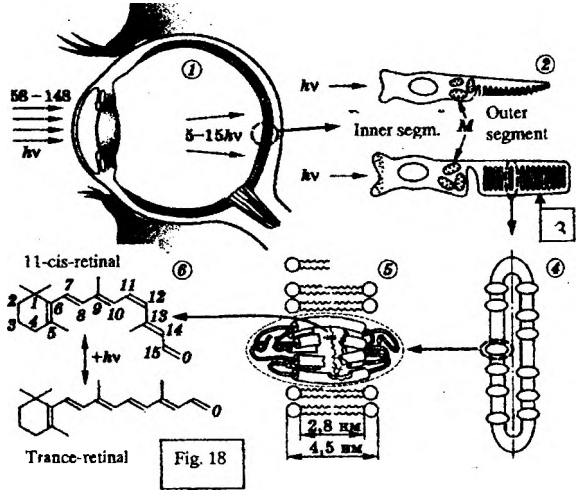


Fig. 18

Such reorganizations of rhodopsin for the first time have been investigated by Wolt, received for it the Nobel Prize in 1966.

Registration of electroretinogramme has allowed to establish that right after illumination of a rod by short flash of light is observed early receptor potential, then approximately through 1ms develops late receptor potential. The nature of these potentials is completely various and is still insufficiently investigated..

These pulses act to the axon of the optic nerve and are transferred to the central nervous system, where the sensation of light is formed.

Color sight is caused by cones. According to theory of Young-Gelmgolts, *there are three types of cones with various curves of spectral sensitivity with maxima of 440, 540 and 590 nanometers* (fig. 19). *Each kind of cones creates sensation only one color: red, green or dark blue. At simultaneous excitation of*

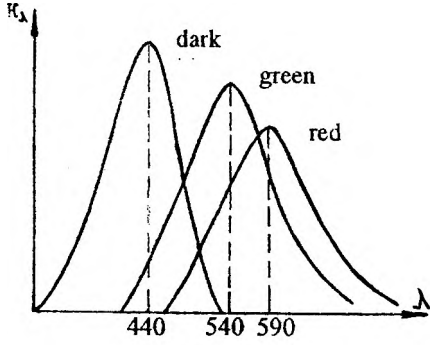


Fig.19

receptors, to the brain enter signals of different intensity that create sensation of intermediate colors. All variety of color sensations is defined by the parity between number of the pulses sent by excited cones.

The pigment of cones contains also 11-cis-retinal, as well as rhodopsin, but the albuminous part of pigment differs, therefore pigments of cones is called **iodineopsin**, they have violet colouring. Everyone cone contains *only one kind of iodineopsin*. Absorption of light by iodineopsin, as well as in the case with rhodopsin, results to occurrence of potentials in cones.

Three componental theory of color vision explains the majority of the facts of physiology and pathology of color vision. At some genetic diseases synthesis of proteins – iodineopsins is broken therefore some pigment of color vision is not formed. The person loses ability to distinguish colors. This illness is known as **daltonism**.

LECTURE №22

OPTICAL AND ELECTRONIC MICROSCOPY. FIBER OPTICS

1. Optical microscope. Path of rays. Magnification

For reception of a big magnification as *magnifier* are used shortphocus lenses. However such lenses have the small sizes and significant aberrations that impose restrictions on their magnification. Substantial magnification can be carried out examining the valid image of a subject, created by additional lens or by system of lenses. Such optical system is the *microscope*, in the elementary case consisting of 2 lenses. The magnifier in this is named the *eyepiece* and the additional lens or system of lenses is called the *objective*. They settle down from each other on distance of 15-20 cm. Usually, the objective of a biological microscope will consist of system of shortphocus lenses which reduces spherical and chromatic aberrations. The eyepiece of a microscope consist from several lenses, it is usual two lenses. Considered object AB (fig. 1) is located on distance **d** near to focus of the objective. Its valid and inverted image A_1B_1 is on distance *f* from the objective. The eyepiece settles down so, that image A_1B_1 was between forward focus F_2 and the eyepiece. The image A_1B_1 descry through the eyepiece as through the magnifier. The final image is imaginary, increased and inverse concerning the considered object.

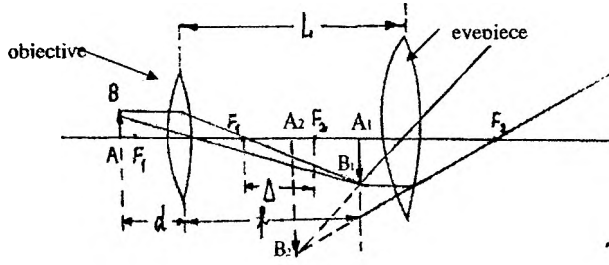


Fig. 1

Position of the objective concerning of the object is selected so, that final image A_2B_2 settled down from the eye on the distance of the best vision a_0=25 cm. The distance between internal focuses of the objective and the eyepiece is known **as optical length of a tube (Δ).** The optical length of a tube usually is shorter than **geometrical length L** for the sum of focal lengths F_1 and F_2. **Magnification of a microscope is equal to product of magnifications of an objective and an eyepiece** $\Gamma=\Gamma_{Ob}\cdot\Gamma_{eyep}$. The eyepiece of the microscope is used as magnifier and its magnification defined under the formula $\boxed{\Gamma_{EYEP.}=\frac{a_0}{F_2}}$. The magnification of the objective can be found taking into account, that the linear magnification of a lens ($\Gamma=\frac{f}{d}$) is equal to the ratio of distance from its optical center up to the image (f) and up to the subject d. Applying this formula to an objective of microscope, it is possible to count that $d=F_1$, $f=F_2+\Delta$, or neglecting by focal length of objective F

$_1$ in comparison with optical length of the tube (last approximately ten times is more), it is possible to count that $f \approx \Delta$. Then magnification of an objective: $\boxed{\Gamma_{ob.} = \frac{F_1 + \Delta}{F_1} \approx \frac{\Delta}{F_1}}$. Hence, magnification of a microscope is equal: $\boxed{\Gamma = \frac{\Delta \cdot a_0}{F_1 \cdot F_2}}$, i.e. magnification of a microscope is equal to the ratio of products of optical length of the tube on distance of the best vision to product of focal lengths of the objective and the eyepiece.

The magnification of objective and eyepiece is called their *own magnifications* and are specified on the frame of lenses.

2. Resolution and useful magnification of a microscope

It is possible to provide rather big magnification of a microscope with corresponding selection of lenses. However in practice seldom use the magnification exceeding of 1500-2000 times. It speaks, that the opportunity to distinguish a fine details of the object is broken by diffraction phenomena that limits useful magnification of a microscope. At passage of light through the smallest details of subject their image owing to diffraction can lose sharpness, there can be infringement of geometrical similarity of the subject and at last probably full disappearance of the image.

Resolution **R (resolving power)** is the ability of a microscope to give separate images of fine details of researched object.

Limit of resolution **(or distinction limit)** (Z) is such least distance between two points of the object, when these points are visible in the microscope separately. Resolution is inversely proprtional to the limit of resolution: $(R \sim \frac{1}{Z})$. Resolution of a microscope is caused by the wave properties of light, therefore expression for limit of the resolution is possible to receive taking into account the diffraction phenomena.

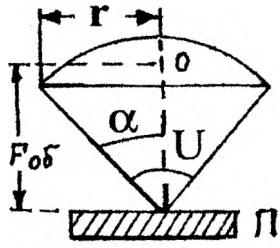


Fig. 2

The diffraction theory of resolution of a microscope is developed by E. Abbey, L. Mandelshtam and D. Rozhdenstvenski. Resolution of microscope as whole is defined by resolution of the objective in which rays of light directly enter, diffracting on an object.

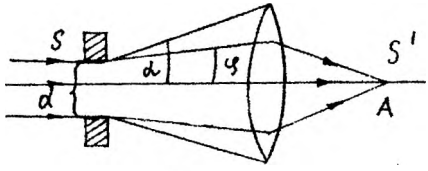


Fig. 3

The basic element causing resolution

of an objective is its **aperture angle** U (fig. 2), equal to the angle, formed by the beams going from the preparation to edges of the objective (angle U) . The aperture angle defines the angular size of the objective and plays the big role at achievement of the big resolution of a microscope. We shall consider formation with help of the objective of the image of luminous aperture S of small diameter d, on which falls the bunch of parallel monochromatic beams (fig. 3). Passing through the aperture, light has diffraction. The objective collects beams and in the connected plane forms in the point A the image of the aperture S'. Two cases are possible:

1) The half of aperture angle α of the objective is more than angle of diffraction φ or is equal to it ($\alpha \geq \varphi$), then *all diffract beams take part in formation of the image and it will be similar to the subject.*

2) The angle $\alpha < \varphi$, then not all starting from aperture beams will take part in formation of image of the subject. The image will not be completely geometrically similar to the subject.

The degree of infringement of the image will depend on what part of diffract beams does not get into objective and does not take part in formation of the image. The angle φ of diffraction is more, than more length of the wave λ and than is less diameter d of the aperture. Then $\varphi \sim \frac{\lambda}{d}$, in the limiting case when $\alpha = \varphi$ it is possible to establish the similar parity $\alpha \sim \frac{\lambda}{d}$, whence $\boxed{d \sim \frac{\lambda}{\alpha}}$.

Thus, *diameter of the aperture at which similarity of image to the subject is kept, can be than less, than more shortly wavelength and than more aperture angle.*

Having transferred reasonings on conditions of work with microscope, it is possible to count, that *diameter d of aperture corresponds to the least size of structural details of a preparation*, i.e. to equate it to the *limit of resolution* of objective of microscope **Z=d**. Then the similar statement can be resulted concerning the resolution of an objective.

In theory of Abbey the diffraction grating undertakes as considered object. In optical devices including a microscope, bunches of light are always limited, therefore it is important to know, how it will affect on distortion of image of a subject, what minimum quantity of beams is capable to transfer the full information about subject. Abbe in the experiments shielded in plane F of converging lens a part of the beams giving the image of diffraction grating. *He has established, that for the resolution of slits in the image of the diffraction grating received with help of a lens on the screen it is necessary, **that for formation of its image participated beams from maxima of zero and first orders, even on the one hand**.* In the limiting case, it agrees to Abbe, extreme beams of the limited conic light bunch will be the beams corresponding to the central (zero) and to the 1-st main maximum. Limit of resolution in this case can be equal to the period of the diffraction grating (d). Then, using the formula of

diffraction grating $k\lambda = d\sin\varphi$, (κ=1; $\varphi = \alpha$; d=Z) for perpendicular incidence beams in air, it is possible to write down: $\boxed{Z = \frac{\lambda}{\sin\alpha}}$; i.e. *distinction limit at direct incidence beams is numerically equal to the ratio of wavelengt of light to the sine of half of the aperture angle of the objective.*

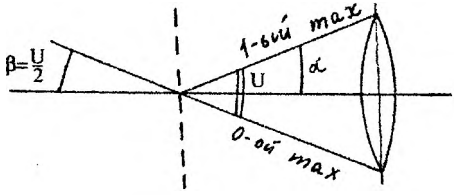


Fig. 4

Let light incident on the diffraction grating under the angle of $\beta = \frac{U}{2} = \alpha$ (fig. 4). In this case the formula of diffraction grating will look like: $d(\sin\beta - \sin\alpha) = \pm k\lambda$. As $\sin(-\alpha) = -\sin\alpha$, from this formula at $\beta = \frac{U}{2}$ also $\alpha = -\frac{U}{2}$, we shall receive $2d\sin\left(\frac{U}{2}\right) = \lambda$, or $Z = \frac{0{,}5\lambda}{\sin\alpha}$. If light is distributed not in air and in the medium with the refractive index n, then $\lambda_n = \frac{\lambda}{n}$ (λ is length of wave of light in air).

For this case the **distinction limit** is equal:

$$\boxed{Z \geq \frac{0{,}5\lambda}{\sin\alpha} = \frac{0{,}5\lambda}{n\cdot\sin\alpha}}.$$

At the other approach to deduce of the formula for definition of the distinction limit at *inclined incident beams* on the objective this formula looks like: $Z = \frac{0{,}61\lambda}{n\cdot\sin\alpha}$, where value $\boxed{A = n\cdot\sin\alpha}$ is named the **numerical aperture.**

From this formula for Z follows, that **one of the ways of reduction of the distinction limit is reduction of wavelength of light.** In this connection the *ultra-violet microscope* in which microobjects are investigated in *UV-light* is applied. In it the optics transparent for UV-beams (quartz optics), and for fixing the image photographic plate, luminescent screens or electron-optical converters is used.

Other way of reduction of limit of resolution is increase of the numerical aperture. It can be increased having increased the aperture angle. It can be made by 1) approaching the subject to the objective. However the distance from subject to lens of the objective cannot change any way, it is constant

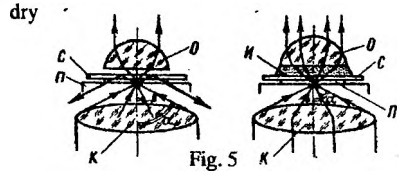


Fig. 5

for each objective.

The numerical aperture can be increased with help of the special liquid medium **(immersion (I))** in space between the objective (O) and integumentary glass (C) of microscope (fig. 5), (K is condenser). In immersion systems in comparison with "dry" systems receive the big numerical aperture. As immersion mediums is used water (n=1.333), cedar oil (n=1.515), monobrominenaphthaline (n=1.66). At immersion light from the subject up to the objective passes on homogeneous optical medium and does not give losses on reflection. It considerably *raises brightness* of the image that has rather essential value especial for the microscope with big magnification. In modern microscopes the aperture angle can have the greatest value of 70^0. In this case the distinction limit of the optical microscope is **0.2 – 0.3 microns**.

Let's estimate **useful magnification** of the optical microscope.

If the subject has the size equal to the distinction limit Z, and the size of its image is Z' and if its image is located on distance of the best vision from eye, magnification of the microscope:

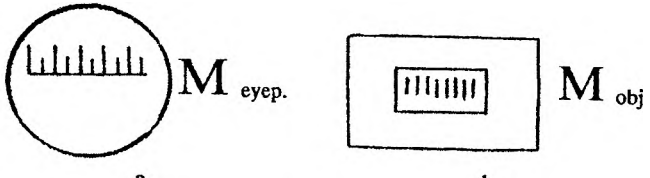
$$\Gamma = \frac{Z'}{Z}.$$

This magnification is known as **useful magnification**.

As $Z = \frac{0.5\lambda}{A}$, then $\boxed{\Gamma = \frac{Z' \cdot A}{0.5\lambda}}$. The magnification of microscope is named *useful* because at it the eye of the person distinguishes all elements of structure of object which gives the microscope.

3. Some special methods of optical microscopy

a) Measurement of the sizes of microscopic objects



a b

Fig. 6

Determination of size of microscopic objects is carried out with help of the eye-piece M_{eyep} (fig. 6a) and objective M_{obj} (fig. 6b) micrometers as glass plates with scales putting on them. The eye-piece micrometer is established in the plane of intermediate image received from the objective. In the eyepiece is observed image of the scale combined with the image of object.

If the scale division value of a scale of the eye-piece micrometer is known, it is possible to determine the size of this image given by the objective; having

divided the received value on known magnification of the objective we can receive the valid sizes of object.

If the scale division value of the eye-piece micrometer is unknown, it can be determined by means of objective micrometer M_{obj} with the known scale division value (usually is 0.01 mm). The objective micrometer is placed on the place of a subject. In the eyepiece observe the combined images of both scales and determine the scale division value of the eye-piece micrometer.

b) Microprojection and microphoto

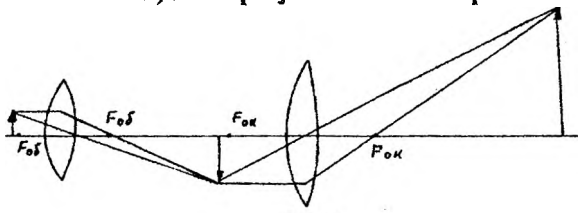


Fig. 7

In the optical microscope the imaginary image originate due to that the intermediate valid image formed by objective settles down between forward focus F_{eyep} and the eyepiece. If to move up the eyepiece so, that the image, which gives the objective there would be before forward focus of eyepiece (fig. 7), last will give the valid image, which can be designed on the screen or photographic plate. The eyepiece in this case serves as a projective lens. It is possible to remove the eyepiece and to project on the screen or a photographic plate the valid image given only by the objective, though thus magnification will be smaller.

Supervision on the screen of the valid image of the subjects received by one the specified ways is known as **microprojection**. Photographing of the valid image received in such way is called **microphoto**. Usually for this purpose is used the special photonozzle (photoadapter) to microscope which represents the camera, putting on the eye-piece end of the tube of microscope. The image of subject is projected on the plane of position of the photographic plate. The nozzle is supplied with visual tube for supervision over the image during shooting.

The linear magnification of photonozzle of the microscope is defined under the formula: $\Gamma_{NOZ} = n_{OB} \cdot n_{eye} \frac{x}{250}$, where x is distance in mm from the eyepiece of microscope up to the photographic plate; 250 is distance of the best sight in mm. n_{OB} and n_{eye} are magnification of the objective and eyepiece.

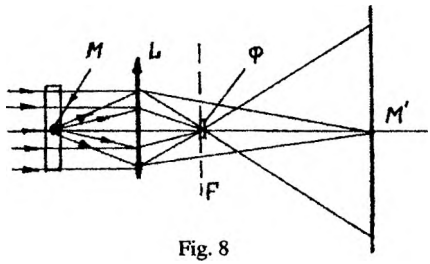


Fig. 8

c) Phase-contrast method

At passage of light wave through

transparent object intensity of light almost do not changes, but phases undergo changes, depending on thickness of object and its refractive index. To see details of such objects by usual way is practically impossible. The phase-contrast method is applied for supervision of not enough contrasting objects and is based on using of phase difference that is formed at passage of light through various structures (sites) of the researched object.

We admit that in the homogeneous transparent medium of object with the refractive index of **n** there is the transparent inclusion M with the refractive index n_1 causing diffraction of light beams (fig. 8). At illumination of the object by parallel bunch of beams, part of it will pass through the medium, will come together in the small site A of focal plane F of objective and then will get on the screen by breaking up bunch. The beams formed owing to diffraction of light on heterogeneity of object fall on the objective as a divergent beam and after objective will not pass through its focus, and will be come together on the screen in the some point M', being the image of inclusion M. Between the beams, falling on the preparation in parallel and beams, diffract on heterogeneity of M be some path difference, which increases with help of the optical device **(phase plate)** up to half of the wavelength ($\lambda/2$). Therefore in the point M' direct and diffract beams interfere and mutually extinguish each other. Therefore the image of inclusion M is observed by blacked out on the light background of the medium environmental it. Phase plate Φ represents the layer of transparent substance of the certain thickness with the certain refractive index. The plate has the form of circle of very small diameter and is established in focus of the objective. Through it pass only beams, which drop on the preparation by parallel bunch. They receive thus the additional path difference in relation to beams diffracting on M. For phase-contrast microscopy applies the special objectives, containing the phase plate and special condensers, which are established in usual biological microscope.

4. Wave properties of particles. Electronic microscopy

First step in creation of quantum mechanics was discovery of wave properties of microparticles.

French *physicist Louis De Broglie in 1924* has drawn the conclusion, *that any moving particle* of substance, as well as quantum of radiation, *has not only* ***corpuscular*** *properties, but also wave,* which can be characterized comparing to particle some wavelength, which is connected with the impulse p of particle by the same ratio, as well as for the photon, i.e.:

$$\lambda = \frac{h}{p} = \frac{h}{m\vartheta},$$

where m is mass of a particle; ϑ is its speed; h is Planck's constant.

This wave is named the **wave of De Broglie.** It characterizes the wave properties of a moving particle. The wavelength of De Broglie is rather small. For electron at $\vartheta=10^{8}$ m/s it has the order of 7A ($7\cdot10^{-10}$ cm), i.e. correspond to wavelength of x-ray radiation. The hypothesis of De Broglie was so unusual, that

many large physics have not given to it any meaning, however several years later presence at moving particles of wave properties has been confirmed experimentally. In 1927 K. Devison and L. Dzhermer observed on the monocrystal of nickel diffraction of electrons.

In later experiments diffraction has been found out at passage of bunch of electrons with high energy through the metal foil (polycrystalline body). Electrons dissipate on the foil and on the photoplate or the fluorescing screen is formed the diffraction picture consisting of lines of concentric dark and light rings (fig. 9). The similar picture takes place at passage through the same foil of X-rays.

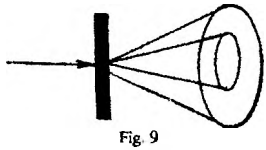


Fig. 9

Wave properties of particles can be used for reception of the increased images of subjects. From told early follows that the distinction limit of optical microscope is defined by value of wavelength. To reduce the distinction limit allows the electronic microscope, in which the *data carrier about a subject* is the *stream of electrons* which passage through substance will dissipate on various directions. We shall find dependence of the wavelength of De Broglie for electrons from accelerating voltage:

$$\frac{m\vartheta^2}{2} = eU, \text{ whence } \vartheta = \sqrt{\frac{2eU}{m}}, \text{ but } \lambda = \frac{h}{m\vartheta}, \text{ then } \boxed{\lambda = \frac{h}{\sqrt{2eUm}}}.$$

From the formula for λ is possible to draw the conclusion, that the *distinction limit of the electronic microscope depends on the accelerating voltage* (other sizes are constants). Having substituted to the formula of the distinction limit of optical microscope length of the wave of De Broglie for electrons, we shall receive for an electronic microscope:

$$\boxed{Z = \frac{0{,}5h}{\sqrt{2emU} \cdot n \cdot \sin\alpha}}.$$

Character of dispersion of electrons depends on structure of the layer of substance through which they pass. Objects of research usually prepare as films, ultrathin cuts and are placed on special frameworks or grids from the thinnest wire, and also on the films-substrates, which are not having own structure. For work with an electronic microscope are suitable very thin objects (5-100nm), because electrons are strongly absorbed and dissipate by substance.

The image which turns out on the screen or photographic plate will display structure of object. Thus it can be considerably increased in comparison with a subject.

The basic difference of an optical microscope from electronic is that with object cooperates the bunch of electrons instead of light beams. Therefore instead of system of optical lenses in the electronic microscope by movement of electrons operate magnetic or electric fields (magnetic or electric lenses). Such fields receive with help of the coil with a current or with the systems of charged electrodes. Magnetic lenses are more frequently used as they give less distortion. On fig.10 the

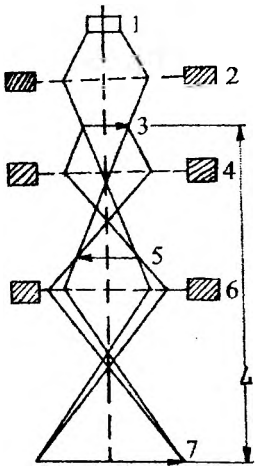


Fig.10

electron-optical system of the elementary electronic microscope with magnetic lenses is schematically shown.

The bunch of fast electrons from the electronic gun (1) gets to the condenser lens (2), directing the bunch of the necessary section on the researched object (3). Due to the different degree of dispersion of electrons by the different sites of object distinguished by thickness, density or the chemical compound, the bunch taking place through the object electrons transfers in itself the information on this object. The *objective lens* (4) gives intermediate (increased in some times) image (5) of object. The *projective lens* (6) forms the final image (7), which is registered by the photographic way, or is observed visually on the luminescent screen in special viewing glass. All these units are connected with each other, forming column of the microscope inside which low pressure (10^{-2}-10^{-3} Pa) is supported. The working voltage for dispersal electrons reaches of 50-100 kV. *Maximal magnification Γ of the microscope having except for the condenser and the objective only one projective lens, is defined by the focal length f_1 and f_2 of objective and projective lenses and by distance L between object and the plane of the final image:* $\Gamma = \frac{L^2}{4 \cdot f_1 \cdot f_2}$ Usually sizes f_1 and f_2 make some millimeters and L is 1-2 m. *The useful magnification in microscopes reaches of 10^6*, and the distinction limit is Z≈0.1 nanometers, that in hundreds times is better, than at optical microscope. It is necessary to note, that application of accelerating voltage greater then 100 κV, though raises resolution of the microscope, but it is connected with destruction of researched object by electrons having the big speed.

The image in the electronic microscope can be formed due to passage of electrons through the object and in this case the microscope is called transmission, if the image is formed by reflected from object electrons the microscope is called reflective. For biological researches is used basically transmission microscope.

With help of the electronic microscope are received unique pictures of various cells, subcellular structures, viruses, bacteria. It is possible to observe large organic molecules (for example, RNA at magnification in 10^5 times). The electronic microscopy allows to study structure of cellular membranes, nervous fibres. With help of electronic microscope the gene for the first time has been seen. The used domestic microscope of the EVM - 100-LM gives the maximal 600000-fold magnification and the distinction limit is about of $3 \cdot 10^{-10}$ m (0.3 nanometers).

5. Fiber optics and its application in endoscopy

In the beginning of 50th years of the last century into various branches of science and especial in medicine began to take root optical fibre elements, which are capable to transfer light on the channels, named *wavebeam guides*.

The section of optics in which is considered transportation of light and the image on wavebeam guides is known as **fiber optics**.

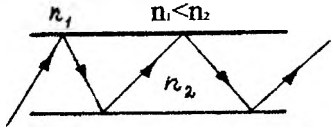


Fig. 11

The fiber optics is based on phenomenon of ***total internal reflection***. Light getting inside of transparent fibre (or core) surrounded with substance with the smaller refractive index, is *repeatedly reflected* and propagated along these fibres (fig. 11). As at total internal reflection coefficient of reflection is rather high (K=0.9999), losses of energy in a fibre basically are caused by absorption of light inside the fibre. So, for example, in the fibre of 1 m in seen area of spectrum is lost from 30 up to 70 % of energy of light. For transfer of the big light streams and preservation of flexibility of wavebeam guide separate fibres gather in bunches (plaits) – optical paths, which in medicine are used for the decision of two problems: 1) transfers of light energy for illumination by cold light of internal cavities; 2) for transfer of the image.

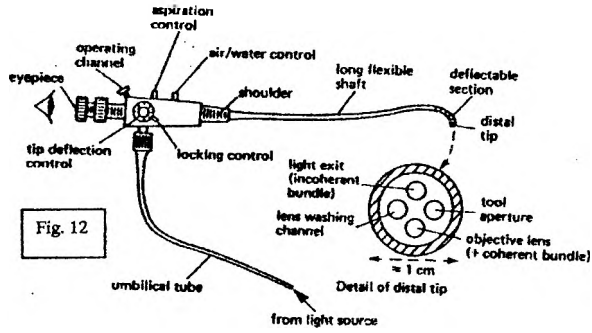


Fig. 12

For decision of the first problem has no value relative position of separate fibres. In the second case very important that the arrangement of fibres in the plait on the input and output was identical, otherwise the image will be deformed. The fiber optics has allowed tomodernize existing early the medical device *endoscope*. *Endoscope* is the special device for inspection of internal cavities (stomach, rectum, bronchial tubes etc.), which will consist of two parts: light source and the viewing part containing system of lenses (fig. 12). The light source (tiny bulb) is placed on the end of endoscope, which is entered inside. Using fiber optics it was possible, at first, light from the bulb to transfer inside of body on optical path and by that to avoid undesirable heating of this body, which arises at premise of light source inside of a cavity in endoscope of old design; second, flexibility of optical fibre system of such endoscope supposes inspection of the most part of cavities of the body of person than rigid endoscopes. Fiber endoscope allows to make necessary pictures with the purpose of diagnostic. With the help of optical paths is possible to transmit laser radiation in internal bodies with the purpose of medical influence on a tumour.

LECTURE №23

THERMAL RADIATION. PHOTOEFFECT

1. Characteristics of thermal radiation. Perfectly black body. Grey bodies. Kirchhoff's law

Electromagnetic radiation of the heated bodies, i.e. the radiation caused by excitation of atoms and molecules of a body at their impact during thermal movement is called the **thermal radiation**. It is inherent to all bodies, which are taking place at any temperature higher than 0 K. Thus each body simultaneously radiates and absorbs radiation falling on it from environmental bodies and, finally, it should come in the condition of thermal (radiant) balance.

The temperature corresponding to this condition is known *as temperature of thermal balance*. For quantitative estimation of processes of radiation and absorption are entered some characteristics.

Radiant emitance (or *emissive ability*) R of bodies is the energy, which is emited in all directions from unit of the area of the body per unit of time. It is measured in $J/s \cdot m^2$ or Wt/m^2.

Ability of a body to absorb energy is estimated by absorptive ability of body A (or **absorptive power**), it is ratio of energy of the electromagnetic radiation absorbed by a body to the energy of radiation falling on it (quantity is dimensionless).

Experiments shows, that radiant emitance and absorptive power of the body depends on its nature, temperature and thus is various for radiations with various wavelength. In this connection is entered the concept of spectral emissivity.

Spectral emissivity R_λ is called the value calculated for the narrow interval of wavelengths $d\lambda$ (from λ up to $\lambda + d\lambda$). The concept of *spectral absorptive power (A_λ)* is similarly entered. Absorptive power of all real bodies is less than 1. So, for example, for the visible region of spectrum absorptive power of aluminium is equal to 0.1; for copper it is 0.5; for water it is 0.67.

The imagined body absorbing at any temperature *all energy* falling on it is called the **perfectly black body**. Absorptive power of such body for all lengths of waves is identical and equal to unit: $A=A_\lambda=1$.

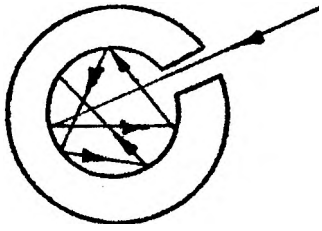


Fig. 1

For visible region of spectrum the body close to perfectly black is soot (A=0.95.) Perfectly black bodies in the nature are not present, this concept is physical abstraction. Model of perfectly black body is the small aperture (fig. 1) in the closed opaque cavity. The beam falling into this aperture is repeatedly reflected from walls and almost will be completely absorbed. The body, which absorptive ability is less than unit and also does not depend on a wavelength of light falling on it is called the **grey body**. Grey

bodies in the nature are not present, however, some bodies in the certain interval of wavelengths radiate and absorb as grey, so, for example, the *body of a person* having absorbing power approximately equal to 0.9 for infra-red area of spectrum.

Distribution of general energy of a composite electromagnetic radiation between waves of various length (distribution of energy of radiation "on spectrum") represents one of the major characteristics of radiation.

The law of distribution can be established experimentally, for example, by decomposition of radiation in the spectrum and establishment with the help of the thermocouple of spectral emissivity $R_{\Delta\lambda}$, falling on each narrow site of $\Delta\lambda$. Then for each site the size is calculated $r_\lambda = \frac{R_{\Delta\lambda}}{\Delta\lambda}$ (r_λ is known as **spectral concentration of radiant emitance** of a body), which is postponed on the graph as function of wavelength. The received curve (fig. 2) characterizes distribution of energy of radiation on spectrum at the given temperature T of a body. Total radiant emitance of body R_T (on all lengths of waves) at temperature T is the area limited by all curve and the axis X:

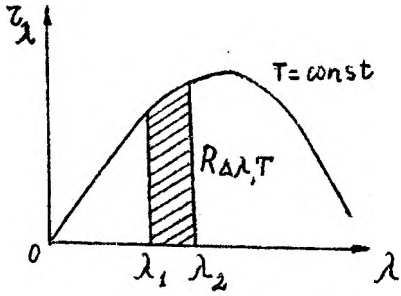


Fig. 2

$$R_T = \int_0^\infty r_\lambda d\lambda .$$

Let's find out connection between radiant emitance and absorptive power of a body. We shall imagine isolated system from two bodies, having various temperatures and exchanging by energy only by emission and absorption. After a while in such system thermal balance will be established. We shall designate radiant emitance and absorptive power of bodies at temperature of radiant balance accordingly as R′, R″ and A′, A″. We shall assume, that the first body emits from unit of the surface for 1 second in n times more energy, than the 2-nd body: $R' = nR''$. But from the condition of thermal balance it should absorb in n times more energy, i.e. $A' = nA''$. This implyies $\frac{R'}{A'} = \frac{R''}{A''}$. If the isolated system will consist of many bodies and one of them is perfectly black body, the similar reasoning will result to the following conclusion: $\frac{R'}{A'} = \frac{R''}{A''} = \frac{R'''}{A'''} = \cdots = \frac{\varepsilon}{1} = \varepsilon$, where ε is the radiant emitance of perfectly black body (A=1). This quantitative connection between radiation and absorption has established in 1859 by Kirchhoff (***Kirchhoff's law***).

For all bodies at the given temperature the ratio of radiant emitance to absorptive power is the constant equal to radiant emitance of the perfectly black body at the same temperature.

3 important consequences follow from Kirchhoff's law:

1. Emissive ability of any body at the given temperature is equal to product of its absorptive power on radiant emitance of perfectly black body at the same temperature: $R=A \cdot \varepsilon$.

2. Radiant emitance of any body is less then radiant emitance of perfectly black body at the same temperature ($R=A \cdot \varepsilon$, but $A<1$, hence, $R<\varepsilon$).

3. If the body does not absorb any waves, it does not emit these waves ($R_\lambda=A_\lambda \cdot \varepsilon_\lambda$, therefore $R_\lambda=0$ at $A_\lambda=0$).

2. Quantum character of radiation. Planck's formula. Laws of radiation of a perfectly black body

Distribution of energy in the spectrum of a perfectly black body at equilibrium radiation and at various temperatures has been investigated by experimental way at the end of the 19 century. Thus two laws of thermal radiation have been formulated.

Radiation of perfectly black body has the continuous spectrum. The experimental curves resulted on fig. 3 $\left(\varepsilon_\lambda = \frac{\varepsilon_\lambda}{\Delta\lambda}\right)$ allow to draw the conclusion, that there is the maximum of spectral concentration of radiant emitance, which with rise of temperature is displaced aside of short wavelengths. From fig. 3 follows, that total radiant emitance of the perfectly black body (the area limited by the curve and axis X) increases with growth of temperature of perfectly black body.

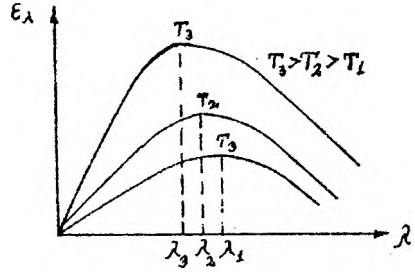


Fig. 3

Long time was difficulty to receive theoretical dependence of ε_λ from wavelength and temperature, which for perfectly black body would correspond to experiment. In 1900 it has been made by M.Planck. In classical physics emission and absorption of radiation by a body is considered as *continuous* process. This position also has not allowed to receive theoretically correct dependence corresponding to experiment. Planck has stated the hypothesis that perfectly black body radiates and absorbs energy *not continuously*, but by discrete portions - **quantums**. Representing a radiating body as set of oscillators, which energy can change on value hν, Planck has received the formula:

$$\varepsilon_\lambda = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/kT\lambda} - 1} \quad \text{or} \quad \boxed{\varepsilon_\nu = \frac{2\pi\nu^2}{c^2} \cdot \frac{h\nu}{e^{h\nu/kT} - 1}}, \qquad (1)$$

where h is Planck's constant; c is speed of light; k is constant of Bolzmann. The formula (1) described the experimental curves represented on fig. 3. From the

formula (1) is possible to receive laws of perfectly black body, which have been established even before Planck's basic work.

Dependence of total (on all spectrum) radiant emitance from temperature is described **by Stephan-Boltzmann's law** (1879г.).

Total radiant emitance of perfectly black body is directly proportional to the fourth power of its absolute temperature T: $\boxed{E=\sigma T^4}$, where σ is constant of Stephan-Boltzmann ($\sigma=5.7\cdot10^{-8}$ Wt/m$^2\cdot$K^4).

The wavelength to which corresponds maximum of energy depends on the body temperature. This dependence is established by the **Wien's law.**

The wavelength on which is necessary the maximum of energy of perfectly black body is inversely proportional to its absolute temperature T: $\boxed{\lambda_m = \frac{b}{T}}$, where $b=2.898\cdot10^{-3}$ m·K is Wien's constant . This law is carried out and for grey bodies. Wien's law is named the *law of displacement* as it shows, that with rise of temperature the maximum of energy of radiation of the black body is displaced aside shorter wavelengths.

3. Radiation of a human body. Bases of thermografy

The human body has the certain temperature due to heat exchange with environment carried out by means of *heat conductivity, convection, evaporation, radiation and absorption.* It is difficult to estimate the percentage parity between the specified kinds of heat exchange, since it depends on many factors: conditions of an organism (temperature, mobility, emotional condition), conditions of environment (temperature, humidity, movement of air), from clothes, etc.

As heat conductivity of air is small, this kind of heat exchange has no of essential value for an organism. However convection in air can considerably strengthen output of heat. The big role for reduction of convection is played with clothes. In conditions of the temperate climate of 15-20 % of heat output of the person is carried out by convection.

Evaporation occurs from surface of skin and lungs of the person (on the average for a day the person allocates 350 g. of water steams), thus loss of heat makes about of 30 %.

Loss of heat by *radiation makes the greatest share* in the general process of heat exchange (50 %). It is carried out from open parts of body and through clothes. The basic part of this radiation concerns to *infra-red* radiation (4-50 microns). The skin of the person, fabric of clothes are accepted for grey bodies and then $R = \alpha\sigma T^4 = \delta T^4$, where $\delta = \alpha\sigma$ is the resulted coefficient of radiation, for the skin of a person it is equal $5.1\cdot10^{-8}$ Wt/m$^2\cdot$K^4; α=0.9 is coefficient of absorption.

If the body temperature of a person is T_1, from the open surface of all body (S=1.5 m^2) is radiated capacity $P_1=S\,\delta T_1^4$. Simultaneously the person absorbs part of radiation from environment. For the dressed person T_1 is the temperature of the surface of clothes.

If the surface of a body of the person would have the temperature equal to temperature of T_0 of air in a room $T_1=T_0$, radiated and absorbed capacities would be equal to each other and equal to $P_0=S\delta T_0^4$. If $T_1 \neq T_0$ the capacity lost by the person at interaction with environment is defined:

$$\boxed{P=P_1-P_0=S\delta(T_1^4-T_0^4)}$$

Maximum of spectral concentration of radiant emitance of a body ($t=32^0$ C is temperature of a skin surface) according to the Wien's law falls at wavelength of 9.5 microns (IR-radiation).

Owing to strong temperature dependence of radiant emitance from T ($R = \delta T^4$), even a little change of body temperature causes significant change of capacity of radiation. If the body temperature of the person will change on 0.3^0 C, i.e. on 1 %, radiant emitance will change on 4 %.

At healthy person distribution of temperature on various sites of a body in various points is rather typically and definitely. However, inflammatory processes, tumours and change of blood circulation can change local distribution of temperature. So, the temperature of veins depends on condition of blood circulation, and also cooling or heating of finitenesses. Thus, *registration of radiation of different sites of a body, determination of their temperature is the diagnostic method. This method is called* ***thermography.*** It is perfectly harmless for a person and finds wide application in clinical practice (exposure of the centers of inflammatory processes, exposure of infringements in vascular system, thrombosis of deep veins, revealing of arterial diseases, revealing of painful zones and traumas, diagnostics of oncological diseases). So, with the diagnostic purpose it is possible to carry out photographing in IR-beams that allows seeing the details invisible by eye in the usual photo. In the photo in IR-beams veins are distinctly visible. Such method is used at diagnostics of skin and vascular diseases.

In some cases at thermography use liquid crystal indicators, which are very sensitive to small changes of temperature. Visually on change of their color it is possible to define local distinctions at temperature.

In medicine can be applied and the method based on using of *thermovisors.* The principle of action of thermovisor with optic-mechanical system of scanning of object consists in the following. During each moment of time the scanning system collects energy of IR-radiation on the high-sensitivity receiver. Due to scanning moving of it is carried out consecutive (as in TV) analysis of the general field of the field of survey. Under action of the stream of radiation, falling on the receiver electric signal is developed, which after amplification and processing moves on the screen of CRT, where the visible image is formed displaying thermal field of researched object. Brightness of the image is proportional to temperature of the scanned sites of the human body.

5. Photo-electric effect

Photo-electric effect (photoeffect) is called group of the phenomena arising at interaction of light with substance, consisting in emission of electrons (external effect), or in change of electroconductivity of substances, or occurrence of EMF (internal effect).

In 1897 in G.Herts and A.G.Stoletov's experiments it has been established that under action of light metals emit electrons. This phenomenon has been named as **photoeffect**.

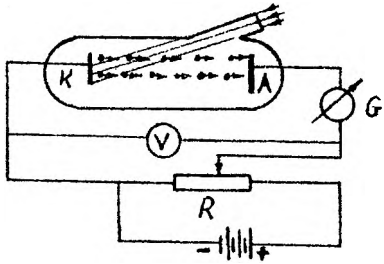


Fig. 4

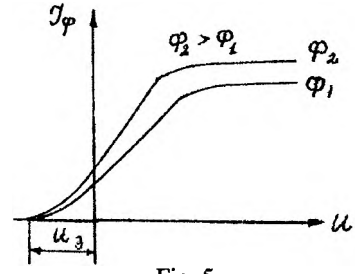


Fig. 5

Detailed study of photoeffect has been carried out on installation (fig. 4), in which electrodes are established in the glass vacuum chamber with quartz window for penetration of UV-beams. The photocurrent formed by the stream of electrons, knocked out by UV-beams from the cathode was fixed by the galvanometer. The voltage on electrodes changed with help of potentiometer R and was fixed with help of voltmeter V. On fig. 5 graphs of dependence of photocurrent I_Φ from the voltage are given at different values of light stream Φ. The current in the beginning grows and then remains the constant (photocurrent of saturation). Value of photocurrent of saturation I_H is defined by quantity of electrons **n** beaten out from the cathode by light per unit of time:

$$I_H = en.$$

Therefore, value of photocurrent of saturation is the measure of photo-electric action of light. If to change polarity of electrodes the electric field will brake moving of electrons, and at some value of $U=U_3$ (detaining voltage) even the fastest electrons do not achieve the anode, the photocurrent will be stopped. From experiment the following **laws of photoeffect** are established:

1. Force of photocurrent of saturation is proportional to falling light stream $I_н=k\Phi$, where k is coefficient of proportionality (photosensivity).

2. The maximal energy of photoelectrons linearly grows with frequency of light and does not depend on intensity.

The photoeffect can be caused by light (irrespective of its intensity), which frequency is not lower than some minimal frequency, characteristic for the given substance of the cathode, and is known as **red border of photoeffect** $\nu_{кр}$ (3-d law of photoeffect). The external photoeffect in metal is energetically described by ***Einstein equation***:

$$\boxed{h\nu = A + \frac{m\vartheta^2}{2}},$$

where A is work of output of electron from metal; hν is energy of photon; $\frac{m\vartheta^2}{2}$ is kinetic energy of electron.

According to Einstein's equation to $\nu_{кр}$ corresponds zero value of kinetic energy. In this case condition of red border of photoeffect is equal $h\nu_{кр}=A$,

$$\nu_{кр} = \frac{A}{h} \quad \text{or} \quad \lambda_{кр} = \frac{hc}{A} \quad \left(\frac{m\vartheta^2}{2} = 0\right).$$

Wavelength $\lambda_{кр}$ (**красный**=red) and work of output for various metals will be various. These data for various metals are usually resulted in the table.

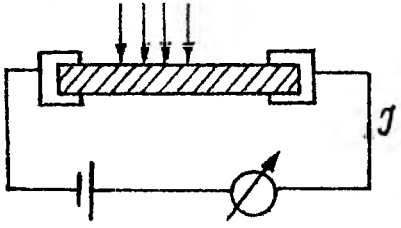


Fig. 6

Internal effect is observed in semiconductors and dielectrics and will consist in increase of concentration of free carriers of charge inside the substance irradiated by light. Thus electroconductivity bodies increases. Due to energy of the absorbed photon connected electron is released and becomes by electron of conductivity. Differently, energy of the photon is spent for transfer of electron from valent zone to the zone of conductivity. All semiconductors are photosensitive, as energy of photon of visible region and even the IR-photon exceeds width of their forbidden zone. The internal photoeffect is easy for finding out on experiment: at switching on of a selenic plate in the circuit of direct current (fig. 6) its conductivity sharply increases at its illumination.

5. Practical application of photoeffect

Action of receivers of radiation (**photo cells**) is based on the phenomenon of photoeffect, transforming light signal to electric. Before others the photo cell with using of external photoeffect (fig. 7) has been created. It will consist of the cathode (source of electrons) and the anode as a loop, disk or a grid. All system is placed into the glass cylinder, from which is pumped out air. The photocathode can be put on the internal surface of glass cylinder as the layer of metal. The important characteristic of the photo cell is its **sensitivity**, expressed by the ratio of photocurrent to corresponding light stream: $k=i/\Phi$. It reaches value of k=100 мсА/lm. Low photosensivity is basic defect of vacuum photo cells.

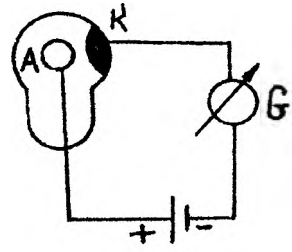


Fig.7

This defect is eliminated in *photoelectronic multipliers* (PEM), in which except of external photoeffect the phenomenon of secondary electronic issue is used. PEM represents vacuum element (fig. 8) with a number of intermediate electrodes $Э_1$, $Э_2$, $Э_3$, ... Under action of light electrons ejecting by cathode K get on dynode $Э_1$, cause secondary emission of electrons (their number at 3-10 time exceeds number of falling electrons). This process of multiplication repeats at the further hit of electrons on the subsequent electrodes.

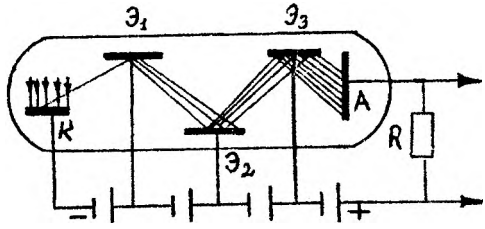


Fig. 8

The increased stream of electrons collects by anode A and forms in the circuit of loading R the current exceeding the photocurrent from cathode (initial photocurrent) in 10^5-10^6 times. Sensitivity of PEM reaches of 10^3 A/lm. PEM is applied mainly for *measurement of small radiant streams*. It registers superweak *bioluminescence*.

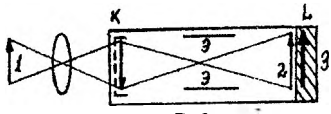


Fig. 9

Work of the *electron-optical converter (EOC)* is based on external photoeffect (fig. 9), intended for *transformation of the image from one area of spectrum to anothers*, and also for amplification of brightness of the image.

The light image of object 1 is projected on translucent photocathode K and transformed to the electronic image. Accelerated and focused by electric field of electrodes Э electrons get on luminescent screen L and the electronic image (2), due to cathodeluminescense again will be transformed into light image (3). *EOC are applied for amplification of brightness of the X-ray image. It allows to* ***reduce the doze*** *of irradiation of the person considerably.* EOC is capable to transform IR-radiation in visible region that it is possible to use for thermographical diagnostics of diseases.

The internal photoeffect in non-uniform semiconductors results to occurrence between p and n semiconductors of EMF under action of light. This phenomenon is known as **photogalvaniceffect** and is used in *valve photo cells* (fig. 10), which will transform light energy to energy of electric current. Valve selenic element will consist of the basic iron plate 3, on which the thin layer of selenium 4 having hole (p) conductivity is rendered. On the surface of selenium is put thin film of gold 1 transparent for light beams. Atoms of gold diffuse inside of selenium and form the connection having electronic (n) conductivity. Between semiconductors with (p) and (n) conductivity forms a locking layer (dashed line), which interferes to

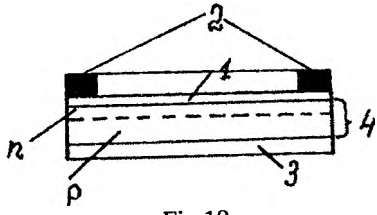


Fig.10

penetration of electrons to the area with p-conductivity. If on the photo cell to direct a stream of light, photons will beat out electrons from atoms of selenium, the way to which aside of iron plate blocks locking layer and which move aside film of gold, charging it negatively. The layer of selenium with p-conductivity and the iron plate are charged positively. Between gold and iron plates there is a potential difference named the *photo - EMF*. If to connect gold and iron plates by a conductor on the circuit there will be photocurrent. The photocurrent is allocated with help of electrodes: the iron plate 3 and the metal ring 2.

Such photo cells are used in ***luxmeters*** for measurement of artificial and natural illumination.

LECTURE №24

LUMINESCENCE. COMPELLED RADIATION

1. Kinds of luminescence. Photoluminescence. Stocks rule

All kinds of self-luminescences except of luminescence of the heated bodies are called ***cold luminescence*** *or* ***luminescence****. As luminescence understand own luminescence of substance arising under influence of external influence*. Example of luminescence: luminescence at electric discharge in gases, at some chemical processes (rotting of organic substances, oxidation of phosphorus), luminescence of glowworms, sea microorganisms and also some substances under action of UV-radiation. This radiation has the duration *considerably exceeding* period (10^{-15} s) of light waves. The luminescence occurs simultaneously with thermal radiation and lays in the optical range.

Depending on kind of excitation distinguish *some kinds of luminescence*.

The luminescence caused by charged particles is **ionluminescence**; by electrons is **cathodeluminescence** (luminescence of a screen of cathode-ray tube); caused by nuclear radiation is **radioluminescence**; caused by x-ray and γ-radiation is **roentgenluminescence**; caused by photons of visible range and UV-radiation is **photoluminescence**; by electric field is **electroluminescence**, which special case is the luminescence of gases at the electric discharge. The luminescence accompanying exothermal reactions (the reactions with allocation of energy) is called **hemiluminescence**. To it concern **bioluminescence:** it is the luminescence of organisms connected with processes of their viability (mushrooms, bacteria and insects).

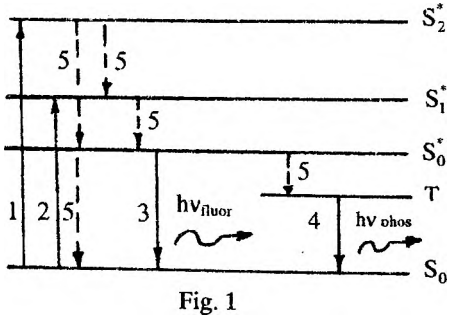


Fig. 1

Let's consider in detail the *photoluminescence which is meant as the secondary luminescence of substance under action of ultra-violet or short-wave part of visible radiation*. Photoluminescence sometimes simply is named *luminescence* and subdivide on **fluorescence** (short-term persistence $\tau=10^{-9}$-10^{-3} s) and **phosphoreccence** (duration of persistence about several seconds and shares of hour). The initial act of photoluminescence is absorption of quantum of light **hν** from the outside and excitation of atom or molecule. On fig. 1 electronic levels of tirosin and electronic transitions in it are represented. If the molecule absorbs quantum of light, electrons of external shells from basic power level S_0 pass to higher power level, for example, S_2^* (1) or S_1^* (2). Thus electrons

shells remains in the single position (all electrons are coupled, also the total spin moment is equal to zero), though the molecule becomes excited. The value of energy of the absorbed quantum is equal to difference of energy of two levels, between which is carried out electronic transition: $h\nu_{abs}=E_2-E_0$, i.e. transition of electron from the basic single level on excited single level will correspond to absorption of light. Molecule can have some such excited single levels (S_1^*, S_2^*, S_0^*).

Time of presence of a molecule in the excited condition is value of the order of 10^{-9}-10^{-7} s. Electronic energy of the excited molecule can be spent as result of course of several processes:1) it can be transferred other molecule (migration of energy), 2) it can be used for increase of thermal energy of the molecule. In all these cases electron comes back to the basic level S_0 or on any level, laying below given excited level. Transitions, which are accompanied by transformation of energy into heat, are called *non - radiating* (5).

Besides there can be process of luminescence of the molecule, accompanying by transition of electrons from the excited levels on the basic S_0. Transition of electron from the excited levels on the basic begins with intermediate transition from the top excited levels on the lowermost excited level ($S_2^* \rightarrow S_0^*$; $S_1^* \rightarrow S_0^*$). Superfluous electronic energy passes thus into heat. Next step is transition from the bottom excited level S_0^* on the basic level S_0 (3), thus radiates quantum of luminescence, which energy always is less than energy of the absorbed quantum on value E_{heat}, i. e. $h\nu_{LUM}=h\nu_{ABS}-E_{heat}$, i. e. $\nu_{LUM}< \nu_{ABS}$ and $\lambda_{LUM} >\lambda_{ABS}$. This dependence is known as **law of Stocks**: *wavelength of light, which is emitted at luminescence* (fig. 2) *always more of wavelength of light which has caused it (the rule of displacement of Stocks)*. Intensity of luminescence is estimated with help of **quantum output of luminescence**: $\varphi = \frac{n}{N}$, where n is number of quantums of luminescence; N is number of the absorbed quantums per unit of time.

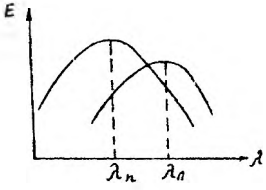


Fig. 2

As the luminescence is always observed at transition from the bottom excited level on the basic, its intensity will not depend on what level has been thrown electron at absorption of light.

The luminescence which is observed at transition of electron from $S_0^* \rightarrow S_0$ is known as **fluorescence** and is observed only directly during of illumination of object.

At some substances the luminescence is observed after deenergizing light, it is caused by transition from the triple level on the basic $T \rightarrow S_0$. The triple level is such level, on which are present two not coupled electrons and their total spin moment can accept one of three values: +1; 0;-1. Level T is located a little bit below of S_0^*, it is named the *forbidden level*, as here electron cannot proceed from level S_0, however, it can get here from excited single level. Its way: $S_0 \rightarrow S_1^* \rightarrow T$.

At transition $S_i^* \rightarrow T$ the part of its energy passes into heat. Thus spin of electron changes on opposite, therefore two electrons become not coupled, and the molecule turns to biradical. Life expectancy of the molecule in the triple condition is from 10^{-3}s up to several seconds. The luminescence, which is accompanied by transition of electrons from T on S_0 is called the **phosphorescence**. As $E_T < E_{S_1^*}$, $\lambda_{phos} > \lambda_{fluor}$. The example: in the molecule of tirosin to transition $S_0 \rightarrow S_2^*$ (1) corresponds maximum in the spectrum of absorption on wavelength of λ_m=217nm. To transition $S_0 \rightarrow S_1^*$ (2) corresponds maximum of λ_m=275 nanometer. The maximum in spectrum of fluorescence is observed at λ_m=304 nanometer (3). To phosphorescence corresponds transition $T \rightarrow S_0$ (4), thus the quantum of λ_m=387 nanometer is radiated.

2. Photoluminescent qualitative and quantitative analysis of biological systems

Phenomenon of luminescence is the basis of the method of detection and determination of the maintenance of chemical components in a mix. This method is known as **luminescent analysis**. Presence of any component (the *qualitative analysis*) determine on *colouring* of luminescent radiation, as to the maximum of spectrum of luminescence corresponds the certain color. Quantity of substance (the *quantitative analysis*) determine on *intensity* of luminescent radiation. At the luminescent analysis for excitation of molecules of substance UV-radiation is used more often.

The luminescent analysis is applied in the most various branches of science and practice. Distinctive feature of the luminescent analysis is the opportunity to find out presence of *insignificant small quantity* of substance (up to 10^{-9} g). The most part of organic connections (acid, dyes) give the characteristic luminescence at absorption of UV-radiation. For example, nicotine gives the dark-violet luminescence. The luminescent analysis is sensitive, it does not demand division of a mix, it can be carried out in biological medium, tissues and other multicomponent systems. On the basis of luminescence in sanitary-and-hygienic practice is applied the method of quality check and sorting of foodstuff (it is used for detection of initial stage of damage of products), sorting and quality check of pharmacological means, vegetative fibre (tissues), skin, detection in them of substitutes or falsifications. The photoluminescence is given tissues of alive organism, especially nails, teeth, non-pigmental (gray-haired) hair, sclera, cornea and especially crystalline lens of eye and other tissues. The luminescent analysis is used for the control of cleanliness of reactants and water.

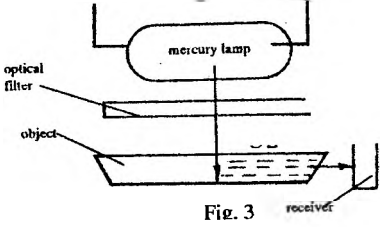


Fig. 3

In criminalistics and forensic medicine the irradiation by UV-radiation allows to find out invisible traces of blood and the

luminescence of blood of the person differs from luminescence of blood of animals and birds.

On color of luminescence distinguish alive and dead cells. Presence of adrenaline in blood of the person is determined on its characteristic green and yellow luminescence. The luminescent analysis is applied at **diagnostics of diseases**, especially skin and other illnesses. So struck by fungus hair, scale of skin under the UV-irradiation give brightly - green luminescence. In many cases as diagnostic reception is used introduction in organism of luminescent paints, which are adsorbed in some tissues. These tissues then investigate under action of UV-radiation. For example, into the vein of a person enter solution of fluorescil and in a few seconds observe the bright green of luminescence of lips and eyes raised by ultraviolet. By this method determines blood circulation in the field of the body with the lowered blood circulation. Permeability of capillaries can be determined, entering hypodermically luminescent painting substances. On fig. 3 the circuit of the luminescent (fluorescent) analysis is shown. UV-radiation from mercury lamp (ML) goes on the object and raises its luminescence. Light of luminescence acts to receiver: an eye, a photo cell, photographic plate, photo multiplier, where it is registered. In order to visible light of a source was not imposed on light of luminescence is applied the optical filter, passing to object only UV-beams invisible for eye.

The luminescent analysis can be subdivided on the macroanalysis and the microanalysis. In the second case supervision is carried out by means of a microscope. In luminescent microscopy the preparations, capable to luminesce are studied in microscope at UV-illumination with corresponding optical filters. By the form of luminescence of the micropreparations prepared from food stuffs, it is possible to distinguish kinds of activators of infectious diseases: tuberculosis, salmonellosis, Siberian ulcer.

It is necessary to note, that if the quantum output of luminescence is more of 1 %, than such connections are easily found out by luminescent method. High quantum output has vitamins A, B_6, E, many medicinal substances. Cancerogenic hydrocarbons in air of cities, smoke of cigarettes, etc. are easily found out by luminescent method.

Some connections which are not having own fluorescence, after special chemical processing give products with high quantum output. By this method can be determined morphine, heroin and other drugs, vitamins C, D, B_{12} and others.

3. Induced radiation of atoms

Searches of management by radiation of atoms or molecules for reception of powerful streams of coherent radiation have resulted in creation of **masers** (or molecular amplifiers) and then **lasers** (Light Amplification by Stimulated Emission of Radiation). These questions are the basic in quantum electronics, which studies methods of amplification and generation of electromagnetic oscillations with use of the compelled radiation of quantum systems.

Let's familiarize with some phenomena underlying of quantum electronics. A. Einstein has proved, that except for two phenomena (absorption and emission) for atom there is one more, it is the **compelled or induced radiation,** which essence consists in the following. Photon of light, flying by the excited atom, transforms it to not excited atom (if energy of the photon coincides with energy of the excited atom), which radiates new photon. As result of the compelled quantum transition, from the atom two identical photons will be distributed: one is initial, external, and the second is secondary. Two photons, flying past by other excited atoms will transfer them also to normal condition with radiation of two photons.

Fig. 4

The number of the compelled transitions, accomplished in second, will depend on number of the photons getting in substance. Besides the compelled transition will be define by filling or population of corresponding power levels. At such radiation there is the avalanche increase in number of photons, i.e. amplification of light. Such radiation also is called **induced**. *Induced radiation identically to falling radiation in every respect including on phase, therefore it is possible to speak about coherent radiation (coherent amplification).* Existence of induced radiation has been predicted by Einstein theoretically. It should be checked up experimentally.

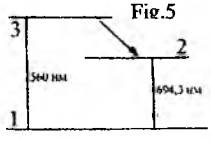
In atom "*density of population*" (number of not exited atoms) of the bottom levels according to distribution of Boltzmann is much more than top shells. The secondary photons, arising as result of induced radiation and also many photons of external influence will be absorbed by the atoms, located at lower levels. As result absorption will be more, than radiation and amplification of light will not take place. For amplification of light it is necessary except for external influence to pick up such *active medium*, in which the number of the excited atoms would be **more** number of not excited atoms, i.e. distribution in atom of electrons should be *opposite to Boltzmann distribution* (**inversion** of density of population). As active medium can be used plasma, some gases and their mixes, crystal bodies, glasses, liquids, many semi-conductor materials. On measure of propagation of light in such medium intensity of light will grow.

4. Optical quantum generators (lasers)

The phenomenon of compelled radiation is used in optical quantum generators (lasers). First generator in the range of the MICROWAVE has been designed in 1955 independently from each other by the Soviet scientists N. Basov, A. Prohorov and by the American scientist I. Towns (they have been awarded for this work of the Nobel Prize). In 1969 the first generator of the visible range has been created with the ruby as working substance.

Let's consider the principle of reception of induced radiation by the example of the ruby laser (fig. 6), which body is the ruby, it is crystal of oxcide of aluminium Al_2O_3 with impurity of trivalent ions of chromium Cr^{3+} (0.03 – 0.05%). As external influence, or so-called **pump**, the Xe pulse lamp spirally located around of ruby core is used. The plasma arising as the result of the discharge in the pulse lamp radiates powerful stream of light, which acts in depth of the ruby core. From all stream of light only green beams (λ =560 nanometers) are useful. They raise atoms of Cr (fig. 5), throwing them from the level 1 on the level 3. At this level many atoms of Cr for a long time are not be located and pass to lower level 2 located close to level 3. This transition is non-radiating (thermal radiation). As the result of such transition the temperature of the crystal lattice of ruby raises. The excited atoms can pass from level 1 to level 3 in time of $t=10^{-6}$; from 3 to 2 - ($t=10^{-8}$); from 2 to 1 - ($t=10^{-3}$).

Fig.5



Apparently, the biggest time is required for transition of atoms from the level 2 on the level 1, therefore level 2 will be the most filling with the excited atoms. This level is called the *metastable* (unstable or temporarily steady). If the photon of external influence flies by the excited atom, which is taking place at level 2, the atom will pass on the level 1 having given photon of red light (λ =694.3 nanometers). There is the coherent induced radiation.

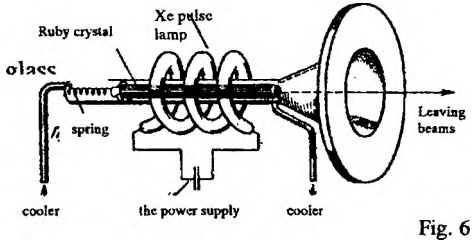


Fig. 6

The crystal of the ruby has the lengthened cylindrical form, with strictly parallel ground end faces (represents the mirror resonator). The forward end face of it is translucent and back is not transparent (fig. 7). The length of the ruby core is limited. Using of the core by length more than 30 cm is not obviously possible, since becomes complicated pump of atoms and focussing of radiation. Therefore for increase of the way of photons them force to be reflected repeatedly from mirror face surfaces. The stream of photons moving in parallel of axis of the crystal, leaves through the translucent end face and is focused by lens and goes on a target as sharply directed coherent beam. The optical quantum generator on ruby works in the *pulse* mode. Energy of generation during one pulse of pump reaches of 1000 J.

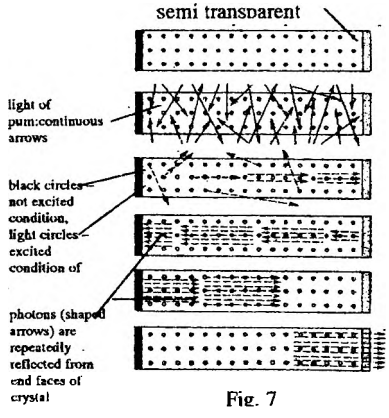


Fig. 7

Alongside with crystal lasers the wide circulation was received gas lasers (fig.9), in which as the active medium a gas is used. Advantage of such lasers is the *continuity of mode* of radiation. The first gas laser represented the quartz tube filled with the mix of gases of helium and neon. Excitation of gas was carried out with help *of high-frequency (HF) generator* with frequency in some tens megahertz (electrodes have been built in the tube). In it atoms of neon were radiating. Atoms of helium plays auxiliary role. On fig. 8 the simplified circuit of power levels of atoms of helium and neon is represented. At the electric discharge the part of Ne atoms from the basic level 1 passes to the excited level 3. Time of life at this level is not enough for the pure neon and atoms pass to levels 1 and 2. For creation of inversion of "density of population" it is necessary to increase "density of population" of the level 3 and to reduce at the level 2. Entering into the mix of helium creates such conditions. The *first excited level of helium coincides with the level 3 of neon.* The excited atoms of helium at not elastic impacts with not excited Ne atoms transfer them to the excited condition and come back to initial condition. As the result it is possible to achieve primary settling of the top levels of the working gas - neon. The top levels (2, 3) have complex structure, they will consist of set of sublevels. *Therefore helium - neon lasers can work on many wavelengths* in the field of visible and infra-red radiation. So, the red helium - neon laser is radiated (transition 3→2) wavelength of 632.8 nanometers. Because at unitary passage of a beam in the active mix the beam amplifies insignificantly, it is used external reflecting plates as resonator mirrors. Conditions for self-excitation and supports of generation are created. Reflecting plates can settle down inside the gas laser also. If end faces of a tube look like the glass plates located under Bruster's angle, the leaving laser beam will be not only *strongly monochromatic and narrowly directed, but also polarized.*

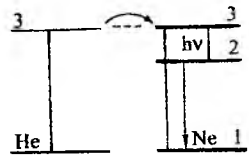


Fig.8

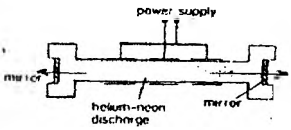


Fig. 9

6. Basic properties of laser radiation, biophysical mechanism of its action, application in biology and medicine

Lasers for short term since time of their creation have found wide application in biology and medicine. Application of lasers is based on **properties of it radiation**: *strong monochromaticity ($\Delta\lambda \approx 0.01$ nanometers), coherency, narrow orientation (the laser beam has property of small divergence), power consumption.* Generally divergence of a beam of the optical quantum generator is defined by the phenomenon of diffraction and depends on diameter of the core of active medium: $\theta = \frac{1{,}22\lambda}{D}$, where θ is angular divergence of beam (in radians); λ is wavelength of radiation; D is diameter of the core.

High coherency of laser beam has allowed tocarry out essentially new method of photographing: reception of the three-dimensional image, which has been named **holographic** (from the Greek word *holos* = *all*). Coherency, the narrow orientation and high concentration of energy of the laser allow to use it in the different areas of science and technics.

The mentioned above properties of laser radiation enable to focus it on rather fine biological structures and to use the laser as the research and microsurgical tool at *cellular level*. The big range of intensity of radiation allow to change character of influence on biological objects from stimulating and therapeutic (10^{-3} Wt/cm^2) up to explosive, accompanying thermal (coagulation), electromagnetic bothacoustic processes and ionization (10^7 Wt/cm^2), see fig 10 and fig. 11.

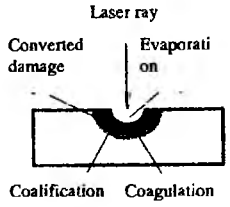


Fig. 10

The basic scopes of lasers in medicine are the **surgery, ophthalmology, oncology, therapy**. In surgery are applied CO_2-lasers with capacity of 30-100 Wt, working in the continuous mode. Properties of laser beam to destroy the biological tissues combined with coagulation of tissue, allows to carry out some bloodless sections. The laser scalpel before a traditional scalpel has a number of advantages. The basic problems of surgery are the pain, bleeding and sterility. These problems are solved at use of the laser very simply: laser radiation, as against a usual scalpel cannot bring in an infection, it sterilizes dissected tissues, even if they are already infected with a suppuration; losses of blood do not occur, as blood vessels instantly coop up by the clotted blood. It is essential, that the laser scalpel does not render on a tissue of mechanical pressure that reduces sensation of pain. Besides with the help of modern endoscopes and flexible optical paths (fiber optics), laser radiation can be entered into internal cavities, due to what there is possible stop of internal bleeding and evaporation of suppurations without opening bodies.

In *ophthalmology* are used pulse ruby lasers (duration of pulses of 30-70 nanoseconds; E=0.1 – 0.3 J), which allow to carry out a number of difficult operations without infringement of integrity of eye: treatment of detachment of retina, welding it to the vascular environment; treatment of the glaucoma by means of piercing of aperture (d=50-100 nm) by laser beam for outflow of intraocular liquid; for treatment of some kinds of cataracts.


Fig.11

Laser radiation is used and for destruction of cells of *malignant tumours and ulcers*. At destruction of malignant tumours is used property of non-uniform absorption of laser pulse radiation by different tissues, histologic structures or cells. For example, some people pigmental tumours absorb laser radiation much more intensively than environmental tissues. Thus in microscopic volumes of tissue is immediately allocated heat with formation of the shock wave extending in the liquid medium with speed about of 1500 km/s. At use of the lasers working in the continuous mode, the temperature

raises up to 100^0 C. For influence on a tumour is used the focused laser radiation (d=1.5-3 mm) on the surface of object, thus I=200-900 Wt/cm^2. It is established, that laser radiation has a number of advantages before used for treatment of skin cancer of X-rays therapy, in particular, number of sessions of irradiation (up to 4 on course of treatment) is essentially removed and in expenses some times decrease. With the help of less intensive radiation it is possible to suppress growth of cancer cells (laser therapy).

LECTURE №25

X-RAY RADIATION

1. Braking and characteristic x-ray radiation, basic properties and characteristics

In 1895 German scientist W. C. Roentgen for the first time has found out luminescence of the fluorescent screen, which has been caused by the radiation invisible to an eye going from the site of glass of the discharge tube, located opposite to the cathode (fig. 1). This kind of radiation had ability to pass through substance, impenetrable for visible light. Roentgen has named it X-beams and has established the basic properties, allowing to apply X-rays in different branches of science and technics, including medicine.

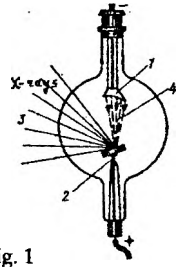


Fig. 1

***X-ray radiation** is called radiation of wavelength 80-10^{-5} nm.* Long-wave x-ray radiation blocks short-wave UV-radiation, short-wave is blocked long-wave γ-radiation. In medicine is used x-ray radiation of wavelength from 10 up to 0.005 nanometers. X-ray radiation is invisible for eye, therefore all supervision with it are made with help of fluorescing screens or films, as it causes luminescence and renders photochemical action.

On the way of excitation x-ray radiation is subdivided on braking and characteristic radiation.

Braking x-ray radiation is caused by braking quickly moving electrons by electric field of atom (of nucleus and electrons) of substance through which they fly. It is possible to explain the mechanism of this radiation. Any moving charge represents the current around of which the magnetic field is created, which induction depends on speed of electron. *At braking of electron its magnetic induction decreases and according to theory of Maxwell there is an electromagnetic wave.*

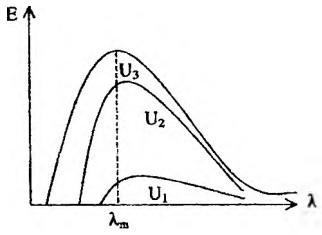


Fig.2

At braking of electrons only *part* of energy is spent on creation of photon of x-ray radiation, other part is spent for heating of the anode. Frequency (wavelength) of photon depends on initial kinetic energy of electron and intensity of its braking. Even if initial kinetic energy of electrons is identical in substance, condition of braking will be various, therefore emitted photons will have the diversified energy and hence wavelength, i.e. *spectrum of x-ray radiation will be continuous*. On fig. 2 the spectrum of brake x-ray radiation is shown for different voltage $U_1 < U_2 < U_3$, where E is energy of photon of brake x-ray radiation.

In each spectrum the most short-wave radiation arises, when all energy got by electron in the accelerating field completely passes into energy of the photon:

$$eU = h\nu_k = \frac{hc}{\lambda_k}.$$

If U to express in kV and to take into account the parity between other values, the formula looks like: $\lambda_к$ = 1.24/U (nm) or $\lambda_к$=1.24/U Å (1Å = 10^{-10} m).

From above mentioned graphs it is possible to establish that wavelength λ_m on which it is necessary the *maximum of radiation energy* is in the constant parity with boundary of wavelength $\lambda_к$:

$$\lambda_m \approx \frac{3}{2}\lambda_k \approx \frac{1{,}86}{U}(nm).$$

The wavelength characterizes energy of a photon from which depends penetrating ability of radiation at its interaction with substance.

Short-wave x-ray radiation usually has the big penetrating ability and is known as *rigid* and long-wave as *soft*. Apparently from the above mentioned formula the wavelength on which is necessary the maximum of energy of radiation is inversely proportional to the voltage between the anode and the cathode of the tube. Increasing the voltage on the anode of x-ray tube we change spectral structure of radiation and increase its rigidity.

At change of glow voltage (the temperature of reheat of the cathode changes) changes the number of electrons emitted by the cathode per unit of time or accordingly the current in the circuit of the anode of tube. Thus capacity of radiation changes proportionally to the first degree of force of current. The spectral structure of radiation will not change.

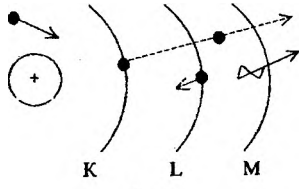


Fig. 3

The general stream (capacity) of radiation Φ, distribution of energy on lengths of waves and also the border of the spectrum on the part of short wavelengths depends on the following three reasons: from voltage U, accelerating electrons and enclosed between the anode and the cathode of a tube; from number of electrons, participating in formation of radiation, i.e. from force of current of reheat of tube; from nuclear number Z of substance of the anode in which there is braking of electron.

The stream of brake x-ray radiation is calculated under the formula: $\Phi = KIU^2Z$, Z is serial number of atom of substance of anode (nuclear number).

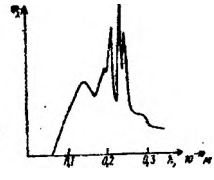


Fig. 4

Increasing the voltage on the x-ray tube, it is possible to notice on the background of continuous brake x-ray radiation occurrence of separate lines (the line spectrum) that corresponds to **characteristic** x-ray radiation (fig 4). It arises at transition of electrons between internal shells of atom in substance (shells K, L, M). Line character of spectrum of characteristic radiation arises because accelerated electrons will penetrate deep into atoms and

from their internal layers beat out electrons for limits of atom. To empty seats pass electrons (fig. 3) from the top layers, therefore photons of x-ray radiation with the frequency corresponding to the difference of levels of energy transition are radiated. Lines in the spectrum of characteristic radiation are united in the series corresponding to transitions of electrons from more high levels to the levels K, L, M.

External influence as result of which electron is beaten out from internal layers should be strong enough. As against optical spectra, characteristic x-ray spectra of different atoms are the same. Uniformity of these spectra is caused by that internal layers at different atoms are identical and differ only energetically, since power influence on the part of nucleus increases on measure of increase of the serial number of element. It results to that characteristic spectra are shifted aside the big frequencies with increase of the charge of nucleus. Such dependence is known as *Moseley's law*: $\sqrt{\nu} = A(Z - B)$, where A and B are constants; Z is serial number of the element.

There is one more difference between x-ray and optical spectra. The characteristic spectrum of atom does not depend on the chemical compound into which the atom enters. So, for example, the x-ray spectrum of atom of oxygen is identical for O, O_2, H_2O, while optical spectra of these connections are essentially different. This feature of x-ray spectra of atoms also has formed the basis for the name of "characteristic".

Characteristic radiation arises always when there are empty seats in internal layers of atom irrespective of the reasons, which have caused it. For example, it accompanies with one of the kinds of radioactive disintegration, which consists in capture by the nucleus of electron from internal layer.

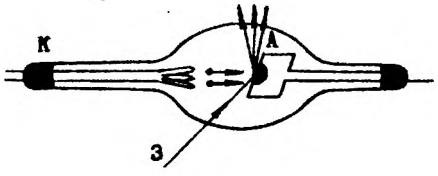


Fig. 5

2. Working principle of x-ray tubes and the elementary x-ray apparatus

The most widespread source of x-ray radiation is the x-ray tube: two-electrode vacuum device (Coolidge tube) (fig. 5). It represents a glass cylinder ($p = 10^{-6} - 10^{-7}$ mm Hg) with two electrodes: the anode A and the cathode K between which the high voltage is created. Heated cathode (K) emits electrons. The anode A frequently is named *anticathode*. It has inclined surface (fig. 6) in order to direct arising x-ray radiation under the angle to the axis of tube. The anode is made from metal with good heat conductivity (copper) for the heat removal formed at impact of electrons. At the oblique end face of the anode there is plate 3 (fig. 5) from refractory metal (tungsten) with the high nuclear number that named the *mirror of the anode*. On occasion the

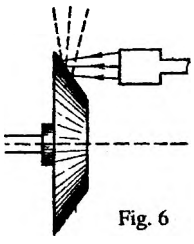


Fig. 6

anode specially cool by water or oil. For diagnostic tubes is important point source of X-rays, that is possible to achieve having focused electrons on one place of the anode. Therefore structurally it is necessary to take into account two opposite problems: on the one hand electrons should get on one place of the anode, on the other hand, to not suppose of overheating, it is desirable distribution of electrons on different sites of the anode. In this connection some x-ray tubes are made with the rotating anode (fig. 6).

In a tube of any construction electrons, accelerated by the voltage between the anode and the cathode get on the mirror of the anode and will penetrate deep into substance, cooperate with atoms and are braked by field of atoms. Thus there is the brake x-ray radiation. Simultaneously with brake the small amount of characteristic radiation is formed. *Only 1-2 % of electrons getting on the anode is cause of the brake radiation and other part is cause of thermal effect.* The part of the tungstic mirror on which falls basic part of electrons is known as focus of the tube.

For feed of a tube is required two sources: the source of high voltage for anodi circuit and low (6-8 V) for the circuit of heat. Both sources should have independent adjustment. *By change of anodi voltage **rigidity** of x-ray radiation is adjusted and by change of filament current the capacity of radiation is adjusted.*

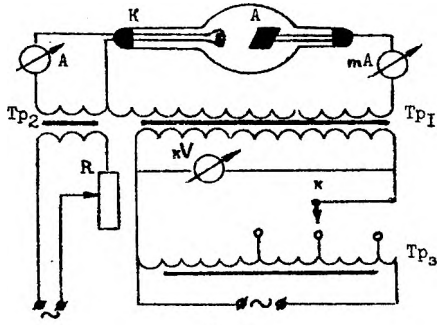


Fig. 7

The basic electric circuit of the elementary x-ray device is resulted on fig. 7. In the circuit are present two transformers: Tp1 of high voltage and Tp2 for feed of heat. The high voltage on the tube is adjusted by autotransformer Tp3 connected to initial winding of transformer Tp1. The switch K adjusts number of coils of winding of the autotransformer. In this connection changes the voltage of the secondary winding of the transformer, submitted on the anode of the tube, i.e. rigidity is adjusted.

The current of reheat of the tube is adjusted by rheostat R swithed to the circuit of initial winding of transformer Tp2. The current of anodi circuit is measured by ammeter mA. Submitted on electrodes of the tube voltage is measured by voltmeter кV. Value of current of reheat adjustable by the rheostat is measured by the ammeter A.

3. Interaction of x-ray radiation with substance (simple scatter, Compton scatter, photoeffect)

At falling of x-ray radiation on any body x-rays in the small amount are reflected from it and basically passes deep into. In mass of the body radiation is in part absorbed, in part dissipates and in part passes through. Passing through the body photons of x-ray radiation cooperate basically with electrons of atoms and

molecules of substance. Registration and using of x-ray radiation and also its influence on biological objects is defined by initial processes of interaction of x-ray photon with electrons. Depending on the parity of energy E of photon and energy of ionization A_i three main processes take place.

a) Coherent dispersion (or simple scatter)

Dispersion of long-wave x-ray radiation occurs basically without change of wavelength and it is named *coherent*. Interaction of photon with electrons of the internal shells strong connected with nucleus, changes only its direction, not changing its energy and so wavelength (fig.8).

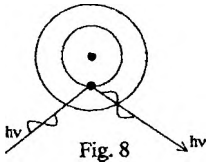


Fig. 8

Coherent dispersion arises, if energy of the photon is less than energy of ionization: $E = h\nu < A_i$. As energy of the photon and energy of atom does not change, coherent dispersion does not cause biological action. However at creation of protection against x-ray radiation it is necessary to take into account opportunity of change of direction of the initial bunch.

b) Compton scatter

In 1922 A. Compton, observing dispersion of rigid X-rays has found out reduction of penetrating ability of the dissipated bunch in comparison with incident. Dispersion of x-ray radiation with change of wavelength is called the Comptoneffect. It arises at interaction of the photon of anyone energy with poorly connected with nucleus electrons of external shells of atoms (fig. 9). Electron comes off atom (this electron is known as electron of recoil). Energy of the photon decreases $h\nu' < h\nu$ (wavelength accordingly increases) and also the direction of its motion changes. Compton effect arises, if energy of the photon of x-ray radiation is more than energy of ionization: $h\nu > A_i$, $h\nu = h\nu' + A_i + E_k$. Thus appear electrons of recoil with kinetic energy of E_K. Atoms and molecules become ions. If E_K is significant, electrons can ionize the next atoms by impact, forming new (secondary) electrons.

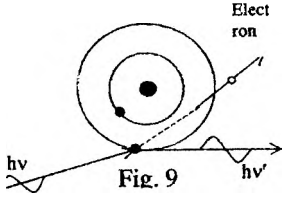


Fig. 9

c) Photoeffect

If energy of the photon hv is sufficient for tearing of electron, at interaction with atom the photon is absorbed and electron comes off atom. This phenomenon is known as *photoeffect*. The atom is ionized. Thus electron gets kinetic energy and if it $\frac{m_e v_e^2}{2} = h\nu - A_i$ is significant, it can ionize the next atoms by impact, forming new (secondary) electrons. If energy of the photon is insufficient for ionization, photoeffect can be shown in excitation of atom or molecule. At some substances it results to the subsequent radiation of photons in the visible region of radiation

(roentgenluminescence) and in tissues it results to activation of molecules and photochemical reactions.

Photoeffect is characteristic for photons of energy about 0.5-1 MeV.

Three basic processes of interaction considered above are initial; they results to the subsequent secondary, tertiary, etc. phenomena. At hit of x-ray radiation in substance there can be a lot of processes before energy of x-ray photon will turn into energy of thermal motion.

As result of the mentioned above processes the initial stream of x-ray radiation is attenuated. This process submits to law of Buger. We shall write down it as: $\Phi = \Phi_0 e^{-\mu x}$, where *μ is the linear coefficient of attenuation,* dependent by nature of substance (mainly from density and nuclear number) and from wavelength of radiation (energy of photon). It can be presented consisting of three composed corresponding to coherent dispersion, Compton effect and photoeffect: $\mu = \mu_c + \mu_{nc} + \mu_{photoef}$.

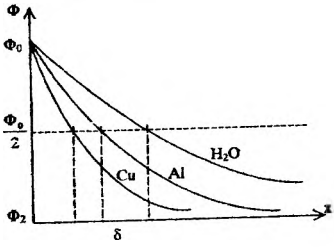


Fig. 10

As the linear coefficient of attenuation depends on density of substance prefer to use *mass coefficient of attenuation,* which is equal to the ratio of linear coefficient of attenuation to density of the absorber and does not depend on density of substance: $\mu_m = \mu/\rho$. Dependence of stream (intensity) of x-ray radiation from thickness of the absorbing filter is submitted on fig. 10 for H_2O, Al and Cu. Calculations show, that the layer of water by thickness of 36 mm, aluminium of 15 mm and copper of 1,6 mm reduce intensity of x-ray radiation in 2 times. This thickness is named *thickness of the half layer δ.* If the substance reduces x-ray radiation by half: $\Phi = 1/2\Phi_0 (x = \delta)$, then $\Phi/\Phi_0 = \frac{1}{2} = e^{-\mu\delta}$ or $e^{\mu\delta} = 2$ $\mu\delta \cdot \lg e = \lg 2$; $\mu\delta \cdot 0{,}4343 = 0{,}3010$; $\delta = 0{,}693/\mu$. Knowing thickness of the half layer, it is possible to determine μ. Dimension of μ is $[m^{-1}]$.

4. Using of x-ray radiation in medicine (roentgenoscopy, roentgenography, x-ray tomography, photoroentgenography, roentgenotherapy)

One of the most widespread applications of x-ray radiation in medicine is influence on internal bodies with the diagnostic purpose (radiodiagnosis).

For diagnostics photons of energy 60-120 keV are used. Thus the mass coefficient of attenuation μ_m is defined basically by photoeffect. Its value is proportional to λ^3 (big penetrating ability of rigid radiation is shown) and it is proportional to the third degree of number of atoms of substance – absorber: $\mu_m = K\lambda^3 Z^3$, where K is coefficient of proportionality.

The human body consists of tissues and the bodies having various absorbing ability in relation to x-ray radiation. Therefore at influence of X-rays the non-uniform shadow image on the screen turns out, which gives the picture of arrangement of internal bodies and tissues. The most dense absorbing radiation of tissues (heart, large vessels, bones) are visible dark, and poorly absorbing tissues (lung) are light (fig. 11).

Fig. 11

In many cases it is possible to judge thus their normal or pathological condition. Radiodiagnosis uses two basic methods: *roentgenoscopy* and *roentgenography* (picture). The block diagram of modern x-ray installation is shown on fig. 12: 1 - x-ray tube, 2-generator of feed of the tube, 3 - control panel, 4 - place for the cartridge with the film, 5 - EOC, 6 - translucent mirror, 7 - movie camera, 8 - television camera, 9 - monitor, 10 - video.

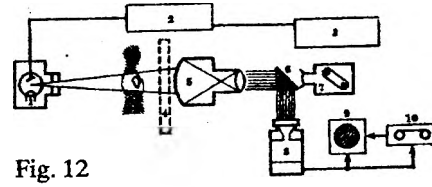


Fig. 12

If the researched body and tissues environmental him approximately equally absorb the stream of x-ray radiation, apply special *contrast substances*. So, for example, before the x-ray research of stomach or intestine give the special kasha of barium sulfate, in this case it is possible to see it shadow image. At roentgenoscopy and roentgenography the x-ray image is the total image of all thickness of object through which pass X-rays. Those details, which are closer to the screen or film are most precisely outlined and removed become indistinct and dim. If in any body there is the pathological site, for example, destruction of lung tissues inside the extensive center of inflammation, than in some cases this site on the roentgenogram in the sum of shadows can "be lost". To make it visible apply the special method *tomography (level-by-level record)*, which allows to receive pictures of separate layers of investigated area. Such level-by-level pictures (tomograms) receive with help of the special device named the *tomograph*, in which periodically in antiphase move x-ray tube (RT) and film (F) concerning of area of research (fig. 13). Thus X-rays at any position of RT will pass through the same point of object (the changed site) being the center concerning which periodic motion of RT and F is made. The shadow image of the site will be fixed on the film. Changing position of « the center of rocking » it is possible to receive level-by-level images of object (fig. 13). Such modern variant of tomography is known as *computer tomography*. The tomography is widely applied at research of lungs, kidneys, bilious bubble, stomach, bones etc.

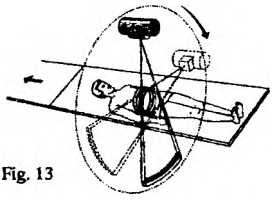


Fig. 13

Other example is *photoroentgenography:* at it on sensitive small format film manages the image from the big roentgenluminescence screen.

The photoroentgenography combines the big opportunity of detection of reticent proceeding diseases (disease of thorax, gastroenteric path, additional bosoms of nose, etc.) with significant throughput (up to 120-150 peoples at hour), in this connection it is rather effective method of mass research.

As photographing of the x-ray image at photoroentgenography is made with help of photographic optics, the image on picture in comparison with x-ray is reduced. In this connection resolution of photoroentgenography (i.e. discernability of fine details) is less than usual roentgenogram.

X-ray radiation is used as well for the medical purposes (*roentgenotherapy*). Biological action of radiation consists in destruction of quickly developing cells of malignant tumours. It is possible to pick up a doze of radiation, sufficient for full destruction of the tumour at rather insignificant damage of environmental healthy tissues, which owing to the subsequent regeneration are restored.

LECTURE №26

RADIO-ACTIVITY

1. Basic law of radioactive decay. Activity. Units of activity

Property of unstable nucleus of some elements spontaneously (i.e. without any influences) to transform to nucleus of other elements with emission of ionizing radiation is called **radio-activity**. The phenomenon is known as **radioactive decay**. Radioactive decay is accompanied by insignificant allocation of heat. Distinguish the artificial and natural radio-activity.

The *natural* radio-activity meets at the unstable nucleus, existing naturally. Radio-activity of the nucleus formed as the result of different nuclear reactions is called the *artificial* radio-activity. Basic distinction between the natural and artificial radio-activity is not present. It is conditional division, since both kinds of radio-activity submit to the same laws.

Feature of radioactive decay is that nucleus of the same element decay not all at once and gradually in various time intervals. The moment of decay of any nucleus cannot be specified beforehand, however, the theory allows to establish *probability* of decay of one nucleus for a time unit, i.e. radioactive decay is a *statistical phenomenon*. At the big set of radioactive nucleus is possible to receive the statistical law expressing dependence of number of not broken nucleus from time. We shall receive this law.

Let for small time interval dt decay dN of nucleus. This number is proportional to the interval of time dt and also to the general number of the radioactive nucleus, which have not decay yet to the beginning of the given time interval:

$$dN = -\lambda N dt, \qquad (1)$$

where λ is *decay constant* (characterizes probability of decay of a nucleus per unit of time and various for different radioactive nucleus). Dimension of decay constant is s^{-1}. The mark minus specifies decrease in time of value N, i.e. $dN < 0$. Expression (1) represents the differential equation of 1-st order with divided variables. We shall divide variables and we shall integrate in view of that the bottom limits of integration correspond to entry conditions: at $t=0$, $N=N_0$, where N_0 is initial number of radioactive nucleus:

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt; \quad \ln\frac{N}{N_0} = -\lambda t; \quad \ln\frac{N}{N_0} = \ln e^{-\lambda t}; \quad \frac{N}{N_0} = e^{-\lambda t};$$

$$\boxed{N = N_0 \cdot e^{-\lambda t}}, \qquad (2)$$

i.e. number of radioactive nucleus, which have not broken up yet decrease on the exponent law. Expression (2) also is the **basic law of radioactive decay.** If there is the necessity to calculate quantity ΔN of nucleus, which decay to some moment of time t: $\Delta N = N_0 - N = N_0(1 - e^{-\lambda t})$.

Rate of decay of various radioactive elements characterize *half-life period* T, it is time during which breaks up half of initial number of radioactive nucleus (fig. 1). We shall establish relationship between T and λ. The half-life period can be determined from the following reasons: at $t = T$, $N = \frac{N_0}{2}$;

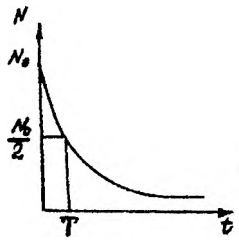


Fig. 1

$\frac{N_0}{2} = N_0 \cdot e^{-\lambda T}$; $\frac{1}{2} = e^{-\lambda T}$; $2 = e^{\lambda T}$; $\ln 2 = \lambda T$;

$0{,}693 = \lambda T$; $T = \frac{0{,}693}{\lambda}$.

The half-life period for various elements matters from shares of second up to millions years. Accordingly radioactive isotopes are divided on shortly living (hours, days) and long-living (years). Examples: half-life period of Uranus $T = 4.51 \cdot 10^9$ years; Lithium T=0.89 seconds. Radioactive elements of Chernobyl emission have the half-life period: ^{239}Pu is 26400 years; ^{137}Cs is 30 years; ^{90}Sr is 29 years. Table 1 lists some common radioisotopes with their half-lives and illustrates, that the naturally-occuring radioisotopes in evidence today have very long half-lives.

Table 1

Radioisotope	*Element*	*Half-Life*
Natural		
^{3}H (Tritium)	Hydrogen	12-26 years
^{14}C	Carbon	5760 years
^{40}K	Potassium	1300 million years
^{226}Ra	Radium	1600 years
^{238}U	Uranium	4500 million years
Artificial		
^{99m}Tc	Technetium	6 hours
^{24}Na	Sodium	15 hours
^{32}P	Phosphorus	14.3 days
^{60}Co	Cobalt	5.3 years
^{132}I	Iodine	60 days
^{131}I	Iodine	8 days
^{137}Cs	Cesium	33 years

In conditions, when radioactive radiation is used for any purposes (for example, in medicine) it is necessary to know total decays per unit of time in the given quantity (mass) of radioactive element. This value is rate of decay and is known as **activity** (A). It is the essential characteristic of a radioactive preparation: $A = -\frac{dN}{dt}$, since $-\frac{dN}{dt} = \lambda N$ and $N = N_0 \cdot e^{-\lambda t}$, then $\boxed{A = \lambda N_0 \cdot e^{-\lambda t}}$. Initial activity (t=0) $A_0 = \lambda N_0$. Then $A = A_0 \cdot e^{-\lambda t} = A_0 \cdot e^{\frac{-0{,}693}{T} t}$. The activity calculated for a mass unit of an isotope is called *specific activity*. For solutions as specific activity understand activity of radioactive solution of 1 ml.

Activity of an isotope is more than is more the radio-activity of nucleus and than is less its half-life period. Activity of the preparation in time decreases on exponent law.

Unit of activity in SI is 1 Becquerel (Bc), that corresponds to activity of a radioactive source in which for 1 second there is 1 act of decay.

The most used unit is *Curie* (Cur):

$$1\text{Cur} = 3.7 \cdot 10^{10}\ \text{Bc} = 3.7 \cdot 10^{10}\ \text{s}^{-1}.$$

Except for it there is one more stand-alone unit: *Rutherford* (R), 1R = 10^6 Bc = 10^6 s^{-1}.

2. Basic kinds of radioactive decay

Under the general name of radioactive radiation are united 3 kinds of radiation, various by the nature, but having some general properties. They have been named historically as alpha, beta, and gamma - beams.

Alpha radiation is the stream of particles with high kinetic energy. Alpha decay will consist in spontaneous transformation of nucleus with emission of α - particles (nucleus of helium). The scheme of α - decay in the view of the rule of displacement:

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}\alpha,$$

where X is the symbol of initial (parent) nucleus; Y is the symbol of the nucleus – product of decay (affiliated or daughter's nucleus).

In connection with emission of α- particles the charge of the nucleus and accordingly nuclear number of the element decreases on two units and mass number on four units. Hence, the secondary element is shifted in Mendeleyev table on two numbers to the left and the nuclear mass of it becomes less on four units. As the example of α- decay can serve decay of radium, at which radon is formed:

$$^{226}_{88}Ra \rightarrow ^{222}_{86}Rn + ^{4}_{2}\alpha.$$

Thus it is radiated γ - photon. At α- decay the affiliated nucleus can be not only in normal, but also in the excited condition and as these conditions are discrete, also values of energy of α – particles, which are taking off from different nucleus of the same radioactive substance are *discrete*. Energy of excitation of the affiliated nucleus is allocated as γ- photon more often. For this reason α- decay is accompanied by γ- radiation. Speed of start of α- particles from a nucleus is (1,4 – 2)·10^7 km/s, that corresponds to initial kinetic energy of 4 – 8.8 MeV. Alpha – particles, which are emitted by certain element, make some groups with close energy, therefore the spectrum of α- radiations will consist of the several close located lines.

β-decay occurs at nucleus, which instability is connected with the certain parity of number of protons and neutrons. If in any nucleus there is surplus of neutrons occurs *electronic β- decay* of nucleus, at which one neutron transforms to proton, thus in the nucleus is formed electron:

$$^{1}_{0}n \rightarrow ^{1}_{1}p + ^{0}_{-1}e + \tilde{\nu},$$

Where $\tilde{\nu}$ is antineutrino (the elementary particle). Electron is thrown out from the nucleus and in it there is steadier complex of nucleons. Electronic β- decay is described by the equation:

$$^{A}_{Z}X \rightarrow ^{A}_{Z+1}Y + ^{0}_{-1}e + \tilde{\nu}.$$

Thus the charge of the nucleus and accordingly nuclear number of the element increases for unit, i.e. the secondary element is shifted in Mendeleyev table on one

number to the right, its mass number remains without change. Example: ${}^{40}_{19}K \rightarrow {}^{40}_{20}Ca + {}^{0}_{-1}e + \bar{\nu}$.

If in a nucleus *surplus of protons occurs positron β-decay*, at which one of protons transforms to neutron, thus in the nucleus is formed positron: ${}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e + \nu$, where ν is neutrino.

The positron is thrown out, in the nucleus steadier complex of nucleons is formed. Positron β- decay is described by the equation:

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}e + \nu . $$

The charge of the nucleus and accordingly nuclear number of an element changes on unit, and the secondary element is shifted in Mendeleyev table on one number to the left, the mass number of it remains constant. Example:

$$ {}^{79}_{37}Rb \rightarrow {}^{79}_{36}Kr + {}^{0}_{+1}e + \nu . $$

Initial speed and accordingly kinetic energy of β -particles can considerably differ. The greatest initial speed has the order of $1.6 \cdot 10^{8}$m/s and energy of β-particles can be in limits from tenth and 100-th shares of MeV till 10-12 MeV. Power spectrum of β- particles *continuous*, i.e. their energy can accept different values. To explain distinction in energy of β- particles at decay of nucleus of the same element, V. Pauli has assumed in 1939, that at β- decay alongside with β – particle from the nucleus are thrown out neutral particles *neutrino and antineutrino* with mass equal about 1/2000 of mass of rest electron and having energy, which in the sum with energy of β- particle make some constant, characteristic for the given substance. And this energy at different nucleus divides between beta and these particles in various parities. It explains the continuous spectrum of β-particles.

At emission of β- particles, as well as at α-decay, nucleus of atoms can be in the excited condition. Their transition to the not excited condition (sometimes in steps) is accompanied by emission of γ-quantums with energy from 0.2 up to 3 MeV. Spectrum of γ- radiation is line. γ-radiation arises not only at α - and β-decays. At collision of the nucleus with a particle it can proceed to the excited condition and then coming back to the basic condition to radiate γ - photon.

There is the third kind of β – decay, that is called **electronic or e-capture**. It consists that the nucleus grasps one of internal electrons, taking place on K, L, M levels, therefore the proton of the nucleus transforms to neutron: ${}^{1}_{1}p + {}^{0}_{-1}e \rightarrow {}^{1}_{0}n + \nu$.

At electronic capture the place in electronic shell is released, therefore this kind of radio-activity is accompanied by characteristic x-ray radiation.

3. Methods of reception of radionuclides

Interaction of a nucleus with an elementary particle or with other nucleus of element as result of which this nucleus transforms to the nucleus of other element is called the *nuclear reaction*. Nuclear reactions allow toreceive from one chemical elements other elements by influence on the nucleus of atom. Effective means of such influence appeared bombardment of nucleus by particles of high energy. For the first time nuclear reaction has carried out by Rutherford in 1919. At

bombardment of nucleus of nitrogen by α - particles formed at decay of radium, occured transformation of nucleus of nitrogen to nucleus of isotope of oxygen with ejection of protons: $^{14}_{7}N+^{4}_{2}\alpha\rightarrow^{1}_{1}p+^{17}_{8}O$.

Brief record of reaction: $^{14}N(\alpha,p)^{17}O$.

Key rule at drawing up of the equation of nuclear reaction is equality in its both parts of the sum top (mass numbers) and bottom (nuclear numbers) indexes. It is expression of laws of preservation of mass and charges of the particles participating in reactions.

The reduced record will consist of four symbols: the initial nucleus (nucleus - target), in brackets is a bombarding particle and other formed particle (or particles), out of brackets is put the symbol of nucleus – product of reaction (nuclear number of the element usually is not put).

Originally as bombarding particles were used α- particles of radioactive radiation. In 1932 by the English physicist D. Chedvik was open the *neutron*. Neutron is the stable, neutral particle, however in a free condition it for a long time does not exist. At collision with the nucleus of any element the neutron is absorbed by it and causes nuclear reaction. For example: $^{14}_{7}N+^{1}_{0}n\rightarrow^{11}_{5}B+^{4}_{2}\alpha$ or $^{14}N(n,\alpha)^{11}B$.

Nuclear reactions under action of neutrons have the greatest probability. Not having electric charge, neutrons freely fly electric shells of atoms and hitting with nucleus more often cause nuclear reactions.

Further began to use and other charged particles, preliminary giving them the big speed (kinetic energy) in special accelerators, for example, in cyclotrons.

All nuclear reactions are accompanied by emission of any elementary particles (including γ- photons). Products of many nuclear reactions are radioactive and products are named *artificial radioactive isotopes (radionuclides).* Phenomenon of artificial radio-activity was open in 1934 by known physicists Frederic and Iren Jolio-Curie.

Isotopes	*Half-Life*	*Uses*
Radio-carbon, C^{14}	4700 years	In the study of protein metabolism and archaeological dating.
Radio-tritium H^{3}	11 years	As a tag in organic substances.
Radio-cobalt, Co^{60}	5·3 years	Emits strong γ-rays. Used in Radiography and Radio therapy as a substitute for radium.
Radio-sulphur S^{35}	87 days	Numerous chemical and industrial applications.
Radio-phosphorus P^{32}	14 days	In the study of bone metabolism and for the treatment of blood diseases.
Radio-iodine I^{131}	8 days	Treatment of thyroid diseases.
Radio-sodium Na^{24}	15 hours	Its chemical property and solubility make it useful in a number of applications, *i.e.* in the study of circulatory disorders in blood vessels.

Table 2

As example of reception of radioactive isotopes (radionuclides) can serve reaction of capture of neutrons by phosphorus $^{31}_{15}P$. At this capture is radiated γ - photon and the radioactive isotope of phosphorus is

formed: $^{31}_{15}P+^{1}_{0}n\rightarrow^{32}_{15}P+\gamma$.

Decay of nucleus of the received isotope $^{32}_{15}P$ is accompanied by emission of β-particle (simultaneously is emitted antineutrino) and formation of stable isotope of silicon:

$$^{32}_{15}P\rightarrow^{32}_{16}Si+^{0}_{-1}e+\tilde{\nu}.$$

As well as to natural radioactive elements to artificial isotopes are peculiar α-, β - and γ- decays.

Radioactive isotopes in small amounts receive in accelerators (for example, in the cyclotron) with help of deutrons (nucleus of heavy hydrogen) d.

In industrial scale artificial radioactive isotopes receive by irradiation (mainly of neutron) of corresponding chemical elements in nuclear reactor.

Except for irradiation by neutrons radioactive isotopes receive in reactors by their allocation from fission products of nucleus of uranium, for example, radioactive iodine $^{131}_{53}I$ is widely used in medicine.

Already are obtained several radioactive isotopes for each chemical element, their general number exceeds of 1500. Many of them are widely applied in quality of labelled atoms in various branches of human activity, including medicine (table 2).

4. Interaction of ionizing radiation with substance

In connection with the general character of initial action on substance α-, β - and γ- radiation, rigid x-ray radiation and also streams of protons and neutrons are united under the general name of *ionizing radiation*. The charged particles and γ - photons being distributed in substance, cooperate with electrons and nucleus, therefore changes condition of substance and particles.

To the basic properties of radioactive radiations are *penetrating* and *ionizing abilities*.

Ionizing ability of radiation is estimated by *linear density of ionization* i: $i = dn/dl$, where dn is number of ions of one mark formed by the particle on the elementary way dl. In practice this value is estimated by number of pairs of ions formed by particle on 1 cm of run.

Ionizing ability is estimated *by linear brake ability of substance* S: $S = dE/dl$, where dE is the energy lost by charged particle at passage of elementary way dl in substance. As ionization of one molecule needs energy of 34 eV, value S can be calculated knowing linear density of ionization.

Penetrating ability of radiation is estimated by *length of free run* or *average linear run* R, it is average distance, which there passes the particle in substance while particle is capable to ionize. More charge and mass of a particle more its ability to ionize substance and the less its average run. Average linear run of α-particles in human organism is 10-100 microns; of β- particles is 10-15 mm; γ-radiation will penetrate on the big depth or penetrates body of the person through. Properties of ionizing particles are resulted in the table 3.

TABLE 3

Kind of radiation	Average energy, MeV	Linear density of ionization i, pair / cm	Average linear run R, m	
			in air	in substance
α	4-8,8	$3 \cdot 10^4$	$(2-8)10^{-2}$	—
β	0,01-10	50-250	10	$1{,}5 \cdot 10^{-2}$
γ	0,2-3	300	300	Near 1

Electrons displaced at ionization can beat out secondary electrons, having energy sufficient for the subsequent ionization of substance. These secondary processes can cause characteristic x-ray radiation, radioluminescense, chemical processes.

γ- photons causing insignificant initial ionization, generate secondary as result of which total ionizing effect can be rather significant.

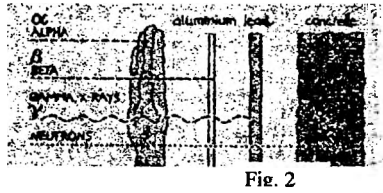


Fig. 2

Owing to various ionizing and penetrating abilities of radioactive radiations **ways of protection** against radiation are different: *for protection from α- particles there is enough layer of a paper, clothes; from β- radiation it is possible to be protected by centimetric layer of a tree, glass or any easy metal; for protection from γ- radiation are applied thick (up to meter) layers of water, concrete, brick walls and also plates of lead by thickness up to 10 cm (fig. 2).*

Except for ionization the particles are capable to cause other processes (fig. 3).

α- particles can cooperate with nucleus, causing nuclear reactions, though this process more rare, than ionization.

β- particles at braking can create braking x-ray radiation.

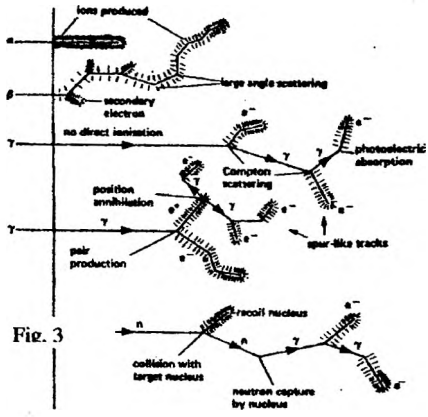


Fig. 3

At hit of positron in substance with high probability there is its interaction with electron, after which there are two γ – photon, which have energy not less energy of rest electron 0.51 MeV (reaction of annihilation): ${}^{0}_{-1}e + {}^{0}_{+1}e \rightarrow \gamma + \gamma$.

For α - and β- particles processes of dispersion are possible, therefore their way in substance is strongly bent.

At hit of γ- radiation in substance alongside with processes characteristic for x-ray radiation (coherent dispersion, Compton effect (scattering), photoeffect) are possible

other processes also.

At interaction of γ- photons of the big energy with nucleus is possible photonuclear reaction. For its occurrence energy of γ - photon should be not less binding energy falling on one nucleon.

At energy of γ - photon more than 1.2 MeV (not less total energy of rest electron and positron) is possible reaction of birth of pair electron-positron: $\gamma \rightarrow {}^{0}_{-1}e + {}^{0}_{+1}e$.

Attenuation of stream of γ- radiation in substance is described by the law: $\Phi = \Phi_0 \cdot e^{-\mu x}$, where μ is linear coefficient of absorption, which can be presented as the sum of corresponding coefficients of absorption, which are taking into account three processes of interaction: photoeffect (μ_{ph}), compton-effect or not coherent dispersion (μ_{nc}) and formation of pairs electron-positron (μ_p): $\mu = \mu_{ph} + \mu_{nc} + \mu_p$.

At action on substance of stream of neutrons can occur: elastic impact with a nucleus and secondary ionization, not elastic impact with a nucleus with emission of γ- quantum, capture of neutron by a nucleus with formation of the radioactive isotope. Last effect can cause formation in organism of radioactive isotopes: ${}^{1}H(n,\gamma){}^{2}H$; ${}^{23}Na(n,\gamma){}^{24}Na$; ${}^{31}P(n,\gamma){}^{32}P$ and some other reactions.

It is necessary to note interaction of radioactive radiations with water, at which there is chemical transformation named the *radiolysis of water*. As result of interaction are possible formation of the excited molecules (H_2O^*), ions (H_2O +), radicals (for example: $\dot{H}$, $\dot{O}H$), peroxide of hydrogen (H_2O_2). These highly active connections in the chemical attitude can cooperate with other molecules of biochemical system that will result to infringement of normal functioning of membranes, cells and bodies.

5. Using of radionuclides in medicine

Medical application of radionuclides can be submitted by two groups of methods: use with the *diagnostic and research purposes (labelled atoms)* and their application with the *therapeutic, medical* purpose. Bactericidal action of radiation concerns to the second group also.

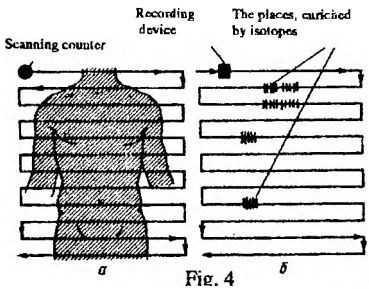


Fig. 4

The method of *labelled atoms*: in organism are entered radionuclides and are determined their location and activity in the bodies and tissues. For example, for diagnostics of disease of thyroid gland in organism enter radioactive iodine ${}^{131}_{53}I$, ${}^{125}_{53}I$, which part concentrates in gland. The counter located near to gland fixes speed of accumulation of iodine, on the basis of which it is possible to make diagnostic conclusions about condition of thyroid

gland. The cancer of thyroid gland can give metastasises in different bodies that can give information about accumulation of radioactive iodine in these bodies.

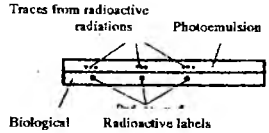


Fig. 5

For detection of distribution of radionuclides in organism it is applied *gammatopography*, which is carried out with help of the γ- topographer. The scanning counter gradually passes the big sites above the body. Intensity of radiation of a preparation, for example, by strokes on the paper (fig. 4) in the places of his presence is automatically fixed. The gamma - topographer gives rather rough distribution of radioactive preparation in the bodies. More exact data gives *autoradiography*. On a biological tissue apply the layer of photoemulsion (fig. 5). Radionuclides contained in the object give the trace on photoemulsion, as though photographing itself. Received picture is called the *autoradiogramm*.

In organism radioactive atoms enter in such small amount that neither atoms nor products of their decay do not exert harmful influence on the body. Applying radioactive isotopes it is possible to study *distribution of blood* and other biological liquids in organism. For this purpose, for example, enter the certain quantity of radioactive indicator into blood and having sustained time for its uniform distribution on blood system, it is possible to find her total amount on activity of unit of volume of blood.

Method of labelled atoms allows todiagnose *diseases of heart* and other bodies also. All researches and supervision are carried out without infringement of normal ability to live of an organism. In it value of method of labelled atoms.

Medical application of radionuclides basically is connected with use of γ-radiations (γ- therapy). Apparatus (cobalt gun) contains the protective lead container with ^{60}Co. Application of γ- radiations with big energy allows to *destroy deeply located tumours*. Superficially located bodies are exposed to smaller pernicious influence. Radioactive cobalt is applied for interstitial irradiation also. The needle containing thin pin from radioactive cobalt is stuck into the tissue.

For treatment of oncological diseases are applied α- particles in combination with streams of neutrons. In a tumour enter elements, which nucleus under action of the stream of neutrons cause nuclear reaction with formation of α- radiation. For example

$$^{6}_{3}Li+^{1}_{0}n\rightarrow^{3}_{1}H+^{4}_{2}\alpha\,.$$

Thus, α-particles and nucleus of feedback are formed in that place of the body, which it is necessary to subject to influence.

In the medical purposes *(treatment of illnesses of blood)* is used radioactive phosphorus $^{32}_{15}P$ (β–particles), which concentrates in the compact substance of tubular bones. $^{32}_{15}P$ irradiates marrow and thus normalizes broken at the certain diseases formation of blood. For similar purposes in relation to thyroid gland use the radioactive iodine $^{131}_{53}I$ giving electronic radiation.

It is applied radonic therapy also, at which are used the mineral waters containing ^{222}Rn and its products for influence on human skin (*radonic baths*), digestive apparatus (*drink*) and respiratory apparatus (*inhalation*).

LECTURE №27

DOSIMETRY OF IONIZING RADIATION

1. Absorbed and exposition doses. Capacity of a dose

Quantitative estimation of action of ionizing radiation on substance of alive and abiocoen nature has resulted to occurrence of the unit of physics named *dosimetry*.

The section of nuclear physics and measuring technics in which are studied the values describing action of ionizing radiation on substance and also methods and devices for their measurement is known as **dosimetry**.

Initial development of dosimetry has been caused, first of all, by necessity of the account of action of x-ray radiation on a human organism.

Action on substance is caused not by all radiation falling on it, but only by part, that cooperates with its atoms and molecules. Part of radiation, which passes the given body through without absorption does not influence on it. Therefore the basic value describing action of ionizing radiation on substance is the *energy of radiation absorbed by mass unit of substance during irradiation*. This value is called the **absorbed dose (D)**. Various effects of ionizing radiations first of all are defined by the absorbed dose. It depends on a kind of ionizing radiation, energy of its particles, structure of irradiated substance and is proportional to time of irradiation. Unit of the absorbed dose for any kind of radiation is **"Gray" (Gy)**(L. Gray is the English radiobiologist). For 1 Gy is accepted the dose of radiation at which to the irradiated substance of mass 1 kg is transferred energy of ionizing radiation of 1 Joule, i.e. 1 Gr = 1J/kg.

The dose of irradiation in time unit is called the *capacity of dose* (P=D/t). Capacity of dose is expressed in Grays per second (Gr/s).

The stand-alone unit of dose of radiation used in radiobiology is ***rad***: it is dose of any kind of ionizing radiation at which 1g of substance absorbs the energy of radiation equal to 100 erg. **1Gy=100 rad; 1rad=**
10^{-2} Gy. Capacity of dose is measured in rad/s.

For finding of the absorbed dose of radiation it is necessary to measure energy of ionizing radiation falling on a body, then the energy past through the body and their difference to divide on mass of the body. However practically in a human body to make it difficultly, as the body is non-uniform, *energy dissipates by a body on every possible directions*. In this connection estimate the dose absorbed by the body on ionizing action of radiation on air environmental the body. For characteristic of a dose on the effect of ionization of air is used so-called **exposition dose** (or **exposure or air dose**) (X) of x-ray or γ- radiation. It is necessary to remember, that the *exposition dose is defined only for air and only for quantum radiation*. The important advantage of this dose is that for its measurement there is the simple physical method, consisting in measurement of total charge of ions formed under action of radiation. For unit of exposition dose is

accepted the **1C/kg:** it is exposition dose of photon radiation at which the total charge of ions of one mark made in 1 kg of irradiated air is equal to 1 Coulomb.

Total ionization of air means effect both from initial action of ionizing radiation and from all secondary processes occuring at it, in particular from action of secondary electrons and nucleus of feedback. In practice is used the old stand-alone unit named **roentgen (R).** The exposition dose of 1R corresponds to formation of $2.08 \cdot 10^9$ pairs of ions in 0.001293g (in $1cm^3$) dry air under normal conditions: 1C/kg = 3876 R, i.e. new unit is much larger then old. It is useful to remember the convenient rule frequently used in the practical dosimetry: ***the dose of 1R collects for 1 hour at distance of 1 m from the radiant of radium by mass of 1 g, that is activity about 1 Curie.***

Power unit of exposition dose is 1 A/kg and stand-alone unit is 1R/s. As the absorbed dose is proportional to falling ionizig radiation between the exposition and absorbed doses, there should be proportional dependence $D=f \cdot X$, where f is the certain transitive coefficient dependent on lines of reasons and, first of all, from irradiated substance and energy of photons. It is easy to count up value of f, if irradiated substance is air. It is established, that for air for the exposition dose of 1R corresponds the absorbed dose equal to 0.88 *rad.* In this case $D=0.88 \cdot X$, f=0.88. For water and soft tissues f=1, hence **the absorbed dose in *rads* is equal to exposition dose in roentgens.**

Action of radiation on tissues of organism depends not only from the general absorbed dose, but also from capacity of radiation. For dot sources of radiations capacity of exposition dose decreases with distance under the law: $P = K_\gamma \frac{A}{R^2}$, where K_γ is ionizing constant or γ–constant of the radioactive isotope, dependent on its nature. Thus, *degree of influence of radiation on a human organism depends on nature of radioactive isotope (K_γ), its activity (A) and distance (R) up to the source.*

Gamma - constant is capacity of dose of radiation in R/s, created by γ - beams of the given radioactive isotope on distance of 1cm from a dot source, if its activity is 1mCr. The exposition dose in this case can be estimated from the parity $X = K_\gamma \frac{A}{R^2} \cdot \Delta t$, where Δt is time of irradiation.

2. Quantitative estimation of biological action of ionizing radiation. Equivalent dose. Equivalent effective dose. Collective dose

For protection against radioactive radiation is important to know its influence on a living tissue. For any kind of radiation biological action is usually than more, than is more absorbed dose. However experiment shows, that action of nuclear radiations on a tissue of living organism is defined not only by the dose, but also by the *nature* of ionizing radiation. Heavy particles (α - particles, protons, neutrons, fast ions) make more physiological infringements, than easy (β, γ and X-rays). Strongly penetrating streams of neutrons are especially dangerous.

In dosimetry it is accepted to compare biological effects of various kinds of radiations with the corresponding effects caused by *x-ray or γ - radiation.*

Distinctions in value of radioactive influence can be taken into account, having attributed to each radiation the *quality coefficient of radiation* (K). X-ray radiation, γ - quantums and β- particles affect an organic tissue approximately equally and for these K=1. For α-particles K=20, i.e. it is considered that α-particles in 20 times more dangerous at hit inside of organism, than γ- radiation. For protons and neutrons K=10, etc.

In radiobiology instead of coefficient of quality is used the *relative biological efficiency* (**RBE**). It is equal to the ratio of the absorbed dose standard (x-ray, γ) radiation causing the certain biological effect to the absorbed dose of the investigated kind of radiation, giving the same effect. This value characterizes also quality of radiation, therefore **RBE=K**.

For account of affecting action on organism of diffeferent kinds of radiations (with their coefficients of quality) is applied the concept of **equivalent dose (D_e),** which is connected with the absorbed dose by parity: $\mathbf{D_e = K \cdot D_{abs}}$.

As K is dimensionless coefficient, the equivalent dose of radiation has the same dimension as the absorbed dose, however is called as **Sievert** (Sv) (R.Sievert is the Swedish radiobiologist). Stand-alone unit of equivalent dose is **ber** (**biological equivalent of roentgen** or **rem**):

$$1 \text{ rem} = 10^{-2} \text{ Sv}; \quad 1 \text{ Sv} = 100 \text{ rem}.$$

The equivalent (biological) dose in rem is equal to the absorbed dose in rad, multiplied on K:

$$D_e \text{ (ber)} = K \cdot D \text{ (rad)}.$$

The equivalent dose is calculated for "average" tissue of human body. But doses should be determined and for separate bodies. In particular, it is necessary in radiation therapy of tumours, when it is not required to irradiate all body.

In relation to ionizing radiations organs and biological tissues have different radiosensitivity. The marrow and genitals are most strongly affected and, for example, nervous tissue is rather steady against radiation.

Account of radiosensitivity make with help of **coefficients of radiating risk (FR)**. Values of these factors for tissues and bodies of the person at uniform irradiation of all body are resulted in the table:

Red marrow – 0.12
Bone tissue – 0.03
Thyroid gland – 0.03
Mammary glands – 0.15
Lungs – 0.12
Ovary or testicle – 0.25
Other bodies – 0.30
Organism as whole – 1.00

If FR for mammary gland is 0.15, it means that the irradiation of mammary gland by the dose of 1Sv results to the same radiation damage of organism, as irradiation by the dose of 0.15 Sv of all body.

Thus, if it is known, what bodies and with what doses are irradiated (it is especially important to know at receiption of radioisotopes with food, water, inhalation of air with the subsequent accumulation in the certain bodies), then, having multyplied equivalent doses on corresponding coefficients of risk and putting on all bodies and tissues, we shall receive the *equivalent effective dose* (EED), reflecting total effect of individual irradiation for an organism. It is measured in Sievert also.

Knowing individual doses and putting them on group of irradiated people it is possible to receive the *collective effective equivalent dose* in *person-Sievert*. The collective dose can be calculated for separate settlement, area and republic. Thus, collective dose is objective estimation of scale of radiating defeat. If any number of people continues to live on polluted by radionuclides territory in conditions of long influence of radiation and laws of change of radiating influence are known, it is possible to calculate *expected collective effective equivalent dose* (it is measured in person-Sievert also) for the certain forthcoming period of time. For example, as result of failure on Chernobyl there was the pollution of significant territory by radio-activity of complex isotope structure. The estimation of expected collective dose in view of breaking up radionuclides is important for forecasting adverse consequences for living and future generations and serves as the reference point for decision-making.

In conclusion of the paragraph we summarize sense of each concept and field of its application (fig. 1).

Radiation hazard of used radiomaterial is convenient to estimate on **activity,** expressed in **Curie or Becquerels** (last unity is very small and consequently is practically inconvenient; normally it is used at measurings of samples of hazardous environment, which specific and aggregate activity insignificantly differs from background, stipulated by natural radioactive nuclides). Knowing the source activity it is possible to calculate **power of air dose (power of exposition dose)** for different distances from it and thus to spot, for example permissible residence time in this field.

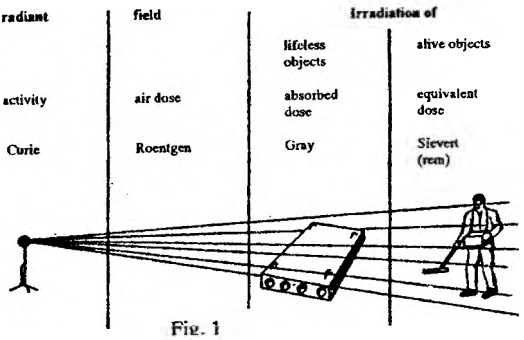


Fig. 1

The air dose (exposure) characterizes *radiation field* on its ionizing power, which is stipulated by character of radiomaterial or other radiant of ionizing radiation. For transition from air dose (characteristic of the field) to the **absorbed dose** (characteristic of interaction of the field and irradiated mediuim) it is necessary to know properties of this mediuim. At the same air dose, i.e. the same field, to water will be transferred smaller energy, than to material of the middle of the table of Mendeleev and the more so to heavy elements. The absorbed dose, i.e. the energy absorbed in mass unit of material, on which the

radiation field reacts, characterizes radiative effect for all kinds of physical and chemical bodies *except for living organisms*.

For estimation of radiation effect on living organisms, first of all for the person, it is offered and is used **the equivalent dose** of irradiation. In some practically frequently meeting events instead of D_e are used D and X. For the mixture of radiations at external and especially at internal irradiation only using of D_e allows to avoid errors in estimation of degree of radiation hazard of irradiation.

3. Doses of natural irradiation

On biosphere of Earth continuously operates space radiation and also streams of α-, β- particles, γ- quantums as result of radiation of various radionuclides, situated in the earth's crust, water of underground sources, in the rivers, seas, oceans, in air. Besides, radionuclides are part of living organisms. Set of radiations of these radioactive sources is called *natural radioactive radiation*. The most widespread on the Earth from radionuclides are ^{220}Rn, ^{222}Rn and ^{40}K and also radionuclides, making the line of uranium.

Isotope of radon ^{222}Rn decays and gives α- radiation, which is accompanied by emission of γ - photon.

Mass of stable ^{40}K always contains about 0.01 % of isotope ^{40}K, which nucleuses after decay forms ^{40}Ca, β - and γ- radiation. Isotope of K contains in ground, fertilizers and also in a brain, muscles, spleen and in a marrow. So, at a person of mass 70 kg contains about 0.021g of radionuclide ^{40}K. The half-life period of ^{40}K is $1.3 \cdot 10^9$ years. It is easy to calculate, that every second in our organism decay $5 \cdot 10^3$ atoms of ^{40}K. But it does not represent for us any danger and, apparently, is necessary for development of organism, as origin and development of life on the Earth were always accompanied by this process.

Space radiation will consist of streams of protons, α- particles, nucleus of some elements, streams of electrons, photons and neutrons. Particles of high energy, cooperating with atmosphere form as result of nuclear reactions series of radionuclides of $^{3}H, ^{7}Be, ^{22}Na$ and streams of neutrons and protons. This secondary radiation will penetrate into the bottom layers of atmosphere and influences biosphere.

As result of this natural both external and internal irradiation average capacity of dose makes about of 2 mSv per year (or *200 mber*). And approximately of 2/3 of these dose (≈135 mber) the person reseives from the radioactive isotopes, which have got in organism with food, water, air (an internal irradiation) and 65 mber from external irradiation. It is important to note, that natural radioactive background, influencing development on life on the Earth is integral part of sphere of dwelling of the person. Infringements of radioactive background are dangerous for existence of biosphere and can result to irreparable consequences.

One of the reasons of increase of radioactive background is human activity. Creation of the large industrial enterprises, power sources, military technics, etc.

can result in local changes of background. But the most dangerous reasons are emissions of radioactive particles, which can arise at nuclear explosions or at operation of atomic power stations. So, for example, at failure on the Chernobyl atomic power station there were emissions of radionuclides: ^{131}I (half-life period T=8 days, it gives γ - radiation), ^{90}Sr (T=29 years, gives β - radiation), ^{137}Cs (T=30 years, gives β - and γ-radiations). These isotopes can collect in organism causing in him infringement of activity, both separate organs and organism as whole. So, ^{131}I collects in the thyroid gland and already 0.35 mg of radioactive iodine is dangerous to life (daily need of stable iodine about 150 mg). The isotope ^{90}Sr collects in a bone tissue and the isotope ^{137}Cs is in regular intervals distributed in cells of organism.

*The maximum permissible biological dose for a person at professional irradiation considers **5 ber** per year. For the population is established the limiting dose in 10 times smaller: **0.5 ber** per year. Minimal lethal dose is conditionally accepted ≈600 ber at irradiation of all body.*

Sometimes the radioactive background is estimated on capacity of radiation. So the *normal natural background should not exceed of **20 mcR/h***. For the areas undergone to radioactive pollution as result of Chernobyl failure norms have been established: for zone of evacuation is 5 mR/h and for zone of alienation is 20 mR/h. Maximum permissible specific activity of the polluted area is considered equal to 15 Cu/km^2. Norms of specific activity of radionuclides in food stuffs are established also: grain for bakeries is $1.6 \cdot 10^{-8}$ Cr/kg; flour, groats is $1 \cdot 10^{-8}$ Cr/kg; children's feed of all kinds is $1 \cdot 10^{-8}$ Cr/kg.

4. Dosimetric devices

Devices for measurement of doses of ionizing radiations or the values connected with doses are called *dosimetric devices or dosimeters*. The principle of action of dosimeters is submitted on the circuit:

DETECTORS --------> THE DEVICE OF PROCESSING OF INFORMATION --------> INDICATION OF RESULTS

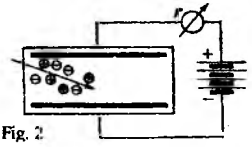


Fig. 2

In these devices for measurement of radiations always there are devices named *detectors*, in which energy of ionizing radiations will be transformed to electric signal. As in absorbing substance the particle or photon spend the energy for formation of a charge from ionization, on value of electric signal it is possible to judge about their energy and about quantity of the registered acts of decay.

There are various detectors of radiations. The most widespread detectors are:1) **ionizing chamber; 2) counter of Geiger-Muller; 3) semi-conductor and scintillometer detectors.**

1) In ionizing chamber as absorbing substance is served certain gas in the space between two electrodes (fig. 2). Particles getting into the chamber and

photons of radiation cause occurrence of a current. The current is proportional to number of ions formed in the chamber per second and, hence, energy flow of transiting ionizing particles. Such chambers use, in particular, in pocket dosimeters (fig. 3).

In practice of radiation monitorings the greatest distribution have received **thimble** ionization chambers though frequently the sizes and the shape of such chambers reminds a thimble a little. Such chamber can be viewed as a small vacuity filled with gas inside a solid body. As the effective atomic number of tissues of a human body $Z_{body}=7.42$ is close to effective atomic number of air $Z_{air}=7.64$, that it allows to determine absorbed energy for tissues of the body by results of measurings the ionization produced by explored radiation in air (walls of the chamber also produce from «tissue – equivalent » materials: polysterene, perspex, etc.). The charge gathered on electrodes of such chamber is directly proportional to the air dose. On fig. 4 the diagram of such chamber is shown: the wall of it is one electrode and the rod included in it is the measuring electrode. If to close key S, then chamber and connected with it in parallel condenser and electrometer (electroscope) are charged up to voltage of U_1 from the source HT. Measuring begins at breaking of key S, then as the result of ionization the voltage of the measuring electrode decreases up to value U_2. Voltage $\Delta U=U_1-U_2$ digitize on the electrometer. The charge Δq arisen owing to ionization is proportional to ΔU: $\Delta q=C\cdot\Delta U$, where C is capacity of the chamber. Corresponding air dose X:

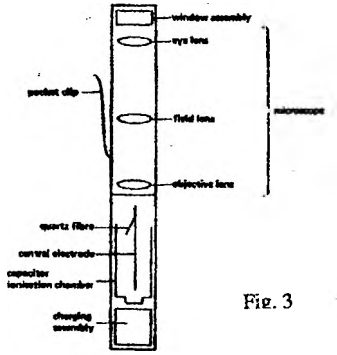


Fig. 3

$$X=\Delta q/V=C\cdot\Delta U/V=k\cdot(U_1-U_2),$$

where V is volume of the chamber, k is coefficient of proportionality determined at graduation of the device. So: $\mathbf{X \sim \Delta q \sim \Delta U}$. The main deficiency of ionization chambers is their rather low sensitivity, therefore them apply in fields of the considerable radiation intensity.

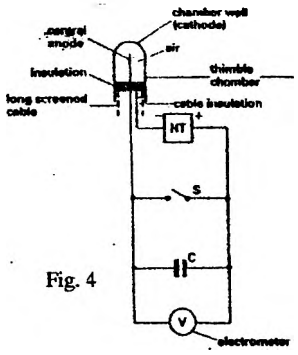


Fig. 4

The gas-discharge counters frequently termed also as Geiger-Muller counters (fig. 5), differ from ionization chambers the greater sensitivity and are capable to register individual pair of ions. By principle of the device such counter does

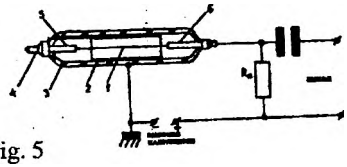


Fig. 5

not differ from ionizing chamber: it also is the condenser, on which the potential difference is applied, however it is so great, that in the gas gap originates new process – gas amplification that is sharp magnification of initial quantity of ions (≈ in 10^7 times). The cylindrical Geiger-Muller counter will consist of coaxially posed electrodes: 1 is the anode (the thin wire tensioned along an axis); 2 is the cathode as sprayed on the glass tube 3 metal; 4 is contact; 5 and 6 are insulators. Pressure of gas inside the counter is about of 100 mm Hg. To electrodes are applied a voltage at some hundreds volts. At hit in the counter of ionizing particle, in gas are formed mobile electrons which move to the anode. As the wire is thin (diameter is about of 0.5 mm), then near to wire the field is strongly nonuniform, also intensity is great. Electrons near to wire are sped up so, that start to ionize gas. As result there is a discharge and on the cicuit the current proceeds. The formed self-sustained discharge is necessary to destroy, as the counter will not react to the following particle. To discharge quenching apply: 1) inserting in series with the counter of high-resistance, on which there is the considerable voltage reduction, therefore the voltage on the counter decreases and then the discharge stops; 2) special gas filling up (argon and alcohol, halogens) giving in interruption of the discharge even at small resistances in the circuit. Limitation of a geiger: sharp dependence of efficiency of recording on energy of incident radiation («course with stiffness»).

γ- quantums in gas seldom make ionization. In this case apply *semi-conductor detectors or scintillometer*. In the semi-conductor detector absorption occurs in a semi-conductor material.

In scintillometer (fig. 6) passage of γ- radiation causes light flashes, which will be transformed to electric pulses and amplify the photoelectronic multiplier 2. For each kind of radiation select the suitable detector 1. In the capacity of detectors use monocrystals: NaJ(Te), CsJ(Na) – γ, β-radiation, ZnS(Ag) – in α- detectors, etc.

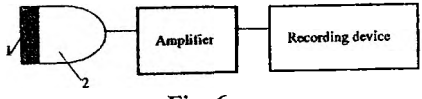


Fig. 6

To **destination** all dosimetric devices are subdivided on the following groups:

1. **Indicators**: elementary devices of radiation survey for detection and rough estimation of dose capacity.
2. **Roentgenometers** are intended for measurement of D_{exp}.
3. **Radiometers** are intended for measurement of activity or density of stream of radiation. With their help determine degree of radioactive pollution of the ground, air by α - β particles.
4. **Dosimeters** are intended for determination of dose and capacity of dose mainly of x-ray and γ - radiations.
5. **Spectrometers** are intended for reception of information on a spectrum of radiation of the person on energy of quantums or particles, to amplitude of signals, etc. parameters. Installation SRP (spectrometer of radiation of the person) is practically unique means of authentic estimation of doses of internal irradiation of a person.

II. Seminars

Seminar №1

Differential and integral calculus.

Basic formulas

Derivative f'(x) $$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Physical sense of a derivative $$\vartheta_{INST} = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t}$$

Geometrical sense of a derivative $$tg\alpha = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

Chain-rule of derivative $$y'_x = y'_u \cdot U'_x$$

Mechanical (physical) sense of the second derivative $$\lim_{\Delta t \to 0} \frac{\Delta \vartheta}{\Delta t} = \vartheta'(t) = a_{INST}$$

Differential of function $$df = f'(x)dx,\ \Delta f \approx df$$

Partial derivative $$\lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} = \frac{\partial f}{\partial x} \quad (y = const)$$

Total differential $$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

Indefinite integral $$\int f(x)dx = F(x) + C$$

Definite integral $$\int_a^b f(x)dx = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

Newton – Labnitz's formula $$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

Differential equation $$F(x, y, y', y'', \ldots, y^{(n)}) = 0$$

Law of dissolution of medicinal forms from tablets $$m = m_0 e^{-kt}$$

Table of derivatives of some elementary functions:

1	$(c = const)' = 0$	5	$(U \cdot \vartheta)' = U'\vartheta + \vartheta'U$	9	$(\cos x)' = -\sin x$
2	$x' = 1$	6	$\left(\frac{U}{\vartheta}\right)' = \frac{U'\vartheta - \vartheta'U}{\vartheta^2}$	10	$(tgx)' = \frac{1}{\cos^2 x}$
3	$(x^n)' = n \cdot x^{n-1}$	7	$(cU)' = c \cdot U'$	11	$(\ln x)' = 1/x$
4	$(U \pm \vartheta)' = U' \pm \vartheta'$	8	$(\sin x)' = \cos x$	12	$(e^x)' = e^x$

Properties of indefinite integral:

1. $\left(\int f(x)dx\right)' = f(x)$;
2. $d\left(\int f(x)dx\right) = f(x)dx$;
3. $\int d\varphi(x) = \varphi(x) + C$;
4. $\int kf(x)dx = k\int f(x)dx$;
5. $\int (f_1(x) \pm f_2(x))dx = \int f_1(x)dx \pm \int f_2(x)dx$

Table of some integrals:

$$\int dx = x + C; \quad \int x^n dx = \frac{x^{n+1}}{n+1} + C; \quad \int \frac{dx}{x} = \ln x + C; \quad \int e^x = e^x + C; \quad \int \sin x dx = -\cos x + C;$$

$$\int \cos x dx = \sin x + C; \quad \int \frac{dx}{\cos^2 x} = tgx + C; \quad \int \frac{dx}{\sin^2 x} = -ctgx + C$$

Integration by parts: $\int udv = uv - \int vdu$

Solving of DE with constant coefficients y'' + py' + qy = 0:

1. $k^2 + pk + q = 0$; 2. $k_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$; 3. Solution of DE:

7) if roots $k_1 \neq k_2$ are real numbers, $y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$;
8) if $k_1 = k_2 = k$, $y = (C_1 x + C_2) e^{kx}$;
9) if roots $k_{1,2} = \alpha \pm \beta i$ are complex numbers, where $i = \sqrt{-1}$ is imaginary unit, than solution is $y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$.

Problems

Find derivative of functions:

1. $y = x^3 - \frac{1}{x}$ 2. $y = x^5 - 2x^3 + 4\ln x - 5e^x$ 3. $y = \frac{e^x}{x^2 + 3}$
4. $y = x \ln x$ 5. $y = e^{3x}$ 6. $y = \cos 4x$ 7. $y = \sin^2 x$
8. $y = \sqrt{\ln x}$ 9. $y = \ln(x^2 + 1)$ 10. $y = tg^5(x^6 + 4)$
11. Displacement of an oscillating body is defined under the formula: $s = 2\sin(\pi t + \pi/2)$. Determine velocity of the body for t=1/6 s.

Find the partial derivatives of the first order:

12. $f(x, y) = x^3 \sin y$ 13. $f(x, y) = (6x^2 y - x^8 + 5)^4$
14. Response of the organism to a dose x of a medicinal preparation later t hours after reception is defined by the equation $f(x,t) = x^2(a - x)t^2 e^{-t}$. At what dose x response of the organism will be maximal and when it will occur?

Find differential of functions:

15. $y = e^{\sin x}$ 16. $y = \ln^2 \cos x$ 17. $y = x \cos x + 2$
18. How will decrease area of a quadrate with length of side 10cm, if the side of the quadrate to reduce on 0.01 cm?
19. The body moved during t=20 s with the constant acceleration of a=0.2 m/s. What path s it will come for the following 0.2 s, if it will prolong to move with the same acceleration?

Find indefinite and definite integrals:

20. $\int 4x^2 dx$ 21. $\int \frac{\sin 2x}{\sin x} dx$ 22. $\int e^{2x+1} dx$ 23. $\int \cos^2 x dx$ 24. $\int \sin^2 x \cos x dx$

25. $\int \frac{\ln^2 x}{x} dx$ 26. $\int_0^{\pi} \sin x dx$ 27. $\int_0^{\frac{\pi}{2}} \sin x \cos x dx$ 28. $\int_0^{\frac{\pi}{6}} \sin 6x$ 29. $\int_0^1 \frac{x dx}{x^2+5}$

30. Calculate the area of the figure limited by a parabola $y = x^2 + 2$ and by a straight line $x + y = 4$.

31. Determine the work expended at squeezing of a spring on 0.03 m, if it is known, that for its squeezing on 0.005 m it is necessary to spend the force of 10 N.

Solve the differential equations:

32. $y' = 8y$ 33. $(x^2+4)y' - 2xy = 0$ at x=1, y=5 34. $y'' - 8y' + 7 = 0$ 35. $y'' + 4y' + 13 = 0$

36. Speed of reduction of concentration of medicinal substance is proportional to concentration of substance at present time. Determine dependence of concentration of the given substance in blood on time, if during the initial moment of time it was equal to 0.2 mg/l and in 28 hours it has decreased twice.

Seminar №2

Elements of probability theory and mathematical statistics. Correlation. Testing of statistical hypotheses

Basic formulas

Classical probability $P(A)=\frac{m}{n}$, $0 \le p(A) \le 1$

Relative frequency $W(A)=\frac{m}{n}$

Probability of a sum of two not combined events $P(A+B)=P(A)+P(B)$

Probability of joint occurrence of two independent events $P(AB)=P(A)\cdot P(B)$

Mean M (x) (or μ) of a discrete random variable X $M(X)=\mu=\sum_{i=1}^{n} X_i P_i = X_1 \cdot p_1 + X_2 \cdot p_2 ... + X_n \cdot p_n$

Variance D(x) of a discrete random variable X $D(X)=M[X_i - M(X)]^2 = \sum_{i=1}^{n}[X_i - M(X)]^2 \cdot p_i$

Standard deviation $\sigma = \sigma(x) = \sqrt{D(x)}$

Normal law (Gauss's law) $f(x)=\frac{1}{\sigma\sqrt{2\Pi}}\ell^{-\frac{(x-\mu)^2}{2\sigma^2}}$, where f(x) is probability density

Standard deviation S_x for X $S_x = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+...+(x_n-\bar{x})^2}{n-1}}$

Standard deviation (error) for $\bar{X}$ $S_{\bar{x}}=\frac{S_x}{\sqrt{n}}$

Absolute error of $\bar{X}$ $\Delta\bar{x}=S_{\bar{x}}\cdot t_{\gamma,n}$, where $t_{\gamma;n}$ is Student's coefficient, it is table value

Interval estimation of $\bar{X}$ $\bar{X}-\Delta\bar{X} < \bar{X} < \bar{X}+\Delta\bar{X}$

Relative error $\varepsilon=\frac{\Delta\bar{x}}{\bar{x}}\cdot 100\%$

Median (Me) is the middle value concerning which in a sample there is identical number of variants

Mode (Mo) is value X_m of a random variable having the greatest probability

Sample correlation coefficient r_s $r_s = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{s_x s_y} = \frac{\overline{xy} - \bar{x}\bar{y}}{\sqrt{\overline{x^2} - (\bar{x})^2}\sqrt{\overline{y^2} - (\bar{y})^2}}$,

where $s_x = \sqrt{\overline{x^2} - (\bar{x})^2}$, $s_y = \sqrt{\overline{y^2} - (\bar{y})^2}$, $\overline{x^2} = \frac{\sum_{i=1}^{n} x_i^2}{n}$, $\overline{y^2} = \frac{\sum_{i=1}^{n} y_i^2}{n}$

Coefficient of regress ρ_{yx} $\rho_{yx} = r\frac{s_y}{s_x}$

Predicted value y $y(x) = ax + b = r\frac{s_y}{s_x}x + (\bar{y} - r\frac{s_y}{s_x}\bar{x})$

Check of significance $t_{experim} = \frac{r_s\sqrt{n-2}}{\sqrt{1-r_s}}$

of correlation coefficient

H_0 is accepted if $p > \alpha$ (α is the accepted significance level, usually is 0.05)

H_0 is rejected if $p < \alpha$, (distinctions are statistically significant at $p < 0.05$)

Student's t-test $t_{obs.} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{(n_X - 1)\cdot S_X^2 + (n_Y - 1)\cdot S_Y^2}{n_X + n_Y - 2}}} \cdot \sqrt{\frac{n_X \cdot n_Y}{n_X + n_Y}}$, $f = n_X + n_Y - 2$

critical region for rejection of H_0: $|t_{obs.}| > t_{cr}$

Fisher F-test $F_{obs.} = \frac{S_L^2}{S_S^2}$, $\kappa_{cr.} = F_{cr.}(\frac{\alpha}{2}; \kappa_1; \kappa_2)$, $\kappa_1 = n_L - 1$ and $\kappa_2 = n_S - 1$,

if $F_{obs.} < \kappa_{cr.}$ H_0 is accepted

Wilcoxon-Mann-Whitney observed values of U-test: $U_1 = R_1 - \frac{n_1(n_1+1)}{2}$ and

$U_2 = R_2 - \frac{n_2(n_2+1)}{2}$, pick $U_{obs.}$ as smaller from U_1

and U_2, if $U_{obs.} > U_{cr.}$ H_0 is accepted,

or U-test $U_{cr.}(\alpha; n_1; n_2)$

Problems

1. One letter at random gets out of letters of word "doctor". What probability that it will be the letter "o"?
2. There are 10 spheres in a box: 2 white, 3 red and 5 green. One sphere is taken randomly. Find probability, that the sphere will be 1) red; 2) white or green.
3. Two marksmen have made on one shot to a target. The probability of hit for the first marksmen is equal to 0.6 for the second is 0.7. To find probability

that: 1) only one of marksmen will get to the target; 2) at least of marksmen will get to the target; 3) both marksmen will get to the target; 4) none will get?

4. There are 514 boys from 1000 newborns. Find a frequency of birth a boy.
5. The law of distribution of a discrete random variable X is set by the table

X_i	1	2	3	4
p_i	0.1	0.2	0.3	0.4

Find the mean, variance and standard deviation. Construct polygon of distribution of probabilities.

6. The law of distribution of a discrete random variable X is set in the table

X_i	2	3	5
p_i	0.1	0.4	0.5

Find the mean, variance and standard deviation. Construct polygon of distribution of probabilities.

7. Height of 30 boys in the age of 2 (cm) is: 92, 91, 96, 93, 97, 93, 91, 92, 90, 97, 95, 94, 92, 98, 96, 90, 95, 93, 94, 89, 91, 89, 96, 94, 94, 92, 93, 95, 87, 94. Generate variational series and find frequency and relative frequency. Find the mean, mode and median of the received statistical series.
8. According to the problem 7 construct the histogram.
9. At measurements in homogeneous groups of patients the following samples have been received: 1) respiration rate: 12, 14, 12; 2) pulse rate: 71, 70, 74. State interval estimation of these values. Confidence probability to accept equal to 0.95.
10. At eight men have been measured height (x) and mass (m=y):

x (cm)	165	176	175	168	167	172	175	180
m (kg)	56	75	70	61	62	63	72	80

a) Determine correlation coefficient r_s.
b) Determine narrowness of correlation relationship.
c) Check up the significance of correlation coefficient.
d) Form equation of regress $y = \rho_{yx}x + b$, construct the graph.

11. At analysis of substance by two ways for two independent samples with volumes of n_X=10 and n_Y=8 taken from two populations X и Y, was found sample means $\bar{x}$=142.3 and $\bar{y}$=145.3 and the sample dispersions S_x^2=2.7 and S_y^2=3.2. For the significance level α =0.05 it is necessary:
   a) testing equality of dispersions by Fisher F- test;
   b) to test a null hypothesis H_0: $M(Y_p)=M(X_p)$ at alternative H_1: $M(Y_p)\neq M(X_p)$; observed value of the Student's t-test is $t_{obs.}$=1.3.
12. Using test of Wilcoxon-Mann-Whitney-test (U-test), estimate significance of distinctions of daily diuresis (ml) in two groups of patients. In group X to patients gave a preparation, in group Y to patients gave placebo. α =0.10. $U_{cr.}$ = 39.
    X: 1000, 1400, 1600, 1000, 1100, 1200, 1700, 1600, 1800, 1100, 1500, 1300, 1600.
    Y: 1200, 1000, 1000, 1100, 1200, 1400, 1450, 1600, 1300, 1250, 1100, 1400, 980.

Seminar №3

Mechanical oscillations and waves

Basic formulas

Angular velocity ω: $\omega = \frac{d\varphi}{dt} = \frac{\Delta\varphi}{\Delta t} = \frac{2\pi}{T} = 2\pi\nu$

Differential equation of free not damped (harmonious) oscillations: $\frac{d^2x}{dt^2} + \omega_0^2 x = 0$

and its solution: $x = A\cos(\omega_0 t + \varphi_0)$

Velocity of a material point at harmonious oscillation

$\vartheta = \frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t + \varphi_0) = \vartheta_0 \cos(\omega_0 t + \varphi_0 + \pi/2)$,

where $\vartheta_0 = A\omega_0$ is the maximal velocity (amplitude of velocity)

Damped harmonious oscillations (DE): $\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0$

and its solution $x = A_0 e^{-\beta t}\cos(\omega t + \varphi_0)$

Decrement of attenuation: $\delta = \frac{A_t}{A_{t+T}} = \frac{Ae^{-\beta t}}{Ae^{-\beta(t+T)}} = e^{\beta T}$

Logarithmic decrement of attenuation: $\lambda = \ln\frac{A_t}{A_{t+T}} = \ln e^{\beta T} = \beta T$.

Addition of harmonious oscillations of identical direction

(method of vector diagrams):

$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)$

$tg\varphi = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}$

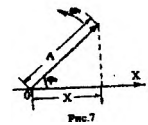


Рис.7

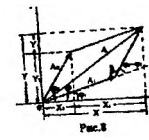
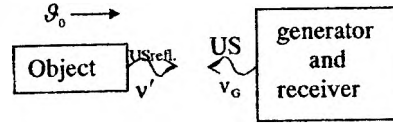


Рис.8

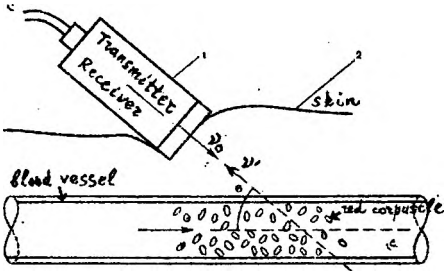
Equation of a flat wave:

$S = A\cos\left[\omega\left(t - \frac{x}{\vartheta}\right)\right]$ X is coordinate of point, S is displacement of the oscillating point

Doppler`s effect $\nu' = \frac{V \pm V_{OBS}}{V \mp V_{sour}}\nu$,



Doppler's frequency shift $v_{SHIFT} = v' - v$



$$v_{SHIFT} = \frac{2\vartheta_0}{\vartheta} v_G$$

v_0 is velocity of moving object,
v is velocity of ultrasound,
v_G is frequency of generator (transmitter)

Problems

1. To write down the equation of harmonious oscillations, if amplitude of velocity is v_m = 63 cm /s, the period of oscillations is T = 1 s, displacement of a point from position of balance during the initial moment of time is equal to zero. Find amplitude of acceleration, frequency of oscillations.

2. Two equally directed oscillations are set by the equations: $x_1 = 3\cos 5(t + 0{,}04\pi)$ and $x_2 = 5\cos 5(t + 0{,}14\pi)$. Write down the equation of resulting oscillations.

3. Logarithmic decrement of attenuation of a tuning fork oscillating with frequency of v =100 Hz is equal λ = 0,002. During what time interval the amplitude of oscillations of the tuning fork will decrease in 100 times?

4. The differential equation of damped oscillations is $0.5\frac{d^2x}{dt^2} + 0.25\frac{dx}{dt} + 8x = 0$. Determine coefficient of attenuation β and circular frequency ω of these oscillations.

5. The source of a sound makes oscillations under the law: $x = \sin 2000\pi t$. Velocity of propagation of sound is equal 340 m/s. To write down the equation of oscillations for a point which are taking place on distance of y = 102 m from the source. To neglect by losses of energy, a wave to count as flat wave.

6. Find the velocity of motion of a forward ventricle wall of heart aside breast, if at sonography by ultrasound with frequency of 800 кHz the reflected signal was accepted on frequency of 800.21 кHz. Velocity of ultrasound is equal to 1540 m / s.

Seminar №4

BASIC FORMULAS FOR THE TOPIC "ACOUSTICS"

Connection of intensity I with sound pressure Δp: $I = \frac{\Delta p_0^2}{2\rho\vartheta} = \frac{\Delta p_{eff}^2}{\rho\vartheta}$

Threshold of audibility I_0: for ν=1kHz, $I_0=10^{-12}$ Wt/m^2 or $p_0=2\cdot10^{-5}$ Pa
Threshold of painful sensation: $I_{max}=10$ Wt/m^2, $p_{max}=60$ Pa

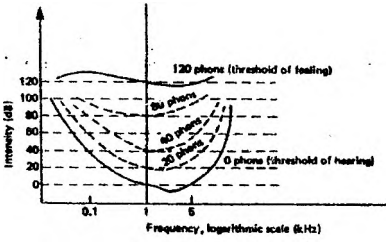
Level of intensity in Bells (B): $L_B = \lg\frac{I}{I_0}$, in decibels $L_{dB} = 10\lg\frac{I}{I_0}$

Physical or objective characteristics of sound:
1) frequency ν,2) harmonious spectrum, 3) intensity I
Physiological characteristic of sound: 1) height, 2) timbre, 3) loudness E
The range of frequencies perceived by an ear
16 Hz-20 kHz

Weber-Fehner's law:

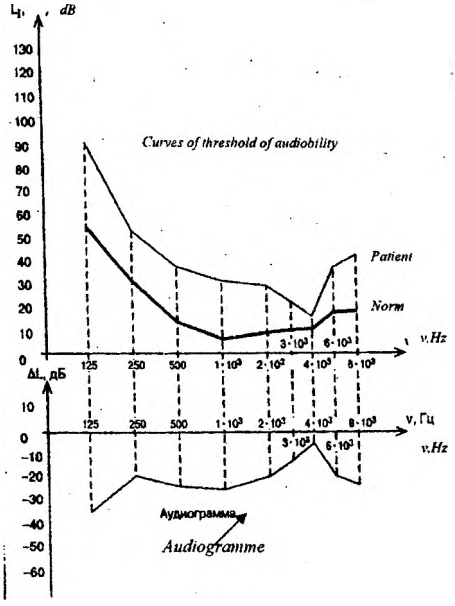
$E = KL = K\lg\frac{I}{I_0}$

Curves of equal loudness



Audiogramme: loss of hearing

$\Delta L = L_{norm} - L_{patient}$



PROBLEMS TO TOPIC "ACOUSTICS"

1. On what physical phenomenon is based the clinical method (percussion) of research of internal bodies by means of knocking on a surface of body and the analysis of sounds arising at it?

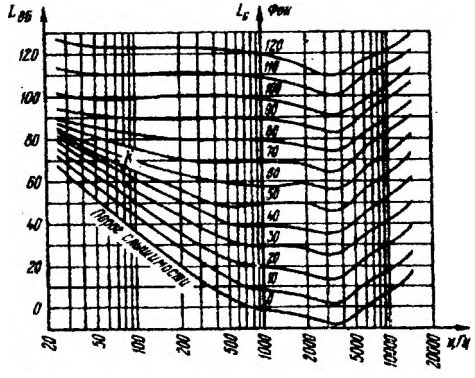
2. Why it is expedient as intermediate transmitting medium between a source of ultrasound and an irradiated tissue to use oil or water?

3. Intensity of the intimate tones perceived through a stethoscope is equal to 10^{-9} mcWt/cm^2. Determine a level of intensity of tones of heart.

4. Determine a level of intensity of sound in decibels, which creates a source of a sound of capacity 10 Wt on distance of 15 m.

5. Each of two speaking students creates a level of intensity of sound 60 dB. Find a total level of intensity of sound.

6. The level of intensity of a sound, measured by noisemeter on frequency of 200 Hz is equal to 40 dB. What is the loudness of the given sound?



7. What sound is louder: with a level of intensity 60 dB on frequency of 50 Hz or 40 dB on frequency of 300 Hz?

8. If in 60th of XIX century on highways of large cities the level of intensity of noise reached 60 dB, now on the same highways it is equal approximately to 110 dB. In how many times intensity of noise has increased?

9. At diagnosing of pathological changes in tissues of organism by ultrasonic method the reflected signal has been accepted through $5 \cdot 10^{-5}$ s after its radiation. What depth in tissues of heterogeneity?

Seminar №5

Hydrodynamics. Hemodynamics.

Basic formulas

Condition of indissolubility of jet $\vartheta_1 S_1 = \vartheta_2 S_2$

Equation of Bernoulli $\frac{\rho v_1^2}{2} + \rho g h_1 + p_1 = \frac{\rho v_2^2}{2} + \rho g h_2 + p_2$

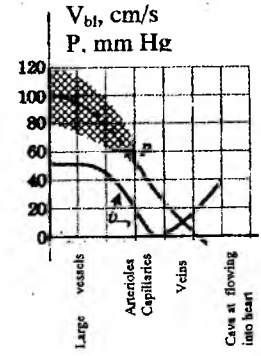
Equation of Newton $F_{fr} = \eta \cdot S \frac{d\vartheta}{dx}$

Unit of viscosity in SI is $N\frac{s}{M^2} = Pa \cdot s$, 1Pa·s=10 P

Relative viscosity $\eta_{rel} = \frac{\eta}{\eta_{water}}$;

Relative viscosity of blood is 4.2-6

Number of Reynolds: $\mathrm{Re} = \frac{2\rho r \vartheta}{\eta}$



Formula of Poiseuille

$Q = \frac{\pi \cdot R^4}{8 \cdot \eta} \cdot \frac{p_1 - p_2}{l}$

Hydraulic resistance ω $Q = \frac{p_1 - p_2}{\omega}$, where

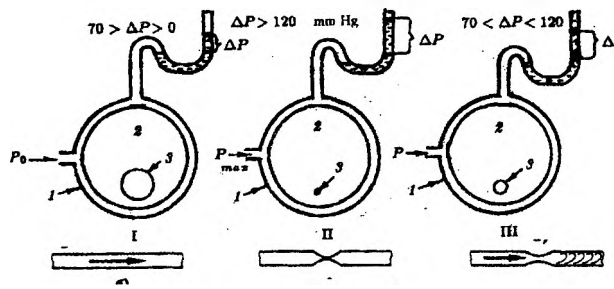
$\omega = \frac{8l\eta}{\pi R^4}$ is hydraulic resistance

Stock's law $F_{fr}=6\,\pi\,\eta\,r\,v$

Speed of pulse wave $\vartheta_{PULSEWAVE} = \sqrt{\frac{Eh}{\rho d}}$; $\vartheta_{PULSEWAVE} = 7\frac{m}{s}$

Work and capacity of heart: $A = A_i + A_r = A_i + 0{,}2A_i = 1{,}2\left(pV_s + \frac{\rho \cdot V_s \cdot \vartheta^2}{2}\right)$, $\overline{N} = \frac{A}{t} = 3{,}3Wt$

Measurement of blood pressure



Problems

1. In the bottom of a vessel of the cylindrical form of diameter 0.5 m is available a round aperture of diameter 1 centimeter. Find the dependence of speed of downturn of water level in the vessel from height of this level. Find value of this speed for height of 0.2 meters.

2. Determine the work at single reduction of the left ventricle, if thus to the aorta are forced 60 ml of blood at pressure of 13 кPa with the velocity of 0.5 m / s. Density of blood is $\rho = 1050$ kg /m^3.

3. Hydrostatic pressure of blood is caused by own weight of blood. Systolic pressure of blood at the level of heart is equal to 16 kPa. Determine total pressure at the level of the head located on 0.4 m above the level of heart, and at the level of the foot located on 1.5 m below heart. Density of blood is $\rho = 1050$ kg/m^3.

4. Determine erythrocyte sedimentation rate in plasma of blood (per mm /hour) at the assumption, that they have the form of balls in diameter of 7 microns. Density of blood is $\rho = 1050$ kg/m^3, density of erythrocytes is $\rho = 1092$ kg /m^3, viscosity of blood is $\eta = 5 \cdot 10^{-3}$ Pa·s.

5. From horizontally located medical syringe of diameter 1.5 cm is squeezed out the physiological solution by force of 10 N. Determine the speed of flowing of solution from the needle. Density of physiological solution is $\rho = 1030$ kg/m^3. The area of cross section of the piston is much more than area of cross section of the needle. Why speed of physiological solution does not depend on section of a needle?

6. Observing under a microscope motion of red corpuscle in a capillary it is possible to measure speed of blood flowing (ϑ = 0.5 mm/s). Average speed of blood in aorta is about ϑ = 40 cm /s. On the basis of these given determine in how many times the sum of area of cross sections of all functioning capillaries is more than area of cross section of aorta.

7. Why at the cut of a finger blood follows from the wound in regular intervals, instead of pulses in a step to heart beat?

8. Why damage of large veins more danger for a person than damage of large arteries?

9. Internal diameters of two needles of the equal length attached to identical syringes concern as 1:2. What is the ratio of volumes of liquids, flowing per unit of time through needles, if on pistons of syringes operates identical force?

10. Sometimes at injection there is a necessity of fast introduction of medicinal substances. In what case and in how many time procedure will pass faster: at increase of pressure twice or at increase of diameter of the needle twice? Lengths of needles are identical.

11. Viscosity or coefficient of internal friction for blood is equal to 0.004 – 0.005 Pa·s, and for plasma of blood is 0.0017 – 0.0022 Pa·s. What is the reason of the big distinction of their viscosity?

12. How to measure blood pressure? Basic physical idea of measurement.

Seminar №6

Direct current. ECG. Galvanization

Basic formulas

1. Intensity E of electric field $\vec{E} = \frac{\vec{F}}{q}$, [E]=N/C or V/m

2. Potential (U or φ) $U = \varphi_1 - \varphi_2 = \frac{A_{1\to 2}}{q}$, [φ]= V

3. Connection of E and φ $E = -\frac{d\varphi}{dl}$

4. Dipole moment P=ql

5. Potential in any point A of medium around of dipole $U_A = \frac{P\cos\alpha}{4\pi\varepsilon\varepsilon_0 r^2}$

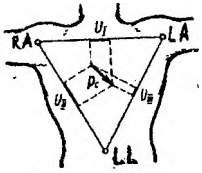
6. Potential difference between two points A and B for the current dipole $U_A = U_A - U_B = \frac{D\rho}{4\pi r^2}(\cos\alpha_A - \cos\alpha_B)$

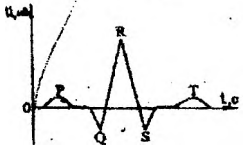
7. Main postulates of Einthoven`s theory:

1) the electric field of a heart is represented as electric field of current dipole $\vec{P}_H$ which is called the integral electric vector of heart;

2) $\vec{P}_H$ is in homogeneous isotropic conducting medium that is tissues of organism;

3) $\vec{P}_H$ permanently changes its direction and value





8. Lead is the difference of biopotentials registered between two points on human body

9. Einthoven`s leads: I- (RA-LA); II - (RA-LL) and III -(LA-LL)

10. Electrocardiogram is time dependence of a voltage in any lead

11. Electromotive force $E=A_{ext.\ force}/q$

12. Ohm's law for biological object $I = \frac{U - E_p(t)}{R}$

13. Galvanizing is the method of medical influence on organism by direct electric current up to 50 mA and the voltage up to 80 V.

14. Medicinal electrophoresis is introduction in tissues of organism through skin or mucous membrane of

medicinal substances (from the electrode of the same polarity)

Problems

1. Between interior part of a cell and outside solution there is a potential difference (a membrane resting potential) about $U = 80$ мV. Let the electric field inside a membrane is unimodal and depth of the membrane is $l = 80$ A. Find the intensity of this field.

2. In the scheme of the defibrillator (device for restitution of heartbeats of stopped or brushing heart by means of electrical irritation) are available two paralleled condensers in capacity of 8 mcF. Determine a charge of capacitor bank and a mean power of discharge if it happens for 10 мs. Voltage on battery is equal to 5000 V.

3. Find the dipole moment of the system electron – nucleus for atom of H, considering this system as the dipole. Distance between the nucleus and electron to is equal to $r = 10^{-8}$ cm.

4. Find the field potential, built by the dipole in the point A removed on distance of $r = 0.5$ m in the direction under the angle of $\alpha = 30°$ concerning the electrical moment p of the dipole. A medium is water. The dipole is formed by charges $q = 2 \cdot 10^{-7}$ Q on distance $l = 0.5$ cm.

5. The direct current 0.05 A represents danger to human life. Determine minimum value of voltage at which the current can reach this value if resistance of human body depending on requirements varies from 1000 up to 100 000 Ohm.

6. Where is case grounding of the electrical equipment is more necessary: а) in dry rooms with a timber floor; б) or in damp rooms with cement floor?

7. Why the approach of a person to the place of a fallen wire of high-voltage power line connected with danger of electrical shock?

8. Resistance of a tissue to the direct current in the circuit between electrodes at galvanization is 2000 Ohm at square of padding of 100 cm^2 and current density of 0.1 mA/cm^2. Determine the voltage with which the means of galvanization should ensure.

9. Determine the value of charge transiting at galvanization through a section of tissue during 2 minutes, if the current density is 0.1 mA/cm^2 and the size of electrodes is 4×6 cm.

10. Why at determination of resistance of tissues of an organism with the help of the ammeter and the voltmeter at use of the constant-current source the calculated outcome happens more than the real resistance value?

11. Electrophoresis is applied for introduction of medicinal materials into the human body. Determine how many ions of Iodine will be injected to the patient for 10 minutes at current density of 0.05 mA/cm^2 from the electrode of square 5 cm^2.

12. Find the interval R-R on the electrocardiogram, if heart rate is equal to 60 per minute.

13. Let the potential difference at present time in some lead is equal to null. Whether it means that value of the dipole moment of heart at this moment also is equal to null?

14. According to Einthoven representations, heart is similar to an electric dipole. The dipole moment of the «heart – dipole» periodically varies as modulo and on direction. ECG (biological potentials) records between vertexes of the conditional equilateral triangle which is formed by two arms and left leg. What form would have ECG in three leads, if the dipole moment of heart was uniformly revolved in the frontal plane? Specify formulas and build three "ECG".

15. At dry skin resistance between palms of hands can reach value of 10^5 Ohm, and for the wet palms this resistance is much less (1000 Ohm). Estimate a current, which will pass through a human body at contact to the network by voltage of 220 V. Compare this current with values of thresholds of perceptible current (1 mA) and not releasing current (10-15 mA) for frequency of 50 Hz.

16. The line voltage to which the medical apparatus is connected is equal to 220 V. Resistance of a human body is 1000 Ohm. The person stands on the ground and touches a body of apparatus. Resistance between conductor and the person is equal to 5000 Ohm. As result of damage of insulation the conductor has incorporated on the body of apparatus («breakdown on a body»). Determine the voltage on the human body, and also a current flowing past through him, if: 1) the apparatus is not grounded; 2) the apparatus is grounded, resistance of ground connection is equal to 4 Ohm. Compare the obtained data to values of threshold of perceptible current and threshold of not releasing current.

Seminar №7

Alternating current.
Nature of capacitive properties of tissues

Basic formulas

Working (effective) or root-mean-square value $I_{ef.} = I_{w.} = \frac{I_m}{\sqrt{2}}$, $U_{ef.} = U_{w.} = \frac{U_m}{\sqrt{2}}$

of alternating current (voltage)

Circuit of alternating current with

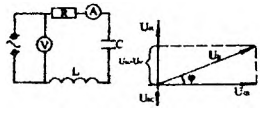
a) active resistance $I = I_0 \sin \omega t$, $U = U_0 \sin \omega t$

b) inductive resistance $I = I_0 \sin \omega t$, $X_L = \omega L$,

$$U_L = I_0 \omega L \cos \omega t = U_{0L} \sin(\omega t + 90^0)$$

c) capacitance $I = I_0 \sin \omega t$, $U_C = U_{0C} \sin(\omega t - 90^0)$, $X_C = 1/\omega C$,

d) total resistance (R,L,C) $I = I_0 \sin \omega t$,



$$U_0 = I_0 \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = I_0 Z$$

Quantity of heat q allocated in 1 m^3 in biological tissues per second at

1) Diathermy $q = \frac{Q}{Vt} = j^2 \rho$

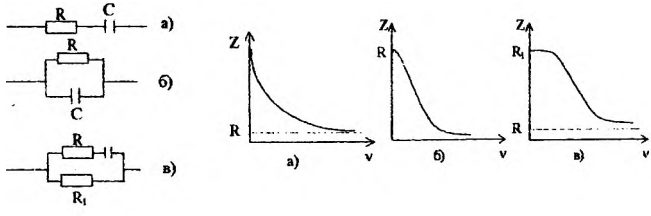
2) Inductothermy $q = j_{ef}^2 \cdot \rho = k \frac{B_{ef}^2 \cdot \omega_0^2}{\rho}$

3) UHF-therapy $q = \frac{P}{V} = \frac{E^2}{\rho} = E^2 \gamma$ (conductors)

$q = \frac{P}{V} = E_{ef}^2 \varepsilon \varepsilon_0 \cdot \omega \cdot tg\delta$ (dielectrics)

Period of oscillations in oscillatory contour $T = 2\pi\sqrt{LC}$

Equivalent electric circuits of living tissue:



Problems

1. Why at equality of direct-current voltage to effective voltage alternating-current last has more strong action on human organism?

2. Whether it is possible to count successful select of frequency 50Hz of alternating-current from the point of view of danger of electrical shock of a person?

3. Instantaneous value of voltage of sine current for the phase of $\pi/6$ equal to 150V. What is amplitude and effective value of voltage?

4. Why with raise of frequency of alternating current its stimulating action on a tissue of organism of a person is reduced?

5. At action of impulse current on a tissue of human organism there is a deformation of impulses compared with the shape of impulses of applied voltage. Why?

6. The finiteness on which electrodes are imposed has ohmic resistance about 1000 Ohm and capacity 0.02 mcF. Determine conductivity of such section, a phase angle between current and voltage for frequency of 50 Hz, considering, that ohmic and capacity resistances are connected in series.

7. Why at location between plates of therapeutic contour of UHF-apparatus of different parts of human body the resonance between anode and therapeutic contours is detuned?

8. The oscillatory circuit of apparatus for therapeutic diathermy will consist of inductance coil and condenser in capacity of 30 picoF. Determine inductance of the coil, if frequency of the generator is 1 MHz.

9. The phase shift between current and voltage at transiting of alternating-current of frequency 30 Hz through a muscle of rabbit makes -65^0. What is the resistance of the resistor in the equivalent circuit connected in series condenser and resistor, if a capacitor capacitance is equal to 3.6 mcF?

Seminar №8

Physical processes in biological membranes

Basic formulas

General equation of transportation $\Delta(N\varphi) = -\frac{1}{3}\lambda\bar{\vartheta}\frac{\Delta(n_0\varphi)}{\Delta X}\Delta S\cdot\Delta t$,

φ is transferable physical value in time Δt through a platform ΔS, V is speed of a molecule, λ is a distance of free run, $\frac{\Delta(n_0\varphi)}{\Delta X}$ is gradient of $(n_0\varphi)$

Equation of diffusion or $\Delta M = -\frac{1}{3}\lambda\bar{\vartheta}\frac{\Delta\rho}{\Delta x}\Delta S\cdot\Delta t = -D\frac{\Delta\rho}{\Delta x}\cdot\Delta S\cdot\Delta t$ or

Fick's law $J = -D\frac{dc}{dx}$, J is density of a stream of a substance

Versions of passive transport 1) simple diffusion, 2) facilitated diffusion, 3) filtration and 4) osmosis

Equation for density of stream at diffusion through a membrane $-\frac{Dk}{l}(C_e - C_i) = -p(C_e - C_i)$

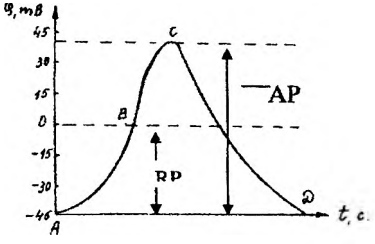
Nernst-Planck's equation (for transport of ions through a membrane) $J = -U_m RT\frac{dc}{dx} - U_m ZFC\frac{d\varphi}{dx}$

Nernst potential φ_m $\varphi_m = \frac{RT}{ZF}\ln\frac{C_1}{C_2}$, R is the molar gas, T is the absolute temperature, F is Faraday's number (96500 C/mole), Z is the valence of the ion, C_1 and C_2 are the molar concentrations of ions on the two sides of the membrane

Equation of resting potential (of Goldman - Hodgkin-Katz) $\varphi_M = -\frac{RT}{F}\ln\frac{P_K[K^+]_i + P_{Na}[Na^+]_i + P_{Cl^-}[Cl^-]_e}{P_K[K^+]_e + P_{Na}[Na^+]_e + P_{Cl^-}[Cl^-]_i}$,

P_K, P_{Na}, P_{Cl} are permeability of ions, $[K^+]_i$, $[K^+]_e$ are concentrations of ions inside and outside of a cell

Action potential



Problems

1. In a solution is supported the stationary condition of distribution of some substance. Thus on distance of l=50 cm the difference of concentration of the dissolved substance makes $2 \cdot 10^{-4}$ mol/l. Coefficient of diffusion is $D=3 \cdot 10^{-12}$ m^2/s. Find density of stream of diffusion and the maximal stream through a platform of area $S=1$ cm^2.

2. Bilayer lipid membrane of thickness 10 nanometers divides the chamber on two parts. The density of a stream of methylene blue through the membrane is constant and equal to $3 \cdot 10^{-4}$ M·cm/s, and concentration of it on the one side of membrane is equal 10^{-2}M and on another side $2 \cdot 10^{-3}$ M. Find the coefficient of diffusion of this substance through the membrane.

3. At preparation of jam is used very high concentration of sugar. It brings, in particular, to death of the bacteria causing botulism. In what will consist one of the reasons of this effect?

4. External surface of a membrane of living intact cell is charged positively and internal is negative. What is the reason of similar distribution of charges on the membrane?

5. The resting potential of a skeletal muscle is equal to 88 mV. Determine the ratio of concentration of potassium ions inside of muscular fibre and in the external medium. A body temperature of the person is equal to 37 °C.

6. Determine the resting potential of a cell at temperature of 20 °C, if the ratio of concentration of potassium ions in the cell and the medium is equal to 10 : 1.

7. Calculate the resting potential of huge axon of squid, if it is known, that concentration of ions of sodium outside is equal to 440 mM, and inside is 49 mM (the temperature is equal 20^0 C).

8. The resting potential of a nervous fibre of a squid is equal to -60 mV, and action potential is equal to +35 mV. Explain such change of membrane potential.

Seminar №9

Optics

Basic formulas

Optical force of lens $D=1/F$

Optical force of system of lenses, located closely $D_{TOTAL}=D_1+D_2+\ldots+D_n$

Connection between the size of a subject B and its image on retina b $b=B(l/L)$, l is the distance from the image on retina up to the central point, L is the distance from a subject up to the central point.

Refracting system of eye cornea, moisture of the forward chamber, crystalline lens, vitreous body

Accommodation is ability of eye to change optical force due to change of curvature of a crystalline lens

Total eye optical force 43(cornea) + 20 (lens) = 63 diopters

Distance of the best vision (≈25cm) (or distinct vision) is minimal distance at which accommodation is made without pressure

Visual acuity V $V=1/\beta_{min}$, where β_{min} is minimal visual angle (≈1')

Defects of vision 1) myopia, 2)hypermetropia, 3) astigmatism, 4) presbyopia, 5) aphakia

Light exposure $E=\frac{d\Phi}{dS}$, (lux)

Magnification of a microscope M $M=\frac{\Delta \cdot a_0}{F_{objec.} \cdot F_{eyepiece}}$, where Δ is optical length of tube, a_0 is the distance of the best vision

Aperture angle α is equal to half of the angle $u=\alpha/2$, formed by the beams going from a subject to objective edges

Distinction limit of a microscope $\ell_{min} \geq \frac{0.5\lambda}{n\sin(\frac{\alpha}{2})}$, where $A=n \cdot \sin(\frac{\alpha}{2})$ is the numerical aperture

Resolving power R $R=1/\ell_{min}$

Useful magnification of microscope M $M=\frac{\ell_{min,eye}}{\ell_{min,micr}}=\frac{\ell_{min,eye} \cdot A}{0,61 \cdot \lambda}$

Path difference $\Delta=r_2-r_1$

Path difference in a medium $L=n \cdot \Delta$

Interference minima condition $\Delta=(2k+1)\frac{\lambda}{2}$, k=0,1,2,3,..., where $\Delta=r_2-r_1$ is geometrical difference of path

Interference maxima condition $\Delta = 2k\frac{\lambda}{2} = k\lambda$

Huygens-Fresnel's principle: points of medium of which were reached front of a wave are sources of elementary secondary coherent waves

Condition of diffraction minima for slit $a\sin\varphi = 2k\frac{\lambda}{2} = k\lambda$, (m=2k)

Formula of a diffraction grating (maxima) $d\sin\varphi = k\lambda$, d is period of grating

Woolf – Bragg`s formula $2d\sin\theta = k\lambda$

Malus's law $I = I_0\cos^2\varphi$, I_0 is light intensity after polarizer, I is the light intensity after analyzer, φ is angle between principal planes of the analyzer and polarizer

Brewster's law $tg\, i_B = n$, $n=n_2/n_1$ is refractive index

Rotation angle of polarization plane in a solution $\Delta\varphi = \alpha \cdot C \cdot l$

Problems

1. Explain, why at artificial expansion of a pupil with the help of medicine the eye starts to distinguish subjects worse?

2. What is caused basically refraction of the light beams penetrating into an eye – by the cornea, crystalline lens or vitreous body?

3. What person (short-sighted or far-sighted) will better see subjects under the water?

4. How can see a person with normal visual acuity, if at him to remove crystalline lens? Explain the answer.

5. At presence of a set of trial spectacled glasses for determination of optical force of unknown glass use the method of neutralization, which essence consists in overlapping glasses and reception of the pair equivalent to a plane-parallel plate. How it is possible to be convinced that neutralization is achieved? What is the optical force of a lens, if neutralization has turned out with the lens of +5 dioptries?

6. The greatest distance from which a person can read the text of the book is 80 cm, and the least is 25. How will changes optical force of his eyes.

7. How should work people using glasses with a microscope: to look in eyepiece through glasses or without them?

8. Find the focal length of objective of the microscope giving magnification of 500 times, if the focal length of eyepiece is 4 cm and length of a tube is 20 cm.

9. Diameter of a bacterium is 7.5 microns. Determine diameter of the image at use of a microscope, if focal lengths of the objective and eyepiece accordingly 4 and 24 mm. Subject glass is located on distance of 4.2 mm from the optical center of the objective.

10. At passage of first beam of monochromatic light through the object of research and the second beam only through the medium between beams there is the path difference of 0.4 λ . Determine a difference of phases of these beams.

11. Difference of phases of two interfering waves is 180°. Determine their path difference.

12. At production of optical systems with «bloomed optics» on a surface of lenses render a thin layer of transparent substance, which refractive index is less, than at glass. As result of interference of light at reflection from border air – layer and layer – glass decreases refractive index of the surface of lens. Estimate thickness of the rendered layer at normal falling of light.

13. Monochromatic light of λ = 700 nanometers falls normally on a slit. The angle of deviation of the beams corresponding to the second minimum is equal to 1 °. Determine width of the slit.

14. Determine the numerical aperture of an immersion objective of a microscope, if its aperture angle is 70 °, and immersion medium is cedar oil (n = 1.51).

15. Concentration of sugar in a solutions determine by polarization method, using light of yellow sodic lines, for which specific rotation of sugar at temperature of 20°C is equal to 66.5 ° sm^3 / (g·dm). What length of a tube it is necessary to use, that the apparent angle of rotation of a polarization plane in degrees was equaled concentration of the solution in grammes on 100 sm^3?

16. For working light is used the lamp of 100 cd. Lamp is placed on height of 2 m. On what height it is necessary to locate the lamp of 50 cd to receive previous illumination intensity?

17. The interferometer is illuminated by monochromatic light by the wave length of 589 nanometers. Length of the cavity is 10 cm. When air in one cavity have exchanged with ammonia, the interference figure on the screen was biased on 17 strips. Refractive index of air is 1.000277. Determine the refractive index of ammonia.

18. The path difference of waves from two interfering sources is 0.2 λ. Determine a phase difference and result of interference.

19. What is the angle between main planes of polarizer and analyzer, if intensity of natural light, past through the polarizer and the analyzer has decreased in 4 times?

20. Determine optical force for two systems of lenses: a) collecting lens of D_1=1 diopter and concave lens of F_2=40 cm; б) convex lens of F_1=1 m and convex lens of D_2=1.5 diopters.

21. The minimal visual angle is equal 5'. In how many times visual acuity is lower than norm?

22. Why the image on retina in twilight is blurred?

23. Determine a distinction limit for dry and immersion objective (n=1.55) for the angular aperture of 140^0.

Seminar №10

Thermal radiation. X-ray radiation

Basic formulas

Some characteristics of thermal radiation

1) Emissive ability $R=\frac{E}{St}$;

2) absorptive ability of body $A=\frac{E_{abs}}{E_{allinc}}$

Perfectly black body (PBB) if A=1

Kirchhoff's law $\frac{R'}{A'}=\frac{R''}{A''}=\frac{R'''}{A'''}=\cdots=\frac{\varepsilon}{1}=\varepsilon$, where ε is emissive ability of PBB

Planck's formula $\varepsilon_\nu=\frac{2\pi\nu^2}{c^2}\cdot\frac{h\nu}{e^{h\nu/kT}-1}$

Stephan-Boltzmann's law $\varepsilon=\sigma T^4$

(Total emissive ability of the perfectly black body ε)

Wien's law $\lambda_m=\frac{b}{T}$

(wavelength to which corresponds maximum of energy)

Capacity P lost by a person at interaction with environment $P=P_1-P_0=S\delta(T_1^4-T_0^4)$

Einstein's equation for the photoeffect $h\nu=A+\frac{m\vartheta^2}{2}$

The border of the most short-wave X-ray radiation $\lambda_{min}=1{,}23/U$ (nm)

Stream of brake x-ray radiation $\Phi=KIU^2Z$

Law of beam attenuation of X- ray radiation in material $\Phi=\Phi_0e^{-\mu x}$

Moseley's law $\sqrt{\nu}=A(Z-B)$

Linear coefficient of attenuation of X-rays $\mu=\mu_m\cdot\rho$

Mass coefficient of absorption $\mu_m=K\lambda^3Z^3$

Relations of energy units $1eV=1.6\cdot10^{-19}$ J

Problems

1. Explain the composite spatial distribution of temperature of a human body.

2. At the fixed stay of a person in water at temperature of 12 °C during 4 minutes is spent about 420 kJ of heat, i.e. as much, as on air at the same temperature is spent for 1 h. Why it happens?

3. The body of a person can be compared to a body, which temperature is 300°K and the absorptive power is 95 %. Determine what quantity of heat is radiated from 1 m^2 of a body surface within one hour, if ambient temperature is 25 °C.

4. Determine the emissive ability of a human body at temperature of t = 36 °C, accepting him as a grey body with absorptive ability of A = 0,9.

5. Determine thermal losses of 1 sm^2 of a human body surface for a minute by radiation, if temperature of the skin surface is 32 °C and temperature surrounding medium is 17 °C.

6. What is the wave length of maximum of emissive ability of a human body, if as a source of radiation it has the same properties as absolutely black body at temperature of 300 ° C?

7. In the lamp of infrared radiation (инфраруж) the maximum of emissive power corresponds to the wave length of 4 microns. Determine the temperature of the lamp spiral.

8. Why a thermal lamp, which cylinder is made of a dark blue glass calls sensation of more intensive heat, than the same power lamp with a cylinder from a pallid glass?

9. On a zinc plate are guided X-rays with wave length of 0.1 nanometers. Determine velocity of photoelectrons. To neglect by the photoelectric work function of electrons from Zincum.

10. Why the brake X-rays has the continuous spectrum and limit boundary on the part of short lengths of waves?

11. Why the increase of the voltage affixed to X-ray tube gives in diminution of a boundary wave length of the spectrum of brake X-rays?

12. Determine velocity of the electrons impinging on the anticathode of a X-ray tube, if the minimal wave length in the continuous spectrum of X-rays is 0.01 nanometers.

13. What radiation will be rigider: X-rays of voltage 150 kV, or γ-radiation of thulium (E_γ = 0.074 MeV)?

14. Considering, that absorption of X-rays does not depend in what compound the atom is submitted in material, determine, in how many times the mass coefficient of absorption of a bone ($Ca_3(PO_4)_2$) more than mass coefficient of absorption of water.

15. For the radiodiagnosis of the soft tissues use contrast agents. For example, the stomach and intestine fill by the mass of barium sulfate $BaSO_4$. Compare mass coefficient of absorption of barium sulfate and the soft tissues (water).

Seminar №11

Radioactivity. Dosimetry

Basic formulas

Basic law of radioactive decay	$N = N_0 \cdot e^{-\lambda t}$
Half-life period T	$T = \frac{0{,}693}{\lambda}$
Activity A	$A = A_0 \cdot e^{-\lambda t} = A_0 \cdot e^{\frac{-0{,}693}{T}t}$
Units of activity	1 Becquerel (Bc) $= \frac{1 decay}{1 \sec ond}$
	1 Curie (Cur) = 1Cur = $3{,}7 \cdot 10^{10}$ Bc
Absorbed doze D, its unit	D=E/m, [D] = Gray (1Gy=1J/1kg)=100rad
Doze capacity	P=D/t, Gr/s
Exposition doze X,	D=fX, for water and soft human tissues
connection of X with absorbed doze	D f=1; unit of X: 1C/kg;
	1 C/kg = 3876 R (roentgen)
Doze rate for dot charge	$P = K_\gamma \frac{A}{R^2}$
Connection of absorbed doze D_{abs} with equivalent doze D_e	$D_e = K \cdot D_{abs}$, where K = RBE is relative biological efficiency (RBE); for x-rays and γ-radiation K=1, for α-radiation K=20;
Units of equivalent doze D_e	1 Sievert (Sv); 1Sv=100rem; 1 rad ≈ 1 R ≈ 1 rem (rem=ber)
Maximum permissible biological doze at a professional irradiation and for population	$D_{max\ prof}$ = 5 ber per year $D_{max\ popul}$ = 0.5 ber per year
Minimal lethal doze (LD50)	≈ 500 ber (rem)
Normal radiation background	P=10 – 20 mcR/h

Problems

1. In organism of a person there are $6 \cdot 10^{-9}$g of $^{226}_{88}Ra$. Determine its activity.

2. Geiger counter near to a preparation of radioactive isotope of argentum at the first measuring noted 5200 β-particles per minute and in day only 300. Determine a half-life period of the isotope.

3. The irradiation of organism of a person by neutrons is more dangerous, than irradiation by other kinds of radiations. Why?

4. For the persons working immediately with radiants of ionizing radiations (category A) sets the maximum permissible dose of 0.001 Gr in a week. Determine the limiting dose capacity of X-ray or γ-radiation (Gr/h) at 25-hour working week.

5. On what distance from the preparation of $^{60}_{27}Co$ of activity 200 mCu it is necessary to be, that the dose for the 6-hour working day did not exceed permissible? Ionization stationary value of the cobalt is 13.5 $R \cdot cm^2 / (h \cdot mCu)$.

6. The body of mass 60 kg for 6 hours had been absorbed energy of 1 J. Determine the absorbed dose and the dose capacity in SI-system.

7. In a tissue of mass 10 g is immersed 10^9 α- particles with energy of 5 MeV. Find the absorbed dose, if coefficient of quality for α- particles is K=20.

8. The mean power of exposition dose in the X-ray room is $6 \cdot 10^{-12}$ Q/(kg·s). A doctor is in the room within 5 hours. What his exposure dose for 5 working days.

9. On how many degrees will increase the temperature of the phantom (model of a human body) of mass 80 kg, if its heat capacity is $4.2 \cdot 10^3$ J/kg at the exposure dose of 600 *ber*.

10. What depth of a material is necessary to take to attenuate a dose of X-ray or γ- radiation in 2 times?

TABLES

SOME PHYSICAL CONSTANTS

Constant	Symbol	Value
Velocity of light in vacuum	c	$3 \cdot 10^{8}$ m/s
Permeability of free space	μ_0	$4\pi \cdot 10^{-7}$ H/m
Permittivity of free space	ε_0	$8.85 \cdot 10^{-12}$ F/m
Universal constant of gravitation	G	$6.67 \cdot 10^{-11}$ Nm^2/kg^2
Planck constant	h	$6.63 \cdot 10^{-34}$ J·s
Rest mass of electron	m_e	$9.1 \cdot 10^{-31}$ kg
Rest mass of proton	m_p	$1.673 \cdot 10^{-27}$ kg
Rest mass of neutron	m_n	$1.674 \cdot 10^{-27}$ kg
Electron charge	e	$1.6 \cdot 10^{-19}$ C
Specific charge of electron	e/m	$1.76 \cdot 10^{11}$ C/kg
Atomic mass unit	u	$1.66 \cdot 10^{-27}$ kg
Avogadro constant	N_A	$6.02 \cdot 10^{23}$ mol^{-1}
Faraday constant	F	$9.65 \cdot 10^{4}$ C/mol
Molar gas constant	R	8.31 J/(mol·K)
Boltzmann constant	$k=R/N_A$	$1.38 \cdot 10^{-23}$ J/K
Acceleration due to gravity	g	9.81 m/s^2
Stefan constant	σ	$5.7 \cdot 10^{-8}$ $W/(m^2 \cdot K^4)$
Bohr magneton	μ_B	$9.27 \cdot 10^{-24}$ J/T
Coefficient in the law of Coulomb	$k=1/(4\pi\varepsilon_0)$	$9.00 \cdot 10^{9}$ m/F
Absolute zero		- 273.16 °C

Units in Physics

Quantity	*Unit*	*Symbol*
length	metre	m
mass	kilogram	kg
atomic mass	atomic mass unit	u
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
luminous intensity	candela	cd
amount of substance	mole	mol
frequency	hertz	Hz
force	newton	N
pressure and stress	pascal	Pa
work, energy, heat	joule	J
power	watt	W
electric charge	coulomb	C
electric potential difference	volt	V
electromotive force	volt	V
electric resistance	ohm	Ω
electric conductance	Siemens	S
electric capacitance	farad	F
magnetic flux	weber	W
magnetic flux density (magnetic induction)	tesla	T
inductance	henry	H
luminous flux	lumen	lm
illuminance	lux	lx
activity (of radioactive source)	becquerel	Bq
specific heat capacity	J/(kg·K)	*c*
momentum	N·s	*P*
moment of a force	N·m	*M*
torque	N·m	*T*
electrical resistivity	Ω·m	ρ
electrical conductivity	S/m	σ
current density	A/m^2	j
permittivity	F/m	ε
electric field strength	N/C or V/m	*E*
capacitance	F	*C*
permeability	H/M	μ
moment of inertia	$kg \cdot m^2$	I
Young modulus	Pa	*E*
surface tension	N/m	α
viscosity	Pa·s	η
thermal conductivity	W/(m·K)	*k*

Student t Distribution

Degrees of freedom f = n-1	Significance level α,%		
	2.5 (one tail)	0.5 (one tail)	0.05 (one tail)
	5 (two tails)	1 (two tails)	0.1 (two tails)
1	12.71	63.66	64.60
2	4.30	9.92	31.6
3	3.18	5.84	12.92
4	2.78	4.60	8.61
5	2.57	4.03	6.87
6	2.45	3.71	5.96
7	2.37	3.50	5.41
8	2.31	3.36	5.04
9	2.26	3.25	4.78
10	2.23	3.17	4.59
11	2.20	3.11	4.44
12	2.18	3.05	4.32
13	2.16	3.01	4.22
14	2.14	2.98	4.14
15	2.13	2.95	4.07
16	2.12	2.92	4.02
17	2.11	2.90	3.97
18	2.10	2.88	3.92
19	2.09	2.86	3.88
20	2.09	2.85	3.85
21	2.08	2.83	3.82
22	2.07	2.82	3.79
23	2.07	2.81	3.77
24	2.06	2.80	3.75
25	2.06	2.79	3.73
26	2.06	2.78	3.71
27	2.05	2.77	3.69
28	2.05	2.76	3.67
29	2.05	2.76	3.66
30	2.04	2.75	3.65
40	2.02	2.70	3.55
60	2.0	2.66	3.46
120	1.98	2.62	3.37
∞	1.96	2.58	3.29

MATHEMATICAL SYMBOLS AND FORMULAS		
SYMBOLS	*ENGLISH*	*RUSSIAN*
+	Plus	Плюс
-	Minus	Минус
±	Plus or minus	Плюс минус
/ or :	Division sign	Знак деления
× or ·	Multiplication sign	Знак умножения
=	1) Sign of equality 2) Equals, (is) equal to	Знак равенства
≠	(is) not equal to	Не равно
>	Greater than	Больше (чем)
≥	Equal or greater than	Больше или равно
<	less than	Меньше (чем)
≈	Approximately equal	Приблизительно равно
$\sqrt{}$	Square root (out) of	Корень квадратный из
∞	Infinity	Бесконечность
.	Point	Точка
$\sqrt[3]{}$	Cube root (out) of	Корень кубический из
$\sqrt[n]{}$	n-th root (out) of	Корень n-ой степени
( )	Round brackets	Круглые скобки
[ ]	Square brackets	Квадратные скобки
||	Parallel to	Параллельно
⊥	Perpendicular to	Перпендикулярно
∠	Angle	Угол
°	Degree	Градус
′	Minute	Минута
″	Second	Секунда
a'	*a* prime	"*a*" штрих или "*a*" "прим"
b_1	*b* sub one or *be* first	"*b*" один или "*b*" с индексом один
c_m	*c* sub *m* or *c*, *m*-th	"*c*" "*m*" или "*c*" с индексом "*m*"
log	Logarithm	Логарифм
sin	Sine	Синус
cos	Cosine	Косинус
tg	Tangent	Тангенс
ctg	Cotangent	Котангенс
Σ	Summation	Знак суммирования

		(сумма)
f(x)		Эф от икс
dx	Differential of x	Дифференциал "x" (дэ икс)
$\frac{dy}{dx}$	dy over dx or The first derivative of y with respect to x	dy по dx или Производная y по x
$\frac{d^2y}{dx^2}$	Second derivative of y with respect to x	Вторая производная y по x или "дэ два y по dx дважды"
$\int$	Integral of	Интеграл от
$\int f(x)dx$	Integral of a function of x over dx	Интеграл от функции f(x) по dx
$\int_a^b$	Integral between limits "a" and "b"	Интеграл в пределах от "a" до "b" или Интеграл от "a" до "b"
$\|x\|$	Absolute value of x	Абсолютное значение x
%	Per cent	Процент
$\frac{3}{5}$	Three fifths	Три пятых
$\frac{2}{7}$	Two sevenths	Две седьмых
$\frac{1}{3}$	One third or A third	Одна треть или Треть
$2\frac{1}{2}$	Two and a half	Две целых одна вторая
0.6	Point six	Ноль шесть или Шесть десятых
0.014	Point 0 one four	Четырнадцать тысячных или Ноль ноль четырнадцать
255,604	Two hundred and fifty five thousand six hundred and four	255604 Двести пятьдесят пять тысяч шестьсот четыре
$\frac{20}{5} = \frac{16}{4}$	The ratio of twenty to five is equal to the ratio sixteen to four	Отношение двадцати к пяти равно отношению шестнадцати к четырём
9^2	Nine square or Nine to the second power	Девять в квадрате или девять во второй степени

10^{-11}	Ten to the minus eleventh (power)	Десять в минус одиннадцатой (степени)
$a = b$	*a* equals *b* or *a* is equal to *b*	*a* равно *b*
$32 + 8 = 40$	Thirty two plus eight is equal to forty	Тридцать два плюс восемь равно сорока
$20 - 5 = 15$	Twenty minus five is equal to fifteen	Двадцать вычесть (отнять) пять равно пятнадцати
$6 \times 10 = 60$	Six multiplied by ten is equal to sixty	Шесть умножить на десять равно шестидесяти
$12 : 3 = 4$	Twelve divided by three equals four	Двенадцать разделить на три равно четырём
$\sqrt{4} = 2$	The square root of four is equal to two	Корень квадратный из четырёх равен двум
$\sqrt[5]{a^2}$	The fifth root of "*a*" square	Корень пятой степени из "*a*" в квадрате
$\int \frac{dx}{\sqrt{a^2 - x^2}}$	Indefinite integral of d*x* divided by the square root out of "*a*" square minus *x* square	Интеграл от d*x* делённый на корень квадратный из разности *a* в квадрате минус *x* в квадрате
$4c + 2m_1 + R_a = 33\frac{1}{3}$	Four "*c*" plus two "*m*" first plus *R* a^{th} is equal to thirty-three one third	Четыре "*c*" плюс два эм один плюс *R* с индексом *a* (или *R* атое) равно тридцати трём и одной третьей

MAIN LAWS AND FORMULAS OF THE COURSE OF MEDICAL AND BIOLOGICAL PHYSICS (I and II term)

Name of the law or physical quantity	Formulas, entering quantities, their units
Elements of mathematics	
Derivative f ' (x)	$f'(x) = \lim\limits_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$
Physical sense of a derivative.	$\vartheta_{INST} = \lim\limits_{\Delta t \to 0} \frac{\Delta S}{\Delta t}$
Geometrical sense of a derivative	$tg\alpha = \lim\limits_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$
Chain-rule of derivative	$y'_x = y'_u \cdot U'_x$
Mechanical (physical) sense of the second derivative	$\lim\limits_{\Delta t \to 0} \frac{\Delta \vartheta}{\Delta t} = \vartheta'(t) = a_{INST}$
Differential of function	$df = f'(x)dx$, $\Delta f \approx df$
Partial derivative	$\lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} = \frac{\partial f}{\partial x}$ (y = const)
Total differential	$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$
Indefinite integral	$\int f(x)dx = F(x) + C$
Definite integral	$\int\limits_a^b f(x)dx = \lim\limits_{\Delta x \to 0} \sum\limits_{i=1}^{n} f(c_i)\Delta x_i$
Newton – Labnitz's formula	$\int\limits_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$
Differential equation	$F(x, y, y', y'', \ldots, y^{(n)}) = 0$
Law of dissolution of medicinal forms from tablets	$m = m_0 e^{-kt}$
Law of free continuous oscillations and its solution	$\frac{d^2S}{dt^2} + \omega_0^2 = 0$ $S = A\sin(\omega_0 t + \varphi_0)$
Law of free damped oscillations and its solution	$\frac{d^2S}{dt^2} + 2\beta\frac{dS}{dt} + \omega_0^2 S = 0$ $S = A\, e^{-\beta t}\sin(\omega\, t + \varphi_0)$

Classical probability	$P(A)=\frac{m}{n}$, $0 \le p(A) \le 1$
Probability of a sum of two not combined events	$P(A+B)=P(A)+P(B)$
Probability of joint occurrence of two independent events	$P(AB)=P(A)\cdot P(B)$
Mean M (x) (or μ) of a discrete random variable X	$M(x)=\mu=\sum_{i=1}^{n} X_i P_i = X_1 \cdot p_1 + X_2 \cdot p_2 ... + X_n \cdot p_n$
Variance D(x) of a discrete random variable X	$D(X)=M[X_i - M(X)]^2 = \sum_{i=1}^{n}[X_i - M(X)]^2 \cdot p_i$ or $D(X)=M(X^2)-[M(X)]^2$
Standard deviation	$\sigma = \sigma(x) = \sqrt{D(x)}$
Normal law (Gauss's law)	$f(x)=\frac{1}{\sigma\sqrt{2\Pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, where f(x) is probability density
Standard deviation S_x for X	$S_x = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+...+(x_n-\bar{x})^2}{n-1}}$
Standard deviation for $\bar{X}$	$S_{\bar{x}}=\frac{S_x}{\sqrt{n}}$
Absolute error of $\bar{X}$	$\Delta\bar{x}=S_{\bar{x}} \cdot t_{\gamma;n}$, where $t_{\gamma;n}$ is Student's coefficient, it is table value
Interval estimation of $\bar{X}$	$\bar{X}-\Delta\bar{X} < \bar{X} < \bar{X}+\Delta\bar{X}$
Relative error	$\varepsilon=\frac{\Delta\bar{x}}{\bar{x}}\cdot 100\%$
Rotation	
Angular velocity ω	$\omega=\frac{d\varphi}{dt}=\frac{\Delta\varphi}{\Delta t}=\frac{2\pi}{T}$
Rotary acceleration ε	$\varepsilon=\frac{d\omega}{dt}=\frac{d^2\varphi}{dt^2}$
Moment of force $\vec{M}$	$\vec{M}=\vec{F}\cdot\vec{R}$
Moment of inertia I_i of mass point and body I	$I_i = m_i \cdot r_i^2$ and $I=\sum_{i=1}^{n} m_i r_i^2$
The basic dynamical equation of a rotation	$M=I\frac{d\omega}{dt}=I\varepsilon$
Law of conservation of angular momentum	$L=I\omega=\text{const}$

Mechanical oscillations Differential equation of free not damped oscillations and its solution	$\frac{d^2x}{dt^2}+\omega_0^2 x=0;$ $x=A\cos(\omega_0 t+\varphi_0)$
Energy of oscillatory movement	$E=E_k+E_p=\frac{1}{2}m\omega_0^2A^2$
Differential equation of forced oscillations and	$\frac{d^2x}{dt^2}+2\beta\frac{dx}{dt}+\omega_0^2x=f_0\cos\omega_B t$, where $f_0=\frac{F_0}{m}$;
its solution	$x = A\cos(\omega_f t + \varphi_0)$, where $A=\frac{f_0}{\sqrt{(\omega_0^2-\omega_B^2)^2+4\beta\omega_B^2}}$
Resonant circular frequency and resonant amplitude	$\omega_{res}=\sqrt{\omega_0^2-2\beta^2}$, $A_{res}=\frac{f_0}{2\beta\sqrt{\omega_0^2-\beta^2}}$
Resonance condition	At $\beta=0$, $\omega_{res}=\omega_0$
Equation of a flat wave	$S=A\cos\left[\omega\left(t-\frac{x}{\vartheta}\right)\right]$, X is coordinate, S is displacement of the oscillating point
Intensity of a wave	$I=\frac{1}{2}\rho w_0^2A^2\vartheta=w\vartheta$, where w is volumetric energy density $w=\frac{1}{2}\rho\omega_0^2A^2$
Umov`s vector	$\vec{I}=w\vec{\vartheta}$
Doppler`s effect	$\nu'=\frac{V\pm V_{OBS}}{V\mp V_{sour}}\nu$
Doppler`s frequency shift	$\nu_{SHIFT}=\frac{2\vartheta_0}{\vartheta}\nu_G$
Connection of intensity I with sound pressure Δp	$I=\frac{\Delta p_0^2}{2\rho\vartheta}=\frac{\Delta p_{eff}^2}{\rho\vartheta}$
Threshold of audibility I_0, Threshold of painful sensation I_{max} Level of intensity in Bells (B)	ν=1kHz, $I_0=10^{-12}$ Wt/m^2 or $p_0=2\cdot10^{-5}$ Pa I_{max}=10 Wt/m^2 , p_{max}=60 Pa $L_B=\lg\frac{I}{I_0}$
Coefficient of penetration $\beta=\frac{I_2}{I_1}$ of a sound wave	$\beta=4\frac{\rho_1\vartheta_1/\rho_2\vartheta_2}{[\rho_1\vartheta_1/\rho_2\vartheta_2+1]^2}$

Physical or objective characteristics of a sound	1)frequency ν,2) harmonious spectrum, 3)intensity I
Physiological characteristic of a sound	1)height, 2) timbre, 3) loudness E
The range of frequencies perceived by an ear	16 Hz-20 kHz
Weber-Fehner's law	$E = KL = K\lg\frac{I}{I_0}$
Condition of indissolubility of a stream	$\Delta V = \vartheta \cdot S = const$
Bernoulli's equation	$p + \rho gh + \frac{1}{2}\rho\vartheta^2 = const.$
Newton`s equation	$F_{fr} = \eta \cdot S\frac{d\vartheta}{dx}$, where $\frac{d\vartheta}{dx}$ is gradient of speed, η is viscosity of a liquid
Unit of viscosity in SI	$N\frac{s}{m^2} = Pa \cdot s$
Relative viscosity	$\eta_{rel} = \frac{\eta}{\eta_{water}}$
Relative viscosity of blood	4.2-6
Raynolds' number Re	$\mathrm{Re} = \frac{2\rho r\vartheta}{\eta}$
Poiseuille's formula	$Q = \frac{\pi \cdot R^4}{8 \cdot \eta} \cdot \frac{p_1 - p_2}{1}$
Hydraulic resistance ω	$\omega = \frac{8l\eta}{\pi R^4}$
Pulse wave speed	$\vartheta_{PULSEWAVE} = \sqrt{\frac{Eh}{\rho d}}$, where E - the module of elasticity ; h - thickness of vessel wall; d - diameter of a vessel; ρ - density of blood
Speed of a pulse waves	$\vartheta_{PULSEWAVE} = 7\frac{m}{s}$
Speed of a sound waves in the system "vessel - blood"	v_{sound}=1500 m/s
Work of the left ventricle	$A_l = pV_s + \frac{\rho V_s\vartheta^2}{2}$
Work of all heart	A≈1J, A=1.2 A_l
<u>Physical processes in biological membranes</u>	
General equation of transportation	$\Delta(N\varphi) = -\frac{1}{3}\lambda\bar{\vartheta}\frac{\Delta(n_0\varphi)}{\Delta X}\Delta S \cdot \Delta t$,

	φ -- transferable physical value in time Δt through a platform ΔS, V is speed of a molecule, λ is a distance of free run, $\frac{\Delta(n_0\varphi)}{\Delta X}$ is a gradient of ($n_0\varphi$)
Equation of diffusion or Fick's law	$\Delta M = -\frac{1}{3}\lambda\bar{\vartheta}\frac{\Delta\rho}{\Delta x}\Delta S\cdot\Delta t = -D\frac{\Delta\rho}{\Delta x}\cdot\Delta S\cdot\Delta t$ or $J = -D\frac{dc}{dx}$, J is density of a stream of a substance $J = \frac{\Delta M}{\Delta t \Delta S}$
Versions of passive transport	1) simple diffusion, 2) facilitated diffusion , 3) filtration and 4) osmosis
Equation for density of a stream at diffusion through a membrane	$J = -\frac{Dk}{l}(C_e - C_i) = -p(C_e - C_i)$
Nernst-Planck's equation (for transport of ions through a membrane)	$J = -U_m RT\frac{dc}{dx} - U_m ZFC\frac{d\varphi}{dx}$
Nernst potential φ_m	$\varphi_m = \frac{RT}{ZF}\ln\frac{C_1}{C_2}$, R is the molar gas, T is the absolute temperature, F is Faraday's number (96500 C/mole), Z is the valence of the ion, C_1 and C_2 are the molar concentrations of the ions on the two sides of the membrane
Equation for the resting potential (of Goldman - Hodgkin-Katz)	$\varphi_M = -\frac{RT}{F}\ln\frac{P_K[K^+]_i + P_{Na}[Na^+]_i + P_{Cl^-}[Cl^-]_e}{P_K[K^+]_e + P_{Na}[Na^+]_e + P_{Cl^-}[Cl^-]_i}$
<u>Electric field</u>	
Intensity E of an electric field	$\vec{E} = \frac{\vec{F}}{q}$, [E]=N/C or V/m
Potential (U or φ)	$U = \varphi_1 - \varphi_2 = \frac{A_{1\to 2}}{q}$, [φ]= V
Connection of E and φ	$E = -\frac{d\varphi}{dl}$
Dipole moment	P=ql
Potential in any point of space around of a dipole	$U_A = \frac{P\cos\alpha}{4\pi\varepsilon\varepsilon_0 r^2}$

Potential difference between two points A and B for the current dipole	$U_A = U_A - U_B = \frac{D\rho}{4\pi r^2}(\cos\alpha_A - \cos\alpha_B)$
The main postulates of Einthoven`s theory are:	1) the electric field of the heart is represented as electric field of current dipole $\vec{P}_H$ which is called the integral electric vector of heart; 2) $\vec{P}_H$ is in homogeneous isotropic conducting medium that is tissues of organism; 3) $\vec{P}_H$ permanently changes its direction and value
Lead is	the difference of biopotentials registered between two points on a body
Einthoven`s leads	I- (RA-LA); II - (RA-LL) and III -(LA-LL)
Electrocardiogram is	time dependence of a voltage in any lead
<u>Direct and alternating current</u>	
Electromotive force	$E=A_{ext.\ force}/q$
Ohm's law for biological object	$I = \frac{U - E_p(t)}{R}$
Working (effective) or root-mean-square value of alternating current (voltage)	$I_{ef.} = I_{w.} = \frac{I_m}{\sqrt{2}}$, $U_{ef.} = U_{w.} = \frac{U_m}{\sqrt{2}}$
Circuit of alternating current with	
a) active resistance	$I = I_0 \sin\omega t$, $U = U_0 \sin\omega t$
b) inductive resistance	$I = I_0 \sin\omega t$, $U_L = I_0\omega L\cos\omega t = U_{0L}\sin(\omega t + 90^0)$
c) capacitance	$I = I_0 \sin\omega t$, $U_C = U_{0C}\sin(\omega t - 90^0)$
d) total resistance (R,L,C)	$I = I_0 \sin\omega t$,

	$U_0 = I_0\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = I_0 Z,$ where Z is impedance
Rheography is	diagnostic method based on registration of impedance of tissues during cardiovascular activity
Quantity of heat q allocated in 1 m^3 in biological tissues for 1 second at	
4) diathermy	$q = \frac{Q}{Vt} = j^2\rho$
5) inductothermy	$q = j_{ef}^2 \cdot \rho = k\frac{B_{ef}^2 \cdot \omega_0^2}{\rho}$
6) UHF-therapy	$q = \frac{P}{V} = \frac{E^2}{\rho} = E^2\gamma$ (conductors)
	$q = \frac{P}{V} = E_{ef}^2 \varepsilon\varepsilon_0 \cdot \omega \cdot tg\delta$ (dielectrics)
Optics	
Refracting system of an eye	cornea, moisture of the forward chamber, crystalline lens, vitreous body
Accommodation is	ability of an eye to change optical force due to change of curvature of a crystalline lens
Total eye optical force	43(cornea) + 20 (lens) = 63 diopters
Distance of the best vision (≈25cm)	is minimal distance at which accommodation is made without a pressure
Visual acuity	$V=1/\beta_{min}$, where β_{min} is minimal visual angle (≈1')
Defects of vision	1) myopia,2)hypermetropia,3) astigmatism, 4) presbyopia, 5) aphakia
Light exposure	$E = \frac{d\Phi}{dS}$, (lux)
Brightness (L)	$L = \frac{d\Phi}{d\Omega dS\cos\alpha}$, (nit)
Magnification of a microscope M	$M = \frac{\Delta \cdot a_0}{F_{objec} \cdot F_{eyepiece}}$, where Δ is optical length of tube, a_0 is the distance of the best vision
Aperture angle α	is equal to the angle α, formed by the beams going from a subject to objective edges
Distinction limit of a microscope	$\ell_{min} \geq \frac{0.5\lambda}{n\sin(\frac{\alpha}{2})}$,

	where $A=n\cdot\sin(\frac{\alpha}{2})$ is the numerical aperture
Resolving power R	$R=1/\ell_{min}$
Useful magnification of a microscope M	$M=\frac{\ell_{min,eye}}{\ell_{min,mic}}=\frac{\ell_{min,eye}\cdot A}{0,61\cdot\lambda}$
Wavelength of De Broglie λ	$\lambda=\frac{h}{p}=\frac{h}{m\vartheta}$
Distinction limit of an electronic microscope Z	$Z=\frac{0.5h}{\sqrt{2emU}\cdot n\cdot\sin\alpha}$
Interference minima condition	$\Delta=(2k+1)\frac{\lambda}{2}$, k=0,1,2,3,…, where $\Delta=r_2-r_1$ is geometrical difference of a course
Interference maxima condition	$\Delta=2k\frac{\lambda}{2}=k\lambda$
Huygens-Fresnel's principle	points of medium of which were reached front of a wave are sources of elementary secondary coherent waves
Condition of diffraction minima for slit	$a\sin\varphi=2k\frac{\lambda}{2}=k\lambda$, (m=2k)
Formula of a diffraction grating (maxima)	$d\sin\varphi=k\lambda$, d is period of grating
Woolf – Bragg`s formula	$2d\sin\theta=k\lambda$, d is distance between the next nuclear layers
Malus's law	$I=I_0\cos^2\varphi$, I_0 is light intensity after polarizer, I is the light intensity after analyzer, φ is angle between principal planes of the analyzer and polarizer
Brewster's law	$tg\,i_B=n$, $n=n_2/n_1$ is refractive index
Rotation angle of polarization plane in a solution	$\Delta\varphi=\alpha\cdot C\cdot l$, α is a specific rotation, l is the thickness of a liquid layer, C is concentration
Some characteristics of thermal radiation	1) emissive ability $R=\frac{E}{St}$

	2) absorptive ability of body $A=\frac{E_{abs}}{E_{allbsf}}$
Perfectly black body (PBB)	if A=1
Kirchhoff's law	$\frac{R'}{A'}=\frac{R''}{A''}=\frac{R'''}{A'''}=\cdots=\frac{\varepsilon}{1}=\varepsilon,$ where ε is emissive ability of PBB
Planck's formula	$\varepsilon_\nu=\frac{2\pi\nu^2}{c^2}\cdot\frac{h\nu}{e^{h\nu/kT}-1}$
Stephan-Boltzmann's law (Full emissive ability of the perfectly black body ε)	$\varepsilon=\sigma T^4$
Wien's law (wavelength to which corresponds maximum of energy)	$\lambda_m=\frac{b}{T}$
Capacity P lost by the person at interaction with environment	$P=P_1-P_0=S\delta(T_1^4-T_0^4)$
Einstein's equation for the Photoeffect	$h\nu=A+\frac{m\vartheta^2}{2}$
Quantum output of a luminescence	$\varphi=\frac{n}{N}$
Stocks's law	$h\nu_{LUM}=h\nu_{ABS}-E_{heat}$, i. e. $\nu_{LUM}<\nu_{ABS}$ and $\lambda_{LUM}>\lambda_{ABS}$
Basic properties of laser radiation	1) strong monochromaticity, 2) coherency, 3) small divergence, 4) power consumption
The border of the most short-wave X-ray radiation	$\lambda_{min}=1.23/U$ (nm)
Stream of brake x-ray radiation	$\Phi=KIU^2Z$
Moseley's law	$\sqrt{\nu}=A(Z-B)$
Linear coefficient of weakening of X-rays μ	$\mu=\mu_m\rho$
Mass coefficient of absorption	$\mu_m=K\lambda^3Z^3$

Basic law of radioactive decay	$N = N_0 \cdot e^{-\lambda t}$
Half-life period T	$T = \frac{0,693}{\lambda}$
Activity A	$A = A_0 \cdot e^{-\lambda t} = A_0 \cdot e^{\frac{-0,693}{T}t}$
Units of activity	$1\ \text{Becquerel (Bc)} = \frac{1 decay}{1 \sec ond}$ 1 Curie (Cur) = 1Cur = $3.7 \cdot 10^{10}$ Bc
Absorbed doze D, its unit	D=E/m, [D] = Gray (1Gy=1J/1kg)=100rad
Doze capacity	P=D/t, Gr/s
Exposition doze X, connection of X with absorbed doze D	D=fX, for water and soft tissues of a person f=1; unit of X: 1C/kg; 1 C/kg = 3876 R(roentgen)
Doze rate for the dot charge	$P = K_\gamma \frac{A}{R^2}$
Connection of absorbed doze D_{abs} with equivalent doze D_e	$D_e = K \cdot D_{abs}$, where K = RBE is relative biological efficiency (RBE); for x-rays and γ-radiation K=1, for α-radiation K=20;
Units of equivalent doze D_e	1 Sievert (Sv); 1Sv=100rem; 1 rad ≈ 1 R ≈ 1 rem (rem=ber)
Maximum permissible biological doze for the person at a professional irradiation and	$D_{max\ prof}$ = 5 ber per year
for population	$D_{max\ popul}$ = 0.5 ber per year
Minimal lethal doze	≈500 ber (rem)
Normal radiation background	10 – 20 mcR/h
Most widespread detectors are	1) ionizing chamber, 2) Geiger-Muller's counter, 3) semi-conductor detector, 4) scintillometer
Groups of dosimetric devices on destination	1) indicators, 2) roentgenometers, 2) radiometers, 4) dosimeters, 5) spectrometers

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