NUMERICAL ANALYSIS OF ORIFICE-TYPE AEROSTATIC THRUST BEARING

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In

MECHANICAL ENGINEERING

By

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Under The Guidance of

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Rourkela, 2011-2015

CERTIFICATE

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

This is to certify that this thesis entitled "Numerical Analysis of Orifice-type Aerostatic Thrust Bearing" submitted by Bapuji Khatua (Roll No. 111ME0302) in the partial fulfilment of the requirements for the degree of Bachelor of Technology in Mechanical Engineering, National Institute of Technology, Rourkela, is an original and authentic work carried out by him under my supervision.

To the best of my knowledge, the data or matter used in this thesis has not been submitted to any other University/Institute for the award of any degree or diploma.

Place:

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Date:

Place:

ABSTRACT

This thesis provides the study about an analysis of aerostatic thrust bearing. In a very high-speed machine, regular bearing cannot be used. At very high speed about 300000 rpm to 600000 rpm, typical bearing gets high friction and wear. Because of this reason thrust bearings are used. In the analysis part, Reynolds equation for aerostatic thrust bearing have been solved by using FDM (Finite Difference Method).The Finite difference method is a numerical method to solve a nonlinear differential equation that is in non-dimensional form. By the principle of discretization with using MATLAB (Matrix Laboratory) software solutions have been found based on convergence condition. After solving the Reynold’s equation, pressure variation over-bearing has been found. Once the pressure distribution was calculated, load carrying capacity, and mass flow rate was calculated.

NOMENCLATURE

A	:	Throat area of feed jet (mm2)
C_d	:	Coefficient of discharge for an orifice
$\overline{D}$	:	Difference coefficient
d_0	:	Feed hole diameter (mm)
$G(\gamma, P_d/P_a)$	:	Recess pressure function
h	:	Bearing clearance (mm)
M	:	Total number of grid in θ direction
N	:	Total number of grid in radial direction
m	:	Mass flow rate of Bering gas through feed hole(Kg/s)
n	:	Number of feed orifice
P	:	Dimensionless bearing pressure = $\left({}^{p}/_{p_a}\right)$
P_a	:	Dimensionless exhaust pressure
P_d	:	Dimensionless recess pressure= $\left({}^{p_d}/_{p_a}\right)$
P_o	:	Dimensionless supply pressure=$\left({}^{p_o}/_{p_a}\right)$
p	:	Bearing pressure (N/mm2)
p_o	:	Supply pressure (N/mm2)
p_d	:	Recess pressure (N/mm2)
$\bar{R}$	:	Specific gas constant (J/K-mol)
r	:	Radial location in thrust bearing
r_0	:	Feed hole pitch circle radius (mm)
r_1	:	Outer radius of thrust bearing (mm)
r_2	:	Inner radius of thrust bearing (mm)
T	:	Ambient temperature of bearing (K)
W_L	:	Load capacity (N)
$\overline{W_L}$	:	Dimensionless load capacity
Λ_r	:	Feed orifice restrictor coefficient
γ	:	Specific heat ratio of bearing gas

Δ	:	An increment
θ	:	Coordinate
$\bar{\theta}$	:	Bearing sector for computation
μ	:	Dynamic viscosity of bearing gas (N-s/mm2)
ρ_o	:	Supply gas density (Kg/m3)
i	:	Grid location in the theta direction
j	:	Grid location in the radial direction
u	:	Velocity component in x direction
v	:	Velocity component in y direction
w	:	Velocity component in z direction

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CHAPTER 1

1.1 INTRODUCTION

Noncontact type's bearings are the most critical components in the design of an ultra-precision machine. One of the major problems of developing such type of devices for plants is the instability of the rotor at very high speed. For stability of rotor system at high rotational speed, better bearings are required. Gas bearings are one of the solutions of maintain stability and prevent contamination of working fluids. Gas bearings are of various types can be used as Aerostatic gas bearings (externally pressurized gas bearings) and Aerodynamic gas bearings (self-acting).

Gas lubricated externally pressurized bearings consume process gas, and they are suitable only up to a medium rotational speed due to whirl speed limitation. Using aerodynamic bearings such as tilting pad journal bearings, spiral groove thrust and journal bearings have the major issue with inability to damp vibrations due to hard supporting surface. Therefore using aerostatic thrust bearings gives the better solution to overcome this types of problem. The focus of this project is to analyses thrust bearings and finding the pressure profile in Non-dimensional form and calculating the bearing load carrying capacity.

1.2 APPLICATION OF AEROSTATIC BEARINGS

1. Gas lubrication bearings are being used for high temperature, high speed and light load conditions. Aerostatic bearings require external pressure source and can support its designed load at zero speed.

2. Aerostatic bearings are bearing used in grinding, machining, and micro-positioning applications, where ever precision required.

3. Air bearings are now commonly used to support LVDT (linear variable differential transformer) cores to improve reliability and reduce gauging forces.

4. An air bearing has been developed recently for improving accuracy for gear checking

5. Other applications include profile projection equipment employing air bearing slides, rotary measuring tables and machine tool lead screw measuring heads.

6. Aerostatic bearings are being used in turbomachinery like aerospace rotating machines and expansion turbines for the cryogenic application.

1.3 DIFFERENT TYPE OF GAS BEARINGS

Gas bearings fall into two main classes: aerostatic bearings, which demand pressure supply for their operation, moreover aerodynamic bearings, which produce internal pressure. A compressor is needed to provide the pressure in an aerostatic bearing, whereas an aerodynamic bearing generates this pressure by the effect of concurrent shearing and squeezing of the gas between the bearing surfaces in relative motion. Both these kinds of bearings can sustain either radial or axial loads or combine the two functions in an individual member. A bearing can run either entirely as aerostatic or aerodynamic or start in one mode and change over to the different as speed changes. A bearing can also run with a compound of the two aerostatic; also aerodynamic pressure produces such a bearing as hybrid bearing.

An inherent disadvantage of aerostatic bearings is that a pressure source and an exhaust sink are needed. On the advantage side, aerostatic bearings can be prepared to untroubled manufacturing tolerances, can give support at low speeds, and support intermittent or fixed loads. An externally pressurized bearing can work in a dust-laden environment because the exhaust gas restricts the entry of solid particles from the atmosphere. On the other hand, in consequence of the low viscosity of the gaseous lubricant, an aerodynamic bearing can support a small load per unit area only depending upon the speed of the rotor. These bearings need careful manufacture and adjustment and are only fit for bearings whose surfaces are constantly in relative motion during loaded. Nevertheless, auxiliary devices for pressure generation is not required, and there is no difficulty of disposing of that exhaust gas.

Aerostatic Bearings:

Bearings have traditionally been classified according to their function. Those providing radial support for a shaft are called journal bearings and are usually of cylindrical geometry. Bearings that provide an axial location of the rotor and carry axial or thrust loads are called thrust bearings, and usually have flat bearing surfaces. Following this tradition, aerostatic bearings can also be classified into journal and thrust bearing categories. Aerostatic bearings, however, in common with other bearing types, can take the form of combined journal and thrust bearings of conical or spherical geometry.

Aerostatic gas bearings known as Externally Pressurized gas bearings, and an external pressurized air, or process gas is used to maintain pressure between bearing sleeve and the journal. Aerostatic bearings utilize a thin film of high-pressure air to support a load. Since air has a very low viscosity, bearings gaps need to be small, on the order of 1-10 μm. There are five basic types of aerostatic bearing geometries: single pad, opposed pad, journal, rotary thrust, and conical journal/thrust bearings similar to hydrostatic bearings.

Aerodynamic Bearings:

Aerodynamic as automated bearings, and an air microfilm created by the relative movement of two mating surfaces divided by a short distance. From rest, as the speed rises, a velocity produced pressure gradient is created across the clearance. The raised pressure between the surfaces generates the load bearing force. The load capability is dependent on the relative pace at which the surface progresses and consequently at zero speed, the bearing carries no load. Zero loads at zero-speed effect cause beginning and preventing friction and results in a few wearing of the bearing surfaces. Despite some of the difficulties, automatic bearings have found popular use in industry. A typical aerostatic thrust bearing is shown in figure 1.1.

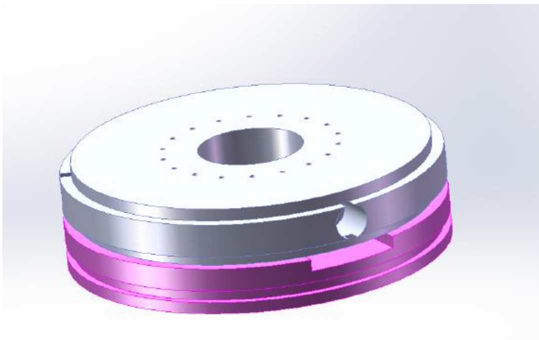


Fig1.1: 3D Model of Aerostatic Thrust Bearing

CHAPTER 2

LITERATURE SURVEY

In an aerostatic bearing examination, the discharge coefficient of a hole is commonly thought to be a consistent and this worth is gotten from test results. There are numerous elements to influence the discharge coefficient. **Belforteetal. [1]** Conducted an analysis study to focus the discharge factor of hole sort restrictors. The outcomes demonstrated that the annular holes and the shallow pocket openings had one discharge factor, and profound pocket openings had to release factors. They likewise introduced estimate capacities for discharge factors in light of the Reynolds number and sustaining framework geometry and analyzed the impact of the pocket profundity (d) on weight dissemination in the pocket. For a given film thickness (h), uniform load dispersion was accomplished in the pocket if d≥h.

Chen and He [2] examined the impact of the broken shape on the bearing execution. They found that the rectangular recessed orientation had higher load limit than the round recessed and non-recessed heading. The mass stream rate was biggest in the rectangular recessed bearing and littlest in the non-recessed bearing. Additionally, they inspected the impact of the hole measurement on the bearing execution. At an individual supply weight and film thickness, load limit expanded with the increment of the opening distance across. **LiandDing [3]** mulled over the impact of the geometrical parameters of the aerostatic push direction with took opening sort restrictors on bearing execution. They demonstrated that course performed well if opening distance across and film thickness were little and air chamber measurement was vast. Disregarding the impact of opening length on bearing execution may bring about expensive slips if hole size was adequately small.

Schenk etal. [4] showed that the whole limit expanded directly and bearing stiffness developed quickly at the working moment that supply pressure grown. **Chenetal. [5]** Studied the impacts of the operational conditions and geometric parameters on the solidness of the aerostatic diary course. They built up a solid hypothetical model to figure trial results accepted the gas-bearing firmness and this model. The outcomes demonstrated that for a given film thickness, solidness enhanced when supply weight expanded. They likewise found that the stiffness of the took hole was higher than that of the innate opening.

Pneumatic flimsiness is a critical issue in the investigation of aerostatic bearing. **Ye et al. [6]** contemplated the impact of the broken shape on pneumatic pounding and found that the non-recessed aerostatic orientation were bringing on less pneumatic pounding than the recessed aerostatic heading. They likewise contemplated the relationship between the supplied weight and the pneumatic precariousness. For a given burden, diminishing supply pressure may increment pneumatic strength. They also inspected the impact of break volume on pneumatic insecurity and found that a bigger break volume brought on more self-energized vibrations. **Talukder and Stowell [7]** contemplated pneumatic pounding in a remotely pressurized hole repair aerostatic bearing. They demonstrated that pneumatic pounding identified with break volume and opening breadth and evaded in log course. Their trial results showed that pneumatic pounding can be forestalled utilizing permit supply pressure little opening distance across, high-load operation and outside damping. **Bhatetal. [8]** Concentrated on the execution of characteristically remunerated level cushion aerostatic orientation subjected to element irritation strengths. They reasoned that pneumatic mallet flimsiness had a tendency to happen at low annoyance frequencies little hole breadths expansive crevice statues and vast supply weights. The consequences of **[6–8]** suggest that the aerostatic course working at small film thicknesses created less pneumatic pounding.

Nakamura and Yoshimoto [9, 10] mulled over the static tilt attributes of the aerostatic rectangular twofold cushion push orientation with compound restrictors. They considered the impacts of connected burden sorts on tilt minute furthermore analyzed the compound restrictor and the food opening restrictor tilt minutes **[9].** The outcomes demonstrated that the aerostatic push heading with compound restrictors had bigger tilt minutes. They additionally demonstrated that the twofold cushion force course had higher firmness than the single cushion drive heading. In a consequent study, they looked at the tilt stiffness of the single column and dual line confirmation push direction **[10].** The outcomes demonstrated that the twofold line affirmation push instruction can enhance tilt stiffness in pitch

CHAPTER 3.

THEORITICAL ANALYSIS OF BEARINGS

3.1: DERIVATION OF REYNOLDS EQUATIONS

In almost all practical bearings, the flow in the bearings clearings is laminar, and pressure losses occur due to viscous shearing in the gas film. Consider a finite volume inside the bearings. We apply and equate the force and taking proper assumptions.

Assumptions:

- Inertia forces are neglected compared with frictional forces.
- Newtonian fluid and Laminar flow conditions.
- The Coefficient of viscosity is constant.
- The Pressure is constant over any section normal to the direction of flow.
- Boundary elements are at the no-slip condition.
- Steady-state system
- No stretching condition

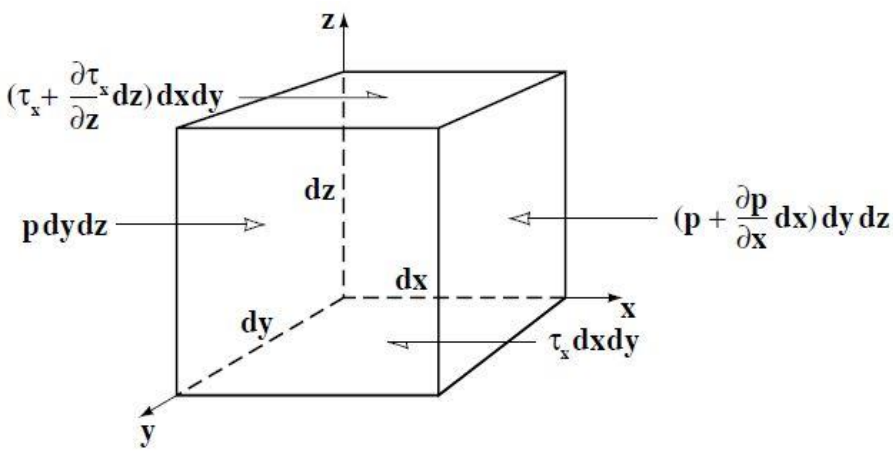


Fig 3.1: Force applied on infinitesimal element in the bearing

Balancing the forces in x direction we get

$$\text{pdydz} + \left(r + \frac{\partial \tau}{\partial y} dy\right) dxdz = \left(p + \frac{\partial p}{\partial x} dx\right) dydz + \tau dydz \quad (3.1)$$

$$\text{i.e.} \frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x} \quad (3.2)$$

For laminar flow of Newtonian Fluid

$$\tau = \eta \frac{\partial u}{\partial y} \tag{3.3}$$

From equations (3.2) and (3.3),

$$\frac{\partial}{\partial x}\left(\eta \frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} \tag{3.4}$$

For constant coefficient of viscosity

$$\eta \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} \tag{3.5}$$

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u}{\partial y^2} \tag{3.6}$$

Similarly force balance in z direction we get

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 w}{\partial y^2} \tag{3.7}$$

Integrating equation (3.6) we get

$$\eta \frac{\partial u}{\partial y} = \frac{\partial p}{\partial x} y + C_1 \tag{3.8}$$

$$\eta u = \frac{\partial p}{\partial x} \frac{y^2}{2} + C_1 y + C_2 \tag{3.9}$$

At y=0, $u=u_2$ and

At y=h, $u=u_1$

For no slip at solid boundary, i.e. liquid has same boundary as the solid plate

$$\eta u_2 = C_2 \tag{3.10}$$

$$\eta u_1 = \frac{\partial p}{\partial x} \frac{h^2}{2} + C_1 h + \eta u_2 \tag{3.11}$$

Solving the above, we get

$$u = \left(\frac{y^2 - yh}{2\eta}\right) \frac{\partial p}{\partial x} + (u_1 - u_2)\frac{y}{h} + u_2 \tag{3.12}$$

Similarly in z direction we get,

$$w = \left(\frac{y^2 - yh}{2\eta}\right)\frac{\partial p}{\partial x} + (w_1 - w_2)\frac{y}{h} + w_2 \tag{3.13}$$

For Incompressible fluid the equation of continuity incompressible flow is given by,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{3.14}$$

Integrating the continuity equation in y direction from y=0 to y=h,

Using Leibnitz Rule,

$$\int_a^b \frac{\partial u(y,x)}{\partial x} dy = \frac{\partial}{\partial x}\left(\int_a^b u dy\right) - u(b,x)\frac{db}{dx} + u(a,x)\frac{da}{dx} \tag{3.15}$$

In our case the Leibnitz equation becomes,

$$\int_0^h \frac{\partial u(y,x)}{\partial x} dy = \frac{\partial}{\partial x}\left(\int_0^h u(x,y) dy\right) - u(x,h)\frac{dh}{dx} + u(x,0)\frac{d0}{dx} \tag{3.16}$$

$$u(x,0)\frac{d0}{dx} = 0 \tag{3.17}$$

$$\int_0^h \frac{\partial u}{\partial x} dy = \frac{\partial}{\partial x}\left(\int_0^h u dy\right) - u_1\frac{dh}{dx} \tag{3.18}$$

$$\int_0^h u dy = \left[\left[\frac{1}{2\eta}\left(\frac{y^3}{3} - \frac{y^2 h}{2}\right)\right]\frac{\partial p}{\partial x} + (u_1 - u_2)\frac{y^2}{2} + u_2 y\right]_0^h = -\frac{h^3}{12\eta}\frac{\partial p}{\partial x} + \frac{h}{2}(u_1 + u_2) \tag{3.19}$$

$$\therefore \int_0^h \frac{\partial u}{\partial x} dy = \frac{\partial}{\partial x}\left(-\frac{h^3}{12\eta}\frac{\partial p}{\partial x} + \frac{h}{2}(u_1 + u_2)\right) - u_1\frac{\partial h}{\partial x} \tag{3.20}$$

$$= \frac{\partial p}{\partial x}\left(-\frac{h^3}{12\eta}\frac{\partial p}{\partial x} + \frac{h}{2}(u_1 + u_2) - u_1 h\right) = -\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right) - \frac{1}{2}\frac{\partial}{\partial x}(u_1 - u_2)h \tag{3.21}$$

Similarly in z direction:

$$\int_0^h \frac{\partial w}{\partial z} dy = -\frac{\partial}{\partial z}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right) - \frac{1}{2}\frac{\partial}{\partial z}\left((w_1 - w_2)h\right) \tag{3.22}$$

Hence, on integrating the continuity equation

$$\int_0^h \frac{\partial u}{\partial x} dy + \int_0^h \frac{\partial v}{\partial y} dy + \int_0^h \frac{\partial w}{\partial z} dy = 0 \tag{3.23}$$

$$\Rightarrow -\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right) - \frac{1}{2}\frac{\partial}{\partial x}\big((u_1 - u_2)h\big) + (v_1 - v_2) - \frac{\partial}{\partial z}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial z}\right) - \frac{1}{2}\frac{\partial}{\partial z}\big((w_1 - w_2)h\big) = 0 \tag{3.24}$$

$$\Rightarrow -\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial z}\right) = \frac{1}{2}\frac{\partial}{\partial x}\big((u_1 - u_2)h\big) + (v_1 - v_2) + \frac{1}{2}\frac{\partial}{\partial z}\big((w_1 - w_2)h\big) \tag{3.25}$$

In the above equation left hand side term is pressure term, right hand side term is source term, $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial z}$ are the stretching terms and $(v_1 - v_2) = \rho\frac{\partial h}{\partial t}$ term is called squeeze term for compressible condition.

Thus, the Reynolds Equation comes out to be:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial z}\right) = \frac{1}{2}\frac{\partial}{\partial x}(\rho(u_1 - u_2)h) + \rho\frac{\partial h}{\partial t} + \frac{1}{2}\frac{\partial}{\partial z}(\rho(w_1 - w_2)h) \tag{3.26}$$

Further for relative tangential velocity only in x-direction and not in z-direction and no stretching condition the final Reynolds equation is

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial z}\right) = \frac{1}{2}(u_1 - u_2)\frac{\partial(\rho h)}{\partial x} + \rho\frac{\partial h}{\partial t} \tag{3.27}$$

3.2: REYNOLDS EQUATION IN POLAR COORDINATE

Usual gas bearing assumptions along with the condition $(u_1 = u_2)$ leads to the following equation in the case of a thrust bearing:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial z}\right) = \frac{1}{2}\rho\frac{dh}{dt} \tag{3.28}$$

Further, if we assume that the gas film is steady, thrust faces are perfectly aligned, isothermal condition $\rho = {}^{p}/_{\bar{R}T}$ and constant viscosity, the Reynolds equation is obtained

$$\frac{\partial^2 p^2}{\partial x^2} + \frac{\partial^2 p^2}{\partial z^2} = 0 \ \ \textbf{[15]} \tag{3.29}$$

Let $p^2 = P$, $x = r\cos\theta$ and $z = r\ \sin\theta$, where r is the function of (x, z), and θ is the angle r makes with the positive x direction

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} = 0 \tag{3.29a}$$

By partial differentials:

$$\frac{\partial P}{\partial r} = \frac{\partial P}{\partial x}\frac{\delta x}{\delta r} + \frac{\partial P}{\partial z}\frac{\delta z}{\delta r} \tag{3.30}$$

As δx and $\delta z \rightarrow 0$:

$$\frac{\partial P}{\partial r} = \frac{\partial P}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial P}{\partial z}\frac{\partial z}{\partial r} = \frac{\partial P}{\partial x}\cos\theta + \frac{\partial P}{\partial z}\sin\theta \tag{3.31}$$

$$\frac{\partial P}{\partial \theta} = \frac{\partial P}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial P}{\partial z}\frac{\partial z}{\partial \theta} = \frac{\partial P}{\partial x}(-r\sin\theta) + \frac{\partial P}{\partial z}\ r\cos\theta \tag{3.32}$$

By multiplying$(r\cos\theta)$ in equation (3.31) and $(\sin\theta)$ in equation (3.32) and after rearrangement, we get:

$$\frac{\partial P}{\partial x} = cos\,\theta\frac{\partial P}{\partial r} - \frac{sin\,\theta}{r}\frac{\partial P}{\partial \theta} \tag{3.33}$$

By multiplying $(r\sin\theta)$ in equation (3.31) and $(\cos\theta)$ in equation (3.32) and after rearrangement, we get:

$$\frac{\partial P}{\partial z} = \sin\theta\frac{\partial P}{\partial r} + \frac{\cos\theta}{r}\frac{\partial P}{\partial \theta} \tag{3.34}$$

$$\frac{\partial}{\partial x} \equiv cos\,\theta \left(\frac{\partial}{\partial r}\right) - \frac{sin\,\theta}{r}\left(\frac{\partial}{\partial \theta}\right) \tag{3.35}$$

$$\frac{\partial}{\partial z} \equiv sin\,\theta \left(\frac{\partial}{\partial r}\right) + \frac{cos\,\theta}{r}\left(\frac{\partial}{\partial \theta}\right) \tag{3.36}$$

Let these two equation (3.35) and (3.36) are the operators**. [16]**

Differentiate equation (3.33) with respect to x and equation (3.34) with respect to z, we get

$$\frac{\partial}{\partial x}\left(\frac{\partial P}{\partial x}\right) = cos\,\theta\ \frac{\partial}{\partial r}\left(\frac{\partial P}{\partial x}\right) - \frac{sin\,\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{\partial P}{\partial x}\right)$$

$$\Rightarrow\ \frac{\partial^2 P}{\partial x^2} = \cos\theta\ \frac{\partial}{\partial r}\left(\cos\theta\frac{\partial P}{\partial r} - \frac{\sin\theta}{r}\frac{\partial P}{\partial \theta}\right) - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\cos\theta\frac{\partial P}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\right)$$

$$\Rightarrow \frac{\partial^2 P}{\partial x^2} = \cos\theta\left(\cos\theta\frac{\partial^2 P}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial P}{\partial \theta}\right) - \frac{\sin\theta}{r}\left(-\sin\theta\frac{\partial P}{\partial r} - \frac{\sin\theta}{r}\frac{\partial^2 P}{\partial \theta^2} - \frac{\cos\theta}{r}\frac{\partial P}{\partial \theta}\right)$$

$$\Rightarrow \frac{\partial^2 P}{\partial x^2} = \cos^2\theta\frac{\partial^2 P}{\partial r^2} + \frac{\cos\theta\sin\theta}{r^2}\frac{\partial P}{\partial \theta} + \frac{\sin^2\theta}{r}\frac{\partial P}{\partial r} + \frac{\sin^2\theta}{r}\frac{\partial^2 P}{\partial \theta^2} + \frac{\cos\theta\sin\theta}{r^2}\frac{\partial P}{\partial \theta}$$

$$\Rightarrow \frac{\partial^2 P}{\partial x^2} = \cos^2\theta\frac{\partial^2 P}{\partial r^2} + \frac{2\cos\theta\sin\theta}{r^2}\frac{\partial P}{\partial \theta} + \frac{\sin^2\theta}{r}\frac{\partial P}{\partial r} + \frac{\sin^2\theta}{r}\frac{\partial^2 P}{\partial \theta^2} \tag{3.37}$$

And

$$\frac{\partial}{\partial z}\left(\frac{\partial P}{\partial z}\right) = \sin\theta\ \frac{\partial}{\partial r}\left(\frac{\partial P}{\partial z}\right) + \frac{\cos\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{\partial P}{\partial z}\right)$$

$$\Rightarrow\ \frac{\partial^2 P}{\partial z^2} = \sin\theta\ \frac{\partial}{\partial r}\left(\sin\theta\frac{\partial P}{\partial r} - \frac{\cos\theta}{r}\frac{\partial P}{\partial \theta}\right) + \frac{\cos\theta}{r}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial P}{\partial r} - \frac{\cos\theta}{r}\frac{\partial P}{\partial \theta}\right)$$

$$\Rightarrow \frac{\partial^2 P}{\partial z^2} = sin\,\theta\left(sin\,\theta\frac{\partial^2 P}{\partial r^2} - \frac{cos\,\theta}{r^2}\frac{\partial P}{\partial \theta}\right) + \frac{cos\,\theta}{r}\left(cos\,\theta\ \frac{\partial P}{\partial r} - \frac{sin\,\theta}{r}\frac{\partial P}{\partial \theta} + \frac{cos\,\theta}{r}\frac{\partial^2 P}{\partial \theta^2}\right)$$

$$\Rightarrow \frac{\partial^2 P}{\partial z^2} = sin^2\,\theta\frac{\partial^2 P}{\partial r^2} - \frac{cos\,\theta sin\,\theta}{r^2}\frac{\partial P}{\partial \theta} + \frac{cos^2\,\theta}{r}\frac{\partial P}{\partial r} + \frac{cos^2\,\theta}{r^2}\frac{\partial^2 P}{\partial \theta^2} - \frac{cos\,\theta sin\,\theta}{r^2}\frac{\partial P}{\partial \theta}$$

$$\Rightarrow \frac{\partial^2 P}{\partial z^2} = sin^2\,\theta\frac{\partial^2 P}{\partial r^2} - \frac{2cos\,\theta sin\,\theta}{r^2}\frac{\partial P}{\partial \theta} + \frac{cos^2\,\theta}{r}\frac{\partial P}{\partial r} + \frac{cos^2\,\theta}{r^2}\frac{\partial^2 P}{\partial \theta^2} \tag{3.38}$$

Putting the value of equations no (3.37) and (3.38) in equation no (3.29a), we get

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial z^2}=\cos^2\theta\frac{\partial^2 P}{\partial r^2}+\frac{2\cos\theta\sin\theta}{r^2}\frac{\partial P}{\partial\theta}+\frac{\sin^2\theta}{r}\frac{\partial P}{\partial r}+\frac{\sin^2\theta}{r}\frac{\partial^2 P}{\partial\theta^2}+\sin^2\theta\frac{\partial^2 P}{\partial r^2}-\frac{2\cos\theta\sin\theta}{r^2}\frac{\partial P}{\partial\theta}+\frac{\cos^2\theta}{r}\frac{\partial P}{\partial r}+\frac{\cos^2\theta}{r^2}\frac{\partial^2 P}{\partial\theta^2}=0$$

$$\Rightarrow \frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial z^2}=\frac{\partial^2 P}{\partial r^2}(\cos^2\theta+\sin^2\theta)+\frac{\partial P}{\partial r}\frac{(\cos^2\theta+\sin^2\theta)}{r}+\frac{\partial^2 P}{\partial\theta^2}\frac{(\cos^2\theta+\sin^2\theta)}{r^2}=0$$

$$\Rightarrow \frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}+\frac{1}{r^2}\frac{\partial^2 P}{\partial\theta^2}=0 \qquad (3.39)$$

The equation (3.39) is called Laplace equation or Reynolds equation in a polar form that will be used for solving aerostatic thrust bearing problem.

3.3: NON-DIMENSONALIZATION

With the help dimensionless parameters

$\bar{P}={}^{P}/_{P_a}=({}^{p}/_{p_a})^2$ And $\bar{r}={}^{r}/_{r_1}$ and putting the value of P and r in equation (3.39).The dimensionless Reynolds equation will be

$$\frac{P_a}{r^2}\frac{\partial^2\bar{P}}{\partial\bar{r}^2}+\frac{P_a}{\bar{r}r^2}\frac{\partial\bar{P}}{\partial\bar{r}}+\frac{P_a}{\bar{r}^2r^2}\frac{\partial^2\bar{P}}{\partial\theta^2}=0$$

$$\Rightarrow \frac{\partial^2\bar{P}}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial\bar{P}}{\partial\bar{r}}+\frac{1}{\bar{r}^2}\frac{\partial^2\bar{P}}{\partial\theta^2}=0 \qquad (3.40)$$

3.4: RELATIONS FOR MASS FLOW

In aerostatic thrust bearings, the gas is supplied to the bearing clearance through feed holes drilled in bearing wall. Consider flow through nozzle of the same throat diameter as the jet, the relationship between supply pressure and at the throat is given by **[14]:**

$$\frac{P_d}{P_o}=\left[1-\frac{\gamma-1}{2}\left(\frac{v}{a}\right)^2\right]^{\frac{\gamma}{\gamma-1}} \qquad (3.41)$$

And mass flow through the jet

$$m=C_d\rho_d Av \qquad (3.42)$$

For isentropic expansion

$$\rho_d = \rho_o \left(\frac{P_d}{P_o}\right)^{\frac{1}{\gamma}} \tag{3.43}$$

Then substituting for the speed of sound in the supply condition $a = (\gamma RT)^{0.5}$ and rearranging, gives the mass flow through a jet or orifice restrictor as:

$$m = C_d A \rho_o (2RT)^{0.5} \left[\frac{\gamma}{\gamma-1}\left\{\left(\frac{P_d}{P_o}\right)^{\frac{2}{\gamma}} - \left(\frac{P_d}{P_o}\right)^{\frac{\gamma+1}{\gamma}}\right\}\right]^{\frac{1}{2}} \tag{3.44}$$

The mass of bearings gas flowing out of the outer edges and inner edges of the annular thrust plate can be expressed as **[11]:**

$$m_1 = \frac{({P_d}^2-1)\pi h^3}{12\mu \bar{R} T {p_a}^2 \ln \overline{r_o}} \tag{3.45}$$

$$m_2 = \frac{({P_d}^2-1)\pi h^3}{12\mu \bar{R} T {p_a}^2 \ln\left(\frac{\overline{r_o}}{\overline{r_2}}\right)} \tag{3.46}$$

Equating the inward and outward flow, the feed hole pitch circle radius can be expressed as **[11]:**

$$\overline{r_o} = \sqrt{\overline{r_2}} \tag{3.47}$$

Equating the mass flow rate of bearing gas through all the orifice to that flowing out through the bearing edges gives an equation involving the dimensionless recess pressure P_d:

$$\left(\frac{{P_d}^2-1}{nP_o}\right)\left[\frac{1}{\ln\left(\frac{\overline{r_o}}{\overline{r_2}}\right)} - \frac{1}{\ln(\overline{r_o})}\right] = \Lambda_r \, G\left(\gamma, \frac{P_d}{P_a}\right) \tag{3.48}$$

Where, the restrictor coefficient

$$\Lambda_r = \frac{12\mu C_d d_o (\bar{R}T)^{0.5}}{p_a h^2} \qquad (3.48)$$

And, the recess pressure function

$$G\left(\gamma, \frac{P_d}{P_a}\right) = \left[\frac{2\gamma}{\gamma-1}\left\{\left(\frac{P_d}{P_o}\right)^{\frac{2}{\gamma}} - \left(\frac{P_d}{P_o}\right)^{\frac{\gamma+1}{\gamma}}\right\}\right]^{0.5} \quad (3.49)$$

CHAPTER 4.

NUMERICAL METHODS

4.1: APPROACH TO THE SOLUTION

Some approximate numerical methods must be adopted to solve the Reynolds' Equation. In order to find the pressure profile, we are using finite difference method. This chapter consists of a detailed description of the Finite Difference Method, which is used for discretization of the Reynolds' Equation. The Reynolds' Equations was written in Finite Difference form and solved by means of an iterative procedure. Radial and theta direction is divided into the small segment, and pressure at each node is obtained by iterative process by solving The Reynold's Equation (3.40).

4.2: FINITE DIFFERENCE FORM

The Reynolds equation is converted into the finite difference form using certain Finite Difference Approximations:

$$\frac{\partial \overline{P}}{\partial \overline{\theta}} = \frac{P_{i+1,j} - P_{i-1,j}}{2\Delta\theta} \tag{4.1}$$

$$\frac{\partial^2 \overline{P}}{\partial \overline{\theta}^2} = \frac{P_{i+1,j} - 2P_{i,j} + P_{i-1,j}}{\Delta\theta^2} \tag{4.2}$$

$$\frac{\partial \overline{P}}{\partial \overline{r}} = \frac{P_{i,j+1} - P_{i,j-1}}{2\Delta\overline{r}} \tag{4.3}$$

$$\frac{\partial^2 \overline{P}}{\partial \overline{r}^2} = \frac{P_{i,j+1} - 2P_{i,j} + P_{i,j-1}}{2\Delta\overline{r}^2} \tag{4.4}$$

Putting these value (4.1), (4.2), (4.3) and (4.4) in equation no (3.40), the equations will be

$$\frac{P_{i,j+1} + P_{i,j-1} - 2P_{i,j}}{\Delta\overline{r^2}} + \frac{1}{r}\frac{P_{i,j+1} - P_{i,j-1}}{2\Delta\overline{r}} + \frac{1}{\overline{\overline{r^2}}}\frac{P_{i+1,j} + P_{i-1,j} - 2P_{i,j}}{\Delta\theta^2} = 0$$

$$\Rightarrow \frac{P_{i,j+1}}{\Delta\overline{r^2}} + \frac{P_{i,j-1}}{\Delta\overline{r^2}} - 2\frac{P_{i,j}}{\Delta\overline{r^2}} + \frac{P_{i,j+1}}{2r\Delta\bar{r}} - \frac{P_{i,j-1}}{2r\Delta\bar{r}} + \frac{P_{i+1,j}}{\overline{\overline{r^2}}\Delta\theta^2} + \frac{P_{i-1,j}}{\overline{\overline{r^2}}\Delta\theta^2} - \frac{2P_{i,j}}{\overline{\overline{r^2}}\Delta\theta^2} = 0$$

$$\Rightarrow \left(\frac{P_{i,j+1}}{\Delta\overline{r^2}} - \frac{P_{i,j-1}}{2r\Delta\bar{r}}\right) + \left(\frac{P_{i,j+1}}{\Delta\overline{r^2}} + \frac{P_{i,j-1}}{\overline{\overline{r^2}}\Delta\theta^2}\right) + \left(\frac{P_{i+1,j}}{\overline{\overline{r^2}}\Delta\theta^2} + \frac{P_{i-1,j}}{\overline{\overline{r^2}}\Delta\theta^2}\right) = P_{i,j}\left\{\frac{2}{\Delta\overline{r^2}} + \frac{2}{\overline{\overline{r^2}}\Delta\theta^2}\right\}$$

$$\Rightarrow P_{i,j} = \left(\frac{P_{i,j+1} + P_{i,j-1}}{\bar{D}\Delta\overline{r^2}}\right) + \left(\frac{P_{i,j+1} - P_{i,j-1}}{2r\Delta\bar{r}\bar{D}}\right) + \left(\frac{P_{i+1,j} + P_{i-1,j}}{\overline{r^2}\Delta\theta^2\bar{D}}\right)$$

$$\Rightarrow P_{i,j} = A + B + C \qquad (4.5)$$

Where

$$A = \frac{P_{i,j+1} + P_{i,j-1}}{\bar{D}\Delta\bar{r}^2} \qquad (4.6)$$

$$B = \frac{P_{i,j+1} - P_{i,j-1}}{2\bar{r}\bar{D}\Delta\bar{r}^2} \qquad (4.7)$$

$$C = \frac{P_{i+1,j} + P_{i-1,j}}{\bar{D}\Delta\bar{r}^2\Delta\theta^2} \qquad (4.8)$$

Where

$$\bar{D} = 2\left[\left(\frac{1}{\Delta\bar{r}}\right)^2 + \left(\frac{1}{\bar{r}\Delta\theta}\right)\right] \qquad (4.9)$$

To solve equation no (4.5), the boundary conditions are

$$\bar{\mathrm{P}}(\bar{\mathrm{r}}_1, \theta) = 1 \qquad (4.10)$$

$$\bar{\mathrm{P}}(\bar{\mathrm{r}}_2, \theta) = 1 \qquad (4.11)$$

$$\left.\frac{\partial\bar{\mathrm{P}}}{\partial\theta}\right|_{\bar{\mathrm{r}},\theta=0} = \left.\frac{\partial\bar{\mathrm{P}}}{\partial\theta}\right|_{\bar{\mathrm{r}},\theta=\bar{\theta}} = 0 \qquad (4.12)$$

$\bar{\mathrm{P}} = (\bar{\mathrm{r}}_o, \bar{\theta}) = \mathrm{P_d}^2$ … **[13]** (4.13)

The first two conditions (4.10) and (4.11)) gives that the pressure at the outer and inner edges is ambient. The third (4.12) is due to the effect of symmetric pressure distribution along the circumferential direction. The fourth (4.13) expresses the input of recess at the location of feed orifice which is calculated from equation (3.48) by bisection method with two end conditions are chocking pressure and supply pressure. The choking pressure of feed holes is given by the relation **[15]:** $P_d = P_o \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ .After finding the pressure distribution over thrust bearing, the dimensionless total load carrying capacity **[12]** can be calculated as given below

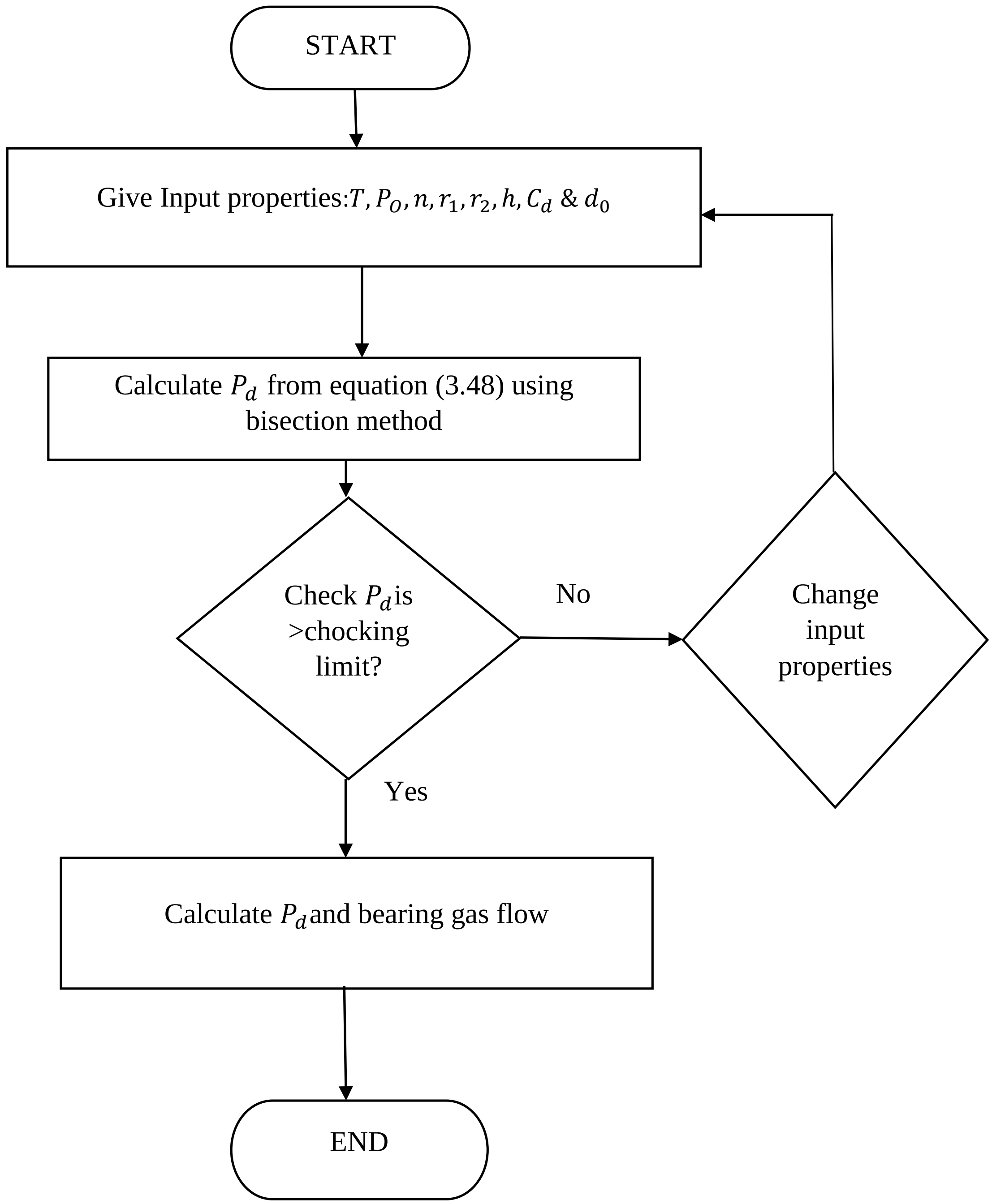
$$\overline{W_L} = 2n \iint_{0\ \overline{r_1}}^{\overline{\theta}\ \overline{r_2}} (P-1)\bar{r}d\bar{r}\, d\theta \qquad (4.14)$$

Where

$$\overline{W_L} = \frac{W_l}{P_a {r_1}^2} \qquad (4.15)$$

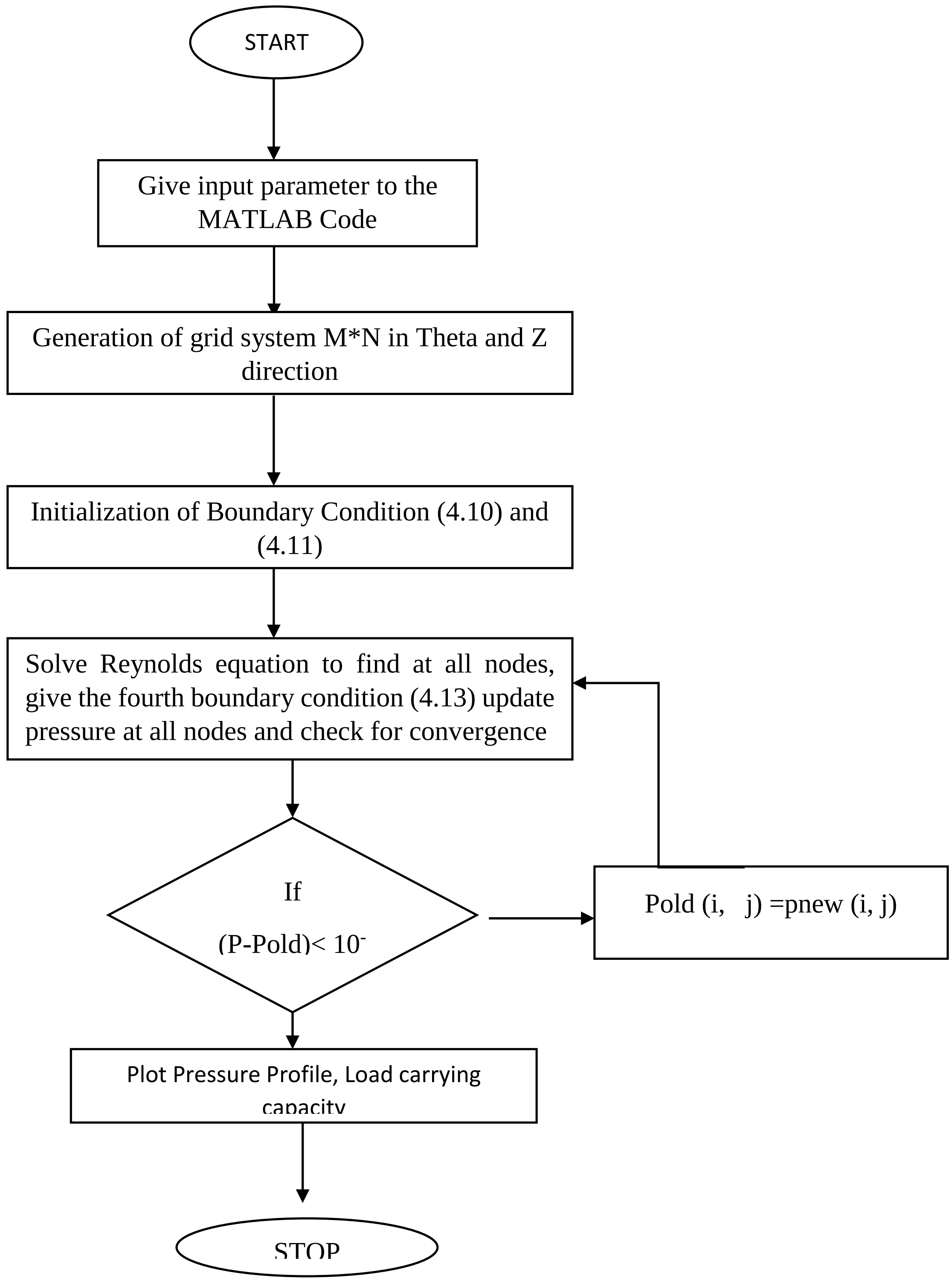
FLOWCHART: 1

A program has been Witten in MATLAB to compute the P_d value which is fourth boundary for solving the dimensionless Reynolds equation and flow rate. The flow chart for the program is given below:



FLOWCHART: 2

After finding the recess pressure now we can move on for finding Pressure profile of aerostatic thrust bearing. An Algorithm is given below:



CHAPTER: 5

RESULTS AND DISSCUSSIONS

We are using air as bearing lubrication and after theoretical analysis code have been written in MATLAB to solve the Non-dimensional equation. By giving various inputs, the results are obtained in terms of Non-dimensional pressure profile, Load carrying capacity and Flow rate. The input parameters and other results are given below.

Sl. No.	Input Property	Value	Unit
1.	Outer Radius of Bearing(r_1)	50	mm
2.	Inner Radius of Bearing(r_2)	30	mm
3.	Feed hole Pitch circle Radius (r_0)	38.73	mm
4.	Bearing Clearance(h)	0.005	mm
5.	Feed hole or orifice diameter(d_0)	0.5	mm
6.	Ambient Temperature of Bearing(T)	300	K
7.	Supply Pressure(P_0)	0.25	N/mm2
8.	Ambient Pressure(P_a)	0.1	N/mm2
9.	Co-efficient Of viscosity(μ)	0.1827	N-s/mm2
10.	Co-efficient Of discharge(C_d)	0.95	-
11.	Specific Gas Constant($\bar{R}$)	8.314	J/k-mol
12.	Specific Heat ratio of Bearing gas(air) (γ)	1.4	-

Table 5.1: Input Properties of Bearing gas (air) and Bearing Specifications.

5.1 PRESSURE PROFILE

The three-dimensional pressure profile under the given input parameters for single orifice aerostatic thrust bearing as shown in the Fig (5.1.1). As we can see that theta varies from 0 to $2\pi/$ where n is 24.Similarly, the z bar axis varies from 0.6 to 1.the pressure profile represented in the form of non-dimensional pressure$\left(\frac{P}{P_a}\right)$.

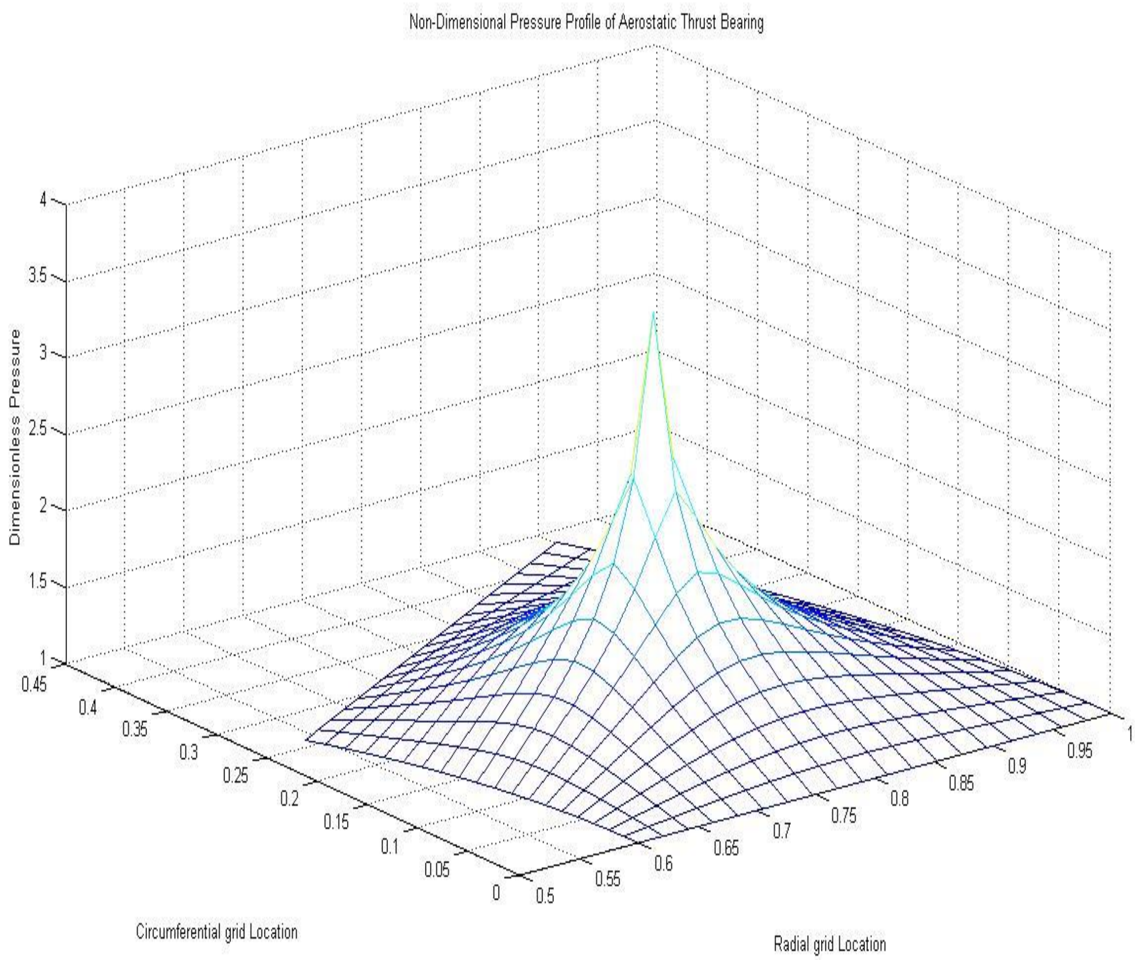


Fig 5.1.1: Dimensionless pressure profile of Aerostatic Thrust Bearing with one orifice

Typically the Pressure variation along the circumferential direction and radial direction are shown in Fig (5.1.2) and Fig (5.1.3). Considerable distance from the maximum non-dimension pressure to the outer edges the pressure profile is atmospheric in nature.

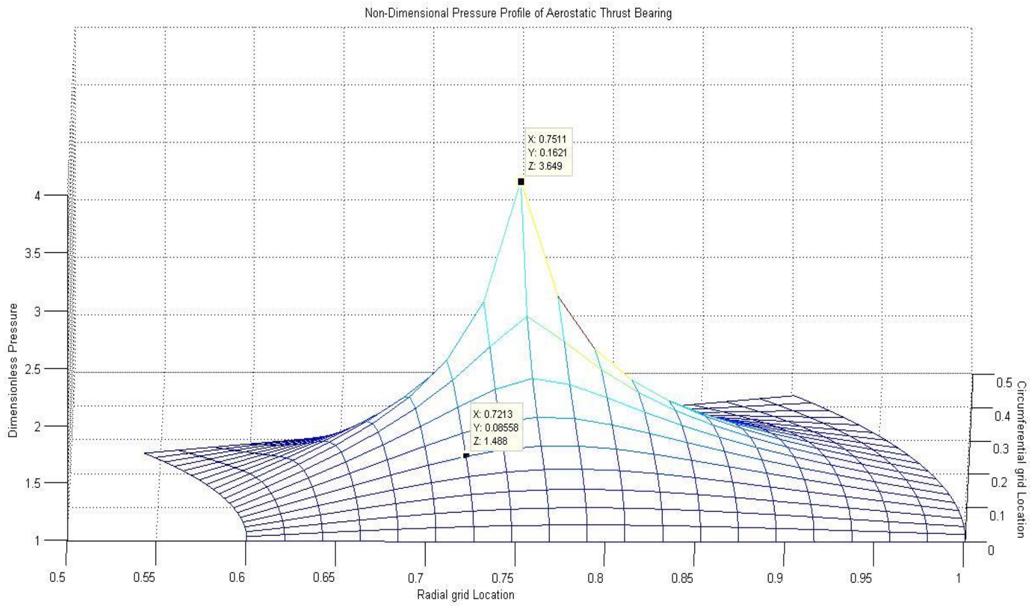


Fig 5.1.2: V**ariation of dimensionless pressure along radial grid location**

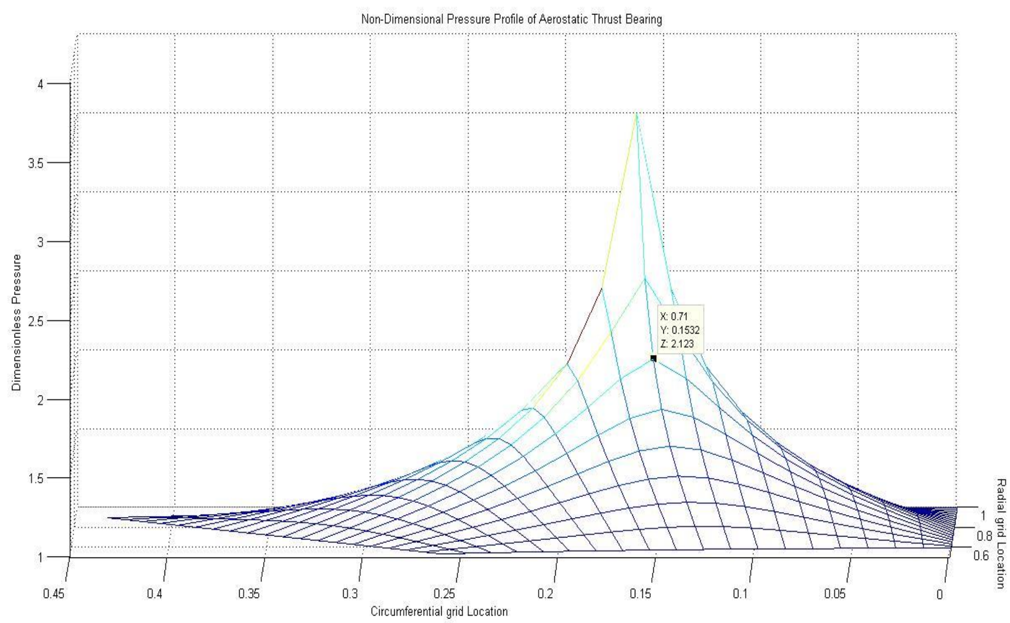


Fig **5.1.3: Variation of dimensionless pressure along circumferential grid location**

Fig (5.1.4) shows the Non-Dimensional pressure profile of Aerostatic Thrust Bearing with 24 feed orifice and supply pressure of 0.35 N/mm2 is given below.

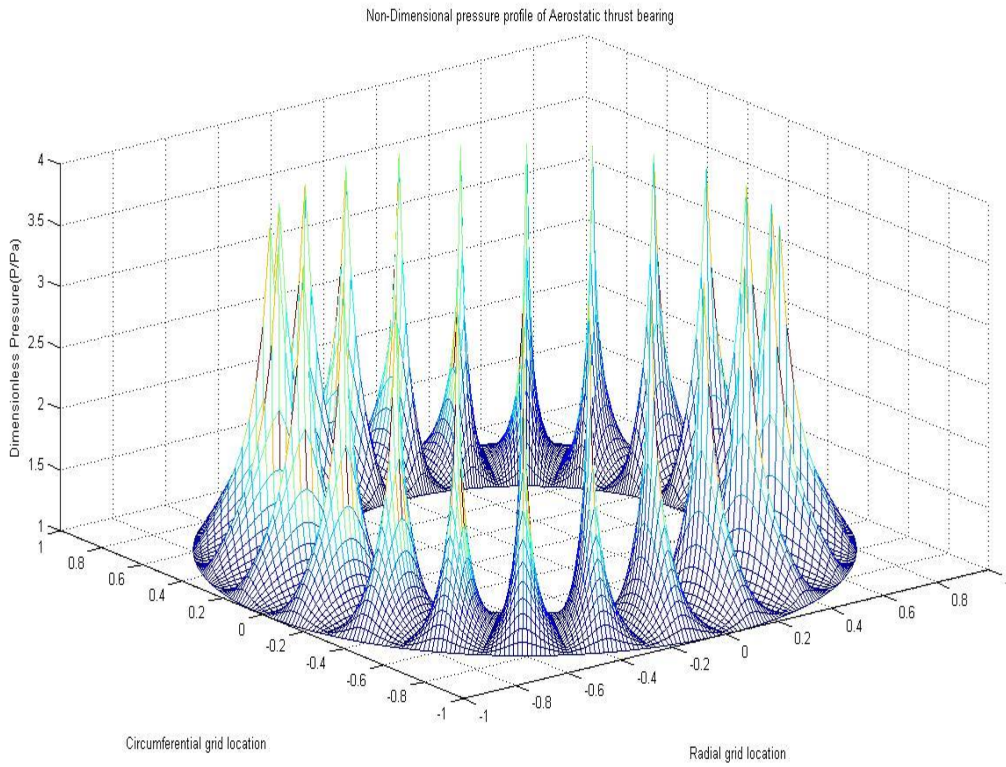


Fig 5.1.4:Non-Dimensional pressure distribution of aerostatic thrust bearing with 24 orifice

5.2 LOAD CARRYING CAPACITY

Variation of load carrying capacity of aerostatic thrust bearing is given below in Fig (5.2.1). We can see that for given input parameters and constant supply pressure the load carrying capacity is first increases and then decreases by varying number of feed hole or orifice. So before using aerostatic thrust bearing one should use optimum number of feed hole for given input supply pressure and bearing parameters. Here we can see that the maximum non-dimensional load carrying capacity 35.09 at 12 number of feed orifice for supply pressure 0.25 $N/_{mm^2}$.

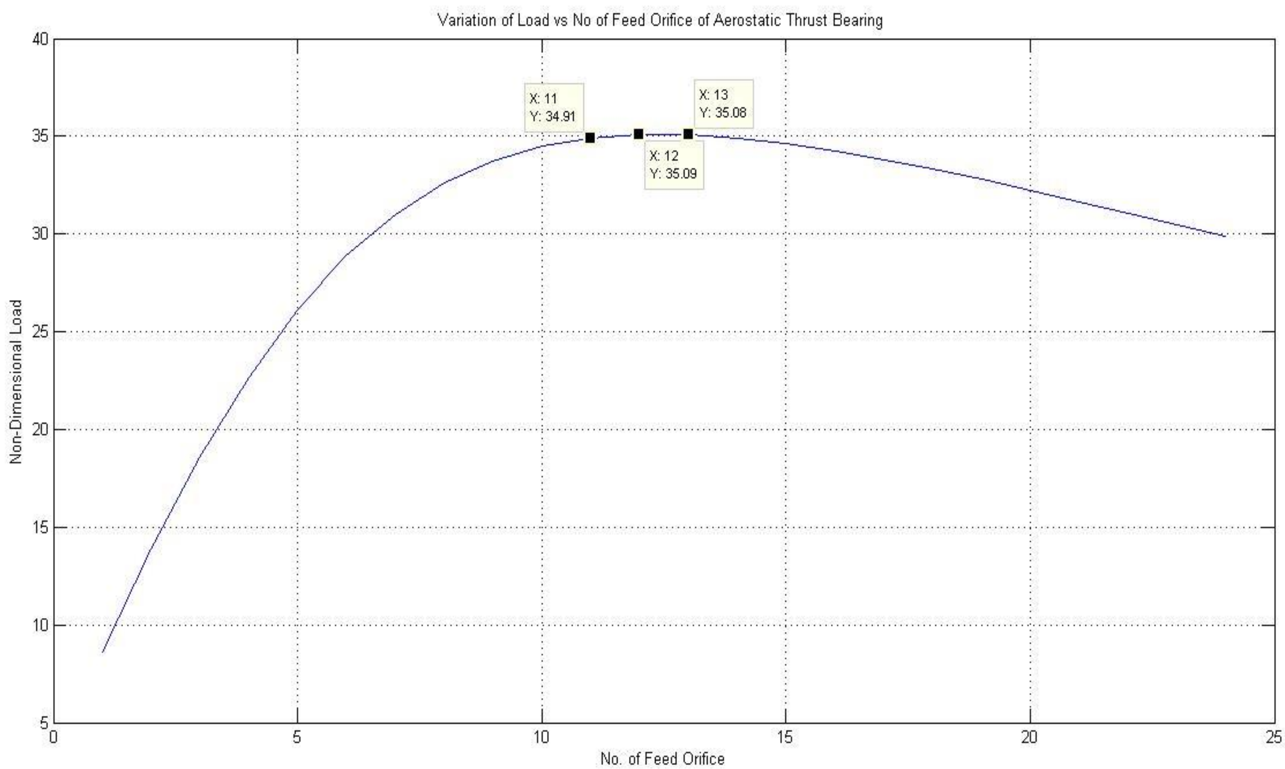


Fig 5.2.1: Variation of Non-Dimension load vs Number of orifice

5.3 FLOW RATE

Flow rate depends on number of feed orifice and diameter of feed hole. If we increase feed hole diameter the flow rate increases in parabolic in nature. In this project diameters are varying from 0.1 to 0.5 mm and the flow rate increases from 0.001172 $\frac{m^3}{s}$ to 0.0293 $\frac{m^3}{s}$. Similarly if we increase the number of feed orifice the flow rate increases linearly. If we increase the number of feed orifice from 1 to 24 the flow rate increases from 0.001221 $\frac{m^3}{s}$ to 0.0293 $\frac{m^3}{s}$.The graphs are given below in Fig (5.3.1) and Fig (5.3.2).

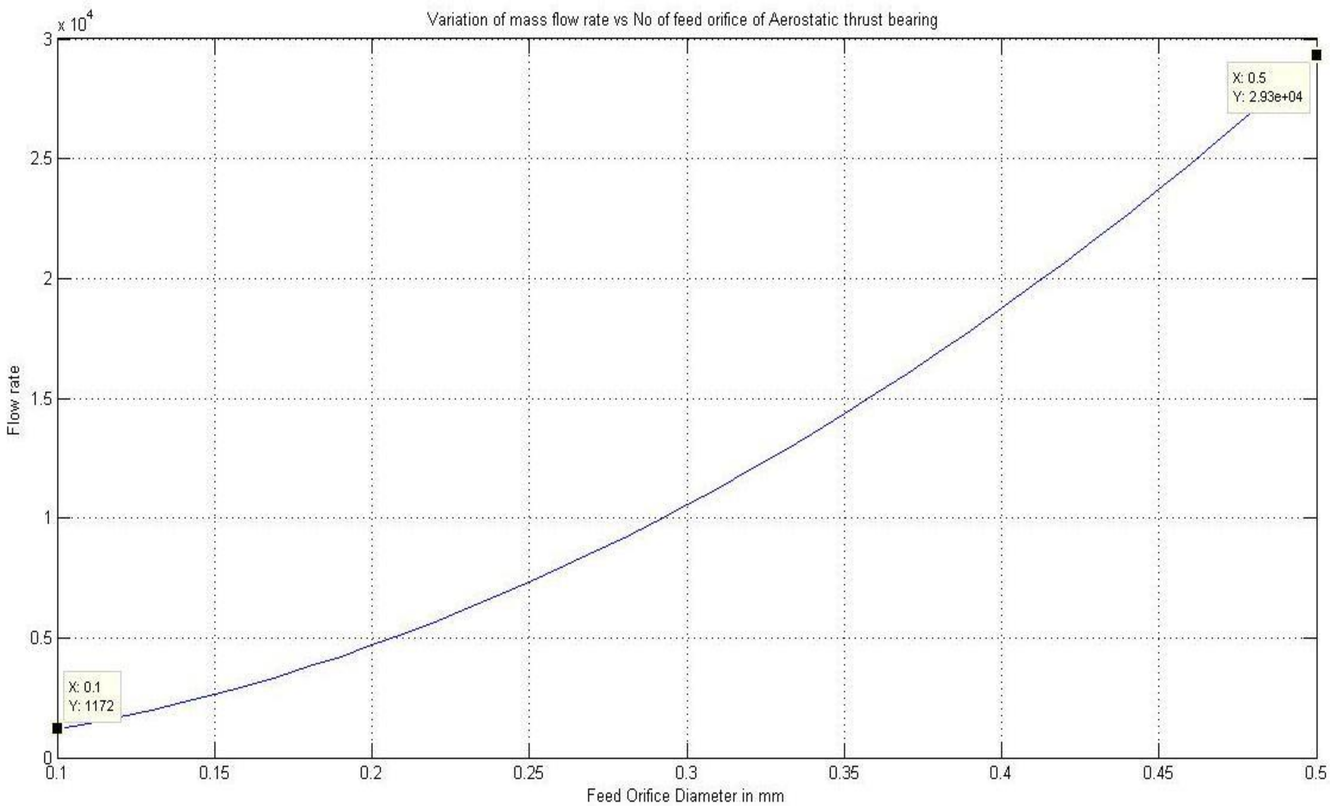


Fig 5.3.1: Variation of Flow rate vs Feed orifice diameter

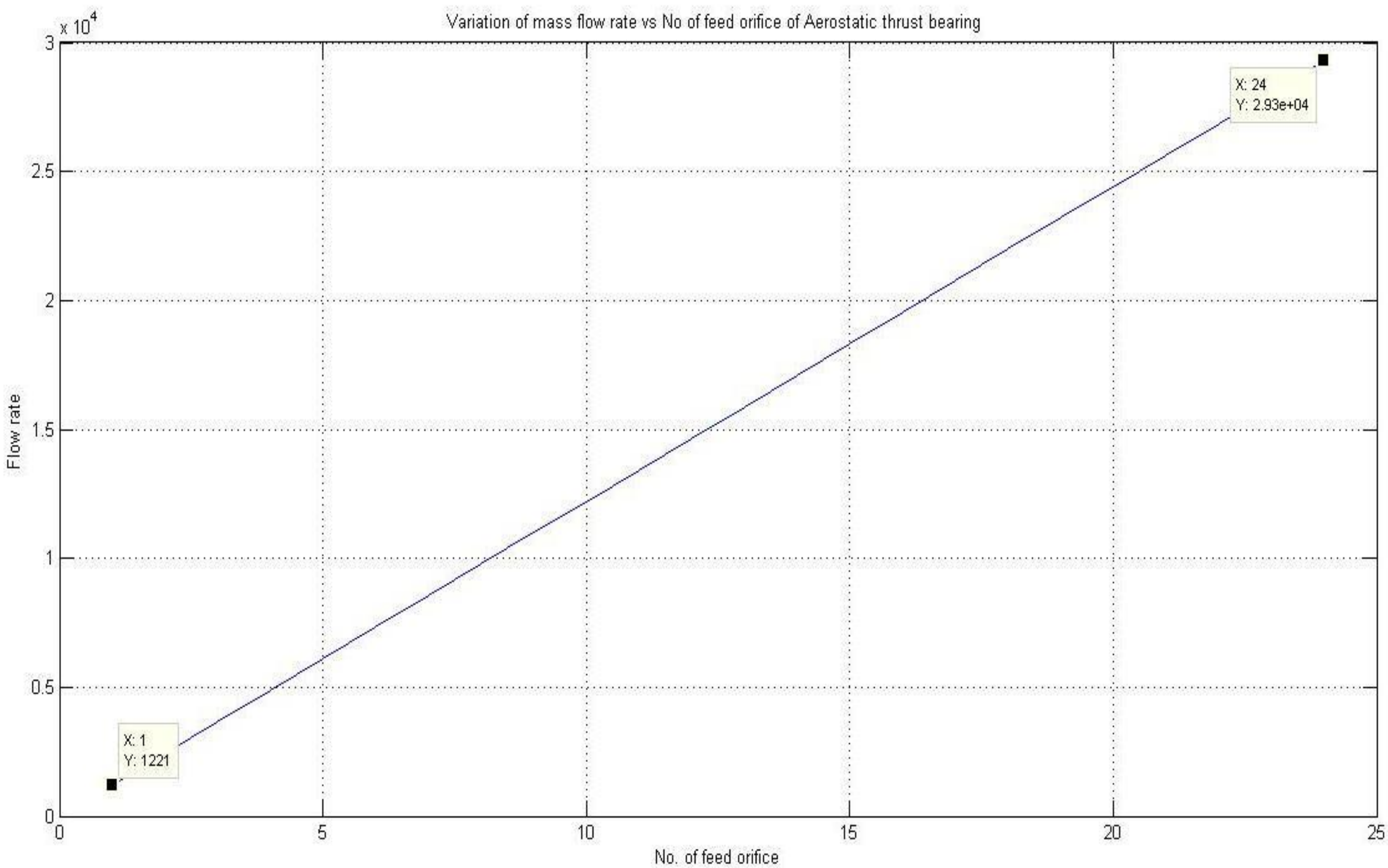


Fig 5.3.2: Variation of Flow rate vs Number of Feed orifice

CONCLUSION

The ever growing needs of the high speed and oil-free turbomachinery like aerospace rotating machinery, micro turbines, expansion turbines for cryogenic application and other forms of turbo machinery requires aerostatic rather aerodynamic. This is because, in many applications where repeated start and stop is necessary, aerostatic bearings are found to be superior to aerodynamic bearings. In current research, the aerostatic thrust bearings have been critically analyzed for their operating parameters, which will be help full to the researchers around the world. The main principle of operation of aerostatic gas bearings is governed by Reynolds equation, which is a non-linear differential equation and there is no close form solution for the same. Current project, a numerical method (Finite Difference Method) is used to solve the non-dimensionalized Reynolds equation. Also, an attempt was made to find the performance parameters like pressure profile over the bearing surface, the variation of load carrying capacity with no of feed holes and flow rate with several assumptions.

REFERENCES

1. **Belforte G,** Raparelli T, ViktorovV Trivella A. Discharge coefficients of orifice-type restrictorforaerostaticbearings.TribologyInternational2007; 40:512–21.
2. **Chen XD, He XM.** The effect of the recess shape on performance analysis of the gas-lubricatedbearinginopticallithography.TribologyInternational2006; 39: 1336–41.
3. **Li Y, Ding H.** Influences of the geometrical parameters of aerostatic thrust bearing with pocketed orifice-type restrictor on its performance. Tribology International2007; 40:1120–6.
4. **Schenk C, Buschmann S**,Risse S, Eberhardt R, Tunnermann A. Comparison between flat aerostatic gas-bearing pads with orifice and porous feedings at high-vacuumconditions.PrecisionEngineering2008;32:319–28.
5. **Chen YS, ChiuCC, ChengYD**. Influences of operational conditions and geometricparametersonthestiffnessofaerostaticjournalbearings.Precision Engineering 2010; 34:722–34.
6. **Ye YX, Chen XD**, Hu YT, Luo X .Effects of recess shapes on pneumatic hammering in aerostatic bearings .Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology2010; 224:231–7.
7. **Talukder HM, Stowell TB**. Pneumatic hammer in an externally pressurized orifice compensated air journal bearing. Tribology International2003; 36: 585–91.
8. **Bhat N, Kumar S**, Tan W, Narasimhan R, Low TC. Performance of inherently compensated flat pad aerostatic bearings subject to dynamic perturbation forces. PrecisionEngineering2012; 36:399–407.
9. **Nakamura T**, Yoshimoto S. Static tilt characteristics of aerostatic rectangular double-pad thrust bearings with compound restrictors .Tribology International 1996;29(2):145–52.
10. **Nakamura T**, Yoshimoto S. Static tilt characteristics of aerostatic rectangular double-pad thrust bearings with double row admissions .Tribology International 1997;30(8):605–11.
11. **Powell, J.W**. Design of Aerostatic Bearings The Machinery Publishing Co. Ltd., New England London (1970)

12. **Chakravarty anindya**, Analytical and Experimental studies on Gas Bearings for Cryogenic Turboexpanders , Thrust bearings governing equation,(2000),63
13. **Chakravarty anindya**, Analytical and Experimental studies on Gas Bearings for Cryogenic Turboexpanders, Thrust bearings governing equation, (2000), 62
14. **J. P. Khatait, W. Lin and W. J. Lin,** Design and development of orifice-type aerostatic thrust bearing, Flow through the orifice,(2005),3
15. **S. H Chang, C. W. Chan and Y.R. Jeng,** Numerical analysis of discharge coefficients in aerostatic bearings with orifice-type restrictors,(2015),6
16. **Gohar ramsey,** Electro hydrodynamics, Imperial college press, London,(2001)