Non Fourier Heat Transfer Across A Gun Barrel

A Thesis Submitted in Partial Fulfilment of the Requirements for the Degree of Bachelor of Technology in Mechanical Engineering

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Certificate

This is to certify that the work in this thesis entitled "**NON FOURIER HEAT TRANSFER ACROSS A GUN BARREL**"by **Jeet Mohapatra** has been strictly carried out under my supervision in partial fulfilment of the requirements for the degree of **Bachelor of Technology** in **Mechanical Engineering** during session **2011- 2015** in the Department of Mechanical Engineering, National Institute of Technology, Rourkela.

To the best of my knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.

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Declaration

I hereby declare that this thesis is my own work and effort. Throughout this documentation wherever contributions of others are involved, every endeavour was made to acknowledge clearly with due reference to literature. This work is being submitted for meeting the partial fulfilment for the Degree of Bachelor of Technology in Mechanical Engineering at National Institute of Technology, Rourkela for the academic session 2011–2015.

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Motivation

Non-Fourier heat transfer has always been an area of interest in this modern era, which finds its applications in numerous research and technological fields. In defence sector, it is used to study the heat transfer phenomena occurring inside rifles and artilleries. Right from my childhood days, I had a proclivity towards weapons and was always instigated with its eccentric designs developed with the state-of-the-art technology. I always had a curiosity that, even after attaining such peak temperatures during firing, how the manufacturers were able to overcome the wear and abrasion related problems. This motivated me to pursue this project and study the behavior of Non-Fourier heat transfer and wear and lubrication problems related with it. My Professor was always a source of motivation for me who exhorted me to take on this project as my final year research project and guided me throughout the year, for which, I am deeply thankful to him.

Abstract

In this project, study of hyperbolic heat transfer equation across a gun barrel is investigated. First, a comparison is made among several research work on parabolic and hyperbolic equations contributed in this field. Then, equations are formulated for Non-Fourier heat equations. These governing equations are normalized after which, a characteristic peclet number is obtained which depends on local sound speed and thermal diffusion speed. The peclet number is varied in the experiment, following which, a transition is seen from the hyperbolic nature to the parabolic curve.

Keywords: Hyperbolic heat transfer; peclet number; relaxation time.

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List of Symbols

∇	Gradient operator
$P_e = \frac{\mu r_o}{\alpha}$	Peclet number
r	Radial distance
r_i	Inner radius of the cylindrical enclosure
r_o	Outer radius of the cylindrical enclosure
T	Temperature
T_l	Temperature at the outer radius
T_h	Temperature at the inner radius
$\xi = \frac{r}{r_o}$	Normalization parameter for radious
$\eta = \frac{\alpha t}{r_o^2}$	Normalization parameter for time
$\theta = \frac{T-T_l}{T_h-T_l}$	Normalization parameter for temperature
q	Heat flux
k	Thermal conductivity
τ	Thermal Relaxation Time
α	Thermal Diffusivity
μ	Speed of sound in that medium
$\frac{dq}{dt}$	Rate of change of heat flux
C_p	Specific heat at constant pressure

Chapter 1

Literature Review

1.1 A Brief Introduction to Non-Fourier Heat Transfer

1.1.1 Fourier Equation

Fouriers classical heat conduction theory states that heat flux linearly depends on temperature gradient and the propagation speed of thermal wave across the given medium is infinite, i.e. there is no lag between the heat flux and temperature gradient. Thus, any thermal disturbance or thermal gradient experienced by a body is instantaneously felt throughout the whole body

The general expression for the Fourier heat transfer equation is

$$q = -k\nabla T \tag{1.1}$$

Where,
k= thermal conductivity
q= heat flux
∇T= temperature gradient

1.1.2 Non-Fourier Equation

But in some situations which involve-

1. extreme thermal gradients [1]
2. very short times
3. high heat flux [2]

4. temperatures near absolute zero condition (such as in NaF at about 4K) [3]

Fourier heat diffusion equation fails in such situations and Non-Fourier heat diffusion equation becomes more reliable and accurate in describing the diffusion process and finding out the temperature distribution profile. Here, we introduce a time constant (τ) or which is also known as *'Thermal Relaxation Time'*. This τ accounts for the phase lag between the heat flux and the temperature gradient.

Cattaneo [4] and Verenotte [5] proposed the first Non-Fourier Heat Transfer Model(also known as CV Model) which is the hyperbolic heat transfer equation, unlike the parabolic heat transfer equation of Fourier Model.

$$q = -k\nabla T - \tau\frac{\partial q}{\partial t} \tag{1.2}$$

But the limitation of this model was the entropy generation calculated which was found to be of negative value for semi-infinite body, under time-dependent heat flux boundary conditions, which was against the 2^{nd} law of thermodynamics. In this case, the assumption of local thermal equilibrium was not valid. Thus, the CV Model was modified and various other models were proposed, such as

Nimr et al. [6] developed the *Hyperbolic 2 Step Model* in which laser heating of metals comprises of two steps of energy transfer, which occur in tandem. In the first step, most of the incident radiation energy is absorbed by the electrons. This excited electron gas transmits its energy to the lattice through an inelastic electron-phonon scattering process. In the second step, the electron gas and the solid lattice maintains a thermal equilibrium. Inside the film, the incident radiation absorbed by the metal diffuses in space in all directions by the electron gas. This finds its applications in laser surgery and laser heating of solids.

Dong et al. [7] proposed the macroscopic model, known as the *Phonon Kinetic Model*, for fast-transient heating and heat transport in naosystems. They showed that, in non- equilibrium system, the static pressure of thermomass is lower than its total pressure. Non-equilibrium temperature is close to that in the extended irreversible thermodynamics, thus providing Non-Fourier conduction law.

Some of the practical applications of Non-Fourier heat transfer in industrial world are-

- Zhao et al. [8] developed a model based on combined radiation and conduction heat transfer to predict the effective thermal conductivity of fibrous insulation at various temperatures and pressures. The spectral extinction coefficients and Rosseland mean extinction coefficients were obtained at various temperatures to investigate the radiative heat transfer in fibrous insulation.

- Collin et al. [9] performed a numerical simulation water curtains which acted as a radiative shield. They combined the Eulerian-Lagrangian code for the dynamics of the mist with the Monte Carlo simulation of the radiative transfer in the air-droplet medium.

- Raj et al. [10] analysed the solidification of semitransparent material. The solidification was assumed to occur at a range of temperatures and thus the presence of a mushy zone was considered. The radiative component of the energy equation in the LBM formulation was computed using the discrete transfer method.

- Kim et al. [11] developed a combined hyperbolic radiation and conduction heat transfer to simulate multi-time-scale heat transfer in turbid tissues exposed to short-pulsed irradiations. They compared the hyperbolic conduction with the traditional parabolic heat diffusion model. It was found that the maximum local temperatures are significantly larger in the hyperbolic prediction than in the parabolic prediction. After 10 thermal relaxation times, thermal waves fade away and hyperbolic and parabolic models shoe similar results.

1.2 Previous Contribution to the Study of Gun Barrel

During firing, gun barrels are subjected to a large amount of heat input at the bore surface. Heat transfer is mostly due to forced convection by the hot combustion gases generated inside the barrel. The temperature reaches to about 1000 Kelvin inside the barrel. There is a high flow of heat flux along with the bullet within a fraction of seconds. Therefore, its temperature distribution follows the non Fourier model. Its study is crucial for us to determine the wear rate of the gun, its overhauling and number of shots required to reach its cook-off temperature. In between the shot fires, the gun barrel is naturally cooled by convection and radiation [12] at its outer surface. However, across the barrel, heat is transferred through conduction, where the outer temperature is the atmospheric temperature and the inner temperature is the temperature of the bullet. The Pressure inside the barrel rises abruptly (about 60,000 Psi) and then decreases with time, thus creating shock waves inside the barrel.

Some of the recent contributions in the field of Non-Fourier heat transfer are:

Hak In Gimm, Ki Up Cha and Chang Ki Cho [13] investigated the shock vibrations characteristics for a medium caliber gun barrel. They carried out a numerical modal analysis, a signaling processing technique and a shock response analysis to calculate mode frequencies using the periodogram. Vibration and shock bearing analysis was also carried out using Fourier Transform method. Avinash Mishra, Amer Hameed and Bryan Lawton computed simulation of gun barrel temperature [14] by setting different operational parameters. He reduced the convective heat input to the bore surface and performed the rise in the bore temperature. Bishri Abdel-Hamid [2] in his experiment showed the transformation of hyperbolic heat conduction into parabolic heat conduction when the relaxation time approaches zero. He used the finite integral method with periodic heat flux (insulated on both the sides) to find the damping heat diffusion coefficient of the Non Fourier conduction. Reza Shirmohammadi also presented the analytical solution of thermal behavior of a hollow sphere with periodic boundary heat flux [15], which showed a hyperbolic trend. He used the method of separation of variables and Duhamel's integral theorem to obtain the solution. By comparing the results of parabolic and hyperbolic model, the transition process from Fourier to Non-Fourier thermal behavior was shown.

Then Subhash C. Mishra made a computational analysis of Non-Fourier combined radiation and conduction heat transfer in a particular medium through concentric cylinders by suddenly perturbing its temperature [16]. Parameters such as the extinction coefficient, the conduction-radiation parameter, the emissivity and the radius ratio were studied.ZHANG Zhe and LIU Dengying [17] conducted the experiment in rapid transient state in a solid sphere to establish Non-Fourier heat transfer. He determined Non-Fourier equations analytically to solve problems involving high-rate change of temperature. Frankel also studied the hyperbolic heat conduction in a finite slab subjected to rectangular heat pulse [18].

Bin Wu, Gang Chen and Wei Xia [19] in their experiment showed the effectiveness of the integral mid-wall cooling channels-which used forced convection- over air cooling-which used convective and radiation heat transfer- with their finite element analysis method. This was also accrued by Pu Qu, Qiang Li, and Zhen Yang [20] in their paper where they used a liquid-solid coupled method to decrease the cook-off temperature of the barrel. In their experiment, they applied an inter-layer cooling method with a fluid with its saturated velocity due to which, temperature decreases at successive intervals,

Chapter 2

Gun Barrel

A gun barrel is a long cylindrical enclosure tube through which hot gases expand rapidly in a short period of time resulting in an abrupt rise in temperature and pressure. This expansion of gases causes the projectile to propel out from the other end at high velocities. A gun barrel must be able to sustain the expanding gas to ensure that optimum muzzle velocity is obtained. It is usually made of Chromium steel alloy, cast iron and wrought iron. Chrome Molybdenum steel is used in high stress applications. It provides resistance to abrasion and corrosion and gives specific load tolerances. In manufacturing industries, chrome vanadium is formed by cold drawing and then it undergoes heat treatment before sending for fabrication, which induces the properties to bear shock loads at elevated temperatures (approx 800-900 °C).

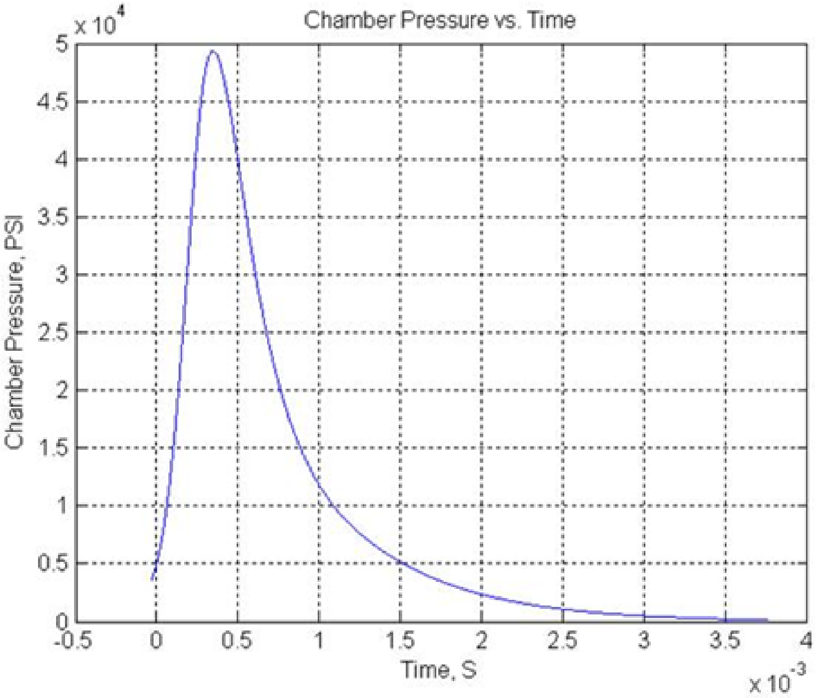

Figure 2.1: Chamber Pressure Vs Time

2.1 Design and manufacturing Process of Gun Barrel

- Straightening of barrel -

  High Precision is required

- Barrel Drilling-

  The centre of the Barrel is drilled by carbine trip drills which rotate with an rpm of 1800-3000.

- Barrel Honing

  It is an abrasive machining process which polishes inside of the barrel to remove any variation in the bore diameter. 3 progressive diamond tip honing spindles are used to incrementally remove the material from the bore. Its main advantages are-

  1. It produces more uniform and controlled barrel geometry, size and shape.
  2. Excellent super-finishing operation.
  3. Improves straightness, cylindricity and roundness. It also corrects taper errors.
  4. Low cost, high performance, easily automated and highly consistence.

- Button Rifling-

  It is the process of carving helical groves in the barrel to impart a spinning motion to the projectile along its axis. This spins gyroscopically improves the aerodynamic stability of the projectile. It is of 2 types-

  1. Muzzle-loader rifling
  2. centre-fire rifling

- Annealing and tempering

  Annealing is the process of heating materials above its recrystallization temperature, maintaining its temperature and then subsequent cooling to the room temperature in ambient condition. It is performed to change the physical and chemical properties of the material. It increases ductility, softens the material, relieves internal stresses, makes it homogeneous and improves its cold working properties.

  Tempering is the process of controlled heating of material below its critical point, which removes the excess hardness present in the material. The amount of hardness removed depends on its temperature.

The barrels are placed in a sealed chamber with an inner atmosphere. After subsequent heating and cooling back to ambient temperature, the toughness of steel increases significantly.

- Tapering crowning-

  Tapering is the process of gradual reduction of the diameter of the bore. The forward portion of the gun barrel is tapered towards the muzzle, since less strength is required for the reduced pressure when the projectile approaches it, thus decreasing its weight and increasing its strength-to-weight ratio.

- The barrel is polished before the taking it to the final production step.

- TIG plasma welding-

  It attaches tenon to the barrel block, using automated computer controlled accuracy. It is used because it provides high precision of welding with low distortion of material.

- Mechanization of barrel-

  The side attachment holes are drilled to provide a passage for the exit of gases. Cooling fins are extruded or machined about the chamber area, which provides excellent cooling to the barrel.

- Sand Blasting-

  This process is used to smooth a rough surface and to remove deposits from the surface. In this, abrasive sand is induced into the steam of compressed air, due to which, it gets accelerated in the outflow from the blasting nozzle.

- Nickel Plating or bluing-

  The final step involves the electroplating a thin layer of nickel onto the barrel. It provides a corrosion resistance, wear resistance and a shiny surface on the metal.

2.2 Properties of Chrome Molybdenum Steel

(AISI 4140- annealed form) [12]-

Table 2.1: Composition of Chrome Molybdenum steel

Element	**Weight %**
Carbon (C)	0.38-0.43
Iron (Fe)	96.8-97.8
Manganese (Mn)	0.75-1.00
Phosphorus (P)	0.035 (max)
Sulphur (S)	0.04 (max)
Silicon (Si)	0.15-0.35
Chromium (Cr)	0.80-1.10
Molybdenum (Mo)	0.15-0.25

Table 2.2: Physical Properties of Chrome Molybdenum steel

Property	**Magnitude**
Density (gm/cc)	7.8
Poisson's Ratio	0.27-0.30
Elastic Modulus (GPa)	210
Tensile Strength (Mpa)	920
Yield Strength (Mpa)	660
Elongation (%)	26.0
Reduction in Area (%)	48.4
Hardness (HB)	207
Impact Strength (J)	27.4
Specific Heat Capacity(J/kg-K)	450
Thermal Expansion ($10^{-6}/^{\circ}C$)	11.8
Thermal diffusivity	12
Thermal Conductivity(W/m-K)	43

Chapter 3

Fourier Heat Transfer

The inner and outer radius, r_i and r_o, of a cylinder are maintained at temperatures T_i and T_o, with $T_i > T_o$. Heat is assumed to be flowing in radial direction only, and there is no conduction of heat along the length of the closure. The rate of heat transfer radially through a thin cylinder of thickness dr is:

$$Q = -kA\frac{dT}{dr} \tag{3.1}$$

$$\Rightarrow q = -k\frac{dT}{dr} \tag{3.2}$$

This is known as Fourier law.

In vector form,

Rate of heat change across the cylinder in radial direction = rate of energy accumulation across the control volume. Thus,

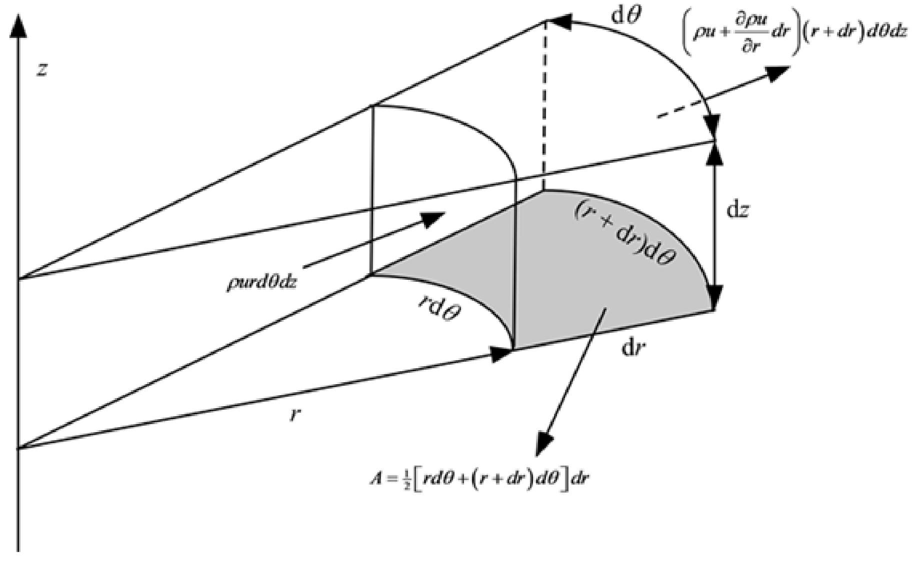

Figure 3.1: Fourier heat transfer across a cylinder

$$dQ_r - dQ_{r+dr} = krd\theta dydzr\frac{\partial^2 T}{\partial r^2} + kd\theta dydzdr\frac{\partial T}{\partial r} \tag{3.3}$$

$$kd\theta dzr\frac{\partial^2 T}{\partial r^2} + kd\theta dzdr\frac{\partial T}{\partial r} = \rho cdr(rd\theta dz)\frac{\partial T}{\partial t} \tag{3.4}$$

$$k\frac{\partial^2 T}{\partial r^2} + k\frac{\partial T}{\partial r} = \rho c\frac{\partial T}{\partial t} \tag{3.5}$$

$$\frac{1}{r}\frac{\partial}{dr}(r\frac{\partial T}{dr}) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3.6}$$

where $\alpha = \frac{k}{\rho c_p}$

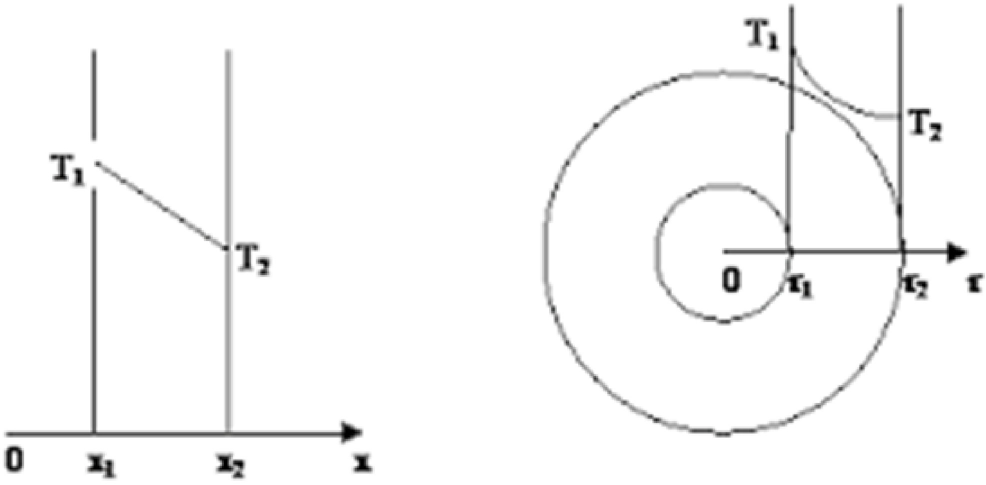

Figure 3.2: Temperature profile of Fourier heat transfer

The above figure shows the steady state variation of temperature and heat flux across the cylinder annulus from inner surface to outer surface. According to Fourier law, the rate of heat flux is directly proportional to its temperature gradient in steady state condition, which can be seen in the figure below. Temperature drops linearly from the inner cylinder to the outer cylinder, thus heat flux diffusing outwards.

Chapter 4

Non-Fourier Heat Transfer Across a Gun Barrel

4.1 Mathematical Formulation

4.1.1 Physical Description and Model

In this mathematical model, a cylinder annulus of finite thickness is considered. The inner and outer cylinder surfaces are first set at a constant temperature condition. It is assumed that the heat transfer is governed by hyperbolic heat conduction. There is no heat exchange due to radiation and convection. There is no heat flow along the cylinder length, i.e. in z-direction. The conduction is assumed to be unidirectional, i.e. heat flows from the inner cylinder surface to the outer surface. The governing equations are given as follows-

$$\rho c_p \frac{\partial T}{\partial t} = \frac{q}{r} + \frac{\partial q}{\partial r} \tag{4.1}$$

$$-k \frac{\partial T}{\partial r} = q + \tau \frac{\partial q}{\partial t} \tag{4.2}$$

par Where ρ is the density, c_p is the specific heat, k is the thermal conductivity, T is the temperature, t is the time, q is the heat flux, r is the radial distance, $\tau = \alpha/\mu^2$ is the relaxation time. The following boundary conditions are applied on the exposed surfaces-

At $r = r_o$, $T = T_l$

At $r = r_i$, $T = T_h$

4.1.2 Normalization

The following normalization parameters are chosen to obtain the non-dimensional equations-

$$\xi = \frac{r}{r_o} \tag{4.3}$$

$$\eta = \frac{\alpha t}{r_o^2} \tag{4.4}$$

$$\theta = \frac{T - T_l}{T_h - T_l} \tag{4.5}$$

$$Q = \frac{q r_o}{k(T_h - T_l)} \tag{4.6}$$

Based on this normalization, the following non-dimensional equations are obtained after calculations-

$$-\frac{\partial \theta}{\partial \eta} = \frac{Q}{\xi} + \frac{\partial Q}{\partial \xi} \tag{4.7}$$

$$-\frac{\partial \theta}{\partial \xi} = Q + \frac{1}{P_e^2}\frac{\partial Q}{\partial \eta} \tag{4.8}$$

Where P_e is the characteristic Peclet Number defined as $P_e = \mu r_o / \alpha$.

4.2 Method Used to Solve the Differential Equation

The equations were formulated and normalized in the previous section and the program was written in MATLAB R2015 (Matrix Laboratory) technical programming language. First, the width of the cylinder was divided into infinitesimally small elements $d\xi$. Then using *Maccormack Predictor and Corrector Algorithm*, values of θ were calculated for corresponding values of ξ. Then a graph was plotted between θ and ξ for a particular value of η. η was varied by multiplying $d\eta$ by number of iterations. In this way, multiple graphs were obtained, showing different stages of heat transfer. The process was repeated for moderate and larger Peclet numbers.

4.2.1 Maccormack Predictor and Corrector Algorithm

It is a discretization scheme used in the solution of hyperbolic partial differential equations. It is a second order finite difference method which is used in 2 steps- the Predictor step and the Corrector step.

Predictor

$$\overline{T_j^{n+1}} = T_j^n - c\frac{\Delta t}{\Delta x}(T_{j+1}^n - T_j^n) \tag{4.9}$$

Corrector

$$T_j^{n+1} = \frac{1}{2}\left[T_j^n + \overline{T_j^{n+1}} - c\frac{\Delta t}{\Delta x}(\overline{T_j^{n+1}} - \overline{T_{j-1}^{n+1}})\right] \tag{4.10}$$

Where T is the provisional value and j-1, j, j+1 denotes the change in x axis. $\frac{\Delta t}{\Delta x}$ represents the slope of the graph at that iteration. The advantage of using this method is that it is elegant and used to solve non-linear equations. It is also easy to understand and program.

4.3 Significance of Peclet Number

Peclet number is a dimensionless number used to distinguish between the convective heat transfer and the conductive heat transfer. The Pclet number is defined as

$$P_e = \frac{vl}{a} = \frac{C_p \rho v l}{k} \tag{4.11}$$

where,

l= Characteristic dimension of the surface of heat exchange

v=Fluid velocity flowing along the heat exchange surface

C_p= Specific heat at constant pressure

a=Thermal diffusivity,

ρ= Density of the medium

Its physical significance is, it is the ratio of the thermal energy transferred by the fluid in bulk due to convection to the thermal energy conducted within the fluid. If P_e is small, conduction predominates and thus the major heat exchange shall occur due to vibration within the medium. Its Mathematical significance is, it is the product of the Reynolds number and the Prandtl number. It depends on the heat capacity, density, velocity, characteristic length and heat transfer coefficient.

Chapter 5

Results and Discussion

5.1 For Small Peclet Number ($P_e = 0.1$)

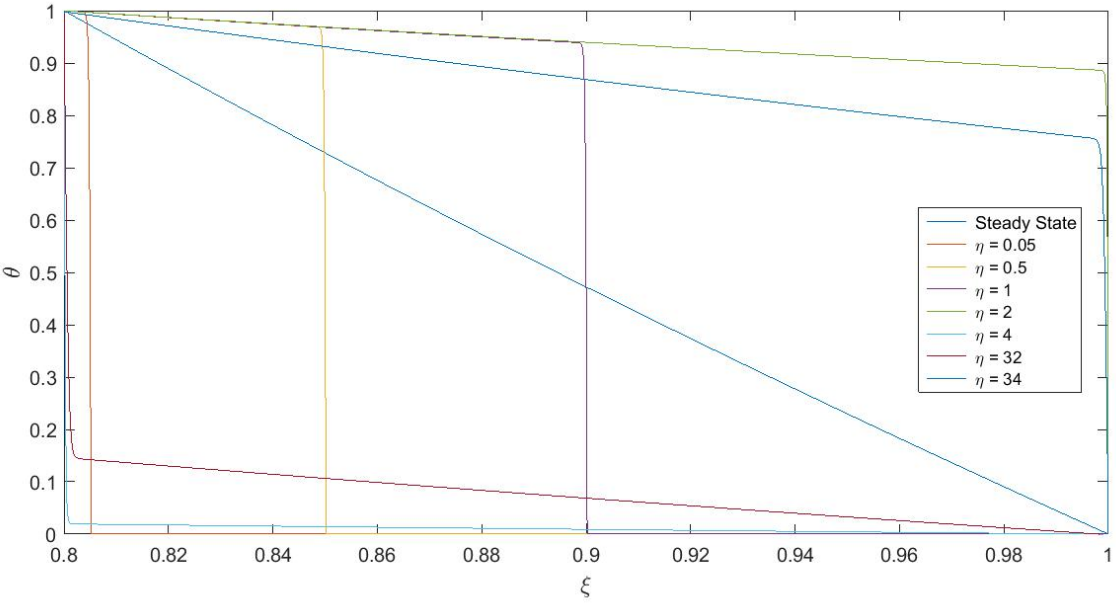

Figure 5.1: Temperature Profile of Small Peclet Number for a Thinner Cylinder

For the thinner cylinder, ξ is first varied from 0.8 to 1.From the figure, we can observe that, in the initial period (i.e. η=0.05), value of θ remains 1 for values of ξ nearly equal to 0.8. After that, there is an abrupt change of θ from 1 to 0. This tells us that, in the initial stage, only the region near to the inner surface has attained the temperature of T_h. The rest of the hollow cylinder still has the atmospheric temperature, T_l. For η=0.5, θ decreases linearly with increase in radius till ξ=0.85. After that, there is a sudden change of θ to 0. This shows the hyperbolic behavior of the thermal wave, i.e. the thermal wave lags the temperature gradient. The wave propagates in the similar manner till η=2, (when the heat flux has almost reached

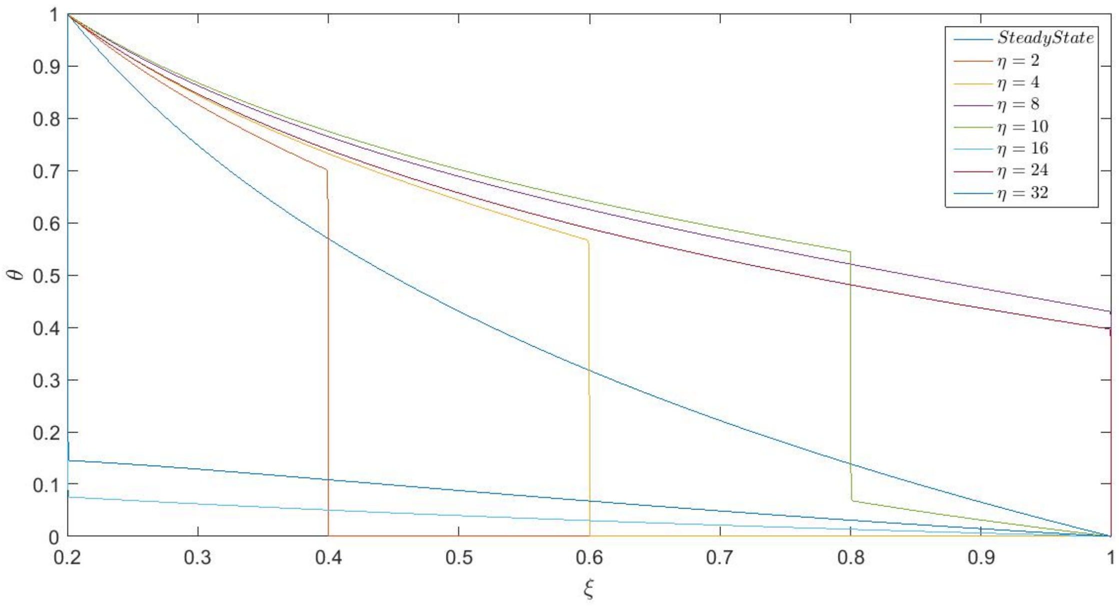

Figure 5.2: Temperature Profile of Small Peclet Number for a Thicker Cylinder

the outer surface and the temperature of the whole slab is nearly equal to T_h). At η=4, the thermal wave reflects back, losing all the energy at the outer surface. The wave after reaching the inner surface again gains energy and reflects back, thus continuing this process until steady state is attained (at large values of η). The slope of the reflecting thermal wave is always less than its previous reflecting wave, which confers that it is gaining energy in each pass and attaining steady state.

For the thicker cylinder, ξ is varied from 0.2 to 1. This shows the variation for a bigger domain. At initial period (η=2), the travelling wave shows the similar behavior that of the thinner slab, except the path it follows is of non-linear type, i.e. an exponential curve. Since the view portion of the thinner slab can be assumed to be a part of the thicker slab, it appears to be a straight line at steady state for the thinner slab. In the figure, for η= 2, there is a sudden drop in temperature of the heat wave at ξ=0.4. For every values of η, there is an abrupt fall in temperature to T_l at a particular ξ. After η=8, the wave starts reflecting back in a linear way, thus increasing energy at every iteration. Initially, the temperature of the reflecting wave remains near to T_l. Gradually when η increases, the slope also increases. Finally a smooth non-linear curve is observed when it attains the steady state condition.

5.2 For moderate Peclet Number ($P_e = 1.0$)

For a moderate Peclet Number, the nature of thermal wave propagation remains similar to that of low Peclet Number for thinner cylinder. For smaller values of η, there is sudden change of temperature in travelling

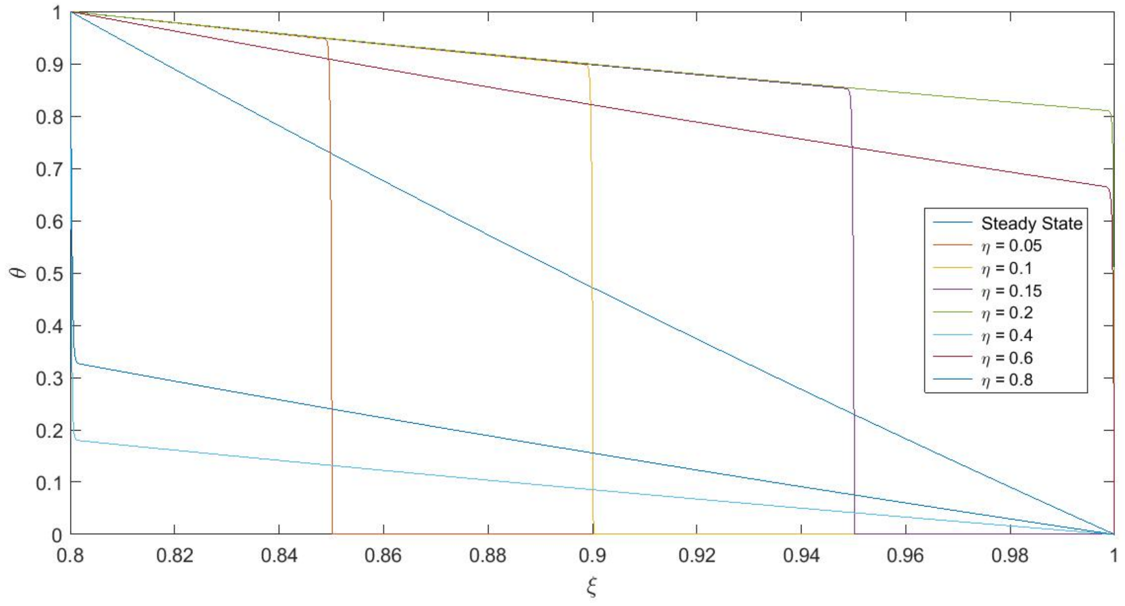

Figure 5.3: Temperature Profile of Moderate Peclet Number for a Thinner Cylinder

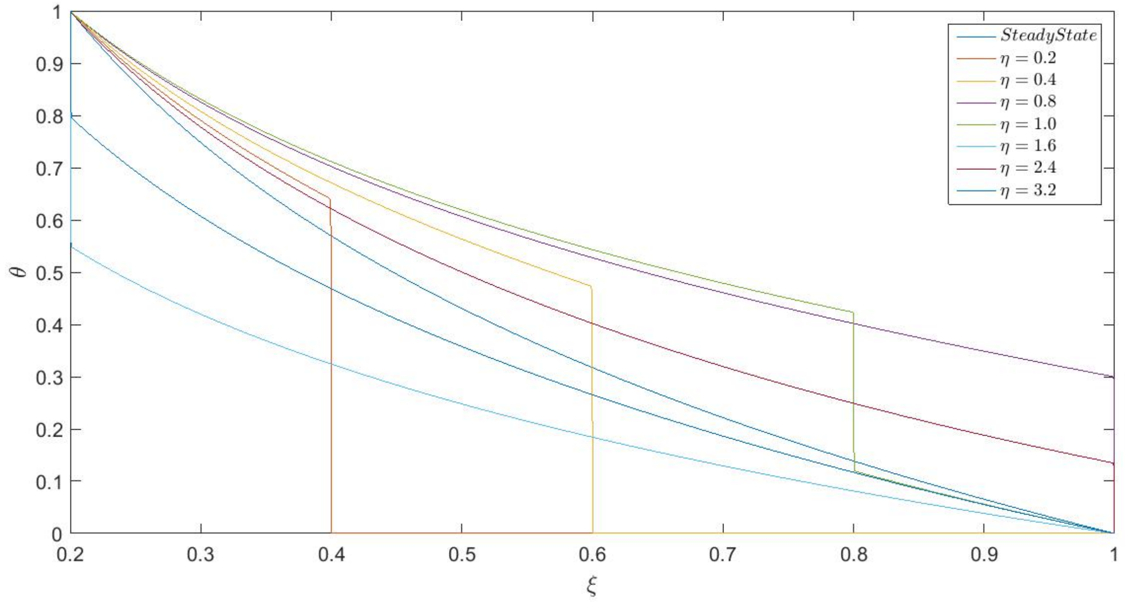

Figure 5.4: Temperature Profile of Moderate Peclet Number for a Thicker Cylinder

wave at specific ξs. For η=0.05, the temperature of the travelling wave drops at ξ=0.85. For η=0.1, the temperature drops to T_l at ξ=0.85. Both travelling and reflecting wave shows linear behavior. The travelling wave drops energy and the reflecting wave absorbs energy in every pass. At every pass, the slope of the wave increases, thus achieving steady state after certain values of η. The frequency of the thermal vibration can be obtained by calculating the number of waves propagated to and fro the outer surfaces between successive intervals of η.

The thicker cylinder slows the similar behavior of that of the higher Peclet numbers. The travelling and reflective waves show the non-linear (exponential) behavior. There is a sudden drop in temperatures until η=1. After that, they follow continuous non-linear path. The wave starts propagating at near steady state after η=1.6. After attaining steady state at a faster pace, the travelling wave and the reflecting wave follows the same path.

5.3 For Large Peclet Number ($P_e = 10.0$)

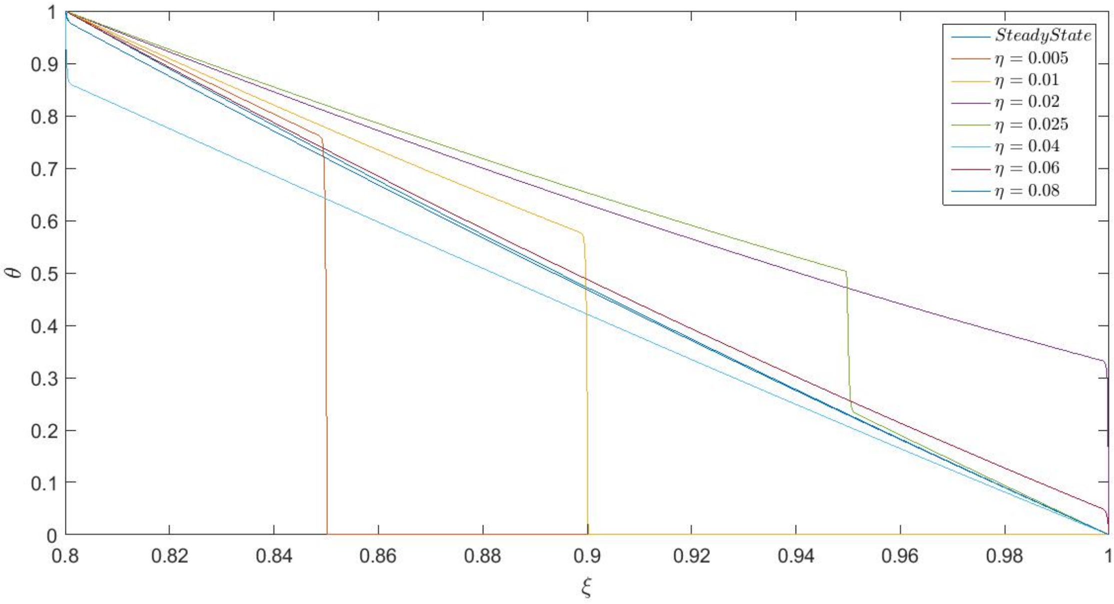

Figure 5.5: Temperature Profile of Large Peclet Number for a Thinner Cylinder

For higher Peclet Number, the trend is similar to that of the lower Peclet Number for thinner cylinder. The travelling and the reflecting wave are both linear in nature, and until η=0.025, we can observe the abrupt drops in temperature. After this, it follows a continuous path without any breaks. The slopesof the reflecting wave is much higher than the graphs obtained in moderate and lower Peclet numbers, thus achieves steady state condition very soon.

For the bigger domain, the trend is completely different. The first wave shows the hyperbolic nature initially. Later on, it diminishes and there is no abrupt rise or fall in temperature. After this, the heat transfer is governed by Fourier law. Both travelling and reflecting wave follow the non-linear path and the transient state is obtained after η=0.08. Thus for higher Peclet Numbers, Hyperbolic nature dwindles and parabolic nature becomes dominating. Thus, in order to study the hyperbolic behavior, Peclet Number of

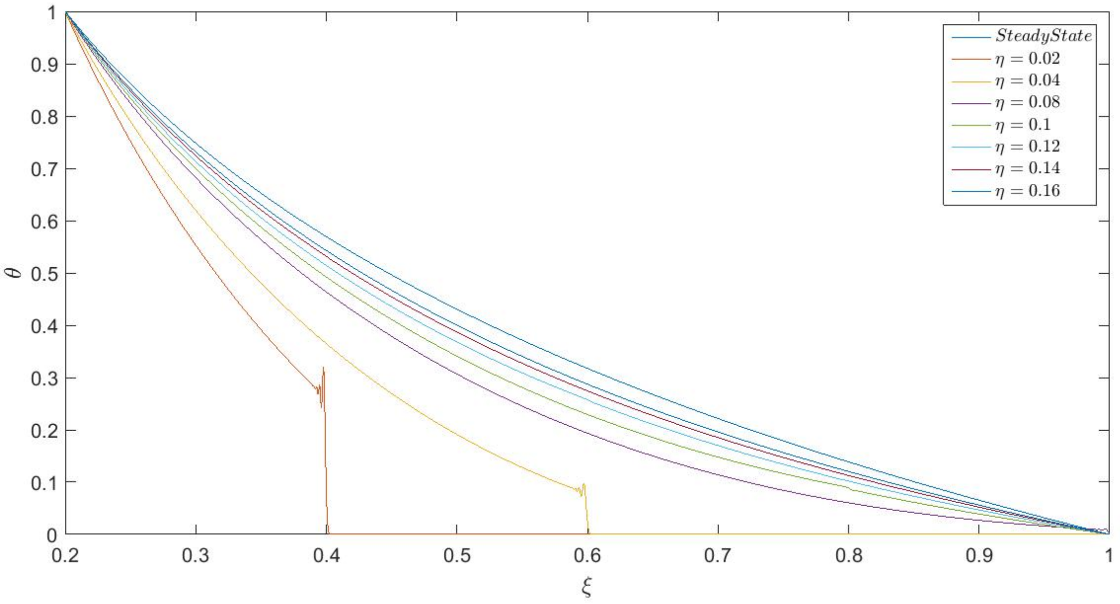

Figure 5.6: Temperature Profile of Large Peclet Number for a Thicker Cylinder

the system has to be decreased which can be done by selecting material with a lower specific heat, lower density or higher thermal conductivity (as mentioned in equation 4.11).

Chapter 6

Conclusion

In this thesis, a comparison was made between Fourier and Non-Fourier heat transfer equation and their behaviors were studied. Properties of the gun barrel were discussed and suitable alloy material was used. Then, equations for Non-Fourier were normalized and formulated. Using the method of Maccormack Predictor and Corrector algorithm, a program was written in Matlab, and plots were obtained for 3 different Peclet numbers using this code. A comparison was made among graphs obtained for these Peclet numbers. It was observed that, at lower Peclet number, the hyperbolic nature is more peculiar, and can be clearly observed for various values of η. While at higher Peclet numbers, hyperbolic nature becomes obscure and parabolic nature becomes significant, thus changing its behavior from Non-Fourier law to the Fourier law.

Bibliography

[1] Mohammad Hossein Babaei and Zengtao Chen. Transient hyperbolic heat conduction in a functionally graded hollow cylinder. *Journal of Thermophysics and Heat Transfer*, 24(2):325–330, 2010.

[2] Da Yu Tzou. Thermal shock waves induced by a moving cracka heat flux formulation. *International journal of heat and mass transfer*, 33(5):877–885, 1990.

[3] Howard E Jackson and Charles T Walker. Thermal conductivity, second sound, and phonon-phonon interactions in naf. *Physical Review B*, 3(4):1428, 1971.

[4] C. Cattaneo. Sur une forme de l'equation de la chaleur elinant le paradoxes d'une propagation instantance. *C.R. Acad. Sci.*, pages 431–432, 1958.

[5] P. Vernotte. Les panadoxes de la theorie continue de l'equatioin de la chaleur. *C.R. Acad. Sci.*, pages 3154–3155, 1958.

[6] MA Al-Nimr and VS Arpaci. The thermal behavior of thin metal films in the hyperbolic two-step model. *International Journal of Heat and Mass Transfer*, 43(11):2021–2028, 2000.

[7] RA Guyer and JA Krumhansl. Thermal conductivity, second sound, and phonon hydrodynamic phenomena in nonmetallic crystals. *Physical Review*, 148(2):778, 1966.

[8] Shu-yuan Zhao, Bo-ming Zhang, and Xiao-dong He. Temperature and pressure dependent effective thermal conductivity of fibrous insulation. *International Journal of Thermal Sciences*, 48(2):440–448, 2009.

[9] Nacer Berour, David Lacroix, Pascal Boulet, and Gérard Jeandel. Radiative and conductive heat transfer in a nongrey semitransparent medium. application to fire protection curtains. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 86(1):9–30, 2004.

[10] Rishi Raj, Amit Prasad, Pritish Ranjan Parida, and Subhash C Mishra. Analysis of solidification of a semitransparent planar layer using the lattice boltzmann method and the discrete transfer method. *Numerical Heat Transfer, Part A: Applications*, 49(3):279–299, 2006.

[11] Kyunghan Kim and Zhixiong Guo. Multi-time-scale heat transfer modeling of turbid tissues exposed to short-pulsed irradiations. *computer methods and programs in biomedicine*, 86(2):112–123, 2007.

[12] Parviz Malekzadeh and Razieh Nejati. Non-fourier heat transfer analysis of functionally graded spherical shells under convection-radiation conditions. *Journal of Oil, Gas and Petrochemical Technology*, 1:73–86, 2014.

[13] Hak In Gimm, Ki Up Cha, and Chang Ki Cho. Characterizations of gun barrel vibrations of during firing based on shock response analysis and short-time fourier transform. *Journal of mechanical science and technology*, 26(5):1463–1470, 2012.

[14] Avanish Mishra, Amer Hameed, and Bryan Lawton. A novel scheme for computing gun barrel temperature history and its experimental validation. *Journal of Pressure Vessel Technology*, 132(6):061202, 2010.

[15] Reza Shirmohammadi and Amin Moosaie. Non-fourier heat conduction in a hollow sphere with periodic surface heat flux. *International Communications in Heat and Mass Transfer*, 36(8):827–833, 2009.

[16] Subhash C Mishra, Anil Stephen, and Man Young Kim. Analysis of non-fourier conduction-radiation heat transfer in a cylindrical enclosure. *Numerical Heat Transfer, Part A: Applications*, 58(12):943–962, 2010.

[17] Yuan Dong, Bing-Yang Cao, and Zeng-Yuan Guo. Temperature in nonequilibrium states and non-fourier heat conduction. *Physical Review E*, 87(3):032150, 2013.

[18] JI Frankel, Brian Vick, and MN Özisik. Flux formulation of hyperbolic heat conduction. *Journal of Applied Physics*, 58(9):3340–3345, 1985.

[19] Bin Wu, Gang Chen, and Wei Xia. Heat transfer in a 155mm compound gun barrel with full length integral midwall cooling channels. *Applied Thermal Engineering*, 28(8):881–888, 2008.

[20] Pu Qu, Qiang Li, and Zhen Yang. Thermal analyses of midwall cooling gun tubes based on liquid-solid coupled method.

[21] Bishri Abdel-Hamid. Modelling non-fourier heat conduction with periodic thermal oscillation using the finite integral transform. *Applied Mathematical Modelling*, 23(12):899–914, 1999.

[22] Arka Bhowmik, Ramjee Repaka, Subhash C Mishra, and Kunal Mitra. Analysis of radiative signals from normal and malignant human skins subjected to a short-pulse laser. *International Journal of Heat and Mass Transfer*, 68:278–294, 2014.

[23] `http://www.efunda.com/materials/alloy`.