

GRAVITATIONAL SEARCH ALGORITHM: A NOVEL OPTIMIZATION FOR ECONOMIC DESIGN IN DISCONTINUOUS MODEL

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CERTIFICATE

*This is to certify that the work in this thesis entitled “**GRAVITATIONAL SEARCH ALGORITHM: A NOVEL OPTIMIZATION FOR ECONOMIC DESIGN IN DISCONTINUOUS MODEL**” by **Abhijeet Mandal**, has been carried out under my supervision in partial fulfillment of the requirements for the degree of **Bachelor of Technology** in *Mechanical Engineering* during session 2014 - 2015 in the *Department of Mechanical Engineering, National Institute of Technology, Rourkela*. To the best of my knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.*

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ABSTRACT

Control charts are generally utilized to monitor and maintain the statistical control of a process. Designing a control chart means selection of three parameters such as sample size n , sampling interval h and width of control limits k . To maintain a control chart we have to incur various types of cost such as prevention costs, appraisal costs, internal failure costs, external failure costs and total cost. In economic design the objective is to minimize the total cost associated with control chart. Thus, the economic design is one type of unconstrained optimization problem. Economic designs of \bar{X} -bar control chart for two types of manufacturing process models namely continuous and discontinuous is provided in the literature. In this project, gravitational search optimization has been utilized for the economic design of \bar{X} -bar chart for discontinuous process. The results were observed to be comparable to that reported by the literature.

LIST OF FIGURES AND TABLE

Figure 1.1: An out of control process	3
Figure 1.2: Type- I error	6
Figure 1.3: Type- II error	6
Figure 3.1: Flowchart of GSA	19
Figure 4.1: Stages of a production cycle	22
Fig. 5.1 Cost vs. Sample size	26
Table 4.2: Input data	24
Table 5.1: Optimum solution	25

CONTENTS

<u>Sl No.</u>	<u>Topic</u>	<u>Page</u>
1	Introduction	1-12
2	Literature review	13-14
3	Gravitational search algorithm	15-19
4	Economic design of control chart	20-24
5	Result and discussion	25-26
6	Conclusion	27
7	References	28

INTRODUCTION

Quality and Quality control

The conventional definition of quality is based on the viewpoint that products and services given to consumers must meet their requirements. Quality is conformance to requirements and specifications. This definition is attributed to Crosby [1]. Quality can be improved by reducing the variability in processes. There are various dimensions of quality. Garvin [2] provides an accurate discussion of eight components or dimensions. The dimensions of quality include:

- 1) Performance
- 2) Reliability
- 3) Durability
- 4) Serviceability
- 5) Aesthetics
- 6) Features
- 7) Perceived quality
- 8) Conformance to standards

Quality control is the set of activities that reduce of variability in processes and products. The two types of quality control are:

- 1) On line quality control
- 2) Off line quality control

On line quality control is the type of quality control which is done without interruption of the production process.

Off line quality control is the type of quality control which is usually done in laboratory or other

production center after stopping the production process.

Quality characteristics

Quality characteristics are elements that define the intended quality level of a product or service.

Quality characteristics fall into two broad categories: variables and attributes. Characteristics that can be measured and expressed on a numerical scale are called variables. Characteristics that cannot be measured on a numerical scale are called attributes.

Statistical process control

Statistical process control is a technique of quality control which monitors and controls a process by use of statistical methods. Monitoring and controlling of a process is necessary for its operation at full potential.

Control chart is one of the seven quality tools. The seven tools of quality is the designation given to a set of graphical methods which are most helpful for solving problems related to quality. The seven tools are:

- 1) Cause and effect diagram
- 2) Check sheet
- 3) Control chart
- 4) Histogram
- 5) Pareto chart
- 6) Scatter diagram
- 7) Flow chart or run chart

Control chart

Control chart is one of the seven tools of quality. It is a graph depicting sample statistics versus sample number or time, which helps in the study of change in a process over time.

Main elements of control chart are:

- 1) Central line(CL),
- 2) Upper control limit(UCL),
- 3) Lower control limit(LCL), and
- 4) Sample statistics plotted on the chart.

When a point falls outside either of the two control limits, the process is said to have gone out-of-control. If all the points are within the control limits, the process is said to be in control. A out-of-control process is shown in Fig. 1.1 below.

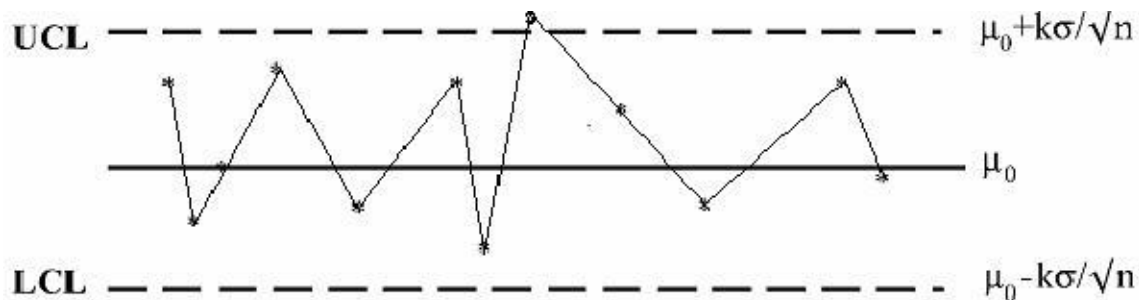


Fig.1.1. An out-of-control process

Types of control charts

Control charts are broadly classified into types: variable charts and attributes charts.

Variable charts

If the process parameter affecting the quality of a process is of variable type, the variable control

chart is used. The shaft diameter, volume of liquid, weight etc. are examples of variable parameters. The following charts come under category of variable charts:

X-bar and Range charts

X-bar chart is used to draw all the mean sample values so as to influence the mean value of variables. R chart is used to draw all the range values of samples.

Standard deviation chart

Standard deviation chart is used to plot all the standard deviation values of samples.

CUSUM chart

A CUSUM Chart is a control chart for variables data which plots the cumulative sum of the deviations from a target.

Exponentially-weighted moving average (EWMA) chart

The idea of moving averages of successive samples can be generalized. In principle, in order to detect a trend we need to weigh successive samples to form a moving average; however, instead of a simple arithmetic moving average, we could compute a geometric moving average.

Attribute charts

Attribute charts are a set of control charts particularly intended for attributes information. The family of attribute charts includes the:

np chart

This chart is used for monitoring the number of times a condition occurs, relative to a constant sample size, when each sample can either have this condition, or not have this condition.

p chart

This chart is used for monitoring the percent of samples having the condition, relative to either a fixed or varying sample size, when each sample can either have this condition, or not have this condition.

c chart

This chart is used for monitoring the number of times a condition occurs, relative to a constant sample size, when each sample can have more than one instance of the condition.

u chart

This chart is used for monitoring the percent of samples having the condition, relative to either a fixed or varying sample size, when each sample can have more than one instance of the condition.

Errors in Control Charts

There are two types of errors in control chart: Type I and Type II. Type I error is when the process is concluded out of control even when the process is in control. Type I error is just a false alarm. The probability of type I error is given by α .

Type II error is when the process appears to be in control even though it is out of control. Type II error is caused by the shift in process mean. The control chart is not good enough to detect process shift. Thus this is a measure of inefficiency of the control chart. The probability of occurrence of type II error is given by β .

$\Phi(z) = (2\pi)^{-1/2} \exp(-z^2/2)$ is the standard normal density

$$\alpha = 2 \int_k^{\infty} \Phi(z) dz \quad (\text{Probability of occurrence of type I error})$$

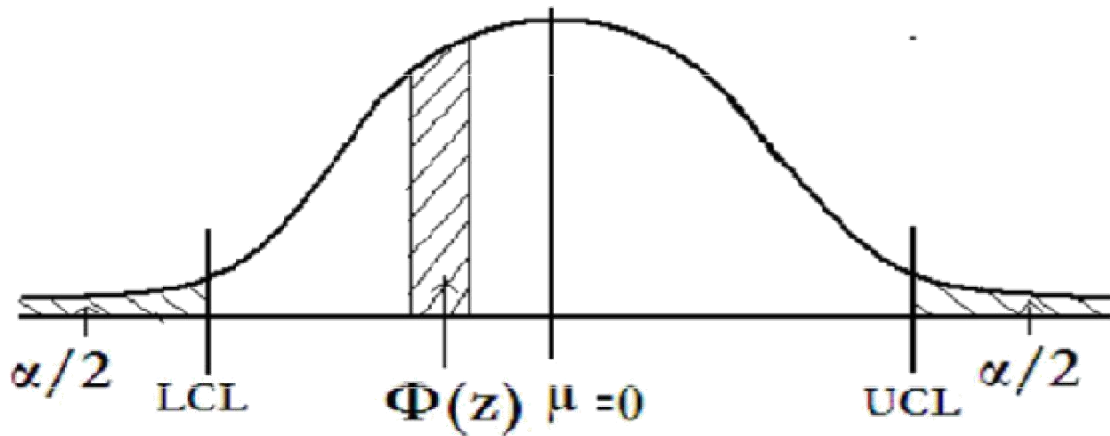


Fig. 1.2. Type I error

$$\beta = 1 - \int_{-\infty}^{-k-\delta/\sqrt{n}} \Phi(z) dz - \int_{k-\delta/\sqrt{n}}^{\infty} \Phi(z) dz \text{ (probability of occurrence of type II error)}$$

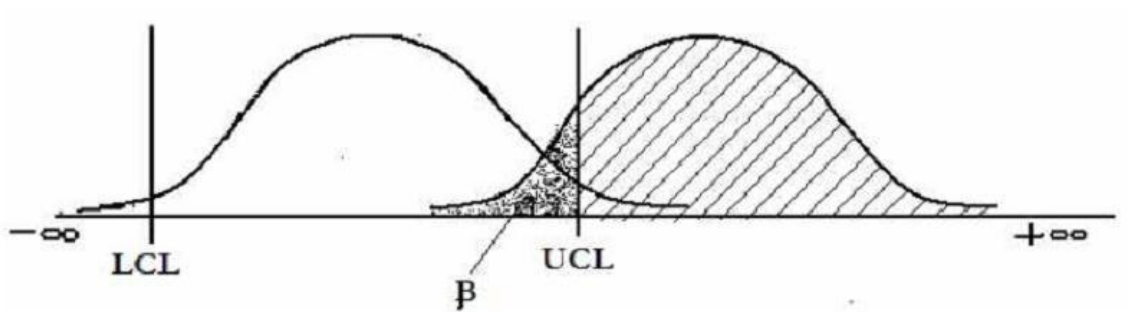


Fig. 1.3. Type II error

Economic Design of Control Charts

Conventionally, the design of control charts is done by only taking statistical criterion into consideration. This usually involves the selection of sample size, the lower control limit and the upper control limit. Another factor like sampling frequency is also taken into consideration.

Thus the design of control chart consists of selecting 3 important parameters:

- 1) Sample size
- 2) Sampling frequency
- 3) Control limits

The design of control graph has various economic outcomes in which the costs of inspecting and testing are connected with exploring signals that are out of control. Redressing the assignable causes and costs of permitting non-adjusting units to achieve the consumer are all influenced by the decision of the control chart from a economic perspective.

Process characteristics

To plan a financial model for the configuration of a control chart, it is important to make certain assumptions about how the process behaves. At the point when no assignable reasons are available, the process is described by a solitary in-control state related to the mean or division non-adjusting relating to quantifiable or an attribute quality feature separately. The process may have, by and large, more than one out of control states. Every out of control state is generally connected with a specific kind of assignable reason. The way the process moves between the in-control and out of control states, having assignable reasons amid an interim of time, have the impression of a Poisson process. The transition of the process between states and the nature of its failure implies the instantaneous nature of the states and signifies that the process has to be interrupted for rectification.

Cost parameters:

Three types of costs are taken into account while designing a control chart:

- 1) Cost incurred on sampling and testing
- 2) Costs relating to investigating an out of control state and the costs of repairing any assignable reasons.
- 3) Costs related to manufacturing and generation of non-adjusting products.

Generally, we assume that the total cost of sampling and testing includes two parts: variable and fixed cost. If fixed and variable costs are denoted by a_1 and a_2 respectively, then the total cost is:

$$a_1 + a_2n$$

The cost of examining and potentially amending the process following an out-of-control signal has been dealt with in various ways. A few authors have recommended that the costs incurred while examining false alarms will change the costs of remedying assignable reasons. Subsequently, these two circumstances must be described in the model by diverse cost coefficients. Besides, the cost of repairing or amending the process could rely on upon the kind of assignable reason available in the process. Generally, it is watched that little shifts are hard to discover yet simple to rectify, though substantial shifts are anything but difficult to discover yet hard to rectify.

Economic models are generally formed utilizing an total cost function, which communicates the relationship between the control chart plan parameters and the three sorts of costs talked about over the generation, checking, and alteration process may be considered as a progression of autonomous cycles over a definite time period. Every cycle starts with generation transform in the in-control state and proceeds until the process checking through the control graph brings about an out-of-control sign. The process is defined to return in the in-control state when some

changes are performed to rectify the process helping in starting of a cycle. Using, the process failure, search, and repair pattern, we can define few expected quantities as follows:

Let

E(T): expected length of a cycle.

E(C): expected total cost induced during a cycle.

E(A): expected cost per unit time.

Thus

$$E(A) = E(C)/E(T) \quad (1)$$

Various optimization processes can be then connected to Eq (1) to focus the ideal control charts drawn economically.

TYPES OF CONTROL CHARTS

Variables Charts

The established kind of control chart is developed by gathering information occasionally and plotting it against time. If more than one information is gathered in the meantime, measurements, for example, the mean, range, or standard deviation is drawn. For signaling unusual large variations, we take the help of the control limits.

X-bar & Range Charts

X-bar chart is used to draw all the mean values of samples so as to influence the mean value of variables. X-bar chart is used to draw all the range values of samples so as to influence the variation of variables.

Attributes Charts

For quality information, for example, emerge from PASS/FAIL testing, the charts utilized frequently draw either rates or extents. At the point when the sample sizes differ, the size of the

samples control variation in control limit values.

Attribute Charts are a amalgamation of control charts particularly intended for Attributes information. Quality charts screen the process area and variety after some time in a solitary chart.

The family of Attribute Charts include the:

c chart: This chart is used to plot the number of defects (per batch, per day, per machine, per 100 feet of pipe, etc.). This chart assumes that defects of the quality attribute are rare, and the control limits in this chart are computed based on the Poisson distribution.

u chart: This chart is used to plot the rate of defects, that is, the number of defects divided by the number of units inspected (the n ; e.g., feet of pipe, number of batches). Unlike the c chart, this chart does not require a constant number of units, and it can be used, for example, when the batches (samples) are of different sizes.

np chart: This chart is used to plot the number of defectives (per batch, per day, per machine) as in the C chart. However, the control limits in this chart are not based on the distribution of rare events, but rather on the binomial distribution.

Therefore, this chart should be used if the occurrence of defectives is not rare. For example, this chart can be used to control the number of units produced with minor flaws.

p chart: This chart is used to plot the fraction of defectives (per batch, per day, per machine, etc.) as in the u chart. However, the control limits in this chart are not based on the distribution of rare events but rather on the binomial distribution (of proportions). Therefore, this chart is most applicable to situations where the occurrence of defectives is not rare. All of these charts can be adapted for short

production runs (short run charts), and for multiple process streams. If the percent of defectives is plotted, it is called 100p chart or percent defective chart.

Exponentially-weighted Moving Average (EWMA) Chart

The idea of moving averages of successive (adjacent) samples can be generalized. In principle, in order to detect a trend we need to weight successive samples to form a moving average; however, instead of a simple arithmetic moving average, we could compute a geometric moving average.

CUSUM Charts

A CUSUM Chart is a control diagram for variables information which plots the total whole of the deviations from an objective. A V-veil is utilized as control cutoff points. Since each plotted point on the CUSUM Chart utilizes data from every single former specimen, it recognizes much littler process shifts than an ordinary control chart would. CUSUM Charts are particularly powerful with a subgroup size of one. Run tests ought not to be utilized following each plotted point is subject to earlier focuses as they contain regular information values.

Types of processes

All processes can be classified into two major groups: continuous and discontinuous. If the process is allowed to continue to operate even if an out-of-control signal is obtained from a control chart, the process is called continuous. The search for assignable cause and then its removal takes place as the process is running. On the other hand, the discontinuous process is stopped during the search and elimination of assignable cause. Here we consider the case of discontinuous model.

Introduction of Economic Models of X-bar Control Charts

The most prominent and broadly utilized control chart is X-bar chart when managing a variable quality trademark. It is typically a standard practice to control σ the mean of value trademark. The development and utilization of x bar chart is effectively charted. In the event that assignable causes are available which bring about out-of-control sign, then the quick activity is to identify and wipe out the cause. Duncan [3] first presented economic model for the optimum economic design of the X-Bar control chart. The model determines the control chart parameters while incorporating formal optimization methodology. Traditional applications of the control charts are based on the assumption of process stability. But this is violated in many cases. In this regard, Cai et al. [4] designed an economic model for a trended process.

Objective

The main objective of this thesis is to minimize the cost function in designing an X-bar control chart for a discontinuous process using Gravitational search algorithm.

Literature Review

Quality is neither intangible nor immeasurable [1]. It is a strategic imperative that can be quantified and put back to work to improve the bottom line. Acceptable quality or defect levels and traditional quality control measures represent evidence of failure rather than assurance of success.

Garvin [2] proposed eight critical dimensions or categories of quality that can serve as a framework for strategic analysis: performance, features, reliability, conformance, durability, serviceability, aesthetics, and perceived quality. He concluded that the most traditional notions: conformance and reliability-remain important, but they are subsumed within a broader strategic framework.

Duncan [3] was the first to propose an economic model for the design of control chart. The model has to be further optimized using various meta-heuristics to obtain an optimal design.

Cai et al. [4] have proposed a economic model for designing a control chart for a process that is trended. Conventional uses of the control charts are based the assumption of process steadiness. Yet, this rule is broken a few times. The creators are of the opinion that the trended output coming about because of a disintegrating factor like instrument wear, material utilization, and power utilization must be analyzed differently. The scientists built up an economic model and tried the outcomes.

Montgomery et al. [5] examined and compared performance of the economic design of X-bar control chart for two production process models: continuous and discontinuous. They concluded that design of discontinuous model will lead to larger sample sizes, wider control limits and a longer sampling interval than the continuous process design.

Nezamabadi-pour et al. [6] proposed a new heuristic algorithm based on law of gravity and mass interaction phenomenon. In order to examine the algorithm, it was tested on various functions and the results when compared to other techniques like PSO, RGA, and CFO were found superior.

Gravitational search algorithm

Gravitational search algorithm (GSA) is a heuristic enhancement calculation which has been increasing enthusiasm among mainstream researchers as of late. GSA is a nature motivated calculation which is taking into account the Newton's law of gravity and the law of motion. GSA is assembled under the population based approach and is accounted for to be more instinctive. The calculation is planned to enhance the execution in the investigation and exploitation capacities of a population based calculation, based on gravity rules. Nonetheless, as of late GSA has been censured for not really in view of the law of gravity. GSA is accounted for to exclude the distance between masses in its formula, though mass and distance is both fundamental parts of the law of gravity.

GSA was presented by Nezamabadi-pour et al. [6] and is expected to clarify enhancement issues. The population based heuristic calculation is situated in light of the law of gravity and mass interactions. The calculation is embodied accumulation of searcher operators that interface with one another through the gravity power. The specialists are considered as items and their execution is measured by their masses. The gravity power causes a worldwide development where all items move towards different items with heavier masses. The moderate development of heavier masses ensures the misuse venture of the calculation and relates to great arrangements. The masses are obeying the law of gravity and the law of motion in Equation (2).

$$F = G (M1.M2 / R^2) \quad (1)$$

$$a = F/M \quad (2)$$

In view to Equation (1), F quantifies the gravitational force, G is gravitational constant, M_1 and M_2 are the mass of the first and second entities and R is the separation distance between the two entities. While for Equation (2), Newton's second law demonstrates that when a power, F , is connected to an item, its increasing speed, a , relies on upon the power and its mass, M .

In GSA, the specialists has four parameters which are position, inertial mass, active gravitational mass, and passive gravitational mass. The position of the mass acts as a solution to the problem, where the gravitational and inertial masses are resolved utilizing a fitness value. The calculation is explored by altering the gravitational and inertial masses, while every mass gives an answer. Masses are pulled in by the heaviest mass. Henceforth, the heaviest mass shows an ideal arrangement in the search space. The steps of GSA are as per the following:

Step 1: Initialization of agents:

The N agents are positioned without any definite sequence. $X_i = (x_i^1, \dots, x_i^d, \dots, x_i^n)$, for $i = 1, 2, \dots, N$.

x_i^d represents the positions of the i th agent in the d th dimension, while n is the space dimension.

Step 2: Fitness evolution and best fitness computation:

For minimization or maximization problems, after each iteration best and worst fitness values of every agent is calculated.

Minimization problems:

$$\text{best}(t) = \min \text{fit}_j(t)$$

$$j = 1, 2, \dots, N$$

$$\text{worst}(t) = \max$$

$$\text{fit}_j(t) \quad j=1, 2, \dots, N$$

Maximization problems:

$$\text{best}(t) = \max \text{fit}_j(t)$$

$$j=1, 2, \dots, N$$

$$\text{worst}(t) = \min$$

$$\text{fit}_j(t) \quad j=1, 2, \dots, N$$

$\text{fit}_j(t)$ signifies the fitness value of the j th agent in iteration t , $\text{best}(t)$ and $\text{worst}(t)$ signifies the best and worst fitness in iteration t .

Step 3: Gravitational constant (G) computation:

$$G(t) = G_0 e^{(-\alpha t/T)}$$

G_0 and α are initialized at the beginning and will be reduced with time to control the search accuracy. T is the total number of iterations.

Step 4: Masses of the agents' calculation:

Gravitational and inertia masses for each agent are calculated at iteration t .

$$M_{ai} = M_{pi} = M_{ji} = M_i, \quad i = 1, 2, \dots, N.$$

$$m_i(t) = \frac{\text{fit}_i(t) - \text{worst}(t)}{\text{best}(t) - \text{worst}(t)}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^N m_j(t)}$$

M_{ai} and M_{pi} are the active and passive gravitational masses respectively, while M_{ii} is the inertia mass of the i^{th} agent.

Step 5: Accelerations of agents' calculation:

Acceleration of the i^{th} agents at iteration t is

computed. $a_i^d(t) = F_i^d(t)/M_{ii}(t)$

$F_i^d(t)$ is the total force acting on i^{th} agent calculated as:

$$F_i^d(t) = \sum_{j \in Kbest, j \neq i} rand_j F_{ij}^d(t)$$

$Kbest$ is the set of first K agents having the best fitness value and largest mass.

$F_{ij}^d(t)$ is computed as the following equation:

$$F_{ij}^d(t) = G(t) \cdot (M_{pi}(t) \cdot M_{aj}(t) / R_{ij}(t) + \epsilon) \cdot (x_j^d(t) - x_i^d(t))$$

$F_{ij}^d(t)$ the force acting on agent i from agent j at d^{th} dimension and t^{th} iteration.

$R_{ij}(t)$ is the Euclidian distance between two agents i and j at iteration t . $G(t)$ is the computed gravitational constant at the same iteration while ϵ is a small constant.

Step 6: Velocity and positions of agents:

Velocity and the position of the agents at next iteration ($t+1$) are computed based on the following equations:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1)$$

Step 7: Repeat steps 2 to 6

Steps 2 to 6 are repeated until the iterations reach their maximum limit. The best fitness value at the final iteration is computed as the global fitness while the position of the corresponding agent at specified dimensions is computed as the global solution of that particular problem.

Flowchart of GSA

The working of gravitational search algorithm is illustrated in Fig. 3.1.

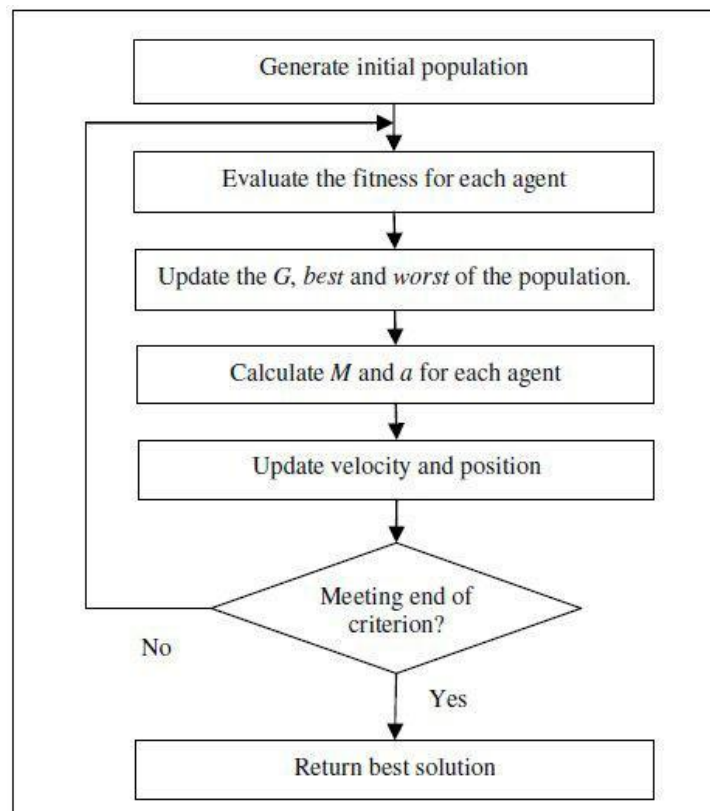


Fig. 3.1. Flowchart of GSA

An Economic Design of the Control Chart

To plot an \bar{x} -bar control chart, we need three parameters in order to design it. The sample size (n), the interval for sampling (h) and the coefficients of control limits (k) come under these parameters.

So it can be concluded that cost is a function of n , h , and k .

Here cost is directly in proportion to size of the sample, number of false alarms and inversely in proportion to sampling interval and width of control limits. The accompanying costs are vital in deciding the choice variables in the configuration of \bar{x} -bar charts economically: examining cost, search cost and the cost of working both in control and out of control. It is expected that output is measured on a continuous scale and follows normal distribution. At the point when the process is in control, the initial mean is μ_0 ; be that as it may, because of the event of an assignable reason, the initial mean may be moved from μ_0 to $\mu_0 + \delta\sigma$ or $\mu_0 - \delta\sigma$ (out-of-control state), where δ is the shift parameter and σ is the standard deviation. The control limit of the \bar{x} -bar control chart is set at $\mu_0 \pm k$ times the standard deviation of the sample means, where k is known as the control limit coefficient.

Here a fixed sampling length and a steady rate of failure over every interval were expected. A fixed sample of size n is taken from yield each h hour. At whatever point the sample mean falls outside the detail furthest reaches of the item, the outcome signals that the process has moved to an out-of-control state.

Consequently, proper activities, for example, recognizing the assignable cause and restoration work is done to bring the process back to an in- control state. Something else, the out of control state will proceed until the end of the production run.

It is assumed that the assignable cause follows Poisson's process having a n intensity of λ occurrence per hour. The process remains in control for a certain period of time having a mean value of $1/\lambda$.

Therefore the appearance of the assignable cause between two consecutive intervals (let j^{th} & $(j+1)^{\text{th}}$ is:

$$\tau = \frac{\int_{jh}^{(j+1)h} \exp(-\lambda t) (t-jh) dt}{\int_{jh}^{(j+1)h} \exp(-\lambda t) \lambda dt} = \frac{1-(1+\lambda h) \exp(-\lambda h)}{\lambda (1-\exp(-\lambda h))}$$

Development of the discontinuous cost model

Cycle time

The production cycle for the discontinuous process contains six periods:

- a) 1 period for the time when the process is in control.
- b) 2 periods for the time when the process is out of control and
- c) 3 periods when the process is stopped.

The estimated interval of time when the process is in control is $1/\lambda$. The estimated interval of time when the process is out of control consists of two periods: the time before the signal is detected [$h (1/P - 1/2 + \lambda h/12)$] and the time required to examine and analyze the signals (gn).

The amount of time for which the process is stopped consists of 3 periods:

- (a) The search and repair time for assignable cause D.
- (b) The restart time of the process S_1
- (c) The search time for a false alarm and after that time to restart the process. This is the sum of the expected search time for a false alarm (D_1) and the setup (S_1) time multiplied by the expected number of false alarms per cycle, or $(D_1 + S_1)\alpha/\lambda h$.

Thus the total time for the discontinuous process to complete a cycle is:

$$E(t) = (1/\lambda) + h(1/P - 1/2 + \lambda h/12) + gn + D + S_1 + ((D_1 + S_1)\alpha)/\lambda h$$

The proportion of time for which the process is in control is:

$$\beta' = (1/\lambda)/E(t),$$

The proportion of time for which the process is out of control is:

$$\gamma' = (h(1/P - 1/2 + \lambda h/12) + gn)/E(t),$$

The proportion of time for which process is halted is:

$$\phi' = (D + S_1 + (D_1 + S_1)\alpha/\lambda h)/E(t).$$

Cost model

The predicted net revenue for one cycle is the probable hourly income while the process is in control times for the estimated interval of time the process is in control $[V_0(1/\lambda)]$ plus the

predicted out of control income times the estimated interval of time the process is out of control $[V_1h(1/P - 1/2 + \lambda h/12) + gn]$ less process costs.

The process costs are:

- (a) The cost of taking a sample times the expected number of samples taken while the process is in operation, in this case $(b + cn) \{ [1/\lambda + h(1/P - 1/2 + \lambda h/12) + gn]/h \}$.
- (b) The cost of investigating a false alarm times the expected number of false alarms that occur before the shift in the process, $\alpha T/\lambda h$, as in the continuous model.
- (c) The cost of finding and repairing an assignable cause W .
- (d) The setup cost S .

Therefore, the expected net income per cycle is:

$$E(C) = V_0(1/\lambda) + V_1[h(1/P - 1/2 + \lambda h/12) + gn] - (b + cn) \times [1/\lambda + h(1/P - 1/2 + \lambda h/12) + gn]/h - \alpha T/\lambda h - W - S.$$

The expected net hourly income is: $E(A) = E(C)/E(t) = V_0 - L$.

Let,

$$B' = ah + gn,$$

$$C' = C' = D + S_1 + (D + S_1)\alpha/\lambda h,$$

$$M = V_0 - V_1.$$

Then the loss cost function for this model is:

$$L = [\lambda MB' + \lambda V_0 C' + (b + cn) (1 + \lambda B')/h + \alpha T/h + \lambda W + \lambda S] / [1 + \lambda(B' + C')].$$

The loss function L is minimized here with respect to n , h and k by gravitational search method.

In this project, the values for optimization solved by Montgomery et al. [5] have been taken, where data given are:

Table 4.1 Input data

Sl	Parameters	Symbol	Value
1	Income loss when process is out of control	M	50
2	Size of shift	δ	2
3	Rate of occurrences of assignable cause per hour	λ	0.05
4	Time to test results	e	0.50
5	Time to find and repair an assignable cause	D	3
6	Fixed cost of sampling	b	0.50
7	Variable cost of sampling	c	0.10
8	Cost to find and repair an assignable cause	W	250
9	Cost of searching a false Alarm	T	50
10	Income while operation in control	V_0	150
11	Setup cost	S	100
12	Setup time	S_1	1.00
13	Time to search a false Alarm	D_1	4

RESULT AND DISCUSSION

In this project work, for each value of sample size n varying from 1 to 10, the cost function has been minimized using the program developed in MATLAB based on Gravitational search algorithm and the results of optimum values for width of control limits k , sampling interval h and corresponding minimum values of cost function have been shown in Table 5.1.

Table 5.1 Optimum solution

n	h	K	Cost
1	0.4793	3.4397	43.7341
2	0.7532	3.6094	42.2719
3	1.0813	3.7221	41.8152
4	1.4266	3.8148	41.6771
5	1.7612	3.9010	41.6894
6	2.0663	3.9868	41.7472
7	2.3357	4.0746	41.8568
8	2.5723	4.1648	41.9881
9	2.7833	4.2570	42.1308
10	2.9762	4.3507	42.2783

As shown in this table, as sample size n increased the optimum cost values first decrease and then increase after $n = 4$.

The minimum cost was found to be 41.6771 for the values of Sample size (n) = 4, Width of the control limits (k) = 3.8148, Sampling interval (h) = 1.4266.

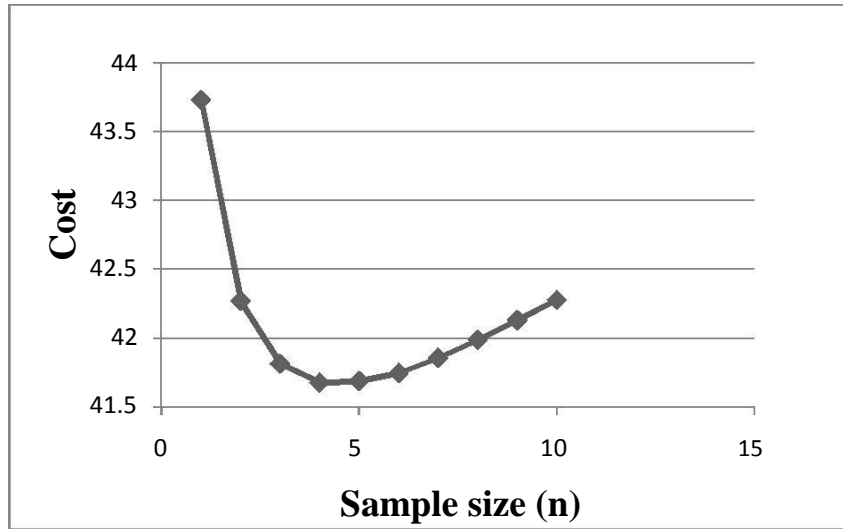


Fig. 5.1 Cost vs. Sample size in economic design of x-bar chart

CONCLUSION

In this thesis, using Gravitational search method, the minimum cost was found to be 41.6771, $n=4$, $k=3.8148$, $h=1.4266$.

The result of Montgomery's [5] economic design was found to be 41.95. Comparing our result with Montgomery's [5] result, the cost is less. So, this report suggests that Gravitational search is an efficient method for the optimization of cost for X-bar control chart.

SCOPE OF FUTURE WORK

The statistical relationship between the response and the variables is unknown and must be estimated. For the tuning of GSA parameters, RSM (Response Surface Methodology) should be used. Response surface methodology is a mathematical-statistical procedure in finding optimal values of some random factors that have significant effects on a response. So RSM can be used in order to find out the best value for the GSA parameters.

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