MODEL PREDICTIVE CONTROL

OF

A TWO-LINK FLEXIBLE MANIPULATOR

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology in Control & Automation
by

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CERTIFICATE

This is to certify that the thesis titled "**Model Predictive Control of a Two link Flexible Manipulator**", submitted to the National Institute of Technology, Rourkela by **Upasana Gogoi**, Roll No. **213EE3303** for the award of **Master of Technology** in **Control & Automation**, is a bona fide record of research work carried out by her under my supervision and guidance. The candidate has fulfilled all the prescribed requirements. The embodiment of the thesis which is based on candidate's own work, has not submitted elsewhere for a degree/diploma. In my opinion, the thesis is of standard required for the award of a Master of Techno logy degree in Control & Automation. To our best knowledge, she bears a good moral character and decent behaviour.

Place: Rourkela **Prof. Bidyadhar Subudhi**

Date:

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List of acronyms

1. AMM- Assumed Mode Method
2. ILC – Iterative Learning Control
3. MF - Membership functions
4. MPC - Model Predictive Control
5. Model Reference Adaptive Control
6. NN – Neural Network
7. PDE – Partial Differential Equation
8. RLS – Recursive Least Square
9. STC- Self Tuning Control
10. SSE- Sum of squared error
11. TLFM-Two Link Flexible Manipulator

ABSTRACT

Flexible manipulators are widely used because of the many advantages it provides like low weight, low power consumption leading to low overall cost. However due to the inherent structural flexibility they undergo vibrations and take time to come to the desired position once the actuating force is removed .The most crucial problems associated while designing a feedback control system for a flexible-link are that the system being non-minimum phase, under-actuated and non-collocated because of the physical separation between the actuators and the sensors. Moreover from mathematical point of view we can say that the dynamics of the rigid link robot can be derived assuming the total mass to be concentrated at centre of gravity of the body hence dynamics of the robot would result in terms of differential equations. On contrary flexible robot position is not constant and hence partial differential equation is used to represent the distributed nature of position which results in large number of equations increasing the computational effort. In this work a two link flexible manipulator is modelled using Assumed Mode Method considering two modes of vibration. Further fuzzy identification is also performed using T-S modelling approach which minimises the computation and takes into account higher modes of vibration. The input spaces consists of the torque inputs to the link and membership function of Gaussian form is chosen. The consequent parameters are calculated using Least Square Algorithm. For controlling the tip vibration a controller is designed using Model Predictive Control. The Model Predictive Control is an optimal control method in which the control law is calculated using the system output. MPC is widely used in the industry due to its better performance. The results are compared with another controller based on Linear Quadratic Regulator.

CHAPTER 1

INTRODUCTION TO FLEXIBLE ROBOT MANIPULATORS

1.1 Background

1.2 Literature review

1.3 Objective

1.4 Motivation

1.5 Organization of the thesis

1.1. Background

Flexible robots consist of manipulators that are made of flexible and lightweight materials. These manipulators are operated by using some actuator that may be a DC motor or electric motors and solenoids as actuators.There are also robots used widely that have a hydraulic system, and some others may use a pneumatic system. Lightweight flexible robots are widely used in space applications as they can carry huge payload and consumes less energy compared to the rigid counterpart. Moreover due to their light weight they can move faster and also the cost of construction is less. However due to light weight they undergo vibrations and hence the control mechanism of the flexible robot becomes more challenging.

1.1.1 Description of flexible robots

Flexible robots consist of manipulators that are made of flexible and lightweight materials such as a wear resistant 1095 spring steel used in the Flexible Manipulator Setup in our experiment. In case of the rigid link robot ordinary differential equations are sufficient to describe the dynamics assuming the total mass to be concentrated at the centre of gravity of the body. However due to the presence of large number of modes of vibration which is said to be infinite, a flexible link undergoes vibration and hence rigid body analysis would be no more valid and so to represent the distributed nature of position along the beam, Partial Differential Equation (PDE), known as Euler's Bernoulli equation is used.

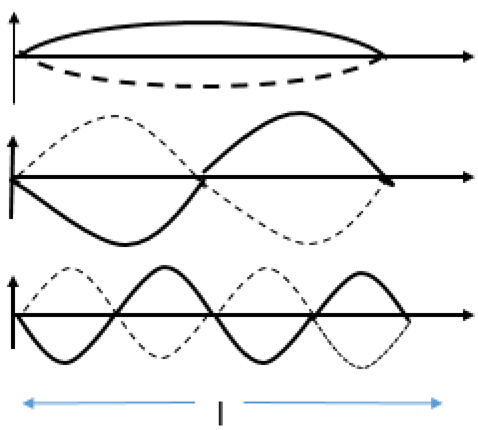

Fig 1.1 Deflection of a flexible link

1.1.2 Advantages of flexible robots

Robots with flexible arms have many advantages in contrast to the conventional rigid counterparts. Fast response and light-weight structure are the two major requirements concerned with robots for industrial use which are not fulfilled by the bulky rigid robots. These requirements are fulfilled by the introduction of flexible robots. They provide faster response, less power consumption, low rated actuators, and less total mass. And these add to low overall cost. But, in addition to these benefits they are associated with serious control problem of vibration. As the structure is flexible when it is provided with an input torque it vibrates with low frequency and it take some time to damp it out. Therefore the control problem for the flexible robot is more complex than rigid link robots.

1.1.3 Applications

Classical rigid link robots cannot perform well in areas like while working in constrained space or operations like assembly in space. These applications require reduced structural mass to allow entering confined spaces. One application is robotic packing and palletising done in food industry. Other applications are in the production of low weight humanoid robots, in civil engineering applications like boring machines, excavators and so on.

1.2 Control Complexities in tip position tracking of flexible robots

When these flexible manipulators are actuated they undergo vibrations due to their flexibility. During the motion of a flexible link, at each point of its trajectory damped vibration exist which cause each point of the link to vibrate and thus the tip position does not come to the desired position quickly and once the torque is removed, the link takes some time to settle down to its final position. The most crucial problems associated while designing a feedback control system for a flexible-link are that the system being non-minimum phase, underactuated and non-collocated.

For mathematical point of view we can say that the dynamics of robots with rigid links can be derived assuming the total mass to be concentrated at centre of gravity of the body hence dynamics of the robot would result in terms of differential equations. On contrary in flexible robots the position is not constant and hence partial differential equation is used to represent the distributed nature of position. Further due to sudden change in payload there may be a large variation in manipulator parameters. Thus control with constant gain controllers is difficult and adaptive methods must be used.

1.3 Literature review on control strategies applied for tip posititition control of a Flexible Manipulator

In late 80's research on flexible manipulator started. The modelling of these flexible manipulators is done by researchers using both assumed mode and finite element methods. Flexible manipulators with single link was discussed using Lagrange's principle and the assumed mode method in the works of Hastings and Book [4], Wang and Wei [18], Wang and Vidyasagar[19]. Finite element approach based dynamical model for single link using is also proposed and compared with experimental results in the works of Tokhi and Mohamed [3]. In the works of Qian a linear model is developed for a single link flexible manipulator. A complete non linear model for single flexible link as well as two link manipulator using assumed mode method is also carried in the works of Luca and Siciliana[1]. In their work two modes of vibration is considered for the links and an inversion based controller design has also been reported in their works .

Several control strategies have been applied for control of the tip position and minimising the deflections of a FLM in the presence of different uncertain conditions ,say changes in payload and friction etc. The structure of the approaches vary depending on i) the technique control structure which is applied, ii) the formation of control law, iii) selection of control parameters which are updated and iv) parameter adaptation law choosen. A brief review of the various adopted approaches is listed in this section**.**

1.3.1 Model Reference Adaptive Control

In MRAC a model is choosen that contains the knowledge regarding the desired behavior of the controlled system and the system performance is based on a reference model defined by the user. The model contains information of the desired behavior of the controlled system. In [5] a model reference adaptive controller is designed for a single link flexible manipulator. A model is choosen on the basis of linearised model of the system. While in [6] better performance is obtained with an non-linear extention of MRAC technique. A fuzzy reference model is introduced in [7]. MRAC approach is suitable for robots with less number of degrees of freedom. With the increase in the number of DOF the performance deteoriates.

1.3.2 Self –tuning control

In [8] a self tuning control law is designed for planar robot with two links and with non-rigid arms. Here the input and output relation is described in terms of a time series model is introduced and an adaptive STC is designed using the model. In [9], a STC has been synthesised for a discrete-time model of a one-link flexible arm when unknown payload is introduced. The identification is done for the unknown payload by using recursive-least-square (RLS) algorithm. In [10] a nonlinear STC for a flexible manipulator with two links is presented which handles unknown payload. In [11] a proportional derivative based STC is introduced in frequency domain for the single-link flexible manipulator. A neural network based approach to adopt the gains of STC is introduced in [12] which simultaneously damps out the vibration with changes in payload

1.3.3 Iterative learning control

Iterative Learning Control (ILC) is a control strategy which is designed for the system showing repetitiveness in its operations. In Iterative learning based control the tracking performance is enhanced, using the error inputs obtained from each trial. Tan, Zhao and Xu [16] used ILC to develop a new approach for tuning the parameters of a proportional integral derivative (PID) controller automatically. They successfully applied ILC approach to a Permanent Magnet Linear Motor (PMLM) in accurate tracking of the desired trajectory.

1.3.4 Intelligent control based on Soft-computing techniques

In [13], a fuzzy controller with adaptive properties is synthesised for flexible link robot arm. Here both time domain and frequency domain techniques are used to design a hybrid controller scheme . The closed loop poles are placed in desired location for the desired performance using feedback gains and the knowledge data base is modified accordingly. In [14],a fuzzy logic controller has been designed which uses minimum number of membership functions (MFs) using a heuristic approach which gives high accuracy of tracking and takes less time for control of a TLFM space robot. In [15] an intelligent-based control method is designed for tracking of the tip position and control of a single-link flexible manipulator. The two neural networks (NNs) with feed-forward are designed using inverse dynamics control strategy

1.4 Motivation

In most robotic applications the ultimate goal is to suppress the vibration more effectively. In this field many approaches have been introduced however because of difficulties and complexities in controller design, further innovation in this field is required. Model predictive control strategy has been widely used in the industry. Recently for highly non-linear systems to avoid complex mathematical computations fuzzy approach is incorporated with MPC. Hence a fuzzy model of the system is developed here and an attempt has been made to apply MPC to control the tip deflection of flexible manipulators.

1.5 Objective of the work

The objectives of the thesis are as follows.

1. To study the dynamics of a flexible beam and have a knowledge of Assumed mode method (AMM), for the modelling of a flexible robot manipulator system.
2. To derive a mathematical model of a physical TLFM set-up and to validate the obtained model .
3. To study fuzzy identification and obtain a fuzzy model of the system.
4. To design and implement control strategies like Linear Quadratic Regulator and MPC.

1.6 Organisation of the thesis

In Chapter 2 a brief description of the experimental setup of the 2-DOF two link flexible manipulator is made.

In Chapter 3 describes the modelling of the system using AMM and fuzzy identification

In Chapter 4 design and analysis of LQR and MPC controllers is discussed.

In Chapter 5 the thesis is concluded and suggestion for future work is discussed.

CHAPTER 2

Experimental Setup of a Flexible Link Manipulator System

2.1 Flexible link manipulator setup

2.2 Flexible links

2.3 Sensors

2.4 Linear Current Amplifier

2.5 Cables

2.6 External power supply

2.1 Flexible link manipulator setup

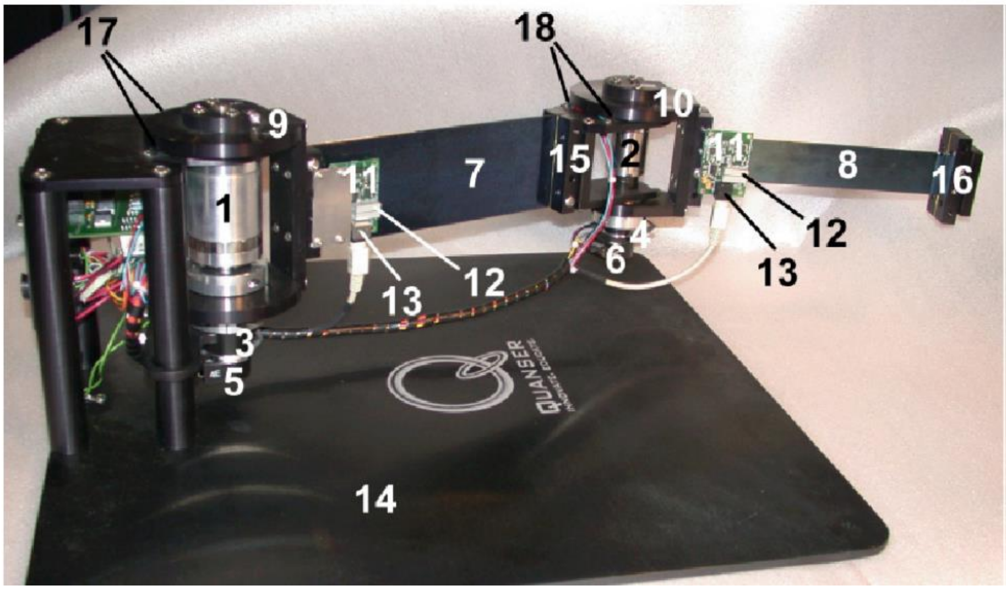

Fig. 2.1 Experimental setup of a two link serial flexible manipulator robot.

The setup consists of two serial flexible links manufactured by Quanser. There are two hubs or joints of the system where separate strain gauges are installed. There is an end effector at the end of link 2 where additional mass or payload mass can be added. The linear amplifier, Q8 terminal board, DAQ system and different sensors like strain gauge, quadrature optical encoder, limit switches are the main components of the setup. The two serial flexible links are actuated by dc motor installed with strain gauges at the clamped end of the links for measurement of tip deflection.

TABLE 2.1 :2DOF Flexible link component nomenclature

SL.No	Description	SL.No	Description
1	Harmonic Drive(link1)	2	Harmonic Drive(link 2)
3	DC Motor (link 1,Shoulder)	4	DC Motor (link2,Elbow)
5	Motor Encoder(link 1)	6	Motor Encoder(link 2)
7	Flexible Link (link 1)	8	Flexible Link (link 2)
9	Rigid Joint (link 1)	10	Rigid Joint (link 2)
11	Strain Gauge Amplifier Board	12	Strain Gauge Offset Potentiometer
13	Strain Gauge Connector	14	Base Plate
15	Link 1 End-Effector	16	Link 2 End-Effector
17	Joint 1 Limit Switches	18	Joint 2 Limit Switches

2.2 Flexible links

The Two link flexible manipulator is provided with one pair of flexible links. This pair is made of one three-inch wide steel beam and another beam which is one –and- a –half-inch wide. Each link has a different thickness. Each beam is made of tough wear-resistant 1095 spring steel. The flexible link dimensions are given in Table 2.2.

Table 2.2 Flexible link dimensions

Link	Width (cm)	Thickness (cm)	Length (cm)
Link 1	7.62	0.127	22
Link 2	3.81	0.089	22

2.3 Sensors

Different sensors are used for measurement of signals for example optical encoder for angular position measurement, strain gauge for strain measurement, limit switches for limiting maximum and minimum positions etc.

2.3.1 Strain Gauge : A strain gauge is used for measurement of strain and uses the principle of change in resistance due to change in strain. The resistance of a body in terms of its dimensions is given by

$$R = \rho \frac{l}{A}$$

where l, A and ρ are the length, area of cross-section of the body and resistivity of the body. Voltage is generated in terms of strain. One strain gauge is mounted at the clamp base of each flexible beam equipping the Two-Degree-Of –Freedom Serial Flexible Link robot which measures the tip deflection. Strain in the tip causes change in dimension which generates a voltage. This strain is calibrated in terms of deflection in m given by

$$y = \frac{2}{3}\frac{E_B {L_B}^2}{T}$$

Where L_B is the length of the link measured up to strain gauge from free end, T is the thickness of the link, E_B is strain at the base .Each strain gauge sensor is connected to its own signal conditioning and amplifier board which is equipped with 2 potentiometers with 20 turns each. The gain potentiometer is set to a fixed maximum gain of 2000. The offset potentiometer and is used for zero tuning and is adjusted manually in order to eliminate any offset voltage present in the strain gauge measurement. A balanced Wheatstone bridge circuit is used with strain gauge forming one of its arms to measure the change in resistance caused due to change in length of the system.

2.3.2 Q-Optical encoder: Quadrature optical encoder measures the angular position. The optical encoder is placed on the top of the shaft of the motor and on the periphery of the disc two digitally encoded signals is placed over it. It consists of two inputs which are 90^0 apart.

2.3.3 Joint Position Limit Switches -Two limit switches are installed at the minimum and maximum rotational positions of each of the two rigid joints. They are magnetically-operated position sensors powered by an external 15VDC. They are the Hamlin 55100 Mini Flange Mount Effect Sensors.

2.4 Linear Current Amplifier-A linear current amplifier with two channels is provided by Quanser. The amplifier gives control signals to the actuators. It is equipped with provision for current measurement and to enable/disable it. The control signal from Q8 terminal board to the motor passes through amplifier. The amplifier has a constant current to voltage gain of 2V/A.

Table-2.3 Properties of the amplifier

Property	Value
Input voltage	27
Maximum Peak current	3 A
Maximum Continuous current	1.2 A

2.5 Cables

Different types of cables are used which perfor different functions like analog, digital, encoder etc. A brief description of details these cables are discussed below.

Motor Cables: These cables consists of four leads two for dc motors, one for ground and other one is left unconnected which carry signals from amplifier to the motor.

fig 2.2: Motor cables

Encoder Cables: These transmit encoded signal generated by the Optical encoder to the Q8 terminal board which is required for the design of the controller.

fig 2.3: Encoder Cables

Analog Cables**:** These carry analog signals like from strain gauge, current sensors which must be converted into digital. So these analog cables carry signals to Q8 terminal board which are then conditioned.

fig 2.4: Analog cables

Digital Cables: These are used for communication with PC for handling digital signals to enable or disable some components for some specific operation of the manipulator .

fig 2.5: Digital cables

2.6 External power supply

The external power supply is provided at at 15± VDC. Some sensors like strain gauge, limit switches require the dc power for operation. It consists of an adapter along with power cable.

fig 2.6: External power supply

2.7 Interfacing with Matlab and Simulink

The control algorithm is implemented using Matlab and Simulink by interfacing the flexible robot with Matlab software.The interfacing is done by Quarc software.Using Quarc various Simulink models can be run in real-time on various targets.

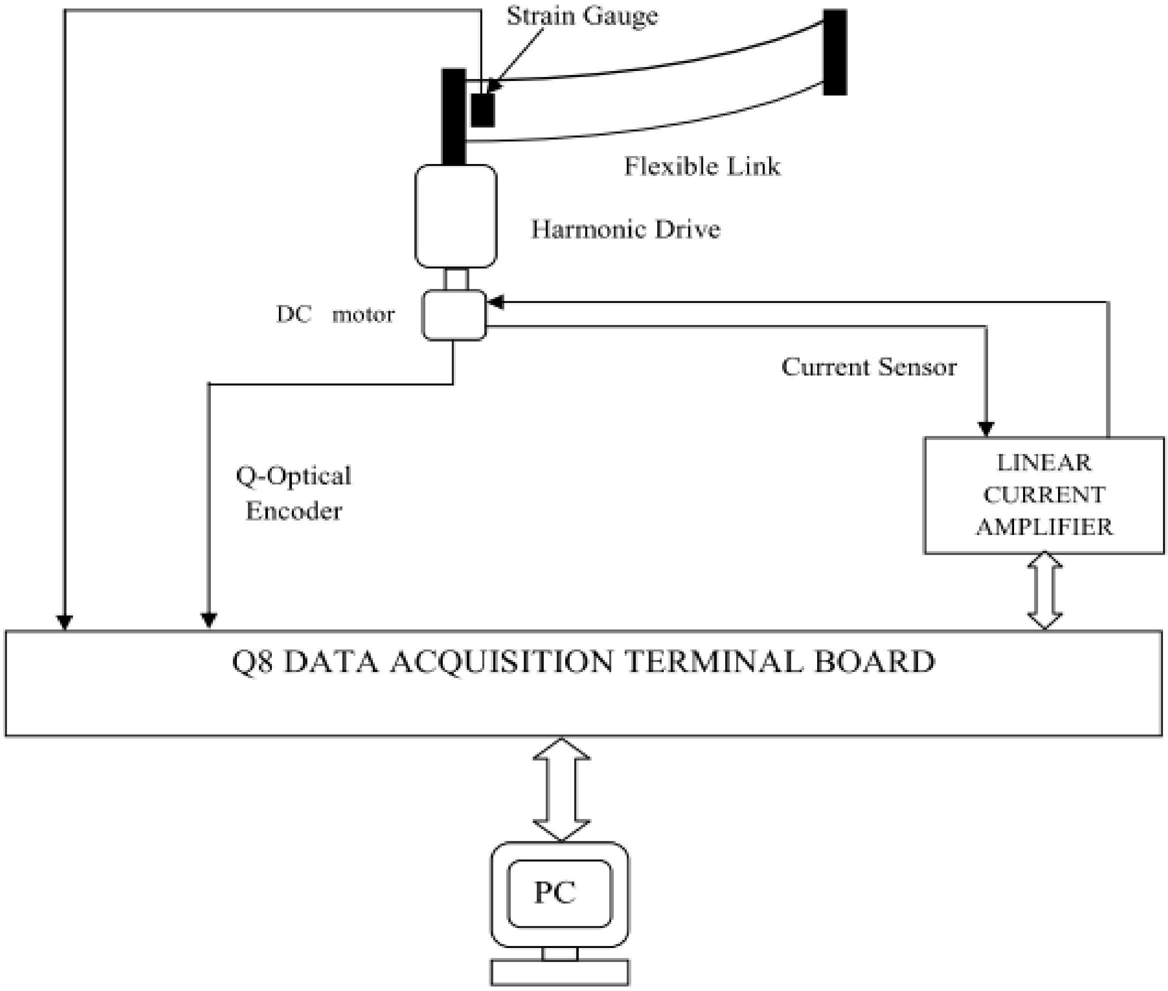

fig 2.7 Interfacing with Matlab

Table 2.4: Physical parameters of TLFM

Parameter	Link-1	Link-2
Link length	0.201m	0.2m
Elasticity	2.0684 x 10^{11}(N/m^2)	2.0684 x 10^{11}(N/m^2)
Rotor moment of Inertia	6.28 x 10^{-6}(kg m^2)	1.03 x 10^{-6}(kg m^2)
Drive moment of Inertia	7.361 x 10^{-4}(kg m^2)	44.55 x 10^{-6}(kg m^2)
Link moment of Inertia	0.17043 (kg m^2)	0.0064387 (kg m^2)
Gear ratio	100	50
Maximum Rotation	(+/- 90 ,+/-90)degrees	(+/- 90 ,+/-90)degrees
Drive Torque constant	0.119(Nm/A)	0.0234(Nm/A)

CHAPTER 3

MODELLING OF A TWO LINK FLEXIBLE ROBOT MANIPULATOR

3.1 Dynamic modelling of a flexible link manipulator

3.2 Fuzzy Identification of Two link flexible manipulator

3.3 Results

3.1 Dynamic modelling of a flexible link manipulator

There are several methods of modelling of a flexible link robot such as Assumed Mode Method and Finite Element Method. A complete nonlinear model for a two flexible link robot using AMM model is also carried out by Luca and Siciliano in [1]. Two modes of vibration analysis has been used in their work. Finite element approach based dynamical model for single link using is also proposed and compared with experimental results in the works of Tokhi and Mohamed [3]. Flexible manipulators with single link , using Lagrange's equation and the assumed mode method ,was studied in the works of Hastings and Book [4], Wang and Vidyasagar [18]. In this work Assumed Mode Method is used in the modelling of the two link flexible manipulator.

3.1.1 Assumed mode method

In Assumed mode method we assume a finite number of modes of vibration for each flexible link.

Before modelling of the single link flexible robot, we need to consider following assumptions for the link:

- The flexible link of the robot is an Euler –Bernoulli beam with uniform density
- The deflection in the beam is small compared to its length
- The payload mass attached is a concentrated mass
- The Flexible link manipulator operates in horizontal plane.

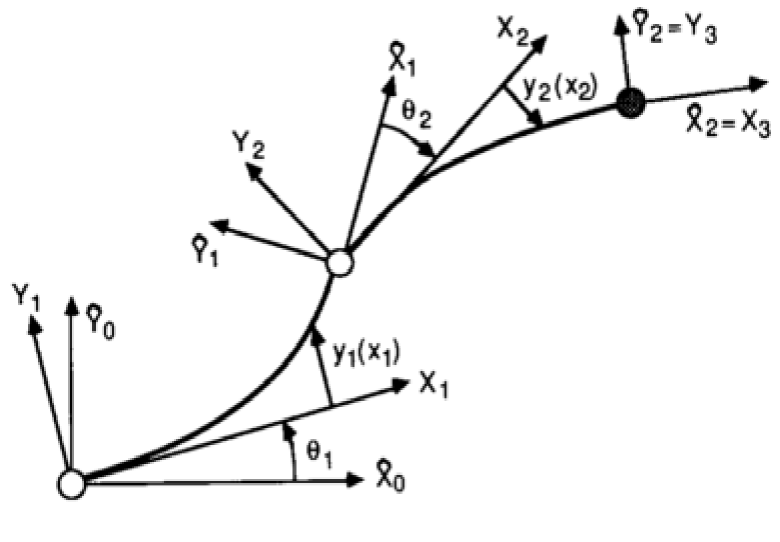

Fig 3.1-Planar two link manipulator

The dynamic equations of a planar robot with n flexible links can be derived by computing the kinetic energy K and potential energy U and then forming the Lagrangian L=K-U and using the Assumed Mode Method. As in [1] the dynamic model is developed which reveals the behaviour of the system using the Lagrangian approach defined as follows

$$\frac{d}{dt}\frac{\partial L}{\partial q_i} - \frac{\partial L}{\partial q_i} = F_i \tag{3.1}$$

$L = (K)_i - (U)_i$: Lagrangian expressed as difference between total kinetic energy and total potential energy of the system

τ_i : Generalized force at the ith joint.

q_i : Generalized coordinate of the ith link.

The generalized coordinate's q_i comprise of joint angles, joint velocities and modal coordinates. The total kinetic energy of the i^{th} link can be expressed as K_i (Total kinetic energy due to i^{th} joint) + (Total kinetic energy due to i^{th} link) + (Total kinetic energy due to payload Mp) and in absence of gravity.

The modelling of the links is done as Euler-Bernoulli beams having uniform density and constant flexural rigidity with deformation $y_i(x_i,t)$, which satisfies the i^{th} link partial differential equation

$$(EI)_i \frac{\partial^4 y_i(x_i,t)}{\partial x_i^4} + p_i \frac{\partial^2 y_i(x_i,t)}{\partial t^2} = 0 \tag{3.2}$$

p_i : Density of the i^{th} link (i=1, 2).

y_i : Deflection of the i^{th} link.

$(EI)_i$: Flexural rigidity of the i^{th} link.

l_i : Length of the i^{th} link.

t : Time.

A solution of equation (2) can be obtained by applying proper boundary conditions at the base and at the end of each link. The three boundary conditions are (a) the clamped-free boundary condition i.e. one end is blocked in both angular and vertical direction and the other end is free. (b) the clamped-inertia boundary condition i.e. one end is blocked clamped-free case but the other end carriers and inertia load. (c) The last boundary condition i.e. pinned.

$$y_i(0,t)=0 \qquad y_i'(0,t)=0 \tag{3.3}$$

$$(EI)_i \frac{\partial^2 y_i(x_i,t)}{\partial x_i^2}\bigg|_{x_i=l} = -J_{Li}\frac{d^2}{dt^2}\left(\frac{\partial y_i(x_i,t)}{\partial x_i}\bigg|_{x_i=l}\right) \tag{3.4}$$

$$(EI)_i \frac{\partial^3 y_i(x_i,t)}{\partial x_i^3}\bigg|_{x_i=l} = -M_{Li}\frac{d^2}{dt^2} y_i(x_i,t)\big|_{x_i=l} \tag{3.5}$$

where J_{Li} and M_{Li} are mass and moment of inertia at the end of ith link

A finite dimensional expression for the link flexibility of ith link can be represented using an Assume mode method. The link deflection can be expressed as

$$y_i(x_i,t)=\sum_{j=1}^{m}\phi_{ij}(x_i)\delta_{ij}(t) \tag{3.6}$$

where

φ_{ij} : jth spatial mode shapes of the ith link.

δ_{ij}: jth modal coordinates (time coordinate) of the ith link.

m : Number of assume modes

Using eqn (3.6)a general solution of (3.2) is derived, which is a product of time harmonic function of the form

$$\delta_{ij}(t)=\exp(j\omega_{ij}t) \tag{3.7}$$

and of a space eigen function of the form

$$\varphi_{ij}(x_{ij})=C_{1,ij}\sin(\beta_{ij}x_i)+C_{2,ij}\cos(\beta_{ij}x_i)+C_{3,ij}\sinh(\beta_{ij}x_i)+C_{4,ij}\cosh(\beta_{ij}x_i) \tag{3.8}$$

where ω_i natural frequency of the ith link and $\beta_i^4=\omega_i^4\rho_i/(EI)_i$. By applying the boundary conditions the constant coefficients in (8) can be determined as

$$C_{3,i}=-C_{1,i} \;;\; C_{4,i}=C_{2,i} \tag{3.9}$$

Now applying the mass boundary conditions (4) we get

$$C_{1,ij}\left[\left(\sin(\beta_{ij}x_i)+\sinh(\beta_{ij}x_i)\right)+\frac{J\beta^3}{\rho}\left(\cos(\beta_{ij}x_i)-\cosh(\beta_{ij}x_i)\right)\right] + C_{2,ij}\left[\left(\cos(\beta_{ij}x_i)\, cosh(\beta_{ij}x_i)\right)+\frac{J\beta^3}{\rho}\left(-\sin(\beta_{ij}x_i)-\sinh(\beta_{ij}x_i)\right)\right]=0 \tag{3.10}$$

Now applying the mass boundary conditions (5) we get

$$C_{1,ij}\left[\left(\cosh(\beta_{ij}x_i)+cos(\beta_{ij}x_i)\right)-\frac{M\beta}{\rho}\left(\sin(\beta_{ij}x_i)-\sinh(\beta_{ij}x_i)\right)\right]+ C_{2,ij}\left[\left(\sin h(\beta_{ij}x_i)-\ \sinh(\beta_{ij}x_i)\right)-\frac{M\beta}{\rho}\left(\cos(\beta_{ij}x_i)-\cosh(\beta_{ij}x_i)\right)\right]=0 \tag{3.11}$$

The above two equations can also be written in matrix

The elements of the F matrix are (3.12)

$$\left[F(\beta_{ij})\right]\begin{bmatrix}C_{1,ij}\\ C_{2,ij}\end{bmatrix}=0$$

$$F_{11} = \left(\sin(\beta_{ij}x_i) + \sinh(\beta_{ij}x_i)\right) + \frac{J\beta^3}{\rho}\left(\cos(\beta_{ij}x_i) - \cosh(\beta_{ij}x_i)\right) \quad (3.13)$$

$$F_{12=}\left(\cos(\beta_{ij}x_i)\, cosh(\beta_{ij}x_i)\right) + \frac{J\beta^3}{\rho}\left(-\sin(\beta_{ij}x_i) - \sinh(\beta_{ij}x_i)\right) \quad (3.14)$$

$$F_{21} = \left(\cosh(\beta_{ij}x_i) + cos(\beta_{ij}x_i)\right) - \frac{M\beta}{\rho}\left(\sin(\beta_{ij}x_i) - \sinh(\beta_{ij}x_i)\right) \quad (3.15)$$

$$F_{22} = \left(\sin h(\beta_{ij}x_i) - \ \sinh(\beta_{ij}x_i)\right) - \frac{M\beta}{\rho}\left(\cos(\beta_{ij}x_i) - \cosh(\beta_{ij}x_i)\right) \quad (3.16)$$

Now $|F(\beta_{ij})| = 0$ leads to the frequency equation

The frequency equation obtained is given by

$$(1+\cos(\beta_{ij}l_i)\cosh(\beta_{ij}l_i)) - \frac{M_{Li}\beta_{ij}}{\rho_i}(\sin(\beta_{ij}l_i)\cosh(\beta_{ij}l_i) - \cos(\beta_{ij}l_i)\sinh(\beta_{ij}l_i))$$
$$- \frac{J_{Li}\beta_{ij}}{\rho_i}(\sin(\beta_{ij}l_i)\cosh(\beta_{ij}l_i) + \cos(\beta_{ij}l_i)\sinh(\beta_{ij}l_i)) + \frac{M_{Li}J_{Li}\beta_{ij}}{\rho_i}(1-\cos(\beta_{ij}l_i)\cosh(\beta_{ij}l_i)) = 0 \quad (3.17)$$

By solving the frequency equation for β_{ij} we get the different modal frequencies of the links. Putting the values in (3.12) we get equations in unknowns of C_{ij}. Hence, a finite solution to the link deformation is obtained.

For link 1 we get $f_{11} = 1.76$, $f_{12} = 2.1857$

For link 2 we get $f_{21} = 3.14$, $f_{22} = 18.11$

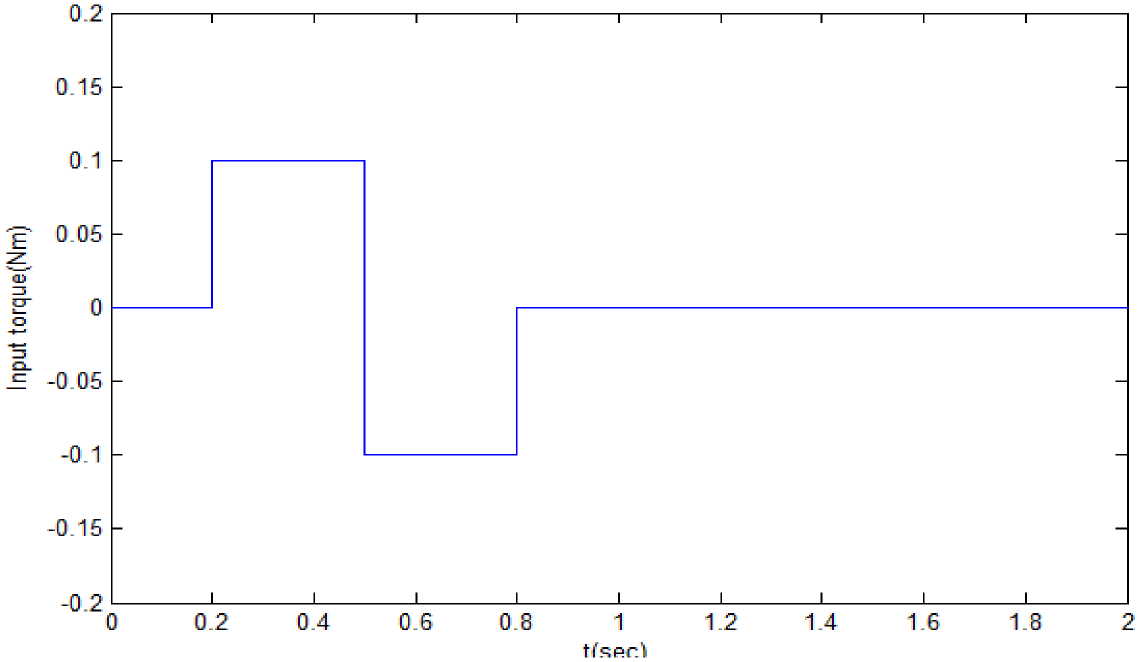

Fig 3.2 Bang-bang torque of 0.1Nm

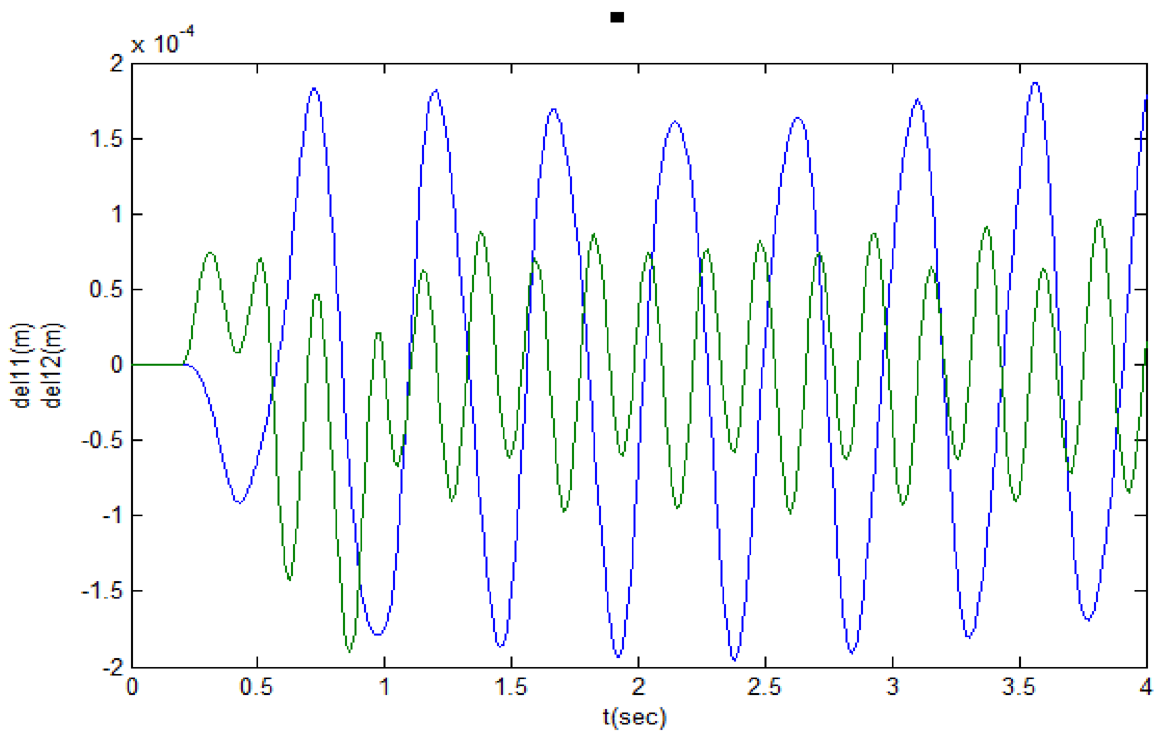

Fig3.3 Deflections of link 1

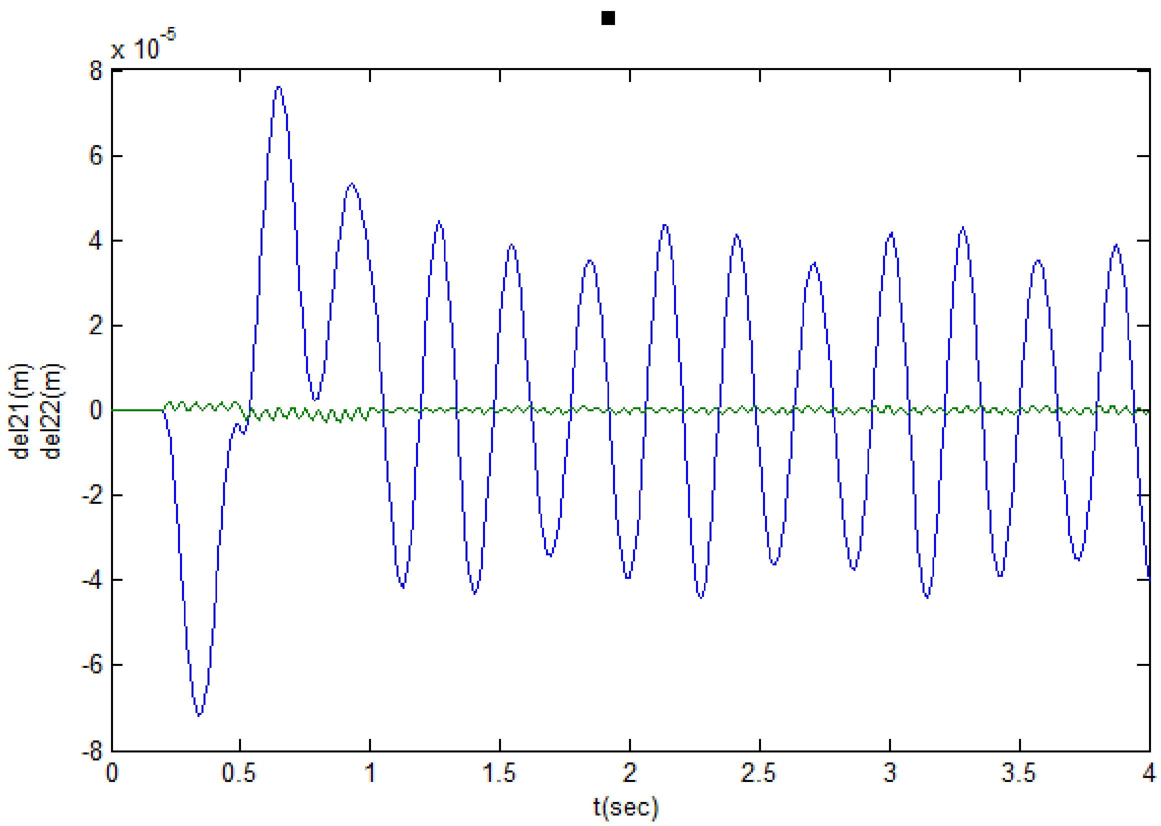

Fig3.4 Deflections of link 2

Hence, a finite solution to the link deformation is obtained. As a result, using the initial Lagrangian equation in (3.1) a matrix representation for the dynamic model of the TLFM is

$$B(q)\ddot{q} + H(q,\dot{q}) + Kq = Qu \qquad (3.18)$$

q: $(\theta_1, \theta_2, \dots, \theta_{n,}\delta_{11\dots\dots}\delta_{1,m\dots\dots}\delta_{n,1} \dots . \delta_{n,m})^T$

B : positive symmetric inertia matrix

H : Coriolis and centrifugal force vector

K : stiffness matrix

Q : input weighing matrix

3.2 FUZZY IDENTIFICATION OF A TWO LINK FLEXIBLE MANIPULATOR

3.2.1. INTRODUCTION

Contemporary industrial applications exhibit certain models which have a high degree of complexity in their dynamic behaviour. In the complete operating range most processes show highly nonlinear behaviour, which cannot be approximately described using conventional linear approach. The dynamics of these models are represented by algebraic equations, partial differential equations and integro-differential equations and hence modelling of such systems requires extensive mathematical computation. To apply different control strategies and to obtain desired performance of the controller an accurate model of the system is necessary. Once a model is developed both linear and non-linear control theory can be used to analyse and control the complex system. However in most cases either the models are not available or are partially understood or if it is available a model showing global behaiviour is very difficult to build. One way of dealing with such a problem is identifying the system using fuzzy logic control. By introducing fuzzy logic , the qualitative and quantitative information is combined mathematically which combines both symbolic and numeric data along with reasoning and computation.

In fuzzy modelling the region of interest is divided into a number of fuzzy regions and a simple model is developed for each region and forms a link between the individual regions in the model input domains and the corresponding output domains. In this way the nonlinearity is handled and the user can have knowledge of the system behaiviour and hence of the original system. Thus in one frame both numerical as well as symbolic processing is brought together. Fuzzy models provide the advantage of combining qualitative data which are represented by IF-THEN rules with quantitative data which are represented by linear models. The rules are constructed using prior knowledge of the experts working with the system related to the particular field. Thus a linguistic interpretation provides a flexible and transparent mathematical approach to the system . Hence model reliability is enhanced and proper insight into the behaviour of the model is provided which is useful for the validation of the model.

3.2.2. T-S MODELLING OVERVIEW

In piecewise linearization method the nonlinear system is linearised about a nominal operating point, and then controller is designed by linear feedback control methods. However since in this method the input space is divided into crisps subsets, a smooth connection between the linear subsets isnot possible and hence a precise global system modal cannot be formed. On the other hand in T-S modelling the input space is divided into fuzzy subspaces and a linear or non-linear model is build from each subspace. The dynamic behaviour of every local region is represented by each sub system. Each local region is then connected with the help of membership functions to form a global dynamic model, then using membership functions the local subsystems are smoothly connected to form a fuzzy model which is global.

3.2.3. FUZZY IDENTIFICATION

The problem of fuzzy identification can be divided to the following two sub problems:

(i) Forming the antecedent part in which the input space is divided into fuzzy regions in which the model has a simple structure which can be represented by a linear model and forming rules.

(ii) Forming the consequent part in which the parameters of linear subsystem models are identified.

The first part can be done using fuzzy clustering. In fuzzy clustering method a set of data is partitioned into a number of overlapping clusters depending on the distance between the data points and the cluster prototypes. Different clustering algorithms like GK fuzzy clustering or fuzzy c-means clustering can be employed. Each cluster represents a rule. Hence the antecedent part of the rule can be identified from the clusters.

3.2.4. CLUSTERING

Clustering is a method of dividing data into a number of clusters on the basis of a similarity function. Clustering is useful in forming rules for T-S fuzzy modelling where each cluster represents a fuzzy IF-THEN rule. Various clustering algorithms can be used depending on the model used and type of data.

Types based on division of data-

The formation of clusters from a given data set depends on the method of clustering choosen.. Clustering methods may be divided into two categories based on their structure. The selection of a particular method depends on the nature of required performance, the size of dataset and the desired type of output. Some of the methods of clustering are discussed below.

1. Method of Partitioning

In this method the data set is divided into a number of clusters represented by a centroid or a cluster representative. In such methods the number of clusters have to be pre-defined by the user.The cluster representative depends on the type of data which are being clustered. In partitioning methods the data points are relocated starting from an initial partitioning and den by moving them from one cluster to another. The main idea behind this is to minimize an error function that measures the distance of each data point to its representative value. Sum of Squared Error (SSE) is the mostly used square error criteria. Here the total squared Euclidian distance of the data points to their representative values is measured.

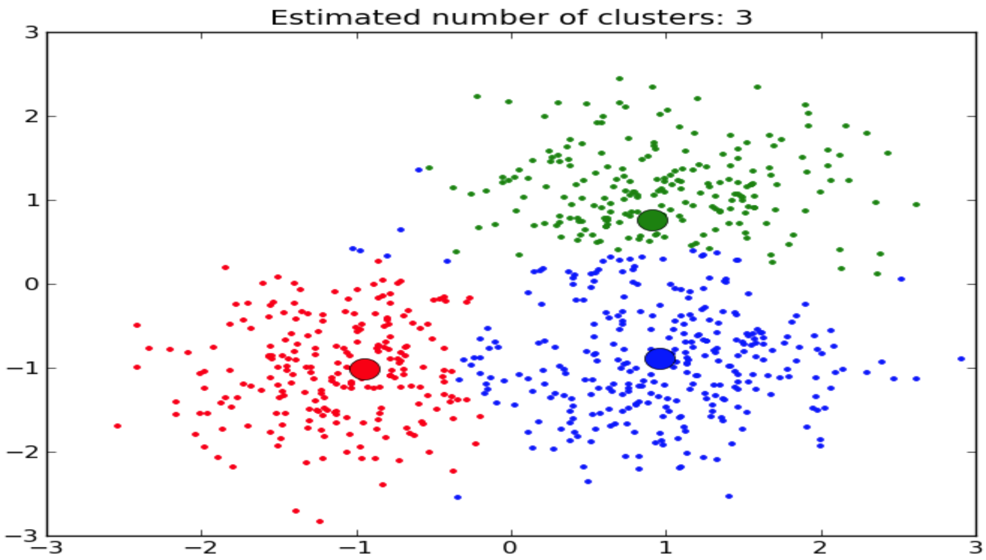

fig 3.5 Cluster formation by partitioning method

K- means clustering- It is the most commonly used algorithm which uses the gradient-decent procedure. In this method the data is partitioned into k clusters employing a square error criteria. Each cluster is represented by their centres. The mean of all data points belonging to that particular cluster is the centre of that cluster. In this method we have to select an initial set of cluster centres in prior. The selection of centres is done randomly or based on some procedure. In each iteration the Euclidean distance between a data point and the centres are calculated and that data point is assigned to that cluster centre having the least diatance. Then the cluster centers are calculated again and the process is repeated. The centre of each cluster is defined as the mean of all the data points that belongs to a cluster.

$$\mu_k = \frac{1}{N}\sum_{q=1}^{N} x_q$$

where N is the number of data points belonging to cluster k and μ_k is its mean.

Input: D (data set), k (number of cluster) Output: clusters

1: Initialize the centers for k clusters.

2: while the process not terminated do

3: Assign data points to the cluster center which is the closest.

4: Update the centres of each cluster.

5: end

Advantages of k-means algorithm-

- The algorithm provides linear complexity which proves to be an added advantage while hierarchical clustering methods exhibit complexities of non-linear nature.
- It can handle large number of instances and is adaptable to sparse data.
- Good speed of convergence
- It is simple to implement and interpret.

Disadvantages

- The number of clusters has to be to be mentioned in advance.

- It is sensitive to noisy data.

2. Hierarchical methods

In these type of methods the clusters are formed by partitioning the data recursively in either a up down or bottom to up manner. These methods can be divided further in following subclasses:

- Agglomerative hierarchical clustering - In this method each data point initially represents a particular cluster . Then the clusters are merged into one another according to some similarity measure until a desired cluster structure is formed.
- Divisive hierarchical clustering - In this method initially all data points belong to one cluster. Then division of the cluster takes place to form sub-clusters, the sub clusters are again divided to form more clusters and the process continues till the formation of desired cluster.

Now based on the similarity measure hierarchical clusters can be further divided into

- Single-link clustering (minimum method) — In this method the shortest distance between any two members belonging to two clusters is considered as the distance between two clusters. And in terms of similarity measure the maximum value of similarity from any member belonging to one cluster to any member belonging to other cluster is considered as the similarity between two clusters. One disadvantage of this method is that two clusters may get united if some of the points form bridge between the clusters.
- Complete-link clustering (maximum method) - In this method the longest distance between any two members belonging to two clusters is considered as the distance between two clusters .This method produces clusters that are more compact than single link cluster.
- Average-link clustering (minimum variance method) -- In this method the average distance between any two members belonging to two clusters is considered as the distance between two clusters .

The main disadvantages of the hierarchical methods are:

- The time complexity of hierarchical algorithms is non-linear with respect to the number of objects. Clustering a large number of objects using these algorithms turns out to be of huge cost.
- Hierarchical methods donot have the capability of back tracking i.e they can never undo what was done previously.

3. Density based clustering method-In this method the clusters are grown until a particular threshold density is reached or within a pre defined radius there exists a minimum number of data points. It assumes that the points in each cluster are taken which follows a specific probability distribution. The component densities are assumed to be of multivariate Gaussian nature. AUTOCLASS , SNOB and MCLUST are some of the density based algorithms.

4. Soft computing based clustering method - Fuzzy clustering is a soft clustering method in which each instance doesnot only belong to one cluster like in partitioning methods but to each of all the clusters with a certain degree or each instance is associated with the clusters with a membership function. Here each cluster can be considered as a fuzzy set formed by all the patterns. The selection of membership function is important in fuzzy clustering .Larger membership values mean that the data point belongs more to that cluster. A hard clustering can be obtained by using a bound to the membership value from a fuzzy partition. Fuzzy c-means (FCM) algorithm is one of the important fuzzy clustering methods. The main advantages over K-means algorithm is that it avoids local minima.

3.2.5 Clustering of tlfm data

400 input output data points are obtained from the model.Out of these 200 are used for identification and 200 for validation.The data points consists of input torques to the two links u_1 and u_2 and output tip position taking one link at a time. Fuzzy c-means clustering is applied and three clusters are formed . Each cluster is used to represent a local linear model. The result of clustering is shown below.

Input data set : $u_1(k), u_1(k-1), u_2(k), u_2(k-1)$

Output data set : $y_1(k), y_1(k-1), y_2(k), y_2(k-1)$

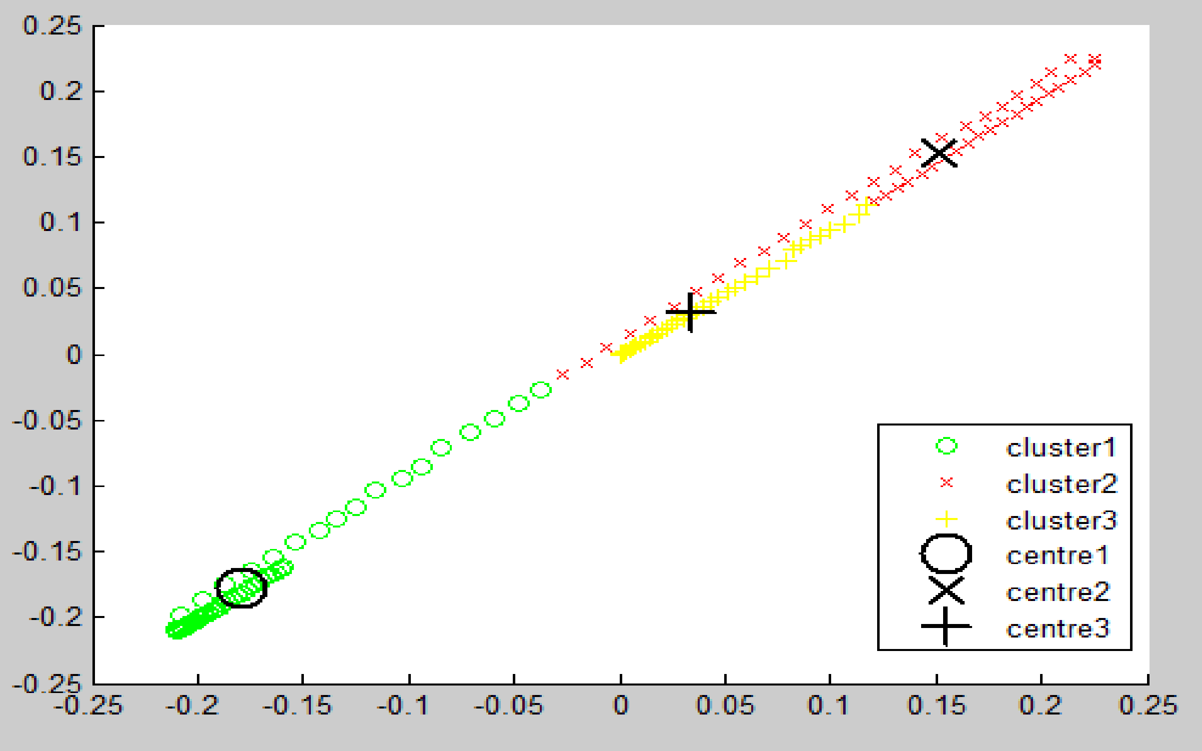

Fig 3.6 .Clustering of TLFM data

3.2.6 MULTIVARIABLE T-S FUZZY MODEL

Let us consider a Multiple Input Multiple Output system with m inputs and p outputs as in [24]. The system can be approximated by a number of MISO systems of ARX type given by

$$y_l(k+1) = R_l\big(\varepsilon_l(k), u(k)\big), \qquad l = 1,2,3,..p$$

where $\varepsilon_l(k) = [y_1(k), \ldots, y_p(k), u_1(k-1), \ldots u_m(k-1)]^T$ ……………………………. (3.19)

and the rules are given by

$$R_i: IF\ \varepsilon_{l1}(k)\ is\ F_{li,1}\ and\ .\ .\ .and\ \varepsilon_{l\rho}(k)\ is\ F_{li,\rho}\ and\ u_1(k)\ is\ F_{li,\rho+1}\ and\ .\ .\ .and\ u_m(k)\ is\ F_{li,\rho+m}$$

$$THEN\ y_{li}(k+1) = \omega_{li}\varepsilon_{li}(k) + n_{li}u(k) + \theta_{li}, \quad i = 1,2,..,K_l \qquad \ldots\ldots\ldots\ldots\ldots\ ..\ .(3.20)$$

Here F_{li} defines the antecedent fuzzy sets of the ith rule, ω_{li} and n_{li} are the vectors which contains the parameters of the consequents and θ_{li} is the offset, K_i is the total number of rules for the lth output. The overall model output is given by the aggregated parameters of the individual model as

$$y_l(k+1) = \omega_l'\varepsilon_l(k) + n_l'u(k) + \theta_l', \quad i = 1,2,..p \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.21)$$

$$w_l' = \frac{\sum_{i=1}^{K_l} \beta_{li}\big(\varepsilon_l(k), u(k)\big).\,\omega_{li}}{\sum_{i=1}^{K_l} \beta_{li}(\varepsilon_l(k), u(k))} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.22)$$

$$n_l' = \frac{\sum_{i=1}^{K_l} \beta_{li}\big(\varepsilon_l(k), u(k)\big).\,n_{li}}{\sum_{i=1}^{K_l} \beta_{li}(\varepsilon_l(k), u(k))} \qquad \ldots\ldots.\ldots\ \ \ldots\ldots\ldots (3.23)$$

$$\theta_l' = \frac{\sum_{i=1}^{K_l} \beta_{li}\big(\varepsilon_l(k), u(k)\big).\,\theta_{li}}{\sum_{i=1}^{K_l} \beta_{li}(\varepsilon_l(k), u(k))} \qquad \ldots\ldots\ldots\ldots\ldots\ldots.\ \ (3.24)$$

The T-S model can be represented by a state space model in the controllable canonical form

$$x_{lin}(k+1) = A(k)x_{lin}(k) + B(k)u(k) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.25)$$

$$y_{lin}(k) = C(k)x_{lin}(k) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.26)$$

A is the state matrix containing the parameters $w_l^{'}$ and $\theta_l^{'}$ and B is the input matrix containing the parameters $n_l^{'}$.The last column of A is the offset for the corresponding output $\theta_l^{'}$.The matrices A,B and C are defined as follows.

$$A = \begin{bmatrix} w_{1,1}^{'} & w_{1,2}^{'} & \cdots & w_{1,p}^{'} & \theta_1^{'} \\ 1 & 0 & \ddots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ w_{2,1}^{'} & w_{2,2}^{'} & \ldots\ \ldots & w_{2,p}^{'} & \theta_2^{'} \\ 0 & \vdots & \ddots & \vdots & \vdots \\ w_{p,1}^{'} & w_{p,2}^{'} & \ldots\ \ldots & w_{p,\rho}^{'} & \theta_p^{'} \\ 0 & 0\ \ 1 & \ldots & 0 & 0 \\ \vdots & \vdots\ \ \vdots & \ddots & \vdots & \vdots \\ 0 & \ldots\ \ 0 & \ldots & 0\ \ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} n_{1,1}^{'} & n_{1,2}^{'} & \cdots & n_{1,m}^{'} \\ 0 & \ldots & \ldots & 0 \\ n_{2,1}^{'} & n_{2,2}^{'} & \cdots & n_{p,m}^{'} \\ 0 & \ldots & \ldots & 0 \\ 1 & \ldots & \ldots & 0 \\ 0 & \ldots & \ldots & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & \ldots & \ldots & \ldots & \ldots & 0 \\ \vdots & & \ddots & & & & \vdots \\ 0 & & \ldots & 1 & 0 & \ldots & 0 \end{bmatrix}$$

3.2.7 APPLICATION TO TWO-LINK FLEXIBLE MANIPULATOR

Flexible robots consist of manipulators that are made of flexible and lightweight materials. These manipulators are operated by using some actuator as for example dc motor or solenoids, while some have a hydraulic or pneumatic system. When these flexible manipulators are actuated they undergo vibrations due to their flexibility. During the motion of a flexible link, at each point of its trajectory damped vibration exist which cause each point of the link to vibrate and thus the tip position does not come to the desired position quickly and once the torque is removed, the link takes some time to settle down to its final position. The most crucial problems associated while designing a feedback control system for a flexible-link are that the system being non-minimum phase, under actuated and non-collocated. The manipulator used here consists of two links having two control inputs u_1 and u_2 i.e the torques to each link and two outputs y_1 and y_2 i.e tip deflection of each link.

Fuzzy Modelling

Using the simulated input-output data a first order T-S fuzzy model of the flexible manipulator is obtained. Rules are extracted by fuzzy clustering based identification. The data is divided into three clusters and hence three fuzzy IF-THEN rules are obtained.

RULE 1: $IF\ u_1(k)\ is\ N\ and\ u_2(k)\ is\ N$

$$THEN\ \ y_1(k+1) = \omega_{1,1}u_1(k) + \omega_{1,2}u_1(k-1) + \omega_{1,3}u_2(k) + \omega_{1,4}u_2(k-1) + \omega_{1,5}y_1(k) + \omega_{1,6}y_1(k-1) + \theta_1 \quad (3.27)$$

RULE 2: $IF\ u_1(k)\ is\ Z\ and\ u_2(k)\ is\ Z$

$$THEN\ \ y_1(k+1) = \omega_{2,1}u_1(k) + \omega_{2,2}u_1(k-1) + \omega_{2,3}u_2(k) + \omega_{2,4}u_2(k-1) + \omega_{2,5}y_1(k) + \omega_{2,6}y_1(k-1) + \theta_2 \quad (3.28)$$

RULE 3: $IF\ u_1(k)\ is\ P\ and\ u_2(k)\ is\ P$

$$THEN\ \ y_1(k+1) = \omega_{3,1}u_1(k) + \omega_{3,2}u_1(k-1) + \omega_{3,3}u_2(k) + \omega_{3,4}u_2(k-1) + \omega_{3,5}y_1(k) + \omega_{3,6}y_1(k-1) + \theta_3 \quad (3.29)$$

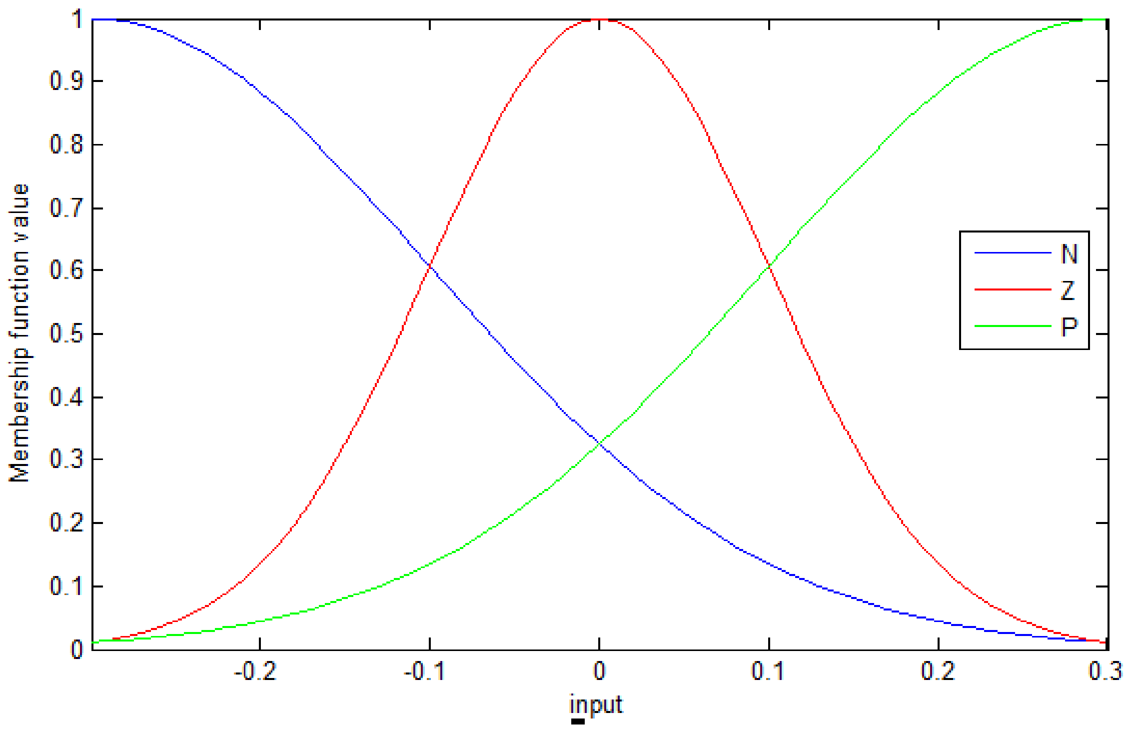

Fig 3.7 Gaussian membership function for T-S modelling

3.2.8 Choosing membership function- Gaussian membership function is choosen and the sets P,Z and N are shown in the fig 3.4. The mean of the sets P,Z and N are 0.3,0,-0.3 and standard deviation is 0.3 is choosen.The set P gives high weightage to the higher values of inputs which are nearer to 0.3 Nm. This value is choosen depending on the maximum torque limit of the actuators.

Table 3.1 Gaussian membership function properties

Membership function	Mean	Standard deviation
P	0.3	0.2
Z	0	0.1
N	-0.3	0.2

3.2.9 Least Square Estimate

Let us consider a linear regression model of the form

$$y(t) = \emptyset^T \theta$$

Where $y(t)$ is a measurable quantity, $\emptyset^T$is a vector of unknown quantities and θ is the vector of unknown parameters. Now the problem is to find an estimate of the unknown parameters $\hat{\theta}$ from the measured quantities $y(1), \emptyset(1), \dots., y(N), \emptyset(N)$. A system of linear equations can be obtained as

$$y(1) = \emptyset^T(1)\theta$$
$$y(2) = \emptyset^T(2)\theta$$
$$\vdots$$
$$y(N) = \emptyset^T(N)\theta$$

In matrix notation it can be written as

$$Y = \varphi\theta \qquad (3.30)$$

where

$$Y = \begin{pmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{pmatrix}$$

And

$$\varphi = \begin{pmatrix} \varphi(1) \\ \varphi(2) \\ \vdots \\ \varphi(N) \end{pmatrix}$$

Now because of the presence of disturbances and noise an exact solution isnot obtained and we get a estimate error given by

$$\varepsilon(t) = y(t) - \emptyset^T \theta$$

Now the least square estimate of θ is defined as the vector $\hat{\theta}$ which minimises the function

$$V(\theta) = \frac{1}{2}||\varepsilon||^2 \qquad (3.31)$$

For positive definite $\varphi^T\varphi$ the minimum point of V is given by

$$\hat{\theta} = (\varphi^T\varphi)^{-1}\varphi^T Y \qquad (3.32)$$

The parameters of the consequent is obtained using Least Square Estimate as given in eqn 3.32. The parameters of both the links are given in Table 3.2 and Table 3.3.

TABLE 3.2: Consequent weights of each rule i=1,2,3 for link1

weights	$\omega_{i,1}$	$\omega_{i,2}$	$\omega_{i,3}$	$\omega_{i,4}$	$\omega_{i,5}$	$\omega_{i,6}$	θ_i
RULE 1	3.75 $X10^{-12}$	-7.63 $X\ 10^{-5}$	3.67 $X\ 10^{-12}$	-7.48 $X\ 10^{-5}$	2.0519	-1.058	3.75 $X\ 10^{-11}$
RULE 2	0.4574	-0.538	-0.467	0.5387	2.004	-1.029	-0.001
RULE 3	-7.78 $X\ 10^{-4}$	-0.009	-0.004	0.004	1.846	-0.845	-2.96 $X\ 10^{-5}$

TABLE 3.3: Consequent weights of each rule i=1,2,3 for link2

weights	$\omega_{i,1}$	$\omega_{i,2}$	$\omega_{i,3}$	$\omega_{i,4}$	$\omega_{i,5}$	$\omega_{i,6}$	θ_i
RULE 1	1.318	-1.278	-1.345	1.304	-0.886	1.870	0.007
RULE 2	3.640 x 10^{-9}	0.001	3.567 x 10^{-9}	0.0012	-1.090	2.081	-5.094 x 10^{-16}
RULE 3	20.161	-13.284	-20.159	13.409	-0.452	1.549	-0.009

3.3 RESULTS

In fig 3.8 the output corresponding to each of the three rules is shown. The performance is validated for a data set which is shown in Fig.3.9 and Fig.3.11 for link 1 and link 2 of the TLFM. The output of the fuzzy identified model is compared with the non-linear model and the error is plotted. From the error plot in fig 3.10 and fig 3.12 it can be observed that the error decreases to zero. Hence it is observed that the fuzzy model can be approximated as the actual model.

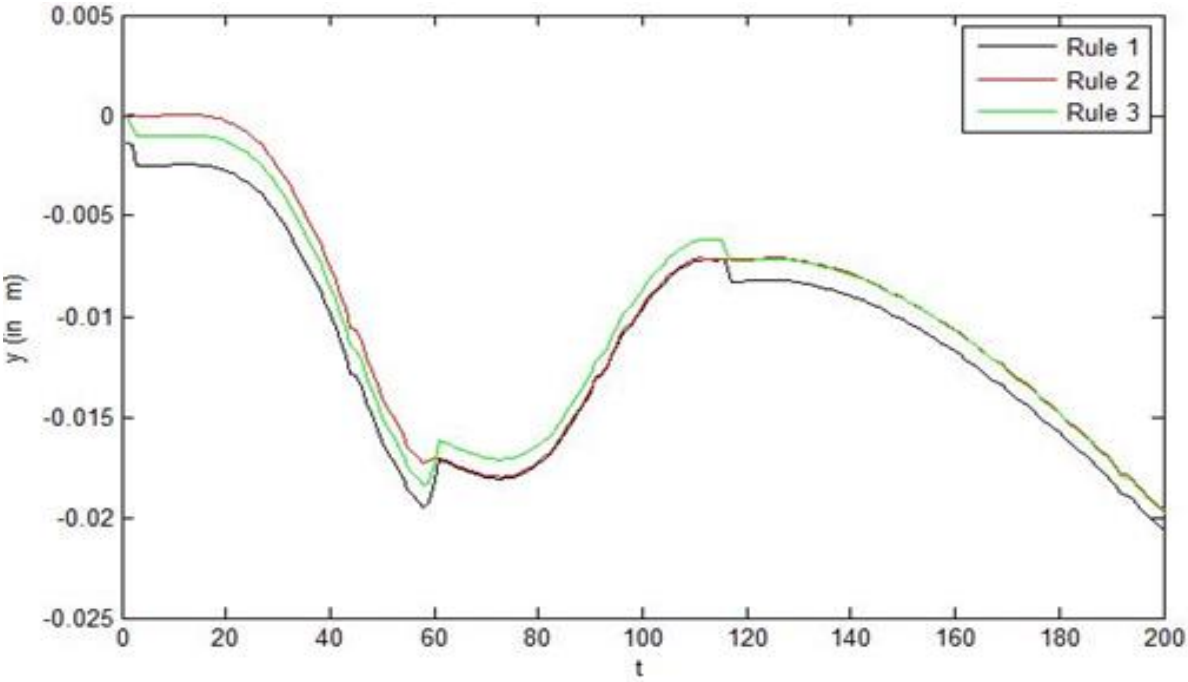

fig 3.8 Output defined by three rules of link 1

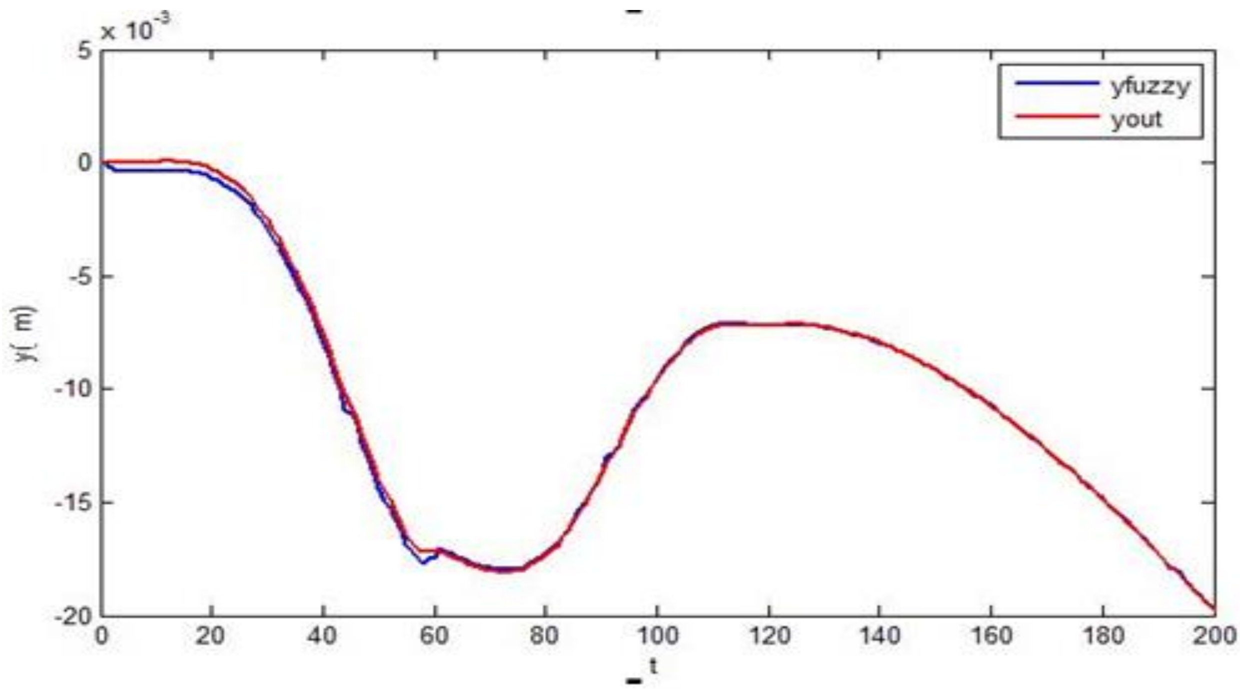

Fig 3.9: plot of tip position of nonlinear model and fuzzy model for link 1

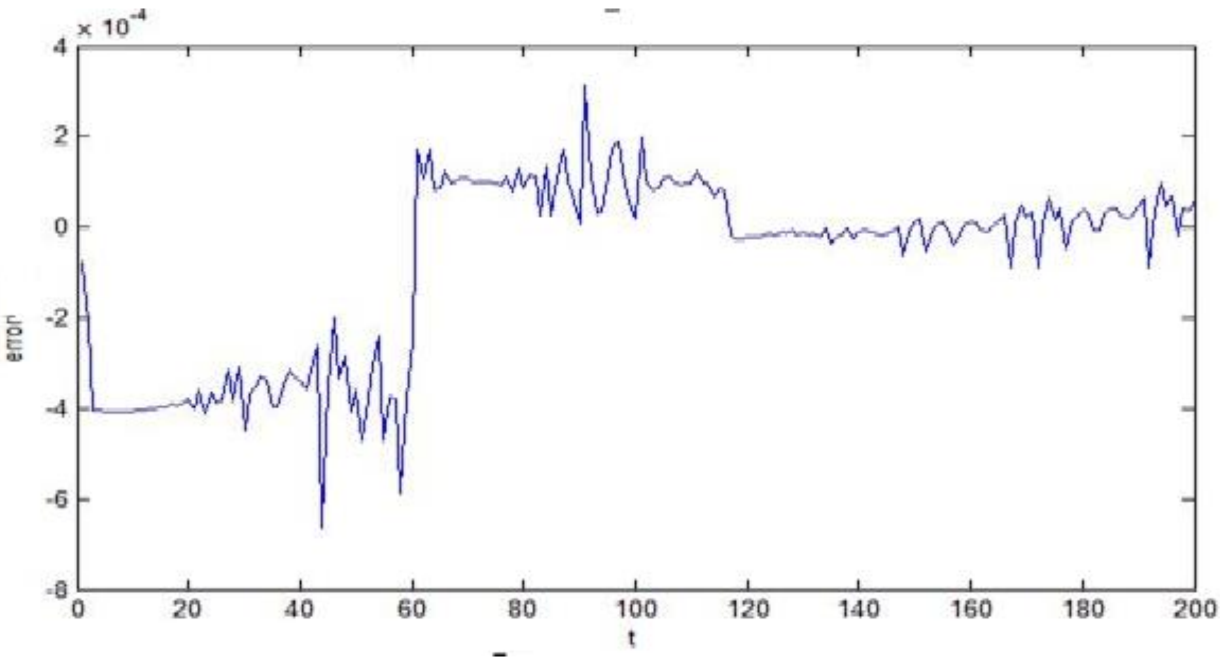

Fig 3.10 : plot of error of link 1

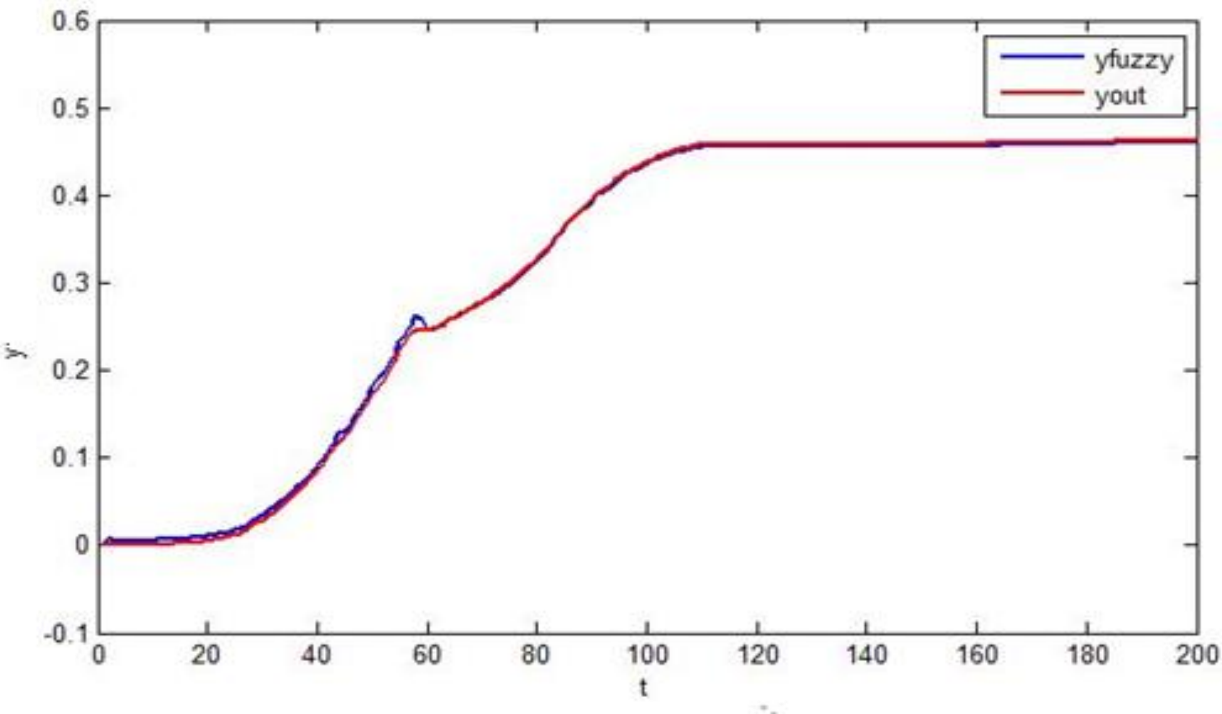

Fig 3.11 : plot of tip position of nonlinear model and fuzzy model for link 2

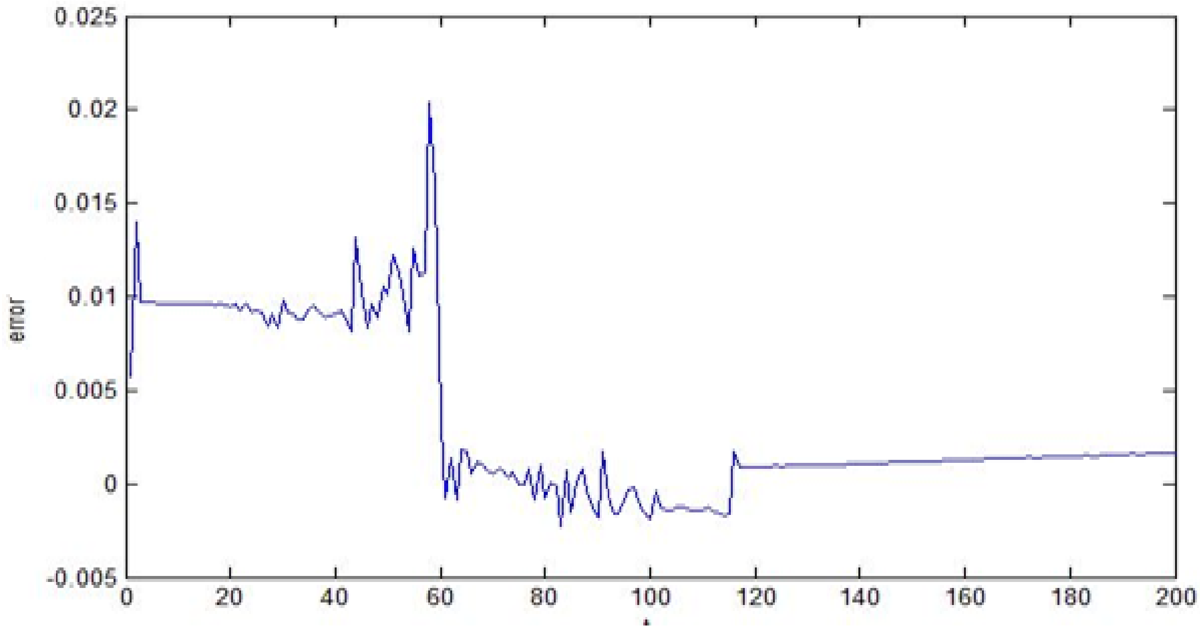

Fig 3.12: plot of error of link 2

3.4. CONCLUSION

In this section fuzzy identification is performed on a set of input-output data obtained from the two-link flexible manipulator. A fuzzy model is obtained which incorporates less computation than mathematical modelling and the results are compared. Moreover it also includes the infinite modes of vibration which are neglected in AMM. Hence a fuzzy model shows more accurate results. The increased number of local models take into account the complexity of the system and also the desired precision.

CHAPTER 4

CONTROLLER DESIGN FOR TIP DEFLECTION CONTROL

4.1 LQR controller design

4.2 MPC controller design

4.1 LQR controller design

4.1.1 Algorithm

LQR is an optimal control method which provides a control law satisfying a quadratic performance index. In this method a feedback gain matrix is obtained which minimizes J in order to achieve desired response. The cost function consists of Q and R ,the weighing factors that are to be tuned.

Let us consider a system

$$\dot{x} = Ax + Bu \tag{4.1}$$

Here we have to find the feeback matrix K for the control law $u = -Kx(t)$ so as to minimize the performance index given by

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt \tag{4.2}$$

where Q and R are positive definite Hermitian or real symmetric matrix.

The performance index can be written as

$$J = \int_0^\infty (x^T Qx + x^T K^T RKx)dt$$

$$= \int_0^\infty x^T (Q + K^T RK)x \ dt \tag{4.3}$$

Let $x^T(Q + K^T RK)x = -\frac{d}{dt}(x^T Px)$

Now $x^T(Q + K^T RK)x = -\dot{x}^T Px - x^T P\dot{x}$

$$= -x^T[(A - BK)^T P + P(A - BK)]x$$

Comparing both sides

$$Q + K^T RK = (A - BK)^T P + P(A - BK)$$

$$=> (A-BK)^T P + P(A - BK) - Q - K^T RK = 0 \tag{4.4}$$

Let $R = T^T T \ \ where\ T\ is\ non\ singular$

$$A^T P + PA + [TK - (T^T)^{-1}B^T P]^T [TK - (T^T)^{-1}B^T P] - PBR^{-1}B^T P + Q = 0$$

Now for minimisation of the cost function

$$\frac{dJ}{dK} = 0$$

$$TK = (T^T)^{-1}B^T P$$

$$K = T^{-1}(T^T)^{-1}B^T P$$

$$= R^{-1}B^T P \tag{4.6}$$

The control law is given by $u = -Kx(t) = -R^{-1}B^{T}Px(t)$, where P satisfies the Ricatti equation.

4.1.1 Results

The two link flexible manipulator is a 2-input 2-output MIMO system having 12 states as given by the state vector $x = [\theta_1, \theta_2, \delta_{11}, \delta_{12}, , \delta_{21}, \delta_{22}, \dot{\theta_1}, \dot{\theta_2}, \dot{\delta_{11}}, \dot{\delta_{12}}, , \dot{\delta_{21}}, \dot{\delta_{22}}]$

The weighing matrices Q and R are choosen to be diagonal. Here the response requirement of the tip deflections of both the links are the same .Initially we set Q as diag[1 1 .1 .1 .1 .1 1 1 1 1 1 1] and R as diag[1 1] and the response is shown in fig.4.1 and fig.4.2. The simulation result for tip position is plotted fig.4.1. Figure 4.2 shows the tip deflection. From the plot it is observed that the tip position of 0.2 rad is reached in 2 secs which is desirable. Now to decrease the overshoot in tip deflection the weights are further modified.

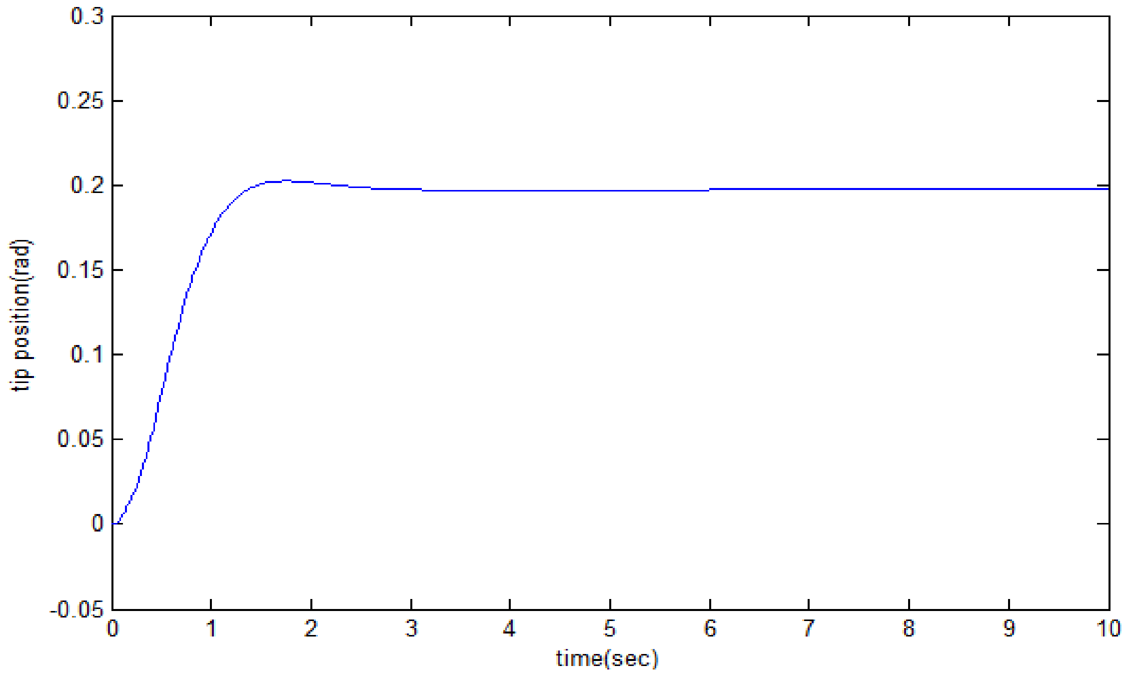

Fig 4.1: Tip position using LQR

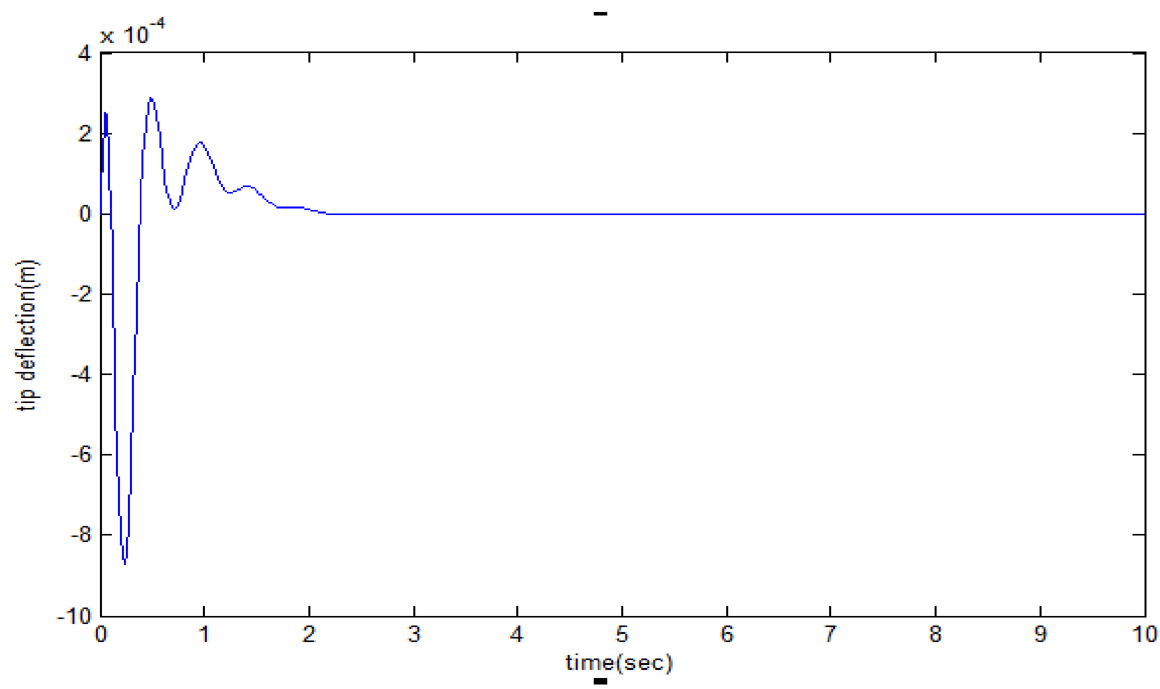

Fig 4.2: Tip deflection using LQR

Taking Q=diag[1 1 .1 .1 .1 .1 .1 .1 2 5 2 5] and R= diag[1 1] it is observed that the tip deflections are further reduced to 0.5 mm which is shown in fig 4.3

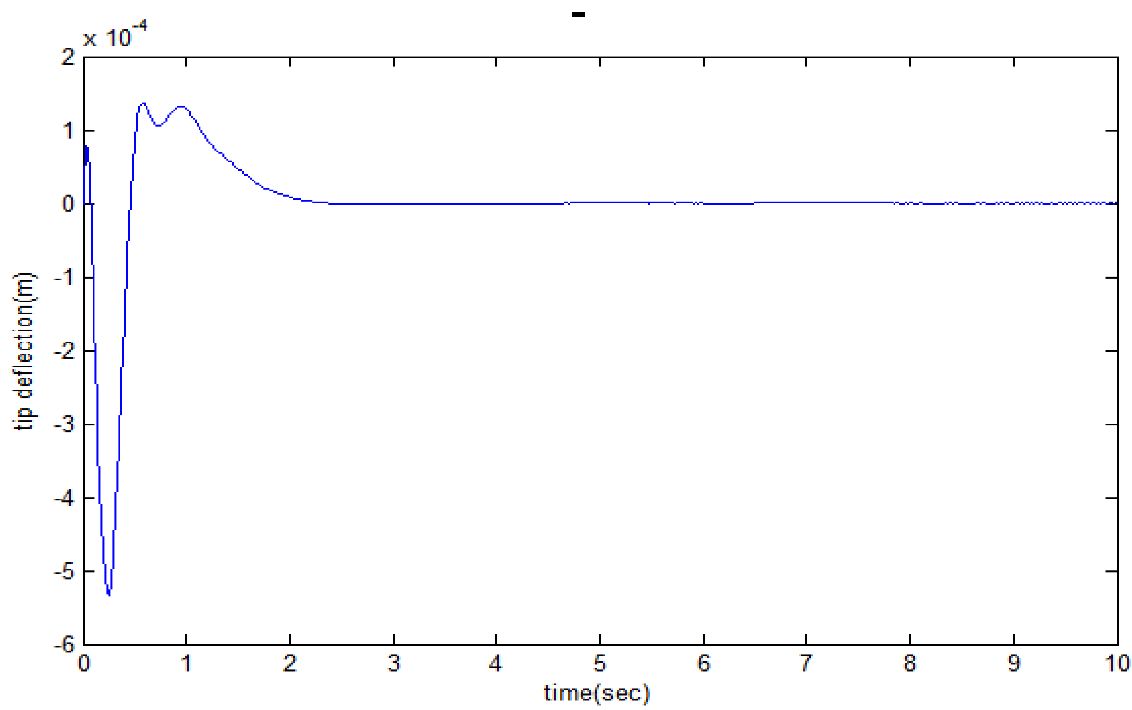

Fig 4.3: Tip deflection using LQR

4.2 Model Predictive Controller design

The Model Predictive Control is an optimal control method in which the control law is calculated using the system output. MPC is widely used in the industry due to its better performance. In MPC_a predictive model is designed which can be linear or nonlinear and using the model the future outputs are predicted at each sampling instant upto a prediction horizon and the predicted output is used in the minimisation of a performance index which results in a sequence of future control input. There are three main concepts lying behind general model predictive control strategy. a) Firstly a model is used which predicts the sequence of future output response of the plant. The output is predicted over a defined prediction horizon .b) Secondly to compute the future control sequence over a control horizon by minimizing a given performance index at the same time instants. c) and thirdly the use of receding horizon strategy. From the sequence of control inputs calculated only the first one in the sequence is applied after which the horizons are moved towards in the future and optimization process is repeated. The manipulation of the control input signal is done only within the control horizon after which it remains constant. The objective is to keep the predicted output closer to the desired path. The parameters to be tunned for the model predictive controller are N_p the prediction horizon, N_u, the control horizon.

The different forms of MPC to make model predictive controller are:

1. GPC(Generalized Predictive Control) ¬

2. Standard MPC

3. Modified MPC

4. Robust MPC

4.2.1 Structure of Model Predictive Controller

The several parts of MPC are shown as interconnected blocks.

The structure consists of two main parts

1. The predictive model

2.The optimiser consisting of cost function and the constraints

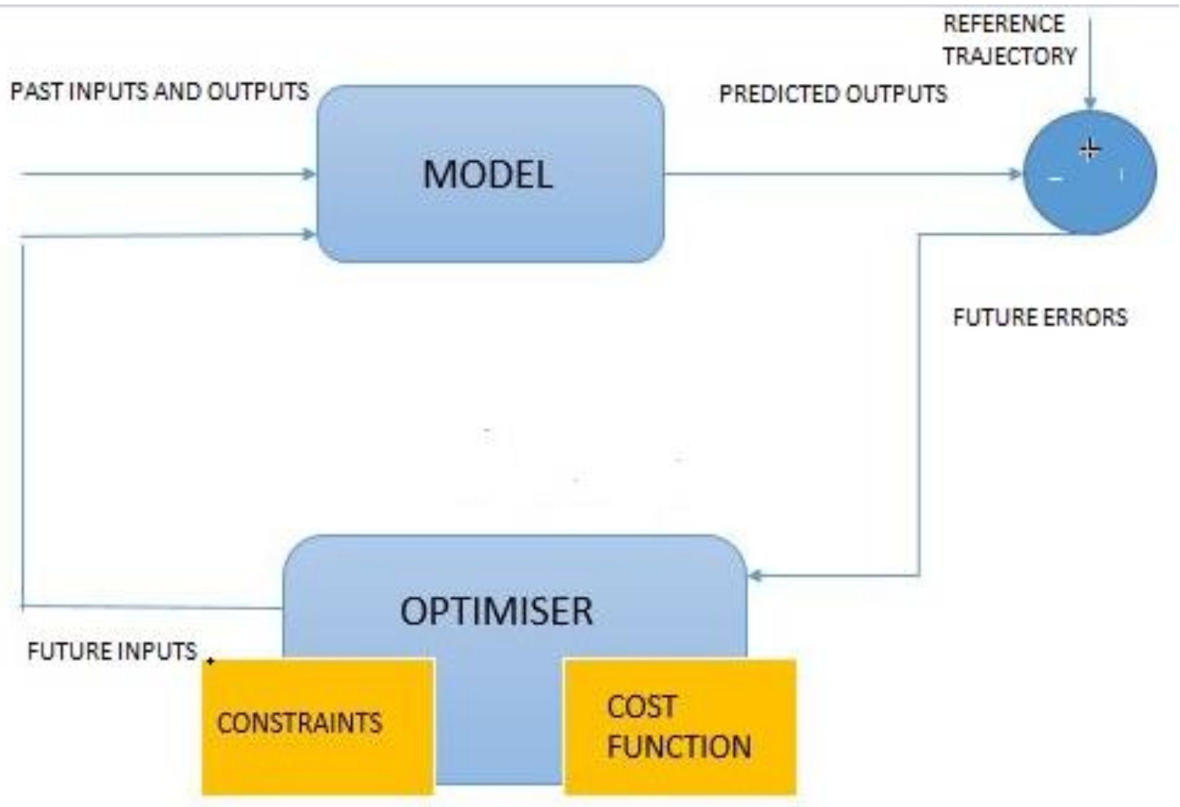

Fig 4.4: Structure of MPC

The controller is based on the model of the plant. The model takes past inputs and outputs of the plant and predicts the future outputs upto a defined furure instant and the error is sent to the optimiser. The optimiser consists of a cost function and constraints. The error is reduced by minimisation of the cost function and satisfying the constraints. The control function is re-optimized at every time step which is reduces the effect of model/plant mismatch, and is also necessary to counteract the effect of unknown disturbances. Different optimisation techniques are used to get a sequence of future inputs. In general, it is desired to control a specific plant by manipulating its inputs and using its outputs to gather state information. Also, it is likely that the measured plant output does not contain all information needed for the controller, thus a state estimator is often used to overcome this problem.

4.2.2 Characteristics

The main features/characteristics of MPC are ¬

- Moving horizon technique implementation with Control horizon, Prediction horizon and Receding horizon control concepts.
- Performance based time domain formulation.
- An explicit system model is used for prediction of future plant dynamics.
- Constraints values can be taken in to consideration

Advantages

- Structural changes are available in this method .
- We can define the prediction horizon.
- Also the number of parameters used to capture the future control trajectory can be predicted. The tuning method is simpler .
- We can handle unstable system and non-minimal phase by this method.

4.2.3 Design of standard MPC

The continuous time model is converted into discrete time model given by the following equations with m inputs and q outputs.

$$x_m(k+1) = A_m x_m(k) + B_m u(k) \tag{4.8}$$

$$y(k) = C_m x_m(k)$$

Now eqn can also be written as

$$x_m(k) = A_m x_m(k-1) + B_m u(k-1)$$

From eqn we can write

$$\Delta x_m(k+1) = A_m \Delta x_m(k) + B_m \Delta u(k) \tag{4.9}$$

where $\Delta x_m(k) = x_m(k) - x_m(k-1)$, $\Delta u(k) = u(k) - u(k-1)$

Now the output y_m can be written in terms of state variable Δx_m as

$$\Delta y(k+1) = C_m \Delta x_m(k+1) = C_m A_m \Delta x_m(k) + C_m B_m \Delta u(k) \tag{4.10}$$

The state-space equations can be written in the augmented form as

$$\begin{bmatrix}\Delta x_m(k+1)\\ y(k+1)\end{bmatrix} = \begin{bmatrix}A_m & 0_m^T\\ C_mA_m & I_{qxq}\end{bmatrix}\begin{bmatrix}\Delta x_m(k)\\ y(k)\end{bmatrix} + \begin{bmatrix}B_m\\ C_mB_m\end{bmatrix}\Delta u(k)$$

$$y(k) = \begin{bmatrix}0_m & I_{qxq}\end{bmatrix}\begin{bmatrix}\Delta x_m\\ y_m\end{bmatrix} \qquad (4.11)$$

Let us represent the sequence of control input and future outputs as

$\Delta U = [\Delta u(k)^T\ \Delta u(k+1)^T\ \ \dots\ \Delta u(k+N_c-1)^T]^T$

$Y = [y(k+1|k)^T\ \ y(k+2|k)^T\ \ y(k+3|k)^T.\ .\ .\ y(k+N_p|k)^T]^T$

Now the future state variables are calculated as follows

$x(k+1|k) = Ax(k) + B\Delta u(k)$

$x(k+2|k) = Ax(k+1|k) + B\Delta u(k+1)$

$=A^2x(k) + AB\Delta u(k) + B\Delta u(k+1)$

$\vdots$

$x\left(k+N_p\middle|k\right) = A^{N_p}x(k) + A^{N_p-1}B\Delta u(k) + A^{N_p-2}B\Delta u(k+1)+..+A^{N_p-N_c}B\Delta u(k+N_c-1)$

Hence using the above we can write

$$Y = Fx(k) + \varphi\Delta u$$

where

$$F = \begin{bmatrix}CA\\ CA^2\\ CA^3\\ \vdots\\ CA^{N_p}\end{bmatrix} \quad ; \qquad \varphi = \begin{bmatrix} CB & 0 & 0 \cdots & 0\\ CAB & CB & 0 \dots & 0\\ CA^2B & CAB & CB \dots & 0\\ \vdots & & & \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \dots CA^{N_p-N_c}B\end{bmatrix}$$

The incremental control law is given by

$$\Delta U = (\varphi^T\varphi + R)^{-1}(\varphi^T R_S - \varphi^T Fx(k)) \qquad (4.12)$$

where the set point signal is given by $r(k) = [r_1(k)\ \ r_2(k)\ \ \dots\ \ r_q(k)]^T$

4.2.4 Results

For the design of the controller the input variables are the control torques u_1 , u_2 and the output variables are the tip positions and tip deflections of the two links i.e θ_{1} , θ_2, y_1 *and* y_2. The plant is discretized and using the linear discrete model a MPC is designed. The reference of angular position is taken as 0.2 rad..The prediction horizon is taken as 7 and control horizon is 2.The simulation results are plotted in fig 4.5.and fig 4.6. Fig shows tip deflection of link 2. From the plots it is observed that link 1 shows a maximum deflection of 0.065 mm with a settling time of 1 secs.

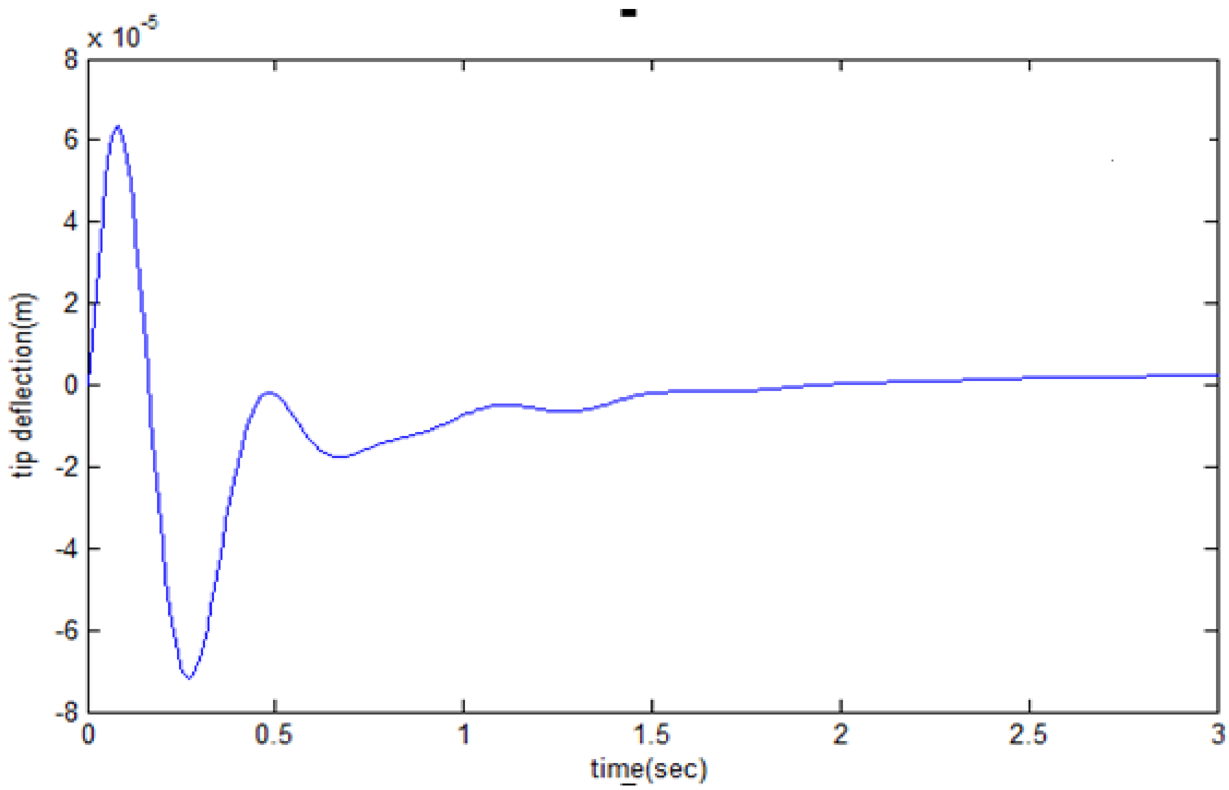

Fig 4.5: Tip deflection using MPC

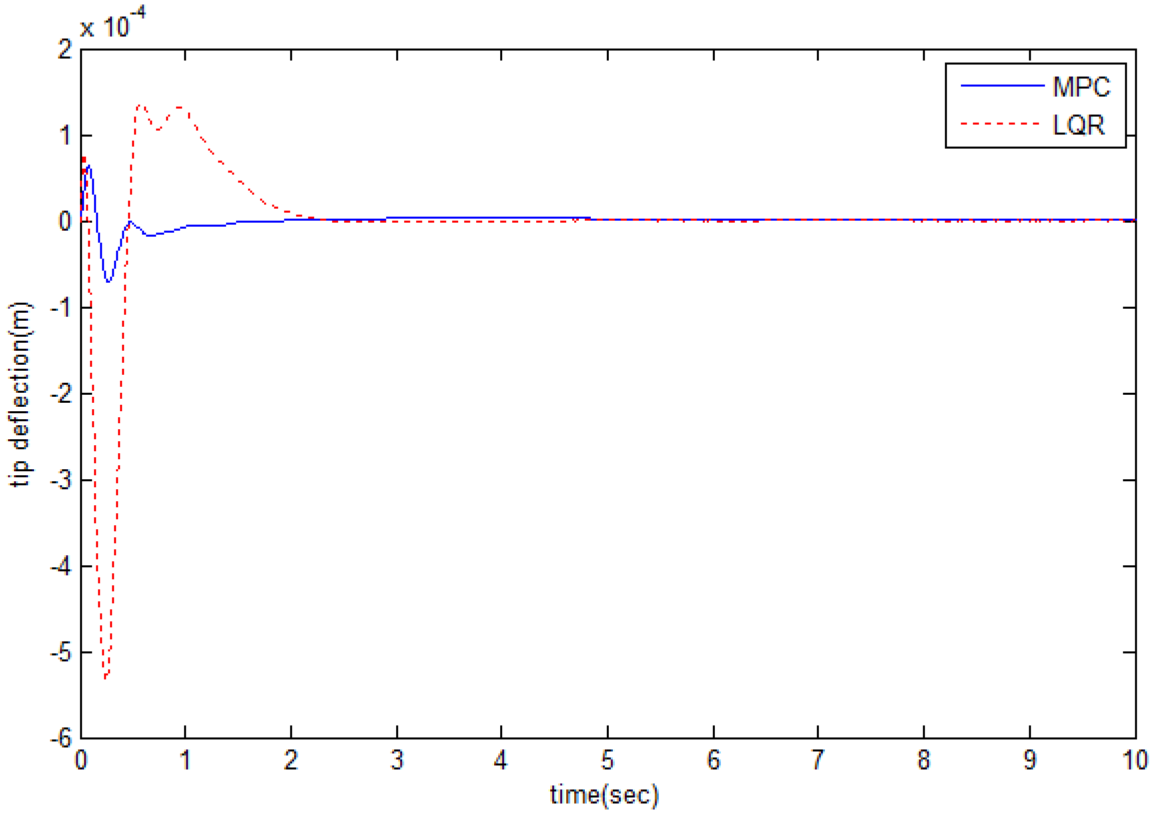

Fig 4.6: Tip deflection using LQR and MPC

4.2.5 Comparision

Nonlinear simulation model is run separately with LQR and MPC type controllers fig 4.5 shows corresponding the tip deflection. Fig 4.6 shows a comparisons of LQR and MPC. In LQR selection of weights is one of the main criteria to get better performance. And care must be taken in choosing proper weights.From the fig 4.6 It is observed that the oscillations are reduced in case of MPC along with maximum overshoot i.e from 0.575mm incase of LQR to 0.065mm incase of MPC. MPC control also improves the settling time to 1 sec while LQR controller takes 3.5 secs. Moreover solution in MPC is much simpler while LQR requires the Ricatti equation to be solved and hence MPC is also better for adaptive purposes.

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1 Conclusion

5.2 Suggestion for future work

5.1 Conclusion

The dynamics of a two link flexible manipulator and its modelling is discussed in chapter 3. A two link flexible manipulator is modelled using both mathematical and measurement approaches. Assumed mode method is used in the mathematical approach where only two modes of vibration are considered .where the frequencies of vibration are calculated using boundary conditions. Another approach i.e fuzzy identification is also applied which minimises the computation of mathematical modelling. Clustering is applied on the input-output responses of the system and local linear ARX models are identified from the clusters. From the results it can be observed that the fuzzy model can be approximated in place of the non-linear model.

The main control complexity ,which is to control the tip position minimising the deflection once the actuating force is removed, is overcome using Model predictive control strategy. The plant is linearised and a the linear model is used to predict the future responses. The prediction and control horizons are adjusted to get better results based on maximum overshoot of the deflection and the settling time. Depending on the maximum current capacity of the actuators the control input is constrained within limits. Simulation results are shown and compared with a LQR controller.

5.2 Suggestion for future work

- In the proposed controller design a linearized model of the plant is used to predict the future control inputs. However the model only considers two modes of vibration on the other hand the modes of vibration are infinite. Hence it may bring decrease the accuracy of control.
- Fuzzy model incorporating infinite modes of vibration can be used in place of a linear or non-linear model of the plant since the fuzzy model takes into account all the modes of vibration and hence it may provide better performance
- A adaptive MPC can be designed to take into account the changes in payload where the parameters of the fuzzy model are updated using RLS algorithm.

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