ECONOMIC DESIGN OF CONTROL CHARTS USING METAHEURISTIC APPROACHES

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This is to certify that the work presented in this dissertation entitled "*Economic Design of Control Charts using Metaheuristic Approaches*" by "*Abhijeet Ganguly*", Roll Number 511ME102, is a record of original research carried out by him under my supervision and guidance in partial fulfillment of the requirements of the degree of *Doctor of Philosophy in Mechanical Engineering*. Neither this dissertation nor any part of it has been submitted for any degree or diploma to any institute or university in India or abroad.

Saroj Kumar Patel

Dedicated to the memory of my beloved

Maa

Declaration of Originality

I, Abhijeet Ganguly, Roll Number 511ME102 hereby declare that this dissertation entitled "*Economic Design of Control Charts using Metaheuristic Approaches*" represents my original work carried out as a doctoral student of NIT Rourkela and, to the best of my knowledge, it contains no material previously published or written by another person, nor any material presented for the award of any other degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the dissertation. Works of other authors cited in this dissertation have been duly acknowledged under the section "References". I have also submitted my original research records to the scrutiny committee for evaluation of my dissertation.

I am fully aware that in case of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present dissertation.

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Abstract

Statistical Process Control (SPC) is a collection of problem solving tools useful in achieving process stability and improving capability through the reduction of variability using statistical methods. It can help industries in reduction of cost, improvement of quality and pursuit of continuous improvement. Among all the SPC tools, the control chart is most widely used in practice. Out of all the control charts, $\bar{X}$ chart is the simplest to use and hence most popularly used for monitoring and controlling processes in an industry. A process may go out-of-control due to shift in process mean and/or process variance. To detect both types of shifts, R chart is often used along with $\bar{X}$ chart.

The design of $\bar{X}$ chart refers to selection of three design variables such as sample size (n), sampling interval (h) and width of control limits (k). On the other hand, the joint design of $\bar{X}$ and R charts involves four design variables i.e., sample size (n), sampling interval (h), and widths of control limits for both charts (i.e., k_1 and k_2). There are four types of control chart designs, namely (i) heuristic design, (ii) statistical design, (iii) economic design, and (iv) economic statistical design. In heuristic design, the values of design variables are selected using some thumb rules. In statistical design, the design variables are selected in such a way that the two statistical errors, namely Type-I error (α), and Type-II error (β) are kept at minimum values. In economic design, a cost function is constructed involving various costs like the cost of sampling and testing, the cost of false alarm, the cost to detect and eliminate the assignable cause(s), and the cost of producing non-conforming products when the process is operating out-of-control. The design parameters of the control chart are then selected so that this cost function is minimized. The design based on combined features of statistical design and economic design is termed as economic statistical design where the cost function is minimized while satisfying the statistical constraints. The effectiveness of economic design or economic statistical design depends on the accuracy of minimization of cost function. So, use of effectively designed control charts is highly essential for ensuring quality control at minimum cost.

Most of the researchers have used either approximate or traditional optimization techniques for minimizing the cost function. With time, more and more efficient optimization methods have been utilized for this purpose. There are a number of metaheuristic algorithms reported in literature for optimization in various types of design problems. Out of them one each from two different groups are selected for the present work i.e., simulated annealing (SA) and teaching-learning based optimization (TLBO). SA is a point to point based metaheuristic technique, whereas TLBO is population based technique. SA is one of the oldest metaheuristic algorithms and proved to be the most robust one, whereas TLBO is one of the most recent and promising techniques. The present work requires optimization techniques that can solve non-linear, non-differentiable, multi-variable, unconstrained as well as constrained type of objective function. Both the above techniques are capable of optimizing this type of objective function. However, from literature review it is observed that neither of these two metaheuristic approaches has been applied in economic or economic statistical design of any type of control chart. In this work, both these metaheuristic techniques (i.e., SA and TLBO) have been applied for minimization of cost function for economic as well as economic statistical design point of view for individual $\bar{X}$ chart, and by taking $\bar{X}$ and R charts jointly in case of continuous as well as discontinuous process. Thus, a total of the following eight distinct design cases have been considered for their optimization.

1. Economic design of $\bar{X}$ chart for continuous process
2. Economic design of $\bar{X}$ chart for discontinuous process
3. Economic statistical design of $\bar{X}$ chart for continuous process
4. Economic statistical design of $\bar{X}$ chart for discontinuous process
5. Joint economic design of $\bar{X}$ and R charts for continuous process
6. Joint economic design of $\bar{X}$ and R charts for discontinuous process
7. Joint economic statistical design of $\bar{X}$ and R charts for continuous process
8. Joint economic statistical design of $\bar{X}$ and R charts for discontinuous process

All the above designs are illustrated through numerical examples taken from literature using two metaheuristics i.e., SA and TLBO separately. These two independent techniques are used to validate their results with each other. Their results are found to be superior to that reported earlier in the literature. Thus, eight types of methodologies based on SA or TLBO approach are recommended in this thesis for designing control charts from economic point of view.

Sensitivity analysis has been carried out using fractional factorial design of experiments and analysis of variance for each of the eight design cases, to examine the effects of all the cost and process parameters on all the output responses such as sample size, sampling interval, width of control limits and expected loss cost per unit time. The process parameters which significantly affect the output responses are identified in each of the eight design cases. These results are expected to be helpful for quality control personnel in identifying the significant factors and thereby taking utmost care in choosing their values while designing the control charts on economic basis.

Keywords: Analysis of Variance; Continuous and Discontinuous Processes; Economic Design; Economic Statistical Design; Simulated Annealing; Teaching-Learning Based Optimization; $\bar{X}$ ***and R Charts.***

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Nomenclature

a	Fixed cost per sample
ARL	Average run length
ARL_0	Average run length when the process is in-control
ARL_1	Average run length when the process is out-of-control
ARL_L	Lower bound on the in-control average run length
ARL_U	Upper bound on the out-of-control average run length
ATS	Average time to signal
ATS_0	Average time to signal that process has gone out-of-control when the process is actually in-control
ATS_1	Average time to signal that process has gone out-of-control when the process is actually out-of-control
ATS_U	Upper bound) on out-of-control average time to signal
$Adj\ SS$	Adjusted sum of squares
$Adj\ MS$	Adjusted mean squares
$Adj\ MS_{error}$	Adjusted mean square error
ANSI	American National Standard Institute
ASQC	American Society for Quality Control
ANOVA	Analysis of variance
b	Variable cost per sample
C_0	Quality cost per hour while producing in-control
C_1	Quality cost per hour while producing out-of-control
$CL_{\overline{X}}$	Centre line of $\overline{X}$ chart
CL_R	Centre line of R chart
CL	Centre line
CTQ	Critical-to-quality
CUSUM	Cumulative sum
DF	Degree of freedom
DS	Double Sapling
DSI	Double Sapling Interval
DSVSI	Double Sapling Variable Sampling Interval
DSVSS	Double Sampling Variable Sample Size
dm_{ij}	Stochastic difference between the existing mean result of the class in TLBO

E(A)	Expected net income per hour
E(C)	Expected net income in one production cycle
E(T)	Expected cycle time
$E(L)_1$	Expected loss cost per unit time for the continuous process
$E(L)_2$	Expected loss cost per unit time for the discontinuous process
E(Q)	Expected total cost per hour
ED	Economic design
ED-C	Economic design - Continuous process
ED-D	Economic design - Discontinuous process
ESD	Economic statistical design
ESD-C	Economic statistical design - Continuous process
ESD-D	Economic statistical design - Discontinuous process
ESD-ARL	Economic statistical design with only average run length constraints
ESD-ATS	Economic Statistical Design with only average time to signal constraint
EPQ	Economic production quantity
EWMA	Exponentially weighted moving average
g	Expected time required to take a sample
h	Sampling interval
H_0	Null hypothesis
H_1	Alternative hypothesis
I	Maximum number of iterations in TLBO
ISO	International organization of standardization
J	Number of subjects taught to the learners (i.e., design variables) in TLBO
JED	Joint economic design
JED-C	Joint economic design - Continuous process
JED-D	Joint economic design - Discontinuous process
JESD	Joint economic statistical design
JESD-C	Joint economic statistical design - Continuous process
JESD-D	Joint economic statistical design - Discontinuous process
k	Width of control limits when the $\overline{X}$ chart is considered alone
k_1	Width of control limits of $\overline{X}$ chart in joint design
k_2	Width of control limits of *R* chart in joint design
K	Number of learners (i.e., population size) in TLBO
K_2	Upper control limit coefficient of *R* chart expressed in multiple of σ_0
K_3	Lower control limit coefficient of *R* chart expressed in multiple of σ_0
LCL	Lower control limit
$LCL_{\overline{X}}$	Lower control limit of $\overline{X}$ chart
LCL_R	Lower control limit of *R* chart
M	Loss of income when process is out-of-control
M_{ij}	Mean result of the class of K learners in a particular subject *j* in iteration *I* in TLBO
MA	Moving average
MR	Moving range
MEWMA	Multivariate exponentially weighted moving average
n	Sample size

P	Power of detecting a shift by the chart
PM	Preventive maintenance
PC	Percentage contribution
$P_{\overline{X}}$	Power of $\overline{X}$ chart in detecting the process shift
P_R	Power of R chart in detecting process shift
QC	Quality control
R chart	Range chart
r	Random number in the range (0, 1) in SA
r_i	Uniformly distributed random number in the range [0, 1] in TLBO
s	Expected number of samples before the process shifts in a cycle
S	Expected cost of restart or setup cost
S_1	Expected time to restart the process
$Seq\ SS$	Sequential sum of squares
$Seq\ SS_{Total}$	Total sequential sum of squares
SA	Simulated annealing
SPC	Statistical Process Control
t	Iteration counter in SA
T_0	Expected search time for one false alarm
T_1	Expected time to search for an assignable cause
T_2	Expected time to repair the assignable cause
T_F	Teaching factor
T_{min}	Termination temperature in SA
TLBO	Teaching-learning based optimization
UCL	Upper control limit
$UCL_{\overline{X}}$	Upper control limit of $\overline{X}$ chart
UCL_R	Upper control limit of R chart
V_0	Net income per hour while the process is in-control
V_1	Net income per hour while the process is out-of-control
VSI	Variable Sampling Interval
VSS	Variable Sample Size
VSSI	Variable Sample Size and Sampling Interval
W	Expected cost of repair including the search for assignable cause
$\overline{X}$ chart	Average or mean chart
X_{ijk}	Result of learner k in subject j in iteration i in TLBO
$X_{ijk\text{-}best}$	Result of the best learner *k-best* in the subject j at any iteration i in TLBO
Y	Cost per false alarm

Greek Symbols

α	Probability of Type-I error
$\alpha_{\overline{X}}$	False alarm rate of $\overline{X}$ chart
α_R	False alarm rate of *R* chart
β	Probability of Type-II error
δ	Shift in process mean
ε	Termination criterion in SA
Υ	Value of shift in standard deviation
λ	Rate of occurrence of assignable cause
μ_0	In-control process mean
μ_1	Shifted process mean
μ_w	Mean of sample statistic *w* when the process is under control
μ_R	Mean of sample range *R*
υ	Smoothing parameter
$\phi(k)$	Area under standard normal distribution curve from $-\infty$ to *k*
$\phi(\bullet)$	Standard normal cumulative distribution function
ρ	Temperature reduction factor in SA
σ	Standard deviation
σ_0	In-control process standard deviation
σ_1	Shifted standard deviation
σ_w	Standard deviation of sample statistic *w* when the process is under control
σ_R	Standard deviation of sample range *R*
τ	Expected time of occurrence of the process shift within the interval between *j*th and (*j+1*)st samples

CHAPTER - 1

Introduction

1.1 Background

An organization must produce its output, whether product or service, with high quality at low cost to survive in the present days of competitive market. Quality means fitness for use (Juran, 1974). According to ISO the *Quality Control* is part of quality management focused on fulfilling requirements (Hoyle, 2001). *Statistical Process Control* (SPC) is a collection of problem solving tools useful in achieving process stability and improving capability through the reduction of variability using statistical methods (Montgomery, 2013). Among all the SPC tools, the control chart is most widely used. It is a graphical tool used for checking whether the process is in-control or out-of-control. $\bar{X}$ chart was first introduced by Shewhart (1931). Many other types of control charts have been subsequently developed and available for use in practice.

1.1.1 Control Chart

The properties on which quality of a product or service is evaluated are known as *quality characteristics*. These are of two types namely, variable data and attribute data. The *variable data* can be measured on a continuous scale, whereas *attribute* data cannot be measured on continuous scale. Attribute data are of discrete type and hence, they are expressed as either acceptable or not acceptable. Samples of some specified size are taken at regular interval of time from the production process. The values of one or more selected quality characteristics for each item in the sample are measured. Using these data for each sample, the value of some sample statistic like mean, range, cumulative sum of means etc. is calculated and then plotted against time or sample number on a control chart for monitoring the process (Montgomery, 2013).

The control chart is a two dimensional graph with horizontal axis representing the time or order of sample collection, and vertical axis representing the sample statistic. A Shewhart control chart has usually three horizontal lines such as one centre line (*CL*) and two other lines representing upper and lower control limits (*UCL* and *LCL*). The centre line represents where the process characteristic is expected to fall if there are no unusual sources of variability. On the other hand, if any unusual source of variability is present, the sample averages will plot outside the control limits (i.e., either *UCL* or *LCL*). Whenever a point falls outside either of the two control limits, the control chart provides strong evidence that the process may have gone out-of-control due to some cause. It is necessary to design the control chart in such a way that it is capable of generating the signal as soon as the process has gone out-of-control. The delay in triggering an out-of-control signal will go on producing more and more non-conforming items and thereby cause loss to the organization. Sometimes, the delay may develop further quality deterioration resulting in a loss of higher magnitude.

As the decision regarding the process status is based on the results obtained from a small size sample data, two types of decision errors are committed in any control chart. When the process is actually in the in-control state but the control chart indicates that process has gone out-of-control then *Type-I or* α *error* is committed. On the other hand, *Type-II or* β *error* is committed if the control chart is unable to provide a signal when the process has really gone out-of-control. The smaller the value of β error, the quicker is the power of detecting the process change (Montgomery, 2013).

1.1.2 $\bar{X}$ and *R* Charts

Among all the control charts, $\bar{X}$ chart is most widely used in industry for monitoring and controlling processes because of its simplicity. But it can detect the process change due to shift in process mean only. However, a process may go out-of-control due to shift in process mean and/or process variability. Therefore, it is often recommended to use both $\bar{X}$ and *R* charts jointly for the statistical process control. Both these charts use variable data. When more than one control chart is used, economic design and economic statistical design are termed as joint economic design and joint economic statistical design respectively.

The quality characteristic *X* usually has normal distribution for almost all production processes. For example, *X* can be external diameter of bolt or internal diameter for a nut. The characteristic on which the quality of the product mainly depends upon is taken as *X*. Values of *X* are measured for all the items in a sample of size *n* and the average of these values is called *sample mean* $\bar{X}$ which is calculated as:

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \ldots\ldots + X_n}{n}$$

The samples are collected at regular time interval. For each sample, one sample mean $\bar{X}$ is calculated and plotted against time. Since $\bar{X}$ is plotted, this control chart plot is called $\bar{X}$ chart. If μ_0 and σ_0 are process mean and process standard deviation of a quality characteristic X for an in-control process respectively and both are known from past experience, then the three horizontal lines for $\bar{X}$ chart are expressed as:

$$CL_{\bar{X}} = \mu_0,$$

$$UCL_{\bar{X}} = \mu_0 + k\frac{\sigma_0}{\sqrt{n}}, \text{ and}$$

$$LCL_{\bar{X}} = \mu_0 - k\frac{\sigma_0}{\sqrt{n}}.$$

When these two process parameters μ_0 and σ_0 are unknown, then at the beginning at least 20 to 25 preliminary samples are taken when the process is in-control. If *m* preliminary samples are taken each of size *n*, then process mean μ_0 is estimated as the grand mean of sample means which is expressed as:

$$\mu_0 \approx \bar{\bar{X}} = \frac{1}{m}\sum_{i=1}^{m}\overline{X_i}$$

The value of $\bar{\bar{X}}$ is taken as the center line on the $\bar{X}$ chart.

Similarly, the value of σ_0 may be estimated from either the standard deviation or the range of the observations within each sample. As sample size is relatively small, there is little loss in efficiency in estimating σ_0 from the sample ranges. Moreover, the calculation of sample range is comparatively easier. In any sample, if X_1 and X_2 are the lowest and highest values respectively, then range R is calculated as $R = X_2 - X_1$. From *m* number of preliminary sample data, average range $\bar{R}$ is calculated as:

$$\bar{R} = \frac{1}{m}\sum_{i=1}^{m}R_i$$

Since, R is a random variable, the quantity $W = R/\sigma_0$, called the relative range, is also a random variable. The mean of the distribution of W is d_2 and the standard deviation of W is d_3. Thus, $\mu_R = d_2\sigma_0$ and $\sigma_R = d_3\sigma_0$. The values of d_2 and d_3 are constants for a particular value of sample size n. The table of d_2 and d_3 values for various values of sample size n is available in any text book of statistical quality control like Montgomery (2013). As the process parameters are unknown, μ_R can be estimated as $\overline{R}$. Thus, $d_2\sigma_0 = \mu_R \approx \overline{R}$ from which σ_0 can be estimated as:

$$\sigma_0 = \frac{\overline{R}}{d_2}$$

Therefore, substituting the values of μ_0 and σ_0, all the three horizontal lines for the $\overline{X}$ chart for unknown process can be written as:

$$CL_{\overline{X}} = \overline{\overline{X}}$$

$$UCL_{\overline{X}} = \overline{\overline{X}} + \frac{k}{d_2\sqrt{n}}\overline{R}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - \frac{k}{d_2\sqrt{n}}\overline{R}$$

For most of the cases in practice, three sigma limits are used. So, putting k = 3, a new constant $A_2 = \frac{3}{d_2\sqrt{n}}$ can be defined and both control limits can be simplified as:

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$$

The values of constant A_2 are tabulated for various sample sizes n in any text book of statistical quality control like Montgomery (2013).

R chart is similar to $\overline{X}$ chart. The only difference is that along y-axis, instead of $\overline{X}$ the sample range R is plotted. Here also, there are three horizontal lines. For the process with unknown parameters, substituting the value of $\sigma_R = d_3\sigma_0 = d_3\frac{\overline{R}}{d_2}$, the three lines are expressed as:

$$CL_R = \overline{R}$$

$$UCL_R = \overline{R} + \frac{kd_3}{d_2}\overline{R} = \left(1 + \frac{kd_3}{d_2}\right)\overline{R}$$

$$LCL_R = \overline{R} - \frac{kd_3}{d_2}\overline{R} = \left(1 - \frac{kd_3}{d_2}\right)\overline{R}$$

For three sigma limits (i.e., k = 3), using two constants $D_3 = \left(1 - \frac{3d_3}{d_2}\right)$ and $D_4 = \left(1 + \frac{3d_3}{d_2}\right)$ the two control limits can be simplified as mentioned below.

$$UCL_R = D_4 \overline{R}$$
$$LCL_R = D_3 \overline{R}$$

The values of constants D_3 and D_4 are also available for various sample sizes n in any text book of statistical quality control like Montgomery (2013). A negative value of R is meaningless. So, the lower control limit LCL_R is set at zero if its calculated value using above expression comes out to be a negative number.

Once the three lines are established, for each sample the values of $\bar{X}$ and R are calculated and then plotted on respective charts. In any chart, if a point falls either above UCL or below LCL, it gives a signal that the process may have gone out-of-control, and necessary steps are then taken for investigation for the causes and its elimination to bring back the process to normalcy. On the other hand, if the point falls within both control limits, it is accepted that the process is running under control and it is allowed to continue as it is. In this way, these two control charts help in ensuring that the process runs under control.

1.1.3 Types of Processes

All the processes can be classified into two major groups i.e., i) continuous process and ii) discontinuous process. In *continuous process* the process is allowed to continue even after the control chart signals that the process has gone out-of-control. On the other hand, the *discontinuous process* is immediately stopped after receiving the out-of-control signal (Panagos et al., 1985). In both the cases, the search for assignable cause begins after the out-of-control signal is obtained from the control chart. After an assignable cause is detected, necessary action is taken for its elimination so as to bring back the process to in-control state. In case of discontinuous process since the process is stopped during repair activity, it requires to be restarted. The shift from the in-control state to the out-of-control state is irreversible. Therefore, once a process has gone out-of-control, it cannot come back to in-control state of its own. It always requires managerial intervention for detecting and eliminating assignable cause so as to bring back the process to in-control state. The quality control personnel always try to complete this remedial action as quickly as possible.

1.1.4 Design of Control Chart

The ability of any control chart for detecting the changes in quality level obtained from a process depends on the effectiveness in design of control chart. The design affects the statistical as well as cost properties. For maintaining a control chart, samples are usually taken from the production process at regular interval. A designer needs to specify when and of what size a sample is to be taken. Further, he needs to decide the distance from the centre line at which the control limits are to be drawn. The number of items in each sample is called *sample size* (*n*). The time interval between two consecutive samples is termed as *sampling interval* (*h*). The two control limits are symmetrically placed from centre line in most of the control charts. The distance of each control limit from the centre line expressed in multiple of standard deviation of sample statistic is called *width of control limits (k)*. The selection of these three parameters *n*, *h* and *k* is termed as *design of a control chart* in most of the cases including the design of $\bar{X}$ or *R* chart (Montgomery, 2013). In addition to the above three, one or more other parameters are also considered depending upon the type of control chart. For example in case of economic design of exponentially weighted moving average (EWMA) chart, a smoothing parameter (υ) is considered in addition to *n*, *h* and *k*.

There are four types of control chart design, namely i) heuristic design, ii) statistical design, iii) economic design, and iv) economic statistical design, details of which are explained below.

i. ***Heuristic design***: The control charts used to be designed on heuristic basis even today on the shop floor. In this design, the width of control limits (*k*) is fixed at 3 and the sample size (*n*) of 4 or 5 is taken.There is no fixed guideline for selection of sampling interval (*h*) and it depends mainly on the production rate. This type of design used to be the most popular one because it is easy to implement and understand with little effort of operator training.

ii. ***Statistical design***: In this design, the control charts are design on statistical basis. The two statistical errors, namely Type-I error (α) and Type-II error (β) are kept at minimum values. These values need to be specified by the user of control chart. Once these two errors are fixed, the next task is to calculate the two control chart design variables i.e., the sample size (*n*) and the width of control limits (*k*). There is no guideline for the selection of sampling interval *h*. This design mainly stresses on how quickly the control chart can detect a process change.

iii. ***Economic design***: The first two methods of design do not consider any cost aspect. However, designing a control chart has economic significance because the costs like the cost of sampling and testing, the cost of false alarm, the cost of detecting and eliminating the assignable cause, and the cost of producing non-conforming product when the process is operating out-of-control, are greatly affected by the choice of the control chart design variables. It is mandatory for all industries to ensure that the total cost of production must be as low as possible to survive in this competitive market. Therefore, the design of control chart from an economic view point has received much attention in the recent past (Montgomery, 1980). In this design procedure, the chart parameters are so selected that the total cost of process control is kept minimum.

iv. ***Economic statistical design*: ** Woodall (1986) criticized the use of economically designed control charts, because this type of design ignores the statistical performance of the control charts (i.e., keeping Type-I and Type-II errors at minimum level). The economic statistical design combines both the economic and statistical methodologies for designing a control chart. Its objective is to minimize the total cost of process control and at the same time satisfying some statistical constraints.

The effectiveness of designing a control chart depends on how accurately the cost function is minimized by selecting proper values of the design variables. Various types of optimization techniques have been adopted to minimize the total cost function. Primarily, researchers have developed heuristic procedures for minimizing the cost. However, most of the heuristics provide approximate solutions. Later, they applied some of the traditional methods for its optimization. With time, more and more efficient optimization methods were also utilized for this purpose. Although numerous efficient optimization algorithms have been developed in the recent past and have been successfully applied in many areas, it is observed that very few of them have been adopted in designing control charts.

All the results of control chart design depend on the assumed values of cost and process parameters for a given manufacturing set up. These values vary from one set up to another. All factors may not be significantly affecting the economic design. Thus, the designer needs to identify the significant factors and accordingly take care to correctly estimate their values. Therefore, a sensitivity analysis is also required to investigate the effect of process and cost parameters on the output results of control chart design.

1.2 Research Gap

There are various types of control charts available for monitoring production processes and they are designed in different ways. These designs are optimized using different procedures. Out of the detailed literature review presented in Chapter 2, the following points are identified as research gap for the present thesis:

1. There are many metaheuristic approaches for optimization in various types of design problems. All these approaches can be broadly classified into two groups on the basis of the number of solution points being considered at any iteration such as:

i) Point to point based approach
ii) Population based approach

In the first type of approach, the number of solution point is only one, whereas in the second type, it is more than one. One each from these two different groups is selected for the present work i.e., simulated annealing (SA) and teaching-learning based optimization (TLBO) respectively. SA is a point to point based metaheuristic technique, whereas TLBO is population based technique. SA mimics the slow cooling phenomenon of hot metal, whereas TLBO mimics the process of teaching a class of students. Moreover, SA is one of the oldest metaheuristic algorithms and proved to be the most robust one, whereas TLBO is one of the most recent and promising techniques. Both the techniques are being popularly used for solving wide range of industrial optimization problems. However, neither of these techniques has been applied so far for the economic design of any control chart.

2. It is also observed that neither of the above two optimization techniques (i.e., SA and TLBO) has been used for joint economic design of control charts.
3. Further, both these optimization methods have never been used for economic statistical design of one control chart or joint economic statistical design of a group of charts.
4. No comparison of results of economic design or economic statistical design obtained using SA or TLBO technique between continuous and discontinuous processes has also been reported in literature.

1.3 Motivation for Research

In view of today's competitive market, it has been always a challenge to produce better quality products and make it available at lower cost. So, the producers always look for all possible means of cost reduction and improvement of quality. A control chart is the most popularly used SPC tool in practice for maintaining the process control at minimum cost. Out of all control charts, $\bar{X}$ and R charts, are the most widely used due to their simplicity. These two charts have attracted attention of majority of researchers. Therefore, these two charts have been considered for the present work.

Moreover, the cost of process control can be minimized through economic design of these control charts. Efficient techniques are used for optimizing the economic design so that the total cost of process control would have the least value. Higher the efficiency of optimization technique, the more will be the reduction in cost. The two metaheuristics, SA and TLBO, are being popularly used for solving wide range of industrial optimization problems. The present work requires optimization techniques that can solve non-linear, non-differentiable, multi-variable, unconstrained as well as constrained type of objective function. Both the techniques are capable of optimizing this type of objective function. But, neither of these techniques has been applied so far for the economic design of any control chart. This provides a strong motivation for exploring these two techniques if they can provide lower value of cost function compared to the techniques already tried by previous researchers in the field of economic and economic statistical designs of these two charts.

1.4 Research Questions

The basic objective of this work is to explore a new optimization technique for economic design of $\bar{X}$ and R charts which are superior to earlier techniques so that the total cost of process control can be further lowered down. For the present work, the basic research questions that arise in the mind are as listed below:

1. How can the two selected metaheuristics, i.e., SA and TLBO, be applied in the following designs and are they superior to the existing methods?
    a) Economic design of $\bar{X}$ chart
    b) Economic statistical design of $\bar{X}$ chart
    c) Joint economic design of $\bar{X}$ and *R* charts
    d) Joint Economic statistical design of $\bar{X}$ and *R* charts
2. What are the most significant cost and process parameters that affect the above mentioned designs of control charts?

1.5 Theme of Work

On the basis of research gap as mentioned in Section 1.2, the main motivation for taking up this research work is to recommend a comprehensive package of designing control charts applicable to a variety of processes (i.e., continuous and discontinuous) subjected to all types of shifts (i.e., shift in process mean and/or process variance) based on economic as well as economic statistical performance. $\bar{X}$ chart has been selected as it is the most popularly used among all types of control charts for statistical process control in real practice due to its simplicity. It is also observed that majority of research on economic design has been focused on this chart. There are also many applications where this chart is used jointly with *R* chart. The detailed objectives of this thesis are outlined below:

1. To develop a cost model by considering all related cost and process parameters for a continuous process in case of economic design of $\bar{X}$ chart as well as joint economic design of $\bar{X}$ and R charts and then optimize these designs using SA and TLBO algorithms.
2. To repeat Step 1 for economic statistical design of $\bar{X}$ chart as well as joint economic statistical design of $\bar{X}$ and R charts.
3. To repeat Steps 1 and 2 for monitoring a discontinuous process.
4. To compare and discuss the results obtained in the above mentioned three steps i.e., among the following eight distinct cases of control chart designs.
    i. Economic design of $\bar{X}$ chart for continuous process
    ii. Joint economic design of $\bar{X}$ and R charts for continuous process
    iii. Economic statistical design of $\bar{X}$ chart for continuous process
    iv. Joint economic statistical design of $\bar{X}$ and R charts for continuous process
    v. Economic design of $\bar{X}$ chart for discontinuous process
    vi. Joint economic design of $\bar{X}$ and R charts for discontinuous process
    vii. Economic statistical design of $\bar{X}$ chart for discontinuous process
    viii. Joint economic statistical design of $\bar{X}$ and R charts for discontinuous process
5. To perform sensitivity analysis in all the above eight cases to investigate the effects of cost and process parameters on the performance of control chart designs.
6. To illustrate the various types of designs of control charts developed in this thesis through numerical examples and compare the results.

1.6 Organization of Thesis

The thesis has been organized in seven chapters. The first chapter provides an introductory overview of the research, research gap, theme of the work, and organization of the thesis.

Chapter 2: An exhaustive review of literature on statistical process control, types of control charts, design of control charts, process failure mechanisms and optimization techniques is presented in this chapter.

Chapter 3: This chapter refers to economic design of $\bar{X}$ chart for both continuous and discontinuous processes. It explains the behavior of both types of processes. It provides the formulation of economic models including the list of all assumptions made for the design of $\bar{X}$ chart. After formulating the objective functions (i.e., the loss cost functions $E(L)_1$ and $E(L)_2$ for continuous and discontinuous processes respectively), they are minimized using two metaheuristic techniques SA and TLBO for finding optimal values of three design variables *n*, *h* and *k*. The economic designs of $\bar{X}$ chart based on these two metaheuristic approaches are then illustrated through numerical examples and the results are compared. Sensitivity analysis is performed using design of experiments and analysis of variance to identify the effects of cost and process parameter on the performance of control chart designs. A summary of results of sensitivity analysis is provided for comparing between continuous and discontinuous processes in terms of significant parameters.

Chapter 4: This chapter deals with development of models for economic statistical designs of $\bar{X}$ chart for both continuous and discontinuous processes and the optimization of these designs with the help of the same two optimization methods (i.e., SA and TLBO). It also provides the sensitivity analysis of these designs. Some problems that deal with economic statistical design are taken from literature and solved using these two techniques, and the results are compared.

Chapter 5: Chapter 3 deals with economic design of $\bar{X}$ chart, whereas this chapter is meant for joint economic design of $\bar{X}$ and *R* charts. Thus, the content of this chapter is similar to that of Chapter 3. The same numerical example of economic design has been considered for joint economic design. The results obtained for joint economic designs of $\bar{X}$ and *R* charts using SA and TLBO are compared for both types of processes. A comparison of results of sensitivity analysis for joint economic designs of $\bar{X}$ and *R* charts for these two processes has also been provided.

Chapter 6: This chapter is similar to Chapter 4 as both are related to economic statistical design. The main difference is that Chapter 4 is meant for $\bar{X}$ chart only, whereas this chapter is for joint design of $\bar{X}$ and *R* charts. This design is also illustrated through the same numerical example considered in all other chapters. The results are obtained for joint economic statistical designs of $\bar{X}$ and *R* charts using SA and TLBO for both continuous and discontinuous processes. Like other chapters, a comparison of results of sensitivity analysis for joint economic design of $\bar{X}$ and *R* charts in both types of processes has also been presented in this chapter.

Chapter 7: Summary and the important managerial implications drawn from the present research work are presented in this chapter. The limitation of this work and possible scope of extension to the present study are also provided.

A list of all the references cited in this thesis is given at the end.

CHAPTER - 2

Literature Review

2.1 Introduction

Nowadays industries are facing a considerable amount of challenges due to stiff competition in both the national and international markets. The competitive market and customer awareness require the production of quality goods and services for survival as well as growth of the company. Therefore, every industry is concerned with quality of their outputs. However, at the same time every industry must aim for maximizing its profit margin for which the production cost is required to be kept at the minimum possible level. There are many types of quality management tools for controlling the quality of process output at minimum cost. Among them, one of the most widely used statistical tools in practice is Shewhart control chart which monitors the process quality by giving signal whenever the process shifts to out-of-control state. Its design involves the selection of three major variables, namely the sample size (n), the sampling interval (h) and the width of control limits (k). The number of variables may vary depending on the chart used. The effectiveness of designing a control chart depends on the technique used for optimizing the design to select the proper values of these design variables. Further, design of control chart has economic consequences.

In today's competitive environment, any technique selected for controlling process quality should also be cost effective. Therefore, design of control chart from economic point of view has been drawing considerable attention from researchers. At the time of introducing the control chart, Shewhart (1931) did not consider the cost aspects for designing the chart. Later, Duncan (1956) included the economic aspects in its design. With due course of time several changes and extensions have occurred in the design procedure. This chapter gives a detailed review of literature related to the design of control charts on economic basis.

2.2 Review Roadmap

An exhaustive literature survey has been conducted to understand the state of art of earlier researches and thereby, identifying the direction of present work. A total number of 204 articles have been reviewed in this thesis and the sources of these articles are classified as shown in Table 2.1.

Table 2.1: Sources of the articles reviewed

Articles	*Number*
Peer reviewed journals	140
Other journals	37
Conference proceedings	2
Books	12
Others	13
Total	204

All of these articles have been thoroughly studied and within this scope, the review of literature has been categorized into three broad groups, namely i) design of control chart, ii) assumptions and iii) optimization techniques. Fig. 2.1 shows the detailed categorization of the articles that have been reviewed in this chapter.

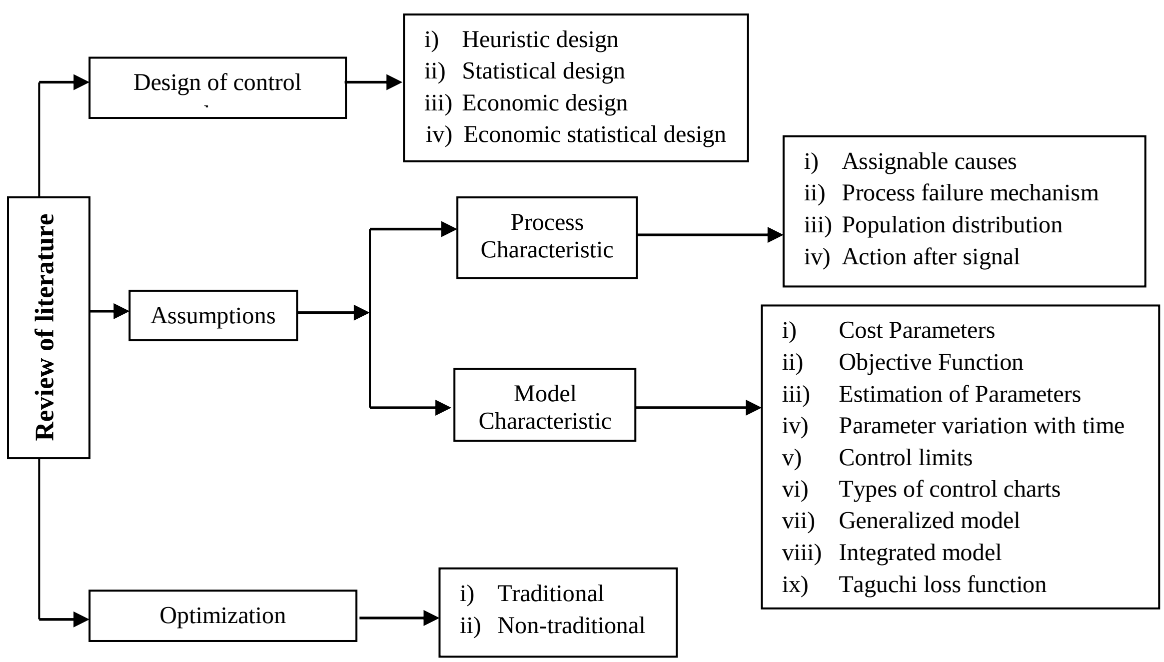

Fig. 2.1: Categorization of review articles

2.3 Quality

Quality is undoubtedly one of the most important decision factors for a customer while purchasing a product or service. He prefers to buy a quality product but it should be available at an affordable price. The term quality has been defined in many ways. From the manufacturer point of view, it means conformance to requirements or specifications (Crosby, 1979). From the consumer point of view, it means fitness for use, or customer satisfaction (Juran, 1974). Taguchi et al. (1989) defined quality as the loss to society caused by the product after it is shipped. Further, quality is inversely proportional to variability (Montgomery, 2013). American National Standard Institute/American Society for Quality Control (ANSI/ASQC A1, 1978) defines quality as totality of features and characteristics of a product or service that bears on its ability to satisfy given needs.

2.3.1 Dimensions of Quality

A product quality can be described in many ways. Garvin (1987) differentiated the quality of products into the following eight dimensions. A product can be said to possess good quality if all these quality dimensions are properly balanced while designing and manufacturing it.

1. *Performance*: A quality product should perform its purpose on the expectations of the customer. If the product shows a poor performance it will disappoint the customer and in the long run the company may face reduction of sales, negative reviews and loss of goodwill in the market.
2. *Reliability*: A product should give efficient and consistent performance in its lifespan. If the product fails frequently its reliability is considered to be poor. Several industries have developed their brands and trust in the market by providing excellent reliability of their products.
3. *Durability*: It quantifies the length of time that a product performs before its replacement. All products should have long life.
4. *Serviceability*: It means that a product should be easy to repair after a breakdown.
5. *Aesthetics*: This is the visual appeal of the product taking some factors like colour, style, shape, packaging, sound, feel etc. depending on the type of product.
6. *Features*: This dimension of quality belongs to the additional features added to the basic operating characteristics of the product.

7. *Perceived quality*: This quality dimension belongs to the past reputation of the company or its product. It refers to the perception of the quality of the product in the mind of customer.
8. *Conformance to standards*: It is the degree up to which any product meets its established specifications. The product should agree with some established national or international standards.

2.3.2 Quality Engineering

A group of engineering, operational and managerial activities that a firm uses to ensure that the quality characteristics of a product are at the nominal or required levels and that the variability around these desired level is minimum is known as *quality engineering* (Montgomery, 2013).

2.3.3 Quality Management

The overall activities and tasks which is required to maintain the quality at a desired level of excellence is known as *quality policy* of an organization (Mitra, 2005). *Quality management* is the overall administration function that decides and executes the quality policy (Mitra, 2005). The achievement of desired quality requires the dedication and cooperation of all members of the organization, whereas the responsibility for quality management belongs to top administration. Quality management comprises of strategic planning, resources allocation, and other systematic activities for quality, such as quality planning, operation, and evaluation.

2.3.4 Quality Assurance

Every customer has some idea about the quality and cost of the product, though he may not be able to define them correctly. It is the manufacturer's responsibility who has to study the requirements of the customers in details, interpret their ideas and make every effort to produce the product that suits the requirements of customers satisfactorily. All those planned or systematic actions required to provide confidence that a product or service will satisfy given needs is known as *quality assurance* (Mitra, 2005). This term deals with the questions of assuring the desired quality, reliability, service and other aspects in the manufactured product.

2.3.5 Quality Aspects

In general there are three quality aspects associated with the definition of quality assurance, namely (i) *quality of design*, (ii) *quality of conformance*, and (iii) *quality of performance* (Mitra, 2005). The *quality of design* of a product is concerned with the tightness of specifications for manufacturing this product. Products are manufactured in various levels of quality. These variations in levels of quality are governed by product type, cost, profit policy of the company, demand of the goods, availability of material and product safety. Higher quality of design means higher cost, quite often it also means higher value. However, human ingenuity often finds way to make design both better and cheaper. The *quality of conformance* is concerned with how well the manufactured product conforms to the design specifications. When a design has been established, it is the task of manufacturer and all responsible personnel engaged in production planning to obtain a high level of quality of conformity. The *quality of performance* is concerned with how well the manufactured product delivers its performance. It depends on both quality of design and quality of conformance. It can be the best possible design, but a poor control on conformance can cause poor product performance. Conversely if the design itself is not correct, even the best conformance control cannot make the product with satisfactory quality (Mitra, 2005). The present work deals with conformance aspect of quality.

2.3.6 Quality Control

Quality Control (QC) is the process through which we measure the actual quality performance, compare it with the standards and take corrective action if there is a deviation (Montgomery, 2013). Therefore major QC functions are inspection, analysis of data, defect analysis, corrective actions, salvage (i.e., scrap or rework) decision methods, maintaining vendor relationships and establishment of quality standards.

2.3.7 Quality Characteristic

The properties on which the quality of goods is evaluated are known as *quality characteristics*. Sometimes, these are also called as critical-to-quality (CTQ) (Montgomery, 2013). Generally, the quality characteristics fall under two broad categories as mentioned below:

1. *Variable data*: The quality characteristics that can be quantified and measurable are known as *variable data*. In other words these are measured and expressed as numbers on

some continuous scale of measurement, such as length, weight, volume, etc. They are also known as *continuous data* (Montgomery, 2013).

2. *Attribute data*: The qualitative characteristics which cannot be measured on a continuous scale are termed as attribute data. They are expressed either as conforming or non-conforming to specifications. These are also called as *discrete data* (Montgomery, 2013).

The data obtained by actual measurement are variable (continuous) data while data obtained by counting are attribute (discrete) data. The present work deals with control charts using variable data.

2.3.8 Causes of Variation

The concept of variation states that no two items will be perfectly identical even if extreme care is taken to make them identical. Variation is a fact of nature. The manufacturing processes are not exception to this. In any manufacturing process, irrespectively how well it is designed or maintained, certain amount of inherent natural variability always exists. Any type of variation that occurs due to a sum total of numerous unavoidable causes of small magnitudes is called as *chance (or, common) cause of variation* (Montgomery, 2013). A process operating with the presence of only chance causes of variation is said to be under statistical control. Such process is said to be in in-control state. Shewhart (1931) mentioned that these causes occur continuously and are economically difficult to identify or eliminate, and do not produce any change in quality levels. On the other hand, there is another type of variability which possesses characteristics like greater magnitude in change of quality level, occasional occurrence, easy detection and economic removal. These sources are called *assignable (or special) causes of variation* (Montgomery, 2013). The major sources of this variability may be due to 3 M's (i.e., man, machine and materials). Any process that is operating in the presence of both chance and assignable causes is said to be out-of-control.

2.4 Statistical Process Control

Statistical Process Control (SPC) is a collection of problem solving tools useful in achieving process stability and improving process capability through the reduction of variability using statistical methods (Montgomery, 2013). It monitors the quality of production processes over a time span and detects changes in process performance. Basically, it consists of the techniques of sampling, inspecting, using sample data to ascertain the extent of control over the production process, and enhancing the manufacturing processes to achieve continuous process improvement. It helps in improvement of quality as well as productivity of industrial firms. It consists of the following seven problem solving tools:

1. Histogram
2. Check sheet
3. Cause and effect diagram
4. Pareto chart
5. Defect concentration diagram
6. Scatter diagram
7. Control chart

These are often known as *magnificent seven* (Montgomery, 2013). Among them, the most popularly and widely used tool in practice is the control chart. The present work deals with design of this chart.

2.4.1 Control Chart

A *control chart* is a graphical representation of information collected from samples taken from a process at some interval of time. Shewhart (1931) pioneered its use. Shewhart control charts are widely used in many fields for maintaining statistical control over a production process. This chart is still being popularly used in practice and extensive research work on its design aspect is being reported in literature. It is only a diagnostic tool. It only indicates whenever a process has gone out-of-control. It cannot rectify an out-of-control process and bring back to in-control state of its own without any external intervention from quality personnel. The prime aim of a control chart is to quickly identify the occurrence of an assignable cause in the production process if it has gone out-of-control so that the process of hunting for assignable causes and then removing them can be initiated. If there is a delay in finding the assignable cause there would be delay in taking the remedial action, and thereby it results in manufacturing of non-conforming products in large numbers. Moreover, the investigation and process correction after getting an alarm from the control chart requires engineering knowledge about the production process. When a process is controlled using a control chart, too much monitoring leads to extra cost or too little monitoring will lead to quality problems. Therefore, there is a need for maintaining process control at minimum possible cost. Economically designed control charts help in achieving this objective.

Two conditions must be satisfied to reduce the proportion of non-conforming items before production starts (Chandra, 2000).

1. The mean of the distribution of the quality characteristic should be as close as possible, if not equal, to the target value.
2. The variance of the distribution of the quality characteristic should be minimum.

Let the values of mean and standard deviation of a quality characteristic for an in-control process are denoted as μ_0 and σ_0 respectively. At any time, the process mean and process standard deviation should remain equal to in-control mean (μ_0) and in-control standard deviation (σ_0) respectively. To achieve this, the process is monitored by taking samples at certain interval of time and inferences are made. Statistically, this is equivalent to hypothesis testing. The two sets of hypotheses that need to be tested are:

1. $H_0 : \mu = \mu_0$

$$H_1 : \mu \neq \mu_0 \tag{2.1}$$

2. $H_0 : \sigma = \sigma_0$

$$H_1 : \sigma \neq \sigma_0 \tag{2.2}$$

In process control when H_0 is accepted, the process is assumed to be in-control and is allowed to run. If H_0 is rejected, the process is assumed to be out-of-control and the process is examined for identifying the corresponding assignable causes. After the assignable causes are identified, they must be removed. Since this procedure has to be carried out till the production process ends, it is convenient to represent the procedure on a control chart. The control chart is a two dimensional graph with horizontal axis representing the time or order of sample collection and vertical axis representing the sample statistic. A Shewhart control chart has three horizontal lines as shown in Fig. 2.2.

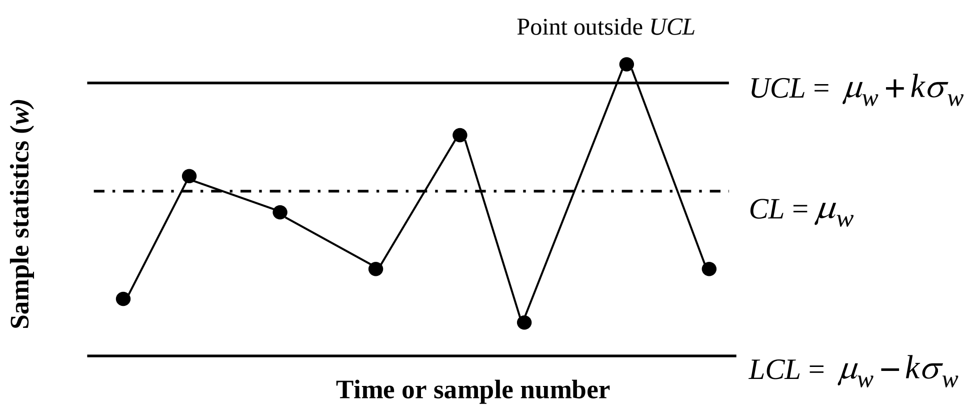

Fig. 2.2: A Shewhart control chart

In case of a general model for a control chart, if w is some sample statistic which is calculated out of all measured values of a quality characteristic of interest for all the items in a sample, then these three lines are calculated as shown below:

i) Center line at $CL = \mu_w$

ii) Upper control limit at $UCL = \mu_w + k\sigma_w$

iii) Lower control limit at $LCL = \mu_w - k\sigma_w$

where k = width of control limits expressed in standard deviation unit, μ_w and σ_w are the mean and standard deviation of sample statistic w when the process is under control. When a point on a control charts falls either above upper control limit or below lower control limit, it indicates that the process may have gone out-of-control.

2.4.2 Types of Control Charts

The control charts are broadly classified into two groups according to the type of quality characteristic under consideration, namely i) *variable control charts*, and ii) *attribute control charts*.

Commonly used variable control charts are mean ($\bar{X}$), range (R), sample standard deviation (S), population standard deviation (σ), sample standard variance (S^2) and population standard variance (σ^2) charts. Similarly, fraction non-conforming (p), number non-conforming (np), non-conformity (c) and non-conformity per unit (u) charts are attribute control charts. In addition, there are also time-weighted charts such as moving average (MA), moving range (MR), exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) charts.

On the basis of number of quality characteristics, the control charts can be classified as i) *univariate charts*, and ii) *multivariate charts*. Univariate charts deal with only one quality characteristic, whereas multivariate charts involve more than one. Examples of multivariate charts are Hotelling T^2 and multivariate exponentially weighted moving average (MEWMA) charts. The present work deals with $\bar{X}$ and R charts which are of univariate type.

A control chart can be classified on the basis of sampling interval as i) *fixed sampling interval chart*, and ii) *variable sampling interval chart*. The time interval between two consecutive samples is fixed in case of a fixed sampling interval control chart. In case of variable sampling charts, the duration of the time interval between two consecutive samples is varied depending upon the severity of effects of assignable causes. The control charts used in this work is of fixed sampling interval type.

2.4.3 Types of Errors

A control chart is based on the principle of hypothesis test. The decision regarding the process is based on the results obtained from a small sample data. Thus, two types of decision errors are committed in any control chart. When the process is actually in-control but the control chart indicates that process has gone out-of-control, then *Type-I* or α *error* is committed (Montgomery, 2013). This is also known as *false alarm*. A false alarm is triggered by the control chart without actual existence of an assignable cause. Therefore, a false alarm leads to needless search for assignable causes.

On the other hand, *Type-II* or β *error* is committed if the control chart is unable to provide an out-of-control signal when the process has really gone out-of-control (Montgomery, 2013). Inability of detecting any process change instantaneously means loss of quality standard. Also, more number of non-conforming items will be produced if β error is high. The probability of detecting the assignable cause(s) when the process has gone out-of-control is called *power* (P) and mathematically $P = (1-\beta)$. Therefore, smaller the value of β error means quicker is the rate of detection of process change. So, a control chart should have both α and β errors as low as possible (Montgomery, 2013). These two errors for a Shewhart control chart are illustrated in Fig. 2.3.

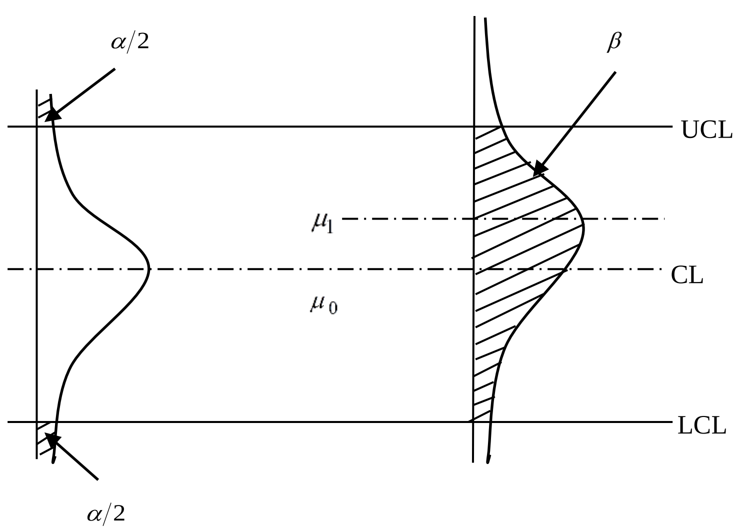

Fig. 2.3: Two types of errors in a Shewhart control chart

The expected number of successive samples taken until a sample point on the control chart triggers an out-of-control signal (i.e., a sample point falls outside either of the two control limits) is called as *average run length* (*ARL*). It is denoted as ARL_0 when the process is in-control and ARL_1 when the process is out-of-control. These two *ARLs* can be expressed as $ARL_0 = \frac{1}{\alpha}$ and $ARL_1 = \frac{1}{(1-\beta)}$ (Montgomery, 2013).

Further, the *average time to signal (ATS)* is defined as the expected time required to get the first signal after the process has really gone out-of-control. It is calculated by multiplying *ARL* with sampling interval *h* i.e., $ATS = h \times ARL$. Average run length depends on sample size *n* and control limit width *k*, whereas *ATS* is a function of all three variables *n*, *h* and *k*. Like ARL_0 and ARL_1, there are two average times to signal i.e., ATS_0 and ATS_1. ATS_0 is defined as the average time to signal that process has gone out-of-control when the process is actually in-control. Similarly, ATS_1 is the average time to signal that process has gone out-of-control when the process is really out-of-control. Thus, ATS_0 should be always high, whereas ATS_1 should be low. Both these *ATSs* are calculated $ATS_0 = h \times ARL_0 = \frac{h}{\alpha}$ and $ATS_1 = h \times ARL_1 = \frac{h}{(1-\beta)} = \frac{1}{P}$ (Montgomery, 2013).

2.5 Design of Control Chart

The performance of a Shewhart control chart varies with the values of chart parameters like sample size (*n*), sampling interval *(h)* and width of control limits *(k).* Other control charts may have their own parameters. The selection of proper values for these control chart parameters is known as *design of control chart* (Montgomery, 2013).

The probability of committing α error is a function of only width of control limit (*k*). By narrowing the width of control limit, α error increases. On the other hand, β error is a function of both sample size (*n*) and width of control limit (*k*). Any change in the value of these two design variables will certainly change the value of β error. Lower the value of β error, higher will be the power of detecting the process shift. The power of detection of an out-of-control process with a control chart can be increased by three ways i.e., (i) increasing the sample size (*n*), (ii) shortening the sampling interval (*h*), or (iii) narrowing the width of control limits (*k*) (Osborn, 1990). The first approach suggests that if the number of items inspected is more, it is easier to detect small size shift but the cost of inspection will be more. In second approach by shortening the sampling interval, the power of detecting the shift increases but the samples are required to be inspected quite often, and thereby inspection cost increases. Narrowing the width of control limits in the third approach helps in early detection of process shift but it increases the rate of occurrence of false alarm (i.e., α error) leading to unnecessary effort in searching for them. However, at the same time it decreases the rate of generation of true out-of-control signal (i.e., β error). In such a case, the process engineer tends to overlook the true alarm, thinking that it may be a false alarm.

Proper selection of control chart parameters is very important because it can affect the cost, the statistical properties of design and finally the confidence of user. Various design approaches exist for different types of control charts. Saniga (1989) classified the design of control chart in four general categories

(i) Heuristic design,
(ii) Statistical design,
(iii) Economic design and
(iv) Economic statistical design.

In the next four sections these design methods are explained in details.

2.6 Heuristic Design

Shewhart control chart are most often designed heuristically in practice because it is easy to understand and implement with little operative training. Table 2.2 shows the different guidelines suggested by various researchers for selection of design variables in case of $\bar{X}$ chart. These guidelines are considered as thumb rules and are often used by many industries (Lorenzen and Vance, 1986).

Table 2.2: Guidelines for heuristic design of $\bar{X}$ chart

Authors	Design variables
Burr (1953)	n = 4 or 5, h =? and k = 3
Feigenbaum (1961)	n = 5, h = 1 and k = 3
Juran et al. (1974)	n = 4, h =? and k = 3
Ishikawa (1976)	n = 5, h = 8 and k = 3

In Table 2.2, a question mark (?) indicates that no particular value has been specified for selection of sampling interval (h). All these guidelines use 3-sigma control limits (i.e., k = 3), but such design does not provide enough power for detecting the process shift, particularly for smaller size shift.

2.7 Statistical Design

This design is based on statistical criteria where both α and β errors are not allowed to exceed beyond some values specified by the user. Based on these two errors, Woodall

(1985) calculated two control chart design variables i.e., the sample size (n) and the width of control limits (k). Various authors have selected different values of α error (or, ARL_0) and β error (or, ARL_1) but no general guidelines are considered for different production situations. No analyses or guidelines are provided for selecting the value of sampling interval which varies from one user to another. However, design of control charts and selection of error probabilities are not justified economically. A mathematical model for designing $\bar{X}$ chart with sampling rate and maximum false alarm rate as inputs was developed by Keats et al. (1995). Sampling rate is ratio of the number of units sampled in a time interval to the quantity produced in the same time interval.

Wu et al. (2002) developed a computer program written in C language in case of joint statistical design of $\bar{X}$ and S charts for minimization of sample size n by taking ARL constraints. De-Magalhaes (2006) applied Markovian approach for the joint statistical design of adaptive $\bar{X}$ and R charts. The process is subjected to two independent assignable causes. One cause changes the process mean and the other changes the process variance. Chen (2007) reported that the adaptive variable parameter Hotelling T^2 chart provides better performance over fixed sampling rate while designing this chart for a specified value of average time to signal and adjusted average time to signal ($AATS$). The various charts of which statistical designs were studied by earlier researchers are summarized in Table 2.3.

Table 2.3: Summary of literature on statistical design

Authors	***Control Charts***
Woodall (1985)	$\bar{X}$ and CUSUM
Keats et al. (1995)	$\bar{X}$
Wu et al. (2002)	$\bar{X}$ and S jointly
De-Magalhaes (2006)	$\bar{X}$ and R jointly
Chen (2007)	Variable parameter T^2 chart

2.8 Economic Design

In last few decades the design of control charts based on cost criteria has gained much popularity. This is called economic design because its major objective is to select the proper values of design variables like sample size (n), sampling frequency (h) and width of the control limits (k) such that the total cost of process control is minimum. The process control cost consists of i) the cost of sampling and testing, ii) the cost of searching false alarm, iii) the cost of searching true signal and repair of assignable cause(s), iv) the cost of producing non-conforming items due to delay in detection and correction, and v) the cost of restart, if process is stopped after getting an out-of-control signal etc.

The economic design of control chart considers all these cost parameters which are not considered in first two approaches i.e., heuristic and statistical designs. Since all these costs are affected by the choice of three control chart parameters, it is reasonable to consider the design of a control chart from an economic viewpoint. Whenever any non-conforming product reaches the customer, the company loses its goodwill in the market. The use of control chart ensures the production of less percentage of non-conforming items. Further, if this control chart is designed on the basis of economic point of view, the cost of production will be less and thereby the profit margin will be higher. Thus, the economic design helps in enhancing the profit as well as goodwill of the firm.

The concept of economic design was first introduced by Girshick and Rubin (1952). Although the optimal control rules in their model are too complex to have practical value, their work provided the real basis for most of the cost based models in control chart design. Their generalized model was further investigated by researchers like Weiler (1952), Savage (1962), Barish and Hauser (1963), Bather (1963), Ross (1971) and White (1974). But their results were basically of theoretical interest. Their models neither considered all relevant costs nor applied any optimization method to minimize cost function. All these work done in earlier years are termed as semi-economic design (Montgomery 2013).

Duncan (1956) first proposed an economic model for design of $\bar{X}$ chart assuming a random shift in process mean due to occurrence of a single assignable cause and exponential distribution for the failure mechanism responsible for shifting the process from in-control to out-of-control state. He assumed that the sample observations are distributed normally and the process is allowed to run even after getting an out-of-control signal (i.e., continuous process). By selecting or neglecting some of his assumptions, most of the researchers developed different versions of cost models. Svoboda (1991) reported that many researchers have tried to adopt more valid and robust assumptions for preparing the cost models. Generally, the economic design models are not easily accepted in the industry due to mathematical complexity of models.

Krishnamoorthi (1985) reported that a cost saving of 48% is obtained by using economically designed $\bar{X}$ chart for monitoring the punching of holes on plastic sheets as compared to the heuristic design. Lorenzen and Vance (1986) reported a cost saving of \$295.69 per hour or \$6, 00,000 per annum using an economically designed control chart over the heuristic design of Ishikawa (1976) in a foundry process. The more detailed approaches, assumptions and models related to economic design of control charts are discussed in Section 2.10. The major contributions related to economic design of control charts are summarized in Table 2.4.

Table 2.4: Summary of literature on economic design

Authors	*Major contributions*
Girshick and Rubin (1952)	Introduced the first cost based models in control chart design
Weiler (1952)	Semi-economic design
Duncan (1956)	First proposed an economic model for design of $\bar{X}$ chart
Savage (1962)	Generalized semi-economic design
Barish and Hauser (1963)	Semi-economic design of Girshick and Rubin (1952) model
Ross (1971)	Suggested renewal reward process
Krishnamoorthi (1985)	Comparison between economic and heuristic designs of $\bar{X}$ chart
Lorenzen and Vance (1986)	Unified approach in economic design model
Svoboda (1991)	Review of literature from 1979-1989

2.8.1 Joint Economic Design

Most of the researchers considered only one control chart at a time for its economic design (Celano, 2011). But, $\bar{X}$ chart is often used along with R chart in practice for simultaneous monitoring of process mean and process variance. Therefore, some researchers have developed economic models for simultaneous design of both these charts. This type of design is known as *joint economic design*.

Saniga (1977) was the first to introduce joint economic design of $\bar{X}$ and R charts. He followed the economic model of Knappenberger and Grandage (1969). Saniga (1979) also investigated the effects of various process models like geometric, Poisson and logarithmic series on the joint economic design of these two charts considering single assignable cause. The occurrence of this single assignable cause results in simultaneous shift in process mean and process variance. Saniga and Montgomery (1981) studied the joint economic design of same two charts for single assignable cause assuming normal distribution. Jones and Case (1981) reported 5% reduction in cost when $\bar{X}$ and R charts are designed jointly in case of both single and multiple assignable cause economic models of Duncan's. They considered all combinations of shift in mean and standard deviation. Chung and Chen (1993) developed an algorithm optimizing the design variables for the joint economic design of these two charts. They reported that their algorithm provided better solution with an improvement up to 9.71% compared to the algorithm presented by Rahim (1989). Costa (1993) proposed joint economic design of these two charts when a process is subject to two independent assignable causes. He assumed that the occurrence of one type of assignable cause does not stop the occurrence of the other type, and out of these two causes, one cause shifts the process mean and the other shifts the process variance. Table 2.5 lists down the major contributions in the area of joint economic design of $\bar{X}$ and R charts.

Table 2.5: Summary of literature on joint economic design

Authors	***Major Contributions***
Saniga (1977)	First to introduce a joint economic design model
Saniga (1979)	Single assignable cause using geometric, Poisson and logarithmic series distributions
Saniga and Montgomery (1981)	Single assignable cause using normal distribution
Jones and Case (1981)	Both single and multiple assignable cause
Chung and Chen (1993)	Developed optimization algorithm
Costa (1993)	Two independent assignable causes

2.8.2 Weaknesses in Economic Design

When a control chart is designed economically it ensures minimum operating cost of process control, whereas it is not possible with heuristically or statistically designed control chart. Gibra (1978) observed on an average 20 false alarms before the process shifts to out-of-control state in case of economic design of *np*-chart. Such a large number of false alarms would lead to over adjustment of the process and as a result it may ignore the true out-of-control signal. Woodall (1986, 1987) pointed out some of the weaknesses of economic design as mentioned below:

1. It is not consistent with Deming's philosophy.
2. Economically designed charts have poor statistical performance.
3. The rate of false alarms rate is often high.
4. Smaller shifts are not detected effectively.
5. Duncan's (1956) economic model has not changed much.

Based on the above mentioned criticisms, Woodall (1987) further suggested that use of Taguchi's loss function (Taguchi et al., 1989) approach may be more appropriate to economically determine the smallest shift with the control charts. Tagaras and Lee (1989) highlighted that unplanned use of economic design without caution and good judgment would lead to the above weaknesses. The simplification of economic model and use of fast computation facility have eliminated these difficulties to some extent, but still the use in shop floor doesn't appear to be easy. Various criticisms against the economic design are listed down in Table 2.6.

Table 2.6: Weaknesses in economic design

Authors	***Design Weakness***
Gibra (1978)	Reported large number of false alarms in np chart
Woodall (1986)	Reported poor statistical performance with high false alarms rate in case of small shifts
Woodall (1987)	For smaller shift, the use of Taguchi's loss function is more accurate
Tagaras and Lee (1989)	Advised not to do economic design without proper planning

2.9 Economic Statistical Design

Being motivated by the criticisms made by Woodall (1986, 1987) regarding the weaknesses of economic design of control charts, Saniga (1989) was the first to propose the *economic statistical design*. In this type of design, the goal is to minimize the cost function subject to some statistical constraints on both α and β errors. This combines the features of both economic design and statistical design. Because of imposition of constraints, this type of design is costlier as compared to economic design (Zhang and Berardi, 1997). However, it provides better statistical performance of the control chart and hence, the additional expense is justified (Saniga, 1989).

McWilliams (1994) compared the performance of economic design, statistical design and economic statistical design of $\bar{X}$ chart. Montgomery et al. (1995) proposed economic model of EWMA chart for controlling the process mean subjected to statistical constraints based on unified approach of Lorenzen and Vance (1986). They compared economic design and economic statistical design, and reported significant increase in cost with the addition of statistical constraints on the cost model. This increase in the cost can be accepted when there is enormous improvement in the statistical performance of the chart. They also suggested that any number of constraints on the out-of-control *ARL* may be added for better protection with a slight increase in the cost. Similarly, Prabhu et al. (1997) proposed a constrained cost model for adaptive $\bar{X}$ charts with dual sample size and dual sampling interval. Morales (2013) considered general failure distribution in case of an integrated model for economic statistical design of joint $\bar{X}$ and S charts with preventive maintenance. Amiri et al. (2014) proposed Taguchi loss function approach for economic statistical design of adaptive $\bar{X}$ chart and compared results with fixed sampling policy. Veljkovic (2015) applied economic statistical design of $\bar{X}$ chart when the quality characteristic has non-normal symmetric distribution. He proposed three non-normal symmetric distributions such as student distribution, standard Laplace distribution and logistic distribution and compared results with that of normal, Pearson and Johnson distributions. The summary of important contributions in the area of economic statistical design of various control charts is given in Table 2.7.

Table 2.7: Summary of literature on economic statistical design

Authors	***Major Contributions***
Saniga (1989)	First to propose ESD for $\bar{X}$ and R charts
McWilliams (1994)	Comparison of ESD with *SD* and *ED* of $\bar{X}$ chart
Montgomery et al. (1995)	Proposed *ESD* of EWMA chart
Zhang and Berardi (1997)	Reported that *ESD* is more costlier than *ED* for $\bar{X}$ chart
Prabhu et al. (1997)	Proposed *ESD* for adaptive $\bar{X}$ chart based on dual sample size sampling interval
Morales (2013)	*ESD* of joint $\bar{X}$ and *S* charts integrated to preventive maintenance
Amiri et al. (2014)	*ESD* of variable sampling $\bar{X}$ chart using Taguchi loss function
Veljkovic (2015)	*ESD* of the $\bar{X}$ chart

SD: Statistical Design; *ED*: Economic Design; *ESD*: Economic Statistical Design

2.10 Assumptions in Economic Models

The basic approach of both economic design and economic statistical design consists of three steps. In the first step, some assumptions on the process behavior and model characteristics are made, and then values of all cost and process parameters associated with the process being controlled are assumed. In the second step, a mathematical model is developed to formulate the cost function in terms of control chart parameters. In the last step, the cost function is minimized to determine optimal values of the chart parameters using some optimization method.

After the pioneer work on economic model development of $\bar{X}$ chart by Duncan (1956), the first review on different control charts was reported by Gibra (1975). Panagos et al. (1985) considered two different scenarios in economic design i.e., i) the process continues in operation while searches for the assignable cause are made (i.e., continuous process), and ii) the process must be shut down during the search (i.e., discontinuous process). Lorenzen and Vance (1986) suggested a unified approach to the economic design of process control charts. They considered various options regarding continuation of production during search for or removal of assignable cause. All the cost based quality control procedures are classified by Menipaz (1978). Montgomery (1980), Svoboda (1991), Ho and Case (1994) and

Celano (2011) provided literature review related to design of control charts on economic basis where it is observed that majority of the researchers have considered $\bar{X}$ chart and Duncan's (1956) single assignable cause model where the loss cost is expressed as a function of three design variables *n*, *h* and *k*. Collani (1988) presented an upgraded bibliography of quality control procedures based on economic design aspects out of which many of the references were cited in the review reported by Svoboda (1991). All review papers available in literature related to control chart design on economic basis are listed in Table 2.8 with their corresponding coverage periods.

Table 2.8: Coverage period of review papers

Authors	*Coverage period*
Montgomery (1980)	1950-1977
Collani (1988)	1975-1987
Svoboda (1991)	1979-1989
Ho and Case (1994)	1981-1991
Celano (2011)	1991-2011

All the assumptions taken by various researchers while developing economic models for design of control chart can be broadly categorized in two groups, namely i) process characteristics, and ii) model characteristics as discussed in next two sections.

2.10.1 Process Characteristics

While formulating an economic model for designing a control chart, certain assumptions are always made regarding the process behavior. Different assumptions considered by various researchers regarding the process behavior and the operation of control chart are discussed below.

i) Assignable Causes

A production process can remain in only two possible states i.e., single in-control state, and multiple out-of-control states. Every time when the process becomes out-of-control it is because of one or more assignable causes. When the process is subjected to only one assignable cause, the economic model developed is called as *single assignable cause model*. On the contrary, in *multiple assignable cause model* the process shifts due to more than one assignable cause.

As the number of causes increases, the mathematical model becomes more complex. Therefore, for simplicity the occurrence of single assignable cause for economic design of $\bar{X}$

chart has been assumed by many researchers like Duncan (1956), Panagos et al. (1985), Lorenzen and Vance (1986) and Banerjee and Rahim (1988). Works on multiple assignable cause model of $\bar{X}$ chart have also been reported by a few other researchers like Knappenberger and Grandage (1969), and Duncan (1971). Some other researchers like Chiu (1976) and Gibra (1981) considered the multiplicity of assignable causes for attribute control charts. Duncan (1971) extended his earlier work on single assignable cause model to multiple assignable cause model. He suggested if a production process is characterized by multiple assignable causes, then it can be adequately approximated to a single assignable cause model and this would also provide good results. Similar types of suggestions were also made by other researchers like Knappenberger and Grandage (1969), Chiu (1976), Gibra (1981), Tagaras and Lee (1988), Chen and Yang (2002), Silver and Bischak (2004) and Yang et al. (2010) developed an economic model with multiple control limits in the occurrence of multiple assignable causes. Gibra (1981) concluded that multiple cause model can be approximated to a single assignable cause model. Tagaras (1989) established approximate solutions to the Duncan's (1971) multiple assignable cause model, using a power approximation method. Arnold (1989) developed an economic model for $\bar{X}$ chart characterized by multiple assignable causes and compared two methods using i) a sampling alternative, and ii) no sampling alternative. Yu and Hou (2006) suggested economic design model for VSI $\bar{X}$ chart with multiple assignable causes. Recently, Yu et al. (2010) and Ahmed et al. (2014) presented a multiple assignable cause economic model subjected to some statistical constraints. The classification of literature on economic design on the basis of number of assignable causes and type of control chart is presented in Table 2.9.

Table 2.9: Classification of literature on the basis of assignable causes

Authors	***Control chart***	***Assignable Cause(s)***
Duncan (1956); Panagos et al. (1985); Lorenzen and Vance (1986); Banerjee and Rahim (1988)	$\bar{X}$ chart	Single
Knappenberger and Grandage (1969); Duncan (1971); Tagaras and Lee (1988); Tagaras (1989); Arnold (1989); Chen and Yang (2002); Yang et al. (2010); Ahmed et al. (2014)	$\bar{X}$ chart	Multiple
Chiu (1976); Gibra (1981)	*np*- chart	Multiple
Yu and Hou (2006)	VSI $\bar{X}$ chart	Multiple
Silver and Bischak (2004); Yu et al. (2010); Ahmed et al. (2014)	$\bar{X}$ chart with statistical constraints	Multiple

VSI: Variable Sampling Interval

ii) Process Failure Mechanism

The *process failure mechanism* is the occurrence of an assignable cause that shifts the process from in-control state to out-of-control state. Most of the earlier research on the control charts considered that assignable causes occur within a given time interval according to Poisson distribution. Therefore, the time duration for which the process remains under control is exponentially distributed. This assumption permits considerable simplification while developing the economic models. Montgomery (1980) suggested that misuse of this model assumption might result unfavorable economic consequences. Baker (1971) considered the sensitivity of optimal parameters of an economically designed $\bar{X}$ chart to different types of process failure mechanism. The designed parameters for joint $\bar{X}$ and R charts widely vary with the assumption of shape of the distribution of occurrence time of assignable cause (Saniga, 1979).

Hu (1986) modified fixed sampling interval model under Poisson to Weibull distributions as process failure mechanism. Researchers like Banerjee and Rahim (1988), Arnold and Collani (1989), and McWilliams (1989) concluded that as long as the mean value of the distribution of process failure mechanism is same, the economic design results are not sensitive to the form of distribution while using a fixed sampling interval. However, if a variable sampling interval is used, the assumptions based on the distribution of process failure mechanism significantly affect the results. Parkhideh and Case (1989) developed a generalized model based on Duncan's single assignable cause model for economically designed dynamic $\bar{X}$ chart with Weibull failure mechanism, where the control chart design variables were allowed to vary with time.

Most of the researchers have assumed that the changeovers between in-control and out-of-control states are sudden. However, any processes that drift gradually from the in-control state (for example, in case of tool wear) have received little attention for economic design. Considering this aspect an economic model of $\bar{X}$ chart under Weibull shock using variable sampling interval for the process with an increasing hazard rate was proposed by Banerjee and Rahim (1988). McWilliams (1989) proposed a unified approach using Weibull failure mechanism that can be applied to various types of distribution suggested by various authors. The Weibull distribution can have increasing, constant and decreasing hazard functions. Shapes closely resembling normal and lognormal are also possible.

Rahim and Banerjee (1993) proposed the economic design of $\bar{X}$ chart for variable sampling interval along with Weibull and Gamma failure mechanisms. Surtihadi and

Raghavachari (1994) used different process failure mechanisms like Weibull, lognormal, folded-normal, folded-logistic and gamma distributions to represent the time to the occurrence of an assignable cause and reported their effects on the optimal solution. The solutions obtained with different failure scenarios are compared with the solutions given by Duncan's (1956) single assignable cause model. They found that in case of small sampling interval their exact method under exponential assumption provide good approximate designs even when the occurrence of the assignable cause follows any non-exponential distribution. This promotes very useful and interesting conclusion that the optimality of the design of $\bar{X}$ chart is insensitive to the assumption on type of distribution when interval of sampling is small. Moskowitz et al. (1994) focused on $\bar{X}$ chart for a continuous process model considering exponential, Weibull and Pareto process failure mechanisms. Their results showed that process failure mechanism has effects on the optimal design variables and the magnitude of these effects is observed to depend on the size of the shift to be detected.

Rahim (1997) introduced an integrated increasing hazard rate and age dependent salvage value of equipment model with Gamma distribution of in-control periods. Zhang and Berardi (1997) extended the work of Rahim and Banerjee (1993) by applying statistical constraints on economic design model with Weibull failure mechanism. Bischak and Silver (2001) used four estimators for process failure rate (λ) for $\bar{X}$ chart. Simulation results shows that among all the four estimators, the maximum likelihood estimator developed on the basis of false alarm information by the control chart performs well in the process failure rate estimation. Al-Oraini and Rahim (2002) proposed their work on the basis of Gamma (λ, 2) failure distribution as failure model with some statistical constraints. They used the cost and process parameters considered by Rahim and Banerjee (1993). Chen and Yeh (2009) suggested economic statistical design of $\bar{X}$ chart under non-normality and Gamma shock process failure. Chen and Yang (2002) extended the work of Banerjee and Rahim (1988) on economic design model of $\bar{X}$ chart from single assignable cause to multiple assignable causes when the process failure mechanism follows Weibull distribution. They reported smaller loss-cost value using multiple assignable cause compared to single assignable cause model under Weibull process failure mechanism. Silver and Bischak (2004) considered the multiple assignable cause model with exponential failure mechanism by introducing Bayesian approach in the estimation of process failure rate (λ) from control chart cycle times. Chen and Cheng (2007) proposed economic statistical design of $\bar{X}$ chart assuming Weibull distribution for non-normality quality measurement. They considered the unified cost model proposed by McWilliams (1989) and performed a sensitivity analysis of the Weibull shape parameter. Yang et al. (2010) proposed a cost model and determined the optimal values of the design parameters for $\bar{X}$ control chart by minimizing the expected cost per unit time with respect to change in exponential parameters. Aghabeig and Moghadam (2013) proposed economic design of $\bar{X}$ chart assuming a generalized exponential shock model with uniform sampling interval scheme. The types of process failure mechanism assumed by various researchers in economic design and the corresponding control charts used by them are listed in Table 2.10.

Table 2.10: Classification of literature on the basis of process failure mechanism

Authors	***Control Chart***	***Process Failure Mechanism***
Hu (1986); Banerjee and Rahim (1988); McWilliams (1989); Parkhideh and Case (1989); Chen and Yang (2002)	$\bar{X}$ chart	Weibull
Rahim and Banerjee (1993)	VSI $\bar{X}$ chart	Weibull and Gamma
Surtihadi and Raghavachari (1994)	$\bar{X}$ chart	Weibull, lognormal, folded-normal, folded-logistic and gamma
Moskowitz et al. (1994)	$\bar{X}$ chart	Exponential, Weibull and Pareto
Rahim (1997); Chen and Yeh (2009)	$\bar{X}$ chart	Gamma
Zhang and Berardi (1997)	$\bar{X}$ chart with and without statistical constraints	Weibull
Al-Oraini and Rahim (2002)	$\bar{X}$ chart with statistical constraints	Gamma
Silver and Bischak (2004)	$\bar{X}$ chart	Exponential and Bayesian
Chen and Cheng (2007)	$\bar{X}$ chart with statistical constraints	Weibull
Aghabeig and Moghadam (2013)	$\bar{X}$ chart	Generalized
Yang (2010)	$\bar{X}$ chart	Exponential and Weibull

VSI: Variable Sampling Interval

iii) Population Distribution

The majority of the work based on economic design considered that the quality characteristic of the process being monitored is independent and identically distributed random normal variable. Burr (1967) considered $\bar{X}$ and R charts jointly, and examined the effect of non-normality on the control limits. He concluded that non-normality is not a serious problem unless there is a considerable deviation from normality. There are numerous process characteristics whose distributions are approximated as normal distribution. Rahim (1985) studied the effects of non-normality and measurement errors in the economic design of $\bar{X}$ chart. James (1989) figured out that some quality characteristics such as roundness, dimensions of mould and time spent in waiting by consumer will be non-normal. Gunter (1991) suggested that the quality characteristics such as flatness and percentage contamination would have skewed distributions. Cox (2013) reported an approximation technique using Burr distribution for economic design of $\bar{X}$ chart. Hsieh and Chen (2013) proposed an economic design of the VSSI $\bar{X}$ chart for positively skewed distributions. Vommi and Kasarapu (2014) considered three types of process shifts, namely positively skewed, uniform and negatively skewed distributions for economic design of $\bar{X}$ chart. The types of population distribution assumed in economic design of various control charts are given in Table 2.11.

Table 2.11: Classification of literature on the basis of population distribution

Authors	***Control Chart***	***Population Distribution***
Burr (1967)	Joint $\bar{X}$ and *R* charts	Non-normal
Rahim (1985)	$\bar{X}$ chart	Non-normal
Cox (2013)	$\bar{X}$ chart	Burr
Hsieh and Chen (2013)	VSSI $\bar{X}$ chart	Skewed
Vommi and Kasarapu (2014)	$\bar{X}$ chart	Positively skewed, uniform and negatively skewed

VSSI: Variable Sample Size and Sampling Interval

iv) Action After Signal

The search for the assignable causes starts, whenever the control chart indicates an out-of-control signal. This signal may be true or false. When the investigation for assignable cause is in progress, the process may be allowed to continue (i.e., continuous process) or stop (i.e., discontinuous process). Panagos et al. (1985) investigated the effect of mis-specifying the process model on control chart design. They reported that in case of discontinuous model the cost is always greater compared to that in continuous model, unless the penalty cost of producing the non-conforming items is very high.

2.10.2 Model Characteristics

Other than the process characteristics, various economic models have been developed considering diverse assumptions on factors like cost parameters, objective function, estimation of parameters, variation of parameters with time, auto-correlation in data, control limits, type of control charts, approximation methods, etc. which are discussed below.

i) Cost Parameters

The economic design of control charts requires the proper consideration of cost parameters. The cost of sampling and testing consists of the wages given to inspectors, the cost of hiring or purchasing the analyzing equipment, and others. Moreover, any destructive testing also includes the unit cost of the item tested. The cost under this head usually consists of both fixed and variable components. The cost of finding the assignable causes and possibly correcting the process after getting an out-of-control alarm has been treated in numerous ways. The cost of investigating false alarms is often assumed to be different from that of investigating and eliminating assignable causes. The cost of repairing the process depends on the number and types of assignable causes involved. Larger process shift may need larger cost of repair. Some of them claimed that a detailed level of modeling is unnecessary, because in various cases small shifts are difficult to identify but easy to correct, whereas large shifts are easy to identify but hard to correct. The costs related to production of non-conforming items contain the costs of scrap or rework in case of internal failures, and the costs of repair or replacement of products covered by warranties in case of external failures. It may also include the cost linked up with loss of goodwill of customers in the market (Montgomery, 2013).

Some other conflicts are also observed regarding the cost parameters like i) simplification of cost structure, and ii) generalization of cost structure. Chiu and Wetherill (1974) applied a simplification method over Duncan's (1956) model. The aim was to simplify the computational effort so that the quality control personnel with small or no programming skill can understand the design of control chart. They adopted a semi-economic scheme to design $\bar{X}$ chart. Collani (1986) simplified the loss cost function of $\bar{X}$ chart by decreasing the number of parameters from economic design model. He discussed the approach of inspection without sampling and introduced a standardized loss cost function which eases the design without the use of a computer. Montgomery and Storer (1986) considered only three types of costs i.e., the cost of sampling, the cost of investigating assignable causes and the cost of producing non-conforming items in their simplified cost model. They reported that simplified model requires only half of the parameters. Also this type of model is easier to optimize and it provides near optimal solutions as compared to the conventional model. Chung (1990) presented a more accurate and simplified procedure compared to the earlier models for the economic design of $\bar{X}$ chart. Baud-Lavigne et al.

(2010) proposed a simplified model for economic design of chart based on Lorenzen and Vance's (1986) model. The problem has been illustrated with an example taken from a semiconductor industry.

ii) Objective Function

Majority of the researchers considered the objective function as minimization of the expected cost per unit time in the field of economic design similar to the work of Duncan (1956). However, Knappenberger and Grandage (1969) developed an economic model by considering the minimization of the expected cost per unit item. Arnold and Collani (1989) assumed maximization of expected profit per unit item. Nikolaidis et al. (1997) considered economic models based on expected cost per unit output. Nantawong et al. (1989) compared $\bar{X}$ chart, CUSUM chart and geometric moving-average charts based on profit as the evaluation criterion instead of cost. Zupancic and Sluga (2008) compared optimum sample sizes in case of economic design of Shewart control charts for process mean assuming process-mean shift as a constant value versus random variable. Moreover, the optimum sample size has been computed on the basis of minimizing the loss function. The types of objective function assumed in the development of economic models are listed in Table 2.12.

Table 2.12: Classification of literature on the basis of objective function

Authors	***Control chart***	***Objective function***
Knappenberger and Grandage (1969)	$\bar{X}$ chart	Minimization of loss cost
Arnold and Collani (1989); Zupancic and Sluga (2008)	$\bar{X}$ chart	Maximization of profit
Nikolaidis et al. (1997)	Joint $\bar{X}$ and R charts	Minimization of loss cost
Nantawong et al. (1989)	$\bar{X}$ CUSUM and Geometric Moving-average charts	Maximization of profit

iii) Estimation of Parameters

The input cost and process parameters for the economic model can be evaluated by observing several production cycles and then taking the weighted averages of the observed readings. Some authors investigated the effect of mis-specification of cost and process parameter data (Duncan, 1956; Montgomery et al., 1975; Chiu, 1977; Panagos et al., 1985; Mortarino, 2010). All of them reported that any errors while estimating the cost and process parameters have some effect on the optimal design results. Pignatiello and Tsai (1988) investigated the optimal economic design of $\bar{X}$ chart when the cost and process parameters are not known. The cost parameters were tested under different noise scenarios. Each noise scenario was tested under a three level orthogonal array. They considered sample estimates using three performance measures, namely i) mean square error, ii) maximum, and iii) mean. A multiple objective approach for economic design of $\bar{X}$ chart was proposed by Del Castillo et al. (1996). This is a semi-economic approach which can be useful where the cost coefficients are either difficult to estimate or not known. For this method only sampling cost is necessary which can be easily estimated. They used multiple objective semi-economic approach by assessing the power of the chart. Also, they applied an interactive algorithm for designing the control chart. Linderman and Choo (2002) discussed the robust economic design of $\bar{X}$ chart when a single process is under three different scenarios with three possible process shifts and the corresponding out-of-control costs are considered while keeping all other parameters as constants. The three different designs proposed by them were based on i) absolute robustness, ii) robust deviation, and iii) relative robustness optimization measures. Various assumptions with regard to cost and process parameters in economic design are summarized in Table 2.13.

Table 2.13: Classification of literature on the basis of cost and process parameters

Authors	***Control chart***	***Cost and process parameters***
Duncan (1956); Panagos et al. (1985); Mortarino (2010)	$\bar{X}$ chart	Mis-specification
Montgomery et al. (1975)	Fraction defective charts	Mis-specification
Chiu (1977)	*np*-charts	Mis-specification
Pignatiello and Tsai (1988); Del Castillo et al. (1996); Linderman and Choo (2002)	$\bar{X}$ chart	Cost parameters are unknown

iv) Variation of Parameters with Time

In control chart design, the design variables are conventionally assumed to remain unchanged with respect to time. Majority of the work on economic design has been done with the assumption of constant parameters. However, Taylor (1965) reported that the control chart with constant parameters do not provide the optimum solutions.

Shewhart control charts are being modified depending upon the severity of effects of assignable causes from fixed sampling interval to variable type of sampling interval. Banerjee and Rahim (1988) reported an economic design of $\bar{X}$ chart with time varying parameters. Their economic design model considers only the sampling interval to change over time while assuming all other parameters as constant. Some authors like Flaig (1991), Daudin (1992), Prabhu et al. (1993), Costa (1994), Costa (1997) and De-Magalhaes et al. (2001) studied the effect of varying sample size while keeping sampling interval fixed for $\bar{X}$ chart. Costa (1998) carried his same research in the field of joint economic design of $\bar{X}$ and R charts with variable parameters. He concluded that variable parameters $\bar{X}$ and R charts detect process shifts faster than the traditional $\bar{X}$ and R chart. Yu and Chen (2005) proposed economic design of $\bar{X}$ chart with variable sampling interval (VSI) using Hooke and Jeeves (1961) pattern search technique and indicated that VSI control chart provided lower cost than the fixed sampling interval (FSI) control chart. Christopher et al. (2010) suggested the economic design of VSI-$\bar{X}$ chart when the sampling interval is calculated with the help of most recent samples. Prajapati (2010) investigated the effect of varying the sampling interval for economic design of $\bar{X}$ chart over Lorenzen and Vance (1986) cost model.

In case of adaptive control chart, some of the control chart parameters are changed during the process based on the sample information. $\bar{X}$ chart with adaptive sampling interval was suggested by Reynolds et al. (1988), and Runger and Pignatiello (1991). An adaptive $\bar{X}$ chart with variable sample size and variable sampling interval has been presented by Prabhu et al. (1994), and Lin and Chou (2005). Park and Reynolds (1994) proposed an economically designed adaptive $\bar{X}$ chart with two possible values for the sample size. They reported that the cost saving over a static $\bar{X}$ chart could be as high as 25%. Tagaras (1994) studied an economic design of Bayesian adaptive $\bar{X}$ chart for a production process that goes out-of-control due to shift in process mean only. He considered sampling interval and control limit coefficients as adaptive parameters. Calabrese (1995) developed an economic design model for partially adaptive *p*-chart. He assumed both the sampling interval and sample size as constant and therefore only the control limits are considered as decision variables. Tagaras (1996) extended his work on Bayesian adaptive model by incorporating adaptive samples in addition to adaptive sampling interval and control limit coefficients. Tagaras (1997) proposed the following two basic principles for adaptive control charts:

i) All available information should be used for effective monitoring of production process.
ii) The process control should be flexible enough to respond to that information by adapting in real time.

Carolan et al. (2010) proposed economic design of $\bar{X}$ chart with continuously variable sampling intervals. Das et al. (1997) suggested an economic model for optimal selection of design parameters for $\bar{X}$ chart with dual sampling interval (DSI) policies, with and without run rules. He and Grigoryan (2005) proposed a multivariate variable sampling control chart. This method is a multivariate extension of double sampling $\bar{X}$ chart. De-Magalhaes et al. (2007) suggested joint statistical design of adaptive $\bar{X}$ and R charts. The process is subjected to two independent assignable causes. One cause changes the process mean and the other changes the process variance. They also assumed that the quality characteristic is normally distributed and the time that the process remains in control has exponential distribution. Results are obtained through Markov chain approach. Chen (2007) proposed an adaptive sampling enhancement for Hotelling's T^2 chart. Numerical comparisons have been made and discussed between adaptive sampling schemes and fixed sampling rate (FSR) T^2 control chart. Montgomery et al. (1990) and Box et al. (1994) studied autoregressive integrated moving average (ARIMA) models and observed that the control chart designed without the assumption of autocorrelation yielded more frequent false alarms for an auto-correlated process. Chen et al. (2007) proposed economic design of the VSSI $\bar{X}$ charts for correlated data. Genetic algorithm has been used to find the optimal values of sample size, sampling interval length, control limit and warning limits. Chou et al. (2008) suggested economic design of variable sampling intervals (VSI) EWMA charts. They considered time of sampling is fixed and on that basis designed a new model VSIFT. Torng et al. (2009) suggested economic design of double sampling $\bar{X}$ chart for correlated data using genetic algorithm. They considered the unified cost model of Lorenzen and Vance (1986). A real life example based on the process of packing integrated circuit is given to illustrate the model. De-Magalhaes et al. (2009) suggested a Markov chain approach for a statistical design of a hierarchy of two-states adaptive parameters $\bar{X}$ chart. The adaptation was combined in such a way that design parameters are allowed to vary when one, two, or all of them were arranged in a hierarchy. Nenes (2011) compared a unified approach for the development of economically designed variable parameter (VP) $\bar{X}$, VP $\bar{X}$-CUSUM and VP $\bar{X}$-EWMA control charts with that of fixed parameter control charts. The comparisons demonstrate the superiority of VP $\bar{X}$-CUSUM and VP $\bar{X}$-EWMA charts over VP $\bar{X}$ which in turn are economically superior to fixed parameter (FP) control charts. Lee et al. (2012) proposed economic design of $\bar{X}$ chart with combined double sampling and variable sampling interval. They constructed economic design model of DSVSI $\bar{X}$ chart for the determination of the design parameters. To study the effect of cost and process parameters, sensitivity analysis has been done. Lee (2013) proposed joint statistical design of $\bar{X}$ and S charts with combined double sampling and variable sampling interval. They applied Markov chain approach to compute the statistical performance. Further, they also reported that with this combination the efficiency of signaling small shift increases. The classification of various control charts on the basis of variation of parameters with time in economic design is summarized in Table 2.14.

Table 2.14: Classification of literature on the basis of variation of parameters with time

Authors	***Control Chart***	***Variation of Parameters with Time***
Taylor (1965)	$\bar{X}$ chart	Fixed
Banerjee and Rahim (1988); Reynolds et al. (1988); Runger and Pignatiello (1991); Christopher et al. (2010); Prajapati (2010)	VSI $\bar{X}$ chart	Variable
Montgomery et al. (1990); Box et al. (1994)	Moving average chart	Autoregressive
Flaig (1991); Prabhu et al. (1993); Costa (1994); Park and Reynolds (1994); Costa (1994)	VSS $\bar{X}$ chart	Variable
Daudin (1992)	DSVSS $\bar{X}$ chart	Variable
Prabhu et al. (1994)	VSS and VSI $\bar{X}$ chart	Adaptive
Calabrese (1995)	*p*-chart	Adaptive
Tagaras (1994); Tagaras (1996)	Bayesian $\bar{X}$ chart	Adaptive
Das et al. (1997)	DSI $\bar{X}$ chart	Adaptive
Costa (1998)	Joint $\bar{X}$ and R charts	Variable
De-Magalhaes et al. (2001)	$\bar{X}$ chart	Adaptive
Yu and Chen (2005)	VSI $\bar{X}$ chart	Variable
He and Grigoryan (2005)	Multivariate DSVSS $\bar{X}$ chart	Variable
Lin and Chou (2005)	VSS and VSI $\bar{X}$ chart	Adaptive
Chen (2007)	Multivariate Hotelling T^2 chart	Adaptive
Chen et al. (2007)	VSSI $\bar{X}$	Variable
De-Magalhaes et al. (2007)	Joint $\bar{X}$ and R charts	Adaptive
Chou et al. (2008)	EWMA chart	Variable
De-Magalhaes et al. (2009)	$\bar{X}$ charts	Adaptive
Torng et al. (2009)	DS $\bar{X}$ chart	Variable
Carolan (2010)	VSS $\bar{X}$ chart	Variable
Nenes (2011)	$\bar{X}$ $\bar{X}$ - CUSUM and $\bar{X}$ - EWMA charts	Variable
Lee et al. (2012)	DSVSI $\bar{X}$ chart	Variable
Lee (2013)	Joint DSVSI $\bar{X}$ and *S* charts	Variable

VSI: Variable Sampling Interval; DSVSS: Double Sampling Variable Sample Size; VSS; Variable Sample Size; DSI: Double Sapling Interval; VSSI; Variable Sample Size and Sampling Interval; DS: Double Sapling; DSVSI: Double Sapling Variable Sampling Interval

v) Control Limits

Traditionally there are three assumptions i.e., i) constant process variance ii) perfect measurement of the quality characteristic, and iii) equal probabilities of shifts from the process mean (i.e., either upward or downward). Considering these three assumptions economic design of $\bar{X}$ chart assumes symmetric control limits. Tagaras (1989) developed economic design of $\bar{X}$ chart with asymmetric control limits and relaxed these three assumptions. Gelinas and Lefrangois (1998) developed a heuristic approach for the joint economic design of $\bar{X}$ and R charts with asymmetric control limits. This approach avoids the use of any non-linear optimization search technique.

vi) Types of Control Charts

Various economic models have been developed considering variety of control charts after pioneer work of Duncan (1956). The economic models of different control charts are similar except the computation of α and β errors. Among all the control charts, $\bar{X}$ chart is the most commonly investigated univariate control chart for economic design purpose. A vast majority of research applied on part manufacturing is based on $\bar{X}$ chart (Montgomery, 1980). Koo and Case (1990) developed economic model for a continuous flow process using $\bar{X}$ chart. Silver and Rohleder (1999) developed an optimal procedure by setting dynamically the values of three design variables of $\bar{X}$ chart for the reduction in the frequency of occurrence of assignable causes due to process improvement. Bai and Lee (1998) reported that economically designed VSI (variable sampling interval) $\bar{X}$ chart is more efficient than the FSI (fixed sampling interval) scheme in terms of the expected cost per unit time. Some authors like Saniga (1977), Jones and Case (1981) Chung and Chen (1993) and De-Magalhaes et al. (2007) carried their research in the field of joint economic design of $\bar{X}$ and R charts

Majority of literature on economic designs is based on monitoring the shift in process mean. Only few are devoted to monitoring the change in process dispersion alone. Collani and Sheil (1989) developed an economic model for the standard deviation or S chart.

The economic design of CUSUM control chart was first proposed by Taylor (1968). He assumed that a single assignable cause occurs according to Poisson distribution and expressed the expected cost per unit time as a function of the sample size (n), sampling interval (h) and two V-mask variables. To solve the model he assumed that n and h were pre-specified. Goel and Wu (1973), and Chiu (1974) reported economic model on a tabular type of CUSUM chart considering normal population distribution. Lashkari and Rahim (1982) reported an economic model of CUSUM chart when the observations are independent but not normally distributed. Pan and Chen (2005) proposed a new way of monitoring and evaluating the environmental performance using the economic design of CUSUM control chart. They estimated the potential loss function using revised inverted normal loss function (RINLF).

Linderman and Love (2000) proposed economic and economic statistical designs for MEWMA control charts and obtained the average run length (*ARL*) using simulation technique. Ho and Case (1994) proposed a fully economic model for EWMA chart. They compared their results with that of CUSUM chart (Goel and Wu, 1973) and of $\bar{X}$ chart Chung (1990). They reported that economic design of EWMA chart provides results which are almost similar to that of CUSUM chart, but superior to that of $\bar{X}$ chart. Serel (2009) presented the case where the assignable cause changes only the process mean or dispersion. The economic design of EWMA mean chart has been extended to the case where the quality related costs are computed based on a loss function. Saghaei et al. (2014) suggested economic design of exponentially weighted moving average (EWMA) control chart where average run length (*ARL*) computed using Markov chain method and design parameters using genetic algorithm with sensitivity analysis. Mohammadian and Paynabar (2008) proposed economic design of acceptance control charts. In this model, control chart parameters, sample size, sampling interval, and the control limits coefficient are determined such that the expected cost of the process is minimized. Table 2.15 shows the summary of literature for various control charts.

Table 2.15: Classification of literature on the basis of control chart

Authors	***Control chart***
Duncan (1956); Montgomery (1980); Koo and Case (1990); Chung (1990); Silver and Rohleder (1999)	$\bar{X}$ chart
Saniga (1977); Jones and Case (1981); Chung and Chen (1993); De-Magalhaes et al. (2007)	Joint $\bar{X}$ and R charts
Taylor (1968); Goel and Wu (1973); Chiu (1974); Lashkari and Rahim (1982); Pan and Chen (2005)	CUSUM chart
Collani and Sheil (1989)	Standard deviation or *S* chart
Ho and Case (1994)	EWMA chart and CUSUM chart
Bai and Lee (1998)	VSI $\bar{X}$ chart
Linderman and Love (2000)	MEWMA chart with and without statistical constraints charts
Mohammadian and Paynabar (2008)	Acceptance control chart
Serel (2009)	EWMA chart and Shewhart charts
Saghaei et al. (2014)	EWMA chart

VSI: Variable Sampling Interval

vii) Generalized Model

There are various approaches and assumptions for economic design considered by different authors. Some assumed continuous model, whereas some other assumed discontinuous process. Some maximized the profit and some minimized the loss. Different quality characteristics like variables or attributes were assumed. Similarly, various cost assumptions were considered. Thus, there was a need of a generalized model which can be applied to all kinds of manufacturing environments. Lorenzen and Vance (1986) proposed a generalized cost model that can be applicable to all types of control charts regardless of the statistic used. The generalized cost models can be applied to wide variety of production situations, whereas the simplified cost models are appropriate for a particular type of application. Murthy and Rambabu (1997) proposed economic design model that is applicable to both $\bar{X}$ chart and np-chart. Their generalized model can be applied to all control charts regardless of the statistics used. For iterating sampling interval *h* they applied Newton Raphson method, whereas Fibonacci search was applied to iterate sample size *n* and width of control limits *k* respectively.

viii) Integrated Models

Simple models consider only the input cost and process parameters for controlling the process and designing of control charts. In case of integrated models, the control chart design is combined with activities like production planning, maintenance schedules, inspection policies, inventory control, labour requirement etc. Generally, the issue of quality control is viewed separately from that of inventory control. Lin et al. (1991) studied the application of a joint economic model for monitoring both quality and inventory for a process producing resistors. Rahim (1994) suggested an integrated design, which considers the design of $\bar{X}$ chart where the in-control period follows a probability distribution of increasing failure rate along with determining the economic production quantity (EPQ) and inventory planning for a production process. The models considered by Lin et al. (1991) and Rahim (1994) were based on a discontinuous process. Later, Rahim and Ben-Daya (1998) generalized the above models to cases where production ceases due to false alarms only. Ben-Daya and Rahim (2000) developed a model which considers the effects of preventive maintenance (PM) on the quality control charts. The model assumed imperfect maintenance and the reduction in the age of the system is proportional to the PM level used. Numerical examples considered by them showed that higher PM levels lead to more reduction in quality control costs. Lam and Rahim (2002) proposed an integrated economic model for the integrated economic design of $\bar{X}$ chart with maintenance schedules. The concepts of production planning, maintenance and quality control are incorporated in the model. Wu et al. (2007) integrated the use of deploying manpower to statistical process control for a multistage manufacturing system. They proposed an algorithm to minimize the total expected cost with the allocation of manpower into the process model. Zhou and Zhu (2008) proposed an integrated model for economic design of control chart in addition to maintenance management. They assumed Weibull failure mechanism and applied grid search approach for determining values of design variables. An integrated model combined with economic design of $\bar{X}$ chart and the preventive maintenance using Taguchi loss function have been suggested by Chen et al. (2011). Rahim and Shakil (2011) suggested Tabu search algorithm for determining the economic design parameters under integrated production planning, quality control and preventive maintenance policy scheme. They assumed general probability distribution with an increasing hazard rate. Further, they considered three different assumptions of the quality control parameters, namely i) n and k uniform, and h as non-uniform, ii) k as uniform and, n and h as non-uniform, and iii) n, h and k as non-uniform. Charongrattanasakul and Pongpullponsak (2011) proposed an integrated model of process control and maintenance using genetic algorithm for economic design of EWMA control chart. Various control charts with regards to type of integration in economic design are summarized in Table 2.16.

Table 2.16: Classification of literature on the basis of type of integration

Authors	***Control Chart***	***Type of Integration***
Lin et al. (1991); Rahim (1994)	$\bar{X}$ chart	Quantity and inventory planning
Ben-Daya and Rahim (2000); Zhou and Zhu (2008); Rahim and Shakil (2011): Chen et al. (2011)	$\bar{X}$ chart	Quantity and preventive maintenance
Lam and Rahim (2002); Wu et al. (2007)	$\bar{X}$ chart	Quantity and manpower schedules
Charongrattanasakul and Pongpullponsak (2011)	EWMA chart	Quantity and preventive maintenance

ix) Taguchi's Loss Function

In case of economic design of control charts, the quality loss is considered as the cost when the quality characteristics are outside the specification limits. All products falling within the range of control limits are considered having the same quality irrespective of their deviation from the target value. According to quality concept of Taguchi, the products close to the target value will have less quality loss as compared to the products far away from the target. Taguchi proposed a quadratic loss function to estimate the quality loss of a product when it deviates from its target value. Taguchi (1984) suggested an economic design model to determine the diagnosis sampling interval and control limits for online production process to minimize the expected cost per unit of production. Kackar (1986) suggested the importance of continuously reducing process variation using quadratic loss concept. Following the same trend Taguchi et al. (1989) provided a detailed explanation of economically designed control system through many practical cases. Elsayed and Chen (1994), Alexander et al. (1995), and Ben-Daya and Duffuaa (2003) incorporated Taguchi's loss function for the economic design of $\bar{X}$ chart. Yang (1998) considered the economic statistical design of standard deviation chart by applying the Taguchi's loss function. The list of control charts in which Taguchi loss function has been applied for economic design is shown in Table 2.17.

Table 2.17: Summary of literature on various control charts based on Taguchi loss function

Authors	***Control Charts***
Taguchi (1984)	$\bar{X}$ chart
Taguchi et al. (1989)	Several practical scenarios
Elsayed and Chen (1994); Alexander et al. (1995); Ben-Daya and Duffuaa (2003)	$\bar{X}$ chart
Yang (1998)	Standard deviation chart or *S* chart with statistical constraints

2.11 Optimization Techniques

Models for economic design of control charts are usually complex. Therefore, their implementation in real scenario is limited in spite of several economic advantages. Therefore, one group of research on economic design is to explore simpler methods to determine optimal values for the control chart design variables. Numerous approximation techniques are considered for the cost model and also for optimization of Duncan's (1956) single assignable cause model. Goel et al. (1968) suggested an iterative procedure which is superior to Duncan's approximate optimization technique. For applying a constraint on the power of detecting the shift, a simple approximate procedure was developed by Chiu and Wetherill (1974). To determine the expected number of samples taken while the process is in-control, Chung (1990) adopted McWilliams' (1989) approximations. Tagaras (1989) proposed a log-power function to approximately estimate the power of detection of $\bar{X}$ chart with asymmetric control limits. An economic design model for $\bar{X}$ chart with a multiple criteria optimization algorithm was proposed by Castillo et al. (1996) where some cost parameters like false alarm costs and cost of running the process in out-of-control state are eliminated. The advantage of his model is that only the cost of sampling is required which is easier to estimate. Chiu and Huang (1996) suggested the economic design of $\bar{X}$ chart with repair cost depending on detection delay in case of both continuous and discontinuous processes.

The effectiveness of economic design depends on how accurately this function is minimized to determine the values of all design variables. Various optimization techniques have been used for its minimization. Traditional optimization techniques like direct search method (Panagos et al., 1985; Ho and Trindade, 2009), Hooke and Jeeves pattern search technique (Banerjee and Rahim, 1988; Rahim, 1989; Rahim and Banerjee, 1993; Rahim, 1993; Rahim, 1994; Rahim and Ben-Daya, 1998; Lam and Rahim, 2002), Newton method (Chiu and Cheung, 1977; Lorenzen and Vance, 1986), Fibonacci search and golden section search (Lorenzen and Vance, 1986) have been used for economic design of $\bar{X}$ chart. McWilliams (1994) and Yu et al. (2010) applied grid search technique for economic statistical design of $\bar{X}$ chart in case of single assignable cause and multiple assignable cause models respectively. Rahim (1989), and Chung and Chen (1993) used pattern search technique for joint economic design of $\bar{X}$ and R charts. Chen and Tirupati (1996) suggested an iterative search method for economic design of $\bar{X}$ chart. They also discussed alternative quick heuristics to determine near optimal values of design parameters for practical use.

The objective function in economic design is of complex nature as it is a multi-variable, non-linear, non-continuous and non-differentiable function. It is difficult to minimize this function without adequate simplification. Therefore traditional optimization techniques cannot be directly applied without simplifying it. Hence, non-traditional optimization techniques are being used in recent years. Kethley and Peters (2004) employed genetic algorithm in the design of *p*-chart, incorporating the constraints such as frequency of inspection, number of defectives allowed and production rate. Vijaya and Murthy (2007) proposed a risk based approach for the economic design of $\bar{X}$ chart for which cost and process parameters were expressed in ranges. In their work genetic algorithm has been used as a search tool for precise estimation of cost and process parameters. Chen and Yeh (2009), and Vommi and Kasarapu (2014) proposed economic statistical design of $\bar{X}$ chart using genetic algorithm. Lee et al. (2012), and Hsieh and Chen (2013) proposed an economic design of the VSSI $\bar{X}$ chart using Markov chain and genetic algorithm. Ahmed et al. (2014) proposed economic and economic statistical designs of $\bar{X}$ chart using genetic algorithm under multiplicity of assignable causes. Results obtained from genetic algorithm have been compared with that of grid search technique for a numerical example. Particle swarm optimization (Chih et al., 2011; Gupta and Patel, 2011) and differential evolution (Kasarapu and Vommi, 2013) have also been suggested for economic design of $\bar{X}$ chart. Kaya (2009) proposed a genetic algorithm approach to determine the sample size for attribute control charts. Chen (2007) applied genetic algorithm on adaptive sampling enhancement for Hotelling's T^2 chart. Torng et al. (2009) suggested economic design of double sampling $\bar{X}$ charts for correlated data using genetic algorithms. Niaki et al. (2010 and 2011) applied genetic algorithm and particle swarm optimization approach respectively for both economic as well as economic statistical design of multivariate exponential weighted moving average (MEWMA) chart. For the same MEWMA chart Niaki et al. (2012) suggested a hybrid ant colony optimization approach for economic statistical design. Genetic algorithm (Chou et al., 2006) and differential evolution (Kasarapu and Vommi, 2011) have been employed for joint economic design of $\bar{X}$ and R charts. Yang et al. (2012) suggested an improved particle swarm optimization approach for economic and economic statistical design of $\bar{X}$ and S charts. The types of optimization techniques used in economic designs are listed in Table 2.18.

Table 2.18: Classification of literature on the basis of optimization techniques

Authors	*Control Chart*	*Optimization Technique*
Duncan's (1956); Chung (1990); Chiu and Wetherill (1974); Tagaras (1989); Chiu and Huang (1996)	$\bar{X}$ chart	Approximation method
Goel et al. (1968); Chen and Tirupati (1996)	$\bar{X}$ chart	Iterative method
Chiu and Cheung (1977)	$\bar{X}$ chart	Newton method
Panagos et al. (1985)	$\bar{X}$ chart	Direct search method
Lorenzen and Vance (1986)	$\bar{X}$ chart	Newton, Fibonacci and Golden section search method
Banerjee and Rahim (1988); Rahim (1989); Rahim and Banerjee (1993); Rahim (1993); Rahim (1994); Rahim and Ben-Daya (1998); Lam and Rahim (2002)	$\bar{X}$ chart	Hooke and Jeeves pattern search
Rahim (1989); Chung and Chen (1993)	Joint $\bar{X}$ and R charts	Pattern search
McWilliams (1994); Yu et al. (2010)	$\bar{X}$ chart	Grid search
Castillo et al. (1996)	$\bar{X}$ chart	Interactive multiple criteria optimization algorithm
Kethley and Peters (2004)	*p*-chart	Genetic algorithm
Chou et al. (2006);	Joint $\bar{X}$ and R charts	Genetic algorithm
Vijaya and Murthy (2007)	$\bar{X}$ chart	Genetic algorithm
Chen (2007)	Hotelling T^2	Genetic algorithm
Ho and Trindade, 2009	$\bar{X}$ chart	Direct search method
Kaya (2009)	u charts	Genetic algorithm
Niaki et al. (2010)	MEWMA chart	Genetic algorithm
Chih et al. (2011); Gupta and Patel (2011)	$\bar{X}$ chart	Particle swarm optimization
Niaki et al. (2011)	MEWMA chart	Particle swarm optimization
Kasarapu and Vommi (2011)	Joint $\bar{X}$ and R charts	Differential evolution
Niaki et al. (2012)	MEWMA chart	Hybrid ant colony optimization
Yang et al. (2012)	Joint $\bar{X}$ and S charts	Improved particle swarm optimization
Kasarapu and Vommi (2013)	$\bar{X}$ chart	Differential evolution
Lee et al. (2012); Hsieh and Chen (2013)	VSSI $\bar{X}$ chart	Markov chain and genetic algorithm
Chen and Yeh (2009); Vommi and Kasarapu (2014)	$\bar{X}$ chart	Genetic algorithm

2.11.1 Computer Program

Montgomery (1982) provided a computer program that determines the optimal values of control chart parameters for economically designed $\bar{X}$ chart subjected to single assignable cause based on Duncan's (1956) model. Rahim (1989) developed a computer program for the joint design of $\bar{X}$ and R charts with a single assignable cause producing a shift in mean and variance. Jaraied and Zhuang (1991) developed program to economically design $\bar{X}$ chart when it is subjected to multiple assignable causes. Torng et al. (1995) presented a FORTRAN program for the statistically constrained economic design of EWMA control chart for controlling process mean. Wu et al. (2002) have written a computer program in C language for joint statistical design of $\bar{X}$ and S charts. Aparisi and Garcia-Diaz (2004) developed computer program for the optimal design of the EWMA and MEWMA chart parameters using genetic algorithm. Sundus (2015) presented R-edcc package for economic design of control charts. This is due to the fact that majority of the applications made up in recent years are carried out using MATLAB, C, SAS etc. The aim is to show how the control charts design can be applied economically on a real life problem. The list of literature in which computer programs have been developed for economic design is given in Table 2.19.

Table 2.19: Summary of literature on computer programs

Authors	***Control Charts***
Montgomery (1982)	$\bar{X}$ chart
Rahim (1989)	Joint $\bar{X}$ and R charts
Jaraied and Zhuang (1991)	$\bar{X}$ chart
Torng et al. (1995)	EWMA chart with constraints
Wu et al. (2002)	Joint $\bar{X}$ and S charts
Aparisi and Garcia-Diaz (2004)	EWMA and MEWMA chart
Sundus (2015)	$\bar{X}$ chart

2.12 Conclusions

After Shewhart proposed the first control chart, various types of charts have been developed as per the need of diversified manufacturing environments over the years. Various methodologies have been evolved for design of control charts. Among them, economic design and economic statistical design have gained the maximum popularity as it helps to minimize the cost of process control so as to cope up with competitive market. It has attracted considerable attention among the researchers to bring down the cost as much as possible. Various types of model characteristics and process characteristics have been considered in the formulation of economic models. Various types of optimization techniques have been tried for optimizing the design of control charts from economic point of view. The extensive review of literature presented in this chapter helped in identifying the research gap and outlining the research objectives as mentioned in Chapter 1.

CHAPTER - 3

Economic Design of $\overline{X}$ Chart

3.1 Introduction

$\overline{X}$ chart is most commonly used for statistical process control in industries. Hence, its design from economic point of view has gained considerable importance. This type of design minimizes the cost of process control and thereby helps in improving profit margin of the industry in competitive market. Researchers are engaged in improving this design by reducing the cost of process control as minimum possible. In this chapter, new methodologies for economic design of $\overline{X}$ chart based on two metaheuristic approaches such as simulated annealing (SA) and teaching-learning based optimization (TLBO) have been developed for both continuous and discontinuous types of processes. The economic design of $\overline{X}$ chart based on simulated annealing algorithm has been illustrated through a numerical example. For validating the design results obtained from SA, the same example has been solved with a comparatively more recent metaheuristic known as teaching-learning based algorithm. The results obtained from both the optimization techniques are found to be better than the results already published in the literature. Sensitivity analysis is also performed using design of experiments and analysis of variance to investigate the effects of cost and process parameters on the output responses of economic design of $\overline{X}$ chart.

3.2 Assumptions

To formulate an economic model for designing a control chart, it is necessary to first make all the assumptions dealing with process behavior, statistical properties of the control chart, control procedure and economic factors. The assumptions listed below are relatively standard in most of the economic models.

- $\bar{X}$ chart is used only to identify whether the process is in the state of in-control or out-of-control but it itself cannot correct the out-of-control process without managerial intervention.
- Only one quality characteristic X which is the most critical for deciding the quality of the product is chosen for the economic model. So, this model is not applicable for any multi-variate control chart.
- The quality characteristic X is a random variable that follows normal distribution with mean μ and variance σ^2.
- The process is considered as a series of production cycles. Each cycle starts with in-control state having process mean μ_0 and process standard deviation σ_0.
- There are three horizontal lines in $\bar{X}$ chart. The centre line (CL) represents the average measure of the quality characteristic corresponding to the in-control state. Other two lines are upper control limit (UCL) and lower control limit (LCL) between which almost all the sample points are supposed to fall when the process is in-control. Thus, the three lines for $\bar{X}$ chart are expressed as

$$CL_{\bar{X}} = \mu_0,$$

$$UCL_{\bar{X}} = \mu_0 + k\frac{\sigma_0}{\sqrt{n}}, \text{ and}$$

$$LCL_{\bar{X}} = \mu_0 - k\frac{\sigma_0}{\sqrt{n}}.$$

where

μ_0 = process mean for in-control process

σ_0 = process standard deviation for in-control process ($\sigma_0 \geq 0$)

k = width of upper or lower control limit expressed in multiple of standard deviation of $\bar{X}$ (i.e., $\sigma_{\bar{X}} = \frac{\sigma_0}{\sqrt{n}}$), $k \geq 0$.

- In this chapter, the standard deviation σ is assumed to be constant throughout i.e., $\sigma = \sigma_0$.
- If a sample point falls outside either of the two control limits, the process is assumed to be out-of-control and a search for the assignable cause is initiated. This is assumed to have occurred due to a single assignable cause that results in shifting of process mean from μ_0 to $\mu_0 \pm \delta\sigma$. Thus, the process shift due to multiple assignable causes or due to change in process standard deviation σ is not considered in this chapter.
- The assignable cause is assumed to occur according to Poisson distribution with a rate of λ occurrences per hour. Thus, the time interval for which the process remains in-control is an exponential random variable with a mean of $1/\lambda$ hour.
- Upon the detection and removal of the assignable cause, the process returns to in-control state again and the new cycle begins.

3.3 Economic Model

All the processes can be classified into two major groups i.e., i) continuous process and ii) discontinuous process. In continuous process the process is allowed to continue even after the control chart signals that the process has gone out-of-control. On the other hand, the discontinuous process is immediately stopped after receiving the out-of-control signal. In both the cases, the search for assignable cause begins after a point falls outside either upper or lower control limits on the control chart. If an assignable cause is detected, necessary action is taken for its elimination so as to bring back the process to in-control state. In case of discontinuous process since the process is stopped during repair activity, it requires to be restarted. It is to be noted that the shift from the in-control state to the out-of-control state is irreversible. Therefore, once a process has gone out-of-control, it cannot come back to in-control state of its own. It always requires managerial intervention for detecting and eliminating assignable cause so as to bring back the process to in-control state. The managers always try to complete this remedial action as quickly as possible. At the same time it is necessary to design the control chart in such a way that it is capable of generating the signal as soon as the process has gone out-of-control. The delay in triggering an out-of-control signal on the control chart will go on producing more and more non-conforming items and thereby causes loss to the organization. Sometimes, the delay may develop further quality deterioration resulting in a loss of higher magnitude.

If sample size is n and k is the width of the control limits for the $\bar{X}$ chart, the centre line will be at μ_0 and the two control limits will be at $\mu_0 \pm k\sigma / \sqrt{n}$. When the process is in-control, the control chart may give a false signal indicating that the process has gone out-of-control even though in reality it is in the in-control state. This is called as Type-I error or α error which is expressed as:

$$\alpha = 2\left[1 - \phi\ (k)\right] \tag{3.1}$$

where $\phi\ (k)$ is the area under standard normal distribution curve from $-\infty$ to k.

The process may be disturbed due to occurrence of an assignable cause randomly at a rate of λ as per Poisson distribution. If the shift in process mean is $\delta\sigma$, the probability that the shift will be detected on any subsequent sample is:

$$P = 1 - \beta = \int_{-\infty}^{(-k-\delta\sqrt{n})} \phi(z)dz + \int_{(k-\delta\sqrt{n})}^{\infty} \phi(z)dz \tag{3.2}$$

where P is the power of detecting the shift.

From Eq. 3.2 it can be easily identified that smaller the β error, more will be the power of detecting the process shift.

The expected number of successive samples taken until a sample point on the control chart triggers an out-of-control signal (i.e., the sample point falls outside either of the two control limits) is called as *average run length* (*ARL*). If ARL_0 and ARL_1 represent in-control and out-of-control average run lengths, then:

$$ARL_0 = 1/\alpha \tag{3.3}$$

$$ARL_1 = 1/(1-\beta) \tag{3.4}$$

The economic models of $\bar{X}$ chart for both the process models (i.e., continuous and discontinuous) have been developed below.

3.3.1 Economic Model for Continuous Process

A continuous process never stops. It is allowed to continue even during the search and removal of identified assignable cause. It remains alternatively in two states in its entire operation i.e., in-control and out-of-control states. In real life, for most of the time the process is expected to run in-control. Occasionally, it gets disturbed due to occurrence of some assignable cause and gets switched to out-of-control state. These two states form one production cycle as shown in Fig. 3.1. Sometimes, no assignable cause may be found out even if an out-of-control signal is obtained from the control chart. In this case, no repair action is necessary and the process continues in the in-control state as before. This type of signal is termed as false signal or false alarm.

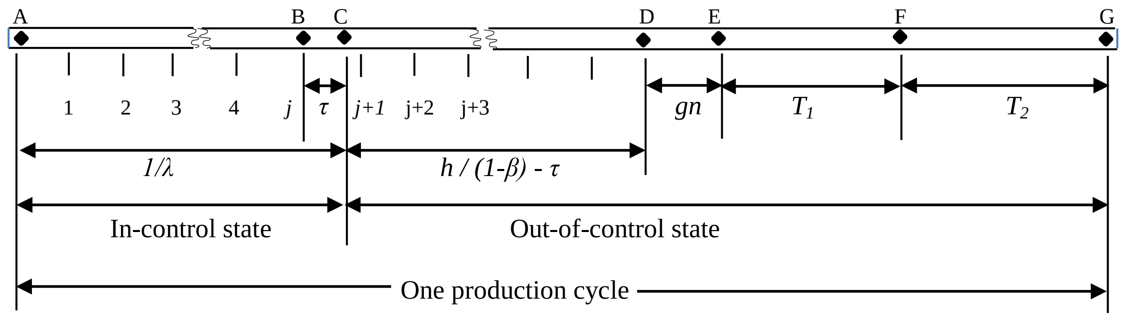

Fig. 3.1: Cycle time for continuous process

Various cardinal points of one production cycle are explained below:

Point **A**: Cycle starts

Point **B**: Last sample just before the assignable cause occurs (i.e., *j*th sample)

Point **C**: Assignable cause occurs (i.e., unknown to operator till Point E)

Point **D:** Sample containing out-of-control signal

Point **E:** Out-of-control signal displayed on control chart

Point **F:** Assignable cause found out

Point **G:** Assignable cause eliminated and process comes back to in-control state

The production cycle consists of the following five components:

i) ***Expected in-control period (AC)***

Fig. 3.1 shows that the production cycle starts at Point A when the process is in-control. The process remains in the same state until some assignable cause occurs at Point C. Thus, the duration from Point A to Point C is the expected in-control time period. After Point C, the process is out-of-control till the end of the cycle. However, the operator is unaware of this condition until an out-of-control signal is indicated on the control chart. As the assignable cause occurs at a rate of λ occurrences per hour as per Poisson distribution, the time interval for which the process remains in-control is an exponential random variable with mean of $1/\lambda$ hour. Thus,

$$\text{Expected in-control time period} = 1/\lambda \qquad (3.5)$$

The assignable cause may occur any time between two consecutive samples. Let it occurs between *j*th and (*j+1*)st samples and after a time interval of τ from *j*th sample (i.e., Point B). Thus, τ represents the fraction of time between these two consecutive samples when the process remains under control. In other words, the in-control period includes the time τ. If the samples are taken at a regular interval of *h* hour, then the expected time of occurrence of the process shift within the interval between *j*th and (*j+1*)st samples (i.e., the expected in-control time within this particular sampling interval) can be mathematically expressed as:

$$\tau = \frac{\int_{jh}^{(j+1)h} e^{-\lambda t} \lambda (t - jh) dt}{\int_{jh}^{(j+1)h} e^{-\lambda t} \lambda dt} = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})}$$

$$= \frac{h}{2} - \frac{\lambda h^2}{12} \text{ plus terms of order } \lambda^3 h^4 \text{ or higher}$$

$$\approx \frac{h}{2} - \frac{\lambda h^2}{12} \tag{3.6}$$

ii) Expected time to signal (CD)

The expected time to signal is the average time until a sample containing the information about out-of-control signal is drawn. In ideal case, this should be the sample immediately after the process has shifted i.e., *(j+1)*st sample. But depending upon the efficiency of control chart in detecting the shift, this usually occurs after a number of samples equal to out-of-control average run length ARL_1. The sample carrying the out-of-control signal should come out as quickly as possible after the shift, otherwise there is a risk of producing unnecessarily more number of non-conforming items resulting in loss of quality as well as productivity.

Expected time to signal = CD = BD – BC = $(ARL_1)h - \tau$

$$= \left(\frac{1}{1-\beta}\right)h - \tau$$

$$= \left(\frac{h}{1-\beta} - \frac{h}{2} + \frac{\lambda h^2}{12}\right)$$

So, expected time to signal = $h\left(\dfrac{1}{1-\beta}-\dfrac{1}{2}+\dfrac{\lambda h}{12}\right)$ (3.7)

iii) The expected time to sample and interpret the result (DE)

Although the sample carrying out-of-control signal is drawn at Point D, the operator cannot know until the quality characteristic X of all the items in the sample are measured and recorded, the value of sample statistic $\bar{X}$ is calculated, and then this value is plotted on $\bar{X}$ chart. All these activities consume some time which is directly proportional to the number of items in a sample called sample size n. If g is the expected time required to take a sample of size 1, the total time for a sample of size n will be gn. Thus,

Expected time to sample and interpret the result = gn (3.8)

In many cases, g may be so small that it can be neglected. However, g can be significantly large when the inspection methodology is complex and time consuming like in a test rig. Therefore, this component of time has not been ignored while developing all the economic models in the entire thesis.

iv) Expected time to search the assignable cause (EF)

After the control chart triggers a signal that process has gone out-of-control, there is always a requirement to identify all possible reasons for the occurrence of assignable cause before taking up any remedial action. This search process consumes some time which is a constant term independent of three design parameters n, h and k. Here, it is assumed as:

Expected time to search for an assignable cause = T_1 (3.9)

v) Expected time to repair an assignable cause (FG)

After identification of assignable cause, actions are taken to eliminate it so as to bring back the process from out-of-control state to the in-control state. Similar to T_1, this action also consumes some time and does not depend on any of the three design parameters. Here, it is assumed as:

Expected time to repair the assignable cause = T_2 (3.10)

After finding all the above five time components, they are added up to calculate the expected cycle time for the continuous process as:

$$E(T) = \frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn + T_1 + T_2 \tag{3.11}$$

After finding the expected cycle time *E(T)*, in the next step all possible income and expenditure components related with these time intervals are considered. Items are produced during both in-control and out-of-control periods. But due to production of non-conforming items during out-of-control period, the rate of earning revenue drops down. The net income is the revenue earned on selling the items minus the cost of producing them. So, there will be two different rates of net incomes during in-control and out-of-control periods. In addition to the cost of production considered while finding out net income, there are many other costs related with sampling, inspection, control chart and process repair. All the above income and cost components are explained below:

i) ***Expected net income when the process is in-control***

As explained earlier, the expected time over which the process remains in-control is $1/\lambda$ hour. Let V_0 be the net income per hour while the process is in-control. Then,

$$\text{Expected net income during in-control state per cycle} = V_0/\lambda \tag{3.12}$$

ii) ***Expected net income when the process is in out-of-control state***

As shown in Fig. 3.1, the expected length of time the process remains out-of-control = CG = CD+DE+EF+FG $= h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn + T_1 + T_2$. Like V_0, let the net income per hour while the process is out-of-control be V_1. Multiplying both,

Expected net income during out-of-control state per cycle

$$= V_1\left[h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn + T_1 + T_2\right] \tag{3.13}$$

iii) ***Expected cost of sampling***

There are two components of cost associated with sampling i.e., fixed and variable. The variable cost is directly proportional to the sample size, whereas fixed cost is not. Let a be the fixed cost per sample and b be the variable cost per one item sampled. If the sample size is n, the variable cost per sample will be bn. Adding both the components, the expected sampling cost per sample $= a + bn$. The expected cost of sampling per cycle is the product of expected sampling cost per sample and the expected number of samples in a production cycle. Thus,

$$\text{Expected number of samples per cycle} = \frac{\text{expected cycle time}}{\text{time for one sample}} = \frac{E(T)}{h}$$

$$\text{So, expected cost of sampling per cycle} = (a+bn)\frac{E(T)}{h}$$

Substituting the value of *E(T)* from Eq. 3.11,

Expected cost of sampling per cycle

$$= \frac{(a+bn)}{h}\left[\frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn + T_1 + T_2\right] \tag{3.14}$$

iv) *Expected cost of false alarm*

When a point falls out of control limits on a control chart, the search for assignable cause begins. If the cause is not found out, it is concluded that the process is still running under control and the signal is a false one. The time and effort spent on searching for the assignable cause become unnecessary and the associated cost is called cost of false alarm. The expected cost of false alarm in a cycle is the product of expected number of false alarms per cycle and the cost of one false alarm. The expected number of false alarms during a cycle is the product of rate of generation of false alarm per sample (i.e., α error) and the expected number of samples taken during the fraction of time the process is in-control in a production cycle. Since, the process shifts due to occurrence of assignable cause as per Poisson distribution, the number of samples taken before the process shift is a random variable and this may vary from 0 to ∞. Thus, the expected number of samples before the process shifts in a cycle can be calculated as:

$$s = \sum_{j=0}^{\infty} j \times \text{Probability that assignable cause occurs between } j\text{th and } (j+1)\text{st samples}$$

$$= \sum_{j=0}^{\infty} j\left[e^{-\lambda hj} - e^{-\lambda h(j+1)}\right]$$

$$= (1-e^{-\lambda h})\sum_{j=0}^{\infty} je^{-\lambda hj} = \frac{e^{-\lambda h}}{(1-e^{-\lambda h})}$$

$$\approx \frac{1}{\lambda h}$$

Thus, expected number of false alarms per cycle is

$$\alpha s = \alpha\left(\frac{1}{\lambda h}\right) = \frac{\alpha}{\lambda h} \tag{3.15}$$

If Y is the cost per false alarm,

Expected cost of false alarm per cycle = $\dfrac{\alpha Y}{\lambda h}$ (3.16)

v) ***Expected cost of search and repair***

If the assignable cause is found out after an out-of-control signal is indicated by the control chart, the signal is true and it is confirmed that the process has really gone out-of-control. In such case, repair activity is carried out till the process is brought back to in-control state. Let W be the expected cost of repair including the search for assignable cause. This cost is constant as it is no way related with design variables n, h and k.

Adding the two types of income and then subtracting the three cost components from them, the expected net income for a continuous process in one production cycle can be expressed as:

$$E(C) = \frac{V_0}{\lambda} + V_1\left[h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn + T_1 + T_2\right]$$
$$- \frac{(a+bn)}{h}\left[\frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn + T_1 + T_2\right] - \frac{\alpha Y}{\lambda h} - W \tag{3.17}$$

The production process is a series of cycles each consisting of in-control and out-of-control states. Applying the renewal reward theorem (Ross, 1972), the expected net income per hour *E(A)* can be written as the ratio of the expected net income for one cycle *E(C)* to the expected time for one cycle *E(T)*. This can be mathematically expressed as:

$$E(A) = \frac{E(C)}{E(T)}$$

Substituting the values of *E(C)* and *E(T)*,

$$E(A)=\frac{\frac{V_0}{\lambda}+V_1\left[h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn+T_1+T_2\right] -\frac{(a+bn)}{h}\left[\frac{1}{\lambda}+h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn+T_1+T_2\right]-\frac{\alpha Y}{\lambda h}-W}{\frac{1}{\lambda}+h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn+T_1+T_2} \quad (3.18)$$

Because of the involvement of various costs, the value of *E(A)* is always less than the maximum possible net income i.e., V_0. This loss of income due to costs is defined as expected loss cost per unit time and if its expected value for a continuous process is $E(L)_1$, then

$$E(L)_1=V_0-E(A)$$

Substituting the value of E(A),

$$E(L)_1=V_0-\left[\frac{\frac{V_0}{\lambda}+V_1\left[h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn+T_1+T_2\right] -\frac{(a+bn)}{h}\left[\frac{1}{\lambda}+h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn+T_1+T_2\right]-\frac{\alpha Y}{\lambda h}-W}{\frac{1}{\lambda}+h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn+T_1+T_2}\right] \quad (3.19)$$

The above equation appears to be complex. So, rewriting the above equation with the introduction of three terms *M*, *C* and *B*, it gets simplified to

$$E(L)_1=\frac{MC+\frac{(a+bn)}{h}\left[\frac{1}{\lambda}+C\right]+\frac{\alpha Y}{\lambda h}+W}{\frac{1}{\lambda}+C} \quad (3.20)$$

where

$$M=V_0-V_1$$

$$C=hB+gn+T_1+T_2$$

$$B=\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)$$

The expression $E(L)_1$ represents the expected value of loss cost per hour incurred by the continuous process. It is a function of three control chart parameters *n*, *h*, and *k*. Since V_0 is constant, maximizing the expected net income per hour *E(A)* is equivalent to minimizing $E(L)_1$.

3.3.2 Economic Model for Discontinuous Process

This model is based on the assumption that the process is shut down once there is an out-of-control signal obtained from the control chart followed by the search for the probable assignable cause responsible for the process shift. If no assignable cause is found out, the process is immediately restarted without the necessity of any repair work. This is a false signal case. But in case of true signal, some assignable cause would be responsible for the shift and the process is not allowed to run till the successful repair of the process is over. Thus, this type of process does not operate continuously for all the time. Therefore, it is termed as discontinuous process. One production cycle of such process is shown in Fig. 3.2.

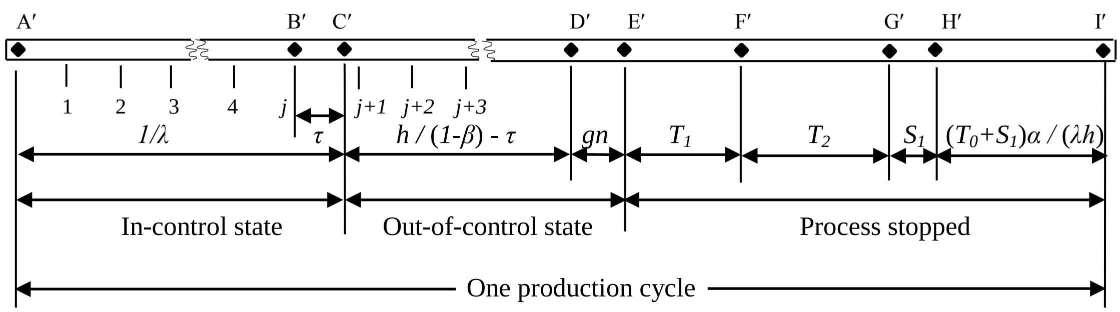

Fig. 3.2: Cycle time for discontinuous process

Various cardinal points of one production cycle are shown below in Fig. 3.2. Each of these points is marked with prime (′) notation as superscript so as to differentiate it from that of continuous process and is explained below:

Point **A′**: Cycle starts

Point **B′**: Last sample just before the assignable cause occurs (i.e., *j*th sample)

Point **C′**: Assignable cause occurs (i.e., unknown to operator till Point E′)

Point **D′:** Sample that contains out-of-control signal

Point **E′:** Out-of-control signal detected and the process is stopped

Point **F′**: Assignable cause found

Point **G′**: Assignable cause eliminated

Point **H′:** Out-of-control process restored to in-control state after restart

Point **I′:** End of one production cycle including stoppage time due to false alarms.

All points in the discontinuous process up to G′ are same as that of continuous process from Point A to Point G. The last two points i.e., H′ and I′ are the additional points compared to continuous process. The process which was discontinued earlier after obtaining an out-of-control signal resumes working at either of these two points. If the process has really gone out-of-control, it resumes working at Point H′ after the completion of repair provided there was no false alarm earlier in the current production cycle. On the other hand if one or more false alarms exist in the current production cycle, the start of next production cycle will be delayed up to Point I′. This delay period is equal to the sum of all the time components for which process was stopped due to those false alarms in that cycle. As shown in Fig. 3.2, one production cycle of a discontinuous process consists of the following seven components out of which the first five are same as that already explained in case of continuous process in Section 3.3.1.

i) ***Expected in-control period (A'C')*** $= 1/\lambda$

ii) ***Expected time to signal (C'D)'*** $= h\left(\frac{1}{P} - \frac{1}{2} + \frac{\lambda h}{12}\right)$

iii) ***Expected time to sample and interpret the result (D'E')*** $= gn$

iv) ***Expected time to search the assignable cause (E'F')*** $= T_1$

v) ***Expected time to repair an assignable cause (F'G')*** $= T_2$

vi) ***Expected time to restart the process (G'H')***

Since the discontinuous process is stopped during the search and elimination of assignable cause, it requires to be restarted after the repair of the process is over. This is also called as set up time. This includes activities like machine set up, job reloading etc. All these activities consume some time say S_1. Thus,

Expected time to restart the process = S_1 (3.21)

vii) ***Expected search time for false alarms and restarting the process (H'I')***

The last portion H′I′ is actually the sum of a number of small intervals before the occurrence of an assignable cause (i.e., during the period A′C′). For each of these small intervals, the process is stopped subsequent to a false alarm during which the search for assignable cause is done. But in none of these cases, any assignable cause is found out which concludes that the signal is a false one. So, there is no need of any repair work and the process is restarted immediately. For the convenience and simplicity, all these small intervals have been grouped together and shown at the end portion of the production cycle. Let T_0 be the expected search time for one false alarm and S_1 be the time to restart the process due to any undue stoppage after each false alarm. Then, the expected time to search for false alarms and restarting the process will be the product of the expected number of false alarms per cycle and the sum of the expected search time for a false alarm and the restart time. Since the expected number of false alarms per cycle is $\frac{\alpha}{\lambda h}$ as mentioned in Eq. 3.15

Expected time to search for false alarms and restarting the process as

$$= (T_0 + S_1)\frac{\alpha}{\lambda h} \tag{3.22}$$

After finding all the above seven time components, they are added to calculate the expected cycle time for the discontinuous process as

$$E(T) = \frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn + T_1 + T_2 + S_1 + (T_0 + S_1)\frac{\alpha}{\lambda h} \tag{3.23}$$

After finding the expected cycle time *E(T)*, the next task is to calculate the expected net income per cycle *E(C)* which is the sum of all incomes minus costs associated with the process quality control as explained below:

i) ***Expected net income during in-control state per cycle*** = V_0 / λ

This is same as that of continuous process.

ii) ***Expected net income when the process is in out-of-control state***

As shown in Fig. 3.2, the expected length of time the process runs in out-of-control state = C′E′ = C′D′+D′E′ $= h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn$. Let the net income per hour while the process is out-of-control be V_1. The process is stopped after the Point E′, i.e., when an out-of-control signal is detected on the control chart. Thus,

Expected net income during out-of-control state per cycle

$$= V_1\left[h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn\right] \tag{3.24}$$

iii) Expected cost of sampling

As discussed in continuous process, the expected cost of sampling is the product of expected sampling cost per sample and the expected number of samples taken in a cycle. Also, the expected cost per sample = (a + bn) where a and b are fixed and variable costs respectively. In discontinuous process, the samples are taken only during the period when the process is running which is less than the cycle time, whereas in continuous process samples are taken throughout the cycle time.

Thus, the expected number of samples per cycle = $\dfrac{\text{Production time per cycle}}{\text{Sampling interval}}$

$$\text{Since, production time per cycle} = \frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn$$

Expected cost of sampling

$$= \frac{(a+bn)}{h}\left[\frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn\right] \quad (3.25)$$

iv) Expected cost of search and repair $= W$ (3.26)

This cost is same as that in continuous process.

v) Expected cost of restart or setup cost = S (3.27)

This cost is not necessary in continuous process.

vi) Expected cost of false alarm $= \dfrac{\alpha Y}{\lambda h}$

This cost is explained earlier in continuous process.

Adding the two types of income and then subtracting the four cost components from them, the expected net income for a discontinuous process in one production cycle can be expressed as:

$$E(C) = \frac{V_0}{\lambda} + V_1\left[h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn\right] - \frac{(a+bn)}{h}\left[\frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}\right) + gn\right] - W - S - \frac{\alpha Y}{\lambda h} \quad (3.28)$$

Substituting the values of *E(T)* and *E(C)*, the expected net income per hour $E(A)$ for the discontinuous process can be written as:

$$E(A)=\frac{\frac{V_0}{\lambda}+V_1\left[h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn\right]-\frac{(a+bn)}{h}\left[\frac{1}{\lambda}+h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn\right]-W-S-\frac{\alpha Y}{\lambda h}}{\frac{1}{\lambda}+h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn+T_1+T_2+S_1+(T_0+S_1)\frac{\alpha}{\lambda h}} \quad (3.29)$$

$$=V_0-E(L)_2$$

where

E(L)$_2$ = expected value of loss cost per hour for the discontinuous process.

Hence,

$$E(L)_2=V_0-E(A)$$

Substituting the value of E(A),

$$E(L)_2=V_0-\left[\frac{\frac{V_0}{\lambda}+V_1\left[h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn\right]-\frac{(a+bn)}{h}\left[\frac{1}{\lambda}+h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn\right]-W-S-\frac{\alpha Y}{\lambda h}}{\frac{1}{\lambda}+h\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)+gn+T_1+T_2+S_1+(T_0+S_1)\frac{\alpha}{\lambda h}}\right] \quad (3.30)$$

Rewriting the above equation with the help of four terms *M*, *B*, *C* and *D*, it gets simplified to

$$E(L)_2=\frac{MC+V_0D+\frac{(a+bn)}{h}\left[\frac{1}{\lambda}+C\right]+W+S+\frac{\alpha Y}{\lambda h}}{\frac{1}{\lambda}+C+D} \quad (3.31)$$

where

$$M=V_0-V_1$$

$$B=\left(\frac{1}{1-\beta}-\frac{1}{2}+\frac{\lambda h}{12}\right)$$

$$C=hB+gn$$

$$D = T_1 + T_2 + S_1 + (T_0 + S_1)\frac{\alpha}{\lambda h}$$

Maximizing the expected net income per hour *E(A)* is equivalent to minimizing the expected loss cost per unit time. Minimizing the expected loss cost per unit time $E(L)_1$ and $E(L)_2$ whose expressions are mentioned in Eqs. 3.20 and 3.31, provides optimum solutions to the economic design of $\bar{X}$ chart for continuous and discontinuous processes respectively.

3.4 Metaheuristic Based Economic Design

The expressions for the expected loss cost per unit time in both types of processes (i.e., $E(L)_1$ and $E(L)_2$) shown in Eqs. 3.20 and 3.31 respectively are functions of only three design variables *n*, *h* and *k* of $\bar{X}$ chart. For the economic design, the values of *n*, *h* and *k* should be so selected that the expected loss cost per unit time is minimum. Thus, it represents an unconstrained multi-variable optimization problem with an objective to minimize the expected loss cost per unit time. The effectiveness of economic design depends on the calculation accuracy of the design variables while minimizing the loss cost function. Different optimization techniques have been earlier tried for its minimization in case of $\bar{X}$ chart as mentioned in Chapter 2. But, as the expressions of $E(L)_1$ and $E(L)_2$ are non-linear and highly complex, the traditional optimization methods have not been effective in providing accurate results. There are many non-traditional optimization techniques or metaheuristics available these days which have not been tried so far for the economic design of $\bar{X}$ chart out of which two techniques, namely simulated annealing and teaching-learning based optimization have been selected in this work and these two are explained below.

3.4.1 Simulated Annealing

Among all the metaheuristics, simulated annealing (SA) is the most robust and effective one. It is a probabilistic random search method proposed by Kirpatrick et al. (1983) that mimics the annealing process wherein a metal is first heated to a high temperature and then very slowly cooled down so as to minimize its free energy. At high temperature, the atoms in the molten metal have more free energy and therefore they can move freely with respect to each other. But as the temperature goes on reducing, the movement of these free moving atoms gets restricted. The atoms start getting arranged and finally form crystals having the minimum possible energy which depend on the cooling rate. If the temperature reduces at a very fast rate, the crystalline state may not be achieved at all and instead the system may end up in a polycrystalline state, which may have a higher energy state than the crystalline state. Therefore, in order to achieve the absolute minimum energy state, the temperature needs to be reduced at a very slow rate.

It is often the most widely used probabilistic metaheuristic for the global optimization. It is commonly used for minimization problems. It allows for uphill movements in order to prevent the algorithm from getting trapped within local minima. In gradient-based minimization algorithms, only downhill moves are allowed. However, this algorithm takes not only downhill moves, but also permits uphill moves with an assigned probability depending on the state temperature. The probability of accepting uphill move is solved by Metropolis algorithm which is based on Boltzmann probability distribution. This helps the solution point to escape from the traps of local minima as illustrated in Fig. 3.3. With minor modification, this algorithm can also be used to search for global maximum solution. Thus, simulated annealing is often most suitable for the optimization problem where the desired global optimal solution is hidden among many local optima.

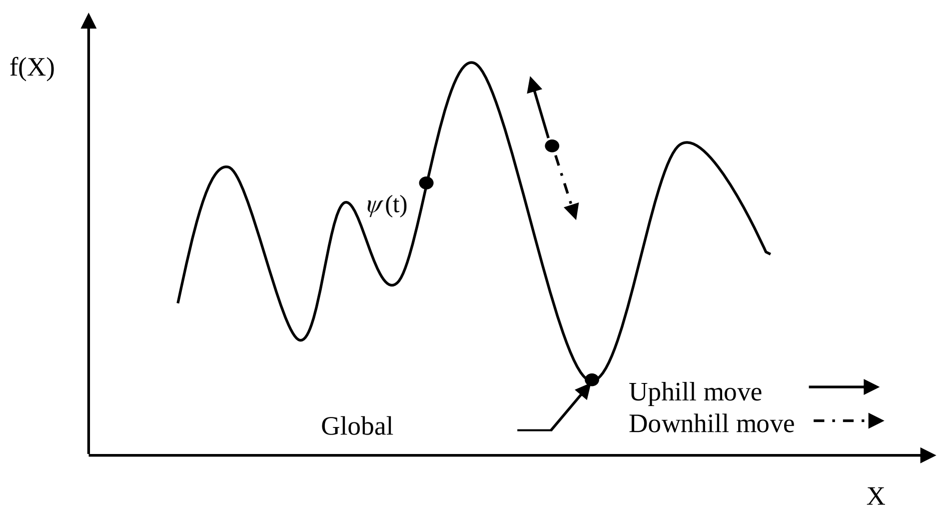

Fig. 3.3: Simulated annealing strategy in a minimization problem

Although it has been used for solving a wide variety of optimization problems, it has not been tried so far for minimization of loss cost function in economic design of $\bar{X}$ chart. Therefore, SA method has been applied in the present work to optimize economic as well as economic statistical design of $\bar{X}$ chart. To facilitate this, a MATLAB computer program has been written. Further, its working has been illustrated by solving a numerical problem of Panagos et al. (1985) and the results obtained are compared.

The steps of SA algorithm are given below:

Step 1: Initialization:

Choose an initial point $\Psi(0)$, a termination criterion ε, termination temperature T_{min}, temperature reduction factor ρ, number of iterations n to be performed at a particular temperature. Set $t = 0$ and initial temperature T at a sufficiently high value.

Step 2: Randomly choose a neighboring point $\Psi(t + 1)$ around the current point $\Psi(t)$ as per normal distribution.

Step 3: Calculate $\Delta E = E[\Psi(t+1))] - E[\Psi(t)]$

If $\Delta E < 0$

set $t = t+1$

Else create a random number r in the range (0, 1)

If $r \leq e^{\frac{-\Delta E}{T}}$

set $t = t+1$

Else go to Step 2

Step 4: If $|\Psi(t+1) - \Psi(t)| < \varepsilon$ and $T < T_{min}$

Terminate

Else If (t mod n) = 0

$T = T/\rho$ and go to Step 2

Else go to Step 2.

Prior to designing the control chart, it is to be noted that the sample size (n) must be a positive integer, whereas the other two design parameters i.e., sampling interval (h) and width of the control chart limits (k) may be taken as real values on continuous scale. Therefore, each time the economic design is done for a particular integer value of n. Thus, the three variable optimization problem is simplified to two variable problem as n is assumed to be constant in each design.

SA being a random search technique, initially the feasible search space for each of the design variables is to be assumed. The lower and upper boundary limits of design variables for economic design of $\bar{X}$ chart are selected as shown in Table 3.1 (Kasarapu and Vommi, 2011).

Table 3.1: Boundary limits of control chart design variables

Design variables	*Boundary limits*
n	2 - 33
h	0.25 - 12.00
k	1.0 – 6.00

The following values are taken for the parameters related to the algorithm of SA in the present work.

i) Termination criterion = ε = 0.001

ii) Iteration counter = t = 300

iii) Temperature reduction factor = ρ = 0.95

The initial temperature T is taken as the average of function values calculated at the extreme corner points of the search space (Deb, 2012). The convergence time is dependent on the values of the parameters T, ε and ρ. T must be sufficiently large for any point within the search space to have a reasonable chance of being visited, but if it is too large then too much of time is spent in the 'molten' state. Increasing ρ increases the reliability of the algorithm in reaching the global optimum, and it corresponds to a slower cooling of the system. A small value of ε gives an accurate solution, but at the expense of convergence time. It is required to run the algorithm for a reasonable number of times, in order to check its consistency in providing the best solution. Accordingly, SA algorithm has been run for 30 times for each set of input data and the best result is accepted (Kuo et al., 2001).

3.4.2 Teaching-Learning Based Optimization

In order to validate the results obtained through SA, the same economic designs have been made with the help of another optimization technique. Teaching-learning based optimization (TLBO) is one of the recently proposed meta-heuristics which are observed to have been popularly used for solving wide variety of industrial optimization problems. But so far no researchers have tested this algorithm in economic design. Therefore, the same numerical problems related to both continuous and discontinuous processes have been solved using this new technique and the results are compared with that obtained using SA. For this, a MATLAB program based on TLBO has been developed for finding out the optimum values of design parameters necessary for the economic design of $\bar{X}$ chart for both continuous and discontinuous processes.

TLBO is a population based algorithm. It is one type of nature inspired optimization techniques that mimic the classroom teaching phenomena for finding the global optimal solution (Rao et al., 2011; 2012). In this algorithm a group of learners is considered as population and different subjects taught to the learners are considered as different design variables of the optimization problem. A learner's overall result is analogous to the value of the objective function. The working of TLBO comprises of two phases, namely teacher phase and learner phase. In teacher phase, the learners learn from the teacher, whereas in the second phase they learn through interaction among themselves. To test the optimization technique a computer program has been developed in MATLAB language based on TLBO algorithm and the results are compared with that obtained using SA.

The steps of TLBO algorithm are given below:

Step 1: Select the number of learners K (i.e., population size), number of subjects J taught to the learners (i.e., design variables) and maximum number of iterations I. Set iteration counter $i = 1$.

Step 2: Generate a random population of results X_{ijk} for all learners ($k = 1, 2, 3,... K$) in each of the subjects ($j = 1, 2, 3,.... J$) at iteration i and calculate the corresponding value of objective function f for each learner.

Step 3: Calculate the mean result of the class of K learners in a particular subject j ($j = 1, 2, 3,.... J$) in iteration i as

$$M_{ij} = \frac{1}{K}\sum_{k=1}^{K} X_{ijk} \qquad (3.32)$$

where X_{ijk} = result of learner k in subject j in iteration i.

Step 4: Out of all the learners (i.e., $k = 1, 2, 3,... K$), there will be one topper securing the best overall result taking all the subjects into consideration and let him be k-best. For example, in case of minimization problem for the k-best solution (i.e., the best learner or topper) the value of objective function would be the lowest.

Step 5: The stochastic difference between the existing mean result of the class (i.e., population) for each subject j and the corresponding result of the best learner k-best in the same subject j at any iteration i is given by,

$$dm_{ij} = r_i \left(X_{ijk\text{-}best} - T_F M_{ij} \right) \tag{3.33}$$

where,

r_i = uniformly distributed random number in the range [0, 1]

$X_{ijk\text{-}best}$ = result of the best learner *k*-best in the subject *j* at any iteration *i*

T_F = Teaching factor which decides the value of mean to be changed and this value varies between 1 or 2.

Step 6: Since there are a total of J subjects, there will be J different values of dm_{ij} at any iteration *i*. For each subject *j* (*j* = 1, 2, 3,…., J) the results of all the learners (*k* = 1, 2, 3,…, K) are updated by adding the value of dm_{ij} as expressed below:

$$X'_{ijk} = X_{ijk} + dm_{ij} \tag{3.34}$$

Evaluate objective function at the updated value and let it be f' =f(X'_{ijk})

If f' gives better result

Accept X'_{ijk}

Else

Retain X_{ijk} as X'_{ijk}

Step 7: Randomly select two learners $k1$ and $k2$ such that $f'_{k1} \neq f'_{k2}$

Step 8: If f'_{k1} is better than f'_{k2}

$$X''_{ijk1} = X'_{ijk1} + r_i \left(X'_{ijk1} - X'_{ijk2} \right) \tag{3.35}$$

Else

$$X''_{ijk1} = X'_{ijk1} + r_i \left(X'_{ijk2} - X'_{ijk1} \right) \tag{3.36}$$

Step 9: Evaluate objective function at the updated value and let it be f'' = f(X''_{ijk})

If f'' is better

Accept X''_{ijk}

Else

Retain X'_{ijk} as X''_{ijk}

Step 10: If $i \geq I$

Terminate and X''_{ijk} is solution

Else

$i = i+1$, $X_{ijk} = X''_{ijk}$, go to Step 3

This new algorithm is gaining more popularity because unlike other evolutionary algorithms, it does not require any algorithm specific parameters. Only population size and number of generations need to be specified (Pawar and Rao, 2013). Other algorithms require proper tuning of algorithm specific parameters in addition to the tuning of common controlling parameters. The effectiveness of such algorithms very much depends on the correctness of tuning. In this thesis, the following values are used while using TLBO:

i) Number of learners (i.e., population size) $K = 150$

ii) Number of subjects (i.e., number of design variables) $J = 2$

iii) Number of iterations $I = 500$

Further, the value of teaching factor T_F is usually taken between 1 and 2 randomly during the optimization process (Rao and Patel, 2013). But in the present work, this aspect has been simplified by taking a constant value for the teaching factor T_F (i.e., 1) throughout the process. The limits of design variables *n*, *h* and *k* are taken same as that were considered in SA as shown in Table 3.1.

3.5 Numerical Illustration: Continuous Process

In order to illustrate the working of optimization methodology, a numerical problem dealing with continuous process has been taken from Panagos et al., (1985) where the cost and process data are as listed in Table 3.2 and has been solved using simulated annealing.

Table 3.2: Cost and process data: continuous process

S. No.	***Cost and process parameter***	***Notation***	***Unit***	***Value***
1	Loss of income when process is out-of-control	M	$	100
2	Shift in process mean	δ	-	1
3	Rate of occurrence of assignable cause	λ	per hour	0.05
4	Time to sample and chart one item	g	hour	0.05
5	Time to find and repair an assignable cause	T_1+T_2	hour	3
6	Fixed cost per sample	a	$	0.5
7	Variable cost per sample	b	$	1.0
8	Cost to locate and repair the assignable cause	W	$	250
9	Cost per false alarm	Y	$	50

3.5.1 Results and Discussion: SA

Table 3.3 shows the results of economic design of $\bar{X}$ chart for a continuous process using SA i.e., the optimal values of two design variables of control chart such as sampling interval (*h*) and width of the control limits (*k*) for each integer value of sample size *n* varying from 2 to 33. It also shows the corresponding optimum values of Type-I error (α), Type-II error (β), power of detecting the shift (*P*), average in-control run length (ARL_0), average out-of-control run length (ARL_1), and finally the expected loss cost per unit time ($E(L)_1$).

Table 3.3: Optimal economic designs of $\bar{X}$ chart using SA: continuous process

n	*h*	*k*	α	β	*P*	ARL_0	ARL_1	$E(L)_1$
2	1.05	1.70	0.0891	0.612	0.388	11.219	2.574	36.521
3	1.32	1.72	0.0854	0.495	0.505	11.705	1.980	35.538
4	1.57	1.74	0.0819	0.397	0.603	12.215	1.659	35.034
5	1.80	1.76	0.0784	0.317	0.683	12.753	1.464	34.795
6	**2.00**	**1.79**	**0.0735**	**0.255**	**0.745**	**13.613**	**1.342**	**34.720**
7	2.18	1.83	0.0673	0.207	0.793	14.868	1.262	34.757
8	2.32	1.88	0.0601	0.172	0.829	16.633	1.207	34.872
9	2.51	1.91	0.0561	0.138	0.862	17.810	1.160	35.044
10	2.65	1.95	0.0512	0.113	0.887	19.534	1.127	35.259
11	2.80	2.00	0.0455	0.094	0.906	21.970	1.104	35.507
12	2.94	2.04	0.0414	0.077	0.923	24.173	1.084	35.779
13	3.06	2.08	0.0375	0.064	0.936	26.635	1.068	36.069
14	3.19	2.13	0.0332	0.033	0.947	30.128	1.057	36.374
15	3.31	2.17	0.0300	0.044	0.956	33.302	1.046	36.689
16	3.42	2.21	0.0271	0.037	0.963	36.864	1.038	37.011
17	3.54	2.26	0.0238	0.031	0.969	41.941	1.032	37.339
18	3.69	2.30	0.0215	0.026	0.974	46.576	1.027	37.672
19	3.76	2.34	0.0193	0.022	0.978	51.799	1.022	38.005
20	3.86	2.39	0.0169	0.019	0.981	59.277	1.019	38.340
21	3.97	2.43	0.0151	0.016	0.984	66.138	1.016	38.675
22	4.06	2.47	0.0135	0.013	0.987	73.900	1.013	39.010
23	4.20	2.51	0.0121	0.011	0.989	82.692	1.011	39.343
24	4.28	2.55	0.0108	0.009	0.991	92.665	1.010	39.675
25	4.38	2.59	0.0096	0.008	0.992	103.992	1.008	40.004
26	4.49	2.64	0.0083	0.007	0.993	120.364	1.007	40.331
27	4.57	2.67	0.0076	0.006	0.994	131.543	1.006	40.656
28	4.66	2.70	0.0070	0.005	0.995	143.879	1.005	40.977
29	4.76	2.75	0.0060	0.004	0.996	167.370	1.004	41.296
30	4.84	2.76	0.0058	0.003	0.997	172.558	1.003	41.612
31	4.95	2.83	0.0047	0.003	0.997	214.210	1.003	41.924
32	5.04	2.86	0.0042	0.003	0.997	235.334	1.003	42.233
33	5.14	2.90	0.0037	0.002	0.002	267.119	1.002	42.539

As shown in Table 3.3, the optimum values of loss cost function $E(L)_1$ decreases as *n* value increases from 2 to 6 and thereafter it increases at higher values of *n*. This trend is also graphically shown in Fig. 3.4.

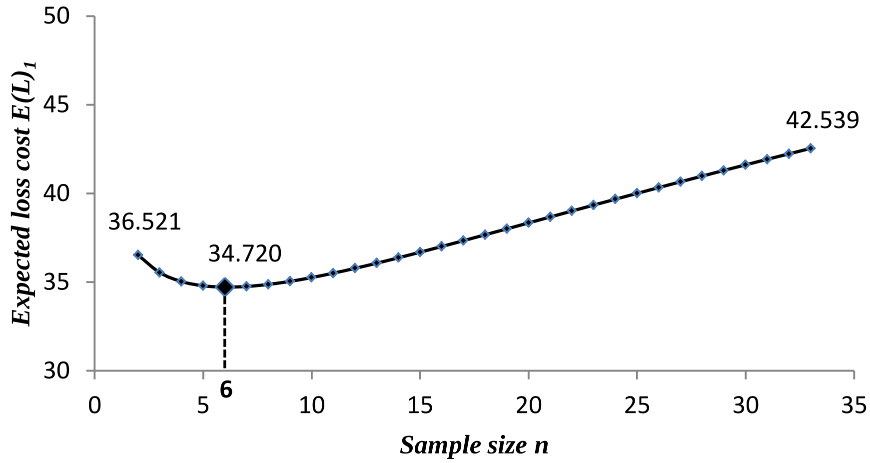

Fig. 3.4: Variation of expected loss cost per unit time with sample size using SA: continuous process

On comparing as many as 32 economic designs, one each for integer value of sample size *n* varying from 2 to 33, the global minimum loss cost is found to be $E(L)_1$ = 34.720 and this occurs at $n = 6$ as shown in Table 3.3. The corresponding values of *h* and *k* at minimum loss cost are 2.00 hour and 1.79 respectively. For the same numerical problem of continuous process, the economic design (i.e., the values of *n*, *h* and *k*) obtained by Panagos et al. (1985) is shown along with that obtained using SA in Table 3.4 for comparison purpose. Further, the value of expected loss cost per unit time $E(L)_1$ corresponding to the economic design suggested by Panagos et al. (1985) is calculated in this work up to 3 decimal places for comparison and it is found to be 35.0107 ≈ 35.011 as shown in the same table. It is observed that for both the cases, the sample size (*n*) is equal to 6 and width of the control limit (*k*) is nearly same. The difference is observed in the optimal values of sampling interval (*h*). In case of simulated annealing, the optimal value of expected loss cost per unit time (i.e., 34.720) is found to be lower than that obtained by Panagos et al. (1985) (i.e., 35.011). Thus, the economic design based on simulated annealing is found to be more effective.

Table 3.4: Comparison of results in continuous process

Methodology	***n***	***h***	***k***	$E(L)_1$
Panagos et al. (1985)	6	1.57	1.78	35.011
SA	6	2.00	1.79	34.720

3.5.2 Results and Discussion: TLBO

The same numerical problem solved using simulated annealing in Section 3.5.1, has been solved using TLBO in this section. The values of relevant data of this problem are already listed in Table 3.2. Similar to Table 3.3, the results of economic design of $\bar{X}$ chart for a continuous process using TLBO are shown in Table 3.5 for each integer value of sample size *n* in the range 2 to 33.

Table 3.5: Optimal economic designs of $\bar{X}$ chart using TLBO: continuous process

n	*h*	*k*	α	β	*P*	ARL_0	ARL_1	$E(L)_1$
2	1.05	1.71	0.0873	0.615	0.385	11.459	2.600	36.521
3	1.32	1.72	0.0854	0.495	0.505	11.705	1.980	35.538
4	1.57	1.74	0.0819	0.397	0.603	12.215	1.659	35.034
5	1.79	1.76	0.0784	0.317	0.683	12.753	1.464	34.795
6	**1.99**	**1.80**	**0.0719**	**0.258**	**0.742**	**13.914**	**1.348**	**34.720**
7	2.18	1.83	0.0673	0.207	0.793	14.868	1.262	34.757
8	2.35	1.87	0.0615	0.169	0.831	16.261	1.203	34.872
9	2.51	1.91	0.0561	0.138	0.862	17.810	1.160	35.044
10	2.66	1.95	0.0512	0.113	0.887	19.534	1.127	35.259
11	2.80	2.00	0.0455	0.094	0.906	21.970	1.104	35.507
12	2.93	2.04	0.0414	0.077	0.923	24.173	1.084	35.779
13	3.06	2.08	0.0375	0.064	0.936	26.635	1.068	36.069
14	3.19	2.13	0.0332	0.054	0.946	30.128	1.057	36.374
15	3.31	2.17	0.0300	0.044	0.956	33.302	1.046	36.688
16	3.43	2.21	0.0271	0.037	0.963	36.864	1.038	37.011
17	3.54	2.25	0.0245	0.031	0.969	40.865	1.032	37.339
18	3.65	2.3	0.0215	0.026	0.974	46.576	1.027	37.671
19	3.76	2.34	0.0193	0.022	0.978	51.799	1.022	38.005
20	3.87	2.38	0.0173	0.018	0.982	57.689	1.019	38.340
21	3.97	2.42	0.0155	0.015	0.985	64.343	1.016	38.675
22	4.08	2.47	0.0135	0.013	0.987	73.900	1.013	39.010
23	4.18	2.51	0.0121	0.011	0.989	82.692	1.011	39.343
24	4.28	2.55	0.0108	0.009	0.991	92.665	1.010	39.675
25	4.38	2.59	0.0096	0.008	0.992	103.990	1.008	40.004
26	4.48	2.63	0.0086	0.007	0.993	116.870	1.007	40.331
27	4.57	2.67	0.0076	0.006	0.994	131.540	1.006	40.656
28	4.67	2.71	0.0067	0.005	0.995	148.270	1.005	40.977
29	4.76	2.75	0.0060	0.004	0.996	167.370	1.004	41.296
30	4.86	2.79	0.0053	0.004	0.996	189.210	1.004	41.611
31	4.95	2.83	0.0047	0.003	0.997	214.210	1.003	41.924
32	5.04	2.86	0.0042	0.003	0.997	235.330	1.003	42.233
33	5.14	2.9	0.0037	0.002	0.998	267.120	1.002	42.539

On comparing all 32 economic designs, one each for integer value of sample size *n* varying from 2 to 33, the optimal expected loss cost per unit time is observed to be $E(L)_1$ = 34.720 and this occurs at *n* = 6 as shown in Table 3.5. Similar to the results of SA, here also the values of expected loss cost per unit time $E(L)_1$ decreases with the increase of *n* value from 2 to 6 and after that it increases at higher values of *n*. The corresponding values of *h* and

k at minimum loss cost are 1.99 hour and 1.80 respectively. This trend is also graphically illustrated in Fig. 3.5.

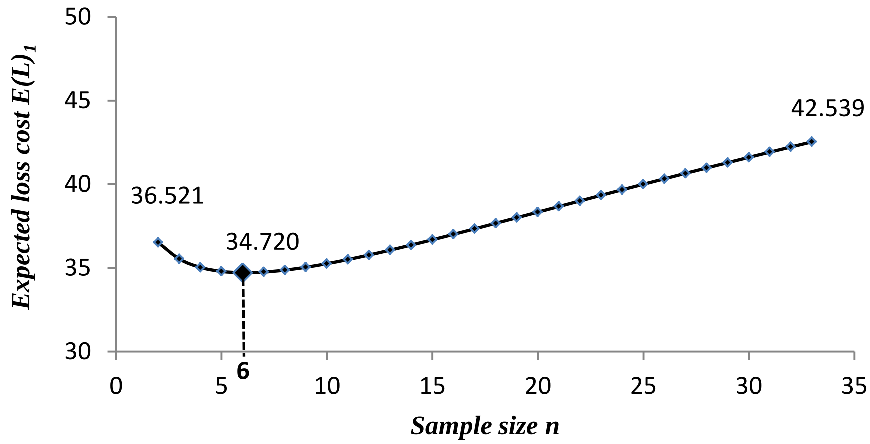

Fig. 3.5: Variation of expected loss cost per unit time with sample size using TLBO: continuous process

Table 3.6 shows the comparison of results of economic design of $\bar{X}$ chart for continuous process using TLBO with that of the results obtained from SA. It is observed that for both the cases, the sample size (n) is same, whereas the sampling interval (h) and control limit width (k) are nearly same. Both the methods give the same value (i.e., 34.720) for loss cost function $E(L)_1$.

Table 3.6: Comparison of results obtained from SA and TLBO in continuous process

Techniques	*n*	*h*	*k*	α	β	*P*	ARL_0	ARL_1	$E(L)_1$
SA	6	2.00	1.79	0.0735	0.255	0.745	13.613	1.342	34.720
TLBO	6	1.99	1.80	0.0719	0.258	0.742	13.914	1.348	34.720

It is observed that both the metaheuristics i.e., SA and TLBO are providing the same results for economic design of $\bar{X}$ chart for continuous process, and thus the results are validated and confirmed to be correct. Moreover, both are providing better results than that of the earlier reported by Panagos et al. (1985).

3.6 Sensitivity Analysis: Continuous Process

The results of economic design depend on the assumed values of cost and process parameters for a given manufacturing set up. These values vary from one set up to another. All factors may not significantly affect the economic design. Thus, the designer needs to identify the significant factors and accordingly take care to correctly estimate their values. Therefore, in this section a sensitivity analysis has been done to investigate the effect of process and cost parameters on the output results of economic design.

In this work, nine cost and process parameters are considered as factors as per the terminology of design of experiments for continuous process model. These factors are denoted with alphabets from A to J as shown in Table 3.7. Each factor has been considered at two levels. This table also shows the low and high values of these factors are taken from Panagos et al. (1985).

Table 3.7: Factor levels: continuous process

S. No.	***Factor***	***Unit***	***Low Level***	***High Level***
1	A = M	$	50.00	100.00
2	B = δ	-	1.00	2.00
3	C = λ	per hour	0.01	0.05
4	D = g	hour	0.05	0.50
5	E = (T_1+T_2)	hour	3.00	20.00
6	F = a	$	0.50	5.00
7	G = b	$	0.10	1.00
8	H = W	$	35.00	250.00
9	J = Y	$	50.00	500.00

Since each of these factors is present at two levels, 2^{k-p} fractional factorial design of resolution *IV* has been conducted to examine the effects of these factors on four output responses i.e., n, h, k and $E(L)_1$. The use of resolution *IV* design ensures that no main effects are aliased with each other, but two factor interactions are aliased with other two factor interactions. The experimental design based on resolution *IV* helps to estimate the main effects of each of the factors. A large number of additional experimental runs would have been required to separate the effects of two-factor interactions. The objective being to identify the significant factors, the study of two-factor interaction is not necessary. The details of 2^{k-p} design are given in Montgomery (2013). A 2_{IV}^{9-4} factorial design with 32 runs is chosen for the continuous process with four independent generators I = ABCF, I = ABDG, I = ABEH and I = ACDJ. For each of 32 runs, a particular set of cost and process parameters values decided as per the fractional factorial design $(=2^{9-4})$ is taken for which the loss cost function $E(L)_1$ is minimized using SA algorithm and the optimal result consisting of the values of n, h, k and $E(L)_1$ is shown in Table 3.8. Thus, this table presents 32 sets of results of economic design of $\bar{X}$ chart for a continuous process using SA. Since both SA and TLBO algorithms provided almost the same results for economic design in a continuous process as observed in Section 3.5, any one of them is sufficient for sensitivity analysis.

Table 3.8: Optimal economic designs of $\bar{X}$ chart: continuous process

S. No.	***Cost and process parameters (factors)***									***Panagos et al. (1985)***				***SA***			
	M	***δ***	***λ***	***g***	***(T_1+T_2)***	***a***	***b***	***W***	***Y***	***n***	***h***	***k***	***$E(L)_1$***	***n***	***h***	***k***	***$E(L)_1$***
1	50	1	0.01	0.05	3	0.5	0.1	35	50	13	2.50	2.65	3.77	13	2.66	2.65	3.769
2	100	1	0.01	0.05	3	5.0	1.0	250	500	5	6.04	2.71	13.25	15	6.50	2.72	13.215
3	50	2	0.01	0.05	3	5.0	1.0	250	50	4	5.95	2.39	7.06	4	6.43	2.38	7.048
4	100	2	0.01	0.05	3	0.5	0.1	35	500	6	1.40	3.67	5.21	6	1.46	3.67	5.210
5	50	1	0.05	0.05	3	5.0	0.1	35	500	19	2.29	3.07	15.34	19	2.90	3.06	15.174
6	100	1	0.05	0.05	3	0.5	1.0	250	50	6	1.57	1.78	35.02	6	1.99	1.79	34.720
7	50	2	0.05	0.05	3	0.5	1.0	250	500	4	1.78	3.06	22.4	4	2.49	3.06	22.157
8	100	2	0.05	0.05	3	5.0	0.1	35	50	5	1.48	2.67	22.24	5	1.77	2.67	22.132
9	50	1	0.01	0.50	3	0.5	1.0	35	500	8	4.65	2.61	8.61	8	5.26	2.60	8.579
10	100	1	0.01	0.50	3	5.0	0.1	250	50	5	3.00	1.81	13.07	5	3.41	1.79	13.051
11	50	2	0.01	0.50	3	5.0	0.1	250	500	5	4.49	3.20	7.49	5	4.96	3.20	7.483
12	100	2	0.01	0.50	3	0.5	1.0	35	50	2	1.96	2.30	7.62	2	2.07	2.29	7.611
13	50	1	0.05	0.50	3	5.0	1.0	35	50	3	2.68	1.29	20.19	3	3.74	1.23	19.666
14	100	1	0.05	0.50	3	0.5	0.1	250	500	5	0.33	3.02	40.72	5	0.44	3.04	40.282
15	50	2	0.05	0.50	3	0.5	0.1	250	50	2	0.61	2.62	21.42	2	0.85	2.63	21.273
16	100	2	0.05	0.50	3	5.0	1.0	35	500	3	1.64	2.75	30.24	3	2.09	2.75	29.903
17	50	1	0.01	0.05	20	0.5	0.1	250	50	13	2.57	2.60	12.05	13	3.16	2.61	12.016
18	100	1	0.01	0.05	20	5.0	1.0	35	500	14	5.97	2.63	23.80	14	7.29	2.62	23.660
19	50	2	0.01	0.05	20	5.0	1.0	35	50	4	6.00	2.35	11.44	4	7.29	2.35	11.38
20	100	2	0.01	0.05	20	0.5	0.1	250	500	6	1.42	3.63	20.39	6	1.73	3.64	20.366
21	50	1	0.05	0.05	20	5.0	0.1	250	500	19	2.62	2.78	35.45	19	5.57	2.95	34.437
22	100	1	0.05	0.05	20	0.5	1.0	35	50	5	1.88	1.41	57.81	5	3.03	1.56	56.762
23	50	2	0.05	0.05	20	0.5	1.0	35	500	4	2.10	2.81	29.48	4	3.72	2.96	28.823
24	100	2	0.05	0.05	20	5.0	0.1	250	50	5	1.57	2.37	61.17	5	3.23	2.56	60.108
25	50	1	0.01	0.50	20	0.5	1.0	250	500	9	5.45	2.56	15.93	9	6.98	2.57	15.782
26	100	1	0.01	0.50	20	5.0	0.1	35	50	6	3.24	1.82	23.35	6	3.98	1.81	23.261
27	50	2	0.01	0.50	20	5.0	0.1	35	500	5	4.56	3.16	11.71	5	5.82	3.17	11.660
28	100	2	0.01	0.50	20	0.5	1.0	250	50	2	2.06	2.24	22.33	2	2.48	2.25	22.274
29	50	1	0.05	0.50	20	5.0	1.0	250	50	4	3.83	0.99	37.45	4	7.74	1.09	36.090
30	100	1	0.05	0.50	20	0.5	0.1	35	500	6	0.56	2.81	59.59	6	0.91	2.90	58.754
31	50	2	0.05	0.50	20	0.5	0.1	35	50	2	0.83	2.35	28.30	2	1.34	2.47	28.009
32	100	2	0.05	0.50	20	5.0	1.0	250	500	3	2.07	2.47	64.64	3	3.87	2.60	63.169

Table 3.8 also shows the result of economic design of $\bar{X}$ chart for continuous process reported by Panagos et al. (1985) for each of 32 sets of various combinations of cost and process data. Compared to their results, it is observed that the expected loss cost per unit time $E(L)_1$ for the economic designs obtained using simulated annealing are less in all the 32 cases. Thus, the simulated annealing is observed to have resulted comparatively superior economic designs.

Further, to find out the statistical significance of all the cost and process parameters (i.e., all the nine factors listed in Table 3.7) on each of the four output responses (i.e., expected loss cost per unit time $E(L)_1$, sample size n, sampling interval h and width of control limit k), analysis of variance (ANOVA) has been performed on the economic design results obtained using simulated annealing shown in Table 3.8. Tables 3.9 - 3.12 show the results of ANOVA at 95% confidence level (i.e., significance level of 5%) for identifying the significant factors affecting the four responses. The significant factors can also be easily identified in the normal plots of standardized effects for four output responses as shown in

Figs. 3.7 - 3.10. These plots and ANOVA tables have been obtained with the help of student version of MINITAB 16.

Table 3.9: Analysis of variance for expected loss cost per unit time $E(L)_1$: continuous process

Source	***DF***	***Seq SS***	***Adj SS***	***Adj MS***	***F***	***p***	***PC(%)***
M	1	1393.060	1393.060	1393.060	31.23	0.000*	16.31
δ	1	51.540	51.540	51.540	1.16	0.294	0.60
λ	1	4165.420	4165.420	4165.420	93.38	0.000*	48.77
g	1	40.200	40.200	40.200	0.90	0.353	0.47
(T_1+T_2)	1	1729.870	1729.870	1729.870	38.78	0.000*	20.25
a	1	0.800	0.800	0.800	0.02	0.895	0.01
b	1	17.780	17.780	17.780	0.40	0.534	0.21
W	1	149.280	149.280	149.280	3.35	0.081	1.75
Y	1	11.860	11.860	11.860	0.27	0.611	0.14
Residual Error	22	981.360	981.360	44.610			
Total	31	8541.180					

* Significant at 5%

The degree of freedom (*DF*) for a factor is one less than the maximum number of levels of values considered for that factor if only main effect of that factor is concerned. In case of interaction effect, the degrees of freedom of corresponding factors are multiplied. For the current statistical models, only main effects of all the factors are considered and only two levels of those factors are taken. Therefore, for each factor the degree of freedom is one as shown in each of Tables 3.9 - 3.12. The sequential sum of squares denoted as *Seq SS* in Tables 3.9 - 3.12 measures the amount of variation in the response that is explained by adding the factors sequentially to the statistical model in the order listed in the ANOVA table. The sequential sum of squares for the factors is specific to the order that the factors are added to the statistical model (Minitab, 2007). The adjusted sum of squares denoted as *Adj SS* in Tables 3.9 - 3.12, for a factor in the statistical model measures the amount of additional variation in the response that is explained by a specific factor, given that all other terms are already in the statistical model. The values of adjusted sum of squares do not depend on the order that the factors are placed in the statistical model (Minitab, 2007). For the current ANOVA model the sequential sum of squares values are equal to the adjusted sum of squares values, which show that the order of the terms in the model do not effect the model results. The adjusted mean squares denoted as *Adj MS* in Tables 3.9 - 3.12 are calculated by dividing the adjusted sum of squares by the corresponding degree of freedom for that factor.

F-test is conducted for each source of variation in ANOVA. The F-value is calculated by dividing the adjusted mean square of a factor by that of the residual error. The p-value is a statistical measure representing the probability of making the mistake of rejecting the null hypothesis when it is actually true. It is graphically represented by the area under the F-distribution curve on the right side of its corresponding F-value as illustrated in Fig. 3.6. Therefore, as F-value of a factor increases, the corresponding p-value decreases. When F becomes $F_{critical}$, p is equal to the value of significance level α. The value of $F_{critical}$ can be obtained from F-table for a given significance level α, and two degrees of freedom (i.e., one for the factor concerned and another for the residual error). The factor having p-value less than the specified value of significance level α, is considered to have statistical significance on a given output response. In other words, for a critical factor its F-value is greater than $F_{critical}$. Thus, $F_{critical}$ divides the entire range into two zones, namely significant and insignificant zones as shown in Fig. 3.6. For the most significant factor, F-value is maximum and p-value is minimum.

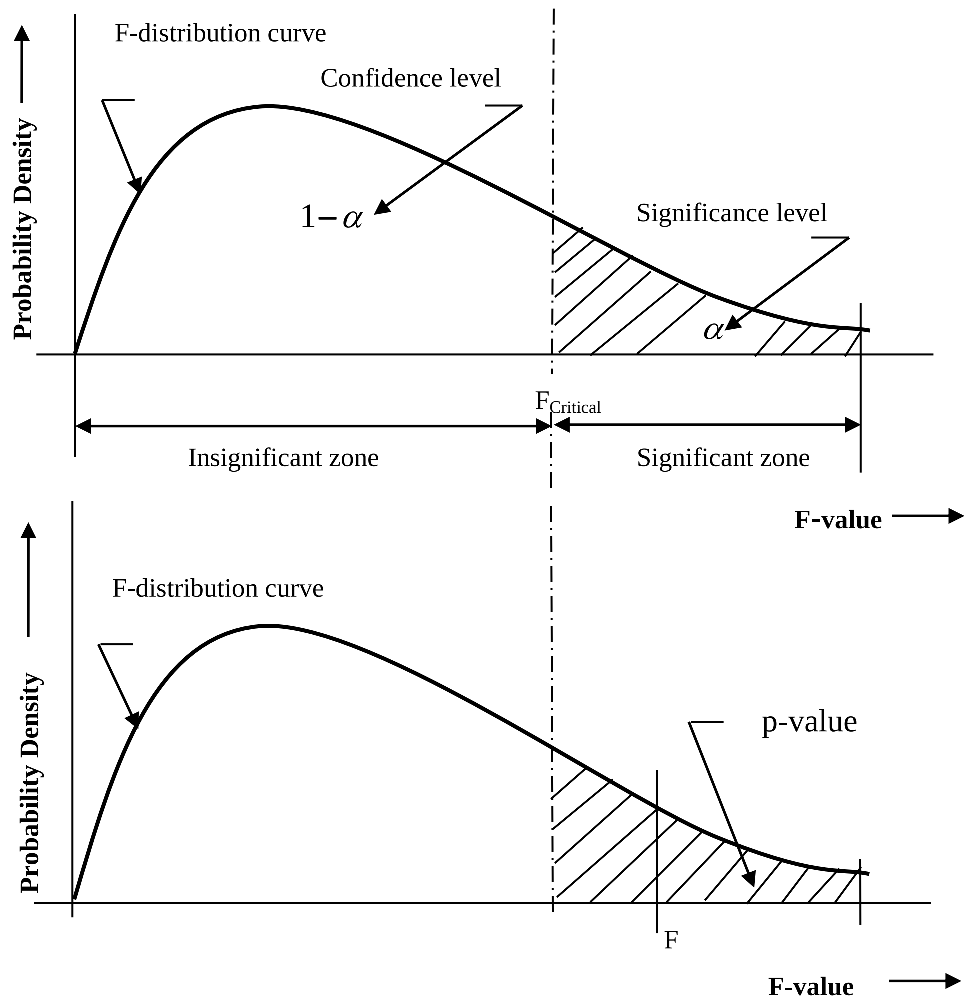

Fig. 3.6: F-value versus p-value

The percentage contribution (*PC*) is defined as the significance rate of a factor on the output response. It reflects the relative portion of the total variation observed in an experiment which is attributed to each factor. It is calculated as:

$$PC = \frac{Seq\ SS}{Seq\ SS_{Total}} \text{x}100 \tag{3.37}$$

where

$Seq\ SS$ = Sequential sum of squares for the given factor

$Seq\ SS_{Total}$ = Total sequential sum of squares

Taking the first row of Table 3.9 as an example, various values are calculated as shown below:

DF = (number of levels – 1) = (2 – 1) = 1

$Seq\ SS$ = 1393.060 (as calculated by student version MINITAB 16)

$Adj\ SS$ = 1393.060 (as calculated by student version MINITAB 16)

$$Adj\ MS = \frac{Adj\ SS}{DF} = \frac{1393.060}{1} = 1393.060$$

$$F\text{-value} = \frac{Adj\ MS}{Adj\ MS_{error}} = \frac{1393.060}{44.61} = 31.23$$

$$PC = \frac{Seq\ SS}{Seq\ SS_{Total}} \text{x}100 = \frac{1393.060}{8541.180} \text{x}100 = 16.31\%$$

In addition to the ANOVA table, the normal plot of the standardized effects is plotted by MINITAB 16 for each response. It is plotted between cumulative probability density function (or, percent) in vertical axis and the standardized effect in horizontal axis. The standardized effect for a factor is the ratio of regression coefficient to its standard error. The straight line in this plot is an imaginary reference line which corresponds to a given value of significance level. On this plot the input parameters having negligible effects on the response will lie near the straight line, whereas the parameters having significant effects will fall away from the straight line on either side. Points falling on the right side of the straight line are considered to have positive effects, whereas that on the left side has negative effects. A positive effect implies that on changing the factor from low to high level it increases the value of response while in case of negative effect for the same change in the factor, the response value decreases. Then relative significance of all the nine factors over the four responses at significance level of 5% are graphically displayed by means of normal plots as shown in Figs. 3.7 - 3.10.

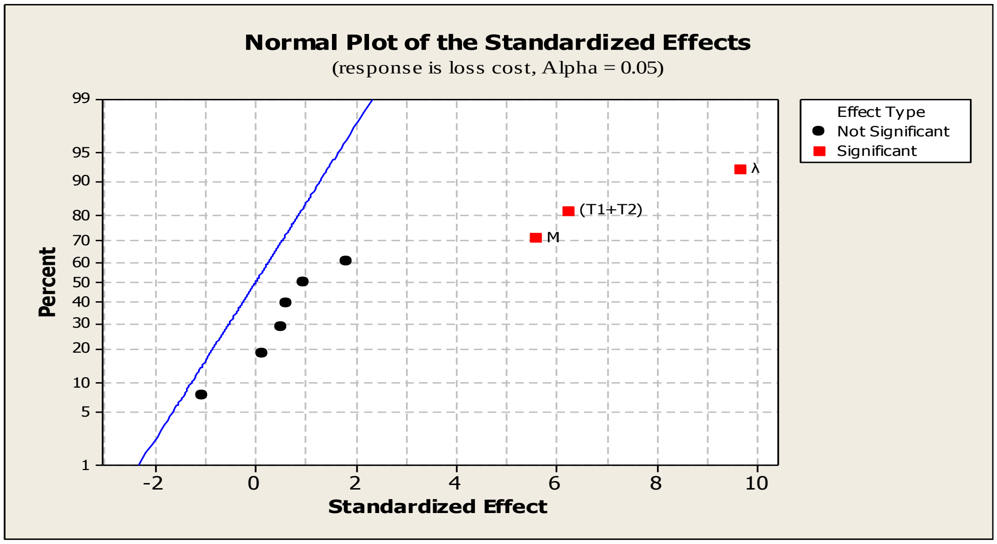

Fig. 3.7: Normal probability plot of standardized effect for expected loss cost per unit time $E(L)_1$: continuous process

Table 3.9 indicates that the expected loss cost per unit time of process control in a continuous process (i.e., $E(L)_1$) is significantly affected by three factors, namely loss of net income when process is out-of-control M, rate of occurrences of assignable causes λ and time to find and repair an assignable cause (T_1+T_2). All these three factors are significant as they all have p-value less than the predetermined significance level of 0.05 and therefore they are marked with asterisks in this table. They are plotted far away from the straight line and therefore marked as "significant" in the normal plot as shown in Fig. 3.7. Among all the factors, λ has the highest significant effect on expected loss cost per unit time $E(L)_1$ since it has the highest F-value i.e., 93.38 as shown in Table 3.9 and plotted at the rightmost location in Fig. 3.7. It can also be observed from this table that λ, (T_1+T_2), and M are the top three percentage contributors which affect the cost by 48.77%, 20.25% and 16.31% respectively. Further, all the nine factors including these three are observed to have positive effects as all the nine points are falling on the right side of the straight line. This implies that as the value of any of the nine factors increases, the expected loss cost per unit time $E(L)_1$ increases.

Table 3.10 presents the results of analysis of variance for the sample size n. There are four factors i.e., the size of the shift in the process mean δ, time to sample and chart one item g, variable cost of sampling b and cost per false alarm Y which have significant effect on sample size. Fig. 3.8 shows that out of these four significant factors, three factors have negative effect on sample size except the cost per false alarm Y. An increase in δ, g or b decreases the optimum sample size, because they all have the negative effects. Moreover, the percentage contributions of these four significant factors δ, g, Y and b affecting the sample size are 34.60%, 23.16%, 11.17% and 4.57% respectively. Thus, the factor δ is the most significant for choosing the value of sample size, in economic design and the effect is of negative type.

Table 3.10: Analysis of variance for sample size n: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	18.000	18.000	18.000	3.03	0.096	2.57
δ	1	242.000	242.000	242.000	40.68	0.000*	34.60
λ	1	15.125	15.125	15.125	2.54	0.125	2.16
g	1	162.000	162.000	162.000	27.23	0.000*	23.16
(T_1+T_2)	1	0.125	0.125	0.125	0.02	0.886	0.02
a	1	21.125	21.125	21.125	3.55	0.073	3.02
b	1	32.000	32.000	32.000	5.38	0.030*	4.57
W	1	0.125	0.125	0.125	0.02	0.886	0.02
Y	1	78.125	78.125	78.125	13.13	0.002*	11.17
Residual Error	22	130.875	130.875	5.949			
Total	31	699.500					

* Significant at 5%

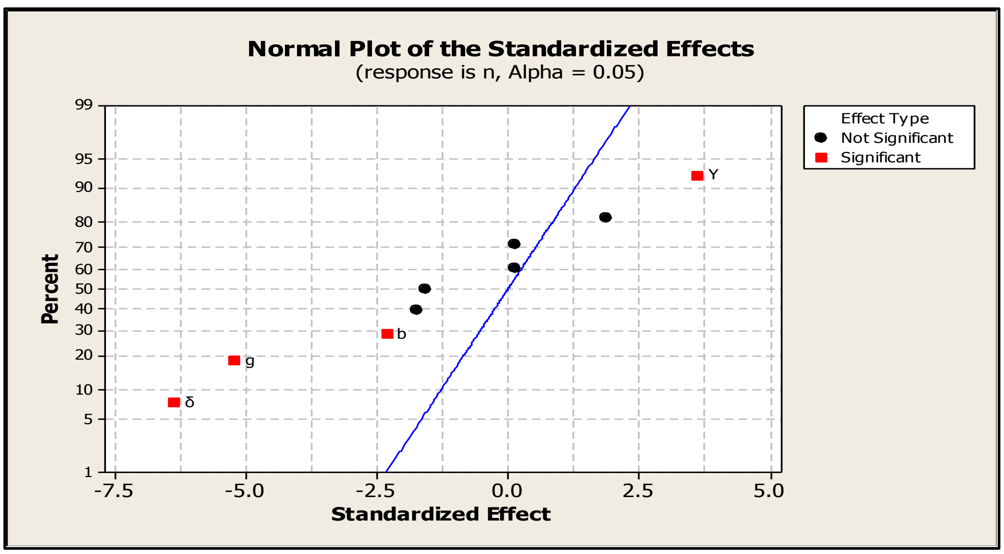

Fig. 3.8: Normal probability plot of standardized effect for sample size n: continuous process

Table 3.11 displays the results of analysis of variance on the sampling interval h. It is significantly affected by six factors, namely loss of net income when process is out-of-control M, the size of the shift in the process mean δ, rate of occurrences of assignable causes λ, the time to find and repair an assignable cause (T_1+T_2), fixed cost per sample a and variable cost per sample b. Out of these six significant factors, three factors i.e., M, λ and δ have negative effects as shown in Fig. 3.9, whereas the factors (T_1+T_2), a and b are significant in terms of positive effect. Moreover, the positive effect parameters such as *a*, *b* and (T_1+T_2) contribute 29.22%, 18.62% and 8.23% respectively, whereas the negative effect parameters like, λ, *M* and δ contribute by 15.01%, 13.66% and 4.39% respectively. Thus, among all the factors, the fixed cost per sample *a* has the highest effect on the sampling interval with a percentage contribution of 29.22% and the effect is in positive direction.

Table 3.11: Analysis of variance for sampling interval *h*: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	18.961	18.961	18.961	36.34	0.000*	13.66
δ	1	6.090	6.090	6.090	11.67	0.002*	4.39
λ	1	20.841	20.841	20.841	39.95	0.000*	15.01
g	1	0.877	0.877	0.877	1.68	0.208	0.63
(T_1+T_2)	1	11.429	11.429	11.429	21.91	0.000*	8.23
a	1	40.567	40.567	40.567	77.76	0.000*	29.22
b	1	25.855	25.855	25.855	49.56	0.000*	18.62
W	1	1.307	1.307	1.307	2.50	0.128	0.94
Y	1	1.450	1.450	1.450	2.78	0.110	1.04
Residual Error	22	11.478	11.478	0.522			
Total	31	138.854					

* Significant at 5%

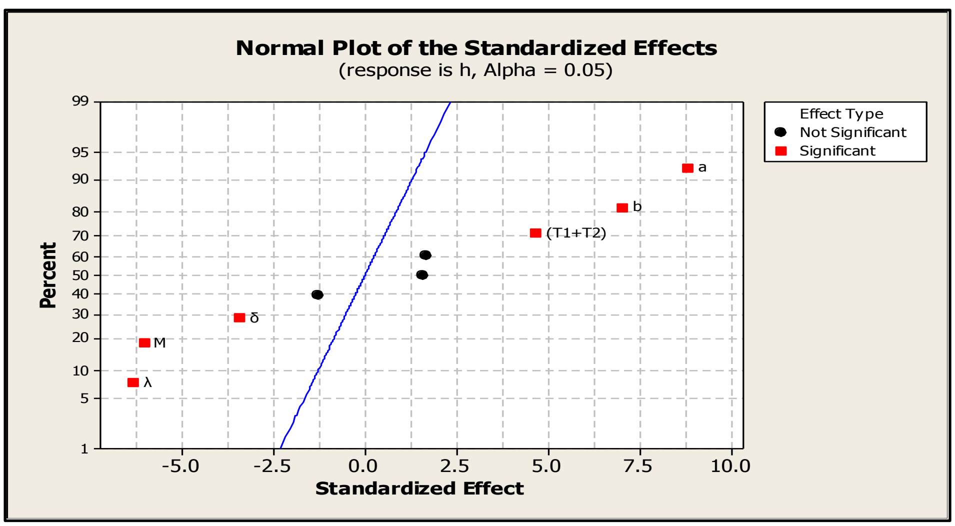

Fig. 3.9: Normal probability plot of standardized effect for sampling interval *h*: continuous process

Table 3.12 presents an analysis of variance on the control limit width *k*. There are six factors δ, λ, *g*, *a*, *b* and *Y* which have significant effects on width of control limits *k*. Fig. 3.10 reveals that out of these significant factors, four factors have negative effect on width of control limits except δ and Y. Further, the percentage contribution of these significant factors Y, b, δ, g, a and λ are 49.27%, 17.45%, 16.20%, 6.55%, 3.83% and 2.45% respectively. Thus, the cost per false alarm Y is observed to have the most significant effect on deciding the value of width of control limits in economic design and the effect is of positive type.

Table 3.12: Analysis of variance for width of control limits k: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	0.003	0.003	0.003	0.18	0.673	0.03
δ	1	1.843	1.843	1.843	98.11	0.000*	16.20
λ	1	0.279	0.279	0.279	14.85	0.001*	2.45
g	1	0.745	0.745	0.745	39.66	0.000*	6.55
(T_1+T_2)	1	0.067	0.067	0.067	3.58	0.072	0.59
a	1	0.436	0.436	0.436	23.20	0.000*	3.83
b	1	1.985	1.985	1.985	105.67	0.000*	17.45
W	1	0.000	0.000	0.000	0.01	0.925	0.00
Y	1	5.605	5.605	5.605	298.39	0.000*	49.27
Residual Error	22	0.413	0.413	0.0188			
Total	31	11.377					

* Significant at 5%

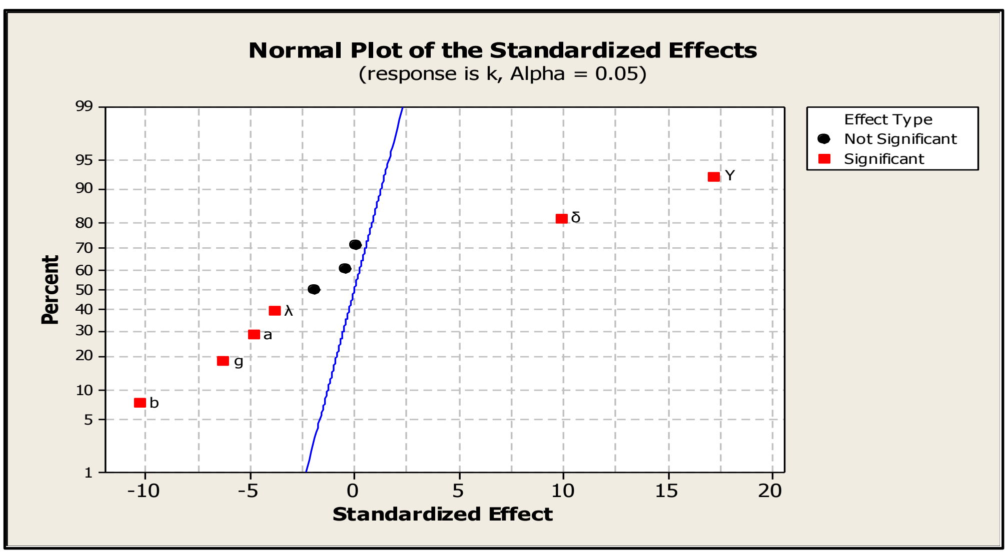

Fig. 3.10: Normal probability plot of standardized effect for width of control limits k: continuous process

It is further observed from Tables 3.9 - 3.12 that the cost to locate and repair the assignable cause W is having no significance on any of the responses n, h, k and $E(L)_1$.

3.6.1 Summary of Results

All the above results related to type of effects of all the nine cost and process parameters on each of the four responses are summarized in Table 3.13. These results are compared with that of Panagos et al. (1985) as shown in this table. The blank spaces denote insignificant factors. Significant factors with positive effects are shown as '+' whereas '–' denotes significant factors with negative effect. The significant factor with the highest percentage contribution for each response is shown bold. The control chart designers must take utmost care on ensuring the correctness of values of significant factors before using them for economic design.

Table 3.13: Summary of significant effects in economic design: continuous process

Output responses	***Methodology***	***Cost and process parameters***								
		M	***δ***	***λ***	***g***	***(T_1+T_2)***	***a***	***b***	***W***	***Y***
n	Panagos et al. (1985)		**–**		–		+	–		+
	Present work		**–**		–			–		+
h	Panagos et al. (1985)	–	–	**–**			+	+		
	Present work	–	–	–		+	**+**	+		
k	Panagos et al. (1985)		+	–	–	–	–	–		**+**
	Present work		+	–	–		–	–		**+**
$E(L)_1$	Panagos et al. (1985)	+		**+**		+				
	Present work	+		**+**		+				

Note:
Blank space : Insignificant factor
\+ : Factor with positive effect
– : Factor with negative effect
+/– in bold : Most significant factor

Table 3.13 shows that the most significant factors are same in both sets of results except in case of sampling interval *h*. Panagos et al. (1985) reported that the rate of occurrence of assignable cause λ is the most significant factor for sampling interval *h*, whereas the results of SA obtained in present work show that the fixed cost *a* is the most significant factor for *h*. Further, most of the other significant factors are observed to be the same in both the results except three instances as discussed in the rest of this paragraph. Panagos et al. (1985) found the fixed cost per sample *a* to be one of the significant factors affecting *n*, while the result of SA does not find the fixed cost (*a*) significantly affecting *n*. As per the results of SA, the time to find and repair an assignable cause (T_1+T_2) significantly affects the sampling interval *h*, whereas it is reported to be insignificant by Panagos et al. (1985) for *h*. On the other hand, as per Panagos et al. (1985) the same factor (T_1+T_2) is significant for width of control limit *k*, whereas this is not significant as per SA results. The sensitivity results for expected loss cost per unit time $E(L)_1$ are found to be the same for the both the cases. The significant parameters are not always same. This depends on the accuracy of calculating the design variables (i.e., *n*, *h* and *k*). Thus, the control charts users must take care while selecting proper optimization techniques.

3.7 Numerical Illustration: Discontinuous Process

A numerical problem related to economic design of $\bar{X}$ chart for discontinuous process already solved by Panagos et al. (1985) has been considered here to investigate the effectiveness of simulated annealing optimization technique. In addition to the nine cost and process parameters corresponding to the continuous process, this process deals with four additional parameters i.e., V_0, *S*, S_1 and T_0. The values of all the thirteen cost and process parameters associated with a discontinuous process in this numerical problem along with their notations are listed in Table 3.14.

Table 3.14: Cost and process data: discontinuous process

S. No.	*Cost and process parameters*	*Notation*	*Unit*	*Value*
1	Loss of income when process is out-of-control	M	$	100
2	Shift in process mean	δ	-	1
3	Rate of occurrences of assignable causes	λ	per hour	0.05
4	Time to sample and chart one item	*g*	hour	0.05
5	Time to find and repair an assignable cause	T_1+T_2	hour	3
6	Fixed cost per sample	*a*	$	0.5
7	Variable cost per sample	*b*	$	1.0
8	Cost to locate and repair the assignable cause	*W*	$	250
9	Cost per false alarm	*Y*	$	50
10	Net income per hour while process is in-control	V_0	$	50
11	Expected cost of restart or setup cost	*S*	$	100
12	Time to restart the process	S_1	hour	1
13	Expected search time for a false alarm	T_0	hour	40

3.7.1 Results and Discussion: SA

Table 3.15 shows the results of optimal economic design of $\bar{X}$ chart for a discontinuous process using simulated annealing i.e., the optimal values of two design variables of control chart such as sampling interval (*h*) and width of the control limits (*k*) for each integer value of sample size *n* varying from 2 to 33. This table also shows the corresponding minimum values of expected loss cost per unit time $E(L)_2$ along with values of other statistical parameters like α, β, *P*, ARL_0 and ARL_1 similar to Table 3.3.

Table 3.15: Optimal economic designs of $\bar{X}$ chart using SA: discontinuous process

n	*h*	*k*	α	β	*P*	ARL_0	ARL_1	$E(L)_2$
2	0.63	2.47	0.0135	0.854	0.146	73.900	6.869	42.846
3	0.80	2.55	0.0108	0.793	0.207	92.665	4.838	40.900
4	1.01	2.57	0.0102	0.716	0.284	98.147	3.517	39.478
5	1.20	2.61	0.0091	0.646	0.354	110.220	2.823	38.429
6	1.41	2.63	0.0086	0.572	0.428	116.870	2.334	37.649
7	1.62	2.64	0.0083	0.498	0.502	120.360	1.991	37.069
8	1.80	2.67	0.0076	0.437	0.563	131.540	1.776	36.645
9	1.99	2.68	0.0074	0.374	0.626	135.520	1.599	36.344
10	2.17	2.71	0.0067	0.326	0.674	148.270	1.483	36.141
11	2.37	2.73	0.0063	0.279	0.721	157.500	1.387	36.021
12	**2.50**	**2.75**	**0.0060**	**0.238**	**0.762**	**167.370**	**1.312**	**35.966**
13	2.65	2.77	0.0056	0.202	0.798	177.920	1.253	35.968
14	2.79	2.79	0.0053	0.171	0.829	189.210	1.206	36.018
15	2.94	2.82	0.0048	0.146	0.854	207.640	1.171	36.108
16	3.06	2.84	0.0045	0.123	0.877	221.010	1.140	36.231
17	3.22	2.86	0.0042	0.103	0.897	235.330	1.115	36.383
18	3.29	2.89	0.0039	0.088	0.912	258.760	1.097	36.559
19	3.42	2.92	0.0035	0.075	0.925	284.740	1.081	36.755
20	3.54	2.94	0.0033	0.063	0.937	303.640	1.067	36.968
21	3.63	2.98	0.0029	0.055	0.945	345.680	1.058	37.195
22	3.73	3.00	0.0027	0.045	0.955	369.030	1.048	37.434
23	3.83	3.03	0.0025	0.039	0.961	407.340	1.040	37.683
24	3.92	3.05	0.0023	0.032	0.968	435.260	1.033	37.939
25	4.02	3.08	0.0021	0.027	0.973	481.110	1.028	38.202
26	4.14	3.10	0.0019	0.023	0.977	514.570	1.023	38.470
27	4.22	3.14	0.0017	0.020	0.980	589.280	1.020	38.741
28	4.29	3.17	0.0015	0.017	0.983	652.990	1.017	39.015
29	4.39	3.19	0.0014	0.014	0.986	699.570	1.014	39.292
30	4.48	3.22	0.0013	0.012	0.988	776.290	1.012	39.569
31	4.54	3.25	0.0012	0.010	0.990	862.150	1.010	39.848
32	4.65	3.28	0.0010	0.009	0.991	958.310	1.009	40.126
33	4.75	3.30	0.0010	0.007	0.993	1028.800	1.007	40.404

Table 3.15 shows that the optimum values of loss cost per unit time $E(L)_2$ decreases as sample size n value increases from 2 to 12 and after that it increases at higher values of n. This trend is also graphically shown in Fig. 3.11.

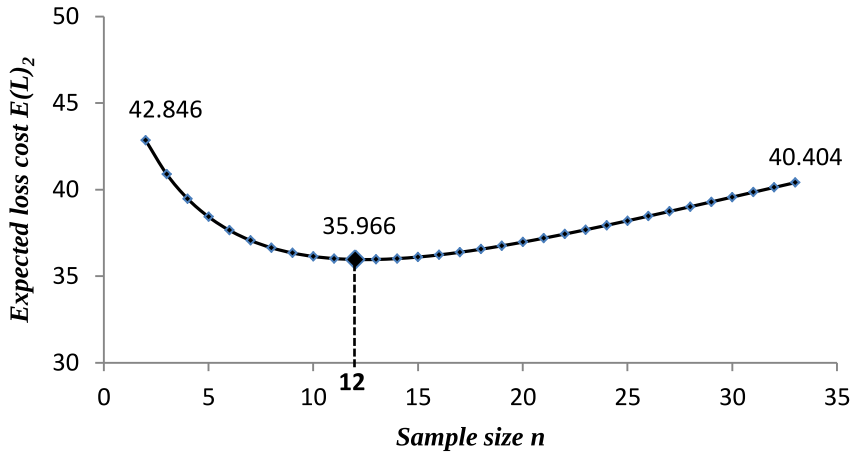

Fig. 3.11: Variation of expected loss cost per unit time with sample size using SA: discontinuous process

Out of all 32 economic designs, one each for integer value of sample size n varying from 2 to 33, the most minimum cost is found to be $E(L)_2$ = 35.966 and this occurs at sample size n = 12 as shown in Table 3.16. The corresponding optimal values of h and k at minimum loss cost are 2.50 hour and 2.75 respectively. For the same numerical problem of discontinuous process, the optimal solution obtained by Panagos et al. (1985) is shown along with that obtained with the use of simulated annealing in Table 3.16 for comparison purpose. The value of expected loss cost per unit time $E(L)_2$ corresponding to the economic design suggested by Panagos et al. (1985) is calculated in this work up to 3 decimal places for comparison and it is found to be 35.9662 ≈ 35.970 as shown in the same table. This table shows that the optimal values of sample size (n) and width of the control limit (k) are same in both the results. There is difference only in the value of sampling interval (h) and that too of very small magnitude. In case of simulated annealing, the optimal value of expected loss cost per unit time is found to be same as that of by Panagos et al. (1985).

Table 3.16: Comparison of results in discontinuous process

Methodology	***n***	***h***	***k***	***E(L)₂***
Panagos et al. (1985)	12	2.48	2.75	35.970
SA	12	2.50	2.75	35.966

3.7.2 Results and Discussion: TLBO

The numerical problem solved using simulated annealing as mentioned in Section 3.7.1, has been considered to illustrate the design methodology based on TLBO for a discontinuous process. The values of relevant data of this problem are already listed in Table 3.14. Similar to Table 3.15, the results of economic design of $\bar{X}$ chart for a discontinuous process using TLBO are shown in Table 3.17 for each integer value of sample size n in the range 2 to 33.

Table 3.17: Optimal economic designs of $\bar{X}$ chart using TLBO: discontinuous process

n	***h***	***k***	***α***	***β***	***P***	***ARL_0***	***ARL_1***	***$E(L)_2$***
2	0.63	2.47	0.0135	0.854	0.146	73.900	6.869	42.846
3	0.81	2.53	0.0114	0.788	0.212	87.521	4.707	40.899
4	1.01	2.57	0.0102	0.716	0.284	98.147	3.517	39.478
5	1.21	2.60	0.0093	0.642	0.358	107.060	2.794	38.429
6	1.41	2.62	0.0088	0.568	0.432	113.500	2.313	37.649
7	1.61	2.65	0.0081	0.502	0.498	123.970	2.007	37.069
8	1.81	2.67	0.0076	0.437	0.563	131.540	1.776	36.645
9	1.99	2.69	0.0072	0.378	0.622	139.630	1.608	36.344
10	2.17	2.71	0.0067	0.326	0.674	148.270	1.483	36.141
11	2.34	2.73	0.0063	0.279	0.721	157.500	1.387	36.020
12	**2.50**	**2.75**	**0.0060**	**0.238**	**0.762**	**167.370**	**1.312**	**35.966**
13	2.65	2.77	0.0056	0.202	0.798	177.920	1.253	35.968
14	2.79	2.79	0.0053	0.171	0.829	189.210	1.206	36.018
15	2.93	2.82	0.0048	0.146	0.854	207.640	1.171	36.108
16	3.06	2.84	0.0045	0.123	0.877	221.010	1.140	36.231
17	3.18	2.87	0.0041	0.105	0.895	242.870	1.117	36.383
18	3.30	2.89	0.0039	0.088	0.912	258.760	1.097	36.559
19	3.41	2.92	0.0035	0.075	0.925	284.740	1.081	36.755
20	3.52	2.95	0.0032	0.064	0.936	313.600	1.068	36.968
21	3.63	2.97	0.0030	0.053	0.947	334.610	1.056	37.195
22	3.73	3.00	0.0027	0.045	0.955	369.030	1.048	37.434
23	3.83	3.03	0.0025	0.039	0.961	407.340	1.040	37.683
24	3.93	3.05	0.0023	0.032	0.968	435.260	1.033	37.939
25	4.02	3.08	0.0021	0.027	0.973	481.110	1.028	38.202
26	4.12	3.11	0.0019	0.023	0.977	532.230	1.024	38.469
27	4.21	3.14	0.0017	0.020	0.980	589.280	1.020	38.741
28	4.30	3.16	0.0016	0.017	0.983	630.970	1.017	39.015
29	4.39	3.19	0.0014	0.014	0.986	699.570	1.014	39.292
30	4.48	3.22	0.0013	0.012	0.988	776.290	1.012	39.569
31	4.56	3.25	0.0012	0.010	0.990	862.150	1.010	39.848
32	4.65	3.27	0.0011	0.009	0.991	925.030	1.009	40.126
33	4.74	3.30	0.0010	0.007	0.993	1028.800	1.007	40.404

On comparing the results of all 32 economic designs, one each for integer value of sample size n varying from 2 to 33, the most minimum expected loss cost per unit time is found to be $E(L)_2$ = 35.966 and this occurs at n = 12 as shown in Table 3.17. The corresponding values of h and k at minimum loss cost are 2.50 hour and 2.75 respectively. This optimum result is exactly same as that obtained earlier with SA in case of discontinuous process as shown in Table 3.18.

Table 3.18: Comparison of results obtained from SA and TLBO: discontinuous process

Techniques	***n***	***h***	***k***	α	β	***P***	ARL_0	ARL_1	$E(L)_2$
SA	12	2.50	2.75	0.0060	0.238	0.762	167.370	1.312	35.966
TLBO	12	2.50	2.75	0.0060	0.238	0.762	167.370	1.312	35.966

Here also the values of expected loss cost per unit time $E(L)_2$ decreases with the increase of n value from 2 to 12 and then it increases for at n greater than 12 as shown in Fig. 3.12.

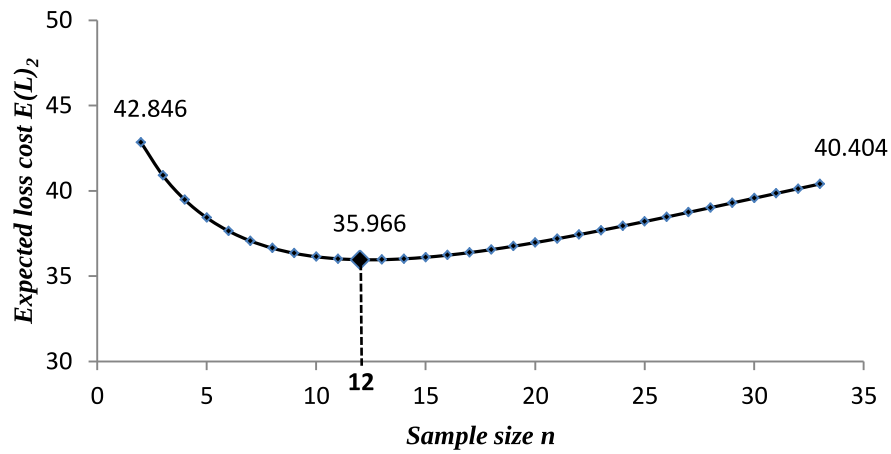

Fig. 3.12: Variation of expected loss cost per unit time with sample size using TLBO: discontinuous process

3.8 Sensitivity Analysis: Discontinuous Process

Similar to continuous process, sensitivity analysis has been done to investigate the effect of process and cost parameters on the output results of economic design in case of discontinuous process. The low and high values of all 13 cost and process parameters (also termed as factors) are taken from Panagos et al. (1985) and listed in Table 3.19. Since the names of factors are already listed in Table 3.14, only their notations are mentioned in Table 3.19.

Table 3.19: Factor levels: discontinuous process

No.	***Factors***	***Unit***	***Low Level***	***High Level***
1	A = M	\$	50.00	100.00
2	B = δ	-	1.00	2.00
3	C = λ	per hour	0.01	0.05
4	D = g	hour	0.05	0.50
5	E = (T_1+T_2)	hour	3.00	20.00
6	F = a	\$	0.50	5.00
7	G = b	\$	0.10	1.00
8	H = W	\$	35.00	250.00
9	J = Y	\$	50.00	500.00
10	K = V_0	\$	50.00	150.00
11	L = S	\$	10.00	100.00
12	M = S_1	hour	0.05	1.00
13	N = T_0	hour	4.00	40.00

A 2_{IV}^{13-8} factorial design with 32 runs has been selected for the discontinuous model with generators I = ABCF, I = ABDG, I = ABEH, I = ACDJ, I = ACEK, I = ADEL, I = BCDM and I = BCEN. Table 3.20 shows a specific combination of the values of thirteen input factors for each of 32 runs. Each such set of input data represents a numerical problem similar to that specified in Table 3.15 for which the optimal economic design of $\bar{X}$ chart has been already found out using SA for the discontinuous process in Section 3.7.1. The result of economic design for each run consists of a set of values of three design variables *n*, *h* and *k* and the corresponding minimum value of objective function $E(L)_2$ and this is shown in Table 3.20. Thus, this table presents 32 sets of results of economic design of $\bar{X}$ chart for a discontinuous process using SA. Since both SA and TLBO algorithms provided almost the same results for economic design in a discontinuous process as observed in Section 3.7, any one of them is sufficient for sensitivity analysis.

Table 3.20: Optimal economic designs of $\bar{X}$ chart: discontinuous process

S. No.	*Cost and process parameters (factors)*													*Panagos et al. (1985)*				*SA*			
	M	***δ***	***λ***	***g***	***(T₁+T₂)***	***a***	***b***	***W***	***Y***	***V_0***	***S***	***S_1***	***T_0***	***n***	***h***	***k***	***$E(L)_2$***	***n***	***h***	***k***	***$E(L)_2$***
1	50	1	0.01	0.05	3	0.5	0.1	35	50	50	10	0.05	4	17	2.75	3.14	4.09	17	2.92	3.12	4.087
2	100	1	0.01	0.05	3	5.0	1.0	250	500	150	100	0.05	4	17	6.33	2.95	15.89	17	6.81	2.95	15.875
3	50	2	0.01	0.05	3	5.0	1.0	250	50	50	10	1.0	40	6	6.46	3.46	7.92	6	6.95	3.45	7.917
4	100	2	0.01	0.05	3	0.5	0.1	35	500	150	100	1.0	40	8	1.54	4.31	9.11	8	1.60	4.32	9.109
5	50	1	0.05	0.05	3	5.0	0.1	35	500	150	10	1.0	40	26	2.41	3.76	32.23	26	3.90	3.79	31.799
6	100	1	0.05	0.05	3	0.5	1.0	250	50	50	100	1.0	40	12	2.48	2.75	35.97	12	2.49	2.75	35.966
7	50	2	0.05	0.05	3	0.5	1.0	250	500	150	10	0.05	4	5	2.01	3.32	35.07	5	3.49	3.34	34.586
8	100	2	0.05	0.05	3	5.0	0.1	35	50	50	100	0.05	4	6	1.50	3.12	19.96	6	1.64	3.14	19.930
9	50	1	0.01	0.50	3	0.5	1.0	35	500	50	100	1.0	4	9	5.03	2.72	10.04	9	5.42	2.74	10.010
10	100	1	0.01	0.50	3	5.0	0.1	250	50	150	10	1.0	4	10	2.79	2.90	17.92	10	3.06	2.90	17.900
11	50	2	0.01	0.50	3	5.0	0.1	250	500	50	100	0.05	40	6	4.53	3.63	8.61	6	4.97	3.64	8.596
12	100	2	0.01	0.50	3	0.5	1.0	35	50	150	10	0.05	40	4	2.34	3.67	11.31	4	2.44	3.68	11.307
13	50	1	0.05	0.50	3	5.0	1.0	35	50	150	100	0.05	40	13	3.96	3.00	39.04	13	7.04	3.11	37.642
14	100	1	0.05	0.50	3	0.5	0.1	250	500	50	10	0.05	40	6	0.40	3.29	36.37	6	0.40	3.29	36.370
15	50	2	0.05	0.50	3	0.5	0.1	250	50	150	100	1.0	4	4	0.79	3.43	41.95	4	1.74	3.52	41.519
16	100	2	0.05	0.50	3	5.0	1.0	35	500	50	10	1.0	4	3	1.61	2.85	25.41	3	1.84	2.86	25.320
17	50	1	0.01	0.05	20	0.5	0.1	250	50	150	100	0.05	40	25	3.27	3.92	29.45	25	4.94	3.93	29.351
18	100	1	0.01	0.05	20	5.0	1.0	35	500	50	10	0.05	40	18	6.54	3.11	15.88	18	6.97	3.11	15.866
19	50	2	0.01	0.05	20	5.0	1.0	35	50	150	100	1.0	4	6	6.59	3.24	29.36	6	9.97	3.27	29.190
20	100	2	0.01	0.05	20	0.5	0.1	250	500	50	10	1.0	4	7	1.50	3.83	12.43	7	1.59	3.83	12.424
21	50	1	0.05	0.05	20	5.0	0.1	250	500	50	100	1.0	4	21	2.39	3.15	36.76	21	4.31	3.19	36.405
22	100	1	0.05	0.05	20	0.5	1.0	35	50	150	10	1.0	4	11	2.54	2.43	82.56	11	4.53	2.59	81.742
23	50	2	0.05	0.05	20	0.5	1.0	35	500	50	100	0.05	40	5	2.20	3.32	30.82	5	3.05	3.42	30.667
24	100	2	0.05	0.05	20	5.0	0.1	250	50	150	10	0.05	40	8	1.67	3.66	84.14	8	3.59	3.90	83.512
25	50	1	0.01	0.50	20	0.5	1.0	250	500	150	10	1.0	40	15	7.13	3.28	32.83	15	11.50	3.32	32.461
26	100	1	0.01	0.50	20	5.0	0.1	35	50	50	100	1.0	40	11	2.90	3.10	18.50	11	3.00	3.11	18.495
27	50	2	0.01	0.50	20	5.0	0.1	35	500	150	10	0.05	4	6	4.60	3.47	27.65	6	6.90	3.47	27.535
28	100	2	0.01	0.50	20	0.5	1.0	250	50	50	100	0.05	4	3	2.43	2.78	15.34	3	2.53	2.79	15.340
29	50	1	0.05	0.50	20	5.0	1.0	250	50	50	10	0.05	4	5	3.93	1.66	36.51	5	5.03	1.83	36.260
30	100	1	0.05	0.50	20	0.5	0.1	35	500	150	100	0.05	4	9	0.63	3.12	83.31	9	1.20	3.26	82.615
31	50	2	0.05	0.50	20	0.5	0.1	35	50	50	10	1.0	40	4	0.86	3.53	28.96	4	1.08	3.61	28.911
32	100	2	0.05	0.50	20	5.0	1.0	250	500	150	100	1.0	40	5	2.52	3.19	89.55	5	5.59	3.43	88.477

For comparison purpose, the result of economic design of $\bar{X}$ chart for discontinuous process reported by Panagos et al. (1985) for each of 32 sets of various combinations of cost and process data is also shown in Table 3.20. It is observed that the expected loss cost per unit time $E(L)_2$ for the economic designs obtained using simulated annealing are less in all the 32 cases. Thus, the simulated annealing is observed to have resulted comparatively better economic designs in discontinuous process too.

Further, to find out the statistical significance of all the factors (i.e., all the thirteen cost and process parameters as listed in Table 3.14) on each of the four output responses (i.e., expected loss cost per unit time $E(L)_2$, sample size, n sampling interval h and width of control chart k), analysis of variance (ANOVA) has been performed on the economic design results obtained using simulated annealing as shown in Table 3.20. Tables 3.21 - 3.24 show the results of ANOVA at significance level of 5% for identifying the significant factors affecting the four responses. The significant factors can be more easily identified in the normal plots of standardized effects for four output responses as shown in Figs. 3.13 - 3.16. The student version of MINITAB 16 is used to obtain all these plots and ANOVA tables.

Table 3.21: Analysis of variance for expected loss cost per unit time $E(L)_2$: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	641.600	641.600	641.65	4.50	0.048*	3.96
δ	1	73.500	73.500	73.46	0.52	0.482	0.45
λ	1	6793.100	6793.100	6793.13	47.63	0.000*	41.93
g	1	50.900	50.900	50.88	0.36	0.558	0.31
(T_1+T_2)	1	2837.700	2837.700	2837.66	19.90	0.000*	17.52
a	1	0.600	0.600	0.57	0.00	0.950	0.00
b	1	12.600	12.600	12.56	0.09	0.770	0.08
W	1	147.600	147.600	147.55	1.03	0.323	0.91
Y	1	0.000	0.000	0.03	0.00	0.989	0.00
V_0	1	3043.500	3043.500	3043.47	21.34	0.000*	18.79
S	1	14.000	14.000	14.00	0.10	0.758	0.09
S_1	1	10.200	10.200	10.22	0.07	0.792	0.06
T_0	1	7.700	7.700	7.69	0.05	0.819	0.05
Residual Error	18	2567.000	2567.000	142.61			
Total	31	16199.900					

* Significant at 5%

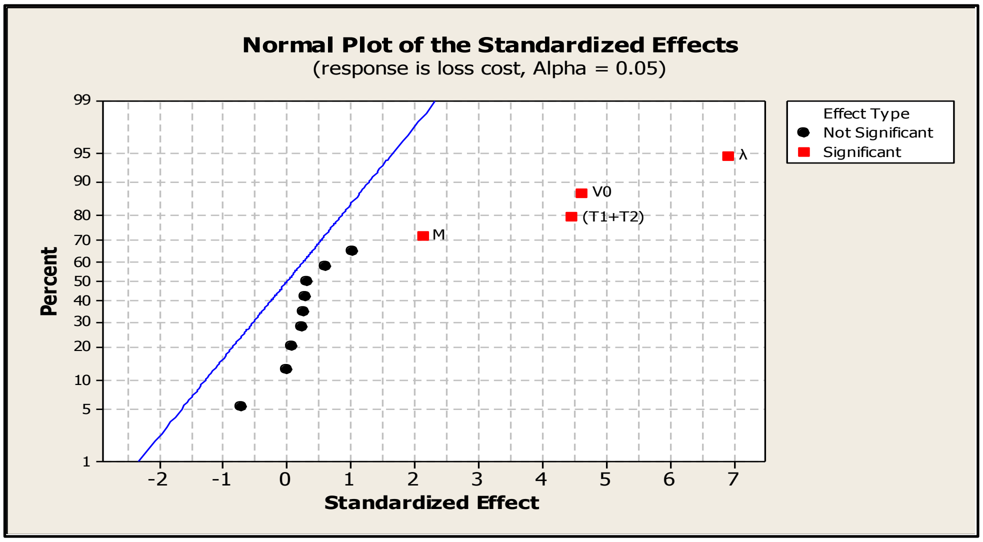

Fig. 3.13: Normal probability plot of standardized effect for expected loss cost per unit time $E(L)_2$: discontinuous process

Table 3.21 indicates that the expected loss cost per unit time of process control $E(L)_2$ in a discontinuous process is significantly affected by four factors, namely loss of net income when process is out-of-control M, rate of occurrences of assignable cause λ, time to find and repair an assignable cause (T_1+T_2), and net income per hour while process is in-control V_0. They are also graphically shown as "significant" in the normal plot shown in Fig. 3.13.

Among all the factors, λ has the highest significant effect on expected loss cost per unit time $E(L)_2$ since it has the highest F-value (i.e., 47.63) as shown in Table 3.21 and plotted at the rightmost location in Fig. 3.13. It can also be observed from this table that λ, V_0, (T_1+T_2), and M are the top four percentage contributors which affect the cost by 41.93%, 18.79%, 17.52% and 3.96% respectively. Further, all the 13 factors including these four are observed to have positive effects as all the thirteen points are lying on the right side of the straight line. This implies that as the value of any of the 13 factors increases, the expected loss cost per unit time $E(L)_2$ increases.

Table 3.22 shows the results of analysis of variance for the sample size n. There are two factors i.e., the size of the shift in the process mean δ**,** and time to sample and chart one item g have significant effect on sample size. Fig. 3.14 shows that both these significant factors have negative effect on sample size. So, an increase in δ or g decreases the optimum sample size. Moreover, the percentage contributions of these two significant factors δ and g affecting the sample size are 49.39% and 18.47% respectively. Thus, the factor δ is the most significant for choosing the value of sample size, in economic design and the effect is of negative type.

Table 3.22: Analysis of variance for sample size n: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	38.28	38.28	38.281	3.64	0.072	3.13
δ	1	603.78	603.78	603.781	57.48	0.000*	49.39
λ	1	19.53	19.53	19.531	1.86	0.189	1.60
g	1	225.78	225.78	225.781	21.50	0.000*	18.47
(T_1+T_2)	1	1.53	1.53	1.531	0.15	0.707	0.13
a	1	16.53	16.53	16.531	1.57	0.226	1.35
b	1	42.78	42.78	42.781	4.07	0.059	3.50
W	1	0.03	0.03	0.031	0.00	0.957	0.00
Y	1	13.78	13.78	13.781	1.31	0.267	1.13
V_0	1	34.03	34.03	34.031	3.24	0.089	2.78
S	1	2.53	2.53	2.531	0.24	0.629	0.21
S_1	1	0.78	0.78	0.781	0.07	0.788	0.06
D_1	1	34.03	34.03	34.031	3.24	0.089	2.78
Residual Error	18	189.06	189.06	10.503			
Total	31	1222.47					

* Significant at 5%

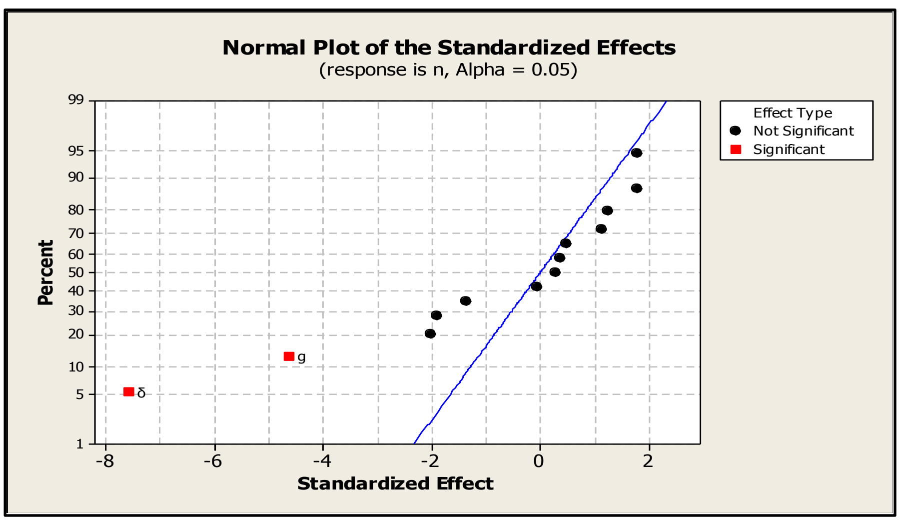

Fig. 3.14: Normal probability plot of standardized effect for sample size n: discontinuous process

Table 3.23 displays an analysis of variance on the sampling interval h. It is significantly affected by seven factors, namely loss of net income when process is out-of-control M, the size of the shift in the process mean δ, rate of occurrences of assignable cause λ, time to find and repair an assignable cause (T_1+T_2), fixed cost per sample a, variable cost per sample b and the net income per hour while process is in-control V_0. Out of these seven significant factors, three factors i.e., M, λ and δ have negative effects as shown in Fig. 3.15, whereas the factors (T_1+T_2), V_0, a and b are significant in terms of positive effect. Among all the factors, the variable cost of sampling b has the highest effect on the sampling interval with a percentage contribution of 22.44% and the effect is in positive direction.

Table 3.23: Analysis of variance for sampling interval *h*: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	35.955	35.955	35.9553	24.34	0.000*	17.16
δ	1	6.641	6.641	6.6414	4.50	0.048*	3.17
λ	1	29.335	29.335	29.3346	19.86	0.000*	14.00
g	1	0.789	0.789	0.7886	0.53	0.474	0.38
(T_1+T_2)	1	11.376	11.376	11.3758	7.70	0.012*	5.43
a	1	29.350	29.350	29.3497	19.87	0.000*	14.01
b	1	47.012	47.012	47.0115	31.82	0.000*	22.44
W	1	0.943	0.943	0.9428	0.64	0.435	0.45
Y	1	1.358	1.358	1.3583	0.92	0.350	0.65
V_0	1	18.179	18.179	18.1792	12.31	0.003*	8.68
S	1	0.000	0.000	0.0002	0.00	0.990	0.00
S_1	1	0.672	0.672	0.6722	0.46	0.509	0.32
T_0	1	1.332	1.332	1.3322	0.90	0.355	0.64
Residual Error	18	26.591	26.591	1.4773			
Total	31	209.532					

* Significant at 5%

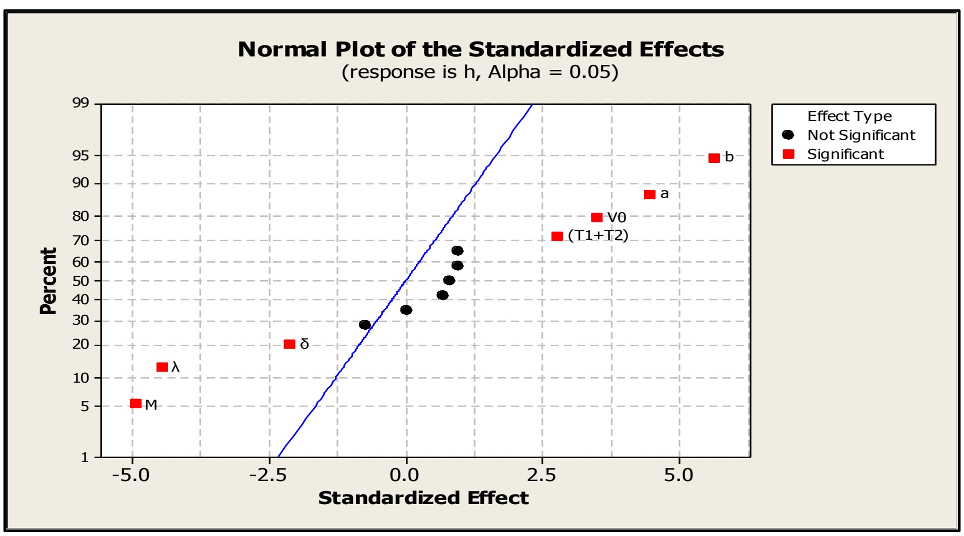

Fig. 3.15: Normal probability plot of standardized effect for sampling interval *h*: discontinuous process

Table 3.24 presents an analysis of variance on the width of control limits k. There are eight factors δ, λ, g, a, b, Y, V_0 and T_0 which have significant effects on width of control limits k. Fig. 3.16 reveals that out of these eight significant factors, four factors i.e., b, g, λ

and a have negative effect on width of control limit, whereas the rest four factors i.e., Y, V_0, δ and T_0 have positive effect. Among all the factors, the variable cost per sample b is observed to be the most significant effect on deciding the value of width of control limit in economic design and the effect is of negative type.

Table 3.24: Analysis of variance for width of control limits k: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	0.025	0.025	0.025	1.14	0.303	0.36
δ	1	1.385	1.385	1.385	61.94	0.000*	19.83
λ	1	0.2085	0.2085	0.2085	9.31	0.007*	2.98
g	1	0.393	0.393	0.393	17.59	0.001*	5.63
(T_1+T_2)	1	0.009	0.009	0.009	0.40	0.535	0.13
a	1	0.177	0.177	0.177	7.91	0.012*	2.53
b	1	1.704	1.704	1.704	76.19	0.000*	24.40
W	1	0.009	0.009	0.009	0.41	0.529	0.13
Y	1	0.335	0.335	0.335	14.96	0.001*	4.79
V_0	1	0.752	0.752	0.752	33.61	0.000*	10.76
S	1	0.007	0.007	0.007	0.32	0.576	0.10
S_1	1	0.016	0.016	0.016	0.69	0.417	0.22
T_0	1	1.562	1.562	1.562	69.85	0.000*	22.37
Residual Error	18	0.403	0.403	0.022			
Total	31	6.983					

* Significant at 5%

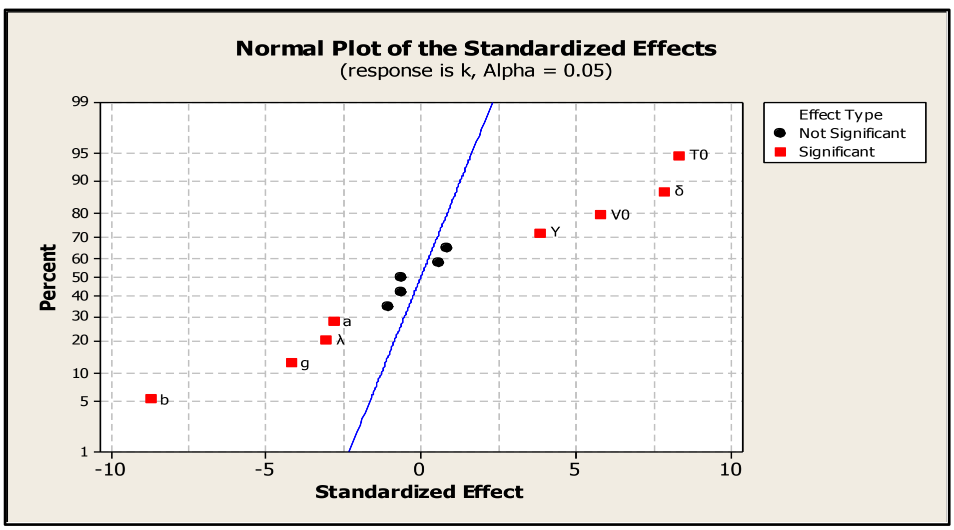

Fig. 3.16: Normal probability plot of standardized effect for width of control limits k: discontinuous process

It is further observed from Tables 3.21 - 3.24 that the cost to locate and repair the assignable cause W, the expected cost of restart or setup cost S and the startup time S_1 are having no significance on any of the responses n, h, k and $E(L)_2$.

3.8.1 Summary of Results

Similar to Table 3.13, all the significant factors in case of economic design for discontinuous process corresponding to each of the four responses are summarized in Table 3.25. This table is expected to be helpful for the control chart designers in case of discontinuous process. This table also compares the results of present work for a discontinuous process with that reported earlier by Panagos et al. (1985).

Table 3.25: Summary of significant effects in economic design: discontinuous process

Output responses	*Methodology*	*Cost and process parameters*												
		M	***δ***	***λ***	***g***	**(T_1+T_2)**	***a***	***b***	***W***	***Y***	**V_0**	***S***	**S_1**	**T_0**
n	Panagos et al. (1985)		**–**		–									
	Present work		**–**		–									
h	Panagos et al. (1985)	–	–	**–**			+	+						
	Present work	–	–	–		+	+	**+**			+			
k	Panagos et al. (1985)		–	–	–		–	**–**		+	+			+
	Present work		+	–	–		–	**–**		+	+			+
$E(L)_2$	Panagos et al. (1985)	+		**+**		+					+			
	Present work	+		**+**		+					+			

Note:

Blank space	: Insignificant factor
+	: Factor with positive effect
–	: Factor with negative effect
+/– in bold	: Most significant factor

Table 3.25 shows that only in case of sampling interval *h*, the most significant factors are different in both sets of results. Panagos et al. (1985) reported that the rate of occurrences of assignable cause λ is the most significant factor for sampling interval *h*, whereas the results obtained in the present work show that the variable cost per sample *b* is the most significant factor for *h*. Further, all other significant factors are observed to be completely same in both sets of results for all the responses except one i.e., sampling interval *h*. Two factors i.e., the time to find and repair an assignable cause (T_1+T_2) and net income per hour while process is in-control V_0 are found in the present work to have significant effect on the sampling interval *h*, whereas they are reported to be insignificant by Panagos et al. (1985) for *h*. The types of effect are same in both sets of results for all the common significant factors

except process shift size δ. The width of control limits k is positively affected by δ, whereas as per Panagos et al. (1985) δ has negative effect on k.

3.9 Numerical Illustration: Total Cost Model

In order to investigate the effectiveness of design methodologies based on two metaheuristics (i.e., SA and TLBO) another numerical problem that was earlier solved by van Deventer and Manna (2009) has been considered. This is related to economic design of $\bar{X}$ chart for a continuous process. In this problem, the economic model is slightly different. Therefore, a brief description of this model is given below.

3.9.1 Total Cost Model

The cost model of van Deventer and Manna (2009) is almost same as the model considered in this thesis which has been earlier explained in Section 3.3.

However, there are a few differences between these two models as discussed below:

1. Here, the objective is to minimize expected total cost per hour. But, in the model discussed earlier in Section 3.3, the objective was to minimize expected loss cost per unit time.
2. The expected time of occurrence of the process shift within the interval between *j*th and *(j+1)*st samples is τ. In this thesis, this value is considered as $\tau = \frac{h}{2} - \frac{\lambda h^2}{12}$ which is shown in Eq. 3.6, whereas van Deventer and Manna (2009) considered it as $\tau = \frac{1}{\lambda} - \frac{h}{e^{\lambda h} - 1}$.
3. The expected number of samples before the process shifts in a cycle is considered as $s \approx \frac{1}{\lambda h}$, whereas they have taken it as $s \approx \frac{1}{e^{\lambda h} - 1}$.
4. Instead of V_0 (i.e., the net income per hour while the process is in-control), they have considered C_0 (i.e., quality cost per hour while producing in-control).
5. Similarly, in place of V_1 (i.e., the net income per hour while the process is out-of-control), C_1 (i.e., quality cost per hour while producing out-of-control) is considered by them.

Incorporating the above five differences, the expected cycle time $E(T)$ and expected cost per cycle $E(C)$ are revised as:

$$E(T) = \frac{1}{\lambda} + h\left(\frac{1}{1-\beta} - \frac{1}{\lambda h} + \frac{1}{e^{\lambda h} - 1}\right) + gn + T_1 + T_2 \quad (3.38)$$

$$E(C) = \frac{C_0}{\lambda} + C_1\left[\frac{h}{1-\beta} - \tau + gn + T_1 + T_2\right] + \alpha sY + W + \frac{(a+bn)}{h}\left[\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + T_1 + T_2\right] \quad (3.39)$$

Dividing Eq. 3.39 by Eq. 3.38, the expected total cost per hour *E(Q)* is written as

$$E(Q) = \frac{E(C)}{E(T)}$$

Thus,

$$E(Q) = \frac{\frac{C_0}{\lambda} + C_1\left[\frac{h}{1-\beta} - \tau + gn + T_1 + T_2\right] + \alpha sY + W + \frac{(a+bn)}{h}\left[\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + T_1 + T_2\right]}{\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + T_1 + T_2} \quad (3.40)$$

The above expression of *E(Q)* is also a function of only three design variables *n*, *h*, and *k*. For economic design, *E(Q)* is minimized using some optimization technique. Since there are no constraints here, it is a case of multi-variable unconstrained minimization problem.

3.9.2 Cost and Process Parameters

The numerical data dealing with a continuous process has been taken from van Deventer and Manna (2009) where the values of cost and process parameters are as listed in Table 3.26.

Table 3.26: Cost and process parameters (van Deventer and Manna, 2009)

S. No.	*Cost and process parameters*	*Notation*	*Unit*	*Value*
1	Quality cost per hour while producing in-control	C_0	ZAR	10
2	Quality cost per hour while producing out-of-control	C_1	ZAR	100
3	Shift size from the mean	δ	-	1
4	Rate of occurrences of assignable causes	λ	per hour	0.01
5	Time to sample and chart one item	g	hour	0.05
6	Time to find the assignable cause	T_1	hour	2
7	Time to repair the assignable cause	T_2	hour	0
8	Fixed cost per sample	a	ZAR	0.5
9	Variable cost per sample	b	ZAR	0.1
10	Cost to locate and repair the assignable cause	W	ZAR	25
11	Cost per false alarm	Y	ZAR	50

This numerical problem is solved using the same two metaheuristics SA and TLBO, and the results obtained are discussed below.

3.9.3 Results and Discussion: SA

Table 3.27 shows the results of economic design of $\bar{X}$ chart for the above mentioned continuous process using SA i.e., the optimal values of two design variables of control chart such as sampling interval (*h*) and the width of the control limits (*k*) for each integer value of sample size *n* varying from 1 to 20. The search domain selected are same for sampling interval *h* (i.e., 0.25 - 12.0) and width of control limit *k* (i.e., 1 - 6) as shown in Table 3.1. For comparison purpose, the range of sample size *n* (i.e., 1 - 20) is taken same as that taken by van Deventer and Manna (2009). The corresponding minimum values of the expected total cost per hour *E*(*Q*) for each of 20 economic designs have been listed in Table 3.27. This table also shows the corresponding optimum values of Type-I error (α), Type-II error (β), power of detecting the shift (*P*), average in-control run length (ARL_0), average out-of-control run length (ARL_1).

Table 3.27: Optimal economic designs: SA

n	*h*	*k*	α	β	*P*	ARL_0	ARL_1	*E(Q)*
1	0.61	2.15	0.0316	0.874	0.126	31.669	7.944	19.204
2	0.68	2.29	0.0220	0.809	0.191	45.365	5.244	17.352
3	0.81	2.35	0.0188	0.732	0.268	53.205	3.727	16.422
4	0.94	2.39	0.0169	0.652	0.348	59.277	2.871	15.870
5	1.08	2.42	0.0155	0.573	0.427	64.343	2.342	15.513
6	1.21	2.44	0.0147	0.496	0.504	67.989	1.985	15.273
7	1.33	2.48	0.0132	0.434	0.566	75.996	1.767	15.107
8	1.43	2.50	0.0124	0.371	0.629	80.390	1.591	14.994
9	1.55	2.54	0.0111	0.323	0.677	90.052	1.477	14.919
10	1.67	2.56	0.0105	0.274	0.727	95.362	1.377	14.871
11	1.76	2.59	0.0096	0.234	0.766	103.992	1.305	14.846
12	**1.85**	**2.61**	**0.0091**	**0.197**	**0.804**	**110.225**	**1.245**	**14.838**
13	1.93	2.65	0.0081	0.170	0.830	123.969	1.204	14.844
14	2.01	2.68	0.0074	0.144	0.856	135.521	1.169	14.861
15	2.09	2.71	0.0067	0.122	0.878	148.271	1.140	14.887
16	2.16	2.75	0.0060	0.106	0.894	167.370	1.118	14.921
17	2.22	2.79	0.0053	0.091	0.909	189.208	1.100	14.961
18	2.29	2.81	0.0050	0.076	0.924	201.284	1.082	15.006
19	2.34	2.83	0.0047	0.063	0.937	214.210	1.067	15.056
20	2.41	2.87	0.0041	0.055	0.945	242.873	1.058	15.109

Table 3.27 shows that the optimum values of expected total cost per hour *E(Q)* decreases as sample size *n* value increases from 1 to 12 and after that it increases at higher values of *n*. This trend is also graphically shown in Fig. 3.17.

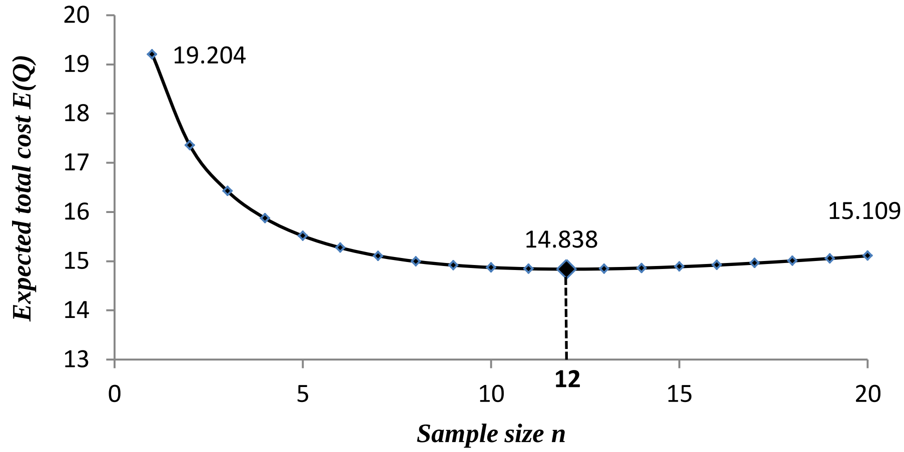

Fig. 3.17: Variation of expected total cost with sample size using SA

Out of all 20 economic designs, one each for integer value of sample size *n* varying from 1 to 20, the most minimum expected total cost per hour is found to be *E(Q)* = 14.838 and this occurs at sample size *n* = 12 as shown in Table 3.27. The corresponding optimal values of *h* and *k* at minimum cost are 1.85 hour and 2.61 respectively. For the same numerical problem, the optimal solution obtained by van Deventer and Manna (2009) is shown along with that obtained with the use of simulated annealing in Table 3.28 for comparison purpose. The value of expected total cost per hour *E(Q)* corresponding to the economic design suggested by van Deventer and Manna (2009) is observed to be 14.840 as shown in the same table.

Table 3.28: Comparison of results obtained from SA

Methodology	***n***	***h***	***k***	α	β	***P***	ARL_0	ARL_1	***E(Q)***
van Deventer and Manna (2009)	12	1.90	2.60	0.0093	0.194	0.806	107.268	1.240	14.840
SA	12	1.85	2.61	0.0091	0.197	0.804	110.225	1.245	14.838

This table shows that the optimal values of sample size (*n*) are same in both the results. There is difference only in the value of sampling interval (*h*) and width of the control limit (*k*) that too of very small magnitude. In case of simulated annealing, the optimal value of expected total cost per hour is found to be slightly lower than that of van Deventer and Manna (2009).

3.9.4 Results and Discussion: TLBO

The same numerical problem related to economic design of a continuous process mentioned in Section 3.9.3 has been again solved using TLBO for cross checking the accuracy of results obtained by SA. Similar to Table 3.27, the results of economic design obtained using TLBO are shown in Table 3.29 for each integer value of sample size *n* in the range 1 to 20.

Table 3.29: Optimal economic designs: TLBO

n	***h***	***k***	***α***	***β***	***P***	ARL_0	ARL_1	***E(Q)***
1	0.60	2.16	0.0308	0.876	0.124	32.474	8.077	19.204
2	0.69	2.29	0.0220	0.809	0.191	45.365	5.244	17.352
3	0.81	2.35	0.0188	0.732	0.268	53.205	3.727	16.422
4	0.94	2.39	0.0169	0.652	0.348	59.277	2.871	15.870
5	1.07	2.42	0.0155	0.573	0.427	64.343	2.342	15.513
6	1.20	2.45	0.0143	0.500	0.500	69.899	2.001	15.273
7	1.33	2.48	0.0132	0.434	0.566	75.996	1.767	15.107
8	1.45	2.50	0.0124	0.371	0.629	80.390	1.591	14.994
9	1.56	2.53	0.0114	0.319	0.681	87.521	1.469	14.919
10	1.66	2.56	0.0105	0.274	0.727	95.362	1.377	14.871
11	1.76	2.59	0.0096	0.234	0.766	103.992	1.305	14.846
12	**1.85**	**2.62**	**0.0088**	**0.199**	**0.801**	**113.495**	**1.249**	**14.838**
13	1.93	2.65	0.0081	0.170	0.830	123.969	1.204	14.844
14	2.01	2.68	0.0074	0.144	0.856	135.521	1.169	14.861
15	2.09	2.71	0.0067	0.122	0.878	148.271	1.140	14.887
16	2.16	2.75	0.0060	0.106	0.894	167.370	1.118	14.921
17	2.23	2.78	0.0055	0.090	0.910	183.470	1.098	14.961
18	2.29	2.81	0.0050	0.076	0.924	201.284	1.082	15.006
19	2.35	2.85	0.0044	0.066	0.934	228.050	1.070	15.056
20	2.41	2.88	0.0040	0.056	0.944	250.677	1.059	15.109

On comparing all 20 economic designs, the minimum value of *E(Q)* is found to be 14.838 and this occurs at sample size *n* = 12 as shown in Table 3.29. Similar to the results of SA, here also the values of expected total cost per hour *E(Q)* decreases with the increase of *n* value from 1 to 12 and after that it increases at higher values of *n*. The corresponding values of *h* and *k* at minimum loss cost are 1.85 hour and 2.62 respectively. This trend is also graphically illustrated in Fig. 3.18.

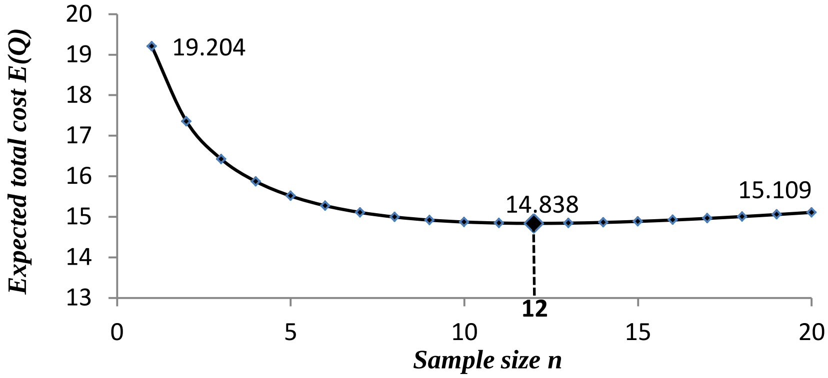

Fig. 3.18: Variation of expected total cost with sample size using TLBO

Table 3.30 shows the comparison of results of economic design of $\bar{X}$ chart for a continuous process using TLBO and SA. It is observed that for both cases, the optimal economic designs provide the same expected total cost per hour *E(Q)* = 14.838. The corresponding sample size (*n*) and the sampling interval (*h*) values are same, whereas width of control limits (*k*) are slightly different.

Table 3.30: Comparison of results obtained from SA and TLBO

Techniques	*n*	*h*	*k*	α	β	*P*	ARL_0	ARL_1	*E(Q)*
SA	12	1.85	2.61	0.0091	0.197	0.804	110.225	1.245	14.838
TLBO	12	1.85	2.62	0.0088	0.199	0.801	113.495	1.249	14.838

3.10 Conclusions

The major contribution of this chapter is development of design methodologies based on two metaheuristics, namely SA and TLBO for economic design of $\bar{X}$ chart for both continuous and discontinuous processes. Both the methodologies have been illustrated through numerical examples taken from literature. It is observed that both are providing nearly the same results and hence any of them can be recommended for use. Moreover, they are providing better results than that reported earlier in the literature. From the results of sensitivity analysis it can be concluded that the shift in process mean δ is the most significant factor affecting the selection of sample size *n* in both continuous and discontinuous processes and its effect is in negative direction. Similarly, the rate of occurrences of assignable cause λ is the most significant factor affecting the expected loss cost per unit time *E(L)* also in both types of processes but its effect is of positive type. For the width of control limits *k*, the cost per false alarm *Y* is observed to have the maximum effect in a continuous process, whereas the variable cost of sampling *b* is the most significant factor in a discontinuous process. For the sampling interval *h*, the fixed cost per sample *a* is the most significant factor in a continuous process, whereas the variable cost per sample *b* is the most significant factor in a discontinuous process.

CHAPTER - 4

Economic Statistical Design of $\overline{X}$ Chart

4.1 Introduction

The same two metaheuristics, namely simulated annealing (SA) and teaching-learning based optimization (TLBO) considered in Chapter 3 are used for the economic statistical design of $\overline{X}$ chart and the results are compared with that of economic design of the same chart in this chapter. In this work, two statistical constraints have been considered i.e., average run length (*ARL*) and average time to signal (*ATS*) while the objective is same as that of economic design i.e., to minimize the expected loss cost function. This overcomes the drawback of frequent false alarms and low power of detecting the process shift observed in economic design.

4.2 Economic Statistical Design Model

The constraints applied in economic statistical design are minimum value (i.e., lower bound) on the in-control average run length (ARL_L), maximum value (i.e., upper bound) on the out-of-control average run length (ARL_U), and maximum value (i.e., upper bound) on out-of-control average time to signal (ATS_U). These constraints are considered in the economic model to yield a design that meets statistical requirements and at the same time the expected loss cost function is minimized (Montgomery et al., 1995). Large value of ARL_0 is always desired when the process is in-control, whereas small ARL_1 value is preferred when the process is out-of-control. Another constraint is *average time to signal (ATS)* which is defined as the average time required to get the first signal that the process has gone out-of-control.

If ATS_0 and ATS_1 represent in-control and out-of-control average time to signal, then

$$ATS_0 = h \times ARL_0 = h/\alpha \quad (4.1)$$

$$ATS_1 = h \times ARL_1 = \frac{h}{(1-\beta)} = \frac{1}{P} \quad (4.2)$$

Thus, the economic statistical design of any control chart can be expressed as

$$\text{Minimize } E(L) \quad (4.3)$$

subject to

$$ARL_0 \geq ARL_L$$

$$ARL_1 \leq ARL_U$$

$$ATS_1 \leq ATS_U$$

where

$E(L)$ is the expected loss cost per unit time that is applicable to both the cost models (i.e., $E(L)_1$ for continuous process and $E(L)_2$ for discontinuous process). Both costs $E(L)_1$ and $E(L)_2$ can be calculated using Eqs. 3.20 and 3.31 respectively.

4.3 Numerical Illustration: Continuous Process

In order to illustrate the economic statistical design of $\bar{X}$ chart for a continuous process, the same numerical problem solved for economic design has been considered of which the input data are already shown in Table 3.2. Further, to meet the statistical requirements the limits on *ARL* and *ATS* are taken as ARL_L = 267, ARL_U = 40 and ATS_U = 1.90 (van Deventer and Manna, 2009). Thus, ARL_0 value should be at least 267 when the process is in-control, whereas ARL_1 and ATS_1 should not exceed 40 and 1.90 respectively when the process is out-of-control.

Thus, the economic statistical design of a continuous process can be modelled as:

$$\text{Minimize } E(L)_1 \tag{4.4}$$

subject to

$$ARL_0 \geq 267$$
$$ARL_1 \leq 40$$
$$ATS_1 \leq 1.90$$

It being a constrained optimization problem, a proper constraint handling technique is to be introduced. Economic design being an unconstrained optimization problem, the objective function is equal to the expected loss cost per unit time $E(L)_1$. But, in case of economic statistical design the objective function is modified by adding a penalty term for each constraint to the expected loss cost per unit time $E(L)_1$. Whenever there is a violation in any of three constraints, the corresponding penalty term is assigned with a very large number which makes the solution worst in itself i.e., the value of the modified objective function is made very high. This constrained optimization problem is solved using simulated annealing (SA) technique and the results are discussed below.

4.3.1 Results and Discussion: SA

Table 4.1 shows the results of economic statistical design of $\bar{X}$ chart for a continuous process using simulated annealing. These results consist of the optimal values of two design variables of control chart such as sampling interval (h) and the width of the control limits (k) for each integer value of sample size n varying from 4 to 33. For sample size n = 2 and 3, the value of modified objective function is highly penalized due to violation of constraints. Thus, no feasible solution is obtained for these two values of sample size. It also shows the corresponding optimum values of Type-I error (α), Type-II error (β), power of detecting the shift (P), average in-control run length (ARL_0), average out-of-control run length (ARL_1), average in-control time to signal (ATS_0), average out-of-control time to signal (ATS_1) and finally the corresponding expected loss cost per unit time ($E(L)_1$).

Table 4.1: Optimal economic statistical designs of $\bar{X}$ chart using SA: continuous process

n	h	k	α	β	P	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_1$
4	0.35	2.90	0.0037	0.816	0.184	267.119	5.433	92.918	1.90	43.082
5	0.48	2.90	0.0037	0.747	0.253	267.119	3.947	127.997	1.89	41.506
6	0.62	2.90	0.0037	0.674	0.326	267.119	3.066	164.873	1.90	40.424
7	0.76	2.90	0.0037	0.600	0.400	267.119	2.502	202.111	1.90	39.712
8	0.89	2.90	0.0037	0.529	0.472	267.119	2.121	238.535	1.89	39.286
9	1.02	2.90	0.0037	0.460	0.540	267.119	1.852	273.240	1.89	38.993
10	1.14	2.90	0.0037	0.397	0.603	267.119	1.657	305.545	1.89	38.848
11	**1.25**	**2.90**	**0.0037**	**0.338**	**0.662**	**267.119**	**1.512**	**335.052**	**1.89**	**38.816**
12	1.35	2.90	0.0037	0.286	0.714	267.119	1.401	361.557	1.89	38.876
13	1.44	2.90	0.0037	0.240	0.760	267.119	1.316	385.015	1.90	39.012
14	1.52	2.90	0.0037	0.200	0.800	267.119	1.250	405.501	1.90	39.215
15	1.58	2.90	0.0037	0.165	0.835	267.119	1.198	423.179	1.89	39.520
16	1.64	2.90	0.0037	0.136	0.864	267.119	1.157	438.268	1.90	39.839
17	1.69	2.90	0.0037	0.111	0.889	267.119	1.124	451.022	1.90	40.211
18	1.73	2.90	0.0037	0.090	0.910	267.119	1.099	461.700	1.90	40.634
19	1.76	2.90	0.0037	0.072	0.928	267.119	1.078	470.549	1.90	41.109
20	1.79	2.90	0.0037	0.058	0.942	267.119	1.062	477.865	1.90	41.588
21	1.81	2.91	0.0036	0.047	0.953	275.778	1.050	499.463	1.90	42.121
22	1.83	2.90	0.0037	0.037	0.963	267.119	1.038	488.748	1.90	42.652
23	1.84	2.93	0.0034	0.031	0.969	294.029	1.032	541.496	1.90	43.244
24	1.86	2.90	0.0037	0.023	0.977	267.119	1.023	495.815	1.90	43.774
25	1.87	2.91	0.0036	0.018	0.982	275.778	1.019	514.344	1.90	44.364
26	1.87	2.93	0.0034	0.015	0.985	294.029	1.015	550.181	1.90	45.013
27	1.88	2.95	0.0032	0.012	0.988	313.603	1.013	588.450	1.90	45.602
28	1.88	2.99	0.0028	0.011	0.989	357.145	1.011	671.239	1.90	46.253
29	1.89	2.91	0.0036	0.007	0.993	275.778	1.007	520.481	1.90	46.848
30	1.89	2.92	0.0035	0.005	0.995	284.744	1.005	538.183	1.90	47.502
31	1.89	3.00	0.0027	0.005	0.995	369.030	1.005	697.600	1.90	48.148
32	1.89	2.95	0.0032	0.003	0.997	313.603	1.003	593.830	1.90	48.812
33	1.88	3.06	0.0022	0.004	0.996	449.993	1.004	851.842	1.90	49.456

Further, a graph is plotted between expected loss cost per unit time $E(L)_1$ and sample size n as shown in Fig. 4.1. It is observed that $E(L)_1$ initially decreases as the value of n increases up to 11 and thereafter it increases. Thus, the most minimum expected loss cost per unit time is found to be $E(L)_1 = 38.816$ occurring at $n = 11$, and the corresponding optimal values of h and k are 1.25 hour and 2.90 respectively as shown in Table 4.1.

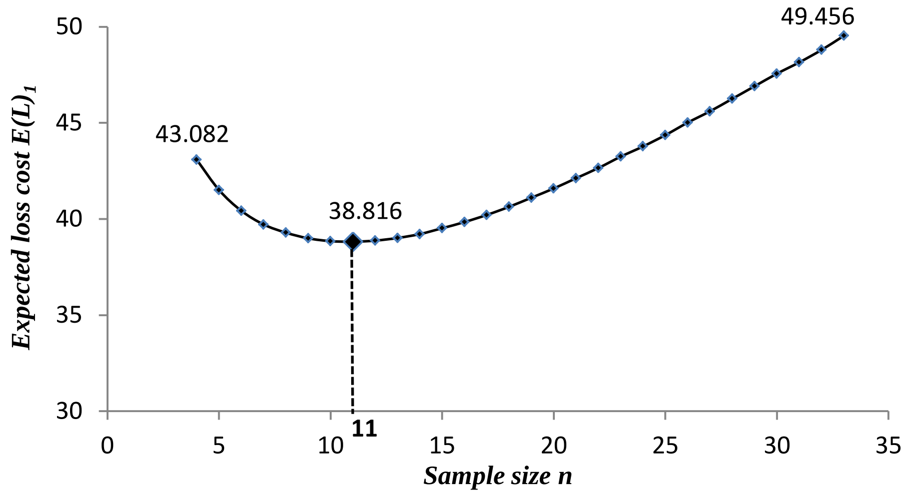

Fig. 4.1: Variation of expected loss cost per unit time with sample size using SA: continuous process

Further, for ease of comparison between economic design and economic statistical design, their optimal results for a continuous process are listed in Table 4.2. The optimal economic design result has been taken from Table 3.3. The corresponding values of two additional parameters ATS_0 and ATS_1 are calculated using Eqs. 4.1 and 4.2, and also listed in Table 4.2.

Table 4.2: Comparison of results between economic design and economic statistical design using SA: continuous process

Design	*n*	*h*	*k*	α	β	*P*	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_1$
ED-C	6	2.00	1.79	0.0735	0.255	0.745	13.613	1.342	27.202	2.68	34.720
ESD-C	11	1.25	2.90	0.0037	0.338	0.662	267.119	1.512	335.052	1.89	38.816

Note:
ED-C : Economic Design - Continuous process
ESD-C : Economic Statistical Design - Continuous process

Table 4.2 reveals that the sample size n is 6 in case of economic design, whereas it is nearly twice for economic statistical design (i.e., n = 11). The value of expected loss cost per unit time $E(L)_1$ is also higher in case of economic statistical design. Even if the cost is higher, it provides the benefit of more satisfactory values of statistical properties like higher value of ARL_0 and lower value of ATS_1. The increase in expected loss cost per unit time compared to that in economic design is:

$$\frac{(38.816-34.720)}{34.720}\times 100 = 11.79\%.$$

In case of economic design, the probability that a point falls outside the control limits for in-control process is α = 0.0735 and the corresponding $ARL_0 = 13.613 \approx 14$ (as shown in Table 4.2). So, when the process remains in-control, an out-of-control signal will be generated on an average after every 14 samples. This means that the false alarm is generated more frequently leading to unnecessary over-adjustment of the process and thereby loss of confidence of quality control personnel on the control chart. Therefore, it is required to keep the value of ARL_0 sufficiently large so that false alarms are avoided as far as possible. In case of economic statistical design, the value of ARL_0 has been increased to 267.119 ≈ 268 which is highly beneficial.

Table 4.2 also compares the effect of average time to signal during out-of-control process (ATS_1) between those two types of design of $\bar{X}$ chart for continuous process. The ATS_1 for the economic statistical design is much better than that for the economic design (i.e., 1.890 < 2.682) because it is able to detect the same magnitude of process shift much earlier. It is further observed that economic statistical design has smaller sampling interval compared to that of economic design (i.e., 1.25 < 2.00). This means samples are taken more frequently compared to economic design. This enhances the sampling cost and thereby the expected loss cost per unit time increases. However, the incorporation of ATS_1 constraint in economic statistical design helps in reduction of ATS_1 compared to that in economic design by:

$$\frac{(2.68-1.89)}{2.68} \times 100 = 29.47\%.$$

Thus, economic statistical design is observed to be costlier than economic design due to the addition of constraints. However, it assures a more satisfactory statistical performance in producing false alarms at less rate while detecting process shift at faster rate.

4.3.2 Results and Discussion: TLBO

In order to validate the results of economic statistical design, the same numerical problem discussed in Section 4.3.1 has been solved by teaching-learning based optimization (TLBO) and the results obtained are discussed below. Similar to Table 4.1, the results of economic statistical design of $\bar{X}$ chart for a continuous process obtained using TLBO are shown in Table 4.3 for all integer values of sample size n from 2 to 33. Here also, no feasible solution is obtained for the initial two sample sizes i.e., n = 2 and 3. Therefore, this table shows the results for sample size starting from 4.

Table 4.3: Optimal economic statistical designs of $\bar{X}$ chart using TLBO: continuous process

n	h	k	α	β	P	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_1$
4	0.35	2.90	0.0037	0.816	0.184	267.119	5.433	92.918	1.90	43.082
5	0.48	2.90	0.0037	0.747	0.253	267.119	3.947	127.997	1.89	41.506
6	0.62	2.90	0.0037	0.674	0.326	267.119	3.066	164.871	1.90	40.424
7	0.76	2.90	0.0037	0.600	0.400	267.119	2.502	202.107	1.90	39.712
8	0.89	2.90	0.0037	0.529	0.472	267.119	2.121	238.534	1.89	39.286
9	1.02	2.90	0.0037	0.460	0.540	267.119	1.852	273.236	1.89	38.993
10	1.14	2.90	0.0037	0.397	0.603	267.119	1.657	305.557	1.89	38.848
11	**1.25**	**2.90**	**0.0037**	**0.338**	**0.662**	**267.119**	**1.512**	**335.047**	**1.89**	**38.816**
12	1.35	2.90	0.0037	0.286	0.714	267.119	1.401	361.545	1.89	38.876
13	1.44	2.90	0.0037	0.240	0.760	267.119	1.316	385.025	1.90	39.012
14	1.52	2.90	0.0037	0.200	0.800	267.119	1.250	405.513	1.90	39.215
15	1.58	2.90	0.0037	0.165	0.835	267.119	1.198	423.169	1.89	39.520
16	1.64	2.90	0.0037	0.136	0.864	267.119	1.157	438.261	1.90	39.839
17	1.69	2.90	0.0037	0.111	0.889	267.119	1.124	451.030	1.90	40.211
18	1.73	2.90	0.0037	0.090	0.910	267.119	1.099	461.180	1.90	40.634
19	1.76	2.90	0.0037	0.072	0.928	267.119	1.078	470.476	1.90	41.109
20	1.79	2.90	0.0037	0.058	0.942	267.119	1.062	477.875	1.90	41.588
21	1.80	2.91	0.0036	0.047	0.953	275.780	1.050	497.424	1.89	42.172
22	1.83	2.92	0.0035	0.038	0.962	284.740	1.040	520.192	1.90	42.656
23	1.84	2.91	0.0036	0.030	0.970	275.780	1.031	508.318	1.90	43.241
24	1.85	2.95	0.0032	0.026	0.974	313.600	1.026	580.505	1.90	43.834
25	1.87	2.91	0.0036	0.018	0.982	275.780	1.019	514.468	1.90	44.364
26	1.87	2.99	0.0028	0.017	0.983	357.150	1.018	666.799	1.90	45.013
27	1.88	2.93	0.0034	0.012	0.988	294.030	1.012	551.571	1.90	45.603
28	1.88	3.01	0.0026	0.011	0.989	381.350	1.011	716.099	1.90	46.252
29	1.89	2.93	0.0034	0.007	0.993	294.030	1.007	554.688	1.90	46.845
30	1.89	2.94	0.0033	0.006	0.994	303.640	1.006	573.455	1.90	47.499
31	1.89	2.92	0.0035	0.004	0.996	284.740	1.004	537.532	1.90	48.159
32	1.89	3.00	0.0027	0.004	0.996	369.030	1.004	698.389	1.90	48.805
33	1.89	2.96	0.0031	0.003	0.997	323.920	1.003	613.699	1.90	49.469

The results of TLBO also show the similar type of variation of expected loss cost per unit time $E(L)_1$ with sample size n. It decreases with the increase of n from 4 to 11 and after that it increases at higher values of n. This variation is also graphically shown in Fig. 4.2. Thus, the optimal solution occurs at n = 11 and the corresponding minimum expected loss cost per unit time is $E(L)_1$ = 38.816 as shown in Table 4.3. The corresponding values of h and k at minimum loss cost are 1.25 hour and 2.90 respectively.

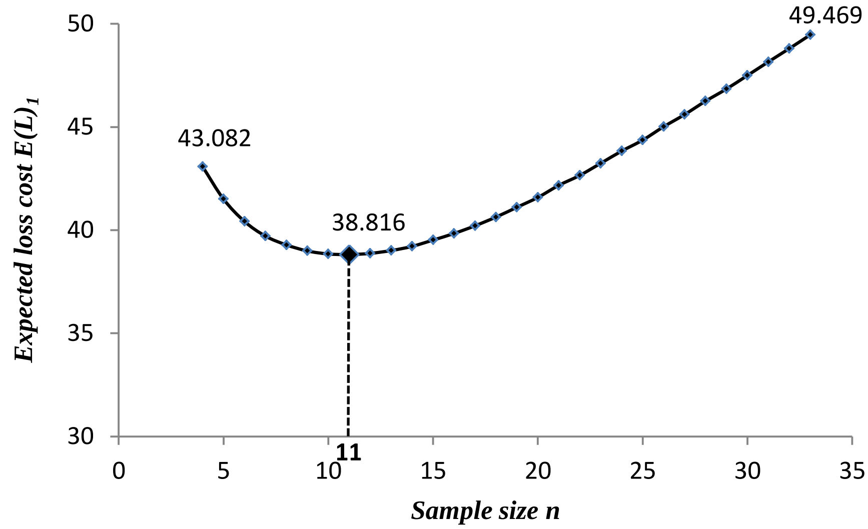

Fig. 4.2: Variation of expected loss cost per unit time with sample size using TLBO: continuous process

Table 4.4 shows the comparison of optimal results of economic statistical design of $\bar{X}$ chart for continuous process by TLBO with that of the results obtained from simulated annealing and it is observed that both the results are same. Thus, it is concluded that the results are validated to be correct.

Table 4.4: Comparison of results obtained from SA and TLBO in continuous process

Techniques	***n***	***h***	***k***	α	β	***P***	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_1$
SA	11	1.25	2.90	0.0037	0.338	0.662	267.119	1.512	335.052	1.89	38.816
TLBO	11	1.25	2.90	0.0037	0.338	0.662	267.119	1.512	335.047	1.89	38.816

4.4 Sensitivity Analysis: Continuous Process

To investigate the statistical significance of all the nine cost and process parameters (i.e., factors) listed in Table 3.7 on each of the four output responses (i.e., expected loss cost per unit time $E(L)_1$, sample size, n sampling interval h and width of control chart k), analysis of variance has been performed. A 2_{IV}^{9-4} factorial design with nine factors and four generators I = ABCF, I = ABDG, I = ABEH and I = ACDJ has been chosen for the model. This design has a total of 32 runs each representing a different combination of values of nine input factors. In addition to that, in each run the limiting values of statistical constraints are taken same as that already considered for a continuous process in Section 4.3 i.e., $ARL_0 \geq$

267, $ARL_1 \leq 40$ and $ATS_1 \leq 1.90$. The optimal values of design parameters *n*, *h* and *k* along with corresponding expected loss cost per unit time value $E(L)_1$ for all the respective 32 runs are found out using SA and listed in Table 4.5. Since both SA and TLBO algorithms provided almost the same results for economic statistical design of continuous process as observed in Section 4.3, any one of them is sufficient for sensitivity analysis.

Table 4.5: Optimal economic statistical designs of $\bar{X}$ chart: continuous process

S No.	n	h	k	α	β	P	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_1$
1	13	1.44	2.90	0.0037	0.240	0.760	267.119	1.316	389.479	1.90	4.010
2	14	1.52	2.90	0.0037	0.200	0.800	267.119	1.250	410.265	1.90	20.677
3	4	1.63	2.92	0.0035	0.140	0.860	285.714	1.163	466.887	1.90	10.076
4	6	1.46	3.67	0.0002	0.110	0.891	4091.151	1.123	5964.871	1.64	5.211
5	20	1.71	3.19	0.0014	0.099	0.900	714.286	1.111	1220.459	1.90	15.915
6	11	1.25	2.90	0.0037	0.339	0.662	267.119	1.512	335.052	1.89	38.816
7	4	1.52	3.15	0.0016	0.197	0.802	625.000	1.246	952.741	1.90	22.595
8	6	1.79	2.90	0.0037	0.023	0.977	267.119	1.023	483.232	1.83	22.184
9	10	1.14	2.91	0.0036	0.400	0.600	275.778	1.668	316.903	1.90	15.253
10	11	1.25	2.90	0.0037	0.339	0.662	267.119	1.512	339.002	1.89	16.191
11	6	1.76	3.45	0.0006	0.074	0.926	1772.704	1.080	3118.269	1.90	8.948
12	3	1.35	2.90	0.0037	0.286	0.714	267.119	1.401	365.560	1.89	8.464
13	10	1.14	2.90	0.0037	0.397	0.603	267.119	1.657	309.150	1.90	30.362
14	5	0.39	3.05	0.0023	0.792	0.208	434.783	4.811	170.713	1.90	40.351
15	3	0.85	3.36	0.0008	0.459	0.542	1275.705	1.847	1086.778	1.57	21.735
16	3	1.35	2.90	0.0037	0.286	0.714	267.119	1.401	365.655	1.89	30.617
17	18	1.54	3.37	0.0008	0.191	0.809	1322.716	1.237	2032.303	1.90	12.591
18	14	1.52	2.91	0.0036	0.203	0.797	275.778	1.254	421.079	1.90	31.697
19	4	1.64	2.90	0.0037	0.136	0.864	267.119	1.157	443.439	1.90	14.644
20	6	1.70	3.64	0.0003	0.104	0.896	3640.467	1.116	6191.565	1.90	20.367
21	22	1.76	3.25	0.0012	0.075	0.925	833.333	1.081	1465.283	1.90	36.448
22	11	1.25	2.90	0.0037	0.339	0.662	267.119	1.512	339.005	1.89	62.286
23	4	1.57	3.06	0.0022	0.174	0.826	454.545	1.210	714.285	1.90	29.857
24	6	1.85	2.95	0.0032	0.026	0.974	312.500	1.026	578.621	1.90	60.684
25	10	1.14	2.90	0.0037	0.397	0.603	267.119	1.657	308.990	1.90	22.908
26	12	1.35	2.90	0.0037	0.286	0.714	267.119	1.401	365.812	1.89	26.372
27	6	1.79	3.32	0.0009	0.057	0.943	1104.850	1.061	1974.924	1.90	13.294
28	3	1.22	3.10	0.0019	0.358	0.642	526.316	1.557	641.041	1.90	23.531
29	12	1.35	2.90	0.0037	0.286	0.714	267.119	1.401	365.823	1.89	46.770
30	7	0.69	3.00	0.0027	0.638	0.362	370.370	2.766	255.634	1.90	59.007
31	3	1.34	2.90	0.0037	0.286	0.714	267.119	1.401	360.856	1.88	28.071
32	4	1.64	2.90	0.0037	0.136	0.864	267.119	1.157	443.304	1.90	65.404

Tables 4.6 - 4.9 show the results of ANOVA conducted on 32 sets of results of economic statistical design listed in Table 4.5 at significance level of 5% for identifying the significant factors affecting the four responses. The significant factors are easily identified in the normal plots of standardized effects for four output responses as shown in Figs. 4.3 - 4.6.

Table 4.6: Analysis of variance for expected loss cost per unit time $E(L)_1$ with constraints: continuous process

Source	DF	Seq SS	Adj SS	Adj MS	F	p	PC(%)
M	1	1229.85	1229.85	1229.85	27.37	0.000*	13.97
δ	1	275.96	275.96	275.96	6.14	0.021*	3.14
λ	1	3979.84	3979.84	3979.84	88.56	0.000*	45.22
g	1	75.70	75.70	75.70	1.68	0.208	0.86
(T_1+T_2)	1	1838.04	1838.04	1838.04	40.90	0.000*	20.88
a	1	38.78	38.78	38.78	0.86	0.363	0.44
b	1	213.10	213.10	213.10	4.74	0.040*	2.42
W	1	156.85	156.85	156.85	3.49	0.075	1.78
Y	1	4.33	4.33	4.33	0.10	0.759	0.05
Residual Error	22	988.69	988.69	44.94			
Total	31	8801.14					

* Significant at 5%

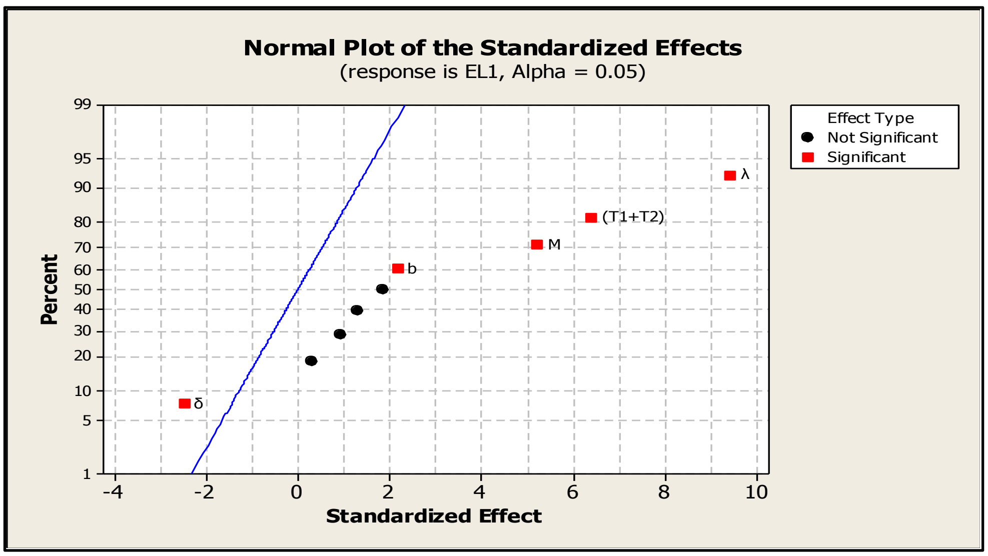

Fig. 4.3: Normal probability plot of standardized effect for expected loss cost per unit time $E(L)_1$ with constraints: continuous process

Table 4.6 indicates that in a continuous process, the expected loss cost per unit time of process control $E(L)_1$ is significantly affected by five factors, namely loss of net income when process is out-of-control M, the shift from the mean δ, rate of occurrence of assignable cause λ, time to find and repair an assignable cause (T_1+T_2), and variable cost per sample b. They are also graphically shown as "significant" in the normal plot as shown in Fig. 4.3.

Among all the factors, λ has the highest significant effect on expected loss cost per unit time $E(L)_1$ since it has the highest F-value i.e., 88.56 as shown in Table 4.6 and plotted at the rightmost location in Fig. 4.3. It can also be observed from this table that λ, (T_1+T_2), M, δ and b are the top five percentage contributors which affect the cost by 45.22%, 20.88%, 13.97%, 3.14% and 2.42% respectively. Out of these five significant factors, all the factors are having positive effect except δ which is having negative effect as shown in Fig. 4.3.

Table 4.7 presents an analysis of variance on sample size *n*. As δ, *g* and *b* get smaller, the optimum sample size increases, because they have the negative effect. The fourth most significant factor is the fixed cost (*a*) which has positive effect. Moreover, it can be observed from Table 4.7 that the factors like δ, *g*, *a* and *b* are the major percentage contributors which affect the sample size by 61.91%, 11.25%, 5.09% and 3.13% respectively. Thus, δ is the most significant factor for choosing the value of sample size, in economic statistical design and the effect is of negative type as shown in Fig. 4.4.

Table 4.7: Analysis of variance for sample size *n* with constraints: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	22.781	22.781	22.781	4.16	0.054	2.71
δ	1	520.031	520.031	520.031	94.99	0.000*	61.91
λ	1	2.531	2.531	2.531	0.46	0.504	0.30
g	1	94.531	94.531	94.531	17.27	0.000*	11.25
(T_1+T_2)	1	5.281	5.281	5.281	0.96	0.337	0.63
a	1	42.781	42.781	42.781	7.81	0.011*	5.09
b	1	26.281	26.281	26.281	4.80	0.039*	3.13
W	1	1.531	1.531	1.531	0.28	0.602	0.18
Y	1	3.781	3.781	3.781	0.69	0.415	0.45
Residual Error	22	120.438	120.438	5.474			
Total	31	839.969					

* Significant at 5%

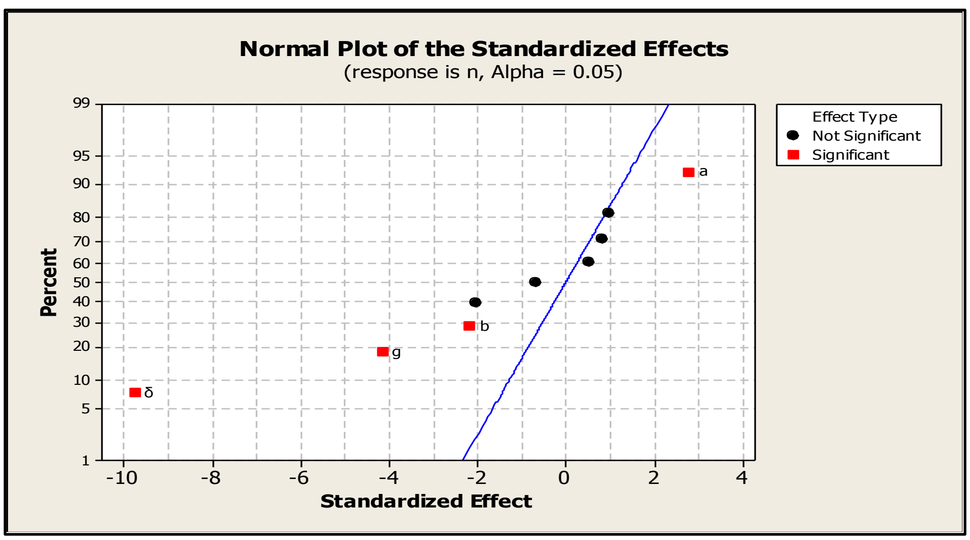

Fig. 4.4: Normal probability plot of standardized effect for sample size *n* with constraints: continuous process

[Table 4.8](#) displays an analysis of variance on the sampling interval *h*. It is significantly affected by three factors i.e., δ, g and a. Out of these three significant factors, two factors i.e., δ and a have positive effects, whereas the factor g is significant in terms of negative effect as shown in Fig. 4.5. Among all the factors, time to sample and chart one item g has the highest effect on the sampling interval with a percentage contribution of 27.08% and the effect is in negative direction.

Table 4.8: Analysis of variance for sampling interval *h* with constraints: continuous process

Source	***DF***	***Seq SS***	***Adj SS***	***Adj MS***	***F***	***p***	***PC(%)***
M	1	0.095	0.095	0.095	2.73	0.113	2.83
δ	1	0.503	0.503	0.503	14.53	0.001*	15.05
λ	1	0.123	0.123	0.123	3.56	0.072	3.69
g	1	0.905	0.905	0.905	26.16	0.000*	27.08
(T_1+T_2)	1	0.099	0.099	0.099	2.86	0.105	2.96
a	1	0.845	0.845	0.845	24.41	0.000*	25.27
b	1	0.005	0.005	0.005	0.15	0.698	0.16
W	1	0.001	0.001	0.001	0.02	0.889	0.02
Y	1	0.005	0.005	0.005	0.15	0.705	0.15
Residual Error	22	0.761	0.761	0.035			
Total	31	3.342					

* Significant at 5%

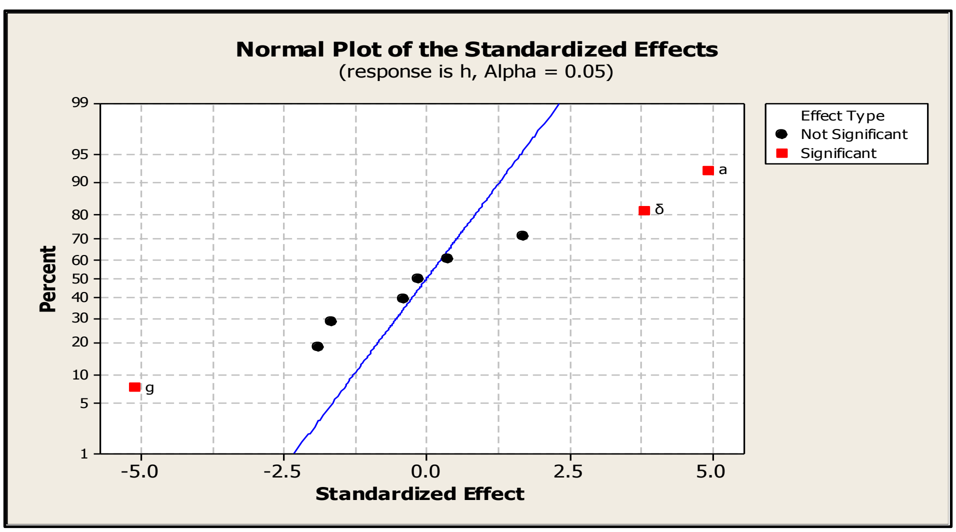

Fig. 4.5: Normal probability plot of standardized effect for sampling interval *h* with constraints: continuous process

Table 4.9 presents an analysis of variance on the width of control limits *k*. There are three factors δ, b and Y which have significant effects on width of control limits *k*. Fig. 4.6 reveals that out of these three significant factors, two factors i.e., δ and Y have positive effect, whereas the remaining one factor i.e., *b* has negative effect. Among all the factors, the variable cost of sampling *b* is observed to be the most significant factor which contributes 26.11% on deciding the value of control limits width in economic statistical design and the effect is of negative type.

Table 4.9: Analysis of variance for width of control limits *k* with constraints: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC (%)*
M	1	0.028	0.028	0.028	1.12	0.302	1.74
δ	1	0.156	0.156	0.156	6.16	0.021*	9.58
λ	1	0.060	0.060	0.060	2.35	0.140	3.65
g	1	0.045	0.045	0.045	1.76	0.198	2.75
(T_1+T_2)	1	0.000	0.000	0.000	0.00	0.981	0.00
a	1	0.080	0.080	0.080	3.16	0.089	4.92
b	1	0.425	0.425	0.425	16.78	0.000*	26.11
W	1	0.069	0.069	0.069	2.71	0.114	4.21
Y	1	0.208	0.208	0.208	8.23	0.009*	12.80
Residual Error	22	0.558	0.558	0.025			
Total	31	1.628					

* Significant at 5%

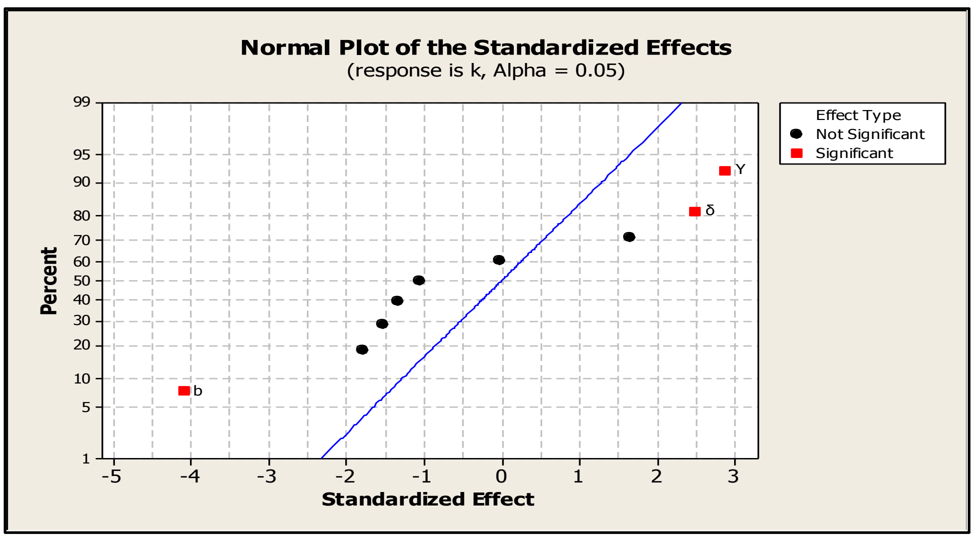

Fig. 4.6: Normal probability plot of standardized effect for width of control limits *k* with constraints: continuous process

It is further observed from Tables 4.6 - 4.9 that the cost to locate and repair the assignable cause *W*, has no significance on any of the responses *n*, *h*, *k* and $E(L)_1$.

4.4.1 Summary of Results

Similar to Table 3.13, all the significant factors in case of economic statistical design for a continuous process with respect to each of the four responses are summarized in Table 4.10. This table also shows the corresponding significant factors in case of economic design already shown in Table 3.13 for the ease of comparison of both the sets of results.

Table 4.10: Comparison of significant effects in economic design and economic statistical design: continuous process

Output responses	***Design***	***Cost and process parameters***								
		M	***δ***	***λ***	***g***	***(T_1+T_2)***	***a***	***b***	***W***	***Y***
n	*ED-C*		**–**		–			–		+
	ESD-C		**–**		–		+	–		
h	*ED-C*	–	–	–		+	**+**	+		
	ESD-C		+		**–**		+			
k	*ED-C*		+	–	–		–	–		**+**
	ESD-C		+					**–**		+
$E(L)_1$	*ED-C*	+		**+**		+				
	ESD-C	+	–	**+**		+		+		

Note:

ED-C	: Economic Design - Continuous process
ESD-C	: Economic Statistical Design - Continuous process
Blank space	: Insignificant factor
+	: Factor with positive effect
–	: Factor with negative effect
+/– in bold	: Most significant factor

From Table 4.10 it is observed that the shift size δ is the most significant factor for selecting the value of sample size *n* in case of economic design (*ED*) as well as economic statistical design (*ESD*) and its effect is of negative type. Similarly, the rate of occurrence of assignable cause λ has the most significant effect over the expected loss cost per unit time $E(L)_1$ in case of both types of the designs. But unlike sample size *n*, the factor λ has the positive effect. On the other hand, the most significant factors are different in these two designs so far as other two output responses are concerned i.e., sampling interval *h* and width of control limits *k*. In case of economic design, the fixed cost *a* is found to be the most significant factor with positive effect on sampling interval *h*, whereas in case of economic statistical design the time to sample and chart one item g is the most significant factor with negative effect. Similarly, so far as the effect on the width of control limits k is concerned, the cost per false alarm Y is the most significant factor with positive effect in economic design while the variable cost of sampling b is the most significant factor with negative effect in case of economic statistical design.

This table also shows that significant parameters are not always same in both economic and economic statistical design. Thus, the users of control charts must take utmost care in ensuring the correctness of values of significant factors before using them in economic design or economic statistical design.

4.5 Numerical Illustration: Discontinuous Process

For illustration of the economic statistical design, the same numerical problem solved for economic design of discontinuous process in Section 3.7 has been considered. Therefore, the values of all the thirteen cost and process parameters are taken same as shown in Table 3.14. The limiting values of statistical constraints are same as that already considered in case of economic statistical design of continuous process as discussed in Section 4.3. But there is a difference in the objective function i.e., to minimize the expected loss cost per unit time for discontinuous process $E(L)_2$ instead of that for continuous process $E(L)_1$. Thus, the economic statistical design of discontinuous process can be modelled as:

$$\text{Minimize } E(L)_2 \tag{4.5}$$

subject to

$$ARL_0 \geq 267$$
$$ARL_1 \leq 40$$
$$ATS_1 \leq 1.90$$

This represents a constrained optimization problem which is solved with the help of simulated annealing and the results are discussed below.

4.5.1 Results and Discussion: SA

Table 4.11 shows the results of optimal economic statistical design of $\bar{X}$ chart for a discontinuous process using SA i.e., the optimal values of two design variables of control chart such as sampling interval (h) and the width of the control limits (k) for each integer value of sample size n varying from 4 to 33. It also shows the corresponding optimum values of Type-I error (α), Type-II error (β), power of detecting the shift (P), average in-control run length (ARL_0), average out-of-control run length (ARL_1), average in-control time to signal (ATS_0), average out-of-control time to signal (ATS_1) and finally the expected loss cost per unit time ($E(L)_2$).

Table 4.11: Optimal economic statistical designs of $\bar{X}$ chart using SA: discontinuous process

n	*h*	*k*	*α*	*β*	*P*	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_2$
4	0.35	2.90	0.0037	0.816	0.184	267.119	5.433	92.918	1.90	42.477
5	0.48	2.90	0.0037	0.747	0.253	267.119	3.947	127.997	1.89	40.838
6	0.62	2.90	0.0037	0.674	0.326	267.119	3.066	164.873	1.90	39.631
7	0.76	2.90	0.0037	0.600	0.400	267.119	2.502	202.111	1.90	38.784
8	0.89	2.90	0.0037	0.529	0.472	267.119	2.121	238.521	1.89	38.229
9	1.02	2.90	0.0037	0.460	0.540	267.119	1.852	273.240	1.89	37.821
10	1.14	2.90	0.0037	0.397	0.603	267.119	1.657	305.542	1.89	37.570
11	1.25	2.90	0.0037	0.339	0.662	267.119	1.512	334.949	1.89	37.439
12	**1.35**	**2.91**	**0.0036**	**0.290**	**0.710**	**275.778**	**1.408**	**372.736**	**1.90**	**37.392**
13	1.44	2.91	0.0036	0.243	0.757	275.778	1.322	397.247	1.90	37.436
14	1.52	2.91	0.0036	0.203	0.797	275.778	1.254	418.374	1.91	37.549
15	1.58	2.90	0.0037	0.165	0.835	267.119	1.198	423.055	1.89	37.773
16	1.63	2.94	0.0033	0.145	0.855	303.644	1.169	493.670	1.91	37.982
17	1.68	2.93	0.0034	0.116	0.884	294.029	1.132	493.423	1.90	38.273
18	1.71	2.96	0.0031	0.100	0.900	323.919	1.111	554.159	1.90	38.599
19	1.72	3.04	0.0024	0.094	0.906	421.046	1.103	725.571	1.90	38.972
20	1.76	3.02	0.0025	0.073	0.927	394.108	1.079	693.742	1.90	39.331
21	1.78	3.05	0.0023	0.063	0.937	435.259	1.067	775.188	1.90	39.735
22	1.78	3.17	0.0015	0.064	0.936	652.992	1.069	1160.258	1.90	40.190
23	1.81	3.12	0.0018	0.047	0.953	550.556	1.049	997.394	1.90	40.607
24	1.82	3.19	0.0014	0.044	0.956	699.575	1.046	1270.458	1.90	41.065
25	1.85	3.10	0.0019	0.029	0.971	514.566	1.030	949.673	1.90	41.509
26	1.84	3.21	0.0013	0.030	0.971	749.759	1.030	1382.989	1.90	42.049
27	1.87	3.08	0.0021	0.017	0.983	481.108	1.018	898.418	1.90	42.517
28	1.87	3.13	0.0018	0.015	0.985	569.563	1.016	1065.716	1.90	43.036
29	1.88	3.15	0.0016	0.013	0.987	609.739	1.013	1143.605	1.90	43.526
30	1.87	3.28	0.0010	0.014	0.986	958.309	1.014	1795.526	1.90	44.095
31	1.88	3.26	0.0011	0.011	0.990	892.997	1.011	1679.072	1.90	44.604
32	1.89	3.16	0.0016	0.006	0.994	630.966	1.006	1191.366	1.90	45.142
33	1.89	3.19	0.0014	0.005	0.995	699.575	1.005	1322.158	1.90	45.694

In case of economic statistical design, as the sample size *n* is varied from 2 to 33, the most minimum cost is found to be $E(L)_2$ = 37.392 and this occurs at *n* = 12 as shown in Table 4.11. Here also the values of expected loss cost per unit time $E(L)_2$ decreases with the increase of *n* from 4 to 12 and after that this value increases at higher values of *n*. The corresponding values of *h* and *k* at minimum loss cost are 1.35 hour and 2.91 respectively. This trend is also graphically shown in Fig. 4.7.

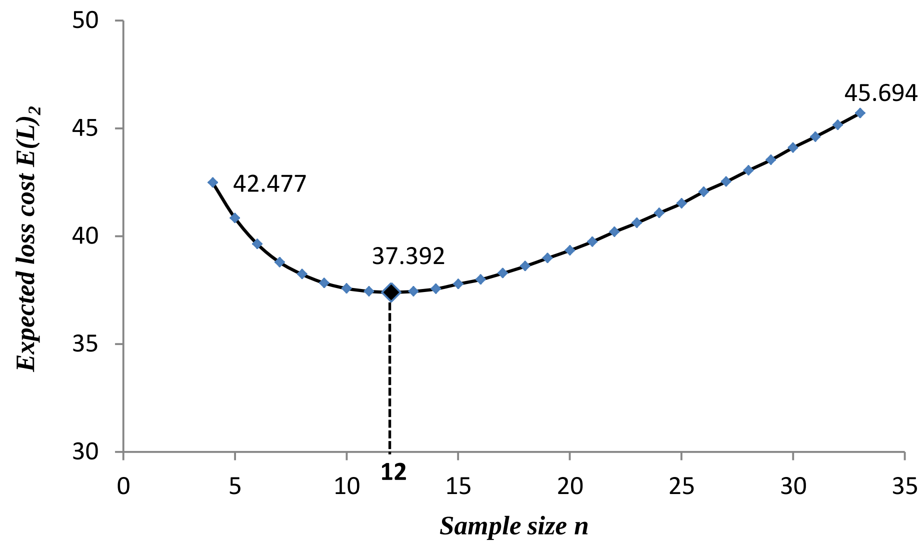

Fig. 4.7: Variation of expected loss cost per unit time with sample size using SA: discontinuous process

Table 4.12 shows a comparison between results of economic design and economic statistical design for a discontinuous process. This table shows the optimal values of three control chart parameters *n*, *h* and *k*, along with the corresponding expected loss cost per unit time $E(L)_2$. It also shows corresponding values of errors α and β, power of detection *P* and *ARLs*. The optimal economic design result has been taken from Table 3.15. The corresponding values of two additional parameters ATS_0 and ATS_1 are calculated using Eqs. 4.1 and 4.2, and also listed in Table 4.12.

Table 4.12: Comparison of results between economic design and economic statistical design using SA: discontinuous process

Design	*n*	*h*	*k*	α	β	*P*	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_2$
ED-D	12	2.50	2.75	0.0060	0.238	0.762	167.37	1.312	418.425	3.28	35.966
ESD-D	12	1.35	2.91	0.0036	0.290	0.710	275.778	1.408	372.736	1.90	37.392

Note:
ED-D : Economic Design - Discontinuous process
ESD-D : Economic Statistical Design - Discontinuous process

Table 4.12 reveals that the sample size *n* is 12 in economic design as well as economic statistical design. This table also shows that the desired statistical properties can be achieved at some higher cost. The increase in overall expected cost in economic statistical design compared to that in economic design is:

$$\frac{(37.392\ -\ 35.966)}{35.966}\times 100 = 3.96\%.$$

In case of economic design, the probability that a point falls outside the control limits when the process is in-control is α = 0.0060 as shown in Table 4.12. So, when the process remains in-control, an out-of-control signal will be generated on an average after every 167.37 ≈ 168 samples. This means that the false alarm is being generated more frequently leading to unnecessary over-adjustment of the process and loss of confidence of quality control personnel. Therefore, it is required to keep the value of ARL_0 comparatively larger so that false alarms are avoided as far as possible. This value has been increased to 275.778 ≈ 276 samples in economic statistical design which is satisfactorily high value.

Table 4.12 shows the effect of average time to signal when the process is out-of-control (ATS_1) compared to the economic design model of discontinuous process. ATS_1 for the economic statistical design is much better than the corresponding ATS_1 for the economic design (i.e., 1.90 against 3.28). It is observed that economic design with statistical constraint has smaller sampling interval (i.e., h = 1.35 hour) compared to economic design (i.e., h = 2.50 hour). The application of ATS constraint assures that the average signaling time for out-of-control process is hugely reduced in economic statistical design compared to that in economic design and the percentage reduction is:

$$\frac{(3.28 - 1.90\ )}{3.28}\times 100 = 42.07\%.$$

Table 4.12 shows that economic statistical design is costlier than economic design due to the addition of constraints. However, the economic statistical design assures a satisfactory statistical performance. It reduces rate of false alarm (i.e., lower ARL_0) and quickly detects the process shift (i.e., lower ATS_1).

4.5.2 Results and Discussion: TLBO

In order to illustrate the TLBO based methodology of economic statistical design of $\bar{X}$ chart for a discontinuous process, the same numerical problem of economic design mentioned in Section 3.7 has been considered. The cost and process parameters are same as that shown in Table 3.14. The results of optimal economic statistical design using TLBO i.e., the optimal values of two design variables of control chart such as sampling interval (h) and the width of the control limits (k) for each integer value of sample size n varying from 1 to 33 are shown in Table 4.13. It also shows the corresponding optimum values of Type-I error (α), Type-II error (β), power of detecting the shift (P), average in-control run length (ARL_0), average out-of-control run length (ARL_1), average in-control time to signal (ATS_0), average out-of-control time to signal (ATS_1) and finally the expected loss cost per unit time ($E(L)_2$).

Table 4.13: Optimal economic statistical designs of $\bar{X}$ chart using TLBO: discontinuous process

n	*h*	*k*	α	β	*P*	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_2$
4	0.35	2.90	0.0037	0.816	0.184	267.119	5.433	92.912	1.90	42.477
5	0.48	2.90	0.0037	0.747	0.253	267.119	3.947	127.995	1.89	40.838
6	0.62	2.90	0.0037	0.674	0.326	267.119	3.066	164.874	1.90	39.631
7	0.76	2.90	0.0037	0.600	0.400	267.119	2.502	202.113	1.90	38.784
8	0.89	2.90	0.0037	0.529	0.472	267.119	2.121	238.526	1.89	38.229
9	1.02	2.90	0.0037	0.460	0.540	267.119	1.852	273.209	1.89	37.821
10	1.14	2.90	0.0037	0.397	0.603	267.119	1.657	305.530	1.89	37.570
11	1.25	2.90	0.0037	0.339	0.662	267.119	1.512	335.020	1.89	37.439
12	**1.35**	**2.90**	**0.0036**	**0.290**	**0.710**	**275.778**	**1.408**	**372.736**	**1.90**	**37.392**
13	1.44	2.90	0.0037	0.240	0.760	267.119	1.316	384.998	1.90	37.449
14	1.52	2.90	0.0037	0.200	0.800	267.119	1.250	405.406	1.90	37.563
15	1.58	2.91	0.0036	0.168	0.832	275.780	1.202	435.981	1.90	37.758
16	1.64	2.91	0.0036	0.138	0.862	275.780	1.160	451.810	1.90	37.987
17	1.68	2.92	0.0035	0.114	0.886	284.740	1.129	478.876	1.90	38.288
18	1.71	2.95	0.0032	0.098	0.902	313.600	1.109	537.416	1.90	38.612
19	1.74	2.98	0.0029	0.084	0.916	345.680	1.092	601.829	1.90	38.950
20	1.77	3.00	0.0027	0.070	0.930	369.030	1.076	651.412	1.90	39.311
21	1.78	3.03	0.0025	0.060	0.940	407.340	1.064	727.020	1.89	39.753
22	1.81	3.05	0.0023	0.050	0.950	435.260	1.053	785.688	1.91	40.128
23	1.82	3.08	0.0021	0.043	0.957	481.110	1.045	874.369	1.90	40.588
24	1.83	3.09	0.0020	0.035	0.965	497.530	1.037	911.972	1.90	41.071
25	1.84	3.11	0.0019	0.029	0.971	532.230	1.030	981.326	1.90	41.551
26	1.85	3.18	0.0015	0.027	0.973	675.850	1.028	1248.903	1.90	42.010
27	1.86	3.15	0.0016	0.020	0.980	609.740	1.021	1134.787	1.90	42.524
28	1.86	3.20	0.0014	0.018	0.982	724.200	1.019	1350.561	1.89	43.056
29	1.87	3.21	0.0013	0.015	0.985	749.760	1.015	1403.476	1.90	43.555
30	1.88	3.20	0.0014	0.011	0.989	724.200	1.012	1360.120	1.90	44.062
31	1.88	3.27	0.0011	0.011	0.989	925.030	1.011	1738.686	1.90	44.601
32	1.88	3.27	0.0011	0.009	0.991	925.030	1.009	1742.757	1.90	45.167
33	1.89	3.27	0.0011	0.007	0.993	925.030	1.007	1745.809	1.90	45.670

In case of economic statistical design, from the results obtained for all values of sample size *n* in the range 2 to 33, the most minimum cost is found to be $E(L)_2$ = 37.392 and this occurs at *n* = 12 as shown in Table 4.13. The values of expected loss cost per unit time $E(L)_2$ decreases with the increase of *n* value from 4 to 11 and after that this cost increases at higher values of *n*. The corresponding values of *h* and *k* at minimum loss cost are 1.35 hour and 2.90 respectively. This trend is also graphically viewed in Fig. 4.8.

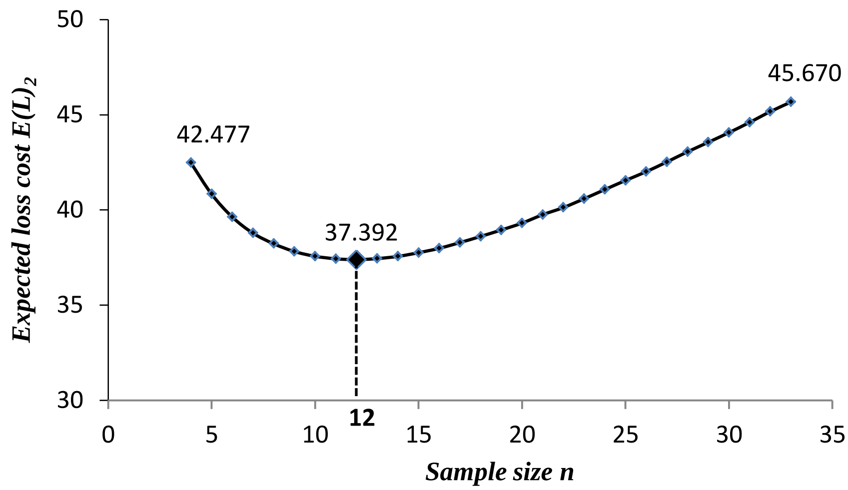

Fig. 4.8: Variation of expected loss cost per unit time with sample size using TLBO: discontinuous process

Table 4.14 shows the comparison of results of economic statistical design of $\bar{X}$ chart for discontinuous process by TLBO with that of the results obtained from SA. It is observed that for both the scenarios the sample size (*n*), the sampling interval (*h*) and width of control limits (*k*) are same. Thus, both the metaheuristics are observed to be providing the same results for economic statistical design of $\bar{X}$ chart for discontinuous process.

Table 4.14: Comparison of results obtained from SA and TLBO in discontinuous process

Techniques	***n***	***h***	***k***	α	β	***P***	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_2$
SA	12	1.35	2.91	0.0036	0.290	0.710	275.778	1.408	372.736	1.90	37.392
TLBO	12	1.35	2.90	0.0036	0.290	0.710	275.778	1.408	372.736	1.90	37.392

4.6 Sensitivity Analysis: Discontinuous Process

Similar to continuous process, sensitivity analysis has been done to investigate the effect of cost and process parameters along with the three statistical constraints on the output results of economic design in case of discontinuous process. The low and high values of all thirteen cost and process parameters (also termed as factors) are taken same as that listed in Table 3.19 for economic design. At each run, the values of thirteen parameters are taken as per the same factorial design considered for economic design as mentioned in Section 3.8 and the expected loss cost function per hour $E(L)_2$ is minimized by running the same MATLAB program developed on the basis of SA. Since both SA and TLBO algorithms provided almost the same results for economic statistical design for discontinuous process as observed in Section 4.5, any one of them is sufficient for sensitivity analysis. A 2_{IV}^{13-8} factorial design with thirteen factors and eight generators I = ABCF, I = ABDG, I = ABEH, I = ACDJ, I = ACEK, I = ADEL, I = BCDM and I = BCEN with 32 runs has been selected for the discontinuous model. Table 4.15 shows all 32 sets of results of optimal economic statistical designs of $\bar{X}$ chart for a discontinuous process.

Table 4.15: Optimal economic statistical designs of $\bar{X}$ chart: discontinuous process

S No.	*n*	*h*	*k*	α	β	*P*	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_1$
1	16	1.48	3.23	0.0012	0.221	0.779	833.333	1.283	1235.713	1.90	4.380
2	15	1.49	3.08	0.0021	0.214	0.786	476.190	1.272	710.928	1.90	23.883
3	6	1.74	3.52	0.0004	0.084	0.916	2301.761	1.092	4004.567	1.90	11.468
4	8	1.60	4.31	0.0000	0.089	0.911	60488.749	1.098	96782.253	1.76	9.109
5	27	1.71	3.92	0.0001	0.101	0.899	11186.183	1.112	19093.686	1.90	33.015
6	12	1.35	2.91	0.0036	0.290	0.710	275.778	1.408	372.736	1.90	37.392
7	5	1.63	3.40	0.0007	0.142	0.858	1475.187	1.165	2407.685	1.90	35.450
8	6	1.65	3.14	0.0017	0.039	0.961	588.235	1.041	967.780	1.72	19.930
9	10	1.13	2.92	0.0035	0.404	0.596	285.714	1.679	322.480	1.90	16.738
10	13	1.33	3.09	0.0020	0.303	0.697	500.000	1.435	662.938	1.90	20.140
11	7	1.76	3.84	0.0001	0.073	0.927	8055.421	1.079	14174.184	1.90	10.058
12	5	1.43	3.80	0.0001	0.251	0.749	6851.662	1.335	9765.060	1.90	12.437
13	15	1.37	3.29	0.0010	0.280	0.720	1000.000	1.389	1365.302	1.90	47.496
14	6	0.38	3.29	0.0010	0.799	0.200	1000.000	4.992	380.476	1.90	36.393
15	4	1.27	3.56	0.0004	0.330	0.670	2677.735	1.493	3414.024	1.90	41.564
16	3	1.34	2.92	0.0035	0.293	0.707	285.714	1.415	383.641	1.90	25.685
17	24	1.53	4.03	0.0001	0.192	0.808	17744.025	1.238	27187.285	1.89	30.118
18	16	1.52	3.15	0.0016	0.198	0.802	625.000	1.246	952.632	1.89	23.114
19	5	1.68	3.26	0.0011	0.113	0.887	909.091	1.127	1531.689	1.89	32.670
20	7	1.59	3.83	0.0001	0.072	0.928	7735.149	1.078	12336.308	1.71	12.424
21	21	1.71	3.30	0.0010	0.099	0.900	1028.785	1.111	1760.764	1.90	37.199
22	12	1.35	2.90	0.0037	0.286	0.714	270.270	1.401	365.809	1.89	84.078
23	5	1.59	3.49	0.0005	0.163	0.837	2056.851	1.195	3275.574	1.90	31.084
24	9	1.84	4.12	0.0000	0.030	0.970	26103.527	1.031	48092.333	1.90	83.909
25	16	1.34	3.46	0.0005	0.295	0.705	1839.588	1.418	2463.748	1.90	41.597
26	14	1.30	3.26	0.0011	0.315	0.685	909.091	1.460	1184.078	1.90	20.513
27	7	1.78	3.74	0.0002	0.060	0.939	5389.673	1.064	9618.989	1.89	29.086
28	3	1.32	2.95	0.0032	0.304	0.696	312.500	1.436	412.556	1.90	15.833
29	11	1.25	2.90	0.0037	0.339	0.661	270.270	1.512	339.005	1.89	41.621
30	9	0.75	3.27	0.0011	0.606	0.394	909.091	2.541	678.598	1.90	82.821
31	4	1.08	3.61	0.0003	0.348	0.652	3242.122	1.534	3490.860	1.66	28.911
32	5	1.57	3.54	0.0004	0.176	0.824	2482.190	1.213	3885.089	1.90	90.268

Tables 4.16 - 4.19 show the results of ANOVA at significance level of 5% for identifying the significant factors affecting the four responses. The significant factors can be more easily identified in the normal plots of standardized effects for four output responses as shown in Figs. 4.9 - 4.12.

Table 4.16: Analysis of variance for expected loss cost per unit time $E(L)_2$ with constraints: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC (%)*
M	1	491.900	491.900	491.900	3.49	0.078	3.07
δ	1	256.600	256.600	256.600	1.82	0.194	1.60
λ	1	6139.500	6139.500	6139.500	43.53	0.000*	38.38
g	1	84.300	84.300	84.300	0.60	0.449	0.53
(T_1+T_2)	1	2814.700	2814.700	2814.700	19.96	0.000*	17.59
a	1	27.600	27.600	27.600	0.20	0.663	0.17
b	1	158.600	158.600	158.600	1.12	0.303	0.99
W	1	145.500	145.500	145.500	1.03	0.323	0.91
Y	1	0.900	0.900	0.900	0.01	0.936	0.01
V_0	1	3298.800	3298.800	3298.800	23.39	0.000*	20.62
S	1	16.500	16.500	16.500	0.12	0.736	0.10
S_1	1	7.200	7.200	7.200	0.05	0.824	0.05
T_0	1	17.100	17.100	17.100	0.12	0.732	0.11
Residual Error	18	2538.700	2538.700	141.040			
Total	31	15997.900					

* Significant at 5%

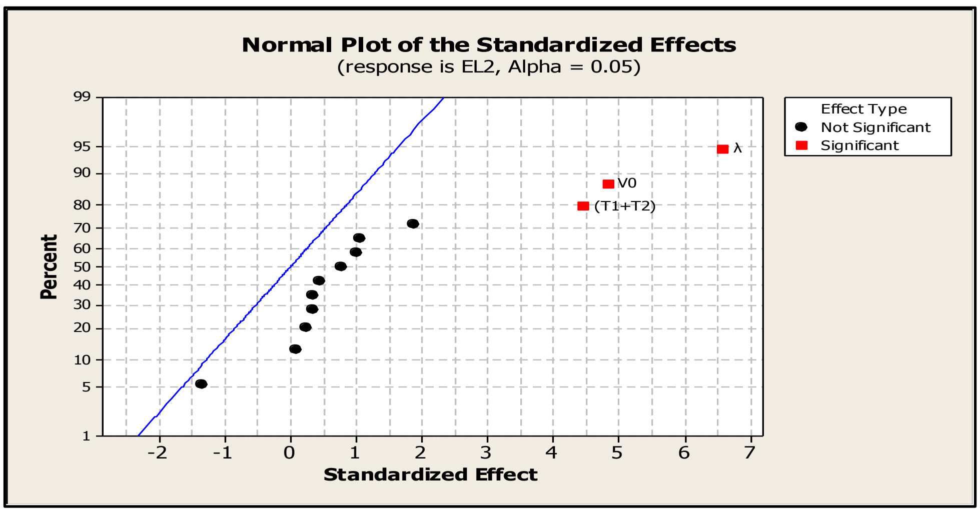

Fig. 4.9: Normal probability plot of standardized effect for expected loss cost per unit time $E(L)_2$ with constraints: discontinuous process

Table 4.16 indicates that the expected loss cost per unit time of process control $E(L)_2$ in a discontinuous process is significantly affected by three factors, namely rate of occurrences of assignable causes λ, time to find and repair an assignable cause (T_1+T_2), and net income per hour while process is in-control V_0. They are also graphically shown as "significant" in the normal plot shown in Fig. 4.9.

Among all the factors, λ has the highest significant effect on expected loss cost per unit time $E(L)_2$ since it has the highest F-value i.e., 43.53 as shown in Table 4.16 and plotted at the rightmost location in Fig. 4.9. It can also be observed from this table that λ, V_0 and

(T_1+T_2) are the top three percentage contributors which affect the cost by 38.38%, 20.62% and 17.59% respectively. All the three factors are having positive effect as shown in Fig. 4.9.

Table 4.17 presents an analysis of variance on the sample size *n*. As the factors M, δ, g and b become smaller, the optimum sample size increases, because they are having the negative effect. This can also be observed from Fig. 4.10. Moreover, it can be observed from Table 4.17 that δ, g, M, and b are the major percentage contributors which affect the sample size by 58.36%, 10.24%, 4.26% and 3.85% respectively. Thus, the factor δ is the most significant for choosing the value of sample size, in economic statistical design and the effect is of negative type.

Table 4.17: Analysis of variance for sample size *n* with constraints: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC (%)*
M	1	50.000	50.00	50.000	5.78	0.027*	4.26
δ	1	684.500	684.50	684.500	79.17	0.000*	58.36
λ	1	10.130	10.13	10.125	1.17	0.293	0.86
g	1	120.120	120.12	120.125	13.89	0.002*	10.24
(T_1+T_2)	1	3.130	3.13	3.125	0.36	0.555	0.27
a	1	36.130	36.13	36.125	4.18	0.056	3.08
b	1	45.120	45.12	45.125	5.22	0.035*	3.85
W	1	0.130	0.13	0.125	0.01	0.906	0.01
Y	1	2.000	2.00	2.000	0.23	0.636	0.17
V_0	1	32.000	32.00	32.000	3.70	0.070	2.73
S	1	0.000	0.00	0.000	-	-	0.00
S_1	1	2.000	2.00	2.000	0.23	0.636	0.17
T_0	1	32.000	32.00	32.000	3.70	0.070	2.73
Residual Error	18	155.620	155.62	8.646			
Total	31	1172.870					

* Significant at 5%

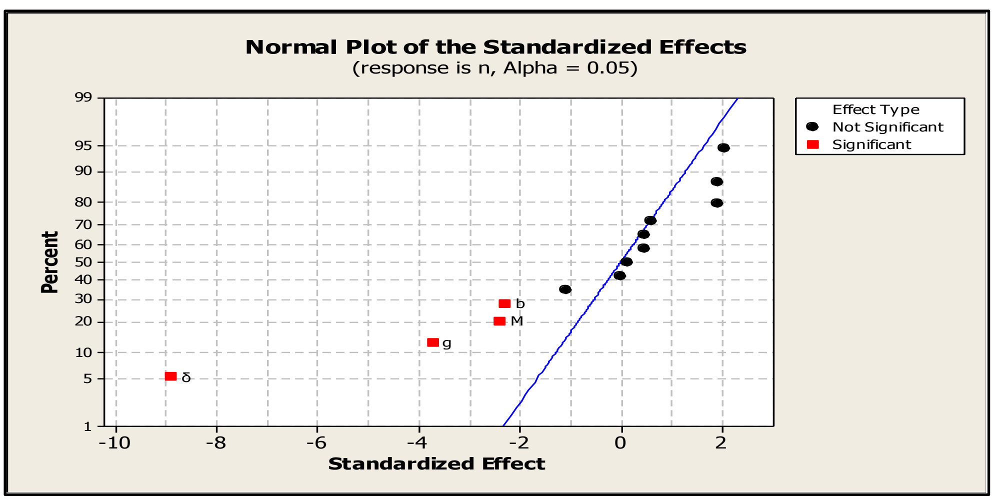

Fig. 4.10: Normal probability plot of standardized effect for sample size *n* with constraints: discontinuous process

Table 4.18 displays the result of analysis of variance on the sampling interval *h*. It is significantly affected by five factors i.e., *M*, δ, λ, *g* and *a*. Out of these five significant factors, two factors i.e., δ and *a* have positive effects as shown in Fig. 4.11, whereas the three factor *M*, λ and *g* are significant in terms of negative effect. Among all the factors, time to sample and chart one item *g* has the highest effect on the sampling interval with a percentage contribution of 28.54% and the effect is in negative direction.

Table 4.18: Analysis of variance for sampling interval *h* with constraints: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC (%)*
M	1	0.158	0.158	0.158	4.71	0.044*	5.57
δ	1	0.471	0.471	0.471	14.00	0.001*	16.54
λ	1	0.150	0.150	0.150	4.48	0.049*	5.29
g	1	0.812	0.812	0.812	24.16	0.000*	28.54
(T_1+T_2)	1	0.010	0.010	0.010	0.30	0.590	0.36
a	1	0.555	0.555	0.555	16.52	0.001*	19.51
b	1	0.004	0.004	0.004	0.11	0.742	0.13
W	1	0.004	0.004	0.004	0.12	0.735	0.14
Y	1	0.000	0.000	0.000	0.01	0.941	0.01
V_0	1	0.067	0.067	0.067	1.99	0.176	2.35
S	1	0.002	0.002	0.002	0.07	0.798	0.08
S_1	1	0.003	0.003	0.003	0.10	0.761	0.11
T_0	1	0.003	0.003	0.003	0.10	0.754	0.12
Residual Error	18	0.605	0.605	0.034			
Total	31	2.845					

* Significant at 5%

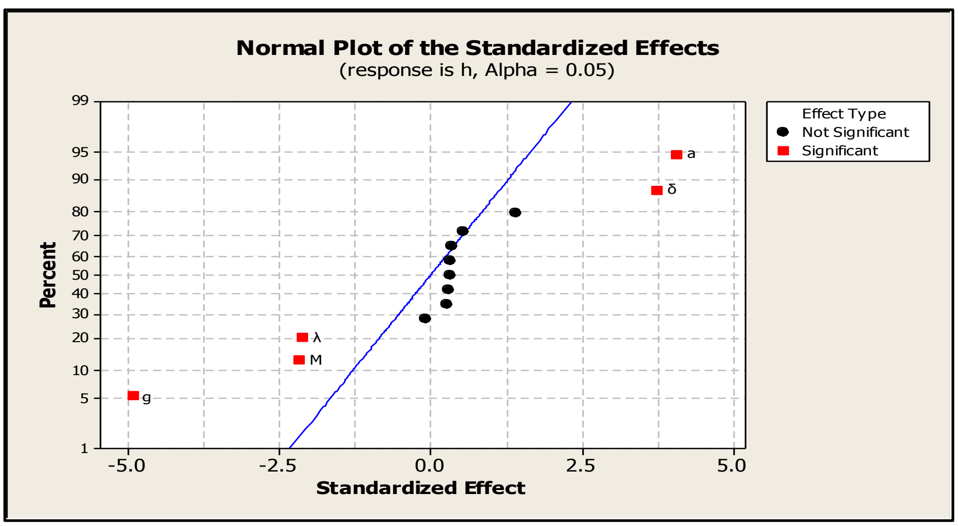

Fig. 4.11: Normal probability plot of standardized effect for sampling interval *h* with constraints: discontinuous process

Table 4.19 presents an analysis of variance on the width of control limits *k*. There are eight factors *M*, δ, λ, *g*, *b*, *Y*, V_0 and T_0 which have significant effects on width of control limits k. Fig. 4.12 reveals that out of these eight significant factors, four factors i.e., δ, Y, V_0 and T_0 have positive effect on width of control limits, whereas the remaining four factors i.e., M, λ, g and b have negative effect. Among all the factors, the expected search time for a false alarm T_0 is observed to have maximum effect on deciding the value of control limit width in economic statistical design with a contribution of 24.66% and the effect is of positive type.

Table 4.19: Analysis of variance for width of control limits k with constraints: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC (%)*
M	1	0.116	0.116	0.116	5.64	0.029*	2.51
δ	1	0.784	0.784	0.784	37.99	0.000*	16.93
λ	1	0.118	0.118	0.118	5.70	0.028*	2.54
g	1	0.141	0.141	0.141	6.82	0.018*	3.04
(T_1+T_2)	1	0.012	0.012	0.012	0.56	0.466	0.25
a	1	0.024	0.024	0.024	1.15	0.298	0.51
b	1	1.135	1.135	1.135	54.98	0.000*	24.50
W	1	0.012	0.012	0.012	0.56	0.463	0.25
Y	1	0.114	0.114	0.114	5.52	0.030*	2.46
V_0	1	0.641	0.641	0.641	31.06	0.000*	13.85
S	1	0.017	0.017	0.017	0.80	0.383	0.36
S_1	1	0.005	0.005	0.005	0.26	0.619	0.11
T_0	1	1.142	1.142	1.142	55.33	0.000*	24.66
Residual Error	18	0.372	0.372	0.021			
Total	31	4.631					

* Significant at 5%

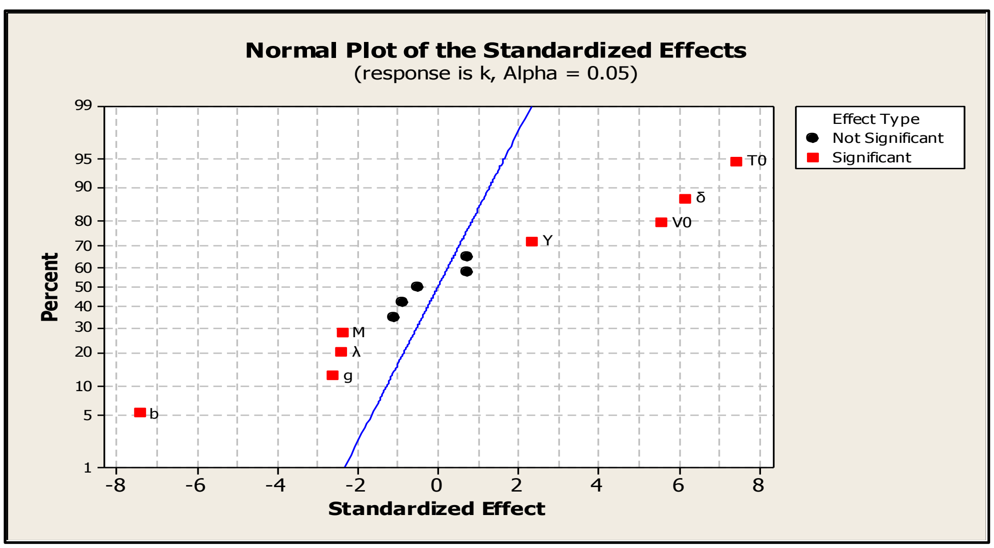

Fig. 4.12: Normal probability plot of standardized effect for width of control limits k with constraints: discontinuous process

It is further observed from Tables 4.16 - 4.19 that the cost to locate and repair the assignable cause *W*, the expected cost of restart or setup cost *S* and the startup time S_1 have no significant effect on any of the responses *n*, *h*, *k* and $E(L)_2$.

4.6.1 Summary of Results

Similar to Table 3.25 as discussed earlier in Section 3.8.1, all the significant factors in case of economic statistical design for discontinuous process with respect to each of the four responses are summarized in Table 4.20. The insignificant factors are shown as blank spaces. This table shows that the shift size δ has negative effect and it is the most significant factor for selecting the value of sample size *n* for a discontinuous process in both economic design (i.e., *ED*) and economic statistical design (i.e., *ESD*) of $\bar{X}$ chart. This result is same as that in the continuous process. But in case of sampling interval *h* and width of control limits *k*, the most significant factors are different in both types of designs for a discontinuous process and their effects are of opposite type. In case of economic design, the variable cost of sampling *b* is found to be the most significant factor affecting *h* and this effect is of positive type. On the other hand in case of economic statistical design, the time to sample and chart one item *g* becomes the most significant parameter for *h* but this effect is of negative type. In case of width of control limits *k*, the variable cost of sampling *b* and the expected search time for a false alarm T_0 are observed to be the most significant factors in economic design and economic statistical design respectively for a discontinuous process. Moreover, the effect of *b* is negative, whereas that of T_0 is positive. So far as the expected loss cost per unit time $E(L)_2$ is concerned, the rate of occurrences of assignable cause λ with positive effect is the most significant factor in case of both types of designs of $\bar{X}$ chart in a discontinuous process. This table also shows that all other significant parameters are not same in both types of designs.

Table 4.20: Comparison of significant effects in economic design and economic statistical design: discontinuous process

Output responses	***Design***	***Cost and process parameters***												
		M	δ	λ	g	(T_1+T_2)	a	b	W	Y	V_0	S	S_1	T_0
n	*ED-D*		**–**		–									
	ESD-D	–	**–**		–			–						
h	*ED-D*	–	–	–		+	+	**+**			+			
	ESD-D	–	+	–	**–**		+							
k	*ED-D*		+	–	–		–	**–**		+	+			+
	ESD-D	–	+	–	–			–		+	+			**+**
E(L)₂	*ED-D*	+		**+**		+					+			
	ESD-D			**+**		+					+			

Note:

ED-D : Economic Design - Discontinuous process
ESD-D : Economic Statistical Design - Discontinuous process
Blank space : Insignificant factor
\+ : Factor with positive effect
– : Factor with negative effect
+/– in bold : Most significant factor

Table 4.21 shows a comparison between economic design and economic statistical design of $\bar{X}$ chart for both continuous and discontinuous processes. Thus, this table summarizes the results of four different cases. All these results have been obtained using SA algorithm. In case of economic design for continuous process nine cost and process parameters are taken, whereas in discontinuous process thirteen parameters are considered. So, the last four columns of this table i.e., the expected net income per hour while the process is in-control V_0, the expected cost of restart or setup cost S, the time to restart the process S_1 and the expected search time for a false alarm T_0 are not applicable for a continuous process.

Table 4.21: Comparison of significant effects in economic design and economic statistical design for both continuous and discontinuous processes

Output responses	***Design***	***All possible cost and process parameters***												
		M	δ	λ	g	(T_1+T_2)	a	b	W	Y	V_0	S	S_1	T_0
n	*ED-C*		**–**		–			–		+				
	ED-D		**–**		–									
	ESD-C		**–**		–		+	–						
	ESD-D	–	**–**		–			–						
h	*ED-C*	–	–	–		+	**+**	+						
	ED-D	–	–	–		+	+	**+**			+			
	ESD-C		+		**–**		+							
	ESD-D	–	+	–	**–**		+							
k	*ED-C*		+	–	–		–	–		**+**				
	ED-D		+	–	–		–	**–**		+	+			+
	ESD-C		+					**–**		+				
	ESD-D	–	+	–	–			–		+	+			**+**
E(L)	*ED-C*	+		**+**		+								
	ED-D	+		**+**		+					+			
	ESD-C	+	–	**+**		+		+						
	ESD-D			**+**		+					+			

Note:
ED-C : Economic Design - Continuous process
ED-D : Economic Design - Discontinuous process
ESD-C : Economic Statistical Design - Continuous process
ESD-D : Economic Statistical Design - Discontinuous process
Blank space : Insignificant factor
\+ : Factor with positive effect
– : Factor with negative effect
+/– in bold : Most significant factor

The significant factors in case of economic design for both continuous and discontinuous processes are already discussed in Sections 3.6.1 and 3.8.1 respectively. These results of economic design are compared with the corresponding results of economic statistical design for each of the four responses (i.e., three design variables *n*, *h*, *k* and the expected loss cost per unit time E*(L)*) below.

i) Effect on sample size n

From Table 4.21 it is observed that the shift in process mean δ is the most significant factor in all the four situations and the effect is of negative type for selecting the value of sample size *n.* Another factor i.e., the time to sample and chart one item *g* is also significant in all the four cases but to a less extent compared to δ and it has negative effect. All significant factors with respect to sample size *n* have negative effects except the cost per false alarm *Y* and the fixed cost *a*.

ii) Effect on sampling interval h

Unlike in case of sample size *n*, the most significant factors are not same with respect to sampling interval *h* in all four situations. However, the time to sample and chart one item *g* is observed to have the most significant effect in both the processes (i.e., in case of economic statistical design) and both effects are of negative type. Only two factors are significant in all the four situations. They are the shift in process mean δ and the fixed cost *a*. There is only one significant factor (i.e., *a*) that has the same type of effect (i.e., positive) in all the four situations. In case of economic design, the lists of significant factors are same in both continuous and discontinuous processes except the expected net income per hour while the process is in-control V_0 which is not applicable in continuous process. In case of economic statistical design, all the three factors which are significant in continuous process are also significant in discontinuous process. But, there are additional factors *M* and λ which are significant only in discontinuous process.

iii) Effect on width of control limits k

Like sampling interval *h*, the most significant factors for the width of control limits *k* are not same in all the four situations. However, the variable cost of sampling *b* is most significant in two situations i.e., economic design for discontinuous process and economic statistical design for continuous process. Three factors i.e., δ, *b* and *Y* are significant in all the four situations. Moreover, each one of these three factors has one type of effect either positive or negative in all those four situations. All the factors which are significant in continuous process are also significant in discontinuous process in economic design as well as economic statistical design except one additional factor M which is significant only in discontinuous process for economic statistical design.

iv) Effect on expected loss cost per unit time E(L)

All the significant factors with respect to the expected loss cost per unit time have positive effect except the shift size δ. Like in case of sample size n, the most significant factor for the expected loss cost per unit time is also same in all the four situations and this factor is λ i.e., the rate occurrence of assignable cause. It has positive type of effect in all the four situations. This means that whenever the value of λ increases the expected loss cost per unit time will also increase. Another factor i.e., the time to find and repair an assignable cause (T_1+T_2) is also significant in all the four cases but to a less extent compared to λ and its type of effect is positive. Moreover, ignoring the expected net income per hour during in-control period V_0 which is not applicable in continuous process, the lists of significant factors are same in both continuous and discontinuous processes in case of economic design.

The objective function equations are not same in continuous and discontinuous processes. The numbers of factors associated with these two of processes are also different. The economic statistical design includes one or more constraints, whereas the economic design does not consider any constraint. These differences in characteristics of the four situations may be the reasons for the differences in results of significant factors as shown in Table 4.21. Therefore, the designers of control charts must ensure the type of process (i.e., continuous process or discontinuous process) and take utmost care in ensuring the correctness of values of significant factors before using them into economic design or economic statistical design.

4.7 Numerical Illustration: ARL Constraints

A numerical problem related to economic statistical design of $\bar{X}$ chart for a continuous process subjected to only *ARL* constraints has been earlier by van Deventer and Manna (2009). This problem is solved with the help of two metaheuristics, namely SA and TLBO in this section. This is a constrained optimization problem as stated below:

$$\text{Minimize } E(Q) \qquad (4.6)$$

subject to

$$ARL_0 \geq 267$$
$$ARL_1 \leq 40$$

where *E(Q)* is the expected total cost per hour whose expression is shown in Eq. 3.40. The objective function of this model *E(Q)* is different from *E(L)* considered in Section 4.3 for a continuous process. The differences between *E(Q)* and *E(L)* are already discussed in Section 3.9.1. *E(Q)* is also a function of three design variables *n*, *h* and *k* out of which *n* must be integer while other two variables can take any real values. Hence, it is a multi-variable constrained optimization problem. Further, the values of all the cost and process parameters are same as that listed in Table 3.26.

4.7.1 Results and Discussion: SA

Table 4.22 shows the results of economic statistical design with *ARL* constraints of $\bar{X}$ chart for a continuous process using SA. These results consist of the optimal values of sampling interval (*h*) and the width of the control limits (*k*) for each integer value of sample size *n* varying from 1 to 20. This table also shows the corresponding values of all the statistical properties mentioned in Section 4.3.1.

Table 4.22: Optimal economic statistical designs with *ARL* constraints of $\bar{X}$ chart using SA

n	*h*	*k*	α	β	*P*	ARL_0	ARL_1	ATS_0	ATS_1	*E(Q)*
1	0.17	2.90	0.0037	0.971	0.029	267.119	34.756	45.970	5.98	21.350
2	0.28	2.90	0.0037	0.931	0.069	267.119	14.562	74.888	4.08	18.490
3	0.40	2.90	0.0037	0.879	0.121	267.119	8.237	105.767	3.26	17.166
4	0.52	2.90	0.0037	0.816	0.184	267.119	5.433	138.587	2.82	16.406
5	0.64	2.90	0.0037	0.747	0.253	267.119	3.947	172.270	2.55	15.920
6	0.78	2.90	0.0037	0.674	0.326	267.119	3.066	207.990	2.39	15.591
7	0.91	2.90	0.0037	0.600	0.400	267.119	2.502	241.953	2.27	15.360
8	1.05	2.90	0.0037	0.529	0.472	267.119	2.121	281.303	2.23	15.196
9	1.19	2.90	0.0037	0.460	0.540	267.119	1.852	318.865	2.21	15.079
10	1.33	2.90	0.0037	0.397	0.603	267.119	1.657	354.573	2.20	14.998
11	1.47	2.90	0.0037	0.339	0.662	267.119	1.512	391.846	2.22	14.944
12	1.58	2.90	0.0037	0.286	0.714	267.119	1.401	421.216	2.21	14.913
13	**1.71**	**2.90**	**0.0037**	**0.240**	**0.760**	**267.119**	**1.316**	**457.087**	**2.25**	**14.899**
14	1.82	2.90	0.0037	0.200	0.800	267.119	1.250	487.472	2.28	14.900
15	1.94	2.90	0.0037	0.165	0.835	267.119	1.198	519.047	2.33	14.914
16	2.05	2.90	0.0037	0.136	0.864	267.119	1.157	546.596	2.37	14.938
17	2.15	2.90	0.0037	0.111	0.889	267.119	1.124	573.344	2.41	14.970
18	2.24	2.90	0.0037	0.090	0.910	267.119	1.099	597.360	2.46	15.011
19	2.34	2.90	0.0037	0.072	0.928	267.119	1.078	625.101	2.52	15.057
20	2.40	2.90	0.0037	0.058	0.942	267.119	1.062	640.687	2.55	15.109

Table 4.22 shows that the optimum value of expected total cost per hour *E(Q)* decreases as sample size *n* increases from 1 to 13 and after that it increases at higher values of *n*. This may also be visualized from Fig. 4.13.

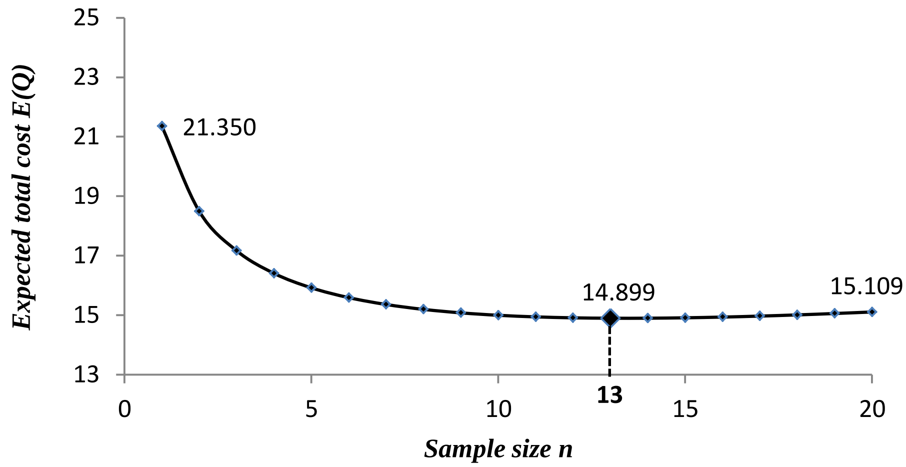

Fig. 4.13: Variation of expected total cost with sample size using SA: *ARL* constraints

On comparing the results of all 20 economic designs, one each for integer value of sample size n varying from 1 to 20, the most minimum expected total cost per hour *E(Q)* is found to be 14.899 and this occurs at *n* = 13 as shown in Table 4.22. The corresponding values of *h* and *k* at minimum total cost are 1.71 hour and 2.90 respectively. The optimal results obtained by van Deventer and Manna (2009) are shown along with that obtained with the use of simulated annealing in Table 4.23 for comparison purpose.

Table 4.23: Comparison of results with *ARL* constraints

Methodology	*n*	*h*	*k*	α	β	*P*	ARL_0	ARL_1	ATS_0	ATS_1	*E(Q)*
van Deventer and Manna (2009)	13	1.70	2.90	0.0037	0.240	0.760	267.970	1.316	455.549	2.24	14.900
SA	13	1.71	2.90	0.0037	0.240	0.760	267.119	1.316	457.087	2.25	14.899

This table shows that the optimal values of sample size (*n*) and width of the control limits (*k*) are same in both the results. There is difference only in the value of sampling interval (*h*) that too of very small magnitude. In case of simulated annealing, the optimal value of expected total cost per hour is found to be slightly lower than that of van Deventer and Manna (2009).

4.7.2 Results and Discussion: TLBO

Similar to Table 4.22, the results of economic statistical design obtained using TLBO for the same numerical problem mentioned in Section 4.7 are shown in Table 4.24 for each integer value of sample size *n* in the range 1 to 20. This cross checks the accuracy of results obtained from SA.

Table 4.24: Optimal economic statistical designs with *ARL* constraints of $\bar{X}$ chart using TLBO

n	*h*	*k*	α	β	*P*	ARL_0	ARL_1	ATS_0	ATS_1	*E(Q)*
1	0.17	2.90	0.0037	0.971	0.029	267.119	34.756	45.677	5.94	21.350
2	0.31	2.90	0.0037	0.931	0.069	267.119	14.562	82.285	4.49	18.520
3	0.41	2.90	0.0037	0.879	0.121	267.119	8.237	110.775	3.42	17.169
4	0.52	2.90	0.0037	0.816	0.184	267.119	5.433	138.320	2.81	16.406
5	0.68	2.90	0.0037	0.747	0.253	267.119	3.947	181.564	2.68	15.925
6	0.80	2.90	0.0037	0.674	0.326	267.119	3.066	212.469	2.44	15.592
7	0.93	2.90	0.0037	0.600	0.400	267.119	2.502	247.683	2.32	15.361
8	1.04	2.90	0.0037	0.529	0.472	267.119	2.121	278.925	2.21	15.196
9	1.19	2.90	0.0037	0.460	0.540	267.119	1.852	316.779	2.20	15.079
10	1.32	2.90	0.0037	0.397	0.603	267.119	1.657	352.714	2.19	14.998
11	1.45	2.90	0.0037	0.339	0.662	267.119	1.512	388.623	2.20	14.944
12	1.59	2.90	0.0037	0.286	0.714	267.119	1.401	424.454	2.23	14.913
13	**1.70**	**2.90**	**0.0037**	**0.240**	**0.760**	**267.119**	**1.316**	**454.222**	**2.24**	**14.899**
14	1.82	2.90	0.0037	0.200	0.800	267.119	1.250	486.821	2.28	14.900
15	1.94	2.90	0.0037	0.165	0.835	267.119	1.198	518.709	2.33	14.914
16	2.06	2.90	0.0037	0.136	0.864	267.119	1.157	551.039	2.39	14.938
17	2.15	2.90	0.0037	0.111	0.889	267.119	1.124	575.435	2.42	14.970
18	2.24	2.90	0.0037	0.090	0.910	267.119	1.099	597.427	2.46	15.011
19	2.32	2.90	0.0037	0.072	0.928	267.119	1.078	619.488	2.50	15.057
20	2.40	2.90	0.0037	0.058	0.942	267.119	1.062	641.116	2.55	15.109

The results of TLBO show the similar pattern of variation of expected total cost per hour *E(Q)* with sample size *n* as that in case of SA. That means *E(Q)* first decreases with the increase of *n* from 1 to 13 and after that it increases at higher values of *n*. This variation is also graphically shown in Fig. 4.14. Thus, the optimal solution occurs at $n = 13$ and the corresponding minimum expected total cost per hour is $E(Q) = 14.899$ as shown in Table 4.23. The corresponding values of *h* and *k* at minimum total cost are 1.70 hour and 2.90 respectively.

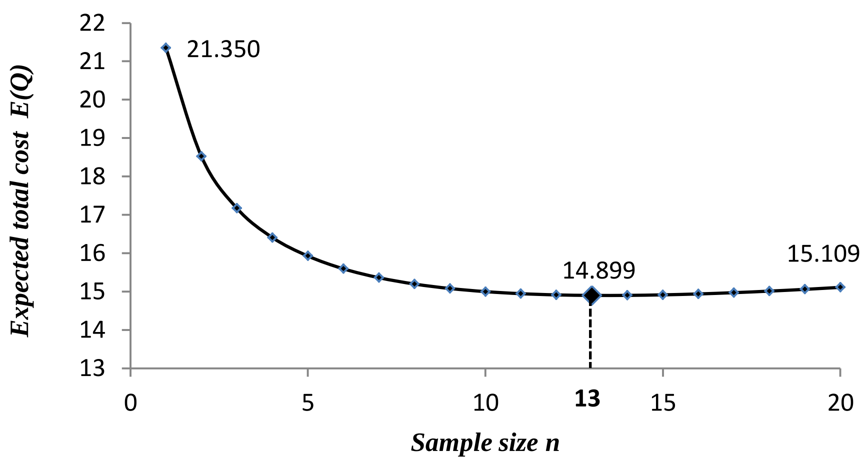

Fig. 4.14: Variation of expected total cost with sample size using TLBO: *ARL* constraints

Table 4.25 shows the comparison of results of economic statistical design with *ARL* constraints of $\bar{X}$ chart for continuous process using TLBO with that of the results obtained using SA. It is observed that for both cases, the minimum values of *E(Q)* are found to be same i.e., 14.899 and this occurs at same values of sample size (n = 13) and width of control limits (k =2.90) with a slight variation in the value of sampling interval (h).

Table 4.25: Comparison of results obtained from SA and TLBO: *ARL* constraints

Techniques	***n***	***h***	***k***	***α***	***β***	***P***	ARL_0	ARL_1	ATS_0	ATS_1	***E(Q)***
SA	13	1.71	2.90	0.0037	0.240	0.760	267.119	1.316	457.087	2.25	14.899
TLBO	13	1.70	2.90	0.0037	0.240	0.760	267.119	1.316	454.222	2.24	14.899

It is observed that both the metaheuristics i.e., SA and TLBO are providing the similar results for economic statistical design with *ARL* constraints of $\bar{X}$ chart for a continuous process. Thus, the results are validated and confirmed to be correct. Moreover, both the techniques have provided better results than that reported by van Deventer and Manna (2009).

4.8 Numerical Illustration: ATS Constraint

van Deventer and Manna (2009) have solved one more numerical example related to economic statistical design of $\bar{X}$ chart for a continuous process but it is it subjected to out-of-control average time to signal (i.e., ATS_1) constraint only. This example is almost same as that discussed in Section 4.7 except the differences in constraints. Here, the constraint is in terms of *ATS*, whereas the example in Section 4.7 deals with *ARL* constraints. The objective is same in both cases i.e. is to minimize the expected total cost per hour *E(Q)*, the details of which has already been explained in Section 3.9.1. The input data regarding all cost and process parameters are also same as listed in Table 3.26. The limiting value of ATS_1 constraint is same as that considered in Section 4.3.1. Thus, the economic statistical design $\bar{X}$ chart for a continuous process subjected to *ATS* constraint can be modelled as

$$\text{Minimize } E(Q) \tag{4.7}$$

subject to

$$ATS_1 \leq 1.90$$

The optimal solutions of the above design problem are obtained separately using SA and TLBO in this section and their results are discussed in next two sections.

4.8.1 Results and Discussion: SA

Similar to Table 4.22, the results of economic statistical design with *ATS* constraint of $\bar{X}$ chart for a continuous process obtained using SA are shown in Table 4.26. It lists down the optimal values of two design variables of control chart such as sampling interval (*h*) and the width of the control limits (*k*) for each integer value of sample size *n* varying from 1 to 20. This table also shows all the statistical properties as mentioned in Section 4.3.

Table 4.26: Optimal economic statistical designs with *ATS* constraint of $\bar{X}$ chart using SA

n	*h*	*k*	α	β	*P*	ARL_0	ARL_1	ATS_0	ATS_1	*E(Q)*
1	0.25	2.12	0.0340	0.868	0.132	29.389	7.561	7.348	1.89	22.495
2	0.36	2.30	0.0215	0.812	0.188	46.576	5.320	16.590	1.90	18.376
3	0.48	2.39	0.0169	0.745	0.255	59.277	3.917	28.588	1.89	16.883
4	0.61	2.47	0.0135	0.681	0.319	73.900	3.133	44.939	1.90	16.082
5	0.79	2.45	0.0143	0.585	0.415	69.899	2.408	54.917	1.89	15.636
6	0.91	2.51	0.0121	0.524	0.476	82.692	2.101	74.886	1.90	15.347
7	1.05	2.52	0.0118	0.450	0.550	85.068	1.818	89.133	1.90	15.161
8	1.15	2.56	0.0105	0.394	0.606	95.362	1.651	109.527	1.90	15.040
9	1.27	2.57	0.0102	0.334	0.666	98.147	1.501	124.348	1.90	14.957
10	1.36	2.59	0.0096	0.284	0.716	103.992	1.396	141.771	1.90	14.910
11	1.44	2.62	0.0088	0.243	0.757	113.495	1.321	163.122	1.90	14.885
12	**1.50**	**2.66**	**0.0078**	**0.211**	**0.789**	**127.694**	**1.267**	**191.351**	**1.90**	**14.879**
13	1.56	2.69	0.0072	0.180	0.820	139.631	1.219	217.574	1.90	14.888
14	1.59	2.76	0.0058	0.163	0.837	172.558	1.195	274.548	1.90	14.911
15	1.64	2.77	0.0056	0.135	0.865	177.922	1.156	292.236	1.90	14.942
16	1.69	2.79	0.0053	0.113	0.887	189.208	1.128	319.019	1.90	14.980
17	1.70	2.86	0.0042	0.103	0.897	235.334	1.115	400.617	1.90	15.031
18	1.74	2.86	0.0042	0.083	0.917	235.334	1.091	409.572	1.90	15.083
19	1.76	2.91	0.0036	0.074	0.926	275.778	1.080	484.945	1.90	15.141
20	1.79	2.92	0.0035	0.060	0.940	284.744	1.064	508.325	1.90	15.203

Table 4.26 shows that the optimum values of expected total cost per hour *E(Q)* decreases as sample size *n* value increases from 1 to 12 and after that it increases at higher values of *n*. This may also be visualized from Fig. 4.15.

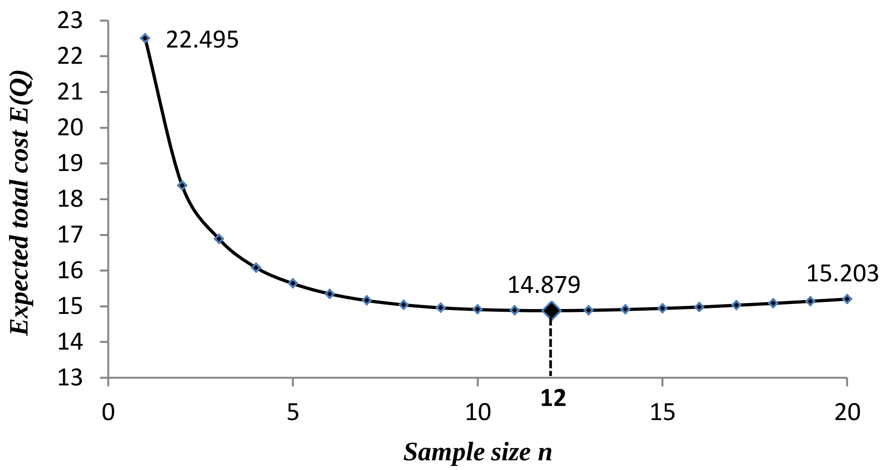

Fig. 4.15: Variation of expected total cost with sample size using SA: *ATS* constraint

On comparing the results of all 20 economic designs, one each for integer value of sample size n varying from 1 to 20, the most minimum expected total cost per hour is found to be $E(Q)$ = 14.879 and this occurs at n = 12 as shown in Table 4.26. The corresponding values of h and k at minimum total cost are 1.50 hour and 2.66 respectively. The optimal results obtained by van Deventer and Manna (2009) are shown along with that obtained with the use of SA in Table 4.27 for comparison purpose. The value of expected total cost per hour $E(Q)$ corresponding to the economic design suggested by van Deventer and Manna (2009) is observed to be 14.879 as shown in the same table.

Table 4.27: Comparison of results with *ATS* constraint

Methodology	n	h	k	α	β	P	ARL_0	ARL_1	ATS_0	ATS_1	$E(Q)$
van Deventer and Manna (2009)	12	1.50	2.60	0.0093	0.194	0.806	107.268	1.240	160.902	1.86	14.900
SA	12	1.50	2.66	0.0078	0.211	0.789	127.694	1.267	191.351	1.90	14.879

This table shows that the optimal values of sample size (n) and of sampling interval (h) are same in both the results. But, there is a difference observed in the value of width of the control limits (k). In case of simulated annealing, the optimal value of expected total cost per hour is found to be lower than that of van Deventer and Manna (2009).

4.8.2 Results and Discussion: TLBO

To cross check the accuracy of the results obtained using SA, the same numerical problem has been solved using TLBO. Table 4.28 shows the results of economic statistical design with *ATS* constraint of $\bar{X}$ chart for a continuous process using TLBO. Here also the value of sample size *n* is varied in the range 1 to 20.

Table 4.28: Optimal economic statistical designs with *ATS* constraint of $\bar{X}$ chart using TLBO

n	*h*	*k*	α	β	*P*	ARL_0	ARL_1	ATS_0	ATS_1	*E(Q)*
1	0.25	2.12	0.0340	0.868	0.132	29.389	7.561	7.347	1.89	22.495
2	0.33	2.36	0.0183	0.828	0.172	54.655	5.807	18.004	1.90	18.365
3	0.48	2.40	0.0164	0.748	0.252	60.915	3.967	29.288	1.90	16.857
4	0.63	2.44	0.0147	0.670	0.330	67.989	3.031	42.507	1.89	16.081
5	0.77	2.48	0.0132	0.596	0.404	75.996	2.478	58.436	1.90	15.632
6	0.91	2.51	0.0121	0.524	0.476	82.692	2.101	75.027	1.90	15.347
7	1.04	2.52	0.0118	0.450	0.550	85.068	1.818	88.803	1.90	15.166
8	1.16	2.55	0.0108	0.390	0.610	92.665	1.640	107.297	1.90	15.039
9	1.26	2.58	0.0099	0.337	0.663	101.023	1.509	126.783	1.89	14.959
10	1.35	2.61	0.0091	0.290	0.710	110.225	1.409	148.847	1.90	14.909
11	1.42	2.65	0.0081	0.253	0.748	123.969	1.338	176.271	1.90	14.885
12	**1.49**	**2.68**	**0.0074**	**0.217**	**0.784**	**135.521**	**1.276**	**201.736**	**1.90**	**14.879**
13	1.56	2.69	0.0072	0.180	0.820	139.631	1.219	217.405	1.90	14.888
14	1.60	2.74	0.0062	0.158	0.842	162.354	1.188	259.409	1.90	14.910
15	1.65	2.77	0.0056	0.135	0.865	177.922	1.156	292.753	1.90	14.939
16	1.69	2.79	0.0053	0.113	0.887	189.208	1.128	318.967	1.90	14.980
17	1.71	2.85	0.0044	0.102	0.899	228.050	1.113	389.007	1.90	15.028
18	1.74	2.88	0.0040	0.087	0.914	250.677	1.095	435.350	1.90	15.081
19	1.76	2.90	0.0037	0.072	0.928	267.119	1.078	470.903	1.90	15.142
20	1.79	2.92	0.0035	0.060	0.940	284.744	1.064	508.325	1.90	15.203

The results of TLBO also show the similar type of variation of expected total cost per hour *E(Q)* with sample size *n*. It decreases with the increase of *n* value from 1 to 12 and after that it increases at higher values of *n*. This variation is also graphically shown in Fig. 4.16. Thus, the optimal solution occurs at *n* = 12 and the corresponding minimum expected total cost per hour is *E(Q)* = 14.879 as shown in Table 4.28. The corresponding values of *h* and *k* at minimum total cost are 1.49 hour and 2.68 respectively.

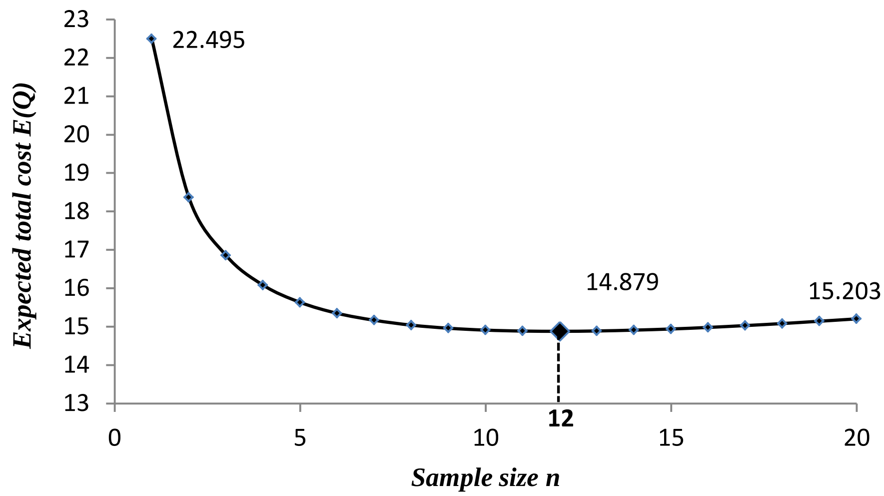

Fig. 4.16: Variation of expected total cost with sample size using TLBO: *ATS* constraint

Table 4.29 shows the comparison of results of economic statistical design with *ATS* constraint of $\bar{X}$ chart for continuous process using TLBO with that of the results obtained from SA. It is observed that for both cases, the sample size (n) and the sampling interval (h) are same, whereas width of control limits (k) are slightly different.

Table 4.29: Comparison of results obtained from SA and TLBO: *ATS* constraint

Techniques	***n***	***h***	***k***	α	β	***P***	ARL_0	ARL_1	ATS_0	ATS_1	***E(Q)***
SA	12	1.50	2.66	0.0078	0.211	0.789	127.694	1.267	191.351	1.90	14.879
TLBO	12	1.49	2.68	0.0074	0.217	0.784	135.521	1.276	201.736	1.90	14.879

It is observed that both the metaheuristics i.e., SA and TLBO are providing the similar results for economic statistical design of $\bar{X}$ chart with *ATS* constraint for a continuous process, and thus the results are validated and confirmed to be correct. Moreover, both are providing better results than that reported by van Deventer and Manna (2009).

4.8.3 Comparison of Results

Table 4.30 summarizes the optimal results of economic statistical design of $\bar{X}$ chart obtained using SA and TLBO when only one type of constraints (i.e., either *ARL* or *ATS*) is present. The results of corresponding economic design already reported in Sections 3.9.3 and 3.9.4 are also reproduced in this table for comparison purpose. This table also compares the results of both economic design and economic statistical design with that of van Deventer and Manna (2009).

Table 4.30: Comparison of results for economic design and economic statistical designs

Methodo-logy	***Design***	***n***	***h***	***k***	α	β	***P***	ARL_0	ARL_1	ATS_0	ATS_1	***E(Q)***
van Deventer and Manna (2009)	*ED*	12	1.90	2.60	0.0093	0.194	0.806	107.268	1.240	203.809	2.36	14.840
	ESD-ARL	13	1.70	2.90	0.0037	0.240	0.760	267.970	1.316	455.549	2.24	14.900
	ESD-ATS	12	1.50	2.60	0.0093	0.194	0.806	107.268	1.240	160.902	1.86	14.900
SA	*ED*	12	1.85	2.61	0.0091	0.197	0.804	110.225	1.245	204.104	2.30	14.838
	ESD-ARL	13	1.71	2.90	0.0037	0.240	0.760	267.119	1.316	457.087	2.25	14.899
	ESD-ATS	12	1.50	2.66	0.0078	0.211	0.789	127.694	1.267	191.351	1.90	14.879
TLBO	*ED*	12	1.85	2.62	0.0088	0.199	0.801	113.495	1.249	209.671	2.31	14.838
	ESD-ARL	13	1.70	2.90	0.0037	0.240	0.760	267.119	1.316	454.222	2.24	14.899
	ESD-ATS	12	1.49	2.68	0.0074	0.217	0.784	135.521	1.276	201.736	1.90	14.879

Note:

ED : Economic Design;

ESD-ARL : Economic Statistical Design with only Average Run Length Constraints

ESD-ATS : Economic Statistical Design with only Average Time to Signal Constraint

Table 4.30 shows the comparison between economic design and economic statistical design of $\bar{X}$ chart with *ARL* and *ATS* constraints. In each type of design, both the metaheuristics (i.e., SA and TLBO) are providing same results and both results are better than that of van Deventer and Manna (2009). The optimal value of sample size *n* in case of economic statistical design with only *ATS* constraint (i.e., *ESD-ATS*) is found to be same as that in economic design (i.e., *ED*). The optimal values of all the design variables (i.e., *n*, *h* and *k*) are found to be less in *ESD-ATS* compared to economic statistical design with only *ARL* constraints (*ESD-ARL*). Further, the total expected cost per hour *E(Q)* of economic statistical design (i.e., both *ESD-ARL* and *ESD-ATS*) is observed to be higher than that of economic design (i.e., *ED*) in all cases. This cost is same for both *ESD-ARL* and *ESD-ATS* as reported by van Deventer and Manna (2009). But, the results of SA and TLBO show that this cost is higher in *ESD-ARL* as compared to *ESD-ATS*.

Table 4.30 also shows that *ESD-ARL* gives higher value of ARL_0 and ATS_0 compared to *ESD-ATS* in all the cases. Therefore, the corresponding value of α error is found to be less in *ESD-ARL*. Thus, the use of *ARL* constraints protects the system from frequent occurrence of false alarms. But, it gives slightly higher value of ARL_1 and β error. This means the number of samples required to detect the process shift is higher. On the other hand, the time required to detect the shift is less in *ESD-ATS*. Thus, the use of *ATS* constraint helps in quick detection of process shift in terms of time. To avail both benefits as discussed above, it is advisable to apply both the *ARL* and *ATS* constraints to obtain the best possible results.

4.9 Conclusions

The main contribution of this chapter is development of new design methodologies based on two metaheuristics, namely SA and TLBO for economic statistical design of $\bar{X}$ chart for both continuous and discontinuous processes. The use of both the methodologies has been illustrated through numerical examples. It is observed that both are providing almost the same results. Therefore, either SA or TLBO can be considered for economic statistical design of $\bar{X}$ chart. Both the methodologies have also been found to be superior compared to that of other researchers. The optimal value of expected loss cost per unit time in economic statistical design of $\bar{X}$ chart is always found to be higher than that of its economic design in both types of processes. But, the economic statistical design provides the benefit of comparatively lower rate of false alarm and quicker detection of process shift. Similar to economic design of $\bar{X}$ chart, the shift in process mean δ and the rate of occurrences of assignable causes λ are found to be the most significant factors affecting the sample size n and the expected loss cost per unit time $E(L)$ respectively in case of its economic statistical design for both continuous and discontinuous processes. For the sampling interval h, the time to sample and chart one item g is observed to have the most significant effect in both types of continuous process for economic statistical design. For the width of control limits k, the variable cost per sample b is the most significant factor in a continuous process, whereas the expected search time for a false alarm T_0 is the most significant factor in a discontinuous process.

CHAPTER - 5

Joint Economic Design of $\bar{X}$ and *R* Charts

5.1 Introduction

Chapter 3 deals with economic design of $\bar{X}$ chart only. But in practice, $\bar{X}$ and R charts are often used together to detect the shift in process mean as well as process standard deviation. This chapter deals with development of design methodologies based on simulated annealing (SA) and teaching-learning based optimization (TLBO) for joint economic design of $\bar{X}$ and R charts. The use of both methodologies is illustrated through numerical examples and their results are found to be superior to that of other researchers.

5.2 Assumptions

All the assumptions listed in Section 3.2 for the economic design of $\bar{X}$ chart are also applicable for the joint economic design of $\bar{X}$ and R charts. So, only the additional assumptions meant for the joint economic design of $\bar{X}$ and R charts are listed below:

a) Mean and standard deviation of sample range R are $\mu_R = d_2\sigma_0$ and $\sigma_R = d_3\sigma_0$ respectively where d_2 and d_3 are constants of R chart. Thus, the three lines for R chart can be expressed as

$$CL_R = \mu_R = d_2\sigma_0,$$

$$\begin{aligned} UCL_R &= \mu_R + k_2\sigma_R \\ &= d_2\sigma_0 + k_2(d_3\sigma_0) \\ &= (d_2 + k_2 d_3)\sigma_0 \\ &= K_2\sigma_0 \text{, and} \end{aligned}$$

$$LCL_R = \mu_R - k_2\sigma_R$$
$$= d_2\sigma_0 - k_2(d_3\sigma_0)$$
$$= (d_2 - k_2 d_3)\sigma_0$$
$$= K_3\sigma_0.$$

where

k_2 = width of control limits of R chart expressed in multiple of standard deviation of sample range σ_R and $k_2 \geq 0$.

K_2 = upper control limit coefficient of R chart expressed in multiple of σ_0

$$= (d_2 + k_2 d_3)$$

K_3 = lower control limit coefficient of R chart expressed in multiple of σ_0

$$= (d_2 - k_2 d_3)$$

The values of d_2 and d_3 depend on the value of sample size n. The value of d_2 is always more than that of d_3. But depending upon the value of width of control limits k_2, the value of d_2 may be sometimes less than $k_2 d_3$. In that case, the value of K_3 will be negative, whereas the value of K_2 is always positive. Therefore, UCL_R is always positive while LCL_R may be sometimes negative. But, the range being highest value minus lowest value of quality characteristic X, it never takes a negative value. Therefore, a control limit having a negative value has no meaning. In such case, the value of LCL_R is taken as zero instead of negative value. In other cases, its value may be positive but it is close to zero. So, in this chapter for simplicity and uniformity LCL_R is assumed to be equal to zero for all cases (i.e., K_3 = 0). This assumption is same as that considered by other researchers like Saniga (1977), Chung and Chen (1993), and Kasarapu and Vommi (2011).

b) The process is disturbed by the occurrence of single assignable cause which shifts both process mean from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0$ and process standard deviation from σ_0 to σ_1 simultaneously where $\sigma_1 > \sigma_0$.

5.3 Joint Probability

The economic models for joint design of $\bar{X}$ and R charts remain same as that for only $\bar{X}$ chart in case of both continuous and discontinuous processes as shown in Chapter 3. The difference is only in the method of calculation of false and true alarms. Since, both $\bar{X}$ and R charts are being used simultaneously for monitoring the process, it is essential to calculate the joint probability of false and true alarms for these two charts as explained below.

i) The joint probability of false alarm (i.e., joint Type-I error) for $\bar{X}$ and R charts is the probability that $\bar{X}$, R or both the charts indicate a false alarm causing Type-I error and it is calculated using the following expression:

$$\alpha = \alpha_{\bar{X}} + \alpha_R - \alpha_{\bar{X}}.\alpha_R \tag{5.1}$$

where

$\alpha_{\bar{X}}$ = False alarm rate of $\bar{X}$ chart $= 2\left[1-\phi\left(k_1\right)\right]$

α_R = False alarm rate of R chart $= 1-\phi(k_2)$

$\phi(\bullet)$ = Standard normal cumulative distribution function

ii) The joint probability of true alarm is same as the power of detecting a shift by $\bar{X}$, R or both the charts and it is calculated as:

$$P = P_{\bar{X}} + P_R - P_{\bar{X}}.P_R \tag{5.2}$$

where

$P_{\bar{X}}$ = Power of $\bar{X}$ chart in detecting the process shift

$$= 1 - \left[\phi\left\{\frac{\left(k_1 - \delta\sqrt{n}\right)}{\Upsilon}\right\} - \phi\left\{\frac{\left(-k_1 - \delta\sqrt{n}\right)}{\Upsilon}\right\}\right]$$

P_R = Power of R chart in detecting process shift

$$= 1 - \phi\left[\left(\frac{d_2}{d_3}\right)\left(\frac{1-\Upsilon}{\Upsilon}\right) + \frac{k_2}{\Upsilon}\right]$$

$\Upsilon = \dfrac{\sigma_1}{\sigma_0}$ (i.e., > 1, as $\sigma_1 > \sigma_0$)

5.4 Numerical Illustration: Continuous Process

In this section the joint economic design of $\bar{X}$ and R charts has been illustrated for which the same numerical problem which was earlier considered for the economic design of $\bar{X}$ chart in Section 3.5 has been taken up. This problem is applicable to a continuous process i.e. the process which continues to operate even if a true or false alarm is obtained in a control chart. All data related to this problem are already shown in Table 3.2 except the value of shift in standard deviation i.e. $\Upsilon = 1.5$.

Compared to the economic design of $\bar{X}$ chart, the joint economic design involves one extra design variable k_2 i.e. the width of control limits of R chart. Thus, the expected loss cost function $E(L)_1$ shown in Eq. 3.20 is a function of four control chart parameters such as the sample size n, the sampling frequency h, and two control limits width parameters k_1 and k_2 in case of joint economic design and the objective is to minimize this function $E(L)_1$ for its optimal solution.

All the above mentioned four design variables are taken as real values on continuous scale except the sample size n which is taken as integer. Thus, it is an example of multi-variable unconstrained minimization problem involving a non-linear and non-differentiable objective function.

The search space defined by the lower and upper boundary limits for each of the four design variables for minimizing the cost function $E(L)_1$ is shown in Table 5.1 (Kasarapu and Vommi, 2011).

Table 5.1: Boundary limits of design variables for $\bar{X}$ and R charts

Design variables	***Boundary limits***
n	2 - 33
h	0.25 - 12.00
k_1	1.0 - 6.0
k_2	1.0 - 6.0

5.4.1 Results and Discussion

Table 5.2 shows the results of economic design of $\bar{X}$ and R charts for a continuous process using two metaheuristics viz., SA and TLBO. In this table, the optimal values of three design variables of control charts such as sampling interval (h) and two widths of control limits (i.e., k_1 and k_2) are shown for each integer value of sample size n varying from 2 to 33. The corresponding minimum values of expected loss cost per unit time $E(L)_1$ are also shown in this table.

Table 5.2: Optimal joint economic designs of $\bar{X}$ and R charts: continuous process

SA					TLBO				
n	***h***	***k_1***	***k_2***	***$E(L)_1$***	***n***	***h***	***k_1***	***k_2***	***$E(L)_1$***
2	0.86	2.21	2.24	34.190	2	0.84	2.25	2.23	34.188
3	**1.16**	**2.20**	**2.09**	**34.050**	**3**	**1.16**	**2.19**	**2.12**	**34.050**
4	1.40	2.18	2.07	34.099	4	1.40	2.18	2.08	34.099
5	1.62	2.17	2.10	34.231	5	1.64	2.16	2.06	34.228
6	1.91	2.15	2.01	34.415	6	1.86	2.15	2.07	34.413
7	2.09	2.15	2.06	34.635	7	2.06	2.14	2.09	34.635
8	2.29	2.15	2.04	34.887	8	2.25	2.15	2.08	34.885
9	2.41	2.17	2.09	35.155	9	2.41	2.17	2.09	35.155
10	2.58	2.18	2.13	35.441	10	2.55	2.19	2.11	35.441
11	2.74	2.20	2.13	35.739	11	2.73	2.17	2.14	35.739
12	2.89	2.21	2.15	36.045	12	2.86	2.22	2.16	36.045
13	2.99	2.23	2.24	36.359	13	2.99	2.23	2.18	36.358
14	3.17	2.22	2.19	36.677	14	3.13	2.25	2.22	36.676
15	3.26	2.27	2.27	36.997	15	3.26	2.26	2.24	36.997
16	3.41	2.29	2.25	37.320	16	3.38	2.28	2.27	37.320
17	3.53	2.29	2.32	37.645	17	3.51	2.30	2.31	37.644
18	3.61	2.34	2.33	37.970	18	3.62	2.32	2.34	37.970
19	3.74	2.35	2.34	38.295	19	3.73	2.35	2.36	38.295
20	3.87	2.38	2.35	38.620	20	3.83	2.38	2.37	38.619
21	3.94	2.38	2.40	38.942	21	3.93	2.39	2.45	38.942
22	4.06	2.41	2.44	39.264	22	4.06	2.41	2.45	39.264
23	4.17	2.42	2.51	39.585	23	4.15	2.43	2.49	39.585
24	4.26	2.46	2.52	39.903	24	4.25	2.44	2.53	39.903
25	4.40	2.50	2.47	40.220	25	4.34	2.48	2.53	40.219
26	4.48	2.54	2.51	40.534	26	4.48	2.50	2.58	40.533
27	4.57	2.55	2.60	40.846	27	4.55	2.52	2.57	40.846
28	4.65	2.54	2.63	41.155	28	4.68	2.55	2.61	41.155
29	4.74	2.54	2.68	41.463	29	4.75	2.55	2.66	41.462
30	4.85	2.59	2.73	41.767	30	4.85	2.59	2.66	41.767
31	4.97	2.57	2.71	42.070	31	4.94	2.59	2.74	42.069
32	5.02	2.66	2.73	42.368	32	5.04	2.64	2.73	42.368
33	5.13	2.67	2.69	42.665	33	5.13	2.64	2.74	42.665

Table 5.2 shows that the results of economic design obtained using SA and TLBO methods are observed to be nearly same for almost all 32 values of sample size *n*. The optimum value of loss cost per unit time $E(L)_1$ initially decreases as *n* value increases from 2 to 3 and thereafter it increases at higher values of *n*. The variation of expected loss cost per unit time $E(L)_1$ with respect to sample size *n* in case of SA and TLBO are graphically shown in Figs. 5.1 and 5.2 respectively. For the sake of showing the optimal point with better clarity, both the graphs are drawn over a limited range of sample size i.e., n = 2 to 20 only. As no other optimal point occurs in the range n = 21 to 33 and also the pattern of variation in this range is not different from that in the range n = 2 to 20, the points beyond n = 20 are not felt to have any worth to be included in these graphs.

Out of all 32 sets of results, the lowest value of expected loss cost per unit time $E(L)_1$ is observed to occur at n = 3 in case of both SA and TLBO as shown in Figs. 5.1 and 5.2.

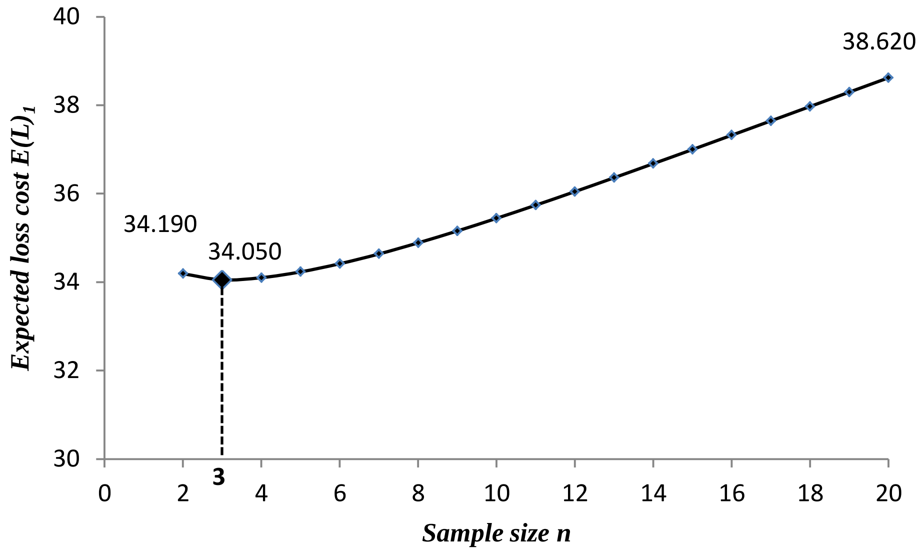

Fig. 5.1: Variation of expected loss cost per unit time with sample size using SA: continuous process

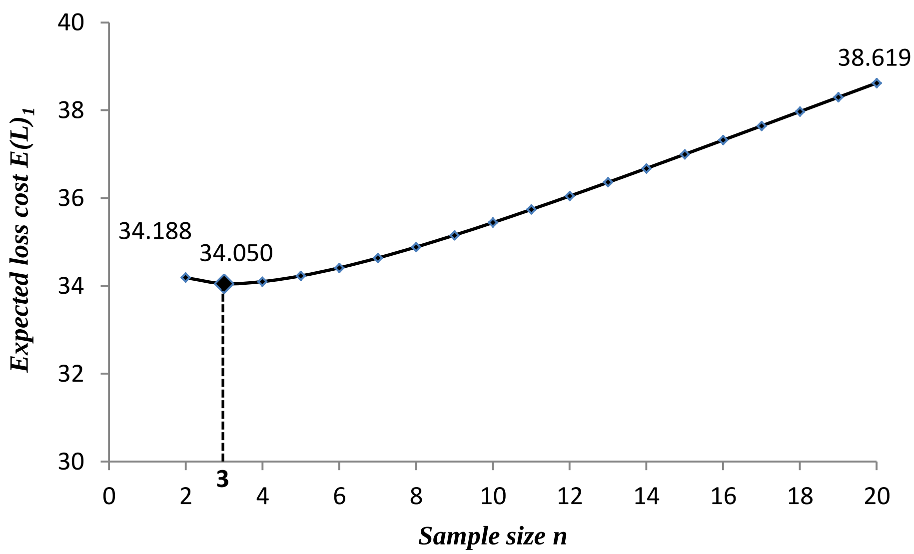

Fig. 5.2: Variation of expected loss cost per unit time with sample size using TLBO: continuous process

Tables 5.3 and 5.4 show a comparison of optimal results of joint economic design of $\bar{X}$ and R charts with that of economic design of $\bar{X}$ chart already shown in Sections 3.5.1 and 3.5.2 using SA and TLBO respectively.

Table 5.3: Comparison of results for economic design of $\bar{X}$ chart with joint economic design of $\bar{X}$ and *R* charts using SA: continuous process

Designs	***n***	***h***	k_1	k_2	α	***P***	ARL_0	ARL_1	$E(L)_1$
ED-C	6	2.00	1.79	-	0.0735	0.745	13.613	1.342	34.720
JED-C	3	1.16	2.20	2.09	0.0456	0.520	21.930	1.922	34.050

Note:
ED-C : Economic Design - Continuous process
JED-C : Joint Economic Design - Continuous process

Table 5.4: Comparison of results for economic design of $\bar{X}$ chart with joint economic design of $\bar{X}$ and *R* charts using TLBO: continuous process

Designs	***n***	***h***	k_1	k_2	α	***P***	ARL_0	ARL_1	$E(L)_1$
ED-C	6	1.99	1.80	-	0.0719	0.742	13.914	1.348	34.720
JED-C	3	1.16	2.19	2.12	0.0453	0.520	22.075	1.924	34.050

Note:
ED-C : Economic Design - Continuous process
JED-C : Joint Economic Design - Continuous process

Both the above tables show that the most minimum value of expected loss cost per unit time $E(L)_1$ in case of joint economic design (*JED*) of $\bar{X}$ and *R* charts is found to be 34.050 and this occurs at $n = 3$. However, in case of economic design (*ED*) of $\bar{X}$ chart the corresponding minimum value of $E(L)_1$ is found to be 34.720 at $n = 6$. In these tables, the corresponding optimal values of design variables *h*, k_1 and k_2 are also shown. Since k_2 is the width of control limits of *R* chart, this is not applicable in case of *ED* of $\bar{X}$ chart. It is to be noted that the symbol k_1 representing the width of control limits of $\bar{X}$ chart in *JED* is same as *k* that was earlier used in case of its economic design in Chapters 3 and 4. Further, these tables show the corresponding values of Type-I error (α), power of detecting a shift (*P*), average in-control run length (ARL_0) and average out-of-control run length (ARL_1). It can be observed that in joint economic design the rate of false alarm (i.e., value of α) is comparatively less than that of *ED* of $\bar{X}$ chart. Accordingly, the *JED* results in higher value of in-control average run length (ARL_0). In addition, the optimal value of expected loss cost per unit time $E(L)_1$ in *JED* is found to be less than that of *ED*. This reduction in cost may be due to comparatively lower value of optimal sample size required in case of joint design. On the other hand, the *JED* is associated with lower power of detecting the process shift (i.e., value of *P*) and corresponding higher value of out-of-control average run length (ARL_1). Moreover, the value of sampling interval *h* is relatively less which means that samples are required to be taken more frequently in *JED*.

5.5 Sensitivity Analysis: Continuous Process

To investigate the effect of cost and process parameters on the output results of joint economic design of $\bar{X}$ and R charts in case of continuous process, sensitivity analysis has been done. Ten cost and process parameters are considered as factors for this analysis. The low and high values of nine of these factors are already listed in Table 3.7. The additional tenth factor i.e., the shift in standard deviation (Υ) has been incorporated with its low and high values of 1.5 and 2.0 respectively. A 2_{IV}^{10-5} factorial design for ten factors with five generators I= ABCDF, I = ABCEG, I = ABDEH, I = ACDEJ and I = BCDEK, and resolution *IV* is chosen for a continuous process for the sensitivity analysis that gives a total of 32 (= 2^{10-5}) runs. For each of 32 runs, a particular set of values of ten factors is taken for which the loss cost function $E(L)_1$ is minimized using TLBO algorithm and the optimal result consisting of the values of five responses viz. n, h, k_1, k_2 and $E(L)_1$ is shown in Table 5.5. Since both SA and TLBO algorithms provided almost the same results for joint economic design in a continuous process as observed in Section 5.4, any one of them is sufficient for sensitivity analysis.

Table 5.5: Optimal joint economic designs of $\bar{X}$ and R charts: continuous process

	Cost and process parameters (*factors*)										*Responses*				
S. No.	***M***	***δ***	***λ***	***g***	***(T₁+T₂)***	***a***	***b***	***W***	***Y***	***ϒ***	***n***	***h***	***k₁***	***k₂***	***E(L)₁***
1	50	2	0.05	0.05	20	5.0	0.1	250	500	1.5	9	5.12	3.39	3.89	33.873
2	100	2	0.01	0.50	20	0.5	0.1	250	50	1.5	2	0.98	2.82	3.40	21.273
3	50	1	0.01	0.05	20	5.0	0.1	35	50	1.5	16	6.09	2.54	2.52	11.308
4	50	2	0.01	0.05	20	0.5	1.0	250	50	2.0	2	3.06	2.61	2.65	12.445
5	100	1	0.05	0.50	3	0.5	1.0	250	50	2.0	2	0.92	2.55	2.33	34.522
6	50	1	0.05	0.50	3	5.0	0.1	35	500	2.0	4	2.02	3.21	2.88	18.311
7	100	1	0.01	0.50	20	5.0	1.0	35	50	2.0	3	4.11	2.25	1.95	22.888
8	100	2	0.05	0.05	20	0.5	1.0	35	50	1.5	2	1.69	2.23	2.70	54.812
9	100	1	0.01	0.05	3	0.5	0.1	35	50	2.0	6	1.24	3.17	2.81	5.632
10	50	1	0.05	0.05	20	0.5	1.0	35	500	2.0	3	2.38	3.14	2.85	29.843
11	50	1	0.01	0.05	3	5.0	1.0	250	500	2.0	8	6.87	3.19	2.84	8.461
12	100	1	0.01	0.05	20	0.5	1.0	250	500	1.5	9	4.33	2.85	2.86	24.746
13	100	2	0.05	0.50	3	5.0	0.1	35	50	1.5	2	1.53	2.14	2.58	27.004
14	100	2	0.05	0.05	3	0.5	0.1	250	500	2.0	5	0.65	3.75	3.78	28.169
15	50	1	0.05	0.50	20	5.0	1.0	250	50	1.5	3	6.14	1.60	1.40	36.007
16	50	2	0.01	0.05	3	0.5	0.1	35	500	1.5	7	2.07	3.71	4.23	3.221
17	100	1	0.05	0.05	3	5.0	1.0	35	500	1.5	10	2.67	2.74	2.75	31.009
18	100	2	0.01	0.50	3	0.5	1.0	35	500	2.0	2	1.47	3.35	3.40	8.328
19	100	2	0.01	0.05	20	5.0	0.1	35	500	2.0	9	4.05	3.57	3.58	20.309
20	50	2	0.05	0.05	3	5.0	1.0	35	50	2.0	4	3.18	2.45	2.43	14.865
21	100	2	0.05	0.50	20	5.0	1.0	250	500	2.0	3	3.41	2.92	2.89	63.391
22	50	2	0.01	0.50	20	5.0	1.0	35	500	1.5	4	6.46	2.93	3.45	12.558
23	50	2	0.05	0.50	3	0.5	1.0	250	500	1.5	2	1.32	3.11	3.62	24.019
24	50	2	0.05	0.50	20	0.5	0.1	35	50	2.0	2	1.17	2.86	2.86	27.946
25	100	1	0.01	0.50	3	5.0	0.1	250	500	1.5	7	2.58	2.98	2.99	14.482
26	50	1	0.01	0.50	20	0.5	0.1	250	500	2.0	3	1.18	3.69	3.44	12.630
27	50	1	0.01	0.50	3	0.5	1.0	35	50	1.5	2	2.14	2.29	2.28	6.127
28	50	1	0.05	0.05	3	0.5	0.1	250	50	1.5	8	1.13	2.73	2.75	20.990
29	100	2	0.01	0.05	3	5.0	1.0	250	50	1.5	4	4.26	2.33	2.77	10.288
30	100	1	0.05	0.05	20	5.0	0.1	250	50	2.0	9	3.23	2.63	2.21	60.773
31	50	2	0.01	0.50	3	5.0	0.1	250	50	2.0	3	4.44	2.55	2.52	7.323
32	100	1	0.05	0.50	20	0.5	0.1	35	500	1.5	3	0.51	3.26	3.29	57.365

To find out the statistical significance of all the ten factors on each of the five output responses, analysis of variance (ANOVA) has been carried out on these 32 sets of results of joint economic design of $\bar{X}$ and R charts for a continuous process. The results of ANOVA at 95% confidence level (i.e., significance level of 5%) on the economic design results are shown in Tables 5.6 - 5.10. To identify the same in graphical manner, the normal plots of standardized effects for five output responses are shown in Figs. 5.3 - 5.7. These plots and ANOVA tables have been obtained with the help of student version of MINITAB 16.

Table 5.6: Analysis of variance for expected loss cost per unit time $E(L)_1$: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	1314.080	1314.080	1314.080	30.60	0.000*	15.87
δ	1	19.950	19.950	19.950	0.46	0.503	0.24
λ	1	4069.780	4069.780	4069.780	94.77	0.000*	49.16
g	1	17.160	17.160	17.160	0.40	0.534	0.21
(T_1+T_2)	1	1791.250	1791.250	1791.250	41.71	0.000*	21.64
a	1	13.500	13.500	13.500	0.31	0.581	0.16
b	1	17.550	17.550	17.550	0.41	0.529	0.21
W	1	119.610	119.610	119.610	2.79	0.110	1.44
Y	1	8.520	8.520	8.520	0.20	0.661	0.10
Υ	1	5.480	5.480	5.480	0.13	0.724	0.07
Residual Error	21	901.780	901.780	42.940			
Total	31	8278.650					

* Significant at 5%

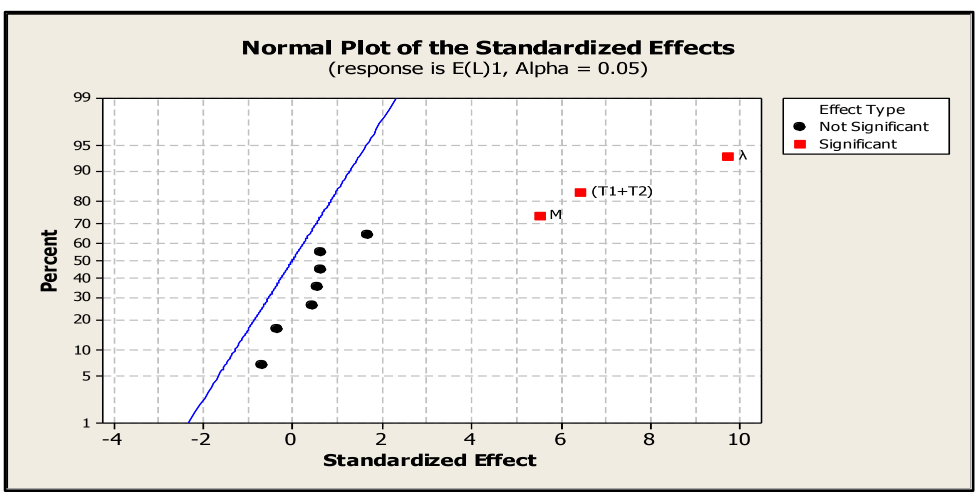

Fig. 5.3: Normal probability plot of standardized effect for expected loss cost per unit time $E(L)_1$: continuous process

Table 5.6 indicates that the expected loss cost per unit time of process control $E(L)_1$ in a continuous process is significantly affected by three factors, namely loss of net income when process is out-of-control *M*, rate of occurrences of assignable causes λ, and time to find and repair an assignable cause (T_1+T_2). All these three factors are significant as they all have p-value less than the predetermined significance level of 0.05. Among all the factors, λ has the highest significant effect on expected loss cost per unit time $E(L)_1$ since it has the highest F-value i.e. 94.77 as shown in Table 5.6 and is also graphically plotted at the rightmost location in the normal probability plot of standardized effect as shown in Fig. 5.3. It can also be observed from this table that λ, (T_1+T_2), and *M* are the top three percentage contributors which affect the cost by 49.16%, 21.64% and 15.87% respectively. All these three factors are observed to have positive effects as all of them are falling on the right side of the straight line in Fig. 5.3. This implies that as the value of any of these factors increases, the expected loss cost per unit time $E(L)_1$ increases.

Table 5.7 shows the results of ANOVA for the sample size *n*. There are four factors, such as the size of the shift in the process mean δ, time to sample and chart one item *g*, fixed cost per sample *a* and variable cost per sample *b* which have significant effect on sample size. Fig. 5.4 shows that out of these four significant factors, three factors have negative effect on sample size except fixed cost of sampling *a*. An increase in δ, *g* or *b* decreases the optimum sample size, because they all have the negative effects. Moreover, the percentage contributions of these four significant factors *g*, *a*, δ and *b* affecting the sample size are 36.58%, 12.90%, 10.33% and 9.15% respectively. Thus, *g* is the most significant factor for choosing the value of sample size, in economic design and the effect is of negative type.

Table 5.7: Analysis of variance for sample size *n*: continuous process

Source	***DF***	***Seq SS***	***Adj SS***	***Adj MS***	***F***	***p***	***PC(%)***
M	1	0.125	0.125	0.125	0.04	0.853	0.04
δ	1	36.125	36.125	36.125	10.23	0.004*	10.33
λ	1	8.000	8.000	8.000	2.27	0.147	2.29
g	1	128.000	128.000	128.000	36.26	0.000*	36.58
(T_1+T_2)	1	1.125	1.125	1.125	0.32	0.578	0.32
a	1	45.125	45.125	45.125	12.78	0.002*	12.90
b	1	32.000	32.000	32.000	9.07	0.007*	9.15
W	1	0.000	0.000	0.000	-	-	0.00
Y	1	10.125	10.125	10.125	2.87	0.105	2.89
Υ	1	15.125	15.125	15.125	4.28	0.051	4.32
Residual Error	21	74.125	74.125	3.530			
Total	31	349.875					

* Significant at 5%

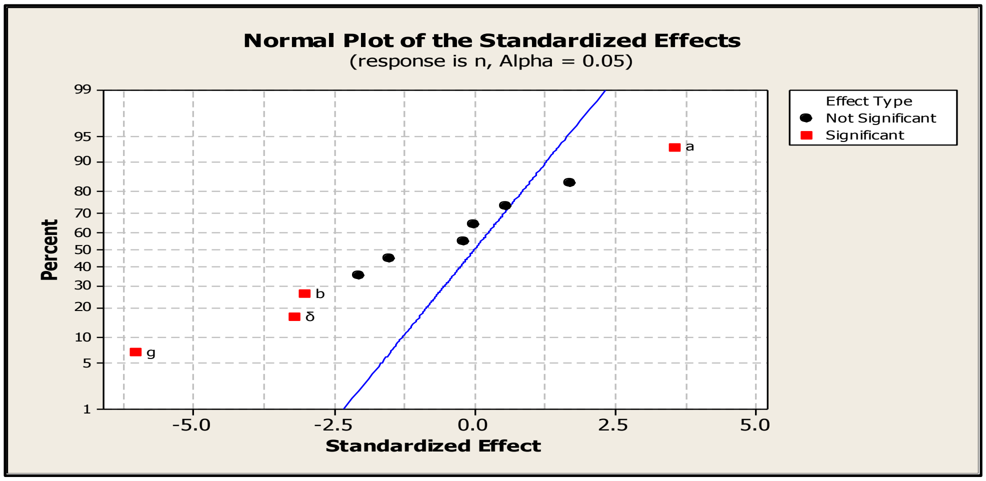

Fig. 5.4: Normal probability plot of standardized effect for sample size *n*: continuous process

Table 5.8 displays the results of analysis of variance on the sampling interval *h*. It is significantly affected by six factors, namely loss of net income when process is out-of-control *M*, rate of occurrences of assignable causes λ, time to sample and chart one item *g*, the time to find and repair an assignable cause (T_1+T_2), fixed cost per sample *a* and variable cost per sample *b*. Out of these six significant factors, three factors i.e., λ, *M* and *g* have negative effects as shown in Fig. 5.5, whereas the factors (T_1+T_2), *b* and *a* are significant in terms of positive effect. Moreover, the positive effect parameters such as *a*, *b* and (T_1+T_2) contribute 47.82%, 8.11% and 7.16% respectively, whereas the negative effect parameters like λ, *M* and *g* contribute by 10.01%, 8.81% and 4.05% respectively. Thus, among all the factors, the fixed cost of sampling *a* has the highest effect on the sampling interval with a percentage contribution and the effect is in positive direction.

Table 5.8: Analysis of variance for sampling interval *h*: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	9.177	9.1765	9.1765	16.24	0.001*	8.81
δ	1	0.222	0.2223	0.2223	0.39	0.537	0.21
λ	1	10.419	10.4193	10.4193	18.44	0.000*	10.01
g	1	4.219	4.2185	4.2185	7.47	0.012*	4.05
(*T1*+*T2*)	1	7.460	7.4597	7.4597	13.21	0.002*	7.16
a	1	49.789	49.7891	49.7891	88.14	0.000*	47.82
b	1	8.441	8.4405	8.4405	14.94	0.001*	8.11
W	1	1.443	1.4431	1.4431	2.55	0.125	1.39
Y	1	0.101	0.1006	0.1006	0.18	0.677	0.10
Υ	1	0.995	0.9948	0.9948	1.76	0.199	0.96
Residual Error	21	11.863	11.8628	0.5649			
Total	31	104.127					

* Significant at 5%

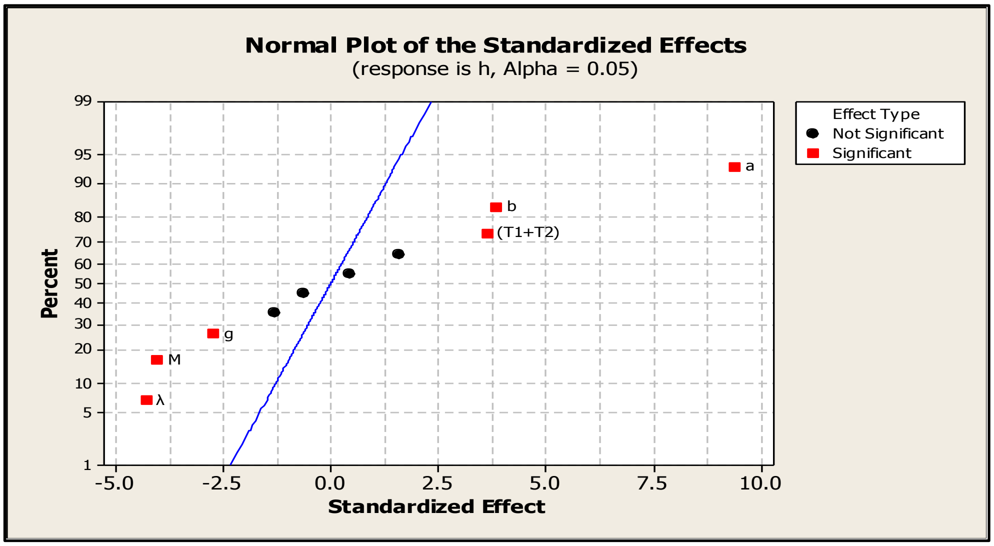

Fig. 5.5: Normal probability plot of standardized effect for sampling interval *h*: continuous process

Table 5.9 presents analysis of variance on the control limits width k_1 of $\bar{X}$ chart. There are seven factors (i.e., δ, λ, g, a, b, Y and Υ) are found to be significant on k_1. Fig. 5.6 reveals that out of these seven significant factors, four factors (i.e., b, a, g and λ) are having negative effect and the rest three factors have positive effect on k_1. Among all factors, the cost per false alarm Y is observed to be the most significant effect with the maximum contribution of 56.91% on deciding the value of k_1 in economic design and its effect is of positive type.

Table 5.9: Analysis of variance for width of control limits k_1 of $\bar{X}$ chart: continuous process

Source	***DF***	***Seq SS***	***Adj SS***	***Adj MS***	***F***	***p***	***PC(%)***
M	1	0.00543	0.00543	0.00543	0.29	0.596	0.07
δ	1	0.11073	0.11073	0.11073	5.90	0.024*	1.39
λ	1	0.13902	0.13902	0.13902	7.41	0.013*	1.75
g	1	0.19350	0.19350	0.19350	10.32	0.004*	2.44
(*T1*+*T2*)	1	0.02956	0.02956	0.02956	1.58	0.223	0.37
a	1	0.68961	0.68961	0.68961	36.76	0.000*	8.69
b	1	1.30032	1.30032	1.30032	69.32	0.000*	16.38
W	1	0.00059	0.00059	0.00059	0.03	0.861	0.01
Y	1	4.51757	4.51757	4.51757	240.84	0.000*	56.91
Υ	1	0.55794	0.55794	0.55794	29.74	0.000*	7.03
Residual Error	21	0.39391	0.39391	0.01876			
Total	31	7.93818					

* Significant at 5%

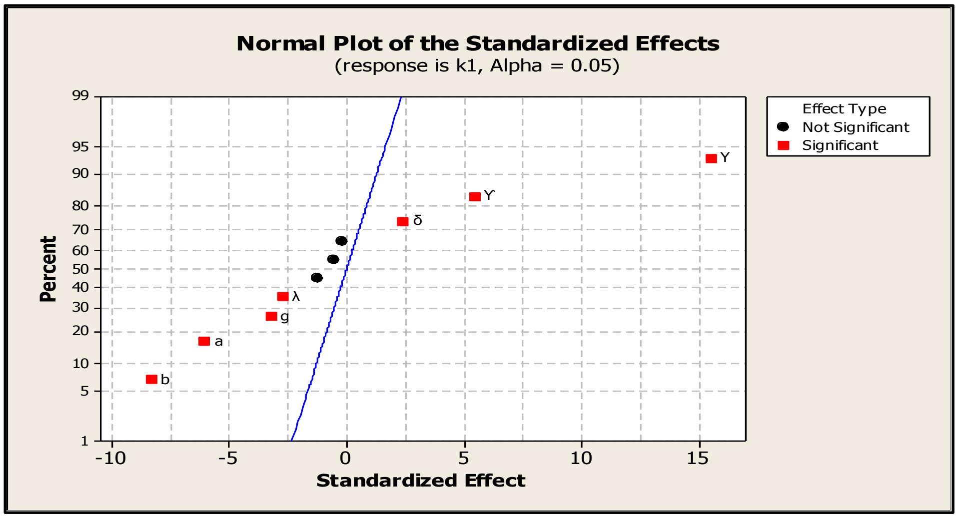

Fig. 5.6: Normal probability plot of standardized effect for width of control limits k_1 of $\bar{X}$ chart: continuous process

Similarly, Table 5.10 shows the ANOVA results on the control limits width k_2 of R chart. There are six factors (i.e., δ, λ, g, a, b and Y) are found to be significant on k_2. Fig. 5.7 reveals that out of these six significant factors, four factors (i.e., b, a, λ and g) have negative effect on width of control limits and two factors (i.e., δ and Y) have positive effect. Further, the percentage contributions of all these significant factors Y, δ, b, a, λ and g are 45.32%, 21.31%, 12.38%, 8.98%, 1.77% and 1.59% respectively. Similar to the ANOVA results for control limits width k_1 of $\bar{X}$ chart, the cost per false alarm Y is found to be the most significant factor with positive effect on deciding the value of k_2. Only one more factor i.e., δ has positive effect, whereas all other significant factors have negative effect as shown in Fig. 5.7.

Table 5.10: Analysis of variance for width of control limits k_2 of R chart: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	0.0031	0.0031	0.00306	0.08	0.776	0.03
δ	1	2.3213	2.3213	2.32126	63.06	0.000*	21.31
λ	1	0.1927	0.1927	0.19267	5.23	0.033*	1.77
g	1	0.1733	0.1733	0.17330	4.71	0.042*	1.59
$(T1+T2)$	1	0.0317	0.0317	0.03171	0.86	0.364	0.29
a	1	0.9783	0.9783	0.97827	26.58	0.000*	8.98
b	1	1.3485	1.3485	1.34845	36.63	0.000*	12.38
W	1	0.0012	0.0012	0.00119	0.03	0.859	0.01
Y	1	4.9364	4.9364	4.93637	134.11	0.000*	45.32
Y'	1	0.1342	0.1342	0.13424	3.65	0.070	1.23
Residual Error	21	0.7730	0.7730	0.03681			
Total	31	10.8935					

* Significant at 5%

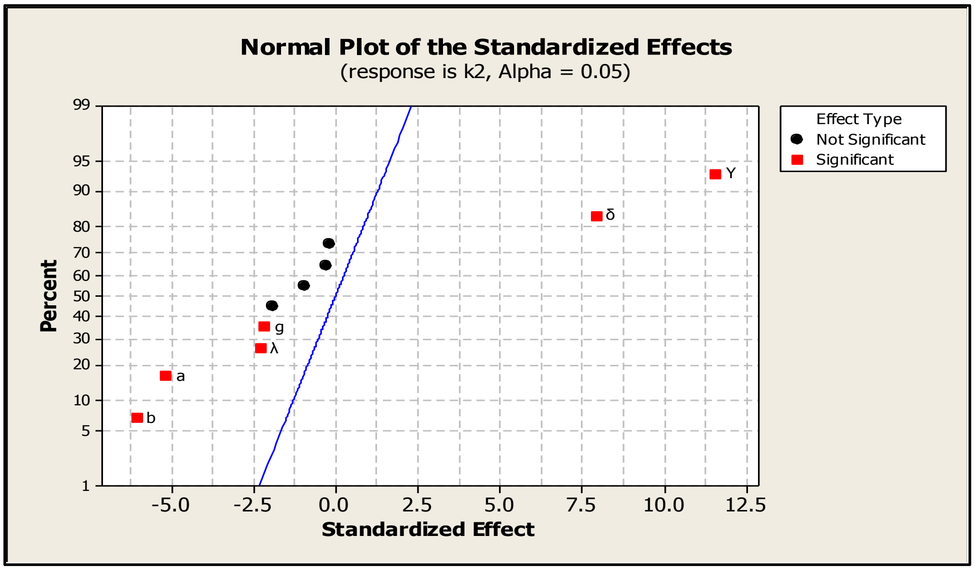

Fig. 5.7: Normal probability plot of standardized effect for width of control limits k_2 of R chart: continuous process

From the AVOVA study it is further observed from Tables 5.6 - 5.10 that the factor W i.e. the cost to locate and repair the assignable cause has no significant effect on any of the five responses n, h, k_1, k_2 and $E(L)_1$ in joint economic design of $\bar{X}$ and R charts. This observation is consistent with that in the results of economic design of $\bar{X}$ chart.

5.5.1 Summary of Results

All the above results related to the types of effects of all the ten cost and process parameters on each of the five responses in case of a continuous process are summarized in Table 5.11. The blank spaces denote insignificant factors. Significant factor with positive effect is shown as '+' whereas '–' denotes significant factor with negative effect. The significant factor with the highest percentage contribution for each response is shown bold. The control chart designers must take utmost care on ensuring the correctness of values of significant factors before using them for joint economic design.

Table 5.11: Summary of significant effects in joint economic design: continuous process

Output responses	*Cost and process parameters*									
	M	***δ***	***λ***	***g***	**(T_1+T_2)**	***a***	***b***	***W***	***Y***	***ϒ***
n		–		**–**		+	–			
h	–		–	–	+	**+**	+			
k_1		+	–	–		–	–		**+**	+
k_2		+	–	–		–	–		**+**	
$E(L)_1$	+		**+**		+					

Note:

Blank space	: Insignificant factor
+	: Factor with positive effect
–	: Factor with negative effect
+/– in bold	: Most significant factor

5.6 Numerical Illustration: Discontinuous Process

In this section, the joint economic design of $\bar{X}$ and *R* charts for a discontinuous process is illustrated through the same numerical problem which was earlier considered for economic design of $\bar{X}$ chart in Section 3.7. The joint economic design deals with fourteen cost and process parameters out of which the values of thirteen factors are mentioned in Table 3.14. The value of fourteenth factor i.e., shift in standard deviation *ϒ* is taken as 1.5 which is same as that considered in case of continuous process in Section 5.4. The primary objective is to select proper values four design variables (i.e., *n*, *h*, k_1 and k_2) so as to minimize the expected loss cost per unit time $E(L)_2$ whose expression is shown in Eq. 3.31. For minimization purpose, the same two metaheuristics i.e., SA and TLBO are also considered in this section.

5.6.1 Results and Discussion

Table 5.12 shows the results of joint economic design of $\bar{X}$ and *R* charts for a discontinuous process using SA and TLBO. This table shows the optimal values of three design variables of control chart such as sampling interval *h*, width of the control limits for

$\bar{X}$ chart k_1 and width of the control limits for R chart k_2 for each integer value of sample size n varying from 2 to 33. It also shows the corresponding minimum value of the expected loss cost per unit time $E(L)_2$.

Table 5.12: Optimal joint economic designs of $\bar{X}$ and R charts: discontinuous process

SA					TLBO				
n	***h***	k_1	k_2	$E(L)_2$	***n***	***h***	k_1	k_2	$E(L)_2$
2	0.50	3.09	3.14	37.435	2	0.50	3.09	3.17	37.435
3	0.71	3.06	3.08	36.726	3	0.71	3.03	3.07	36.725
4	0.91	3.03	3.04	36.321	4	0.92	3.02	3.03	36.321
5	1.11	3.00	3.02	36.060	5	1.12	3.00	3.00	36.059
6	1.32	2.98	2.99	35.896	6	1.32	2.98	2.99	35.896
7	1.50	2.97	2.98	35.805	7	1.51	2.97	2.98	35.805
8	**1.69**	**2.96**	**2.97**	**35.770**	**8**	**1.69**	**2.96**	**2.97**	**35.770**
9	1.87	2.95	2.97	35.780	9	1.86	2.96	2.97	35.780
10	2.04	2.95	2.97	35.825	10	2.04	2.94	2.98	35.825
11	2.21	2.94	2.98	35.902	11	2.21	2.95	2.96	35.902
12	2.39	2.94	2.99	36.006	12	2.36	2.95	2.98	36.006
13	2.52	2.95	2.98	36.132	13	2.51	2.95	3.00	36.132
14	2.63	2.95	3.00	36.279	14	2.66	2.96	3.00	36.278
15	2.81	2.95	3.00	36.440	15	2.80	2.95	3.02	36.440
16	2.95	2.96	3.02	36.617	16	2.94	2.96	3.02	36.617
17	3.06	2.97	3.03	36.807	17	3.07	2.96	3.05	36.807
18	3.20	2.96	3.04	37.009	18	3.19	2.97	3.05	37.008
19	3.28	2.99	3.05	37.219	19	3.31	2.97	3.06	37.219
20	3.40	3.00	3.08	37.438	20	3.43	2.99	3.06	37.437
21	3.56	2.98	3.09	37.663	21	3.53	2.99	3.11	37.663
22	3.65	3.01	3.09	37.896	22	3.63	3.00	3.11	37.896
23	3.75	3.01	3.14	38.134	23	3.76	3.02	3.10	38.133
24	3.85	3.04	3.13	38.376	24	3.86	3.03	3.13	38.375
25	3.94	3.06	3.11	38.622	25	3.97	3.04	3.14	38.621
26	4.04	3.06	3.17	38.870	26	4.04	3.05	3.16	38.870
27	4.18	3.05	3.18	39.124	27	4.15	3.08	3.15	39.124
28	4.23	3.09	3.18	39.378	28	4.25	3.08	3.17	39.378
29	4.36	3.09	3.21	39.634	29	4.32	3.10	3.18	39.634
30	4.44	3.09	3.24	39.891	30	4.43	3.10	3.22	39.891
31	4.54	3.10	3.18	40.151	31	4.52	3.12	3.23	40.150
32	4.63	3.14	3.26	40.409	32	4.62	3.12	3.25	40.409
33	4.70	3.13	3.26	40.668	33	4.72	3.14	3.26	40.668

Table 5.12 reveals that the results of joint economic design obtained using SA and TLBO techniques are observed to be nearly same for almost all 32 values of sample size n. The optimum value of expected loss cost per unit time $E(L)_2$ initially decreases as n value increases from 2 to 8 and thereafter it increases at higher values of n. The variations of $E(L)_2$ with respect to n in case of SA and TLBO are graphically shown in Figs. 5.8 and 5.9 respectively. Out of all 32 sets of results, the lowest value of expected loss cost per unit time

$E(L)_2$ is observed to occur at n = 8 in case of both SA and TLBO as shown in Figs. 5.8 and 5.9. Like in Section 5.4.1, both these graphs are drawn over a limited range of n = 2 to 20 for better clarity.

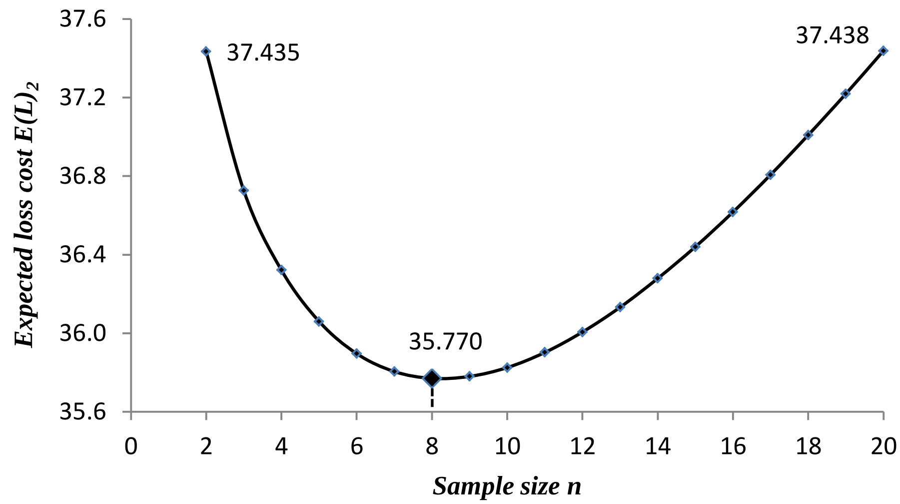

Fig. 5.8: Variation of expected loss cost per unit time with sample size using SA: discontinuous process

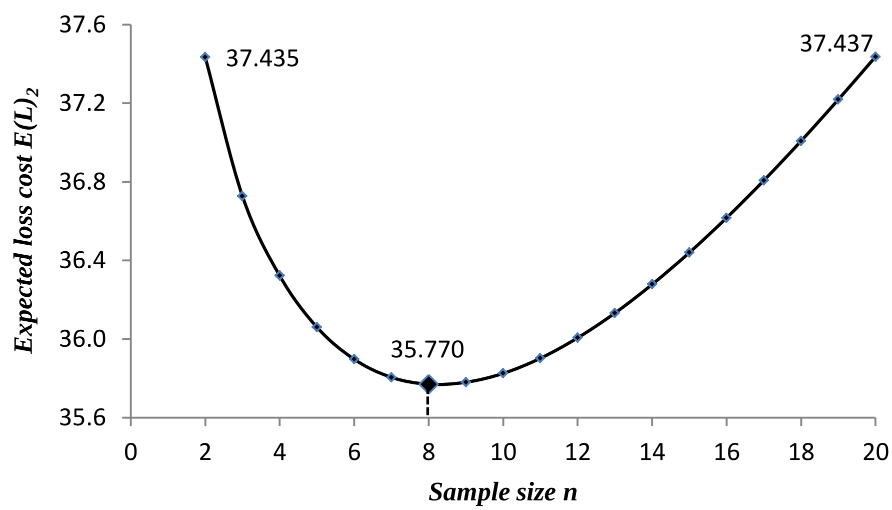

Fig. 5.9: Variation of expected loss cost per unit time with sample size using TLBO: discontinuous process

Table 5.13 shows a comparison of optimal results of joint economic design of $\bar{X}$ and R charts with that of economic design of $\bar{X}$ chart (i.e., already shown in Table 3.15) for a discontinuus process using SA. Similarly, Table 5.14 shows the comparion of both sets of results obtained using TLBO. The results of economic design shown in Table 5.14 have been reproduced from Table 3.17.

Table 5.13: Comparison of results for economic design of $\bar{X}$ chart with joint economic design of $\bar{X}$ and *R* charts using SA: discontinuous process

Designs	***n***	***h***	k_1	k_2	α	***P***	ARL_0	ARL_1	$E(L)_2$
ED-D	12	2.50	2.75	-	0.0060	0.762	167.370	1.312	35.966
JED-D	8	1.69	2.96	2.97	0.0046	0.575	217.813	1.738	35.770

Note:
ED-D : Economic Design - Discontinuous process
JED-D: Joint Economic Design - Discontinuous process

Table 5.14: Comparison of results for economic design of $\bar{X}$ chart with joint economic design of $\bar{X}$ and *R* charts using TLBO: discontinuous process

Designs	***n***	***h***	k_1	k_2	α	***P***	ARL_0	ARL_1	$E(L)_2$
ED-D	12	2.50	2.75	-	0.0060	0.762	167.370	1.312	35.966
JED-D	8	1.69	2.96	2.97	0.0046	0.575	217.813	1.738	35.770

Note:
ED-D : Economic Design - Discontinuous process
JED-D: Joint Economic Design - Discontinuous process

Tables 5.13 - 5.14 show that the most minimum value of expected loss cost per unit time $E(L)_2$ in case of joint economic design of $\bar{X}$ and *R* charts is found to be 35.770 and this occurs at n = 8. However, in case of economic design of $\bar{X}$ chart the corresponding minimum value of $E(L)_2$ is found to be 35.966 occurring at n = 12. Further, these tables show the values of Type-I error (α), power of detecting a shift (*P*), average in-control run length (ARL_0) and average out-of-control run length (ARL_1) corresponding to optimal designs. It is observed that the rate of false alarm (i.e., α-error) in joint economic design is comparatively less than that of economic design of $\bar{X}$ chart. Accordingly, the joint economic design results in higher value of in-control average run length. In addition, the optimal value of expected loss cost per unit time $E(L)_2$ in joint economic design is found to be less than that of economic design. This reduction in cost may be due to comparatively lower value of optimal sample size required in case of joint design. On the other hand, the joint economic design is associated with lower power of detecting the process shift *P* and corresponding higher value of out-of-control average run length ARL_1. Moreover, the value of sampling interval h is relatively less which means that samples are required to be taken more frequently in joint economic design.

5.7 Sensitivity Analysis: Discontinuous Process

To investigate the effect of process and cost parameters on the output results of joint economic design of $\bar{X}$ and R charts in case of discontinuous process, sensitivity analysis has been done. Each of fourteen cost and process parameters are considered as a factor for this analysis. The low and high values of thirteen of these factors are already listed in Table 3.19. For the additional fourteenth factor i.e., the shift in standard deviation (Υ), the low and high values are taken as 1.5 and 2.0 respectively. A 2_{IV}^{14-9} factorial design for fourteen factors with nine generators I = ABCF, I = ABDG, I = ABEH, I = ACDJ, I = ACEK, I = ADEL, I = BCDM, I = BCEN and I = BDEO, and resolution *IV* is chosen for a discontinuous process for the sensitivity analysis that gives a total of 32 (= 2^{14-9}) runs. For each of 32 runs, a particular set of values of fourteen factors is taken for which the loss cost function $E(L)_2$ is minimized using TLBO algorithm and the optimal result consisting of the values of five responses viz., n, h, k_1, k_2 and $E(L)_2$ is shown in Table 5.15. Since both SA and TLBO algorithms provided almost the same results for joint economic design in a discontinuous process as observed in Section 5.6, any one of them is sufficient for sensitivity analysis.

Table 5.15: Optimal joint economic designs of $\overline{X}$ and R charts: discontinuous process

S. No.	Cost and process parameters (factors)														Responses				
	M	δ	λ	g	(T_1+T_2)	a	b	W	Y	V_0	S	S_1	T_0	Υ	n	h	k_1	k_2	$E(L)_2$
1	50	2	0.05	0.05	20	0.5	1.0	35	500	50	100	0.05	40	1.5	5	2.83	3.43	3.95	30.865
2	100	1	0.01	0.05	20	5.0	1.0	35	500	50	10	0.05	40	2.0	9	4.72	3.55	3.23	14.772
3	100	2	0.05	0.05	3	5.0	0.1	35	50	50	100	0.05	4	2.0	6	1.56	3.14	3.12	20.392
4	50	1	0.01	0.50	20	0.5	1.0	250	500	150	10	1.00	40	1.5	10	7.25	3.50	3.57	32.326
5	50	1	0.01	0.05	3	0.5	0.1	35	50	50	10	0.05	4	1.5	14	2.37	3.24	3.31	4.133
6	100	2	0.05	0.05	20	5.0	0.1	250	50	150	10	0.05	40	1.5	10	3.64	3.93	4.50	83.603
7	100	2	0.01	0.05	20	0.5	0.1	250	500	50	10	1.00	4	1.5	7	1.48	3.78	4.24	12.549
8	100	1	0.01	0.50	3	5.0	0.1	250	50	150	10	1.00	4	2.0	5	2.56	3.42	3.10	15.461
9	100	2	0.05	0.50	3	5.0	1.0	35	500	50	10	1.00	4	1.5	3	1.65	2.96	3.45	25.919
10	100	1	0.05	0.05	3	0.5	1.0	250	50	50	100	1.00	40	1.5	8	1.69	2.96	2.97	35.770
11	50	2	0.01	0.05	3	5.0	1.0	250	50	50	10	1.00	40	2.0	7	6.95	3.61	3.61	8.191
12	100	1	0.01	0.05	3	5.	1.0	250	500	150	100	0.05	4	1.5	14	5.79	3.03	3.09	16.130
13	50	1	0.05	0.05	3	5.0	0.1	35	500	150	10	1.00	40	1.5	26	3.72	3.82	3.98	32.028
14	50	2	0.05	0.05	3	0.5	1.0	250	500	150	10	0.05	4	2.0	4	2.71	3.49	3.47	34.755
15	50	2	0.01	0.05	20	5.0	1.0	35	50	150	100	1.00	4	1.5	7	10.13	3.24	3.81	29.346
16	100	2	0.01	0.05	3	0.5	0.1	35	500	150	100	1.00	40	2.0	7	1.34	4.35	4.42	9.266
17	50	1	0.05	0.05	20	5.0	0.1	250	500	50	100	1.00	4	2.0	14	3.86	3.53	3.26	36.245
18	50	1	0.05	0.50	3	5.0	1.0	35	50	150	100	0.05	40	2.0	5	3.54	3.69	3.38	34.816
19	100	2	0.01	0.50	3	0.5	1.0	35	50	150	10	0.05	40	1.5	4	2.20	3.78	4.40	11.480
20	100	1	0.05	0.50	3	0.5	0.1	250	500	50	10	0.05	40	2.0	2	0.25	3.93	3.76	27.995
21	100	1	0.05	0.50	20	0.5	0.1	35	500	150	100	0.05	4	1.5	4	0.60	3.50	3.56	81.898
22	50	2	0.05	0.50	20	0.5	0.1	35	50	50	10	1.00	40	2.0	2	0.63	3.80	3.91	28.676
23	50	2	0.05	0.50	3	0.5	0.1	250	50	150	100	1.00	4	1.5	3	1.28	3.54	4.09	41.580
24	100	1	0.01	0.50	20	5.0	0.1	35	50	50	100	1.00	40	1.5	8	2.44	3.29	3.32	17.997
25	50	2	0.01	0.50	3	5.0	0.1	250	500	50	100	0.05	40	1.5	6	4.58	3.62	4.17	8.829
26	50	2	0.01	0.50	20	5.0	0.1	35	500	150	10	0.05	4	2.0	6	6.34	3.58	3.57	27.659
27	50	1	0.01	0.50	3	0.5	1.0	35	500	50	100	1.00	4	2.0	2	1.64	3.45	3.26	7.581
28	100	1	0.05	0.05	20	0.5	1.0	35	50	150	10	1.00	4	2.0	4	2.09	3.25	2.93	80.789
29	50	1	0.05	0.50	20	5.0	1.0	250	50	50	10	0.05	4	1.5	4	4.15	2.19	2.08	35.993
30	100	2	0.05	0.50	20	5.0	1.0	250	500	150	100	1.00	40	2.0	4	4.50	3.58	3.59	88.577
31	100	2	0.01	0.50	20	0.5	1.0	250	50	50	100	0.05	4	2.0	2	1.61	3.11	3.14	15.110
32	50	1	0.01	0.05	20	0.5	0.1	250	50	150	100	0.05	40	2.0	12	3.10	4.29	4.03	29.118

To find out the statistical significance of all the fourteen factors on each of the five output responses, analysis of variance (ANOVA) has been carried out on these 32 sets of results of joint economic design of $\bar{X}$ and R charts for a discontinuous process. The results of ANOVA at 95% confidence level (i.e., significance level of 5%) on the economic design results are shown in Tables 5.16 - 5.20. The significant factors are also graphically identified in the normal plots of standardized effects for five output responses as shown in Figs. 5.10 - 5.14. These normal plots and ANOVA tables have been obtained with the help of student version of MINITAB 16.

Table 5.16: Analysis of variance for expected loss cost per unit time $E(L)_2$: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	574.4	574.4	574.35	3.85	0.066	3.55
δ	1	21.5	21.5	21.54	0.14	0.709	0.13
λ	1	6611.1	6611.1	6611.13	44.29	0.000*	40.91
g	1	17.9	17.9	17.92	0.12	0.733	0.11
(T_1+T_2)	1	3026.4	3026.4	3026.40	20.27	0.000*	18.73
a	1	4.6	4.6	4.55	0.03	0.863	0.03
b	1	19.5	19.5	19.52	0.13	0.722	0.12
W	1	130.5	130.5	130.48	0.87	0.363	0.81
Y	1	0.8	0.8	0.80	0.01	0.942	0.00
V_0	1	3156.5	3156.5	3156.48	21.14	0.000*	19.53
S	1	23.1	23.1	23.11	0.15	0.699	0.14
S_1	1	19.1	19.1	19.14	0.13	0.725	0.12
T_0	1	2.4	2.4	2.40	0.02	0.901	0.01
Y'	1	13.8	13.8	13.84	0.09	0.764	0.09
Residual Error	17	2537.8	2537.8	149.28			
Total	31	16159.5					

* Significant at 5%

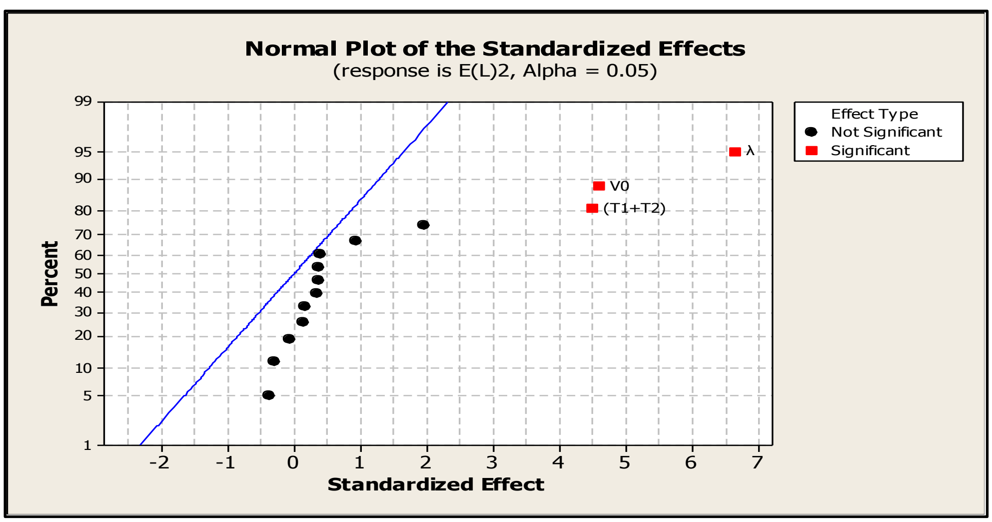

Fig. 5.10: Normal probability plot of standardized effect for expected loss cost per unit time $E(L)_2$: discontinuous process

Table 5.16 indicates that the expected loss cost per unit time of process control $E(L)_2$ in a discontinuous process is significantly affected by three factors, namely rate of occurrence of assignable cause (λ), time to find and repair an assignable cause (T_1+T_2), and net income per hour while process is in-control (V_0). They are also graphically shown as "significant" in the normal plot shown in Fig. 5.10. Among all the factors, λ has the highest significant effect on expected loss cost per unit time $E(L)_2$ since it has the highest F-value i.e. 44.29 as shown in Table 5.16 and plotted at the rightmost location in Fig. 5.10. It can also be observed from this table that the factors λ, V_0 and (T_1+T_2) are the top three percentage contributors which affect the cost by 40.91%, 19.53% and 18.73% respectively. All these three factors have positive effect as shown in Fig. 5.10.

Table 5.17 shows the results of ANOVA for the sample size *n*. There are five factors (i.e., δ, *g*, *a*, *b* and Υ) which have significant effect on sample size. Fig. 5.11 shows that out of these five significant factors, four factors have negative effect and the fixed cost of sampling *a* has positive effect. An increase in *g*, δ, Υ and *b* decreases the optimum sample size n, because they all have the negative effects. Moreover, the percentage contributions of these five significant factors *g*, δ, *a*, Υ and *b* affecting the sample size are 29.09%, 13.87%, 7.98%, 7.27% and 6.60% respectively. Thus, the factor *g* is the most significant for choosing the value of sample size, in joint economic design and the effect is of negative type.

Table 5.17: Analysis of variance for sample size *n*: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	28.125	28.125	28.125	2.90	0.107	3.71
δ	1	105.125	105.125	105.125	10.83	0.004*	13.87
λ	1	8.000	8.000	8.000	0.82	0.377	1.06
g	1	220.500	220.500	220.500	22.72	0.000*	29.09
(T_1+T_2)	1	2.000	2.000	2.000	0.21	0.656	0.26
a	1	60.500	60.500	60.500	6.23	0.023*	7.98
b	1	50.000	50.000	50.000	5.15	0.037*	6.60
W	1	0.000	0.000	0.000	-	-	0.00
Y	1	15.125	15.125	15.125	1.56	0.229	2.00
V_0	1	21.125	21.125	21.125	2.18	0.158	2.79
S	1	3.125	3.125	3.125	0.32	0.578	0.41
S_1	1	3.125	3.125	3.125	0.32	0.578	0.41
T_0	1	21.125	21.125	21.125	2.18	0.158	2.79
Υ	1	55.125	55.125	55.125	5.68	0.029*	7.27
Residual Error	17	165.000	165.000	9.706			
Total	31	758.000					

* Significant at 5%

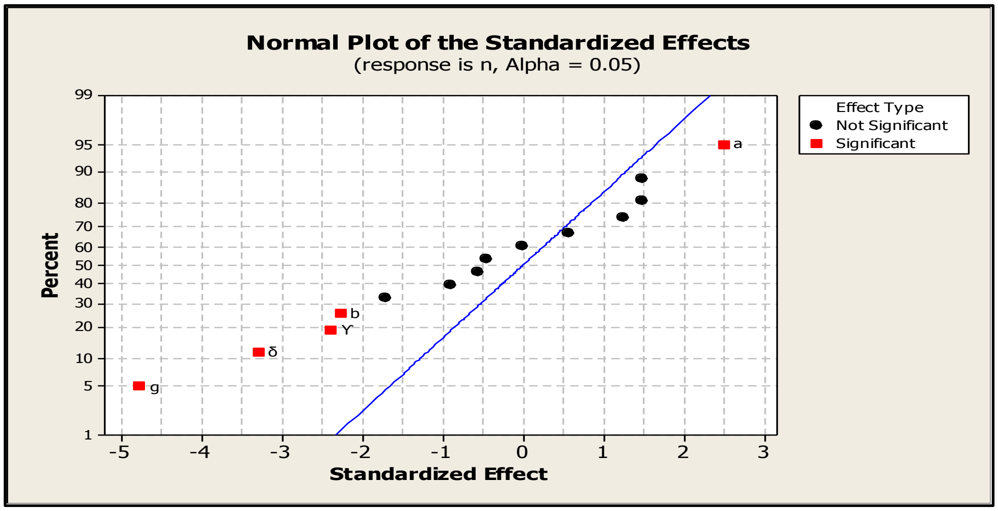

Fig. 5.11: Normal probability plot of standardized effect for sample size *n*: discontinuous process

Table 5.18 displays the results of analysis of variance on the sampling interval *h*. It is significantly affected by seven factors out of which three factors i.e., *M*, λ and *g* have negative effects, whereas the remaining four factors (T_1+T_2), V_0, *b* and *a* are significant in terms of positive effect as shown in Fig. 5.12. Moreover, the positive effect parameters such as *a*, *b*, V_0, and (T_1+T_2), V_0 contribute 28.30%, 11.56%, 6.96% and 4.97% respectively, whereas the negative effect parameters like, *M*, λ and *g* contribute by 14.99%, 13.71% and 3.36% respectively. Thus, among all the factors, the fixed cost of sampling '*a*' has the highest effect on the sampling interval with a percentage contribution of 28.30% and the effect is in positive direction.

Table 5.18: Analysis of variance for sampling interval *h*: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	22.708	22.708	22.7079	20.51	0.000*	14.99
δ	1	0.420	0.420	0.4201	0.38	0.546	0.28
λ	1	20.779	20.779	20.7790	18.77	0.000*	13.71
g	1	5.089	5.089	5.0890	4.60	0.047*	3.36
(T_1+T_2)	1	7.525	7.525	7.5255	6.80	0.018*	4.97
a	1	42.892	42.892	42.8923	38.75	0.000*	28.30
b	1	17.522	17.522	17.5223	15.83	0.001*	11.56
W	1	1.806	1.806	1.8061	1.63	0.219	1.19
Y	1	0.348	0.348	0.3479	0.31	0.582	0.23
V_0	1	10.548	10.548	10.5477	9.53	0.007*	6.96
S	1	0.154	0.154	0.1543	0.14	0.714	0.10
S_1	1	0.327	0.327	0.3268	0.30	0.594	0.22
T_0	1	0.393	0.393	0.3930	0.36	0.559	0.26
Υ	1	2.205	2.205	2.2047	1.99	0.176	1.46
Residual Error	17	18.819	18.819	1.1070			
Total	31	151.536					

* Significant at 5%

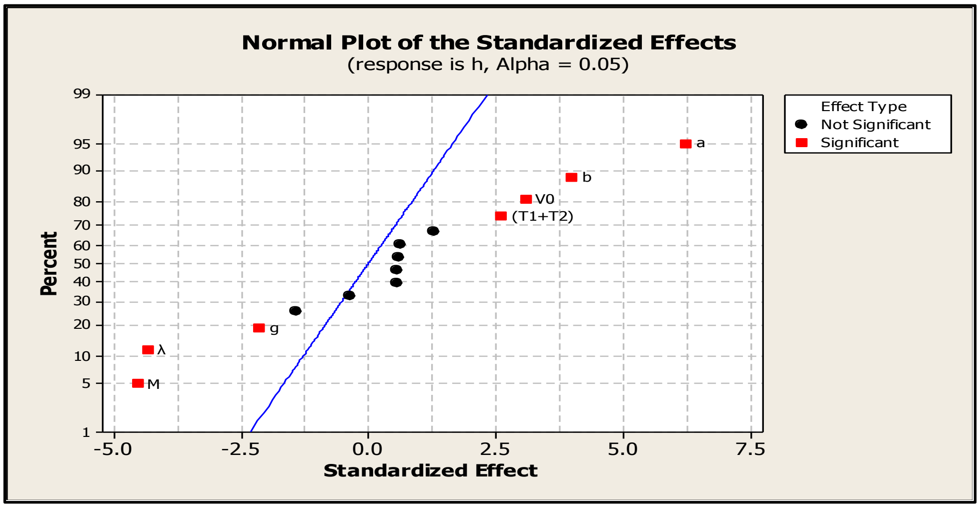

Fig. 5.12: Normal probability plot of standardized effect for sampling interval *h*: discontinuous process

Table 5.19 presents the results of analysis of variance on the control limits width k_1 of $\bar{X}$ chart. Seven factors (i.e., δ, a, b, Y, V_0, T_0 and Υ) are found to be significant on k_1. Fig. 5.13 reveals that out of these seven significant factors, two factors (i.e., b and a) have negative effect and the rest five factors have positive effect on k_1. Among all the factors, the expected search time for a false alarm T_0 is observed to have the most significant effect with a maximum contribution of 26.70% on deciding the value of k_1 in joint economic design and its effect is of positive type.

Table 5.19: Analysis of variance for width of control limits k_1 of $\bar{X}$ chart: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	0.00651	0.00651	0.00651	0.17	0.683	0.12
δ	1	0.16801	0.16801	0.16801	4.46	0.050*	3.22
λ	1	0.13709	0.13709	0.13709	3.64	0.073	2.63
g	1	0.09237	0.09237	0.09237	2.45	0.136	1.77
(T_1+T_2)	1	0.00740	0.00740	0.00740	0.20	0.663	0.14
a	1	0.32291	0.32291	0.32291	8.57	0.009*	6.19
b	1	1.10994	1.10994	1.10994	29.46	0.000*	21.27
W	1	0.00993	0.00993	0.00993	0.26	0.614	0.19
Y	1	0.22003	0.22003	0.22003	5.84	0.027*	4.22
V_0	1	0.61004	0.61004	0.61004	16.19	0.001*	11.69
S	1	0.00025	0.00025	0.00025	0.01	0.936	0.00
S_1	1	0.01067	0.01067	0.01067	0.28	0.602	0.20
T_0	1	1.39357	1.39357	1.39357	36.99	0.000*	26.70
Υ	1	0.48923	0.48923	0.48923	12.99	0.002*	9.37
Residual Error	17	0.64050	0.64050	0.03768			
Total	31	5.21847					

* Significant at 5%

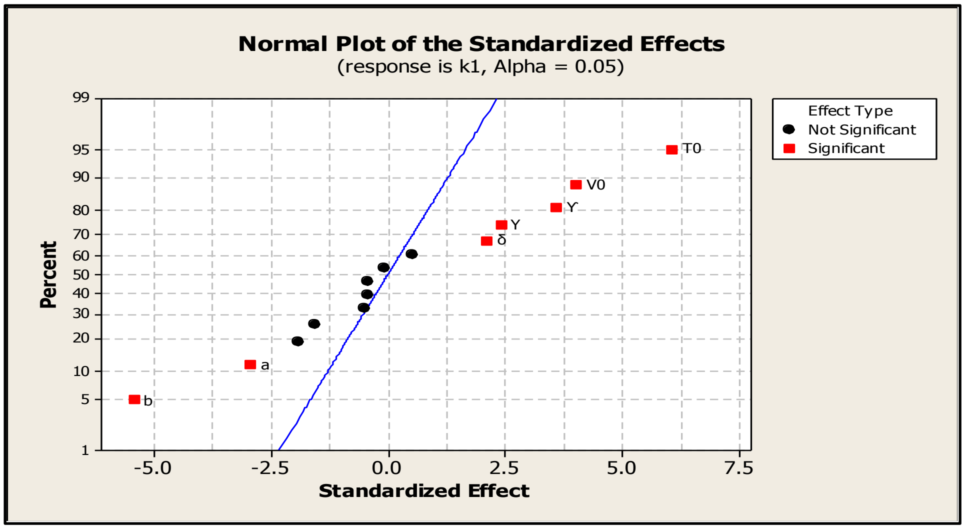

Fig. 5.13: Normal probability plot of standardized effect for width of control limits k_1 of $\overline{X}$ chart: discontinuous process

Similarly, Table 5.20 shows the ANOVA results for the control limits width k_2 of R chart. Five factors (i.e., δ, a, b, V_0 and T_0) are found to be significant on k_2. Fig. 5.14 reveals that out of these five significant factors, two factors (i.e., b and a) have negative effect and three factors (i.e., V_0, T_0 and δ) have positive effect. Further, the percentage contributions of all these five significant factors δ, T_0, b, V_0 and a are 27.46%, 19.84%, 15.24%, 8.27% and 5.28% respectively. The shift in process mean δ is found to be the most significant factor with positive effect on deciding the value of k_2.

Table 5.20: Analysis of variance for width of control limits k_2 of R chart: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	0.01281	0.01281	0.01281	0.18	0.675	0.15
δ	1	2.30329	2.30329	2.30329	32.67	0.000*	27.46
λ	1	0.15900	0.15900	0.15900	2.26	0.151	1.90
g	1	0.07552	0.07552	0.07552	1.07	0.315	0.90
(T_1+T_2)	1	0.02587	0.02587	0.02587	0.37	0.553	0.31
a	1	0.44249	0.44249	0.44249	6.28	0.023*	5.28
b	1	1.27867	1.27867	1.27867	18.14	0.001*	15.24
W	1	0.02827	0.02827	0.02827	0.40	0.535	0.34
Y	1	0.26175	0.26175	0.26175	3.71	0.071	3.12
V_0	1	0.69337	0.69337	0.69337	9.84	0.006*	8.27
S	1	0.00011	0.00011	0.00011	0.00	0.969	0.00
S_1	1	0.01645	0.01645	0.01645	0.23	0.635	0.20
T_0	1	1.66462	1.66462	1.66462	23.61	0.000*	19.84
Υ	1	0.22760	0.22760	0.22760	3.23	0.090	2.71
Residual Error	17	1.19841	1.19841	0.07049			
Total	31	8.38823					

* Significant at 5%

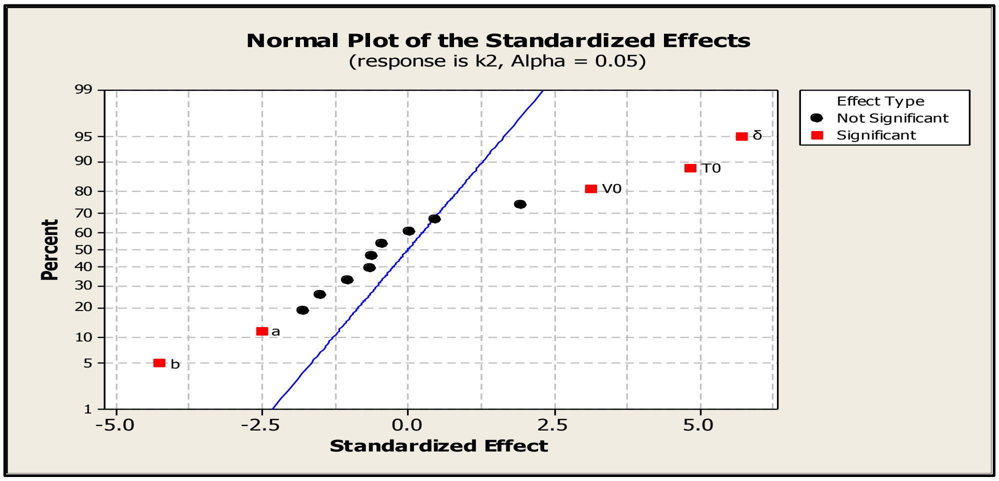

Fig. 5.14: Normal probability plot of standardized effect for width of control limits k_2 of R chart: discontinuous process

It is further observed from Tables 5.16 - 5.20 that the cost to locate and repair the assignable cause W, the expected cost of restart or setup cost S and the startup time S_1 have no significance on any of the responses n, h, k_1, k_2 and $E(L)_2$.

5.7.1 Summary of Results

Similar to Table 5.11 for continuous process, all the significant factors in case of joint economic design for discontinuous process for each of the five output responses are summarized in Table 5.21.

Table 5.21: Summary of significant effects in joint economic design: discontinuous process

Output responses	***Cost and process parameters***													
	M	***δ***	***λ***	***g***	***(T_1+T_2)***	***a***	***b***	***W***	***Y***	***ϒ***	***V_0***	***S***	***S_1***	***T_0***
n		–		**–**		+	–			–				
h	–		–	–	+	**+**	+				+			
k_1		+				–	–		+	+	+			**+**
k_2		**+**				–	–				+			+
$E(L)_2$			**+**		+						+			

Note:
Blank space : Insignificant factor
\+ : Factor with positive effect
– : Factor with negative effect
+/– in bold : Most significant factor

Table 5.22 shows a comparison between joint economic designs of continuous and discontinuous processes. All these results have been obtained using TLBO technique. In case of joint economic design of $\bar{X}$ and R charts for continuous process ten cost and process parameters are taken, whereas in discontinuous process fourteen parameters are considered. So, the factors V_0, S, S_1 and T_0 shown in last four columns of this table are applicable only for discontinuous process and not for a continuous process.

Table 5.22: Comparison of significant effects for both continuous and discontinuous processes

Output responses	***Process***	***Cost and process parameters***													
		M	δ	λ	g	(T_1+T_2)	a	b	W	Y	Υ	V_0	S	S_1	T_0
n	*JED-C*		–		**–**		+	–							
	JED-D		–		**–**		+	–			–				
h	*JED-C*	–		–	–	+	**+**	+							
	JED-D	–		–	–	+	**+**	+				+			
k_1	*JED-C*		+	–	–		–	–		**+**	+				
	JED-D		+				–	–		+	+	+			**+**
k_2	*JED-C*		+	–	–		–	–		**+**					
	JED-D		**+**				–	–				+			+
E(L)	*JED-C*	+		**+**		+									
	JED-D			**+**		+						+			

Note:
JED-C : Joint Economic Design - Continuous process
JED-D : Joint Economic Design - Discontinuous process
Blank space : Insignificant factor
\+ : Factor with positive effect
– : Factor with negative effect
+/– in bold : Most significant factor

The significant factors in joint economic design for a continuous process are already discussed in Sections 5.5.1. These results of a continuous process are compared below with that of a discontinuous process for each of the five responses (i.e., four design variables n, h, k_1, k_2 and the expected loss cost per unit time E(L)).

i) Effect on sample size n

From Table 5.22 it is observed that the time to sample and chart one item g is the most significant factor in both the processes, and the effect is of negative type for selecting the value of sample size n. All the factors which are significant in continuous process are also significant in discontinuous process except the factor Υ which is significant only in discontinuous process and it has negative effect.

ii) Effect on sampling interval h

Similar to sample size n, the lists of significant factors are same in both continuous and discontinuous processes for sampling interval h except the factor V_0 which is not applicable in continuous process. Among all those factors, the fixed cost of sampling (a) is observed to have the most significant effect in both the processes and both the effects are of positive type.

iii) Effect on the control limits width k_1 of $\bar{X}$ chart

Unlike n and h, the most significant factors for the width of control limits k_1 are not same in both the processes. In case of k_1, the cost per false alarm Y and the expected search time for a false alarm T_0 are the most significant factors in continuous and discontinuous process respectively. Here, there are five significant factors i.e., δ, a, b, Y and Υ which are common to both the processes with same type of effects (i.e. either positive or negative). Besides these five common factors, there are another two factors (i.e., λ and g) which are significant only for continuous process and both have negative effects. On the other hand, the factors V_0 and T_0 are significant only for discontinuous process and both are with positive effects. These last two factors are not applicable to a continuous process.

iv) Effect on the control limits width k_2 of R chart

Like the control limits width k_1 of $\bar{X}$ chart, the most significant factors are found to be different in case of the control limits width k_2 of R chart. The cost per false alarm Y and the shift in process mean δ are observed to be the most significant factors for k_2 in case of continuous and discontinuous process respectively. Three factors δ, a and b are significant and common to both the processes. But three other factors λ, g and Y are found to have significant effect only for continuous process, whereas factors V_0 and T_0 are significant only for discontinuous process. As such V_0 and T_0 are not relevant in continuous process.

v) Effect on expected loss cost per unit time E(L)

All the significant factors with respect to the expected loss cost per unit time $E(L)$ have positive effects whether the process is continuous or discontinuous. Similar to n and h, the most significant factor for $E(L)$ is also same in both the processes and this factor is λ i.e., the rate occurrence of assignable cause. There is one more significant factor i.e., the time to find and repair an assignable cause (T_1+T_2) which is common to both the processes but to a less extent compared to λ. Moreover, the expected net income per hour during in-control period V_0 is significant for only discontinuous process. As such this factor is not relevant in continuous process. On the other hand, the factor M representing the loss of income when process is out-of-control is significant only for continuous process.

The objective function equations are not same in continuous and discontinuous processes as shown in Eqs. 3.20 and 3.31. The numbers of factors associated with these two of processes are also different. The joint economic design for discontinuous process includes fourteen factors, whereas the joint economic design for continuous process considers only ten factors. These differences associated with the two processes may be the reasons for the differences in results of significant factors as shown in Table 5.22. Thus, the designers of control charts must ensure the type of process (i.e., continuous or discontinuous) and take utmost care in ensuring the correctness of values of significant factors before using them in joint economic design.

5.8 Another Numerical Illustration

Another numerical example has been considered in this section for the illustration of design methodologies based on SA and TLBO for joint economic design of $\bar{X}$ and R charts. This problem has been solved by most of the researchers in the area of joint economic design of $\bar{X}$ and R charts (Rahim, 1989; Chung and Chen, 1993; Kasarapu and Vommi, 2011). This problem is related to discontinuous process which is stopped during search and repair of assignable cause. The cost model considered by them is same as that shown in Eq. 3.31 except the following two points.

i) They have considered a set of two factors V_0 and V_1 instead of V_0 and M considered in this thesis. Both assumptions are equivalent as $M = V_0 - V_1$.

ii) Further, they have not considered the following four factors in their cost models:

a) Expected time to repair the assignable cause T_2

b) Time to sample and chart one item g

c) Expected cost of restart or setup cost S, and

d) Time to restart the process S_1.

Hence, the values of all these four factors are assumed as zero in Eq. 3.31 in this example.

5.8.1 Cost and Process Parameters

The numerical data dealing with a discontinuous process has been taken from Kasarapu and Vommi (2011). They have considered 160 data sets of 12 cost and process parameters (i.e., δ, λ, V_0, V_1, W, Y, T_1, T_0, a, b, σ_0 and σ_1) as shown in Table 5.23. As this table also contains the results of joint economic designs, this table has been shown in Section 5.8.2. The value of M is calculated using the expression $M = V_0 - V_1$ and is also added in this table since this value is required for calculating the expected loss cost per unit time $E(L)_2$ using Eq. 3.31.

For each of 160 data sets, the optimal values of four design variables (i.e., n, h, k_1 and k_2) are required to be found out with an objective to minimize the expected loss cost per unit time $E(L)_2$ in joint economic design of $\bar{X}$ and R charts.

All the four design variables are taken as real values on continuous scale except the sample size n which is taken as integer. The joint economic design is an example of multi-variable unconstrained minimization problem with a non-linear and non-differentiable objective function. The search space defined by the lower and upper boundary limits for each of the four design variables for minimizing the cost function $E(L)_2$ is same as that mentioned in Table 5.1. All the above mentioned 160 sets of joint economic design problems are solved using both SA and TLBO, and the results obtained are discussed below.

5.8.2 Results and Discussion

Table 5.23 shows the results of joint economic design of $\bar{X}$ and R charts for a discontinuous process for each of the 160 sets of numerical data related to various cost and process parameters using SA. To cross check the accuracy of the results obtained by SA, all those 160 design problems have been again solved using TLBO and the results are included in the same table. After comparing the results of both SA and TLBO, it is observed that the results obtained from TLBO are either superior or same as that of the results of SA in almost all cases except three (i.e., serial numbers 59, 127 and 147).

Further, the results obtained in the present work using SA are then compared with that of Kasarapu and Vommi (2011) and Chung and Chen (1993) in Table 5.24. Similarly, the results of TLBO are also compared with theirs in Table 5.26. It is observed from both the tables that the optimal value of sample size n is same in all the results obtained for each of 160 design problems. For R chart, the upper control limit coefficient K_2 is expressed as $K_2 = d_2 + k_2 d_3$ where k_2 is the width of control limits, and d_2 and d_3 are control chart constants. Since other researchers have reported the value of K_2 instead of k_2 for R chart, for comparison purpose the value of K_2 is calculated for each of 160 design problems and mentioned in addition to the values of four design variables in Tables 5.24 and 5.26. After comparing the results with that of other authors, the percentage reductions in the output $E(L)_2$ in the present work for all the 160 design problems are shown in the respective tables. The positive value in the percentage reduction means that the present work yields comparatively lower cost in terms of $E(L)_2$.

Table 5.23: Optimal joint economic designs using SA and TLBO

S. No.	***Cost and process parameters***													***SA***					***TLBO***				
	δ	λ	V_0	V_1	M	W	Y	T_1	T_0	a	b	σ_0	σ_1	n	h	k_1	k_2	$E(L)_2$	n	h	k_1	k_2	$E(L)_2$
1	0.5	0.01	50	25	25	2	1	0.3	0.1	0.5	0.1	0.02	0.03	10	3.20	2.12	1.53	1.450	10	3.24	2.13	1.51	1.450
2	0.5	0.05	50	25	25	2	1	0.4	0.1	0.5	0.1	0.02	0.03	10	1.56	2.13	1.44	3.779	10	1.53	2.11	1.48	3.778
3	0.5	0.01	50	25	25	15	5	0.3	0.1	0.5	0.2	0.02	0.03	8	3.88	2.09	1.49	1.927	8	3.76	2.13	1.52	1.927
4	0.5	0.01	50	25	25	15	5	0.3	0.1	0.5	0.1	0.02	0.03	13	3.58	2.29	1.75	1.670	13	3.51	2.31	1.75	1.670
5	0.5	0.05	50	25	25	15	5	0.6	0.1	0.5	0.8	0.02	0.03	2	2.13	1.40	1.11	6.531	2	2.08	1.44	1.13	6.531
6	0.5	0.05	50	25	25	15	5	0.5	0.1	0.5	0.4	0.02	0.03	5	2.14	1.87	1.20	5.844	5	2.09	1.85	1.23	5.844
7	0.5	0.05	50	25	25	15	5	0.4	0.1	0.5	0.2	0.02	0.03	8	1.81	2.09	1.49	5.087	8	1.83	2.09	1.49	5.087
8	0.5	0.05	50	25	25	15	5	0.4	0.1	0.5	0.1	0.02	0.03	13	1.68	2.30	1.71	4.575	13	1.66	2.30	1.74	4.575
9	0.5	0.01	150	50	100	2	1	0.4	0.1	0.5	0.4	0.02	0.03	6	2.10	2.11	1.51	4.852	6	2.10	2.10	1.50	4.852
10	0.5	0.01	150	50	100	2	1	0.4	0.1	0.5	0.2	0.02	0.03	10	2.01	2.30	1.71	4.159	10	1.96	2.29	1.72	4.159
11	0.5	0.01	150	50	100	2	1	0.6	0.1	0.5	0.1	0.02	0.03	16	1.87	2.46	1.95	3.845	16	1.85	2.46	1.95	3.845
12	0.5	0.05	150	50	100	2	1	0.6	0.1	0.5	0.4	0.02	0.03	6	0.99	2.07	1.48	13.286	6	0.99	2.07	1.46	13.286
13	0.5	0.05	150	50	100	2	1	0.6	0.1	0.5	0.2	0.02	0.03	10	0.92	2.26	1.69	11.887	10	0.93	2.27	1.69	11.887
14	0.5	0.05	150	50	100	2	1	0.6	0.1	0.5	0.1	0.02	0.03	15	0.84	2.45	1.93	10.645	15	0.83	2.45	1.93	10.645
15	0.5	0.01	150	50	100	15	5	0.6	0.1	0.5	0.4	0.02	0.03	7	2.27	2.19	1.57	5.439	7	2.24	2.18	1.59	5.439
16	0.5	0.01	150	50	100	15	5	0.3	0.1	0.5	0.2	0.02	0.03	11	2.07	2.34	1.82	4.265	11	2.03	2.37	1.82	4.264
17	0.5	0.01	150	50	100	15	5	0.6	0.1	0.5	0.1	0.02	0.03	17	1.86	2.54	2.05	4.053	17	1.87	2.53	2.04	4.052
18	0.5	0.05	150	50	100	15	5	0.6	0.1	0.5	0.4	0.02	0.03	7	1.09	2.13	1.53	14.274	7	1.08	2.15	1.54	14.273
19	0.5	0.05	150	50	100	15	5	0.6	0.1	0.5	0.2	0.02	0.03	11	0.96	2.33	1.79	12.752	11	0.95	2.35	1.79	12.751
20	0.5	0.05	150	50	100	15	5	0.6	0.1	0.5	0.1	0.02	0.03	17	0.88	2.51	2.02	11.442	17	0.87	2.51	2.04	11.442
21	0.5	0.01	150	50	100	2	1	0.6	0.1	5.0	0.2	0.02	0.03	17	4.12	2.00	1.43	5.916	17	4.10	2.03	1.41	5.916
22	0.5	0.01	150	50	100	2	1	0.6	0.1	5.0	0.1	0.02	0.03	27	3.94	2.20	1.69	5.402	27	3.93	2.21	1.70	5.401
23	0.5	0.05	150	50	100	2	1	0.6	0.1	5.0	0.2	0.02	0.03	17	1.93	2.03	1.36	14.982	17	1.93	2.00	1.37	14.982
24	0.5	0.05	150	50	100	2	1	0.4	0.1	5.0	0.1	0.02	0.03	26	1.79	2.17	1.67	12.676	26	1.81	2.19	1.65	12.676
25	0.5	0.01	150	50	100	15	5	0.3	0.1	5.0	0.2	0.02	0.03	19	4.15	2.12	1.54	5.725	19	4.15	2.12	1.55	5.725
26	0.5	0.01	150	50	100	15	5	0.3	0.1	5.0	0.1	0.02	0.03	29	3.97	2.30	1.78	5.170	29	3.95	2.29	1.82	5.170
27	0.5	0.05	150	50	100	15	5	0.4	0.1	5.0	0.2	0.02	0.03	18	1.93	2.10	1.51	14.559	18	1.92	2.09	1.50	14.559
28	0.5	0.05	150	50	100	15	5	0.4	0.1	5.0	0.1	0.02	0.03	29	1.84	2.29	1.80	13.426	29	1.84	2.28	1.79	13.425
29	0.5	0.05	150	50	100	15	5	0.6	0.1	5.0	0.1	0.02	0.03	28	1.86	2.26	1.77	14.677	28	1.84	2.26	1.78	14.677
30	1.0	0.01	50	25	25	2	1	0.3	0.1	0.5	0.4	0.02	0.03	4	3.89	1.70	1.54	1.694	4	3.93	1.70	1.50	1.694
31	1.0	0.01	50	25	25	2	1	0.3	0.1	0.5	0.2	0.02	0.03	6	3.36	1.95	1.80	1.446	6	3.48	1.94	1.80	1.446
32	1.0	0.01	50	25	25	2	1	0.3	0.1	0.5	0.1	0.02	0.03	9	3.21	2.17	2.08	1.233	9	3.18	2.15	2.09	1.233

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S. No.	Cost and process parameters													SA					TLBO				
	δ	***λ***	V_0	V_1	***M***	***W***	***Y***	T_1	T_0	***a***	***b***	σ_0	σ_1	***n***	***h***	k_1	k_2	$E(L)_2$	***n***	***h***	k_1	k_2	$E(L)_2$
33	1.0	0.05	50	25	25	2	1	0.6	0.1	0.5	0.8	0.02	0.03	2	2.06	1.28	1.16	5.160	2	2.05	1.28	1.16	5.160
34	1.0	0.05	50	25	25	2	1	0.6	0.1	0.5	0.4	0.02	0.03	3	1.67	1.61	1.49	4.670	3	1.66	1.64	1.46	4.669
35	1.0	0.05	50	25	25	2	1	0.6	0.1	0.5	0.2	0.02	0.03	6	1.68	1.90	1.74	4.194	6	1.66	1.91	1.76	4.194
36	1.0	0.05	50	25	25	2	1	0.6	0.1	0.5	0.1	0.02	0.03	9	1.48	2.14	2.08	3.767	9	1.50	2.13	2.06	3.767
37	1.0	0.01	50	25	25	15	5	0.6	0.1	0.5	0.4	0.02	0.03	5	4.22	1.91	1.74	1.981	5	4.18	1.92	1.76	1.981
38	1.0	0.01	50	25	25	5	5	0.3	0.1	0.5	0.2	0.02	0.03	8	3.97	2.11	2.04	1.553	8	3.88	2.15	2.05	1.552
39	1.0	0.01	50	25	25	5	5	0.4	0.1	0.5	0.2	0.02	0.03	7	3.57	2.15	2.05	1.597	7	3.62	2.13	2.05	1.596
40	1.0	0.01	50	25	25	5	5	0.3	0.1	0.5	0.1	0.02	0.03	11	3.42	2.34	2.30	1.309	11	3.40	2.35	2.32	1.309
41	1.0	0.05	50	25	25	5	5	0.5	0.1	0.5	0.8	0.02	0.03	3	2.22	1.63	1.45	5.507	3	2.23	1.63	1.45	5.506
42	1.0	0.05	50	25	25	5	5	0.3	0.1	0.5	0.4	0.02	0.03	5	2.00	1.90	1.74	4.462	5	1.99	1.89	1.74	4.462
43	1.0	0.05	50	25	25	5	5	0.3	0.1	0.5	0.2	0.02	0.03	7	1.69	2.12	2.00	3.860	7	1.70	2.11	2.01	3.860
44	1.0	0.05	50	25	25	5	5	0.3	0.1	0.5	0.1	0.02	0.03	11	1.59	2.33	2.34	3.362	11	1.59	2.34	2.30	3.362
45	1.0	0.01	50	25	25	5	5	0.5	0.1	5.0	0.2	0.02	0.03	13	7.97	2.00	1.81	2.404	13	7.99	1.97	1.89	2.404
46	1.0	0.01	50	25	25	5	5	0.6	0.1	5.0	0.1	0.02	0.03	18	7.51	2.23	2.17	2.263	18	7.57	2.21	2.20	2.263
47	1.0	0.05	50	25	25	5	5	0.6	0.1	5.0	0.2	0.02	0.03	12	3.79	1.94	1.79	6.055	12	3.81	1.92	1.81	6.054
48	1.0	0.05	50	25	25	5	5	0.6	0.1	5.0	0.1	0.02	0.03	17	3.59	2.16	2.14	5.687	17	3.57	2.16	2.14	5.687
49	1.0	0.01	150	50	100	2	1	0.4	0.1	0.5	0.4	0.02	0.03	6	2.18	2.11	1.95	4.142	6	2.17	2.09	1.98	4.142
50	1.0	0.01	150	50	100	2	1	0.4	0.1	0.5	0.2	0.02	0.03	9	1.98	2.30	2.25	3.470	9	1.98	2.31	2.25	3.470
51	1.0	0.01	150	50	100	2	1	0.4	0.1	0.5	0.1	0.02	0.03	13	1.78	2.54	2.49	2.932	13	1.77	2.53	2.55	2.932
52	1.0	0.05	150	50	100	2	1	0.5	0.1	0.5	0.8	0.02	0.03	4	1.16	1.84	1.67	12.851	4	1.15	1.84	1.68	12.851
53	1.0	0.05	150	50	100	2	1	0.5	0.1	0.5	0.4	0.02	0.03	6	1.01	2.08	1.95	11.222	6	1.00	2.08	1.96	11.222
54	1.0	0.05	150	50	100	2	1	0.5	0.1	0.5	0.2	0.02	0.03	9	0.92	2.26	2.22	9.829	9	0.91	2.30	2.22	9.829
55	1.0	0.05	150	50	100	2	1	0.5	0.1	0.5	0.1	0.02	0.03	12	0.79	2.51	2.45	8.691	12	0.79	2.50	2.47	8.691
56	1.0	0.01	150	50	100	5	5	0.5	0.1	0.5	0.4	0.02	0.03	7	2.35	2.18	2.08	4.417	7	2.34	2.17	2.07	4.417
57	1.0	0.01	150	50	100	5	5	0.5	0.1	0.5	0.2	0.02	0.03	10	2.07	2.40	2.35	3.707	10	2.07	2.39	2.35	3.707
58	1.0	0.01	150	50	100	5	5	0.6	0.1	0.5	0.1	0.02	0.03	13	1.78	2.59	2.58	3.285	13	1.78	2.58	2.59	3.285
59	1.0	0.05	150	50	100	5	5	0.6	0.1	0.5	0.8	0.02	0.03	4	1.13	1.93	1.79	13.930	4	1.15	1.91	1.76	13.932
60	1.0	0.05	150	50	100	5	5	0.5	0.1	0.5	0.4	0.02	0.03	6	1.00	2.16	2.05	11.570	6	1.00	2.15	2.05	11.570
61	1.0	0.05	150	50	100	5	5	0.5	0.1	0.5	0.2	0.02	0.03	9	0.90	2.36	2.33	10.099	9	0.90	2.38	2.33	10.099
62	1.0	0.05	150	50	100	5	5	0.5	0.1	0.5	0.1	0.02	0.03	13	0.82	2.56	2.60	8.919	13	0.82	2.57	2.58	8.919
63	1.0	0.01	150	50	100	2	1	0.6	0.1	5.0	0.8	0.02	0.03	6	4.45	1.65	1.42	6.511	6	4.44	1.65	1.43	6.511
64	1.0	0.01	150	50	100	2	1	0.6	0.1	5.0	0.4	0.02	0.03	10	4.27	1.91	1.83	5.814	10	4.24	1.92	1.80	5.814

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S. No.	Cost and process parameters													SA					TLBO				
	δ	λ	V_0	V_1	M	W	Y	T_1	T_0	a	b	σ_0	σ_1	n	h	k_1	k_2	$E(L)_2$	n	h	k_1	k_2	$E(L)_2$
65	1.0	0.01	150	50	100	2	1	0.6	0.1	5.0	0.2	0.02	0.03	15	4.01	2.18	2.14	5.255	15	4.01	2.16	2.15	5.255
66	1.0	0.01	150	50	100	2	1	0.3	0.1	5.0	0.1	0.02	0.03	20	3.75	2.36	2.44	4.409	20	3.74	2.40	2.41	4.409
67	1.0	0.05	150	50	100	2	1	0.6	0.1	5.0	0.8	0.02	0.03	6	2.10	1.61	1.41	16.188	6	2.10	1.61	1.39	16.188
68	1.0	0.05	150	50	100	2	1	0.3	0.1	5.0	0.4	0.02	0.03	10	1.98	1.90	1.75	12.919	10	1.96	1.93	1.79	12.919
69	1.0	0.05	150	50	100	2	1	0.3	0.1	5.0	0.2	0.02	0.03	14	1.83	2.13	2.07	11.737	14	1.81	2.13	2.09	11.737
70	1.0	0.05	150	50	100	2	1	0.3	0.1	5.0	0.1	0.02	0.03	20	1.72	2.38	2.40	10.853	20	1.73	2.37	2.41	10.853
71	1.0	0.01	150	50	100	5	5	0.4	0.1	5.0	0.4	0.02	0.03	11	4.29	2.03	1.93	5.638	11	4.30	2.03	1.95	5.638
72	1.0	0.01	150	50	100	5	5	0.4	0.1	5.0	0.2	0.02	0.03	15	4.02	2.23	2.23	5.042	15	3.99	2.24	2.22	5.042
73	1.0	0.01	150	50	100	5	5	0.5	0.1	5.0	0.1	0.02	0.03	21	3.81	2.46	2.58	4.747	21	3.79	2.48	2.52	4.746
74	1.0	0.05	150	50	100	5	5	0.6	0.1	5.0	0.8	0.02	0.03	7	2.17	1.75	1.55	16.610	7	2.16	1.75	1.56	16.609
75	1.0	0.05	150	50	100	5	5	0.5	0.1	5.0	0.4	0.02	0.03	10	1.97	1.97	1.85	14.472	10	1.97	1.97	1.87	14.472
76	1.0	0.05	150	50	100	5	5	0.5	0.1	5.0	0.2	0.02	0.03	15	1.85	2.21	2.21	13.251	15	1.86	2.22	2.22	13.251
77	1.0	0.05	150	50	100	5	5	0.5	0.1	5.0	0.1	0.02	0.03	21	1.75	2.47	2.48	12.343	21	1.74	2.45	2.53	12.343
78	1.5	0.01	50	25	25	2	1	0.6	0.1	0.5	0.4	0.02	0.04	3	3.37	2.14	1.93	1.527	3	3.37	2.14	1.95	1.527
79	1.5	0.01	50	25	25	2	1	0.6	0.1	0.5	0.2	0.02	0.04	4	3.04	2.32	2.21	1.325	4	2.96	2.36	2.13	1.325
80	1.5	0.01	50	25	25	2	1	0.3	0.1	0.5	0.1	0.02	0.04	6	2.83	2.63	2.33	1.027	6	2.83	2.59	2.39	1.026
81	1.5	0.05	50	25	25	2	1	0.6	0.1	0.5	0.4	0.02	0.04	3	1.59	2.11	1.94	4.080	3	1.60	2.12	1.92	4.080
82	1.5	0.05	50	25	25	2	1	0.3	0.1	0.5	0.2	0.02	0.04	4	1.39	2.36	2.13	3.014	4	1.38	2.35	2.13	3.014
83	1.5	0.05	50	25	25	2	1	0.4	0.1	0.5	0.1	0.02	0.04	6	1.29	2.63	2.33	2.909	6	1.31	2.57	2.39	2.909
84	1.5	0.01	50	25	25	5	5	0.4	0.1	0.5	0.4	0.02	0.04	3	3.26	2.32	2.11	1.515	3	3.23	2.33	2.15	1.514
85	1.5	0.01	50	25	25	5	5	0.4	0.1	0.5	0.2	0.02	0.04	5	3.24	2.57	2.35	1.298	5	3.22	2.57	2.37	1.298
86	1.5	0.01	50	25	25	5	5	0.3	0.1	0.5	0.1	0.02	0.04	6	2.81	2.73	2.51	1.076	6	2.78	2.74	2.55	1.076
87	1.5	0.05	50	25	25	5	5	0.6	0.1	0.5	0.4	0.02	0.04	3	1.53	2.35	2.15	4.326	3	1.55	2.32	2.12	4.326
88	1.5	0.05	50	25	25	5	5	0.3	0.1	0.5	0.2	0.02	0.04	5	1.50	2.56	2.37	3.241	5	1.48	2.58	2.35	3.241
89	1.5	0.05	50	25	25	5	5	0.3	0.1	0.5	0.1	0.02	0.04	6	1.30	2.73	2.53	2.871	6	1.30	2.71	2.53	2.871
90	1.5	0.05	50	25	25	2	1	0.6	0.1	5.0	0.1	0.02	0.04	10	3.31	2.39	2.20	5.230	10	3.36	2.38	2.16	5.230
91	1.5	0.01	50	25	25	5	5	0.4	0.1	5.0	0.2	0.02	0.04	8	7.37	2.39	2.10	2.148	8	7.37	2.37	2.14	2.148
92	1.5	0.01	50	25	25	5	5	0.4	0.1	5.0	0.1	0.02	0.04	11	7.19	2.60	2.43	2.024	11	7.15	2.59	2.40	2.024
93	1.5	0.05	50	25	25	5	5	0.5	0.1	5.0	0.4	0.02	0.04	6	3.73	2.11	1.87	5.800	6	3.72	2.14	1.88	5.800
94	1.5	0.05	50	25	25	5	5	0.5	0.1	5.0	0.2	0.02	0.04	8	3.49	2.36	2.07	5.444	8	3.51	2.36	2.12	5.444
95	1.5	0.05	50	25	25	5	5	0.6	0.1	5.0	0.1	0.02	0.04	11	3.40	2.57	2.41	5.392	11	3.40	2.59	2.39	5.392
96	1.5	0.01	150	50	100	2	1	0.6	0.1	0.5	0.2	0.02	0.04	5	1.57	2.71	2.47	3.076	5	1.54	2.70	2.53	3.076

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S. No.	Cost and process parameters													SA					TLBO				
	δ	λ	V_0	V_1	M	W	Y	T_1	T_0	a	b	σ_0	σ_1	n	h	k_1	k_2	$E(L)_2$	n	h	k_1	k_2	$E(L)_2$
97	1.5	0.01	150	50	100	2	1	0.3	0.1	0.5	0.1	0.02	0.04	7	1.47	2.89	2.73	2.273	7	1.44	2.90	2.73	2.273
98	1.5	0.05	150	50	100	2	1	0.6	0.1	0.5	0.8	0.02	0.04	3	0.97	2.30	2.13	11.434	3	0.98	2.30	2.12	11.434
99	1.5	0.05	150	50	100	2	1	0.4	0.1	0.5	0.4	0.02	0.04	4	0.84	2.50	2.32	8.780	4	0.84	2.52	2.30	8.780
100	1.5	0.05	150	50	100	2	1	0.3	0.1	0.5	0.2	0.02	0.04	5	0.71	2.71	2.51	7.034	5	0.71	2.71	2.51	7.034
101	1.5	0.05	150	50	100	2	1	0.3	0.1	0.5	0.1	0.02	0.04	7	0.66	2.93	2.70	6.243	7	0.66	2.92	2.72	6.243
102	1.5	0.01	150	50	100	5	5	0.4	0.1	0.5	0.2	0.02	0.04	5	1.55	2.81	2.60	2.845	5	1.55	2.78	2.59	2.844
103	1.5	0.01	150	50	100	5	5	0.4	0.1	0.5	0.1	0.02	0.04	7	1.46	2.95	2.77	2.466	7	1.43	2.97	2.79	2.466
104	1.5	0.05	150	50	100	5	5	0.6	0.1	0.5	0.4	0.02	0.04	4	0.83	2.59	2.42	10.338	4	0.83	2.59	2.39	10.338
105	1.5	0.05	150	50	100	5	5	0.6	0.1	0.5	0.2	0.02	0.04	5	0.72	2.77	2.59	9.266	5	0.72	2.77	2.59	9.266
106	1.5	0.05	150	50	100	5	5	0.3	0.1	0.5	0.1	0.02	0.04	7	0.67	2.99	2.77	6.426	7	0.66	2.96	2.78	6.425
107	1.5	0.01	150	50	100	2	1	0.5	0.1	5.0	0.4	0.02	0.04	7	3.89	2.41	2.12	5.029	7	3.91	2.38	2.14	5.029
108	1.5	0.01	150	50	100	2	1	0.3	0.1	5.0	0.2	0.02	0.04	9	3.67	2.58	2.31	4.351	9	3.67	2.57	2.36	4.351
109	1.5	0.01	150	50	100	2	1	0.3	0.1	5.0	0.1	0.02	0.04	12	3.53	2.80	2.57	4.079	12	3.53	2.77	2.60	4.079
110	1.5	0.05	150	50	100	2	1	0.5	0.1	5.0	0.4	0.02	0.04	7	1.82	2.36	2.10	12.853	7	1.80	2.36	2.13	12.853
111	1.5	0.05	150	50	100	2	1	0.5	0.1	5.0	0.2	0.02	0.04	9	1.71	2.54	2.31	12.029	9	1.70	2.55	2.33	12.029
112	1.5	0.05	150	50	100	2	1	0.4	0.1	5.0	0.1	0.02	0.04	12	1.61	2.76	2.58	10.809	12	1.62	2.77	2.54	10.809
113	1.5	0.01	150	50	100	5	5	0.6	0.1	5.0	0.4	0.02	0.04	7	3.90	2.47	2.23	5.229	7	3.90	2.44	2.21	5.229
114	1.5	0.01	150	50	100	5	5	0.4	0.1	5.0	0.2	0.02	0.04	10	3.74	2.67	2.51	4.547	10	3.74	2.68	2.49	4.547
115	1.5	0.01	150	50	100	5	5	0.5	0.1	5.0	0.1	0.02	0.04	12	3.52	2.87	2.66	4.403	12	3.54	2.83	2.66	4.403
116	1.5	0.05	150	50	100	5	5	0.6	0.1	5.0	0.8	0.02	0.04	5	1.94	2.20	1.95	14.833	5	1.91	2.21	1.96	14.833
117	1.5	0.05	150	50	100	5	5	0.5	0.1	5.0	0.4	0.02	0.04	7	1.81	2.43	2.19	13.057	7	1.81	2.43	2.20	13.057
118	1.5	0.05	150	50	100	5	5	0.5	0.1	5.0	0.2	0.02	0.04	9	1.70	2.61	2.42	12.209	9	1.70	2.62	2.39	12.209
119	1.5	0.05	150	50	100	5	5	0.6	0.1	5.0	0.1	0.02	0.04	12	1.66	2.82	2.65	12.267	12	1.64	2.78	2.64	12.267
120	2.0	0.01	50	25	25	2	1	0.6	0.1	0.5	0.4	0.02	0.04	3	3.45	2.24	2.18	1.433	3	3.44	2.25	2.19	1.433
121	2.0	0.01	50	25	25	2	1	0.3	0.1	0.5	0.2	0.02	0.04	4	3.08	2.48	2.42	1.102	4	3.07	2.46	2.43	1.102
122	2.0	0.01	50	25	25	2	1	0.3	0.1	0.5	0.1	0.02	0.04	5	2.69	2.65	2.57	0.959	5	2.73	2.65	2.60	0.959
123	2.0	0.05	50	25	25	2	1	0.5	0.1	0.5	0.4	0.02	0.04	3	1.65	2.24	2.14	3.679	3	1.63	2.22	2.16	3.679
124	2.0	0.05	50	25	25	2	1	0.3	0.1	0.5	0.2	0.02	0.04	4	1.43	2.46	2.32	2.848	4	1.42	2.45	2.39	2.848
125	2.0	0.05	50	25	25	2	1	0.3	0.1	0.5	0.1	0.02	0.04	5	1.27	2.66	2.56	2.543	5	1.26	2.65	2.60	2.543
126	2.0	0.01	50	25	25	5	5	0.4	0.1	0.5	0.4	0.02	0.04	3	3.36	2.41	2.36	1.403	3	3.39	2.41	2.37	1.403
127	2.0	0.01	50	25	25	5	5	0.4	0.1	0.5	0.2	0.02	0.04	4	3.04	2.60	2.58	1.201	4	3.02	2.62	2.57	1.202
128	2.0	0.01	50	25	25	5	5	0.5	0.1	0.5	0.1	0.02	0.04	5	2.67	2.86	2.80	1.099	5	2.70	2.79	2.76	1.099

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S. No.	Cost and process parameters													SA					TLBO				
	δ	λ	V_0	V_1	M	W	Y	T_1	T_0	a	b	σ_0	σ_1	n	h	k_1	k_2	$E(L)_2$	n	h	k_1	k_2	$E(L)_2$
129	2.0	0.05	50	25	25	5	5	0.6	0.1	0.5	0.4	0.02	0.04	3	1.57	2.43	2.37	4.103	3	1.60	2.40	2.35	4.103
130	2.0	0.05	50	25	25	5	5	0.5	0.1	0.5	0.2	0.02	0.04	4	1.42	2.60	2.58	3.474	4	1.42	2.60	2.57	3.474
131	2.0	0.05	50	25	25	5	5	0.5	0.1	0.5	0.1	0.02	0.04	5	1.27	2.77	2.81	3.161	5	1.26	2.80	2.74	3.161
132	2.0	0.05	50	25	25	2	1	0.6	0.1	5.0	0.1	0.02	0.04	8	3.32	2.44	2.42	5.132	8	3.30	2.47	2.41	5.132
133	2.0	0.01	50	25	25	5	5	0.6	0.1	5.0	0.2	0.02	0.04	7	7.26	2.52	2.50	2.166	7	7.31	2.50	2.43	2.166
134	2.0	0.01	50	25	25	5	5	0.3	0.1	5.0	0.1	0.02	0.04	9	7.04	2.80	2.55	1.925	9	7.02	2.72	2.67	1.925
135	2.0	0.05	50	25	25	5	5	0.5	0.1	5.0	0.4	0.02	0.04	5	3.62	2.22	2.14	5.599	5	3.61	2.21	2.13	5.599
136	2.0	0.05	50	25	25	5	5	0.3	0.1	5.0	0.2	0.02	0.04	7	3.42	2.50	2.45	4.893	7	3.42	2.48	2.42	4.893
137	2.0	0.05	50	25	25	5	5	0.3	0.1	5.0	0.1	0.02	0.04	9	3.30	2.71	2.68	4.679	9	3.30	2.70	2.67	4.679
138	2.0	0.01	150	50	100	2	1	0.4	0.1	0.5	0.2	0.02	0.04	4	1.45	2.75	2.71	2.578	4	1.47	2.75	2.72	2.578
139	2.0	0.01	150	50	100	2	1	0.4	0.1	0.5	0.1	0.02	0.04	6	1.43	2.95	2.95	2.263	6	1.42	2.98	2.96	2.263
140	2.0	0.05	150	50	100	2	1	0.5	0.1	0.5	0.4	0.02	0.04	3	0.75	2.56	2.54	8.868	3	0.75	2.54	2.53	8.868
141	2.0	0.05	150	50	100	2	1	0.5	0.1	0.5	0.2	0.02	0.04	4	0.67	2.74	2.71	7.952	4	0.68	2.74	2.71	7.952
142	2.0	0.05	150	50	100	2	1	0.6	0.1	0.5	0.1	0.02	0.04	6	0.66	2.97	2.95	7.965	6	0.65	2.97	2.95	7.965
143	2.0	0.01	150	50	100	5	5	0.6	0.1	0.5	0.2	0.02	0.04	5	1.64	2.91	2.84	2.932	5	1.62	2.88	2.86	2.932
144	2.0	0.01	150	50	100	5	5	0.3	0.1	0.5	0.1	0.02	0.04	6	1.43	3.03	3.03	2.158	6	1.39	3.03	3.03	2.158
145	2.0	0.05	150	50	100	5	5	0.4	0.1	0.5	0.4	0.02	0.04	3	0.74	2.62	2.59	8.415	3	0.74	2.62	2.59	8.415
146	2.0	0.05	150	50	100	5	5	0.3	0.1	0.5	0.2	0.02	0.04	4	0.66	2.84	2.77	6.779	4	0.67	2.81	2.78	6.778
147	2.0	0.05	150	50	100	5	5	0.3	0.1	0.5	0.1	0.02	0.04	6	0.64	3.04	3.02	6.076	6	0.63	3.05	3.00	6.077
148	2.0	0.01	150	50	100	5	5	0.6	0.1	5.0	0.4	0.02	0.04	6	3.83	2.54	2.47	4.995	6	3.83	2.53	2.46	4.995
149	2.0	0.01	150	50	100	5	5	0.6	0.1	5.0	0.2	0.02	0.04	7	3.56	2.67	2.60	4.653	7	3.57	2.68	2.63	4.653
150	2.0	0.01	150	50	100	5	5	0.4	0.1	5.0	0.1	0.02	0.04	9	3.44	2.93	2.85	4.141	9	3.45	2.87	2.88	4.141
151	2.0	0.05	150	50	100	5	5	0.5	0.1	5.0	0.4	0.02	0.04	6	1.76	2.53	2.44	12.571	6	1.78	2.49	2.48	12.571
152	2.0	0.05	150	50	100	5	5	0.4	0.1	5.0	0.2	0.02	0.04	7	1.64	2.66	2.64	11.201	7	1.64	2.69	2.62	11.201
153	2.0	0.05	150	50	100	5	5	0.4	0.1	5.0	0.1	0.02	0.04	9	1.59	2.89	2.87	10.718	9	1.58	2.88	2.84	10.718
154	2.0	0.01	150	50	100	5	5	0.6	0.1	5.0	0.4	0.02	0.04	6	3.84	2.53	2.47	4.995	6	3.83	2.53	2.46	4.995
155	2.0	0.01	150	50	100	5	5	0.4	0.1	5.0	0.2	0.02	0.04	8	3.64	2.76	2.71	4.373	8	3.64	2.77	2.73	4.373
156	2.0	0.01	150	50	100	5	5	0.4	0.1	5.0	0.1	0.02	0.04	10	3.50	2.97	2.95	4.143	10	3.49	2.96	2.94	4.143
157	2.0	0.05	150	50	100	5	5	0.6	0.1	5.0	0.8	0.02	0.04	4	1.84	2.24	2.18	14.198	4	1.84	2.24	2.20	14.198
158	2.0	0.05	150	50	100	5	5	0.5	0.1	5.0	0.4	0.02	0.04	6	1.77	2.52	2.47	12.571	6	1.78	2.49	2.48	12.571
159	2.0	0.05	150	50	100	5	5	0.3	0.1	5.0	0.2	0.02	0.04	8	1.67	2.77	2.72	10.559	8	1.67	2.72	2.68	10.559
160	2.0	0.05	150	50	100	5	5	0.3	0.1	5.0	0.1	0.02	0.04	10	1.59	2.98	2.94	10.066	10	1.60	2.94	2.94	10.066

Table 5.24: Comparison of results of joint economic design with that of SA

S. No.	CC					KV					SA						% Reduction	
	n	*h*	k_1	K_2	$E(L)_2$	*n*	*h*	k_1	K_2	$E(L)_2$	*n*	*h*	k_1	k_2	K_2	$E(L)_2$	SA with CC	SA with KV
1	10	3.46	2.07	4.14	1.4869	10	3.4682	2.0792	4.15	1.4780	10	3.20	2.12	1.53	4.29	1.450	2.448	1.861
2	10	1.66	2.04	4.09	3.8380	10	1.6720	2.0428	4.10	3.8130	10	1.56	2.13	1.44	4.23	3.779	1.542	0.897
3	8	4.04	2.05	3.96	1.9794	8	4.0837	2.0605	3.95	1.9690	8	3.88	2.09	1.49	4.07	1.927	2.646	2.132
4	13	3.71	2.24	4.58	1.7224	13	3.7201	2.2551	4.60	1.7110	13	3.58	2.29	1.75	4.68	1.670	3.027	2.380
5	2	4.10	0.56	1.00	6.5776	2	3.5593	1.0000	1.00	6.5420	2	2.13	1.40	1.11	2.12	6.531	0.711	0.171
6	5	2.15	1.70	2.96	5.9003	5	2.4010	1.7160	3.15	5.8770	5	2.14	1.87	1.20	3.36	5.844	0.948	0.556
7	8	1.88	2.00	3.79	5.1683	8	2.0247	2.0132	3.90	5.1390	8	1.81	2.09	1.49	4.07	5.087	1.572	1.011
8	13	1.74	2.21	4.48	4.6643	13	1.8238	2.2248	4.55	4.6340	13	1.68	2.30	1.71	4.65	4.575	1.908	1.267
9	6	2.26	2.01	3.70	5.0149	6	2.2664	2.0140	3.70	5.0000	6	2.10	2.11	1.51	3.81	4.852	3.245	2.956
10	10	2.08	2.22	4.36	4.3160	10	2.0644	2.2274	4.40	4.2910	10	2.01	2.30	1.71	4.44	4.159	3.632	3.071
11	16	1.93	2.39	4.93	3.9883	16	1.9385	2.4002	4.95	3.9620	16	1.87	2.46	1.95	4.99	3.845	3.586	2.946
12	6	1.09	1.97	3.64	13.5670	6	1.0899	1.9688	3.65	13.5260	6	0.99	2.07	1.48	3.79	13.286	2.070	1.773
13	10	0.99	2.19	4.32	12.1752	10	0.9861	2.1973	4.35	12.1150	10	0.92	2.26	1.69	4.43	11.887	2.365	1.880
14	15	0.88	2.36	4.86	10.9316	15	0.8891	2.3798	4.85	10.8710	15	0.84	2.45	1.93	4.93	10.645	2.618	2.075
15	7	2.39	2.10	3.93	5.6217	7	2.3826	2.1022	3.95	5.6010	7	2.27	2.19	1.57	4.01	5.439	3.247	2.889
16	11	2.14	2.29	4.53	4.4449	11	2.1406	2.3013	4.55	4.4160	11	2.07	2.34	1.82	4.60	4.265	4.051	3.423
17	17	1.95	2.45	5.06	4.2163	17	1.9467	2.4577	5.10	4.1880	17	1.86	2.54	2.05	5.12	4.053	3.881	3.231
18	7	1.15	2.06	3.88	14.5951	7	1.1498	2.0636	3.90	14.5400	7	1.09	2.13	1.53	3.98	14.274	2.201	1.831
19	11	1.02	2.26	4.49	13.0856	11	1.0282	2.2744	4.50	13.0180	11	0.96	2.33	1.79	4.58	12.752	2.552	2.046
20	17	0.92	2.43	5.04	11.7576	17	0.9268	2.4410	5.05	11.6910	17	0.88	2.51	2.02	5.09	11.442	2.687	2.133
21	17	4.29	2.03	4.46	5.9667	17	4.2836	2.0393	4.50	5.9340	17	4.12	2.00	1.43	4.65	5.916	0.846	0.300
22	27	4.06	2.21	5.06	5.4536	27	4.0557	2.2132	5.10	5.4270	27	3.94	2.20	1.69	5.18	5.402	0.955	0.470
23	17	2.01	1.99	4.35	15.0411	17	2.0395	2.0057	4.45	14.9860	17	1.93	2.03	1.36	4.60	14.982	0.390	0.023
24	26	1.89	2.18	4.99	12.7627	26	1.8889	2.1786	5.03	12.6720	26	1.79	2.17	1.67	5.14	12.676	0.679	-0.032
25	19	4.35	2.12	4.67	5.7943	19	4.3497	2.1236	4.70	5.7590	19	4.15	2.12	1.54	4.82	5.725	1.196	0.591
26	29	4.12	2.29	5.24	5.2376	29	4.0814	2.2797	5.25	5.2090	29	3.97	2.30	1.78	5.30	5.170	1.287	0.745
27	18	2.04	2.08	4.57	14.6625	18	2.0437	2.0815	4.60	14.5490	18	1.93	2.10	1.51	4.76	14.559	0.703	-0.072
28	29	1.93	2.26	5.18	13.5381	29	1.9142	2.2500	5.18	13.4430	29	1.84	2.29	1.80	5.31	13.426	0.831	0.129
29	28	1.93	2.25	5.14	14.7888	28	1.9277	2.2472	5.18	14.6950	28	1.86	2.26	1.77	5.27	14.677	0.754	0.120
30	4	4.07	1.62	3.36	1.7241	4	4.0816	1.6157	3.35	1.7210	4	3.89	1.70	1.54	3.41	1.694	1.732	1.555
31	6	3.55	1.87	4.09	1.4732	6	3.5525	1.8706	4.10	1.4690	6	3.36	1.95	1.80	4.06	1.446	1.823	1.542
32	9	3.23	2.09	4.73	1.2563	9	3.2484	2.1011	4.70	1.2520	9	3.21	2.17	2.08	4.65	1.233	1.849	1.512

Note:

CC: Chung and Chen (1993)

KV: Kasarapu and Vommi (2011)

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S. No.	CC					KV					SA						% Reduction	
	n	***h***	***k_1***	***K_2***	***$E(L)_2$***	***n***	***h***	***k_1***	***K_2***	***$E(L)_2$***	***n***	***h***	***k_1***	***k_2***	***K_2***	***$E(L)_2$***	***SA with CC***	***SA with KV***
33	2	2.37	1.06	1.90	5.1965	2	2.5118	0.9910	1.75	5.1750	2	2.06	1.28	1.16	2.17	5.160	0.706	0.294
34	3	1.78	1.52	2.97	4.7272	3	2.0257	1.5556	3.25	4.7120	3	1.67	1.61	1.49	3.02	4.670	1.212	0.893
35	6	1.72	1.84	4.03	4.2393	6	1.7376	1.8332	4.00	4.2240	6	1.68	1.90	1.74	4.01	4.194	1.058	0.700
36	9	1.54	2.07	4.69	3.8083	9	1.5466	2.0643	4.70	3.7940	9	1.48	2.14	2.08	4.65	3.767	1.073	0.700
37	5	4.29	1.85	3.90	2.1147	5	4.2883	1.8429	3.90	2.0150	5	4.22	1.91	1.74	3.83	1.981	6.314	1.678
38	8	3.93	2.08	4.62	1.6806	8	3.9160	2.0784	4.65	1.5790	8	3.97	2.11	2.04	4.52	1.553	7.608	1.663
39	7	3.67	2.07	4.49	1.7273	7	3.9200	2.0782	4.65	1.6260	7	3.57	2.15	2.05	4.41	1.597	7.569	1.811
40	11	3.44	2.29	5.15	1.4326	11	3.4388	2.2896	5.15	1.3310	11	3.42	2.34	2.30	4.98	1.309	8.625	1.650
41	3	2.44	1.50	2.93	6.0073	3	2.4454	1.4787	2.90	5.5480	3	2.22	1.63	1.45	2.98	5.507	8.337	0.748
42	5	2.10	1.81	3.84	4.9751	5	2.0810	1.7980	3.85	4.5030	5	2.00	1.90	1.74	3.83	4.462	10.311	0.908
43	7	1.77	2.04	4.45	4.3811	7	1.7604	2.0339	4.45	3.9050	7	1.69	2.12	2.00	4.37	3.860	11.886	1.144
44	11	1.64	2.27	5.12	3.8778	11	1.6192	2.2627	5.15	3.3960	11	1.59	2.33	2.34	5.02	3.362	13.292	0.991
45	13	8.18	1.92	4.78	2.5157	13	8.1724	1.9169	4.80	2.4050	13	7.97	2.00	1.81	4.73	2.404	4.428	0.029
46	18	7.71	2.15	5.32	2.3725	18	7.6950	2.1447	5.35	2.2640	18	7.51	2.23	2.17	5.24	2.263	4.599	0.027
47	12	4.01	1.85	4.61	6.5125	12	3.9831	1.8372	4.60	6.0100	12	3.79	1.94	1.79	4.65	6.055	7.032	-0.741
48	17	3.76	2.08	5.20	6.1509	17	3.7267	2.0771	5.20	5.6510	17	3.59	2.16	2.14	5.18	5.687	7.538	-0.641
49	6	2.19	2.03	4.32	4.2405	6	2.2043	2.0336	4.30	4.2330	6	2.18	2.11	1.95	4.19	4.142	2.313	2.140
50	9	1.98	2.25	4.94	3.5493	9	1.9831	2.2467	4.95	3.5420	9	1.98	2.30	2.25	4.79	3.470	2.227	2.026
51	13	1.79	2.46	5.50	2.9911	13	1.7926	2.4605	5.50	2.9840	13	1.78	2.54	2.49	5.26	2.932	1.964	1.730
52	4	1.20	1.76	3.57	13.0558	4	1.2062	1.7507	3.55	13.0320	4	1.16	1.84	1.67	3.52	12.851	1.569	1.389
53	6	1.03	2.00	4.28	11.4064	6	1.0321	1.9982	4.30	11.3800	6	1.01	2.08	1.95	4.19	11.222	1.616	1.388
54	9	0.92	2.22	4.91	9.9825	9	0.9290	2.2250	4.90	9.9570	9	0.92	2.26	2.22	4.76	9.829	1.535	1.283
55	12	0.80	2.42	5.40	8.8167	12	0.8034	2.4303	5.35	8.7960	12	0.79	2.51	2.45	5.17	8.691	1.425	1.193
56	7	2.36	2.11	4.56	4.6163	7	2.3638	2.1168	4.55	4.5100	7	2.35	2.18	2.08	4.44	4.417	4.309	2.053
57	10	2.07	2.33	5.13	3.8869	10	2.0707	2.3257	5.15	3.7810	10	2.07	2.40	2.35	4.95	3.707	4.616	1.945
58	13	1.77	2.52	5.58	3.4476	13	1.7720	2.5200	5.60	3.3430	13	1.78	2.59	2.58	5.32	3.285	4.707	1.726
59	4	1.17	1.85	3.72	14.6262	4	1.1711	1.8444	3.70	14.1490	4	1.13	1.93	1.79	3.63	13.930	4.757	1.545
60	6	1.01	2.08	4.39	12.2456	6	1.1167	2.0905	4.50	11.7530	6	1.00	2.16	2.05	4.27	11.570	5.515	1.555
61	9	0.91	2.29	5.01	10.7404	9	0.9765	2.3124	5.05	10.2450	9	0.90	2.36	2.33	4.85	10.099	5.976	1.429
62	13	0.82	2.50	5.56	9.5223	13	0.8196	2.5018	5.60	9.0280	13	0.82	2.56	2.60	5.33	8.919	6.332	1.203
63	6	4.57	1.59	3.66	6.5753	6	4.5848	1.5935	3.65	6.5540	6	4.45	1.65	1.42	3.74	6.511	0.978	0.656
64	10	4.31	1.87	4.50	5.8736	10	4.3180	1.8749	4.50	5.8510	10	4.27	1.91	1.83	4.54	5.814	1.012	0.629

Note:

CC: Chung and Chen (1993)

KV: Kasarapu and Vommi (2011)

Contd...

S. No.	CC					KV					SA						% Reduction	
	n	***h***	k_1	K_2	$E(L)_2$	***n***	***h***	k_1	K_2	$E(L)_2$	***n***	***h***	k_1	k_2	K_2	$E(L)_2$	***SA with CC***	***SA with KV***
65	15	4.06	2.12	5.15	5.3024	15	4.0627	2.1247	5.15	5.2830	15	4.01	2.18	2.14	5.09	5.255	0.902	0.538
66	20	3.79	2.33	5.63	4.4447	20	3.7959	2.3407	5.60	4.4290	20	3.75	2.36	2.44	5.51	4.409	0.799	0.447
67	6	2.19	1.55	3.59	16.2683	6	2.1967	1.5357	3.60	16.1780	6	2.10	1.61	1.41	3.73	16.188	0.493	-0.062
68	10	2.02	1.84	4.46	13.0180	10	2.0266	1.8418	4.45	12.9290	10	1.98	1.90	1.75	4.47	12.919	0.761	0.077
69	14	1.86	2.06	5.03	11.8297	14	1.8618	2.0598	5.05	11.7530	14	1.83	2.13	2.07	4.99	11.737	0.781	0.134
70	20	1.76	2.31	5.61	10.9214	20	1.7605	2.3124	5.60	10.8560	20	1.72	2.38	2.40	5.48	10.853	0.623	0.024
71	11	4.38	1.97	4.72	5.8022	11	4.3926	1.9796	4.70	5.6820	11	4.29	2.03	1.93	4.70	5.638	2.822	0.766
72	15	4.03	2.18	5.23	5.1946	15	4.0337	2.1806	5.25	5.0780	15	4.02	2.23	2.23	5.16	5.042	2.929	0.700
73	21	3.82	2.41	5.77	4.8813	21	3.8152	2.4120	5.80	4.7690	21	3.81	2.46	2.58	5.64	4.747	2.759	0.469
74	7	2.25	1.68	3.93	17.1665	7	2.2671	1.6771	3.90	16.6160	7	2.17	1.75	1.55	3.99	16.610	3.245	0.039
75	10	2.03	1.91	4.56	15.0487	10	2.0727	1.9375	4.70	14.5030	10	1.97	1.97	1.85	4.56	14.472	3.831	0.213
76	15	1.90	2.15	5.20	13.8127	15	1.8990	2.1563	5.20	13.2740	15	1.85	2.21	2.21	5.14	13.251	4.064	0.171
77	21	1.79	2.39	5.75	12.8792	21	1.7847	2.3933	5.75	12.3492	21	1.75	2.47	2.48	5.58	12.343	4.165	0.050
78	3	3.40	2.06	3.53	1.5699	3	3.3936	2.0511	3.55	1.5690	3	3.37	2.14	1.93	3.40	1.527	2.705	2.649
79	4	3.00	2.29	4.06	1.3562	4	3.0059	2.2867	4.05	1.3550	4	3.04	2.32	2.21	4.01	1.325	2.282	2.195
80	6	2.84	2.52	4.67	1.0477	6	2.8517	2.5254	4.65	1.0450	6	2.83	2.63	2.33	4.51	1.027	2.009	1.756
81	3	1.64	2.02	3.47	4.1584	3	1.6448	2.0124	3.45	4.1500	3	1.59	2.11	1.94	3.42	4.080	1.873	1.675
82	4	1.40	2.27	4.03	3.0758	4	1.4116	2.2612	4.00	3.0680	4	1.39	2.36	2.13	3.93	3.014	2.019	1.769
83	6	1.33	2.50	4.64	2.9502	6	1.3318	2.4992	4.65	2.9400	6	1.29	2.63	2.33	4.51	2.909	1.390	1.048
84	3	3.26	2.25	3.80	1.6657	3	3.2515	2.2437	3.80	1.5690	3	3.26	2.32	2.11	3.57	1.515	9.077	3.473
85	5	3.25	2.50	4.51	1.4283	5	3.2506	2.4997	4.50	1.3280	5	3.24	2.57	2.35	4.36	1.298	9.099	2.233
86	6	2.79	2.66	4.86	1.2008	6	2.9859	2.7120	5.00	1.1000	6	2.81	2.73	2.51	4.66	1.076	10.389	2.177
87	3	1.59	2.22	3.76	4.8826	3	1.5763	2.2095	3.75	4.4210	3	1.53	2.35	2.15	3.60	4.326	11.402	2.151
88	5	1.54	2.48	4.48	3.7728	5	1.5236	2.4739	4.50	3.2940	5	1.50	2.56	2.37	4.37	3.241	14.088	1.601
89	6	1.32	2.65	4.84	3.3970	6	1.3120	2.6524	4.80	2.9160	6	1.30	2.73	2.53	4.68	2.871	**15.483**	1.542
90	10	3.47	2.34	4.76	5.2434	10	3.4761	2.3351	4.75	5.1920	10	3.31	2.39	2.20	4.83	5.230	0.250	-0.738
91	8	7.52	2.32	4.59	2.2593	8	7.5071	2.3229	4.60	2.1510	8	7.37	2.39	2.10	4.57	2.148	4.921	0.133
92	11	2.55	5.11	7.25	2.1304	11	7.2428	2.5600	5.10	2.0230	11	7.19	2.60	2.43	5.09	2.024	5.010	-0.033
93	6	3.92	2.08	4.04	6.2683	6	3.9039	2.0686	4.00	5.7670	6	3.73	2.11	1.87	4.12	5.800	7.474	-0.569
94	8	3.68	2.29	4.55	5.9134	8	3.6457	2.2840	4.55	5.4130	8	3.49	2.36	2.07	4.54	5.444	7.930	-0.581
95	11	3.55	2.52	5.07	5.8564	11	3.5145	2.5298	5.05	5.3590	11	3.40	2.57	2.41	5.07	5.392	7.922	-0.624
96	5	1.56	2.64	4.70	3.1579	5	1.5621	2.6399	4.70	3.1550	5	1.57	2.71	2.47	4.46	3.076	2.591	2.502

Note:
CC: Chung and Chen (1993)
KV: Kasarapu and Vommi (2011)

Contd...

S. No.	CC					KV					SA						% Reduction	
	n	*h*	k_1	K_2	$E(L)_2$	*n*	*h*	k_1	K_2	$E(L)_2$	*n*	*h*	k_1	k_2	K_2	$E(L)_2$	*SA with CC*	*SA with KV*
97	7	1.45	2.83	5.18	2.3313	7	1.4539	2.8396	5.15	2.3270	7	1.47	2.89	2.73	4.98	2.273	2.481	2.301
98	3	0.99	2.20	3.73	11.7212	3	0.9978	2.1921	3.70	11.7210	3	0.97	2.30	2.13	3.58	11.434	2.447	2.445
99	4	0.84	2.43	4.27	8.9925	4	0.8471	2.4304	4.25	8.9820	4	0.84	2.50	2.32	4.10	8.780	2.358	2.243
100	5	0.72	2.63	4.68	7.2060	5	0.7203	2.6276	4.65	7.1950	5	0.71	2.71	2.51	4.49	7.034	2.385	2.235
101	7	0.66	2.82	5.17	6.3645	7	0.6646	2.8250	5.15	6.3500	7	0.66	2.93	2.70	4.95	6.243	1.904	1.680
102	5	1.54	2.71	4.79	3.0331	5	1.6906	2.7338	5.00	2.9310	5	1.55	2.81	2.60	4.57	2.845	6.213	2.945
103	7	1.43	2.89	5.26	2.6275	7	1.4485	2.9001	5.20	2.5250	7	1.46	2.95	2.77	5.01	2.466	6.156	2.347
104	4	0.84	2.50	4.36	11.0358	4	0.8414	2.5011	4.35	10.5580	4	0.83	2.59	2.42	4.19	10.338	6.321	2.081
105	5	0.72	2.69	4.77	9.9209	5	0.7188	2.6893	4.75	9.4390	5	0.72	2.77	2.59	4.56	9.266	6.602	1.834
106	7	0.66	2.88	5.25	7.0397	7	0.6622	2.8874	5.20	6.5450	7	0.67	2.99	2.77	5.01	6.426	8.721	1.822
107	7	3.96	2.33	4.50	5.0818	7	3.9637	2.3312	4.50	5.0640	7	3.89	2.41	2.12	4.47	5.029	1.034	0.686
108	9	3.71	2.51	4.92	4.3916	9	3.7073	2.5124	4.95	4.3760	9	3.67	2.58	2.31	4.84	4.351	0.920	0.567
109	12	3.56	2.71	5.38	4.1063	12	3.5687	2.7342	5.35	4.0920	12	3.53	2.80	2.57	5.26	4.079	0.673	0.326
110	7	1.85	2.30	4.47	12.9502	7	1.8533	2.2923	4.50	12.8790	7	1.82	2.36	2.10	4.46	12.853	0.748	0.199
111	9	1.73	2.49	4.89	12.1054	9	1.7352	2.4911	4.90	12.0410	9	1.71	2.54	2.31	4.84	12.029	0.631	0.099
112	12	1.65	2.70	5.35	10.8632	12	1.6557	2.7068	5.35	10.8050	12	1.61	2.76	2.58	5.26	10.809	0.496	-0.040
113	7	3.94	2.39	4.59	5.3874	7	3.9438	2.3889	4.60	5.2730	7	3.90	2.47	2.23	4.56	5.229	2.933	0.827
114	10	3.77	2.62	5.14	4.6830	10	3.7734	2.6291	5.15	4.5710	10	3.74	2.67	2.51	5.08	4.547	2.899	0.520
115	12	3.56	2.77	5.44	4.5323	12	3.5641	2.7749	5.45	4.4200	12	3.52	2.87	2.66	5.33	4.403	2.845	0.377
116	5	1.99	2.13	3.99	15.4402	5	1.9874	2.1226	4.00	14.9110	5	1.94	2.20	1.95	4.01	14.833	3.934	0.525
117	7	1.85	2.36	4.56	13.6345	7	1.8521	2.3648	4.55	13.1000	7	1.81	2.43	2.19	4.53	13.057	4.234	0.327
118	9	1.73	2.55	4.97	12.7639	9	1.7348	2.5553	4.95	12.2340	9	1.70	2.61	2.42	4.93	12.209	4.344	0.201
119	12	1.67	2.75	5.42	12.7929	12	1.6647	2.7485	5.45	12.2700	12	1.66	2.82	2.65	5.32	12.267	4.108	0.021
120	3	3.48	2.16	3.87	1.4699	3	3.4822	2.1594	3.85	1.4720	3	3.45	2.24	2.18	3.63	1.433	2.489	2.628
121	4	3.08	2.40	4.41	1.1217	4	3.0800	2.3930	4.40	1.1200	4	3.08	2.48	2.42	4.19	1.102	1.716	1.567
122	5	2.73	2.59	4.82	0.9735	5	2.7355	2.5876	4.80	0.9720	5	2.69	2.65	2.57	4.55	0.959	1.456	1.304
123	3	1.65	2.13	3.83	3.7483	3	1.6541	2.1253	3.80	3.7510	3	1.65	2.24	2.14	3.59	3.679	1.848	1.918
124	4	1.44	2.38	4.38	2.8859	4	1.4410	2.3707	4.35	2.8770	4	1.43	2.46	2.32	4.10	2.848	1.300	0.995
125	5	1.27	2.57	4.80	2.5722	5	1.2686	2.5659	4.80	2.5650	5	1.27	2.66	2.56	4.54	2.543	1.117	0.840
126	3	3.38	2.34	4.12	1.5341	3	3.3633	2.3314	4.15	1.4360	3	3.36	2.41	2.36	3.79	1.403	8.540	2.292
127	4	3.03	2.55	4.62	1.3224	4	3.0261	2.5483	4.60	1.2240	4	3.04	2.60	2.58	4.33	1.201	9.144	1.839
128	5	2.70	2.73	5.01	1.2140	5	2.6997	2.7276	5.00	1.1150	5	2.67	2.86	2.80	4.75	1.099	9.437	1.396

Note:

CC: Chung and Chen (1993)

KV: Kasarapu and Vommi (2011)

Contd...

S. No.	CC					KV					SA						% Reduction	
	n	***h***	***k_1***	***K_2***	***$E(L)_2$***	***n***	***h***	***k_1***	***K_2***	***$E(L)_2$***	***n***	***h***	***k_1***	***k_2***	***K_2***	***$E(L)_2$***	***SA with CC***	***SA with KV***
129	3	1.63	2.31	4.08	4.6250	3	1.6338	2.3054	4.05	4.1580	3	1.57	2.43	2.37	3.80	4.103	11.279	1.315
130	4	1.44	2.53	4.59	3.9854	4	1.4287	2.5241	4.60	3.5120	4	1.42	2.60	2.58	4.33	3.474	12.820	1.069
131	5	1.28	2.71	4.99	3.6644	5	1.2708	2.7127	4.95	3.1880	5	1.27	2.77	2.81	4.75	3.161	13.727	0.835
132	8	3.40	2.40	4.90	5.1444	8	3.4021	2.3901	4.90	5.0970	8	3.32	2.44	2.42	4.83	5.132	0.232	-0.696
133	7	7.42	2.43	4.84	2.2725	7	7.4081	2.4278	4.85	2.1660	7	7.26	2.52	2.50	4.78	2.166	4.685	0.003
134	9	7.12	2.65	5.29	2.0284	9	7.1077	2.6478	5.30	1.9220	9	7.04	2.80	2.55	5.03	1.925	5.077	-0.178
135	5	3.78	2.13	4.20	6.0691	5	3.7478	2.1139	4.20	5.5710	5	3.62	2.22	2.14	4.17	5.599	7.743	-0.505
136	7	3.57	2.40	4.81	5.3637	7	3.5394	2.3942	4.80	4.8600	7	3.42	2.50	2.45	4.75	4.893	8.767	-0.689
137	9	2.62	5.26	3.43	5.1463	9	3.3910	2.6201	5.25	4.6440	9	3.30	2.71	2.68	5.14	4.679	9.077	-0.757
138	4	1.46	2.69	4.81	2.6876	4	1.4608	2.6891	4.80	2.6890	4	1.45	2.75	2.71	4.44	2.578	4.070	4.120
139	6	1.41	2.92	5.38	2.2963	6	1.4170	2.9204	5.35	2.2940	6	1.43	2.95	2.95	5.04	2.263	1.438	1.339
140	3	0.75	2.48	4.32	9.3800	3	0.7456	2.4684	4.35	9.4000	3	0.75	2.56	2.54	3.95	8.868	5.456	**5.660**
141	4	0.67	2.67	4.79	8.1658	4	0.6738	2.6704	4.80	8.1750	4	0.67	2.74	2.71	4.45	7.952	2.617	2.726
142	6	0.65	2.90	5.36	8.0331	6	0.6538	2.9040	5.35	8.0240	6	0.66	2.97	2.95	5.03	7.965	0.844	0.732
143	5	1.62	2.81	5.12	3.0774	5	1.6191	2.8042	5.15	2.9770	5	1.64	2.91	2.84	4.78	2.932	4.714	1.501
144	6	1.41	2.97	5.45	2.2925	6	1.4061	2.9752	5.45	2.1920	6	1.43	3.03	3.03	5.10	2.158	5.861	1.545
145	3	0.74	2.55	4.42	9.0710	3	0.7403	2.5489	4.35	8.5920	3	0.74	2.62	2.59	3.99	8.415	7.228	2.056
146	4	0.66	2.74	4.88	7.3852	4	0.7403	2.7908	5.15	6.8980	4	0.66	2.84	2.77	4.50	6.779	8.212	1.729
147	6	0.64	2.96	5.44	6.6344	6	0.6426	2.9638	5.45	6.1430	6	0.64	3.04	3.02	5.09	6.076	8.413	1.086
148	6	3.87	2.41	4.71	5.0919	6	3.8698	2.4122	4.70	5.0030	6	3.83	2.54	2.47	4.63	4.995	1.895	0.152
149	7	3.60	2.55	5.01	4.6586	7	3.6066	2.5572	5.00	4.6280	7	3.56	2.67	2.60	4.87	4.653	0.110	-0.550
150	9	3.48	2.76	5.44	4.1543	9	3.4763	2.7622	5.45	4.1130	9	3.44	2.93	2.85	5.27	4.141	0.320	-0.681
151	6	1.80	2.39	4.68	12.6804	6	1.8086	2.3864	4.65	12.3990	6	1.76	2.53	2.44	4.60	12.571	0.860	-1.390
152	7	1.67	2.54	4.98	11.2577	7	1.6718	2.5291	5.00	11.0350	7	1.64	2.66	2.64	4.91	11.201	0.501	-1.507
153	9	1.61	2.74	5.41	10.7751	9	1.6106	2.7373	5.45	10.5490	9	1.59	2.89	2.87	5.29	10.718	0.532	-1.600
154	6	3.85	2.47	4.79	5.1348	6	3.8550	2.4703	4.80	5.0240	6	3.84	2.53	2.47	4.63	4.995	2.714	0.569
155	8	3.67	2.70	5.28	4.4982	8	3.6661	2.6981	5.30	4.3880	8	3.64	2.76	2.71	5.07	4.373	2.774	0.333
156	10	3.51	2.89	5.67	4.2590	10	3.5133	2.8990	5.65	4.1500	10	3.50	2.97	2.95	5.43	4.143	2.716	0.161
157	4	1.88	2.17	4.09	14.7881	4	1.8844	2.1604	4.05	14.2700	4	1.84	2.24	2.18	3.98	14.198	3.993	0.507
158	6	1.81	2.45	4.77	13.1163	6	1.8037	2.4474	4.75	12.5910	6	1.77	2.52	2.47	4.63	12.571	4.157	0.158
159	8	1.70	2.68	5.26	11.0839	8	1.6966	2.6744	5.30	10.5560	8	1.67	2.77	2.72	5.07	10.559	4.740	-0.024
160	10	1.63	2.88	5.65	10.5750	10	1.6227	2.8784	5.65	10.0510	10	1.59	2.98	2.94	5.42	10.066	4.812	-0.150

Note:

CC: Chung and Chen (1993)

KV: Kasarapu and Vommi (2011)

The comparison of results in Table 5.24 reveals that the results of present work are superior to that of Chung and Chen (1993) in all the 160 design problems, whereas compared to Kasarapu and Vommi (2011) it is superior in 137 out 160 cases. In the remaining 23 (i.e., 160-137 = 23) cases, the values of percentage reduction are found to be negative which means that present work has yielded comparatively inferior results. Therefore to check the accuracy of results in these 23 cases, the values of output $E(L)_2$ reported by Kasarapu and Vommi (2011) have been recalculated taking the same optimal values of input variables (i.e., n, h, k_1 and K_2) reported by them. The correct values of loss cost per unit time $E(L)_2$ obtained after recalculation and the corresponding percentage reduction in cost are shown in Table 5.25. The serial numbers shown in this table are same as that of all those 23 cases for which the percentage reduction values are found to be negative in Table 5.24. Table 5.25 shows that value of percentage reduction is not found to be negative in any of 23 cases after using the correct values. This suggests that in true sense, the results of SA are found to be better than that of Kasarapu and Vommi (2011) in all the 160 design problems.

Further, compared to the results of Chung and Chen (1993), SA has provided a maximum reduction of 15.483% in the expected loss cost per unit time (i.e., serial number 89 in Table 5.24). Similarly, compared to the results of Kasarapu and Vommi (2011), a maximum reduction of 5.660% has been obtained using SA (i.e., serial number 140 in Table 5.24).

Table 5.25: Recalculated expected loss cost per unit time and percentage reduction with SA

S. No.	*Cost and process parameters*													*Kasarapu and Vommi (2011)*					*SA*						% Reduction Recalculated
	δ	λ	V_0	V_1	M	W	Y	T_1	T_0	a	b	σ_0	σ_1	n	h	k_1	K_2	$E(L)_2$	n	h	k_1	k_2	K_2	$E(L)_2$	
24	0.5	0.05	150	50	100	2	1	0.4	0.1	5	0.1	0.02	0.03	26	1.8889	2.1786	5.03	12.6950	26	1.79	2.17	1.67	5.14	12.676	0.149
27	0.5	0.05	150	50	100	15	5	0.4	0.1	5	0.2	0.02	0.03	18	2.0437	2.0815	4.60	14.6007	18	1.93	2.10	1.51	4.76	14.559	0.283
47	1.0	0.05	50	25	25	5	5	0.6	0.1	5	0.2	0.02	0.03	12	3.9831	1.8372	4.60	6.0603	12	3.79	1.94	1.79	4.65	6.055	0.095
48	1.0	0.05	50	25	25	5	5	0.6	0.1	5	0.1	0.02	0.03	17	3.7267	2.0771	5.20	5.6907	17	3.59	2.16	2.14	5.18	5.687	0.061
67	1.0	0.05	150	50	100	2	1	0.6	0.1	5	0.8	0.02	0.03	6	2.1967	1.5357	3.60	16.2066	6	2.10	1.61	1.41	3.73	16.188	0.114
90	1.5	0.05	50	25	25	2	1	0.6	0.1	5	0.1	0.02	0.04	10	3.4761	2.3351	4.75	5.2319	10	3.31	2.39	2.20	4.83	5.230	0.030
92	1.5	0.01	50	25	25	5	5	0.4	0.1	5	0.1	0.02	0.04	11	7.2428	2.5600	5.10	2.0239	11	7.19	2.60	2.43	5.09	2.024	0.011
93	1.5	0.05	50	25	25	5	5	0.5	0.1	5	0.4	0.02	0.04	6	3.9039	2.0686	4.00	5.8068	6	3.73	2.11	1.87	4.12	5.800	0.121
94	1.5	0.05	50	25	25	5	5	0.5	0.1	5	0.2	0.02	0.04	8	3.6457	2.2840	4.55	5.4471	8	3.49	2.36	2.07	4.54	5.444	0.048
95	1.5	0.05	50	25	25	5	5	0.6	0.1	5	0.1	0.02	0.04	11	3.5145	2.5298	5.05	5.3945	11	3.40	2.57	2.41	5.07	5.392	0.038
112	1.5	0.05	150	50	100	2	1	0.4	0.1	5	0.1	0.02	0.04	12	1.6557	2.7068	5.35	10.8123	12	1.61	2.76	2.58	5.26	10.809	0.028
132	2.0	0.05	50	25	25	2	1	0.6	0.1	5	0.1	0.02	0.04	8	3.4021	2.3901	4.90	5.1343	8	3.32	2.44	2.42	4.83	5.132	0.036
134	2.0	0.01	50	25	25	5	5	0.3	0.1	5	0.1	0.02	0.04	9	7.1077	2.6478	5.30	1.9258	9	7.04	2.80	2.55	5.03	1.925	0.052
135	2.0	0.05	50	25	25	5	5	0.5	0.1	5	0.4	0.02	0.04	5	3.7478	2.1139	4.20	5.6029	5	3.62	2.22	2.14	4.17	5.599	0.067
136	2.0	0.05	50	25	25	5	5	0.3	0.1	5	0.2	0.02	0.04	7	3.5394	2.3942	4.80	4.8962	7	3.42	2.50	2.45	4.75	4.893	0.056
137	2.0	0.05	50	25	25	5	5	0.3	0.1	5	0.1	0.02	0.04	9	3.3910	2.6201	5.25	4.6818	9	3.30	2.71	2.68	5.14	4.679	0.056
149	2.0	0.01	150	50	100	5	5	0.6	0.1	5	0.2	0.02	0.04	7	3.6066	2.5572	5.00	4.6568	7	3.56	2.67	2.60	4.87	4.653	0.072
150	2.0	0.01	150	50	100	5	5	0.4	0.1	5	0.1	0.02	0.04	9	3.4763	2.7622	5.45	4.1440	9	3.44	2.93	2.85	5.27	4.141	0.072
151	2.0	0.05	150	50	100	5	5	0.5	0.1	5	0.4	0.02	0.04	6	1.8086	2.3864	4.65	12.5808	6	1.76	2.53	2.44	4.60	12.571	0.075
152	2.0	0.05	150	50	100	5	5	0.4	0.1	5	0.2	0.02	0.04	7	1.6718	2.5291	5.00	11.2109	7	1.64	2.66	2.64	4.91	11.201	0.086
153	2.0	0.05	150	50	100	5	5	0.4	0.1	5	0.1	0.02	0.04	9	1.6106	2.7373	5.45	10.7261	9	1.59	2.89	2.87	5.29	10.718	0.078
159	2.0	0.05	150	50	100	5	5	0.3	0.1	5	0.2	0.02	0.04	8	1.6966	2.6744	5.30	10.5690	8	1.67	2.77	2.72	5.07	10.559	0.099
160	2.0	0.05	150	50	100	5	5	0.3	0.1	5	0.1	0.02	0.04	10	1.6227	2.8784	5.65	10.0732	10	1.59	2.98	2.94	5.42	10.066	0.070

Similar to Table 5.24, the results of all 160 design problems related to the joint economic design of $\bar{X}$ and R charts for a discontinuous process using TLBO are shown in Table 5.26. The comparison of results in this table shows that the results obtained using TLBO are found to be superior to that of Chung and Chen (1993) in all the 160 design problems, whereas compared to Kasarapu and Vommi (2011) it is superior in 137 out 160 cases. This observation is same as that with the results of SA including the same serial numbers where the values of percentage reduction are found to be negative. The corrected values of loss cost per unit time $E(L)_2$ and corresponding percentage reduction of cost compared to TLBO for all these 23 cases are shown in Table 5.27. The serial numbers shown in this table are same as that of all those 23 cases for which the percentage reduction values are found to be negative in Table 5.26. Table 5.27 shows that value of percentage reduction is not found to be negative in any of these 23 cases. This suggests that in true sense, the results of TLBO are found to be better than that of Kasarapu and Vommi (2011) in all the 160 design problems.

Further, compared to the results of Chung and Chen (1993), TLBO has also provided a maximum percentage reduction of 15.483% in the expected loss cost per unit time $E(L)_2$ (i.e., serial number 89 Table 5.26). Similarly, compared to the results of Kasarapu and Vommi (2011), a maximum reduction of 5.660% has been obtained using TLBO (i.e., serial number 140 in Table 5.26). Thus, both these values obtained by TLBO are found to be the same as that obtained by SA.

Table 5.26: Comparison of results with that of TLBO

S. No.	***CC***					***KV***					***TLBO***						***% Reduction***	
	n	***h***	k_1	K_2	$E(L)_2$	***n***	***h***	k_1	K_2	$E(L)_2$	***n***	***h***	k_1	k_2	K_2	$E(L)_2$	***TLBO with CC***	***TLBO with KV***
1	10	3.46	2.07	4.14	1.4869	10	3.4682	2.0792	4.15	1.4780	10	3.24	2.13	1.51	4.28	1.450	2.455	1.867
2	10	1.66	2.04	4.09	3.8380	10	1.6720	2.0428	4.10	3.8130	10	1.53	2.11	1.48	4.26	3.778	1.558	0.913
3	8	4.04	2.05	3.96	1.9794	8	4.0837	2.0605	3.95	1.9690	8	3.76	2.13	1.52	4.09	1.927	2.667	2.153
4	13	3.71	2.24	4.58	1.7224	13	3.7201	2.2551	4.60	1.7110	13	3.51	2.31	1.75	4.68	1.670	3.042	2.396
5	2	4.10	0.56	1.00	6.5776	2	3.5593	1.0000	1.00	6.5420	2	2.08	1.44	1.13	2.14	6.531	0.716	0.176
6	5	2.15	1.70	2.96	5.9003	5	2.4010	1.7160	3.15	5.8770	5	2.09	1.85	1.23	3.39	5.844	0.963	0.570
7	8	1.88	2.00	3.79	5.1683	8	2.0247	2.0132	3.90	5.1390	8	1.83	2.09	1.49	4.07	5.087	1.577	1.016
8	13	1.74	2.21	4.48	4.6643	13	1.8238	2.2248	4.55	4.6340	13	1.66	2.30	1.74	4.67	4.575	1.908	1.267
9	6	2.26	2.01	3.70	5.0149	6	2.2664	2.0140	3.70	5.0000	6	2.10	2.10	1.50	3.81	4.852	3.246	2.958
10	10	2.08	2.22	4.36	4.3160	10	2.0644	2.2274	4.40	4.2910	10	1.96	2.29	1.72	4.45	4.159	3.640	3.079
11	16	1.93	2.39	4.93	3.9883	16	1.9385	2.4002	4.95	3.9620	16	1.85	2.46	1.95	4.99	3.845	3.591	2.951
12	6	1.09	1.97	3.64	13.5670	6	1.0899	1.9688	3.65	13.5260	6	0.99	2.07	1.46	3.77	13.286	2.071	1.774
13	10	0.99	2.19	4.32	12.1752	10	0.9861	2.1973	4.35	12.1150	10	0.93	2.27	1.69	4.43	11.887	2.367	1.882
14	15	0.88	2.36	4.86	10.9316	15	0.8891	2.3798	4.85	10.8710	15	0.83	2.45	1.93	4.93	10.645	2.622	2.079
15	7	2.39	2.10	3.93	5.6217	7	2.3826	2.1022	3.95	5.6010	7	2.24	2.18	1.59	4.03	5.439	3.252	2.894
16	11	2.14	2.29	4.53	4.4449	11	2.1406	2.3013	4.55	4.4160	11	2.03	2.37	1.82	4.60	4.264	4.063	3.435
17	17	1.95	2.45	5.06	4.2163	17	1.9467	2.4577	5.10	4.1880	17	1.87	2.53	2.04	5.10	4.052	3.887	3.238
18	7	1.15	2.06	3.88	14.5951	7	1.1498	2.0636	3.90	14.5400	7	1.08	2.15	1.54	3.99	14.273	2.207	1.836
19	11	1.02	2.26	4.49	13.0856	11	1.0282	2.2744	4.50	13.0180	11	0.95	2.35	1.79	4.58	12.751	2.557	2.051
20	17	0.92	2.43	5.04	11.7576	17	0.9268	2.4410	5.05	11.6910	17	0.87	2.51	2.04	5.10	11.442	2.684	2.130
21	17	4.29	2.03	4.46	5.9667	17	4.2836	2.0393	4.50	5.9340	17	4.10	2.03	1.41	4.64	5.916	0.855	0.308
22	27	4.06	2.21	5.06	5.4536	27	4.0557	2.2132	5.10	5.4270	27	3.93	2.21	1.70	5.19	5.401	0.957	0.472
23	17	2.01	1.99	4.35	15.0411	17	2.0395	2.0057	4.45	14.9860	17	1.93	2.00	1.37	4.61	14.982	0.393	0.027
24	26	1.89	2.18	4.99	12.7627	26	1.8889	2.1786	5.03	12.6720	26	1.81	2.19	1.65	5.13	12.676	0.679	-0.032
25	19	4.35	2.12	4.67	5.7943	19	4.3497	2.1236	4.70	5.7590	19	4.15	2.12	1.55	4.83	5.725	1.196	0.590
26	29	4.12	2.29	5.24	5.2376	29	4.0814	2.2797	5.25	5.2090	29	3.95	2.29	1.82	5.33	5.170	1.294	0.753
27	18	2.04	2.08	4.57	14.6625	18	2.0437	2.0815	4.60	14.5490	18	1.92	2.09	1.50	4.75	14.559	0.706	-0.069
28	29	1.93	2.26	5.18	13.5381	29	1.9142	2.2500	5.18	13.4430	29	1.84	2.28	1.79	5.31	13.425	0.835	0.134
29	28	1.93	2.25	5.14	14.7888	28	1.9277	2.2472	5.18	14.6950	28	1.84	2.26	1.78	5.27	14.677	0.756	0.122
30	4	4.07	1.62	3.36	1.7241	4	4.0816	1.6157	3.35	1.7210	4	3.93	1.70	1.50	3.38	1.694	1.740	1.563
31	6	3.55	1.87	4.09	1.4732	6	3.5525	1.8706	4.10	1.4690	6	3.48	1.94	1.80	4.06	1.446	1.867	1.586
32	9	3.23	2.09	4.73	1.2563	9	3.2484	2.1011	4.70	1.2520	9	3.18	2.15	2.09	4.66	1.233	1.855	1.518

Note:

CC: Chung and Chen (1993)

KV: Kasarapu and Vommi (2011)

Contd...

S. No.	CC					KV					TLBO						% Reduction	
	n	*h*	k_1	K_2	$E(L)_2$	*n*	*h*	k_1	K_2	$E(L)_2$	*n*	*h*	k_1	k_2	K_2	$E(L)_2$	TLBO with CC	TLBO with KV
33	2	2.37	1.06	1.90	5.1965	2	2.5118	0.9910	1.75	5.1750	2	2.05	1.28	1.16	2.17	5.160	0.708	0.296
34	3	1.78	1.52	2.97	4.7272	3	2.0257	1.5556	3.25	4.7120	3	1.66	1.64	1.46	2.99	4.669	1.223	0.904
35	6	1.72	1.84	4.03	4.2393	6	1.7376	1.8332	4.00	4.2240	6	1.66	1.91	1.76	4.03	4.194	1.061	0.703
36	9	1.54	2.07	4.69	3.8083	9	1.5466	2.0643	4.70	3.7940	9	1.50	2.13	2.06	4.64	3.767	1.084	0.712
37	5	4.29	1.85	3.90	2.1147	5	4.2883	1.8429	3.90	2.0150	5	4.18	1.92	1.76	3.85	1.981	6.318	1.682
38	8	3.93	2.08	4.62	1.6806	8	3.9160	2.0784	4.65	1.5790	8	3.88	2.15	2.05	4.53	1.552	7.634	1.691
39	7	3.67	2.07	4.49	1.7273	7	3.9200	2.0782	4.65	1.6260	7	3.62	2.13	2.05	4.41	1.596	7.578	1.820
40	11	3.44	2.29	5.15	1.4326	11	3.4388	2.2896	5.15	1.3310	11	3.40	2.35	2.32	5.00	1.309	8.628	1.653
41	3	2.44	1.50	2.93	6.0073	3	2.4454	1.4787	2.90	5.5480	3	2.23	1.63	1.45	2.98	5.506	8.338	0.750
42	5	2.10	1.81	3.84	4.9751	5	2.0810	1.7980	3.85	4.5030	5	1.99	1.89	1.74	3.83	4.462	10.311	0.908
43	7	1.77	2.04	4.45	4.3811	7	1.7604	2.0339	4.45	3.9050	7	1.70	2.11	2.01	4.38	3.860	11.887	1.145
44	11	1.64	2.27	5.12	3.8778	11	1.6192	2.2627	5.15	3.3960	11	1.59	2.34	2.30	4.99	3.362	13.301	1.001
45	13	8.18	1.92	4.78	2.5157	13	8.1724	1.9169	4.80	2.4050	13	7.99	1.97	1.89	4.79	2.404	4.452	0.054
46	18	7.71	2.15	5.32	2.3725	18	7.6950	2.1447	5.35	2.2640	18	7.57	2.21	2.20	5.26	2.263	4.603	0.031
47	12	4.01	1.85	4.61	6.5125	12	3.9831	1.8372	4.60	6.0100	12	3.81	1.92	1.81	4.67	6.054	7.036	-0.737
48	17	3.76	2.08	5.20	6.1509	17	3.7267	2.0771	5.20	5.6510	17	3.57	2.16	2.14	5.18	5.687	7.539	-0.641
49	6	2.19	2.03	4.32	4.2405	6	2.2043	2.0336	4.30	4.2330	6	2.17	2.09	1.98	4.21	4.142	2.323	2.150
50	9	1.98	2.25	4.94	3.5493	9	1.9831	2.2467	4.95	3.5420	9	1.98	2.31	2.25	4.79	3.470	2.229	2.027
51	13	1.79	2.46	5.50	2.9911	13	1.7926	2.4605	5.50	2.9840	13	1.77	2.53	2.55	5.30	2.932	1.962	1.729
52	4	1.20	1.76	3.57	13.0558	4	1.2062	1.7507	3.55	13.0320	4	1.15	1.84	1.68	3.54	12.851	1.569	1.389
53	6	1.03	2.00	4.28	11.4064	6	1.0321	1.9982	4.30	11.3800	6	1.00	2.08	1.96	4.20	11.222	1.617	1.388
54	9	0.92	2.22	4.91	9.9825	9	0.9290	2.2250	4.90	9.9570	9	0.91	2.30	2.22	4.76	9.829	1.543	1.291
55	12	0.80	2.42	5.40	8.8167	12	0.8034	2.4303	5.35	8.7960	12	0.79	2.50	2.47	5.18	8.691	1.431	1.199
56	7	2.36	2.11	4.56	4.6163	7	2.3638	2.1168	4.55	4.5100	7	2.34	2.17	2.07	4.43	4.417	4.309	2.053
57	10	2.07	2.33	5.13	3.8869	10	2.0707	2.3257	5.15	3.7810	10	2.07	2.39	2.35	4.95	3.707	4.618	1.947
58	13	1.77	2.52	5.58	3.4476	13	1.7720	2.5200	5.60	3.3430	13	1.78	2.58	2.59	5.33	3.285	4.708	1.726
59	4	1.17	1.85	3.72	14.6262	4	1.1711	1.8444	3.70	14.1490	4	1.15	1.91	1.76	3.61	13.932	4.746	1.534
60	6	1.01	2.08	4.39	12.2456	6	1.1167	2.0905	4.50	11.7530	6	1.00	2.15	2.05	4.27	11.570	5.517	1.557
61	9	0.91	2.29	5.01	10.7404	9	0.9765	2.3124	5.05	10.2450	9	0.90	2.38	2.33	4.85	10.099	5.972	1.425
62	13	0.82	2.50	5.56	9.5223	13	0.8196	2.5018	5.60	9.0280	13	0.82	2.57	2.58	5.32	8.919	6.335	1.206
63	6	4.57	1.59	3.66	6.5753	6	4.5848	1.5935	3.65	6.5540	6	4.44	1.65	1.43	3.75	6.511	0.978	0.656
64	10	4.31	1.87	4.50	5.8736	10	4.3180	1.8749	4.50	5.8510	10	4.24	1.92	1.80	4.52	5.814	1.018	0.636

Note:

CC: Chung and Chen (1993)

KV: Kasarapu and Vommi (2011)

Contd...

S. No.	CC					KV					TLBO						% Reduction	
	n	***h***	***k_1***	***K_2***	***$E(L)_2$***	***n***	***h***	***k_1***	***K_2***	***$E(L)_2$***	***n***	***h***	***k_1***	***k_2***	***K_2***	***$E(L)_2$***	***TLBO with CC***	***TLBO with KV***
65	15	4.06	2.12	5.15	5.3024	15	4.0627	2.1247	5.15	5.2830	15	4.01	2.16	2.15	5.10	5.255	0.901	0.538
66	20	3.79	2.33	5.63	4.4447	20	3.7959	2.3407	5.60	4.4290	20	3.74	2.40	2.41	5.49	4.409	0.805	0.454
67	6	2.19	1.55	3.59	16.2683	6	2.1967	1.5357	3.60	16.1780	6	2.10	1.61	1.39	3.71	16.188	0.494	-0.062
68	10	2.02	1.84	4.46	13.0180	10	2.0266	1.8418	4.45	12.9290	10	1.96	1.93	1.79	4.51	12.919	0.760	0.077
69	14	1.86	2.06	5.03	11.8297	14	1.8618	2.0598	5.05	11.7530	14	1.81	2.13	2.09	5.00	11.737	0.784	0.136
70	20	1.76	2.31	5.61	10.9214	20	1.7605	2.3124	5.60	10.8560	20	1.73	2.37	2.41	5.49	10.853	0.626	0.028
71	11	4.38	1.97	4.72	5.8022	11	4.3926	1.9796	4.70	5.6820	11	4.30	2.03	1.95	4.71	5.638	2.823	0.767
72	15	4.03	2.18	5.23	5.1946	15	4.0337	2.1806	5.25	5.0780	15	3.99	2.24	2.22	5.15	5.042	2.932	0.703
73	21	3.82	2.41	5.77	4.8813	21	3.8152	2.4120	5.80	4.7690	21	3.79	2.48	2.52	5.60	4.746	2.768	0.478
74	7	2.25	1.68	3.93	17.1665	7	2.2671	1.6771	3.90	16.6160	7	2.16	1.75	1.56	4.01	16.609	3.248	0.042
75	10	2.03	1.91	4.56	15.0487	10	2.0727	1.9375	4.70	14.5030	10	1.97	1.97	1.87	4.57	14.472	3.832	0.214
76	15	1.90	2.15	5.20	13.8127	15	1.8990	2.1563	5.20	13.2740	15	1.86	2.22	2.22	5.15	13.251	4.067	0.173
77	21	1.79	2.39	5.75	12.8792	21	1.7847	2.3933	5.75	12.3492	21	1.74	2.45	2.53	5.61	12.343	4.165	0.050
78	3	3.40	2.06	3.53	1.5699	3	3.3936	2.0511	3.55	1.5690	3	3.37	2.14	1.95	3.42	1.527	2.707	2.651
79	4	3.00	2.29	4.06	1.3562	4	3.0059	2.2867	4.05	1.3550	4	2.96	2.36	2.13	3.94	1.325	2.337	2.251
80	6	2.84	2.52	4.67	1.0477	6	2.8517	2.5254	4.65	1.0450	6	2.83	2.59	2.39	4.56	1.026	2.033	1.780
81	3	1.64	2.02	3.47	4.1584	3	1.6448	2.0124	3.45	4.1500	3	1.60	2.12	1.92	3.40	4.080	1.881	1.682
82	4	1.40	2.27	4.03	3.0758	4	1.4116	2.2612	4.00	3.0680	4	1.38	2.35	2.13	3.93	3.014	2.022	1.773
83	6	1.33	2.50	4.64	2.9502	6	1.3318	2.4992	4.65	2.9400	6	1.31	2.57	2.39	4.56	2.909	1.413	1.071
84	3	3.26	2.25	3.80	1.6657	3	3.2515	2.2437	3.80	1.5690	3	3.23	2.33	2.15	3.60	1.514	9.089	3.486
85	5	3.25	2.50	4.51	1.4283	5	3.2506	2.4997	4.50	1.3280	5	3.22	2.57	2.37	4.37	1.298	9.102	2.236
86	6	2.79	2.66	4.86	1.2008	6	2.9859	2.7120	5.00	1.1000	6	2.78	2.74	2.55	4.69	1.076	10.401	2.191
87	3	1.59	2.22	3.76	4.8826	3	1.5763	2.2095	3.75	4.4210	3	1.55	2.32	2.12	3.58	4.326	11.408	2.158
88	5	1.54	2.48	4.48	3.7728	5	1.5236	2.4739	4.50	3.2940	5	1.48	2.58	2.35	4.36	3.241	14.085	1.597
89	6	1.32	2.65	4.84	3.3970	6	1.3120	2.6524	4.80	2.9160	6	1.30	2.71	2.53	4.68	2.871	**15.483**	1.543
90	10	3.47	2.34	4.76	5.2434	10	3.4761	2.3351	4.75	5.1920	10	3.36	2.38	2.16	4.80	5.230	0.259	-0.728
91	8	7.52	2.32	4.59	2.2593	8	7.5071	2.3229	4.60	2.1510	8	7.37	2.37	2.14	4.60	2.148	4.922	0.135
92	11	2.55	5.11	7.25	2.1304	11	7.2428	2.5600	5.10	2.0230	11	7.15	2.59	2.40	5.06	2.024	5.013	-0.030
93	6	3.92	2.08	4.04	6.2683	6	3.9039	2.0686	4.00	5.7670	6	3.72	2.14	1.88	4.12	5.800	7.477	-0.565
94	8	3.68	2.29	4.55	5.9134	8	3.6457	2.2840	4.55	5.4130	8	3.51	2.36	2.12	4.58	5.444	7.936	-0.575
95	11	3.55	2.52	5.07	5.8564	11	3.5145	2.5298	5.05	5.3590	11	3.40	2.59	2.39	5.05	5.392	7.923	-0.623
96	5	1.56	2.64	4.70	3.1579	5	1.5621	2.6399	4.70	3.1550	5	1.54	2.70	2.53	4.51	3.076	2.593	2.504

Note:

CC: Chung and Chen (1993)

KV: Kasarapu and Vommi (2011)

Contd...

S. No.	CC					KV					TLBO						% Reduction	
	n	*h*	k_1	K_2	$E(L)_2$	*n*	*h*	k_1	K_2	$E(L)_2$	*n*	*h*	k_1	k_2	K_2	$E(L)_2$	TLBO with CC	TLBO with KV
97	7	1.45	2.83	5.18	2.3313	7	1.4539	2.8396	5.15	2.3270	7	1.44	2.90	2.73	4.98	2.273	2.488	2.308
98	3	0.99	2.20	3.73	11.7212	3	0.9978	2.1921	3.70	11.7210	3	0.98	2.30	2.12	3.57	11.434	2.450	2.449
99	4	0.84	2.43	4.27	8.9925	4	0.8471	2.4304	4.25	8.9820	4	0.84	2.52	2.30	4.08	8.780	2.360	2.246
100	5	0.72	2.63	4.68	7.2060	5	0.7203	2.6276	4.65	7.1950	5	0.71	2.71	2.51	4.50	7.034	2.386	2.236
101	7	0.66	2.82	5.17	6.3645	7	0.6646	2.8250	5.15	6.3500	7	0.66	2.92	2.72	4.97	6.243	1.907	1.683
102	5	1.54	2.71	4.79	3.0331	5	1.6906	2.7338	5.00	2.9310	5	1.55	2.78	2.59	4.57	2.844	6.221	2.955
103	7	1.43	2.89	5.26	2.6275	7	1.4485	2.9001	5.20	2.5250	7	1.43	2.97	2.79	5.02	2.466	6.162	2.352
104	4	0.84	2.50	4.36	11.0358	4	0.8414	2.5011	4.35	10.5580	4	0.83	2.59	2.39	4.16	10.338	6.323	2.084
105	5	0.72	2.69	4.77	9.9209	5	0.7188	2.6893	4.75	9.4390	5	0.72	2.77	2.59	4.56	9.266	6.602	1.834
106	7	0.66	2.88	5.25	7.0397	7	0.6622	2.8874	5.20	6.5450	7	0.66	2.96	2.78	5.02	6.425	8.735	1.837
107	7	3.96	2.33	4.50	5.0818	7	3.9637	2.3312	4.50	5.0640	7	3.91	2.38	2.14	4.49	5.029	1.041	0.693
108	9	3.71	2.51	4.92	4.3916	9	3.7073	2.5124	4.95	4.3760	9	3.67	2.57	2.36	4.88	4.351	0.924	0.571
109	12	3.56	2.71	5.38	4.1063	12	3.5687	2.7342	5.35	4.0920	12	3.53	2.77	2.60	5.28	4.079	0.675	0.327
110	7	1.85	2.30	4.47	12.9502	7	1.8533	2.2923	4.50	12.8790	7	1.80	2.36	2.13	4.48	12.853	0.751	0.202
111	9	1.73	2.49	4.89	12.1054	9	1.7352	2.4911	4.90	12.0410	9	1.70	2.55	2.33	4.85	12.029	0.631	0.100
112	12	1.65	2.70	5.35	10.8632	12	1.6557	2.7068	5.35	10.8050	12	1.62	2.77	2.54	5.23	10.809	0.499	-0.037
113	7	3.94	2.39	4.59	5.3874	7	3.9438	2.3889	4.60	5.2730	7	3.90	2.44	2.21	4.55	5.229	2.938	0.833
114	10	3.77	2.62	5.14	4.6830	10	3.7734	2.6291	5.15	4.5710	10	3.74	2.68	2.49	5.06	4.547	2.900	0.521
115	12	3.56	2.77	5.44	4.5323	12	3.5641	2.7749	5.45	4.4200	12	3.54	2.83	2.66	5.33	4.403	2.851	0.382
116	5	1.99	2.13	3.99	15.4402	5	1.9874	2.1226	4.00	14.9110	5	1.91	2.21	1.96	4.02	14.833	3.933	0.523
117	7	1.85	2.36	4.56	13.6345	7	1.8521	2.3648	4.55	13.1000	7	1.81	2.43	2.20	4.54	13.057	4.236	0.328
118	9	1.73	2.55	4.97	12.7639	9	1.7348	2.5553	4.95	12.2340	9	1.70	2.62	2.39	4.90	12.209	4.347	0.204
119	12	1.67	2.75	5.42	12.7929	12	1.6647	2.7485	5.45	12.2700	12	1.64	2.78	2.64	5.31	12.267	4.111	0.024
120	3	3.48	2.16	3.87	1.4699	3	3.4822	2.1594	3.85	1.4720	3	3.44	2.25	2.19	3.64	1.433	2.490	2.629
121	4	3.08	2.40	4.41	1.1217	4	3.0800	2.3930	4.40	1.1200	4	3.07	2.46	2.43	4.20	1.102	1.721	1.571
122	5	2.73	2.59	4.82	0.9735	5	2.7355	2.5876	4.80	0.9720	5	2.73	2.65	2.60	4.58	0.959	1.472	1.320
123	3	1.65	2.13	3.83	3.7483	3	1.6541	2.1253	3.80	3.7510	3	1.63	2.22	2.16	3.61	3.679	1.857	1.927
124	4	1.44	2.38	4.38	2.8859	4	1.4410	2.3707	4.35	2.8770	4	1.42	2.45	2.39	4.16	2.848	1.317	1.011
125	5	1.27	2.57	4.80	2.5722	5	1.2686	2.5659	4.80	2.5650	5	1.26	2.65	2.60	4.57	2.543	1.124	0.846
126	3	3.38	2.34	4.12	1.5341	3	3.3633	2.3314	4.15	1.4360	3	3.39	2.41	2.37	3.80	1.403	8.546	2.298
127	4	3.03	2.55	4.62	1.3224	4	3.0261	2.5483	4.60	1.2240	4	3.02	2.62	2.57	4.32	1.202	9.142	1.838
128	5	2.70	2.73	5.01	1.2140	5	2.6997	2.7276	5.00	1.1150	5	2.70	2.79	2.76	4.71	1.099	9.481	1.444

Note:

CC: Chung and Chen (1993)

KV: Kasarapu and Vommi (2011)

Contd...

S. No.	CC					KV					TLBO						% Reduction	
	n	*h*	*k_1*	*K_2*	*$E(L)_2$*	*n*	*h*	*k_1*	*K_2*	*$E(L)_2$*	*n*	*h*	*k_1*	*k_2*	*K_2*	*$E(L)_2$*	*TLBO with CC*	*TLBO with KV*
129	3	1.63	2.31	4.08	4.6250	3	1.6338	2.3054	4.05	4.1580	3	1.60	2.40	2.35	3.78	4.103	11.291	1.328
130	4	1.44	2.53	4.59	3.9854	4	1.4287	2.5241	4.60	3.5120	4	1.42	2.60	2.57	4.32	3.474	12.822	1.071
131	5	1.28	2.71	4.99	3.6644	5	1.2708	2.7127	4.95	3.1880	5	1.26	2.80	2.74	4.69	3.161	13.738	0.847
132	8	3.40	2.40	4.90	5.1444	8	3.4021	2.3901	4.90	5.0970	8	3.30	2.47	2.41	4.83	5.132	0.235	-0.693
133	7	7.42	2.43	4.84	2.2725	7	7.4081	2.4278	4.85	2.1660	7	7.31	2.50	2.43	4.73	2.166	4.695	0.009
134	9	7.12	2.65	5.29	2.0284	9	7.1077	2.6478	5.30	1.9220	9	7.02	2.72	2.67	5.13	1.925	5.107	-0.146
135	5	3.78	2.13	4.20	6.0691	5	3.7478	2.1139	4.20	5.5710	5	3.61	2.21	2.13	4.16	5.599	7.744	-0.504
136	7	3.57	2.40	4.81	5.3637	7	3.5394	2.3942	4.80	4.8600	7	3.42	2.48	2.42	4.72	4.893	8.770	-0.685
137	9	2.62	5.26	3.43	5.1463	9	3.3910	2.6201	5.25	4.6440	9	3.30	2.70	2.67	5.13	4.679	9.078	-0.756
138	4	1.46	2.69	4.81	2.6876	4	1.4608	2.6891	4.80	2.6890	4	1.47	2.75	2.72	4.45	2.578	4.082	4.132
139	6	1.41	2.92	5.38	2.2963	6	1.4170	2.9204	5.35	2.2940	6	1.42	2.98	2.96	5.04	2.263	1.446	1.347
140	3	0.75	2.48	4.32	9.3800	3	0.7456	2.4684	4.35	9.4000	3	0.75	2.54	2.53	3.94	8.868	5.458	**5.660**
141	4	0.67	2.67	4.79	8.1658	4	0.6738	2.6704	4.80	8.1750	4	0.68	2.74	2.71	4.44	7.952	2.621	2.730
142	6	0.65	2.90	5.36	8.0331	6	0.6538	2.9040	5.35	8.0240	6	0.65	2.97	2.95	5.04	7.965	0.844	0.732
143	5	1.62	2.81	5.12	3.0774	5	1.6191	2.8042	5.15	2.9770	5	1.62	2.88	2.86	4.79	2.932	4.731	1.518
144	6	1.41	2.97	5.45	2.2925	6	1.4061	2.9752	5.45	2.1920	6	1.39	3.03	3.03	5.11	2.158	5.863	1.547
145	3	0.74	2.55	4.42	9.0710	3	0.7403	2.5489	4.35	8.5920	3	0.74	2.62	2.59	3.99	8.415	7.229	2.057
146	4	0.66	2.74	4.88	7.3852	4	0.7403	2.7908	5.15	6.8980	4	0.67	2.81	2.78	4.50	6.778	8.218	1.735
147	6	0.64	2.96	5.44	6.6344	6	0.6426	2.9638	5.45	6.1430	6	0.63	3.05	3.00	5.08	6.077	8.408	1.081
148	6	3.87	2.41	4.71	5.0919	6	3.8698	2.4122	4.70	5.0030	6	3.83	2.53	2.46	4.62	4.995	1.895	0.152
149	7	3.60	2.55	5.01	4.6586	7	3.6066	2.5572	5.00	4.6280	7	3.57	2.68	2.63	4.90	4.653	0.114	-0.547
150	9	3.48	2.76	5.44	4.1543	9	3.4763	2.7622	5.45	4.1130	9	3.45	2.87	2.88	5.30	4.141	0.327	-0.673
151	6	1.80	2.39	4.68	12.6804	6	1.8086	2.3864	4.65	12.3990	6	1.78	2.49	2.48	4.63	12.571	0.863	-1.387
152	7	1.67	2.54	4.98	11.2577	7	1.6718	2.5291	5.00	11.0350	7	1.64	2.69	2.62	4.88	11.201	0.504	-1.504
153	9	1.61	2.74	5.41	10.7751	9	1.6106	2.7373	5.45	10.5490	9	1.58	2.88	2.84	5.27	10.718	0.530	-1.602
154	6	3.85	2.47	4.79	5.1348	6	3.8550	2.4703	4.80	5.0240	6	3.83	2.53	2.46	4.62	4.995	2.715	0.569
155	8	3.67	2.70	5.28	4.4982	8	3.6661	2.6981	5.30	4.3880	8	3.64	2.77	2.73	5.08	4.373	2.774	0.333
156	10	3.51	2.89	5.67	4.2590	10	3.5133	2.8990	5.65	4.1500	10	3.49	2.96	2.94	5.42	4.143	2.717	0.161
157	4	1.88	2.17	4.09	14.7881	4	1.8844	2.1604	4.05	14.2700	4	1.84	2.24	2.20	3.99	14.198	3.990	0.505
158	6	1.81	2.45	4.77	13.1163	6	1.8037	2.4474	4.75	12.5910	6	1.78	2.49	2.48	4.63	12.571	4.157	0.159
159	8	1.70	2.68	5.26	11.0839	8	1.6966	2.6744	5.30	10.5560	8	1.67	2.72	2.68	5.04	10.559	4.736	-0.028
160	10	1.63	2.88	5.65	10.5750	10	1.6227	2.8784	5.65	10.0510	10	1.60	2.94	2.94	5.42	10.066	4.813	-0.149

Note:

CC: Chung and Chen (1993)

KV: Kasarapu and Vommi (2011)

Table 5.27: Recalculation of expected loss cost per unit time and percentage reduction with TLBO

S. No.	Cost and process parameters													Kasarapu and Vommi (2011)					TLBO						% Reduction Recalculated
	δ	λ	V_0	V_1	M	W	Y	T_1	T_0	a	b	σ_0	σ_1	n	h	k_1	K_2	$E(L)_2$	n	h	k_1	$k2$	K_2	$E(L)_2$	
24	0.5	0.05	150	50	100	2	1	0.4	0.1	5	0.1	0.02	0.03	26	1.8889	2.1786	5.03	12.6950	26	1.81	2.19	1.65	5.13	12.676	0.150
27	0.5	0.05	150	50	100	15	5	0.4	0.1	5	0.2	0.02	0.03	18	2.0437	2.0815	4.60	14.6007	18	1.92	2.09	1.50	4.75	14.559	0.286
47	1.0	0.05	50	25	25	5	5	0.6	0.1	5	0.2	0.02	0.03	12	3.9831	1.8372	4.60	6.0603	12	3.81	1.92	1.81	4.67	6.054	0.099
48	1.0	0.05	50	25	25	5	5	0.6	0.1	5	0.1	0.02	0.03	17	3.7267	2.0771	5.20	5.6907	17	3.57	2.16	2.14	5.18	5.687	0.062
67	1.0	0.05	150	50	100	2	1	0.6	0.1	5	0.8	0.02	0.03	6	2.1967	1.5357	3.60	16.2066	6	2.10	1.61	1.39	3.71	16.188	0.115
90	1.5	0.05	50	25	25	2	1	0.6	0.1	5	0.1	0.02	0.04	10	3.4761	2.3351	4.75	5.2319	10	3.36	2.38	2.16	4.80	5.230	0.040
92	1.5	0.01	50	25	25	5	5	0.4	0.1	5	0.1	0.02	0.04	11	7.2428	2.5600	5.10	2.0239	11	7.15	2.59	2.40	5.06	2.024	0.015
93	1.5	0.05	50	25	25	5	5	0.5	0.1	5	0.4	0.02	0.04	6	3.9039	2.0686	4.00	5.8068	6	3.72	2.14	1.88	4.12	5.800	0.124
94	1.5	0.05	50	25	25	5	5	0.5	0.1	5	0.2	0.02	0.04	8	3.6457	2.2840	4.55	5.4471	8	3.51	2.36	2.12	4.58	5.444	0.055
95	1.5	0.05	50	25	25	5	5	0.6	0.1	5	0.1	0.02	0.04	11	3.5145	2.5298	5.05	5.3945	11	3.40	2.59	2.39	5.05	5.392	0.039
112	1.5	0.05	150	50	100	2	1	0.4	0.1	5	0.1	0.02	0.04	12	1.6557	2.7068	5.35	10.8123	12	1.62	2.77	2.54	5.23	10.809	0.031
132	2.0	0.05	50	25	25	2	1	0.6	0.1	5	0.1	0.02	0.04	8	3.4021	2.3901	4.90	5.1343	8	3.30	2.47	2.41	4.83	5.132	0.039
134	2.0	0.01	50	25	25	5	5	0.3	0.1	5	0.1	0.02	0.04	9	7.1077	2.6478	5.30	1.9258	9	7.02	2.72	2.67	5.13	1.925	0.052
135	2.0	0.05	50	25	25	5	5	0.5	0.1	5	0.4	0.02	0.04	5	3.7478	2.1139	4.20	5.6029	5	3.61	2.21	2.13	4.16	5.599	0.068
136	2.0	0.05	50	25	25	5	5	0.3	0.1	5	0.2	0.02	0.04	7	3.5394	2.3942	4.80	4.8962	7	3.42	2.48	2.42	4.72	4.893	0.059
137	2.0	0.05	50	25	25	5	5	0.3	0.1	5	0.1	0.02	0.04	9	3.3910	2.6201	5.25	4.6818	9	3.30	2.70	2.67	5.13	4.679	0.058
149	2.0	0.01	150	50	100	5	5	0.6	0.1	5	0.2	0.02	0.04	7	3.6066	2.5572	5.00	4.6568	7	3.57	2.68	2.63	4.90	4.653	0.075
150	2.0	0.01	150	50	100	5	5	0.4	0.1	5	0.1	0.02	0.04	9	3.4763	2.7622	5.45	4.1440	9	3.45	2.87	2.88	5.30	4.141	0.080
151	2.0	0.05	150	50	100	5	5	0.5	0.1	5	0.4	0.02	0.04	6	1.8086	2.3864	4.65	12.5808	6	1.78	2.49	2.48	4.63	12.571	0.078
152	2.0	0.05	150	50	100	5	5	0.4	0.1	5	0.2	0.02	0.04	7	1.6718	2.5291	5.00	11.2109	7	1.64	2.69	2.62	4.88	11.201	0.088
153	2.0	0.05	150	50	100	5	5	0.4	0.1	5	0.1	0.02	0.04	9	1.6106	2.7373	5.45	10.7261	9	1.58	2.88	2.84	5.27	10.718	0.076
159	2.0	0.05	150	50	100	5	5	0.3	0.1	5	0.2	0.02	0.04	8	1.6966	2.6744	5.30	10.5690	8	1.67	2.72	2.68	5.04	10.559	0.095
160	2.0	0.05	150	50	100	5	5	0.3	0.1	5	0.1	0.02	0.04	10	1.6227	2.8784	5.65	10.0732	10	1.60	2.94	2.94	5.42	10.066	0.071

5.9 Conclusions

In this chapter two new design methodologies have been developed based on metaheuristics viz., SA and TLBO for joint economic design of $\bar{X}$ and R charts for both continuous and discontinuous processes. The use of both the methodologies has been illustrated through numerical examples. Both are observed to have yielded nearly the same results. Therefore, either SA or TLBO can be recommended for joint economic design of $\bar{X}$ and R charts. Both the methodologies have also been found to be superior compared to that of other researchers. The optimal value of expected loss cost per unit time in joint economic design of $\bar{X}$ and R charts is found to be less than that of its economic design of $\bar{X}$ chart in both types of processes. The time to sample and chart one item g, the fixed cost per sample a and the rate of occurrences of assignable cause λ are found to be the most significant factors affecting sample size n, sampling interval h and expected loss cost per unit time $E(L)$ respectively in joint economic design of $\bar{X}$ and R charts for both types of processes. The cost per false alarm Y is the most significant for the widths of control limits both the charts i.e., k_1 and k_2 in a continuous process. On the other hand in case of discontinuous process, the most significant factors are the expected search time for a false alarm T_0 and shift in process mean δ for the widths of control limits k_1 and k_2.

CHAPTER - 6

Joint Economic Statistical Design of $\bar{X}$ and *R* Charts

6.1 Introduction

The previous chapter was related to joint economic design of $\bar{X}$ and *R* charts, whereas this chapter deals with joint economic statistical design of those two charts. In this type of design, the values of design variables of these two charts are so selected that the expected loss cost per unit time is minimized and at the same time some statistical constraints are satisfied. Due to these constraints, economic statistical design is usually costlier than economic design. But considering the better statistical performance of the control charts in economic statistical design, its additional expense is justified (Saniga, 1989). The use of constraints helps in overcoming the drawback of control charts like frequent false alarms and low power of detecting the process shift. In this chapter, two design methodologies based on simulated annealing (SA) and teaching-learning based optimization (TLBO) have been developed for joint economic statistical design of both $\bar{X}$ and *R* charts in continuous as well as discontinuous process and illustrated through numerical examples. These two techniques were also used in previous chapters. This chapter also includes sensitivity analysis for identifying significant factors affecting the design results.

6.2 Joint Economic Statistical Design Model

The same three constraints which were used earlier in economic statistical design of $\bar{X}$ chart in Chapter 4 have been considered in this chapter for the joint economic statistical design of $\bar{X}$ and R charts. These three constraints are lower bound on in-control average run length (ARL_L), upper bound on out-of-control average run length (ARL_U) and upper bound on out-of-control average time to signal (ATS_U).

Thus, the joint economic statistical design can be modelled as

$$\text{Minimize } E(L) \tag{6.1}$$

subject to

$$ARL_0 \geq ARL_L$$

$$ARL_1 \leq ARL_U$$

$$ATS_1 \leq ATS_U$$

where

$E(L)$ is the expected loss cost per unit time which can be either $E(L)_1$ for a continuous process or $E(L)_2$ for a discontinuous process as shown in Eqs. 3.20 and 3.31 respectively,

ARL_0 is the average run length when the process is in-control,

ARL_1 is the average run length when the process is out-of-control, and

ATS_1 is the average time to signal when the process is out-of-control.

6.3 Numerical Illustration: Continuous Process

The joint economic statistical design of $\bar{X}$ and R charts is illustrated in this section through the same numerical example that was earlier considered in the joint economic design of $\bar{X}$ and R charts in Section 5.4. This problem is related to a continuous process i.e., the process which continues to operate even if a true or false alarm is obtained in a control chart. In addition to the numerical data provided in Section 5.4, limiting values of three constraints are taken as ARL_L = 267, ARL_U = 40 and ATS_U = 1.90 (van Deventer and Manna, 2009). Thus, ARL_0 value should be at least 267 when the process is in-control, whereas ARL_1 and ATS_1 should not exceed 40 and 1.90 respectively when the process is out-of-control.

Thus, the joint economic statistical design of continuous process can be modelled as:

$$\text{Minimize } E(L)_1 \tag{6.2}$$

subject to

$$ARL_0 \geq 267$$
$$ARL_1 \leq 40$$
$$ATS_1 \leq 1.90$$

As discussed in Section 5.4 the expected loss cost per unit time $E(L)_1$ is a function of four design variables such as the sample size n, the sampling frequency h, and two control limit width parameters k_1 and k_2. In economic statistical design, the objective is to minimize the objective function $E(L)_1$ for its optimal solution while satisfying all the constraints. All these four design variables are taken as real values on continuous scale except the sample size n which is taken as integer. Thus, it is an example of multi-variable constrained minimization problem including a non-linear and non-differentiable objective function. The lower and upper boundary limits of all these four design variables are already shown in Table 5.1 for minimizing $E(L)_1$. The solution of this optimization problem is obtained with the help of both SA and TLBO and their results are discussed below.

6.3.1 Results and Discussion

Table 6.1 shows the results of joint economic statistical design of $\bar{X}$ and R charts for a continuous process using two metaheuristics viz., SA and TLBO. In this table, the optimal values of three design variables such as sampling interval (h) and two widths of control limits (i.e., k_1 and k_2) are shown for each integer value of sample size n varying from 2 to 33. The corresponding minimum values of expected loss cost per unit time $E(L)_1$ obtained from both the metaheuristics are also shown in this table.

Table 6.1: Optimal joint economic statistical designs of $\bar{X}$ and R charts: continuous process

SA					TLBO				
n	***h***	***k_1***	***k_2***	***$E(L)_1$***	***n***	***h***	***k_1***	***k_2***	***$E(L)_1$***
2	0.36	3.00	3.08	36.828	2	0.36	3.01	3.07	36.845
3	**0.50**	**3.03**	**3.02**	**36.756**	**3**	**0.50**	**3.02**	**3.04**	**36.757**
4	0.62	3.03	3.03	36.896	4	0.62	3.03	3.03	36.881
5	0.74	3.04	3.01	37.012	5	0.74	3.03	3.03	37.015
6	0.85	3.02	3.04	37.162	6	0.85	3.03	3.03	37.171
7	0.96	3.01	3.06	37.331	7	0.96	3.02	3.04	37.340
8	1.05	3.03	3.02	37.599	8	1.05	3.03	3.03	37.609
9	1.14	3.02	3.04	37.848	9	1.14	3.02	3.05	37.858
10	1.22	3.03	3.03	38.152	10	1.22	3.02	3.04	38.137
11	1.30	3.02	3.04	38.432	11	1.30	3.02	3.05	38.440
12	1.36	3.01	3.06	38.801	12	1.36	3.02	3.05	38.806
13	1.42	3.02	3.05	39.173	13	1.42	3.01	3.06	39.174
14	1.48	3.01	3.06	39.536	14	1.48	3.01	3.07	39.541
15	1.53	3.01	3.07	39.945	15	1.53	3.01	3.06	39.945
16	1.57	3.01	3.07	40.394	16	1.57	3.01	3.06	40.394
17	1.61	3.01	3.06	40.842	17	1.61	3.00	3.08	40.839
18	1.64	3.01	3.07	41.336	18	1.64	3.01	3.07	41.336
19	1.67	3.01	3.07	41.825	19	1.67	3.00	3.09	41.826
20	1.70	3.02	3.04	42.312	20	1.70	3.00	3.09	42.310
21	1.73	2.99	3.13	42.796	21	1.73	3.00	3.08	42.794
22	1.75	3.00	3.10	43.331	22	1.75	3.00	3.08	43.330
23	1.77	3.02	3.05	43.864	23	1.77	3.00	3.10	43.864
24	1.78	2.99	3.11	44.454	24	1.78	3.01	3.07	44.455
25	1.80	3.00	3.10	44.982	25	1.80	3.00	3.09	44.981
26	1.81	3.00	3.10	45.570	26	1.81	3.00	3.08	45.570
27	1.82	3.00	3.11	46.159	27	1.82	2.99	3.14	46.159
28	1.83	3.00	3.10	46.744	28	1.83	2.98	3.15	46.744
29	1.84	3.02	3.06	47.329	29	1.84	3.01	3.08	47.328
30	1.85	3.04	3.01	47.911	30	1.85	3.00	3.09	47.910
31	1.86	3.02	3.05	48.490	31	1.86	2.99	3.14	48.489
32	1.86	3.02	3.04	49.145	32	1.86	2.97	3.20	49.145
33	1.86	2.99	3.42	49.803	33	1.86	2.99	3.39	49.802

This table shows that the results of joint economic statistical design obtained using SA and TLBO methods are observed to be nearly same for almost all the values of sample size *n*. The optimum value of expected loss cost per unit time $E(L)_1$ initially decreases as *n* value increases from 2 to 3 and thereafter it increases at higher values of *n*. The variation of expected loss cost per unit time $E(L)_1$ with respect to sample size *n* in case of SA and TLBO are graphically shown in Figs. 6.1 and 6.2 respectively. For the sake of displaying the optimal point with better clarity, both the graphs are drawn over a limited range of sample size i.e., *n* = 2 to 20. As no other optimal point occurs or no change is observed in pattern of variation in the range *n* = 21 to 33, the points in this range are not felt to have any worth to be included in these two graphs. Out of all 32 sets of results, the lowest value of expected loss cost per unit time $E(L)_1$ is observed to occur at n = 3 in case of both SA and TLBO as shown in Figs. 6.1 and 6.2.

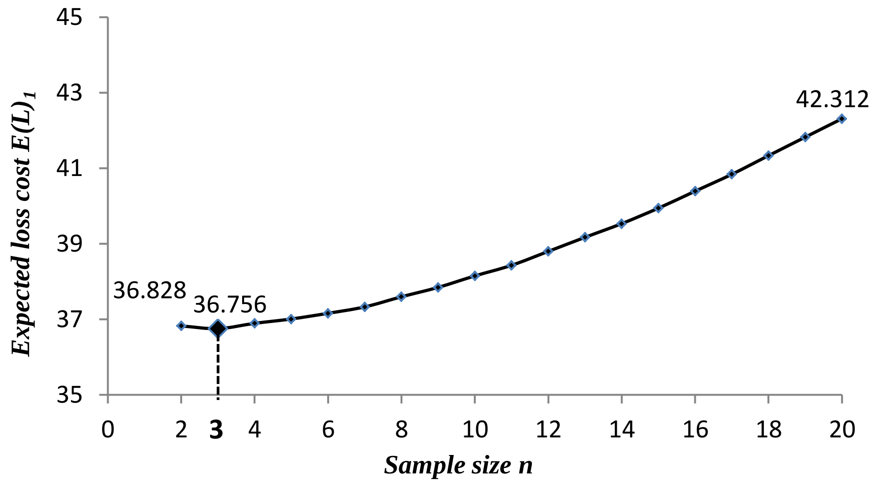

Fig. 6.1: Variation of expected loss cost per unit time with sample size using SA: continuous process

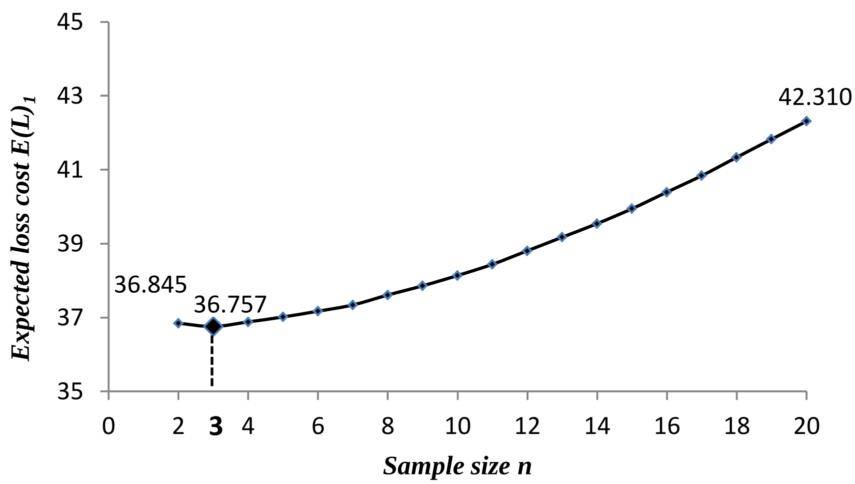

Fig. 6.2: Variation of expected loss cost per unit time with sample size using TLBO: continuous process

Table 6.2 shows a comparison of optimal results of joint economic statistical design of $\bar{X}$ and R charts with that of joint economic design of same two charts obtained using SA for a continuous process. Similarly, Table 6.3 shows the comparison of results of above two types of designs obtained using TLBO for the same continuous process. The results of joint economic design of $\bar{X}$ and R charts obtained from SA and TLBO for the continuous process are earlier shown in Table 5.3 and 5.4 respectively. The corresponding values of two additional parameters ATS_0 and ATS_1 are calculated using Eqs. 4.1 and 4.2 and are also listed in Tables 6.2 and 6.3 respectively.

Table 6.2: Comparison of results of joint economic design with their joint economic statistical design of $\overline{X}$ and R charts using SA: continuous process

Design	*n*	*h*	k_1	k_2	α	*P*	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_1$
JED-C	3	1.16	2.20	2.09	0.0456	0.520	21.930	1.922	25.439	2.23	34.050
JESD-C	3	0.50	3.03	3.02	0.0037	0.262	270.270	3.817	135.135	1.90	36.756

Note:
JED-C : Joint Economic Design - Continuous process
JESD-C : Joint Economic Statistical Design - Continuous process

Table 6.3: Comparison of results of joint economic design with their joint economic statistical design of $\overline{X}$ and R charts using TLBO: continuous process

Design	*n*	*h*	k_1	k_2	α	*P*	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_1$
JED-C	3	1.16	2.19	2.12	0.0453	0.520	22.075	1.924	25.607	2.23	34.050
JESD-C	3	0.50	3.02	3.04	0.0037	0.262	270.270	3.817	135.135	1.90	36.757

Note:
JED-C : Joint Economic Design - Continuous process
JESD-C : Joint Economic Statistical Design - Continuous process

Tables 6.2 and 6.3 reveal that the sample size n is same in joint economic design (*JED*) as well as joint economic statistical design (*JESD*) of $\overline{X}$ and R charts (i.e., $n = 3$) in continuous process. The value of expected loss cost per unit time $E(L)_1$ is found to be higher in case of *JESD*. The increase in expected loss cost per unit time in *JESD* compared to that in *JED* using SA is:

$$\frac{(36.756 - 34.050)}{34.050} \times 100 = 7.94\%$$

Similarly, the increase in expected loss cost per unit time in *JESD* compared to that in *JED* using TLBO is 7.95%. Even if the cost is higher, it provides the benefit of more satisfactory values of statistical properties like higher value of ARL_0 and lower value of ATS_1.

In case of joint economic design, the probability that a point falls outside the control limits for in-control process is obtained as α = 0.0456 and 0.0453 using SA and TLBO techniques respectively as shown in Tables 6.2 and 6.3. The corresponding values of ARL_0 are 21.930 and 22.075 (i.e., around 22) as obtained from SA and TLBO techniques respectively. So, when the process remains in-control, an out-of-control signal will be generated on an average after every 22 samples. This means that the false alarm is generated more frequently leading to unnecessary over-adjustment of the process and thereby loss of confidence of quality control personnel. Therefore, it is required to keep the value of ARL_0 sufficiently larger so that false alarms are avoided as far as possible. In case of *JESD*, the value of ARL_0 is found to be 270.270 (≈ 270) which is comparatively much higher and hence more beneficial.

Both these tables also compare the effect of average time to signal during out-of-control process (ATS_1) between those two types of joint design of $\bar{X}$ and R charts for continuous process. ATS_1 for the *JESD* (i.e., 1.90) is much less than that for the *JED* (i.e., 2.23) using both the metaheuristics. That means it is able to detect the same magnitude of process shift much earlier when the process is out-of-control. It is further observed that JESD has smaller sampling interval compared to that of *JED* (i.e., h = 0.50 hour < 1.16 hour). This means that samples are taken more frequently in *JESD* compared to *JED*. This increases the sampling cost and thereby the expected loss cost per unit time. However, the incorporation of ATS_1 constraint in *JESD* helps in reduction of ATS_1 compared to that in *JED* by an amount equal to:

$$\frac{(2.23-1.90)}{2.23}\times 100 = 14.80\%.$$

This percentage reduction is found to be same in the results of both the metaheuristics.

Thus, joint economic statistical design is observed to be costlier than joint economic design due to the addition of constraints. However, it assures a more satisfactory statistical performance in terms of lower α and β errors.

6.4 Sensitivity Analysis: Continuous Process

To investigate the effect of cost and process parameters on the output results of joint economic statistical design of $\bar{X}$ and R charts in case of continuous process, sensitivity analysis has been done. Ten cost and process parameters are considered as factors for this analysis. The low and high values of nine of these factors are already listed in Table 3.7. The additional tenth factor i.e., the shift in standard deviation (Υ) has been incorporated with its low and high values of 1.5 and 2.0 respectively. A 2_{IV}^{10-5} factorial design for 10 factors with 5 generators I= ABCDF, I = ABCEG, I = ABDEH, I = ACDEJ and I = BCDEK, and resolution *IV* is chosen for a continuous process for the sensitivity analysis that gives a total of 32 (= 2^{10-5}) runs. In each run the limiting values of statistical constraints are taken same as that already considered for a continuous process in Section 4.4 i.e., $ARL_0 \geq 267$, $ARL_1 \leq 40$ and $ATS_1 \leq 1.90$. For each of 32 runs, a particular set of values of ten factors is taken for which the loss cost function $E(L)_1$ is minimized using TLBO algorithm subject to the above mentioned constraints and the optimal result consisting of the values of five responses viz. *n*, *h*, k_1, k_2 and $E(L)_1$ is shown in Table 6.4. Since both SA and TLBO algorithms provided almost the same results for joint economic statistical design for a continuous process as observed in Section 6.3.1, any one of them is sufficient for sensitivity analysis.

Table 6.4: Optimal joint economic statistical designs of $\bar{X}$ and *R* charts: continuous process

S. No.	Cost and process parameters (factors)										Responses				
	M	***δ***	***λ***	***g***	(T_1+T_2)	***a***	***b***	***W***	***Y***	Υ	***n***	***h***	k_1	k_2	$E(L)_1$
1	50	2	0.05	0.05	20	5.0	0.1	250	500	1.5	4	1.42	3.06	3.64	36.087
2	100	2	0.01	0.50	20	0.5	0.1	250	50	1.5	2	0.92	2.93	3.47	21.282
3	50	1	0.01	0.05	20	5.0	0.1	35	50	1.5	14	1.47	3.05	2.98	13.714
4	50	2	0.01	0.05	20	0.5	1.0	250	50	2.0	2	1.07	3.02	3.04	13.373
5	100	1	0.05	0.50	3	0.5	1.0	250	50	2.0	2	0.67	3.11	2.91	35.236
6	50	1	0.05	0.50	3	5.0	0.1	35	500	2.0	5	1.15	3.33	3.00	19.540
7	100	1	0.01	0.50	20	5.0	1.0	35	50	2.0	4	1.08	3.18	2.85	27.690
8	100	2	0.05	0.05	20	0.5	1.0	35	50	1.5	3	1.25	2.93	3.42	55.447
9	100	1	0.01	0.05	3	0.5	0.1	35	50	2.0	6	1.22	3.19	2.84	5.632
10	50	1	0.05	0.05	20	0.5	1.0	35	500	2.0	3	0.88	3.24	2.95	31.554
11	50	1	0.01	0.05	3	5.0	1.0	250	500	2.0	6	1.29	3.25	2.93	14.151
12	100	1	0.01	0.05	20	0.5	1.0	250	500	1.5	6	0.85	3.05	3.01	29.339
13	100	2	0.05	0.50	3	5.0	0.1	35	50	1.5	3	1.25	2.93	3.41	28.142
14	100	2	0.05	0.05	3	0.5	0.1	250	500	2.0	5	0.66	3.74	3.76	28.168
15	50	1	0.05	0.50	20	5.0	1.0	250	50	1.5	8	1.04	3.05	2.99	46.045
16	50	2	0.01	0.05	3	0.5	0.1	35	500	1.5	7	1.62	3.75	4.34	3.257
17	100	1	0.05	0.05	3	5.0	1.0	35	500	1.5	10	1.22	3.02	3.05	34.257
18	100	2	0.01	0.50	3	0.5	1.0	35	500	2.0	2	0.89	3.42	3.44	8.801
19	100	2	0.01	0.05	20	5.0	0.1	35	500	2.0	9	1.78	3.69	3.70	21.350
20	50	2	0.05	0.05	3	5.0	1.0	35	50	2.0	4	1.52	3.03	3.02	16.386
21	100	2	0.05	0.50	20	5.0	1.0	250	500	2.0	3	1.33	3.04	3.06	65.681
22	50	2	0.01	0.50	20	5.0	1.0	35	500	1.5	2	0.90	2.99	3.52	18.602
23	50	2	0.05	0.50	3	0.5	1.0	250	500	1.5	2	0.80	3.17	3.73	24.512
24	50	2	0.05	0.50	20	0.5	0.1	35	50	2.0	2	1.07	3.01	3.07	27.960
25	100	1	0.01	0.50	3	5.0	0.1	250	500	1.5	10	1.15	3.14	3.16	17.025
26	50	1	0.01	0.50	20	0.5	0.1	250	500	2.0	4	0.80	3.77	3.49	12.873
27	50	1	0.01	0.50	3	0.5	1.0	35	50	1.5	2	0.36	3.02	3.06	10.463
28	50	1	0.05	0.05	3	0.5	0.1	250	50	1.5	9	1.04	3.03	3.03	21.085
29	100	2	0.01	0.05	3	5.0	1.0	250	50	1.5	5	1.63	2.93	3.48	12.792
30	100	1	0.05	0.05	20	5.0	0.1	250	50	2.0	11	1.65	3.19	2.84	61.718
31	50	2	0.01	0.50	3	5.0	0.1	250	50	2.0	5	1.65	3.07	2.97	8.871
32	100	1	0.05	0.50	20	0.5	0.1	35	500	1.5	4	0.50	3.26	3.30	57.522

To find out the statistical significance of all the ten factors on each of the five output responses, analysis of variance (ANOVA) has been carried out on these 32 sets of results of joint economic statistical design of $\bar{X}$ and *R* charts for a continuous process. The results of ANOVA at 95% confidence level (i.e., significance level of 5%) on the joint economic statistical design results are shown in Tables 6.5 - 6.9. To identify the same in graphical manner, the normal plots of standardized effects for five output responses are shown in Figs.

6.3 - 6.7. These plots and ANOVA tables have been obtained with the help of student version of MINITAB 16.

Table 6.5: Analysis of variance for expected loss cost per unit time $E(L)_1$ with constraints: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	1182.34	1182.34	1182.34	28.11	0.000*	14.17
δ	1	77.09	77.09	77.09	1.83	0.190	0.92
λ	1	3857.82	3857.82	3857.82	91.72	0.000*	46.23
g	1	30.18	30.18	30.18	0.72	0.407	0.36
(T_1+T_2)	1	1937.81	1937.81	1937.81	46.07	0.000*	23.22
a	1	86.61	86.61	86.61	2.06	0.166	1.04
b	1	109.68	109.68	109.68	2.61	0.121	1.31
W	1	149.44	149.44	149.44	3.55	0.073	1.79
Y	1	6.44	6.44	6.44	0.15	0.700	0.08
Υ	1	23.95	23.95	23.95	0.57	0.459	0.29
Residual Error	21	883.30	883.30	42.06			
Total	31	8344.64					

* Significant at 5%

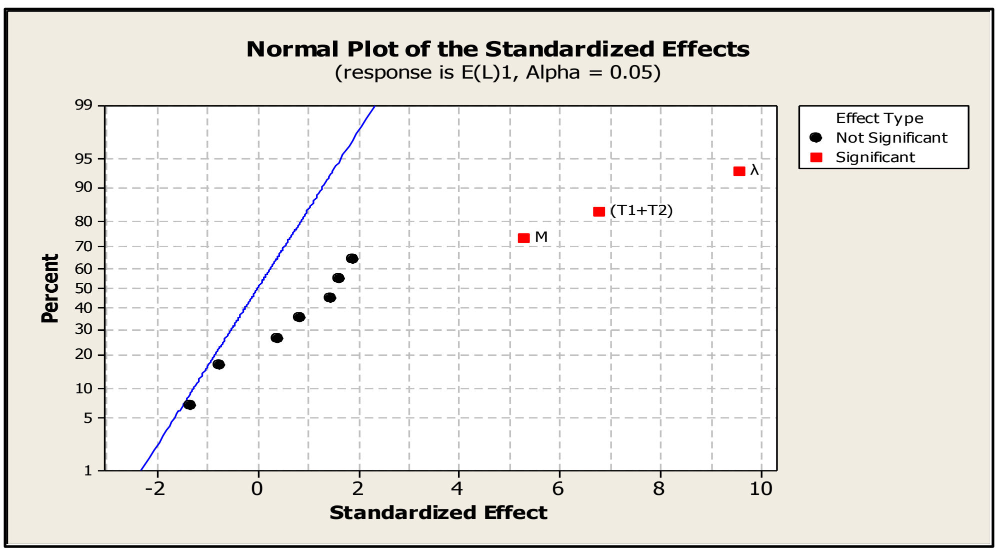

Fig. 6.3: Normal probability plot of standardized effect for expected loss cost per unit time $E(L)_1$ with constraints: continuous process

Table 6.5 indicates that the expected loss cost per unit time $E(L)_1$ in a continuous process is significantly affected by three factors, namely loss of net income when process is out-of-control M, rate of occurrences of assignable causes λ, and time to find and repair an assignable cause (T_1+T_2). All these three factors are significant as they all have p-value less than the predetermined significance level of 0.05. Among all the factors, λ has the highest significant effect on $E(L)_1$ since it has the highest F-value i.e., 91.72 as shown in Table 6.5

and is also graphically plotted at the rightmost location in the normal probability plot of standardized effect as shown in Fig. 6.3. It can also be observed from this table that λ, (T_1+T_2), and M are the top three percentage contributors which affect the cost by 46.23%, 23.22% and 14.17% respectively. All these three factors are observed to have positive effects as all of them are falling on the right side of the straight line in Fig. 6.3. This implies that as the value of any of these factors increases, the expected loss cost per unit time $E(L)_1$ increases.

Table 6.6 shows the results of ANOVA for the sample size *n*. There are five factors i.e., the size of the shift in the process mean δ, time to sample and chart one item *g*, fixed cost per sample *a*, variable cost per sample *b* and value of shift in standard deviation Υ which have significant effect on sample size *n*. Fig. 6.4 shows that out of these five significant factors, four factors have negative effect on sample size except fixed cost per sample *a*. An increase in δ, *g*, *b* or Υ decreases the optimum sample size, because they all have the negative effects. Moreover, the percentage contributions of these five significant factors *a*, *g*, δ, *b* and Υ affecting the sample size are 22.01%, 18.87%, 13.32%, 13.32% and 6.79% respectively. Thus, *a* is the most significant factor for choosing the value of sample size in joint economic statistical design and its effect is of positive type.

Table 6.6: Analysis of variance for sample size *n* with constraints: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	1.125	1.125	1.1250	0.24	0.628	0.27
δ	1	55.125	55.125	55.1250	11.87	0.002*	13.32
λ	1	4.500	4.500	4.5000	0.97	0.336	1.09
g	1	78.125	78.125	78.1250	16.83	0.001*	18.87
(T_1+T_2)	1	3.125	3.125	3.1250	0.67	0.421	0.75
a	1	91.125	91.125	91.1250	19.63	0.000*	22.01
b	1	55.125	55.125	55.1250	11.87	0.002*	13.32
W	1	0.000	0.000	0.0000	-	-	0.00
Y	1	0.125	0.125	0.1250	0.03	0.871	0.03
Υ	1	28.125	28.125	28.1250	6.06	0.023*	6.79
Residual Error	21	97.500	97.500	4.6429			
Total	31	414.000					

* Significant at 5%

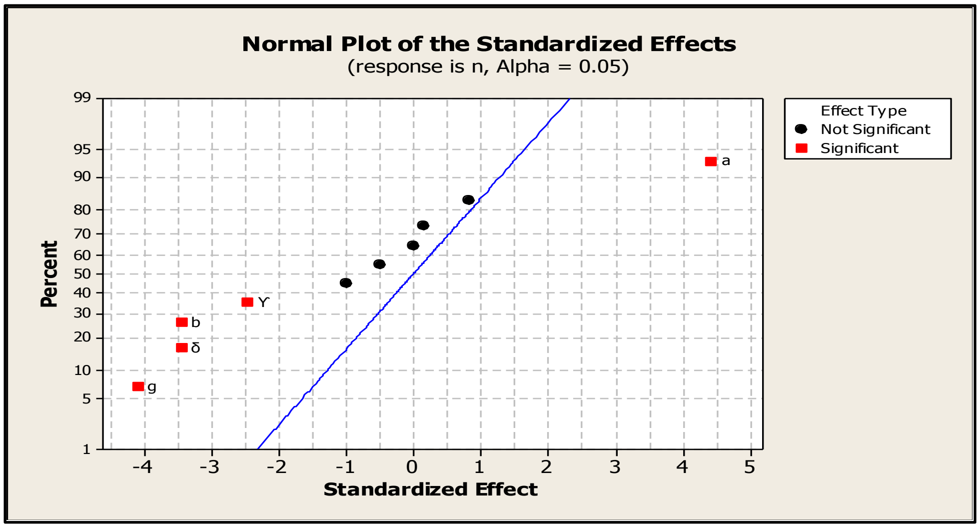

Fig. 6.4: Normal probability plot of standardized effect for sample size *n* with constraints: continuous process

Table 6.7 displays the results of analysis of variance on the sampling interval *h*. It is significantly affected by four factors, namely the size of the shift in the process mean δ, time to sample and chart one item *g*, fixed cost per sample *a* and variable cost of sampling *b*. Out of these four significant factors, two factors i.e., *g* and *b* have negative effects as shown in Fig. 6.5, whereas the factors δ and a are significant in terms of positive effect. Moreover, the positive effect parameters such as *a* and δ contribute 46.55% and 12.28% respectively, whereas the negative effect parameters like *g* and *b* contribute by 16.83% and 4.13% respectively. Thus, among all the factors, the fixed cost of sampling *a* has the highest effect on the sampling interval and the effect is in positive direction.

Table 6.7: Analysis of variance for sampling interval *h* with constraints: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	0.04698	0.04698	0.04698	1.42	0.246	1.04
δ	1	0.55199	0.55199	0.55199	16.74	0.001*	12.28
λ	1	0.09569	0.09569	0.09569	2.90	0.103	2.13
g	1	0.75678	0.75678	0.75678	22.95	0.000*	16.83
(T_1+T_2)	1	0.03838	0.03838	0.03838	1.16	0.293	0.85
a	1	2.09299	2.09299	2.09299	63.46	0.000*	46.55
b	1	0.18555	0.18555	0.18555	5.63	0.027*	4.13
W	1	0.01470	0.01470	0.01470	0.45	0.512	0.33
Y	1	0.02063	0.02063	0.02063	0.63	0.438	0.46
Υ	1	0.00010	0.00010	0.00010	0.00	0.958	0.00
Residual Error	21	0.69259	0.69259	0.03298			
Total	31	4.49637					

* Significant at 5%

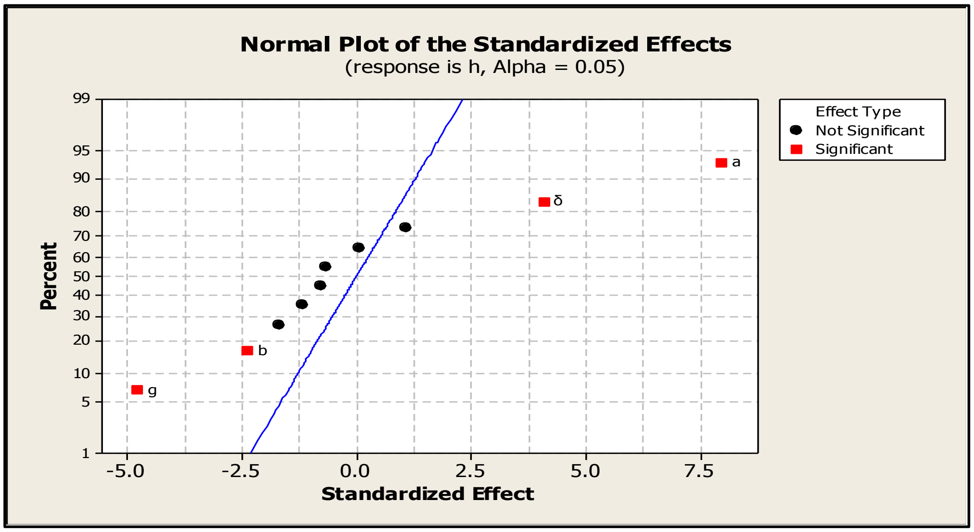

Fig. 6.5: Normal probability plot of standardized effect for sampling interval *h* with constraints: continuous process

Table 6.8 presents analysis of variance on the control limits width k_1 of $\bar{X}$ chart. There are three factors (i.e., *b*, *Y* and Υ) which are found to be significant on k_1. Fig. 6.6 reveals that out of these three significant factors, two factors (i.e., Υ and *Y*) have positive effect, whereas the factor *b* is having negative effect on k_1. Among all the factors, the cost per false alarm *Y* is observed to have the most significant effect with the maximum contribution of 36.91% on deciding the value of k_1 in joint economic statistical design and its effect is of positive type.

Table 6.8: Analysis of variance for width of control limits k_1 of $\bar{X}$ chart with constraints: continuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	0.00911	0.00911	0.009109	0.32	0.578	0.49
δ	1	0.00263	0.00263	0.002634	0.09	0.764	0.14
λ	1	0.03675	0.03675	0.036754	1.29	0.269	1.96
g	1	0.02750	0.02750	0.027501	0.97	0.337	1.47
(T_1+T_2)	1	0.00152	0.00152	0.001517	0.05	0.820	0.08
a	1	0.04669	0.04669	0.046688	1.64	0.214	2.49
b	1	0.25764	0.25764	0.257636	9.05	0.007*	13.75
W	1	0.00309	0.00309	0.003091	0.11	0.745	0.16
Y	1	0.69146	0.69146	0.691459	24.30	0.000*	36.91
Υ	1	0.19916	0.19916	0.199159	7.00	0.015*	10.63
Residual Error	21	0.59762	0.59762	0.028458			
Total	31	1.87317					

* Significant at 5%

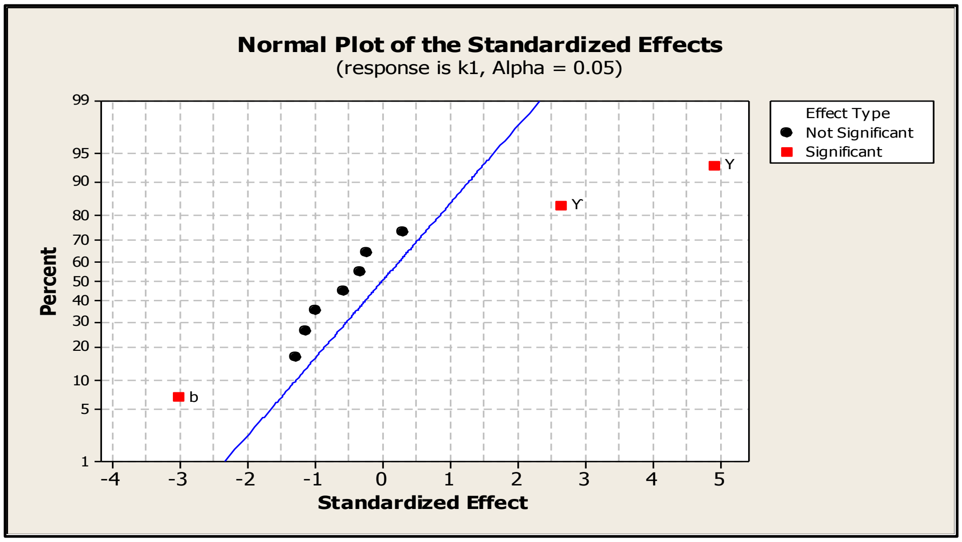

Fig. 6.6: Normal probability plot of standardized effect for width of control limits k_1 of $\bar{X}$ chart with constraints: continuous process

Similarly, Table 6.9 shows the ANOVA results on the control limits width k_2 of R chart. There are three factors (i.e., δ, Y and Υ) which are found to be significant on k_2. Fig. 6.7 reveals that out of these three significant factors, one factor (i.e., Υ) has negative effect on k_2 and two factors (i.e., Y and δ) have positive effect. Further, the percentage contributions of all these significant factors δ, Y and Υ are 30.18%, 18.42% and 14.28% respectively. The ANOVA result shows that for control limits width k_2 of R chart, the size of the shift in the process mean δ is found to be the most significant factor with positive effect.

Table 6.9: Analysis of variance for width of control limits k_2 of R chart with constraints: continuous process

Source	***DF***	***Seq SS***	***Adj SS***	***Adj MS***	***F***	***p***	***PC(%)***
M	1	0.23488	0.23488	0.23488	1.84	0.189	2.66
δ	1	2.66758	2.66758	2.66758	20.92	0.000*	30.18
λ	1	0.09771	0.09771	0.09771	0.77	0.391	1.11
g	1	0.00001	0.00001	0.00001	0.00	0.994	0.00
(T_1+T_2)	1	0.10817	0.10817	0.10817	0.85	0.368	1.22
a	1	0.00428	0.00428	0.00428	0.03	0.856	0.05
b	1	0.12121	0.12121	0.12121	0.95	0.341	1.37
W	1	0.03695	0.03695	0.03695	0.29	0.596	0.42
Y	1	1.62772	1.62772	1.62772	12.76	0.002*	18.42
Υ	1	1.26172	1.26172	1.26172	9.89	0.005*	14.28
Residual Error	21	2.67794	2.67794	0.12752			
Total	31	8.83817					

* Significant at 5%

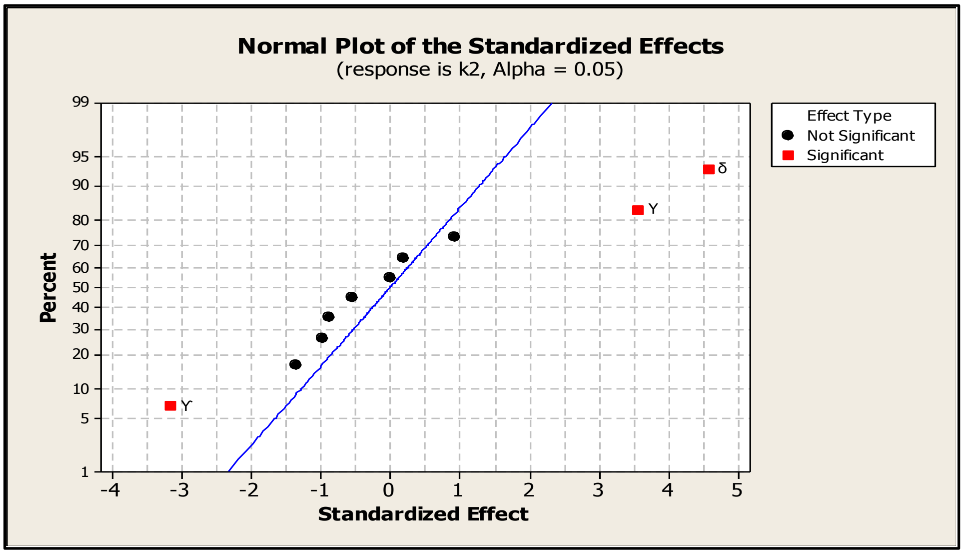

Fig. 6.7: Normal probability plot of standardized effect for width of control limits k_2 of R chart with constraints: continuous process

From the ANOVA study it is further observed from Tables 6.5 - 6.9 that the factor W i.e., the cost to locate and repair the assignable cause has no significant effect on any of the five responses n, h, k_1, k_2 and $E(L)_1$ in joint economic statistical design of $\bar{X}$ and R charts. This observation is consistent with that in the results of joint economic design.

6.4.1 Summary of Results

Similar to Table 5.11, all the significant factors in case of joint economic statistical design of $\bar{X}$ and R charts for a continuous process with respect to each of the five responses are summarized in Table 6.10. This table also shows the corresponding significant factors in case of joint economic design of $\bar{X}$ and R charts which are already shown in Table 5.11 for the ease of comparing both sets of results.

Table 6.10: Comparison of significant effects in joint economic design and joint economic statistical design: continuous process

Output responses	***Design***	***Cost and process parameters***									
		M	***δ***	***λ***	***g***	***(T_1+T_2)***	***a***	***b***	***W***	***Y***	***ϒ***
n	*JED-C*		−		**—**		+	−			
	JESD-C		−		−		**+**	−			−
h	*JED-C*	−		−	−	+	**+**	+			
	JESD-C		+		−		**+**	−			
k_1	*JED-C*		+	−	−		−	−		**+**	+
	JESD-C							−		**+**	+
k_2	*JED-C*		+	−	−		−	−		**+**	
	JESD-C		**+**							+	−
$E(L)_1$	*JED-C*	+		**+**		+					
	JESD-C	+		**+**		+					

Note:

JED-C	: Joint Economic Design - Continuous process
JESD-C	: Joint Economic Statistical Design - Continuous process
Blank space	: Insignificant factor
+	: Factor with positive effect
–	: Factor with negative effect
+/– in bold	: Most significant factor

From Table 6.10 it is observed that the time to sample and chart one item *g* which is having negative effect and fixed cost per sample *a* which is having positive effect are the most significant factors for selecting the value of sample size *n* in case of joint economic design (*JED*) and joint economic statistical design (*JESD*) respectively. All the factors which are significant in *JED* are also significant in *JESD* for continuous process except the factor *ϒ* which is significant only in *JESD* for continuous process and it has negative effect.

On the other hand in case of sampling interval *h*, the most significant factor is same in both types of designs (i.e., *JED* and *JESD*) in a continuous process and this factor is fixed cost of sampling *a*. This factor has positive type of effect in both the designs. In case of *JED*, six factors are found to be significant (i.e., *M*, λ, *g*, (T_1+T_2), *a* and *b*), whereas only four factors (i.e., δ, *g*, *a* and *b*) are found to be significant in case of *JESD*. Out of these significant factors, g, *a* and *b* are common to both types of designs. Factors *g* and *a* have same type of effects, whereas factor *b* has opposite type of effects in both the designs.

Like the sampling interval *h*, the most significant factor affecting the control limits width k_1 of $\bar{X}$ chart is found to be the same in both types of designs (i.e., *JED* and *JESD*) of continuous process and this factor is the cost per false alarm *Y* having positive type of effect. There seven and three significant factors in case of *JED* and *JESD* respectively. All the three significant factors (i.e., *b*, *Y* and Υ) in *JESD* are also significant in *JED*.

Like sample size *n*, the most significant factors for the width of control limits of *R* chart (k_2) are not same in both types of design. In case of k_2, the cost per false alarm *Y* and the shift in the process mean δ are the most significant factors in *JED* and *JESD* respectively for a continuous process. Both these factors have positive effect. Here, there are two significant factors (i.e., δ and *Y*) which are common to both the designs with same type of effect (i.e., positive). There are additional four factors (i.e., λ, *g*, *a* and *b*) for *JED* and one factor Υ in case of *JESD* which are significant and all of them have negative effects.

All the significant factors with respect to the expected loss cost per unit time $E(L)_1$ have positive effects whether the design is *JED* or *JESD* for a continuous process. The most significant factor for $E(L)_1$ is also same in both the cases and this factor is λ i.e., the rate occurrence of assignable cause. There are two more significant factors i.e., the loss of income when process is out-of-control *M*, and the time to find and repair an assignable cause (T_1+T_2) which are common to both the designs and less significant compared to λ.

This table also shows that significant parameters are not always same in both joint economic and joint economic statistical designs. Thus, the users of control charts must take utmost care in ensuring the correctness of values of significant factors before using them in joint economic design or joint economic statistical design.

6.5 Numerical Illustration: Discontinuous process

In this section, the joint economic statistical design of $\bar{X}$ and R charts for a discontinuous process is illustrated through the same numerical problem which was earlier considered for joint economic design of $\bar{X}$ and R charts in Section 5.6. The joint economic statistical design deals with fourteen cost and process parameters out of which the values of thirteen factors are mentioned in Table 3.14. The value of fourteenth factor i.e., shift in standard deviation Υ is taken as 1.5 which is same as that considered in case of joint economic statistical design of continuous process in Section 6.3. Similarly, the limiting values of statistical constraints are also same as that mentioned in Section 6.3. Thus, the joint economic statistical design of a discontinuous process can be modelled as

$$\text{Minimize } E(L)_2 \tag{6.3}$$

subject to

$$ARL_0 \geq 267$$
$$ARL_1 \leq 40$$
$$ATS_1 \leq 1.90$$

The objective is to select proper values four design variables (i.e., n, h, k_1 and k_2) so as to minimize the expected loss cost per unit time $E(L)_2$ whose expression is shown in Eq. 3.31 and at the same time all the constraints are satisfied. For minimization purpose, the same two metaheuristics i.e., SA and TLBO are considered in this section too.

6.5.1 Results and Discussion

Table 6.11 shows the results of joint economic statistical design of $\bar{X}$ and R charts for a discontinuous process using SA and TLBO. This table shows the optimal values of three design variables such as sampling interval h, width of the control limits for $\bar{X}$ chart k_1 and width of the control limits for R chart k_2 for each integer value of sample size n varying from 2 to 33. It also shows the corresponding minimum value of the expected loss cost per unit time $E(L)_2$.

Table 6.11: Optimal joint economic statistical designs of $\bar{X}$ and R charts: discontinuous process

SA					***TLBO***				
n	***h***	***k1***	***k2***	***E(L)₂***	***n***	***h***	***k1***	***k2***	***E(L)₂***
2	0.33	3.09	3.16	38.294	2	0.33	3.10	3.16	38.262
3	0.47	3.09	3.10	37.439	3	0.47	3.07	3.09	37.502
4	0.60	3.07	3.06	37.058	4	0.61	3.06	3.07	37.005
5	0.74	3.04	3.05	36.758	5	0.73	3.04	3.05	36.808
6	0.85	3.04	3.06	36.666	6	0.85	3.04	3.04	36.654
7	**0.96**	**3.04**	**3.02**	**36.612**	**7**	**0.96**	**3.03**	**3.03**	**36.617**
8	1.05	3.04	3.03	36.672	8	1.05	3.02	3.05	36.683
9	1.14	3.02	3.08	36.763	9	1.14	3.02	3.06	36.770
10	1.22	3.00	3.09	36.927	10	1.22	3.01	3.06	36.925
11	1.29	3.04	3.07	37.083	11	1.30	3.02	3.06	37.076
12	1.36	3.01	3.10	37.313	12	1.36	3.03	3.06	37.308
13	1.42	3.02	3.06	37.574	13	1.42	3.02	3.08	37.561
14	1.48	3.02	3.06	37.833	14	1.47	3.02	3.09	37.858
15	1.52	3.04	3.06	38.145	15	1.52	3.02	3.07	38.166
16	1.57	3.01	3.12	38.458	16	1.56	3.02	3.10	38.492
17	1.59	3.05	3.10	38.846	17	1.60	3.04	3.10	38.820
18	1.63	3.04	3.13	39.196	18	1.63	3.03	3.12	39.211
19	1.66	3.06	3.10	39.575	19	1.66	3.04	3.12	39.588
20	1.69	3.03	3.16	39.974	20	1.69	3.04	3.15	39.969
21	1.72	3.03	3.15	40.367	21	1.71	3.05	3.13	40.403
22	1.73	3.08	3.17	40.803	22	1.74	3.06	3.14	40.780
23	1.75	3.06	3.18	41.255	23	1.76	3.05	3.14	41.223
24	1.77	3.07	3.19	41.683	24	1.77	3.08	3.16	41.684
25	1.78	3.10	3.19	42.149	25	1.79	3.07	3.19	42.117
26	1.80	3.10	3.12	42.606	26	1.80	3.09	3.17	42.600
27	1.81	3.09	3.25	43.071	27	1.81	3.06	3.19	43.104
28	1.82	3.08	3.22	43.572	28	1.82	3.08	3.19	43.578
29	1.83	3.07	3.20	44.073	29	1.83	3.09	3.21	44.050
30	1.84	3.12	3.12	44.552	30	1.84	3.09	3.24	44.543
31	1.85	3.07	3.34	45.030	31	1.85	3.09	3.25	45.030
32	1.86	3.09	3.18	45.527	32	1.85	3.09	3.25	45.581
33	1.85	3.15	3.32	46.109	33	1.85	3.14	3.33	46.111

Table 6.11 reveals that the results of joint economic statistical design obtained using SA and TLBO techniques are observed to be nearly same for almost all 32 values of sample size *n*. The optimum value of expected loss cost per unit time $E(L)_2$ initially decreases as *n* value increases from 2 to 7 and thereafter it increases at higher values of *n*. The variations of $E(L)_2$ with respect to *n* in case of SA and TLBO are graphically shown in Figs. 6.8 and 6.9 respectively. Out of all 32 sets of results, the lowest value of expected loss cost per unit time $E(L)_2$ is observed to occur at $n = 7$ in case of both SA and TLBO as shown in Figs. 6.8 and 6.9. As explained earlier in Section 6.3.1, both these graphs are drawn over a limited range of $n = 2$ to 20 for better clarity.

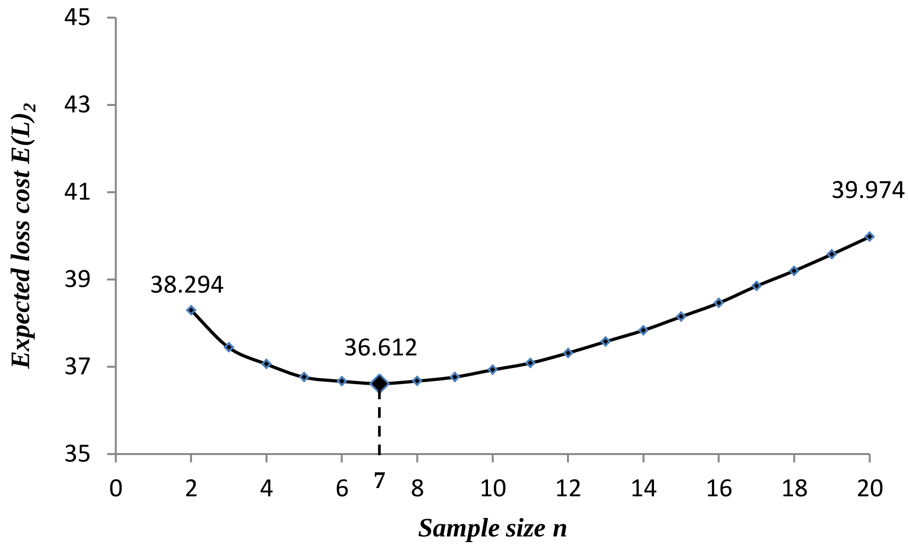

Fig. 6.8: Variation of expected loss cost per unit time with sample size using SA: discontinuous process

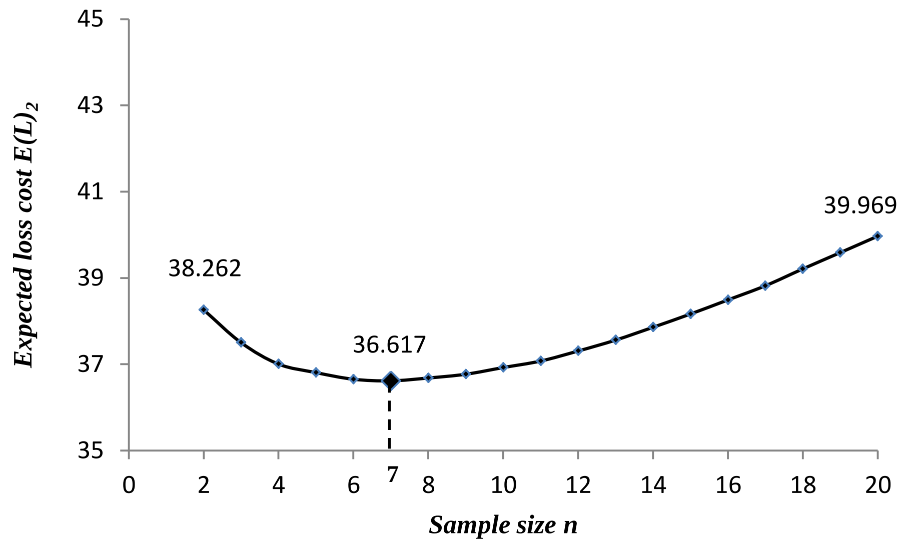

Fig. 6.9: Variation of expected loss cost per unit time with sample size using TLBO: discontinuous process

Tables 6.12 and 6.13 show a comparison of optimal results of joint economic statistical design of $\bar{X}$ and R charts with that of their joint economic design using SA and TLBO respectively. The optimal joint economic design results have been taken from Tables 5.13 and 5.14 respectively. The corresponding values of two additional parameters ATS_0 and ATS_1 are calculated using Eqs. 4.1 - 4.2 and are also listed in these two tables.

Table 6.12: Comparison of results of joint economic design with their joint economic statistical design of $\bar{X}$ and R charts using SA: discontinuous process

Design	***n***	***h***	***k_1***	***k_2***	***α***	***P***	***ARL_0***	***ARL_1***	***ATS_0***	***ATS_1***	***$E(L)_2$***
JED-D	8	1.69	2.96	2.97	0.0046	0.575	217.813	1.738	368.104	2.94	35.770
JESD-D	7	0.96	3.04	3.02	0.0036	0.503	277.778	1.987	266.667	1.90	36.612

Note:
JED-D : Joint Economic Design - Discontinuous process
JESD-D : Joint Economic Statistical Design - Discontinuous process

Table 6.13: Comparison of results of joint economic design with their joint economic statistical design of $\bar{X}$ and R charts using TLBO: discontinuous process

Design	***n***	***h***	***k_1***	***k_2***	***α***	***P***	***ARL_0***	***ARL_1***	***ATS_0***	***ATS_1***	***$E(L)_2$***
JED-D	8	1.69	2.96	2.97	0.0046	0.575	217.813	1.738	368.104	2.94	35.770
JESD-D	7	0.96	3.03	3.03	0.0037	0.503	270.270	1.987	258.751	1.90	36.617

Note:
JED-D : Joint Economic Design - Discontinuous process
JESD-D : Joint Economic Statistical Design - Discontinuous process

Tables 6.12 - 6.13 reveal that optimal values of sample size n in joint economic design (i.e., n = 8) and joint economic statistical design (i.e., n = 7) are not same in discontinuous process. The values of expected loss cost per unit time $E(L)_2$ are found to be higher in case of joint economic statistical design. The increase in expected loss cost per unit time in joint economic statistical design compared to that in joint economic design is 2.35% and 2.37% with SA and TLBO respectively. Even if the cost is higher, it provides the benefit of more satisfactory values of statistical properties like higher value of ARL_0 and lower value of ATS_1.

In case of economic design, the probability that a point falls outside the control limits for in-control process is α = 0.0046 and the corresponding ARL_0 = 217.813 (≈ 218) as shown in Tables 6.12 - 6.13. So, when the process remains in-control, an out-of-control signal will be generated on an average after every 218 samples. This means that the false alarm is generated more frequently leading to unnecessary over-adjustment of the process and thereby loss of confidence of quality control personnel. Therefore, it is desirable to keep the value of ARL_0 sufficiently higher to reduce the rate of false alarm. Compared to joint economic design, the values of ARL_0 in joint economic statistical design are higher i.e., 277.778 ≈ 278 in SA and 270.270 ≈ 270 in TLBO.

Both these tables also compare the effect of average time to signal during out-of-control process (ATS_1) between those two types of joint design of $\bar{X}$ and R charts for discontinuous process. The ATS_1 for the joint economic statistical design (i.e., 1.90) is much better than that for the joint economic design (i.e., 2.94) because it is able to detect the same magnitude of process shift much earlier. It is further observed that joint economic statistical design has smaller sampling interval compared to that of joint economic design (i.e., h = 0.96 hour < 1.69 hour). This means that samples are taken more frequently compared to joint economic design. This enhances the sampling cost and thereby the expected loss cost per unit time. However, the incorporation of ATS_1 constraint in joint economic statistical design helps in reduction in ATS_1 compared to that in joint economic design by 35.37% in both the results of SA and TLBO. Thus, joint economic statistical design is observed to be costlier than joint economic design due to the addition of constraints. However, it provides a more satisfactory statistical performance.

6.6 Sensitivity Analysis: Discontinuous Process

Similar to continuous process, sensitivity analysis has been done to investigate the effect of cost and process parameters on the output results of joint economic statistical design in case of discontinuous process. There are a total of 14 cost and process parameters, each of which is termed as a factor for this analysis. The low and high values of 13 factors are already listed in Table 3.19. For the additional fourteenth factor i.e., the shift in standard deviation (Υ), the low and high values are taken as 1.5 and 2.0 respectively. A 2_{IV}^{14-9} factorial design for 14 factors with 9 generators I = ABCF, I = ABDG, I = ABEH, I = ACDJ, I = ACEK, I = ADEL, I = BCDM, I = BCEN and I = BDEO, and resolution *IV* is chosen for a discontinuous process for the sensitivity analysis that gives a total of 32 (= 2^{14-9}) runs. For each of 32 runs, a particular set of values of fourteen factors is taken for which the expected loss cost function $E(L)_2$ is minimized using TLBO algorithm and the optimal result consisting of the values of five responses viz., n, h, k_1, k_2 and $E(L)_2$ is shown in Table 6.14. Since both SA and TLBO algorithms provided almost the same results for joint economic statistical design for a discontinuous process as observed in Section 6.5, any one of them is adequate for sensitivity analysis.

Table 6.14: Optimal joint economic statistical designs of $\bar{X}$ and R charts: discontinuous process

S. No.	***Cost and process parameters (factors)***														***Responses***				
	M	δ	λ	g	(T_1+T_2)	a	b	W	Y	V_0	S	S_1	T_0	Υ	n	h	k_1	k_2	$E(L)_2$
1	50	2	0.05	0.05	20	0.5	1.0	35	500	50	100	0.05	40	1.5	4	1.22	3.49	4.01	31.340
2	100	1	0.01	0.05	20	5.0	1.0	35	500	50	10	0.05	40	2.0	7	1.24	3.61	3.29	18.981
3	100	2	0.05	0.05	3	5.0	0.1	35	50	50	100	0.05	4	2.0	6	1.56	3.16	3.15	20.391
4	50	1	0.01	0.50	20	0.5	1.0	250	500	150	10	1.00	40	1.5	6	0.53	3.62	3.59	42.601
5	50	1	0.01	0.05	3	0.5	0.1	35	50	50	10	0.05	4	1.5	13	1.26	3.31	3.40	4.409
6	100	2	0.05	0.05	20	5.0	0.1	250	50	150	10	0.05	40	1.5	10	1.78	4.04	4.57	84.045
7	100	2	0.01	0.05	20	0.5	0.1	250	500	50	10	1.00	4	1.5	7	1.48	3.79	4.35	12.549
8	100	1	0.01	0.50	3	5.0	0.1	250	50	150	10	1.00	4	2.0	6	1.17	3.54	3.20	17.143
9	100	2	0.05	0.50	3	5.0	1.0	35	500	50	10	1.00	4	1.5	3	1.20	3.02	3.52	26.324
10	100	1	0.05	0.05	3	0.5	1.0	250	50	50	100	1.00	40	1.5	7	0.96	3.03	3.04	37.526
11	50	2	0.01	0.05	3	5.0	1.0	250	50	50	10	1.00	40	2.0	3	1.14	3.50	3.55	12.693
12	100	1	0.01	0.05	3	5.0	1.0	250	500	150	100	0.05	4	1.5	11	1.24	3.11	3.18	24.020
13	50	1	0.05	0.05	3	5.0	0.1	35	500	150	10	1.00	40	1.5	27	1.60	3.93	4.08	33.382
14	50	2	0.05	0.05	3	0.5	1.0	250	500	150	10	0.05	4	2.0	3	1.10	3.58	3.62	35.422
15	50	2	0.01	0.05	20	5.0	1.0	35	50	150	100	1.00	4	1.5	9	3.44	3.50	4.15	31.033
16	100	2	0.01	0.05	3	0.5	0.1	35	500	150	100	1.00	40	2.0	7	1.34	4.37	4.41	9.265
17	50	1	0.05	0.05	20	5.0	0.1	250	500	50	100	1.00	4	2.0	14	1.64	3.65	3.35	36.908
18	50	1	0.05	0.50	3	5.0	1.0	35	50	150	100	0.05	40	2.0	6	1.04	3.81	3.50	40.158
19	100	2	0.01	0.50	3	0.5	1.0	35	50	150	10	0.05	40	1.5	4	1.04	3.84	4.40	12.744
20	100	1	0.05	0.50	3	0.5	0.1	250	500	50	10	0.05	40	2.0	2	0.25	3.93	3.76	27.995
21	100	1	0.05	0.50	20	0.5	0.1	35	500	150	100	0.05	4	1.5	5	0.47	3.53	3.60	82.088
22	50	2	0.05	0.50	20	0.5	0.1	35	50	50	10	1.00	40	2.0	2	0.62	3.82	3.91	28.675
23	50	2	0.05	0.50	3	0.5	0.1	250	50	150	100	1.00	4	1.5	4	1.14	3.65	4.21	41.633
24	100	1	0.01	0.50	20	5.0	0.1	35	50	50	100	1.00	40	1.5	11	1.08	3.39	3.47	20.240
25	50	2	0.01	0.50	3	5.0	0.1	250	500	50	100	0.05	40	1.5	8	1.69	3.85	4.44	10.463
26	50	2	0.01	0.50	20	5.0	0.1	35	500	150	10	0.05	4	2.0	15	1.86	4.07	4.37	30.016
27	50	1	0.01	0.50	3	0.5	1.0	35	500	50	100	1.00	4	2.0	3	0.75	3.50	3.26	9.931
28	100	1	0.05	0.05	20	0.5	1.0	35	50	150	10	1.00	4	2.0	3	0.83	3.33	3.06	81.352
29	50	1	0.05	0.50	20	5.0	1.0	250	50	50	10	0.05	4	1.5	6	0.85	3.03	3.04	40.663
30	100	2	0.05	0.50	20	5.0	1.0	250	500	150	100	1.00	40	2.0	4	1.28	3.67	3.71	90.506
31	100	2	0.01	0.50	20	0.5	1.0	250	50	50	100	0.05	4	2.0	2	1.00	3.16	3.22	15.438
32	50	1	0.01	0.05	20	0.5	0.1	250	50	150	100	0.05	40	2.0	11	1.21	4.37	4.12	29.510

Tables 6.15 - 6.19 show the results of ANOVA at significance level of 5% for identifying the significant factors affecting the five responses. The significant factors can be more easily identified in the normal plots of standardized effects for all the responses as shown in Figs. 6.10 - 6.14.

Table 6.15: Analysis of variance for expected loss cost per unit time $E(L)_2$ with constraints: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC (%)*
M	1	463.4	463.4	463.38	3.22	0.091	2.91
δ	1	92.4	92.4	92.37	0.64	0.434	0.58
λ	1	5978.0	5978.0	5977.95	41.51	0.000*	37.58
g	1	35.7	35.7	35.68	0.25	0.625	0.22
(T_1+T_2)	1	3050.7	3050.7	3050.71	21.19	0.000*	19.18
a	1	37.2	37.2	37.17	0.26	0.618	0.23
b	1	120.2	120.2	120.20	0.83	0.374	0.76
W	1	194.0	194.0	193.98	1.35	0.262	1.22
Y	1	0.5	0.5	0.54	0.00	0.952	0.00
V_0	1	3411.2	3411.2	3411.24	23.69	0.000*	21.45
S	1	14.4	14.4	14.39	0.10	0.756	0.09
S_1	1	18.1	18.1	18.12	0.13	0.727	0.11
T_0	1	13.5	13.5	13.53	0.09	0.763	0.08
Υ	1	29.4	29.4	29.41	0.20	0.657	0.18
Residual Error	17	2448.0	2448.0	144.00			
Total	31	15906.7					

* Significant at 5%

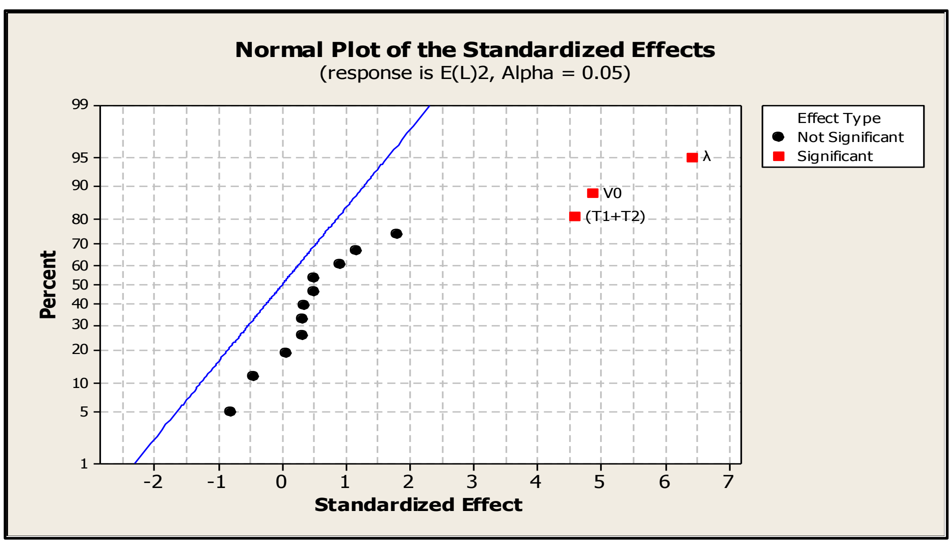

Fig. 6.10: Normal probability plot of standardized effect for expected loss cost per unit time $E(L)_2$ with constraints: discontinuous process

Table 6.15 indicates that the expected loss cost per unit time of process control $E(L)_2$ in a discontinuous process is significantly affected by three factors, namely rate of occurrences of assignable causes λ, time to find and repair an assignable cause (T_1+T_2), and net income per hour while process is in-control V_0. They are also graphically displayed as significant in the normal plot as shown in Fig. 6.10.

Among all the factors, λ has the highest significant effect on expected loss cost per unit time $E(L)_2$ since it has the highest F-value i.e., 41.51 as shown in Table 6.15 and plotted at the rightmost location in Fig. 6.10. It is also observed from this table that λ, V_0 and (T_1+T_2) are the top three percentage contributors in affecting the cost by 37.58%, 21.45% and 19.18% respectively. All these three factors have positive effect as they are located on the right side of the straight line as shown in Fig. 6.10.

Table 6.16 presents an analysis of variance on the sample size *n*. There are four factors that significantly affect sample size out of which factors *b*, *g* and δ have negative effect and the fourth factor *a* has positive effect as shown Fig. 6.11. Moreover, it can be observed from Table 6.16 that *b*, *a*, g and δ are the major percentage contributors in affecting the sample size by 17.31%, 15.31%, 11.67% and 8.52% respectively. Thus, the factor *b* is the most significant for choosing the value of sample size, in joint economic statistical design and its effect is of negative type.

Table 6.16: Analysis of variance for sample size *n* with constraints: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC (%)*
M	1	47.531	47.531	47.531	3.94	0.063	5.87
δ	1	69.031	69.031	69.031	5.72	0.029*	8.52
λ	1	9.031	9.031	9.031	0.75	0.399	1.11
g	1	94.531	94.531	94.531	7.84	0.012*	11.67
(T_1+T_2)	1	0.281	0.281	0.281	0.02	0.880	0.03
a	1	124.031	124.031	124.031	10.28	0.005*	15.31
b	1	140.281	140.281	140.281	11.63	0.003*	17.31
W	1	13.781	13.781	13.781	1.14	0.300	1.70
Y	1	16.531	16.531	16.531	1.37	0.258	2.04
V_0	1	34.031	34.031	34.031	2.82	0.111	4.20
S	1	0.781	0.781	0.781	0.06	0.802	0.10
S_1	1	0.281	0.281	0.281	0.02	0.880	0.03
T_0	1	2.531	2.531	2.531	0.21	0.653	0.31
Υ	1	52.531	52.531	52.531	4.36	0.052	6.48
Residual Error	17	205.031	205.031	12.061			
Total	31	810.219					

* Significant at 5%

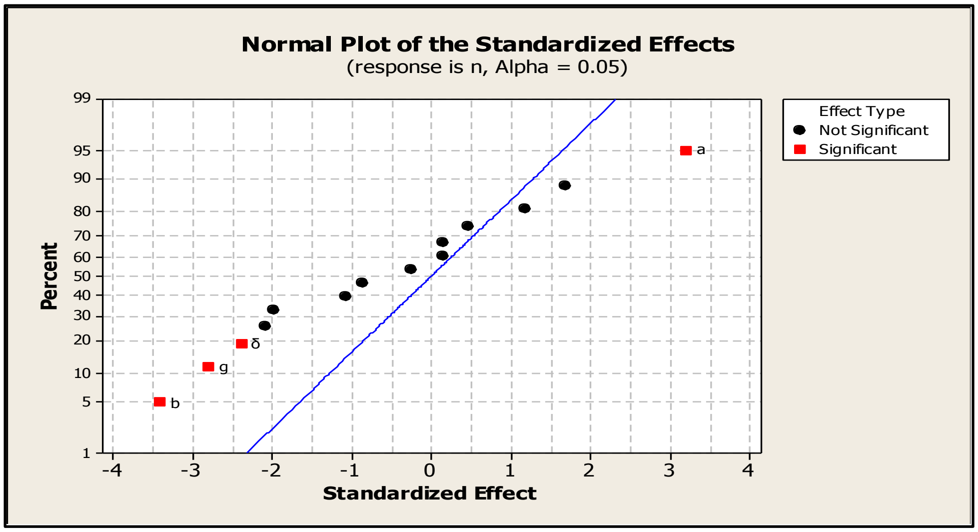

Fig. 6.11: Normal probability plot of standardized effect for sample size *n* with constraints: discontinuous process

Table 6.17 displays an analysis of variance on the sampling interval *h*. It is significantly affected by four factors i.e., δ, λ, *g* and *a*. Out of these four significant factors, two factors i.e., δ and *a* have positive effect, whereas the other two factor *g* and λ have negative effect as shown in Fig. 6.12. Among all the factors, the fixed cost per sample *a* has the highest effect on the sampling interval with a percentage contribution of 24.36% and the effect is in positive direction.

Table 6.17: Analysis of variance for sampling interval *h* with constraints: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC (%)*
M	1	0.31108	0.31108	0.31108	2.97	0.103	3.29
δ	1	1.44411	1.44411	1.44411	13.78	0.002*	15.26
λ	1	0.48043	0.48043	0.48043	4.58	0.047*	5.08
g	1	1.55444	1.55444	1.55444	14.83	0.001*	16.43
(T_1+T_2)	1	0.13043	0.13043	0.13043	1.24	0.280	1.38
a	1	2.30479	2.30479	2.30479	21.99	0.000*	24.36
b	1	0.05272	0.05272	0.05272	0.50	0.488	0.56
W	1	0.13787	0.13787	0.13787	1.32	0.267	1.46
Y	1	0.04884	0.04884	0.04884	0.47	0.504	0.52
V_0	1	0.30764	0.30764	0.30764	2.93	0.105	3.25
S	1	0.29938	0.29938	0.29938	2.86	0.109	3.16
S_1	1	0.05739	0.05739	0.05739	0.55	0.469	0.61
T_0	1	0.27992	0.27992	0.27992	2.67	0.121	2.96
Υ	1	0.27190	0.27190	0.27190	2.59	0.126	2.87
Residual Error	17	1.78196	1.78196	0.10482			
Total	31	9.46288					

* Significant at 5%

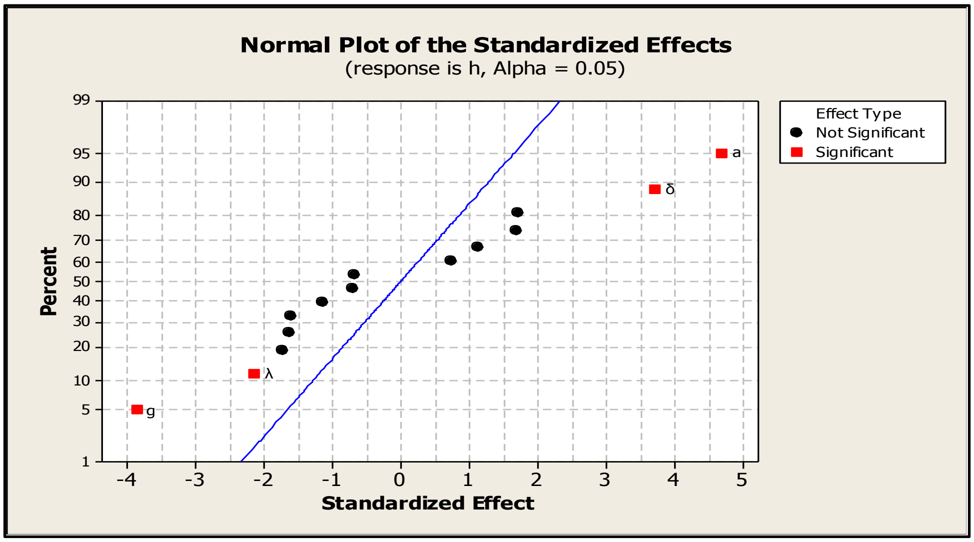

Fig. 6.12: Normal probability plot of standardized effect for sampling interval h with constraints: discontinuous process

Table 6.18 presents the results of analysis of variance on the control limits width k_1 of $\bar{X}$ chart. Six factors (i.e., M, b, Y, V_0, T_0 and Υ) are found to be significant on k_1. Fig. 6.13 reveals that out of these six significant factors, two factors (i.e., b and M) have negative effect and the rest five factors have positive effect on k_1. Among all the factors, variable cost of sampling b is observed to have the most significant effect with a maximum contribution of 24.91% on deciding the value of k_1 in joint economic design and its effect is of negative type.

Table 6.18: Analysis of variance for width of control limits k_1 of $\bar{X}$ chart with constraints: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	0.14431	0.14431	0.144305	5.19	0.036*	3.66
δ	1	0.10284	0.10284	0.102842	3.70	0.071	2.61
λ	1	0.10547	0.10547	0.105467	3.79	0.068	2.67
g	1	0.00422	0.00422	0.004221	0.15	0.702	0.11
(T_1+T_2)	1	0.02829	0.02829	0.028292	1.02	0.327	0.72
a	1	0.06445	0.06445	0.064449	2.32	0.146	1.63
b	1	0.98312	0.98312	0.983117	35.33	0.000*	24.91
W	1	0.00141	0.00141	0.001414	0.05	0.824	0.04
Y	1	0.15802	0.15802	0.158020	5.68	0.029*	4.00
V_0	1	0.69399	0.69399	0.693989	24.94	0.000*	17.59
S	1	0.01612	0.01612	0.016124	0.58	0.457	0.41
S_1	1	0.01004	0.01004	0.010036	0.36	0.556	0.25
T_0	1	0.89108	0.89108	0.891079	32.02	0.000*	22.58
Υ	1	0.26987	0.26987	0.269874	9.70	0.006*	6.84
Residual Error	17	0.47308	0.47308	0.027829			
Total	31	3.94631					

* Significant at 5%

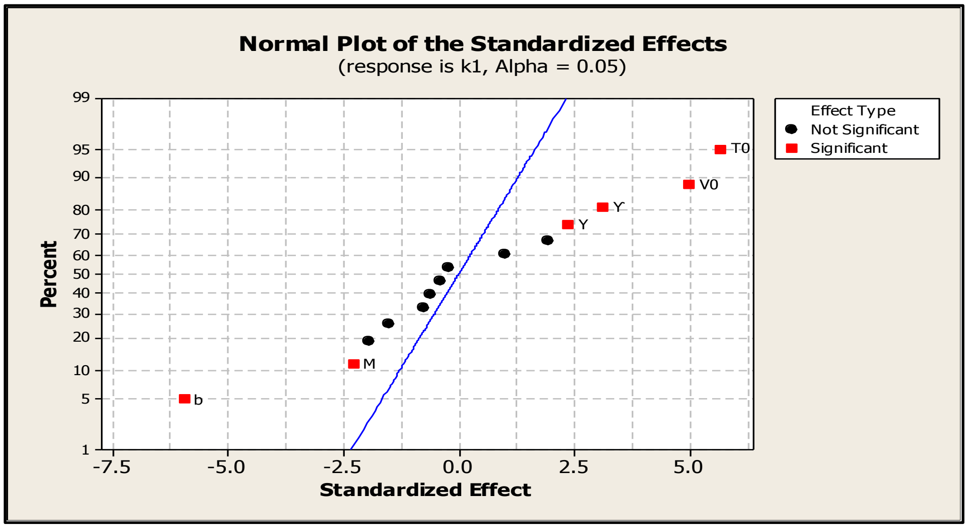

Fig. 6.13: Normal probability plot of standardized effect for width of control limits k_1 of $\overline{X}$ chart with constraints: discontinuous process

Similarly, Table 6.19 shows the ANOVA results for the control limits width k_2 of R chart. Five factors (i.e., δ, b, V_0, T_0 and Υ) are found to be significant on k_2. Fig. 6.14 reveals that out of these five significant factors, two factors (i.e., b and Υ) have negative effect and three factors (i.e., V_0, T_0 and δ) have positive effect. The shift in process mean δ is found to be the most significant factor with positive effect on deciding the value of k_2 and its percentage contribution is also the highest i.e., 32.40%.

Table 6.19: Analysis of variance for width of control limits k_2 of R chart with constraints: discontinuous process

Source	*DF*	*Seq SS*	*Adj SS*	*Adj MS*	*F*	*p*	*PC(%)*
M	1	0.22530	0.22530	0.22530	4.41	0.051	3.13
δ	1	2.33570	2.33570	2.33570	45.70	0.000*	32.40
λ	1	0.16271	0.16271	0.16271	3.18	0.092	2.26
g	1	0.00048	0.00048	0.00048	0.01	0.924	0.01
(T_1+T_2)	1	0.03688	0.03688	0.03688	0.72	0.407	0.51
a	1	0.05989	0.05989	0.05989	1.17	0.294	0.83
b	1	1.23193	1.23193	1.23193	24.10	0.000*	17.09
W	1	0.01192	0.01192	0.01192	0.23	0.635	0.17
Y	1	0.20231	0.20231	0.20231	3.96	0.063	2.81
V_0	1	0.79393	0.79393	0.79393	15.53	0.001*	11.01
S	1	0.02582	0.02582	0.02582	0.51	0.487	0.36
S_1	1	0.01958	0.01958	0.01958	0.38	0.544	0.27
T_0	1	0.83614	0.83614	0.83614	16.36	0.001*	11.60
Υ	1	0.39710	0.39710	0.39710	7.77	0.013*	5.51
Residual Error	17	0.86882	0.86882	0.05111			
Total	31	7.20850					

* Significant at 5%

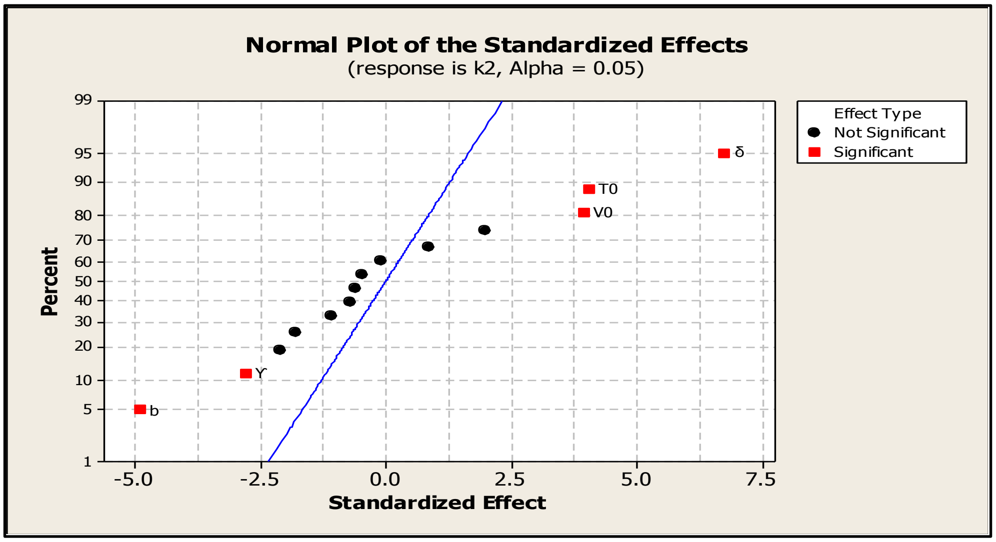

Fig. 6.14: Normal probability plot of standardized effect for width of control limits k_2 of *R* chart with constraints: discontinuous process

It is further observed from Tables 6.15 - 6.19 that the cost to locate and repair the assignable cause *W*, the expected cost of restart or setup cost *S* and the startup time S_1 have no significance on any of the responses *n*, *h*, k_1, k_2 and $E(L)_2$.

6.6.1 Summary of Results

Similar to Table 6.10, all the significant factors in case of joint economic statistical design for a discontinuous process are summarized for each of the five output responses in Table 6.20. This table also shows the corresponding significant factors of joint economic design earlier shown in Table 5.22 for the ease of comparison of both the sets of results. The insignificant factors are shown as blank spaces.

Table 6.20: Comparison of significant effects in joint economic design and joint economic statistical design: discontinuous process

Output responses	***Design***	***Cost and process parameters***													
		M	***δ***	***λ***	***g***	**(T_1+T_2)**	***a***	***b***	***W***	***Y***	***ϒ***	***V_0***	***S***	***S_1***	***T_0***
n	*JED-D*		–		**–**		+	–			–				
	JESD-D		–		–		+	**–**							
h	*JED-D*	–		–	–	+	**+**	+				+			
	JESD-D		+	–	–		**+**								
k_1	*JED-D*		+				–	–		+	+	+			**+**
	JESD-D	–						**–**		+	+	+			+
k_2	*JED-D*		**+**				–	–				+			+
	JESD-D		**+**					–			–	+			+
$E(L)_2$	*JED-D*			**+**		+						+			
	JESD-D			**+**		+						+			

Note:
JED-D : Joint Economic Design - Discontinuous process
JESD-D : Joint Economic Statistical Design - Discontinuous process
Blank space : Insignificant factor
\+ : Factor with positive effect
– : Factor with negative effect
+/– in bold : Most significant factor

Table 6.20 shows that time to sample and chart one item *g*, and variable cost per sample *b* are the most significant factors for selecting the value of sample size *n* in case of joint economic design (*JED*) and joint economic design with constraints (*JESD*) respectively. Both of them have negative effect. All other significant factors in *JED* are also significant in *JESD* except the factor *ϒ*. The type of effect (i.e., positive or negative) is also same for each of those significant factors which are common to *JED* and *JESD*.

On the other hand in case of sampling interval *h*, the fixed cost of sampling *a* is observed to have the most significant effect in both the designs (i.e., *JED* and *JESD*) of a discontinuous process and both the effects are of positive type. In case of *JED*, there are seven significant (i.e., *M*, λ, *g*, (T_1+T_2), *a*, *b* and V_0), whereas JESD has four significant factors (i.e., δ, λ, *g* and *a*) out of which three factors i.e., λ, *g* and *a* are common to both the designs and with same type of effects (i.e., either positive or negative).

Similar to the sample size n, the most significant factors for the width of control limits of $\bar{X}$ chart k_1 are not same in both the designs. In case of k_1, expected search time for a false alarm T_0 and variable cost per sample b are the most significant factors with positive and negative effects in *JED* and *JESD* for discontinuous process respectively. There are seven and six factors which are found significant in case of *JED* and *JESD* respectively. Out of them, five factors (i.e., b, Y, Υ, V_0 and T_0) are found significant and common to both the cases.

Similar to the sampling interval h, the most significant factor for the width of control limits of R chart k_2 is same in both types of designs (i.e., *JED* and *JESD*) for a discontinuous process and this is the shift in process mean δ Its type of effect is also same in both the designs and it is of positive type. Here, there are four significant factors i.e., δ, b, V_0 and T_0 which are common to both the designs with same type of effects (i.e., either positive or negative). On the other hand, the factors a and Υ are significant only in *JED* and *JESD* respectively and both have negative effect.

There are three significant factors with respect to the expected loss cost per unit time $E(L)_2$ and they all have positive effects in both types of design for a discontinuous process. The most significant factor for $E(L)_2$ is also same in both cases and this factor is λ i.e., the rate occurrence of assignable cause. There are two more significant factors i.e., the time to find and repair an assignable cause (T_1+T_2), and net income per hour while process is in-control V_0 which are also common to both types of design but to a less extent compared to λ.

This table also shows that significant parameters are not always same in both joint economic design and joint economic statistical design. Thus, the users of control charts must be careful in ensuring the correctness of values of significant factors before using them in joint economic design or joint economic statistical design.

Table 6.21 shows a comparison between joint economic design and joint economic statistical design of $\bar{X}$ and R charts for both continuous and discontinuous processes. Thus, this table summarizes the results of four different cases. All these results have been obtained using TLBO optimization technique. In case of joint economic design for continuous process ten cost and process parameters are taken, whereas in discontinuous process fourteen parameters are considered. So, the last four columns for the four factors such as expected net income per hour while the process is in-control V_0, the expected cost of restart or setup cost S, the time to restart the process S_1 and the expected search time for a false alarm T_0 in this table are not applicable for a continuous process.

Table 6.21: Comparison of significant effects in joint economic design and joint economic statistical design for both continuous and discontinuous processes

Output responses	***Design***	***Cost and process parameters***													
		M	***δ***	***λ***	***g***	**(T_1+T_2)**	***A***	***b***	***W***	***Y***	***ϒ***	**V_0**	***S***	**S_1**	**T_0**
n	*JED-C*		–		**–**		+	–							
	JED-D		–		**–**		+	–			–				
	JESD-C		–		–		**+**	–			–				
	JESD-D		–		–		+	**–**							
h	*JED-C*	–		–	–	+	**+**	+							
	JED-D	–		–	–	+	**+**	+				+			
	JESD-C		+		–		**+**	–							
	JESD-D		+	–	–		**+**								
k_1	*JED-C*		+	–	–		–	–		**+**	+				
	JED-D		+				–	–		+	+	+			**+**
	JESD-C							–		**+**	+				
	JESD-D	–						**–**		+	+	+			+
k_2	*JED-C*		+	–	–		–	–		**+**					
	JED-D		**+**				–	–				+			+
	JESD-C		**+**							+	–				
	JESD-D		**+**					–			–	+			+
E(L)	*JED-C*	+		**+**		+									
	JED-D			**+**		+						+			
	JESD-C	+		**+**		+									
	JESD-D			**+**		+						+			

Note:

JED-C : Joint Economic Design - Continuous process
JED-D : Joint Economic Design - Discontinuous process
JESD-C : Joint Economic Statistical Design - Continuous process
JESD-D : Joint Economic Statistical Design - Discontinuous process
Blank space : Insignificant factor
\+ : Factor with positive effect
– : Factor with negative effect
+/– in bold : Most significant factor

The significant factors in case of joint economic design for both continuous and discontinuous processes are already discussed in Sections 5.5.1 and 5.7.1 respectively. All these results of joint economic design (*JED*) are compared with the corresponding results of joint economic statistical design (*JESD*) for each of the five responses (i.e., four design variables n, h, k_1, k_2 and the expected loss cost per unit time E(L)) below.

i) Effect on sample size n

From Table 6.21 it is observed that the most significant factors are not same in all four situations for selecting the value of sample size n. However, the time to sample and chart one item g is observed to have the most significant effect in case of *JED* for continuous and discontinuous processes, and both effects are of negative type. Four factors i.e., δ, g, a and b are found to be significant with same type of effect, positive or negative, in all the four cases.

ii) Effect on sampling interval h

Unlike in case of sample size n, the most significant factor for sampling interval h is same (i.e., fixed cost per sample a) in all the four situations and its effect is of positive type. Only two significant factors are common in all the four situations and these two are the time to sample and chart one item g, and the fixed cost a. But unlike a, the factor g has negative type of effect. Two factors i.e., M and (T_1+T_2) are found significant in case of only *JED* and not in *JESD* for both continuous and discontinuous processes. On the other hand, the factor δ is found to be significant only in both the cases of JESD and not in any type of *JED*.

iii) Effect on width of control limits k_1 of $\bar{X}$ chart

Three factors i.e., b, Y and Υ are observed to be significant in all the four situations. Moreover, each one of these three factors has one type of effect either positive or negative in all those four situations. But none of them is most significant for the width of control limits k_1 of $\bar{X}$ chart in any of the four situations. However, the cost per false alarm Y is most significant in both types of designs for a continuous process.

iv) Effect on width of control limits k_2 of R chart

There is only one factor (i.e., the shift in process mean δ) which is observed to be significant in all the four situations. The nature of its effect is also same in all these cases and it is of positive type. This factor also happens to be the most significant one for the width of control limits k_2 of R chart in all the situations except joint economic design of $\bar{X}$ and R charts in continuous process (*JED-C*). In case of *JED-C*, the cost per false alarm Y is the most significant factor with a positive effect. Factor a is significant in case of only *JED* for both continuous and discontinuous processes and not in any of these processes for *JESD*. On the other hand, the factor Υ is found to be significant only in both the cases of *JESD* and not in any of the cases of *JED*. Further it observed that the factor *Y* is found to be significant in both types of designs (i.e., *JED* and *JESD*) for a continuous process only but not in any type of design for a discontinuous process. Out of four factors which are applicable only in discontinuous process and not in continuous process, only two factors i.e., V_0 and T_0 are found to be significant in both types of designs.

v) Effect on expected loss cost per unit time E(L)

Two factors i.e., λ and (T_1+T_2) are observed to be significant for expected loss cost per unit time *E(L)* and both these factors have positive effect in all the four situations. The most significant factor is the rate occurrence of assignable cause λ in all the four situations. The loss of income when process is out-of-control *M* is a significant factor in both types of designs for continuous process and not significant in discontinuous process. Out of four factors which are applicable only in discontinuous process and not in continuous process, only one factor (i.e., the expected net income per hour during in-control period V_0) is found to be significant in both types of designs.

The objective function equations are not same in continuous and discontinuous processes. The numbers of factors associated with these two types of processes are also different (i.e., 10 factors in continuous and 14 factors in discontinuous processes). The economic statistical design deals with one or more constraints, whereas the economic design does not consider any constraint. These differences in characteristics of the four situations may be the reasons for the differences in results of significant factors as shown in Table 6.21. Therefore, the designers of control charts must ensure the type of process (i.e., continuous process or discontinuous process) and take utmost care while ensuring the correctness of values of significant factors before using them into economic design or economic statistical design.

6.7 Another Numerical Illustration

In order to validate and to investigate the effectiveness of two metaheuristics used in this thesis (i.e., SA and TLBO) another numerical problem from the literature of Saniga (1989) has been considered. This problem is related to joint economic statistical design of $\bar{X}$ and *R* charts for a discontinuous process (i.e., the process stopped during search and repair of assignable cause). This is a constrained optimization problem as stated below:

Minimize $E(L)_2$

subject to

$$\alpha \leq 0.0052$$

$$P \geq 0.95$$
$$ATS_1 \leq 4$$

where

$E(L)_2$ is the expected loss cost per unit time,

α is the joint value of Type-I error for both $\bar{X}$ and R charts,

P is the joint value of power for both $\bar{X}$ and R charts, and

ATS_1 is the average time to signal when the process is out-of-control.

6.7.1 Cost and Process Parameters

The cost model considered in this problem is same as that shown in Eq. 3.31 as explained in Section 5.8. The values of all the related cost and process parameters are taken as listed in Table 6.22 along with their corresponding notations followed in this thesis.

Table 6.22: Cost and process parameters (Saniga, 1989)

S. No.	*Cost and process parameters*	*Notation*	*Unit*	*Value*
1	Shift in process mean	δ	-	1.5
2	Rate of occurrences of assignable causes	λ	per hour	0.05
3	Time to find and repair an assignable cause	(T_1+T_2)	hour	0.30
4	Fixed cost per sample	a	hour	0.5
5	Variable cost per sample	b	hour	0.1
6	Cost to locate and repair the assignable cause	W	$	2
7	Cost per false alarm	Y	$	1
8	Net income per hour while process is in-control	V_0	$	50
9	Net income per hour while process is out-of-control	V_1	$	25
10	Expected search time for a false alarm	T_0	$	0.1
11	Value of shift in standard deviation	Υ	-	2

From Table 6.22 it is observed that the value of loss of income when process is out-of-control M is not given but this value is required for calculating the expected loss cost per unit time $E(L)_2$ in Eq. 3.31. So, this value is calculated as $M = V_0 - V_1 = 50 - 25 = 25$.

For the above mentioned data set, the optimal values of four design variables (i.e., n, h, k_1 and k_2) are required to be found out with an objective to minimize the expected loss cost per unit time $E(L)_2$ in joint economic statistical design of $\bar{X}$ and R charts. All the four design variables are taken as real values on continuous scale except the sample size n which is taken as integer. The search space defined by the low and high limits for each of these four design variables for minimizing $E(L)_2$ is already mentioned in Table 5.1.

This numerical problem is then solved using the same two metaheuristics SA and TLBO for each integer value of sample size n varying from 2 to 33, and the results obtained are discussed below.

6.7.2 Results and Discussion

Table 6.23 shows the results of joint economic statistical design of $\bar{X}$ and R charts for a discontinuous process using both SA and TLBO. The results consist of the optimal values of three design variables of control chart such as sampling interval h and width of the control limits for $\bar{X}$ chart k_1 and width of the control limits for R chart k_2 for each integer value of sample size n in the range 11 to 33. It also shows the corresponding minimum value of the expected loss cost per unit time $E(L)_2$. For sample size n = 2 to 10, the value of objective function is highly penalized due to violation of constraints. Thus, no feasible solution is obtained in this range of sample size and therefore the results are not shown in this table. The optimal values of $E(L)_2$ obtained from SA and TLBO techniques are nearly found to be same for each value of sample size from 11 to 33.

Table 6.23: Optimal joint economic statistical designs of $\bar{X}$ and R charts: discontinuous process

SA					***TLBO***				
n	***h***	***k₁***	***k₂***	***E(L)₂***	***n***	***h***	***k₁***	***k₂***	***E(L)₂***
11	**1.61**	**2.98**	**2.83**	**2.849**	**11**	**1.62**	**2.99**	**2.83**	**2.849**
12	1.68	2.98	2.83	2.888	12	1.69	2.99	2.82	2.888
13	1.75	2.99	2.82	2.930	13	1.75	2.98	2.83	2.930
14	1.80	3.02	2.81	2.973	14	1.81	2.98	2.83	2.973
15	1.88	2.98	2.83	3.017	15	1.86	2.98	2.83	3.017
16	1.91	3.00	2.89	3.062	16	1.92	3.00	2.86	3.062
17	1.97	3.04	2.91	3.107	17	1.96	3.04	2.91	3.107
18	2.01	3.05	2.98	3.151	18	2.01	3.08	2.95	3.151
19	2.06	3.12	3.01	3.195	19	2.06	3.12	2.99	3.195
20	2.10	3.15	3.05	3.239	20	2.10	3.15	3.04	3.239
21	2.14	3.21	3.07	3.283	21	2.15	3.19	3.08	3.283
22	2.19	3.21	3.14	3.326	22	2.19	3.23	3.12	3.326
23	2.24	3.32	3.10	3.368	23	2.23	3.26	3.16	3.368
24	2.27	3.25	3.31	3.410	24	2.27	3.30	3.20	3.410
25	2.31	3.36	3.24	3.451	25	2.31	3.34	3.24	3.451
26	2.35	3.37	3.28	3.492	26	2.35	3.37	3.28	3.492
27	2.39	3.44	3.27	3.533	27	2.39	3.40	3.32	3.533
28	2.43	3.45	3.34	3.573	28	2.43	3.44	3.36	3.573
29	2.46	3.49	3.35	3.612	29	2.47	3.47	3.40	3.612
30	2.50	3.46	3.40	3.651	30	2.51	3.50	3.43	3.651
31	2.54	3.60	3.49	3.690	31	2.54	3.54	3.47	3.690
32	2.58	3.62	3.54	3.728	32	2.58	3.57	3.51	3.728
33	2.61	3.61	3.60	3.765	33	2.62	3.60	3.54	3.765

The variations of $E(L)_2$ with respect to n in case of SA and TLBO are graphically shown in Figs. 6.15 and 6.16 respectively. Out of all 23 sets of results, the lowest value of expected loss cost per unit time $E(L)_2$ is observed to occur at n = 11 in case of both SA and TLBO as shown in these two figures.

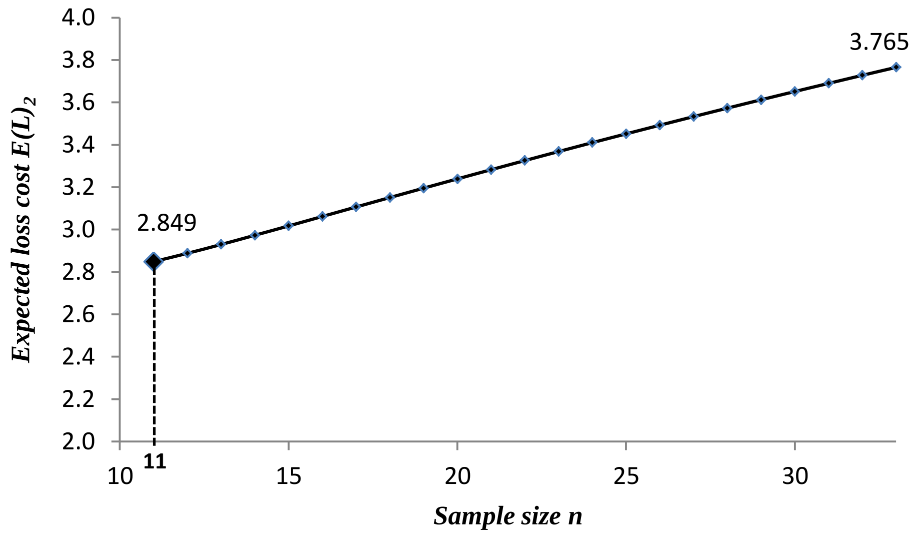

Fig. 6.15: Variation of expected loss cost per unit time with sample size using SA: discontinuous process

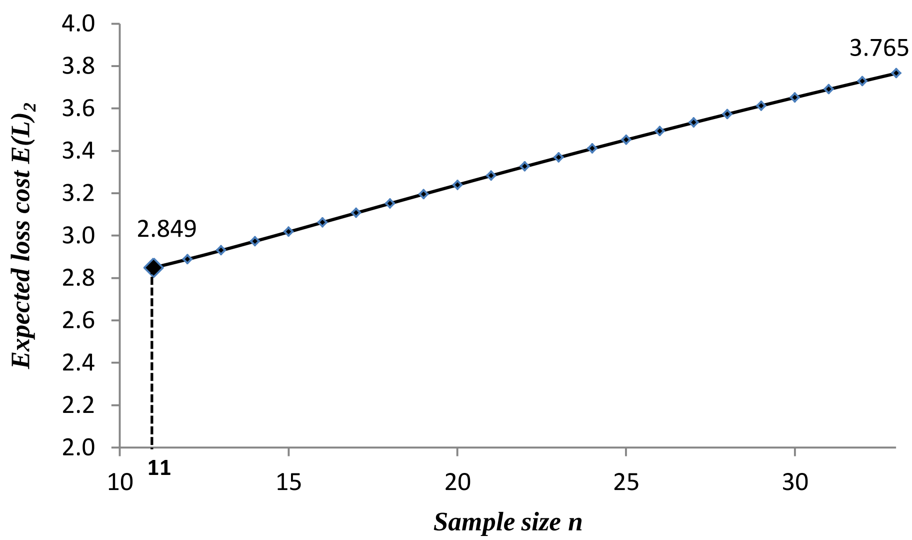

Fig. 6.16: Variation of expected loss cost per unit time with sample size using TLBO: discontinuous process

Tables 6.24 and 6.25 show comparison of optimal results of joint economic statistical design of $\bar{X}$ and R charts using SA and TLBO respectvely for a discontinous process with that reported by Saniga (1989).

Table 6.24: Comparison of results: SA

Methodology	***n***	***h***	k_1	k_2	K_2	α	***P***	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_2$
Saniga (1989)	12	1.667	3.013	-	5.579	0.0052	0.952	192.308	1.05	320.577	1.750	2.916
SA	11	1.61	2.98	2.83	5.40	0.0052	0.956	192.308	1.05	310.190	1.69	2.849

Table 6.25: Comparison of results: TLBO

Methodology	***n***	***h***	k_1	k_2	K_2	α	***P***	ARL_0	ARL_1	ATS_0	ATS_1	$E(L)_2$
Saniga (1989)	12	1.667	3.013	-	5.579	0.0052	0.952	192.308	1.05	320.577	1.750	2.916
TLBO	11	1.62	2.99	2.83	5.40	0.0051	0.956	196.078	1.05	318.529	1.70	2.849

Saniga (1989) has considered K_2 instead of k_2 for the width of control limits for R chart. Actually, K_2 represents upper control limit coefficient and is expressed as $K_2 = d_2 + k_2 d_3$ where k_2 is the width of control limits, and d_2 and d_3 are control chart constants which depend on sample size. In this thesis, the width of control limits k_2 is considered and not K_2. Therefore, for comparison purpose the value of K_2 is calculated from k_2 and reported in addition to the optimal values of four design variables in Tables 6.24 and 6.25. Further, these tables show the values of Type-I error (α), power of detecting a shift (P), average in-control run length (ARL_0), average out-of-control run length (ARL_1), average in-control time to signal (ATS_0), average out-of-control time to signal (ATS_1) corresponding to optimal designs and the global minimum value of expected loss cost per unit time $E(L)_2$.

Tables 6.24 and 6.25 show that the most minimum value of expected loss cost per unit time $E(L)_2$ is found to be 2.849 and this occurs at n = 11 in case of both SA and TLBO. On the other hand, Saniga (1989) reported the corresponding minimum value of $E(L)_2$ to be 2.916 occurring at n = 12. So, the reduction in overall cost is:

$$\frac{(2.916 - 2.849)}{2.916} \times 100 = 2.30\%.$$

Saniga (1989) reported the probability that a point falls outside the limits when the process is in-control is α = 0.0052 and the same result has been obtained using SA as shown in Table 6.24. Similarly, Table 6.25 shows the value of Type-I error (α) is found as 0.0051 using TLBO. So, when the process remains in-control, an out-of-control signal will be generated on an average after every 192.308 ≈ 192 and 196.078 ≈ 196 samples using SA and TLBO respectively. On the other hand, SA provides much better value for the average time to signal during out-of-control process (ATS_1) compared to Saniga (1989) (i.e., 1.69 against 1.75), whereas TLBO provides this value as 1.70. Also, the joint power of detecting any shift has been found slightly better in case of both the metaheuristics (i.e., 0.956 against 0.952).

6.8 Conclusions

In this chapter two new design methodologies have been developed based on metaheuristics viz., SA and TLBO for joint economic statistical design of $\bar{X}$ and R charts for both continuous and discontinuous processes. The use of both the methodologies has been illustrated through numerical examples. Both are observed to have yielded nearly the same results. Therefore, either SA or TLBO can be recommended for joint economic statistical design of $\bar{X}$ and R charts. Both the methodologies have also been found to be superior to that reported in the literature. The optimal value of expected loss cost per unit time in joint economic statistical design of $\bar{X}$ and R charts is found to be less than that of its economic statistical design of $\bar{X}$ chart in both types of processes. The fixed cost per sample a, the shift in process mean δ and the rate of occurrences of assignable cause λ are found to be the most significant factors affecting sampling interval h, width of control limits of R chart k_2 and expected loss cost per unit time $E(L)$ respectively in joint economic statistical design of $\bar{X}$ and R charts for both types of processes. The fixed cost per sample a and variable cost per sample b are the most significant factors found in case of continuous process and discontinuous process respectively for the sample size n. Similarly, the cost per false alarm Y and the variable cost per sample b are the most significant factors for the width of control limits of $\bar{X}$ chart k_1 in case of continuous process and discontinuous process respectively.

CHAPTER - 7

Conclusions and Future scope of Research

A process may shift to out-of-control state due to shift in process mean or process variance or both. The objective of the thesis is to design control charts for detecting the process shift as quickly as possible and at the same time with minimum possible cost. In the present work the key contribution is the development of design methodologies based on two metaheuristics, namely simulated annealing (SA) and teaching-learning based optimization (TLBO) for the following eight distinct design problems:

i. Economic design of $\bar{X}$ chart for continuous process
ii. Joint economic design of $\bar{X}$ and R charts for continuous process
iii. Economic statistical design of $\bar{X}$ chart for continuous process
iv. Joint economic statistical design of $\bar{X}$ and R charts for continuous process
v. Economic design of $\bar{X}$ chart for discontinuous process
vi. Joint economic design of $\bar{X}$ and R charts for discontinuous process
vii. Economic statistical design of $\bar{X}$ chart for discontinuous process
viii. Joint economic statistical design of $\bar{X}$ and R charts for discontinuous process

Each of the above design methodologies has been illustrated with numerical examples taken from literature. The comparisons of their results are given in the respective chapters. Sensitivity analysis has been carried out using design of experiment and analysis of variance for each of the eight distinct design cases to identify the cost and process parameters that affect the design responses significantly.

7.1 Conclusions

On the basis of the results obtained out of the present work, the following conclusions are drawn.

1. The results of simulated annealing and teaching-learning based optimization are observed to be almost same in all the eight design cases considered in the present work. The differences in their results in all cases are limited within 0.01%. Further, these two metaheuristics are providing superior results than that of the corresponding results reported earlier in the literature. Therefore, design methodology based on any one of these two metaheuristics can be recommended for use in any of the eight design cases mentioned above.

2. The optimal values expected loss cost per unit time in case of joint design (i.e., joint economic design or joint economic statistical design) of $\bar{X}$ and R charts is observed to be always less than that of its corresponding design of $\bar{X}$ chart alone.

3. The optimal values of sample size (n) and sampling interval (h) in a continuous process are always found to be less as compared to that of discontinuous process. It is true for economic design as well as economic statistical design. It is also true for individual design of $\bar{X}$ chart as well as joint design of $\bar{X}$ and R charts.

4. In all eight types of design, the value of power of detecting the shift (P) is more in case of discontinuous process compared to continuous process.

5. In economic design, the values of in-control average run length (ARL_0) are significantly high in case of discontinuous process than that of continuous process. It is true for $\bar{X}$ chart. It is also true when $\bar{X}$ and R charts are jointly. However, due to incorporation of constraints on ARL_0, this disparity is eliminated in economic statistical design.

6. Another statistical parameter i.e., out-of-control average run length (ARL_1) is always desired to be as low as possible. In the present work, it is observed to be marginally less in case of discontinuous process than that of continuous process when $\bar{X}$ chart is used alone and also when $\bar{X}$ and R charts are used jointly. This is true for both economic design and economic statistical design. The control charts are able to detect the shift quicker in almost all cases except in joint economic statistical design of $\bar{X}$ and R charts for continuous process.

7. The out-of-control average time to signal (ATS_1) is slightly less in continuous process than that of discontinuous process in case of economic design of $\bar{X}$ chart, and joint economic design of $\bar{X}$ and R charts. This suggests that the economic design of control chart for continuous process triggers a quicker out-of-control signal than that of discontinuous process. The main reason behind this is frequent rate of sampling in case of continuous process.

8. The list of all the significant factors affecting the output responses in all the eight design environments are summarized below in Tables 7.1 - 7.5. The notations shown at the bottom of Table 7.1 are also applicable to Tables 7.2-7.5.

Table 7.1: All results of sensitivity analysis for expected loss cost per unit time *E(L)*

Output responses	Process	Cost and process parameters													
		M	***δ***	***λ***	***g***	**(T_1+T_2)**	***a***	***b***	***W***	***Y***	***ϒ***	**V_0**	***S***	**S_1**	**T_0**
E(L)	*ED-C*	+		**+**		+									
	ED-D	+		**+**		+						+			
	ESD-C	+	–	**+**		+		+							
	ESD-D			**+**		+						+			
	JED-C	+		**+**		+									
	JED-D			**+**		+						+			
	JESD-C	+		**+**		+									
	JESD-D			**+**		+						+			

Note:

+ : Factor with positive effect; – : Factor with negative effect; +/– in bold : Most significant factor
Blank space : Insignificant factor
ED-C : Economic Design - Continuous process
ED-D : Economic Design - Discontinuous process
ESD-C : Economic Statistical Design - Continuous process
ESD-D : Economic Statistical Design - Discontinuous process
JED-C : Joint Economic Design - Continuous process
JED-D : Joint Economic Design - Discontinuous process
JESD-C : Joint Economic Statistical Design - Continuous process
JESD-D : Joint Economic Statistical Design - Discontinuous process

Table 7.2: All results of sensitivity analysis for sample size *n*

Output responses	Process	Cost and process parameters													
		M	***δ***	***λ***	***g***	**(T_1+T_2)**	***a***	***b***	***W***	***Y***	***ϒ***	**V_0**	***S***	**S_1**	**T_0**
n	*ED-C*		**–**		–			–		+					
	ED-D		**–**		–										
	ESD-C		**–**		–		+	–							
	ESD-D	–	**–**		–			–							
	JED-C		–		**–**		+	–							
	JED-D		–		**–**		+	–			–				
	JESD-C		–		–		**+**	–			–				
	JESD-D		–		–		+	**–**							

Table 7.3: All results of sensitivity analysis for sampling interval *h*

Output responses	***Process***	***Cost and process parameters***													
		M	δ	λ	g	(T_1+T_2)	a	b	W	Y	Υ	V_0	S	S_1	T_0
h	*ED-C*	–	–	–		+	**+**	+							
	ED-D	–	–	–		+	+	**+**				+			
	ESD-C		+		**—**		+								
	ESD-D	–	+	–	**—**		+								
	JED-C	–		–	–	+	**+**	+							
	JED-D	–		–	–	+	**+**	+				+			
	JESD-C		+		–		**+**	–							
	JESD-D		+	–	–		**+**								

Table 7.4: All results of sensitivity analysis for width of control limits k_1

Output responses	***Process***	***Cost and process parameters***													
		M	δ	λ	g	(T_1+T_2)	a	b	W	Y	Υ	V_0	S	S_1	T_0
k_1	*ED-C*		+	–	–		–	–		**+**					
	ED-D		+	–	–		–	**—**		+		+			+
	ESD-C		+					**—**		+					
	ESD-D	–	+	–	–			–		+		+			**+**
	JED-C		+	–	–		–	–		**+**	+				
	JED-D		+				–	–		+	+	+			**+**
	JESD-C							–		**+**	+				
	JESD-D	–						**—**		+	+	+			+

Table 7.5: All results of sensitivity analysis for width of control limits k_2

Output responses	***Process***	***Cost and process parameters***													
		M	δ	λ	g	(T_1+T_2)	a	b	W	Y	Υ	V_0	S	S_1	T_0
k_2	*JED-C*		+	–	–		–	–		**+**					
	JED-D		**+**				–	–				+			+
	JESD-C		**+**							+	–				
	JESD-D		**+**					–			–	+			+

Note: The output response k_2 is only valid for joint design of $\overline{X}$ and R charts not for design of $\overline{X}$ chart

9. From the sensitivity analysis it can be concluded that for both economic design and economic statistical design of $\bar{X}$ chart for both the processes, the shift in process mean (i.e., δ) is the most significant factor for selection of sample size (*n*).

10. The sensitivity analysis also suggests that in case of both joint economic design and joint economic statistical design of $\bar{X}$ and *R* charts for both types of processes, the fixed cost of sampling (*a*) is the most significant factor for selection of sampling interval (*h*).

11. In case of all eight design cases, the rate of occurrence of assignable cause (λ) is the most significant factor that affects the loss cost function (i.e., $E(L)_1$ or $E(L)_2$).

12. Three parameters, namely the cost to locate and repair the assignable cause *W*, the expected cost of restart or setup cost *S*, and the startup time S_1 have no significance in any of the eight design cases. However out of them, two factors (i.e., *S* and S_1) are as such not applicable in a continuous process.

7.2 Managerial Implications

The results obtained from the present work are expected to be helpful to the managers in the following directions:

1. The mangers should use SA and TLBO techniques for the economic and economic statistical design of $\bar{X}$ chart as well as joint $\bar{X}$ and *R* charts for better results in case of both continuous and discontinuous processes.
2. Tables 7.1-7.5 are expected to be highly helpful for the quality control personnel to take utmost care in assuming the values of the significant cost and process parameters while designing for $\bar{X}$ as well as joint $\bar{X}$ and *R* charts.

7.2 Limitations of the Work

The present work is not applicable to the following applications:

- any multi-variate or attribute control chart
- for the quality characteristic *X* having non-normal distribution model
- for the process shift due to multiple assignable causes
- for failure mechanisms other than Poisson
- for control chart having time varying design parameters *n*, *h* or *k*

7.3 Scope for Future Work

In the present work, economic and economic statistical design concept has been carried out for an individual $\bar{X}$ chart and joint $\bar{X}$ and R charts using simulated annealing and teaching-learning based optimization techniques under two different process environments. The same can be extended to other control charts like CUSUM, EWMA and attribute control charts when these charts are used individually or few of them are used jointly. The control charts studied in the present work monitors only one quality characteristic. Hence, similar work can be extended to the design of multivariate control charts from economic point of view. Further, similar economic models can be developed by taking various assumptions like non-exponential process failure mechanism and non-normal distribution of quality characteristic. The economic design can also be done by taking objective function as expected profit per unit product.

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Dissemination

Journal Articles

1. **Ganguly, A.** and Patel S.K. (2012) “Economic design of $\bar{X}$ control chart using simulated annealing”, Procedia Engineering, Vol. 38, pp. 1037-1043.

2. **Ganguly, A.** and Patel S.K. (2014) “Joint economic design of X-bar and *R* charts using simulated annealing”, International Journal of Process Management and Benchmarking, Vol. 4, No. 3, pp. 305-323.

3. **Ganguly, A.** and Patel S.K. (2014) “A teaching-learning based optimization approach for economic design of X-bar control chart”, Applied Soft Computing, Vol. 24, pp. 643-653.

4. **Ganguly, A.** and Patel S.K. (2015) “Computer aided design of $\bar{X}$ and *R* charts using teaching-learning-based optimization algorithm”, International Journal of Productivity and Quality Management, Vol. 16, No.3, pp. 325-346.

5. **Ganguly, A.** and Patel S.K. (2016) “Metaheuristic approach for economic statistical design of $\bar{X}$ chart” (Communicated)

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Conference Presentation

1. **Ganguly, A.** and Patel S.K. (2013) “A global optimization approach for economic design of X-bar control chart”, National Conference on Advanced Manufacturing Technology, (NCAMT-2013), NITTTR, Chandigarh.

2. **Ganguly, A.** and Patel S.K. (2013) “Teaching learning based optimization: a novel approach for economic design of X-bar control chart”, International Conference on Advances in Mechanical Engineering, (ICAME-2013), College of Engineering, Pune, Maharashtra.