FREE VIBRATION OF LAMINATED COMPOSITE PLATES WITH CUT-OUT

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

Master of Technology
In
Structural Engineering

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CERTIFICATE

This is to certify that the thesis entitled, ***"FREE VIBRATION OF LAMINATED COMPOSITE PLATES WITH CUT-OUT"*** *submitted by* ***BISWAJIT MAJHI*** *in partial fulfilment of the requirements for the award of* ***Master of Technology*** *degree in Civil Engineering with specialization in "***Structural Engineering***" during 2013-2015 session at the National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.*

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institution for the award of any Degree or Diploma.

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ABSTRACT

The laminated composite plate are basics components of structure used in various field of engineering such as turbine blades, airplane wing and helicopter blades as well as many others in civil, automotive and ship industries etc. due to their excellent high stiffness to weight ratio and strength to weight ratio. Cut-outs are provided in structure for venting, reducing weight and passage of electrical wires. Most of the structures are subjected to severe dynamic loading during their service life. This may lead to change the dynamics response of the structure. The presences of cut-outs not only reduce the strength of composite plate but also alter the dynamics characteristics of composite plate. Therefore, it necessitates predicting the dynamics responses of laminated composite plates with cut-outs with cost effective and good accuracy of these complex structures.

This present paper deals with combined numerical and experimental approach on dynamics characteristics of laminated composite plate with square cut-outs. The laminated composite plates are made by using hand lay-up method. Bidirectional glass fibres are used as reinforcement and polyester resin as matrix for composite plate. The experimental dynamics test has been carried out by using different dimensions of plate with various design parameters such as cut out ratio (D/d ratio), position of cut out, aspect ratio (a/b ratio), no of layers, ply orientations under different boundary conditions. The natural frequencies of composite plate with cut-outs are determined numerically using ANSYS 14.5 software. The convergence study is done for numerically obtained results and compare with other existing literature. The experimental values are also compared with the result obtained from ANSYS 14.5 software. It was seen that the fundamental frequency decreases with increase the cut-out ratio (d/D ratio) under CFFF and SFSF boundary conditions. But fundamental frequency decreases with increase the cut-out ratio (d/D ratio) up to 0.2 under CFCF boundary condition. Further fundamental frequency increases on increase of cut-out ratio (d/D ratio).

KEYWORDS: Natural frequency, Laminated composite plate, Bi-directional glass fibre, Square cut-outs.

CONTENTS

LIST OF SYMBOLS

a , b	Dimensions of plate in X and Y axis
h	Thickness of plate
$[A_{ij}]$	Extensional stiffness
$[B_{ij}]$	Coupling stiffness
$[D_{ij}]$	Bending stiffness
[D]	Flexural rigidity or elasticity matrix
$[S_{ij}]$	Shear stiffness
$[E_{11}]$, $[E_{22}]$	Elasticity moduli of lamina in both 1 & 2
$[G_{12}]$	Shear modulus of rigidity
[K]	Global elastic stiffness matrix
[M]	Global mass matrix
$\left[N_x\right],\left[N_y\right],\left[N_{xy}\right]$	Plane internal stress resultants of the plate
$N_x^0\ ,\ N_y^0$	External loading in the X and Y direction
$\left[M_x\right],\left[M_y\right],\ \left[M_{xy}\right]$	Moment resultants of the plate
n	Number of layer of the laminated panel
$Rx,\ R_y\ ,Rxy$	Radii of curvature in the X and Y direction
$\left[Q_x\right],\ \left[Q_y\right]$	Transverse shearing forces
[T]	Transformation matrix
u, v, w	Displacements in X, Y, Z direction
X, Y, Z	Global coordinate axis system
$\sigma_x,\sigma_y\ ,\tau_{xy}$	Stresses at a point

ε_x , ε_y , γ_{xy}	Bending strains
υ	Poisson’s ratio
θ_x , θ_y , θ_z	Slopes with respect to X, Y and Z axes
ω	Natural frequency
$\{\phi\}$	Eigen vector

LIST OF TABLES

LIST OF FIGURES

CHAPTER- 1

INTRODUCTION

1.1 OVERVIEW

Composite materials are structural materials which are obtained by combination of two or more different constituents on a macroscopic scale. There are two phases of composite such as reinforcing phase and matrix phase. The materials of reinforcing phase are in the form of fibres, particles or flakes and embedded in the matrix phase. The reinforcing material and the matrix material can be metal, ceramic, or polymer. The properties of composite materials are derived from its constituents, geometry and distributions of phases. Some of the composite materials such as plywood and reinforced concrete are being used for a long time. In general, composite materials may be fibrous, laminated and particulate. The composite materials inherit the superior qualities of the combining materials such as excellent high strength to weight ratio, high stiffness to weight ratio, low weight, long fatigue life, resistance to corrosion, good thermal conductivity and low specific density. So that fibre reinforced laminated are being increasing extensively in many engineering application. The elements such as plates and shell have been successfully implementation in real structures. For designers and engineers composites act as a solution for structural problems such as crack prevention.

1.2 IMPORTANCE OF CUT-OUT

Cut-out is used almost every structural element such as civil, aerospace and automotive industry. In aircraft components cut-outs are used to reduce the weight, to lay fuel lines and electrical lines etc. For doors and windows, cut-outs are provided in structure. In water retaining structure cut-outs are provided at the bottom of the structure for passage of liquid. Cut-outs are also needed for ventilation. Cut-outs in plate change the dynamics characteristics of plates. Sometimes designers use the cut-outs of different shapes and sizes to alter the natural frequency of the structure to make them safe. Most of the structures such as beams, columns and plates fail due to vibration. Hence vibration analysis of laminated composite plate with cut-out has been a major concern for designers and researcher etc.

CHAPTER- 2

REVIEW OF LITERATURE

2.1 INTRODUCTION

Cut-outs may reduce the strength and stiffness of structure. Also, it influences on vibrational behaviour of composite structure. That's why presence of cut-outs in composite structure is an important factor during the structural design of composite structure. So many researchers are investigated on dynamics analysis of laminated composite plate. But the literatures related to vibration are very limited. In this study reviews on vibration analysis on composite plate are discussed. The related literatures are critically revised so as to provide contextual information on the complications to be considered in the proposed work and to highlight the importance of the present study.

Paramasivam (1973) used a technique to investigate the influence of square cut-out on the natural frequencies of isotropic plates under different end conditions (simply supported and clamped) using the finite difference method. Aksu and Ali (1976) formulated a theory to examine the vibration behaviours of isotropic and orthotropic plates with one or two rectangular cut-outs. The rectangular plate is used for this study. They used a method based on variational principles in addition with finite difference technique. Rajamani and Phrabhakaran (1977) studied in orthotropic laminated composite plates. He assumed composite plate to be homogeneous and symmetrical about the mid-plane. He analysed the influence of centrally located circular and square cut-outs on the natural frequency of the laminated composite plate. He considered two different end constrains such as simply supported and clamped-clamped. Ali and Atwal (1980) developed a method by using Rayleigh's principle for the free vibration analysis of plates with square and rectangular cut-outs under simple supported end condition. Laura *et al.* (1981) obtained an approximate solution of natural frequency of a rectangular plate with corner cut-out based on Ritz method. It is assumed that the sides of the plate are elastically retrained against rotation and translation. The amplitude is approached in terms of a polynomial coordinate function which satisfied the prescribed end conditions along the orthogonal edges but not along the corner cut-out. The analytical values are in good agreement with experimental predictions performed on clamped laminated square plate. Reddy (1982) analysed the dynamics analysis of anisotropic rectangular composite plates. He considered different aspect ratio, side-to-thickness ratio and side of plate to cut-out side ratio and examined the effect on natural frequency. Lee *et al.* (1987) analysed the rectangular composite plates with centrally located rectangular shape cut-outs under simply supported condition. He predicted the natural frequency of composite plate with cut-out by using Rayleigh principle. Bicos and Spring (1989) used finite element method to derive equations for vibration analysis of shells and plates with and without cut-outs. Also, he developed a computer program to calculate the natural frequencies of rectangular plates and cylindrical panels with three different end conditions such as free, clamped and simply supported. Ramakrishna *et al.* (1992) analysed the laminated composite plates with a centrally located circular cut-outs. A computer program is established for determining the fundamental frequencies of composite plate by using a hybrid-stress finite element. Also, he considered the influence of ply orientations, width-to-thickness ratio, hole-size and aspect ratio on the fundamental frequencies. Lee and Lim (1992) analysed free vibration behaviours of isotropic and orthotropic square plates with a square cut-out under simply supported condition which is subjected to an in plane force based on the Rayleigh method. They determined that the in-plane tensile force increases the natural frequency of plate and compressive forces decreases the natural frequencies until buckling state is reached. Boay (1996) presented numerical and experimental results on vibration analysis of laminated plates with centrally located circular cut-outs. The following varying parameters are considered in this study for example hole sizes and aspect ratio under different constrained conditions. Sabir and Davies (1997) determined the natural frequencies of flat square composite plates with eccentrically located square cut-outs based on finite element method. Laminated plates are subjected to in-plane uniaxial or biaxial compression or uniformly distributed shear along the four outer edges under simply supported or clamped end conditions. Sivakumar *et al.* (1999) examined the vibration characteristics of laminated plates with cut-outs under large oscillations. He used Ritz finite element model and got results for laminated composite plates with holes of various shapes such as circle, square, rectangular and ellipse. Chen *et al.* (2000) studied the vibration analysis of symmetrically thick laminated, doubly connected plates of arbitrary plate perimeter for the outer boundary and a hole defined by a super-elliptical equation which is accomplished to define a rectangular and an ellipse. They considered Rayleigh Ritz method in addition with Reddy's higher order theory. The influence of no of layers, length-to-thickness ratio, aspect ratio, fibre angle, cut-out sizes under various end conditions on natural frequency are investigated. Turvey *et al.* (2000) determined the free vibration characteristics of 3.2 mm thick pultruded GRP square plates under six combinations of simply supported (S), clamped (C) and free (F) edge boundary conditions. The comparison of experimental and theoretical natural frequencies established that thin homogeneous orthotropic or anisotropic plate theory provides a realistic model for calculating the free vibration behaviours of pultruded GRP plates. Also, they investigated the free vibration experiments on composite plates with centrally located circular cut-outs. The hole size ratio are varied from 0.1 to 0.4 under different end conditions. Based on orthotropic plate theory, numerical values are shown good agreement with the experimental natural frequencies and mode shapes. Liew *et al* (2001) developed a semi analytical method to determine the natural frequencies of square composite plate with discontinuities in cross-section. He used Ritz procedure to evaluate the natural frequencies and mode shapes. Ram and Babu (2002) inspected the free vibration analysis of composite spherical shell cap with cut-out and without a cut-out. The study is carried out using the finite element method by using higher-order shear deformation theory. Eight nodded degenerated isoparametric shell panel with nine degrees of freedom at each node are considered. The results are shown for axisymmetric free vibration analysis of laminated composite spherical shell cap with cut-out and without a cut-out. The effects of number of plies, cut-out size, radius to thickness ratio under different boundary conditions on the fundamental frequency of orthotropic and laminated composite spherical shell cap were studied. Liew *et al* (2003) investigated the vibration behaviours of isotropic rectangular plate with central cut-outs. Ritz discrete method is used to extract the fundamental frequency of plates. Two different types of end condition are considered for example simply supported and two opposite edges clamped (CSCS) case. Hota and Padhi (2007) studied the example related to vibration of laminated plate with holes and results obtained from this study are presented together with published literature. Udar and Datta (2007) investigated on dynamics instability responses of simply supported laminated composite doubly curved panel with circular cut-out. They considered various design parameters such as no of layers, non-uniform edge loading, damping and width to thickness ratio etc. Poore *et al* (2008) examined a semi analytical solution method for evaluating the natural frequency of laminated cylindrical shells with centrally located circular cut-out. Jhung *et al.* (2009) examined the vibration response of circular plate with eccentric hole. He assumed the plate was submerged in fluid. He developed an analytical method, by using finite Fourier-Bessel series expansion and Rayleigh-Ritz method. He considered hole-size as variable and studied its influence on the dynamics characteristics of the plate with hole. Aly *et al* (2010) analysed the effect of ply orientation on natural frequency of laminated composite beams. The experimental results are also compared with results obtained from ANSYS software. Lee and Chung (2010) developed a finite element delamination model for composite shell with centrally located hole based on third order shear deformation theory. He also studied the influence on natural frequency of composite shell panels by considering the location of delamination, no of layers and delamination size. Ovesy and Fazilati (2012) modelled internal cut-outs based on two different modelling approaches and investigated the influence on buckling critical analysis and vibration response due to presence of cut-out in composite plate. Gaira *et al* (2012) determined the buckling load factors of laminated plates with circular hole for different aspect ratio. They also examined nature of buckling load of composite plate with central cut-out with multiple holes. Kalita *et al* (2013) investigated the influence of distance of auxiliary holes from Central Square cut-out of orthotropic plate on mitigation of stress concentration. Auxiliary holes of circular shape are considered. A popular finite element package, ANSYS has been used for comparing with experimental results. The orthotropic plate having four side clamped is presented for the analysis. Sahoo (2014) employed finite element methodology to investigate the vibration related problems of composite stiffened shallow spherical shell panel with cut-out. They used eight nodded quadratic isoparametric element for shell and three nodded beam for stiffened formulation. Kalita and Halder (2014) analysed the deflection and stresses induced due to cut-outs for orthotropic and isotropic plate under transverse loading using finite element method. They considered the plate having central circular and square cut-out under four different end conditions. Kalita (2014) studied the stress concentration of clamped steel plates with cut-outs. The present proposed work aims to reduce the stress concentrations around cut-outs by providing auxiliary holes. Yin *et al* (2015) developed a new iso-geometric analysis for modelling of free vibration and buckling related problem of thin plates with cut-out and adopted classical plate theory for this formulation. They considered numerical examples with complicated cut-out shapes and studied the influence of fibre orientation, cut-out geometry on buckling behaviour and natural frequency of laminated plate under different boundary conditions. Bhardwaj *et al* (2015) discussed the effect of different parameters such as no of layers, fibre orientation, size of cut-out, aspect ratio and distance between cut-out on vibration behaviour of laminated structure with triangular shape cut-out. A finite element model is established using ANSYS parametric design language (APDL) code for comparing the experimental results. They concluded that the end conditions of plate has been played important role for dynamics response of the plate with cut-outs.

2.2 OBJECTIVE AND SCOPE OF THE PRESENT INVESTIGATION

From the above review of literature it is noted that most of the work done on laminated composite plates are analytical based on central cut-out. Almost all works are related to unidirectional fibre for fabrication of laminated composite. But now a day’s woven glass fibre are used for fabrication of laminated composite plates. The present work deals with an experimental investigation on vibration analysis of laminated composite plate with cut-out by considering the effect of cut-out ratio (d/D ratio), position of cut-out, aspect ratio (a/b ratio), ply orientation, no of layers under different boundary conditions. The results obtained from experimental works are compared with computational package ANSYS.

CHAPTER- 3

MATHEMATICAL FORMULATION

3.1 GOVERNING DIFFERENTIAL EQUATIONS

The differential equations of motion are obtained by considering a differential element of shell panel as shown in Fig: 1. This figure indicates internal forces of an element such as membrane forces N_x, N_y and N_{xy}, shearing forces are Q_x and Q_y and the moment of resultants are M_x, M_y and M_{xy}.

The governing differential equations of equilibrium for a shear deformable doubly curved panel subjected to external in-plane loading can be expressed as (Chandrashekhara (1989) , Sahu and Datta (2003)):

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=P_1\frac{\partial^2 u}{\partial t^2}+P_2\frac{\partial^2 \theta_x}{\partial t^2}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=P_1\frac{\partial^2 v}{\partial t^2}+P_2\frac{\partial^2 \theta_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+N_x^0\frac{\partial^2 w}{\partial x^2}+N_y^0\frac{\partial^2 w}{\partial y^2}=P_1\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=P_1\frac{\partial^2 \theta_x}{\partial t^2}+P_2\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y=P_1\frac{\partial^2 \theta_y}{\partial t^2}+P_2\frac{\partial^2 v}{\partial t^2} \qquad \text{……….…………………………} \qquad (1)$$

N_x^0 and N_y^0 are the external loading in the ‘x’ and ‘y’ directions respectively.

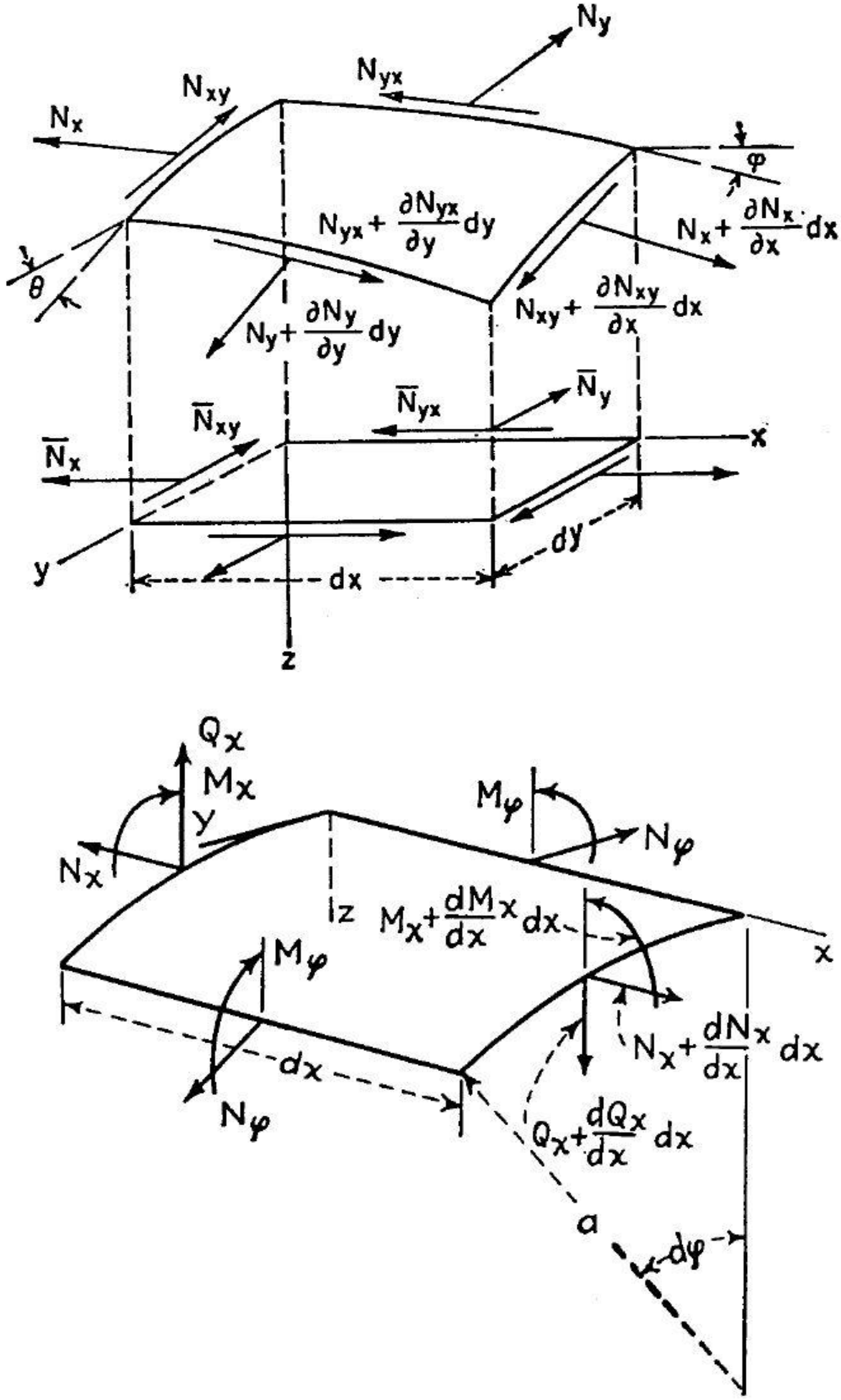


Fig.1: Element of a shell panel

The constants Rx, Ry and Rxy are the radii of curvature in the x and y directions and the radius of twist.

$$(P1,P2,3)=\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}(\rho)_k(1,z,z^2)dz \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Where, n = number of layers of the laminated composite plate and $(\rho)_k$ = density of kth layer from the mid-plane. In this present study only flat plates have been analysed. Hence Rx, Ry and Rxy are all infinity.

3.2 CONSTITUTIVE EQUATIONS

The composite panel is constituted of thin layers composite laminates. The matrix materials are embedded with fibres. Each layer is considered as homogeneous and orthotropic. The laminated fibre plate is consisting of a number of thin laminates as shown in Fig: 2.The principle material axes are shown by 1 and 2 and modulus of elasticity of a laminated plate along these directions are E_1 and E_2 respectively. The stress strain relationship is given as

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{21} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0\\ 0 & 0 & 0 & 0 & Q_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

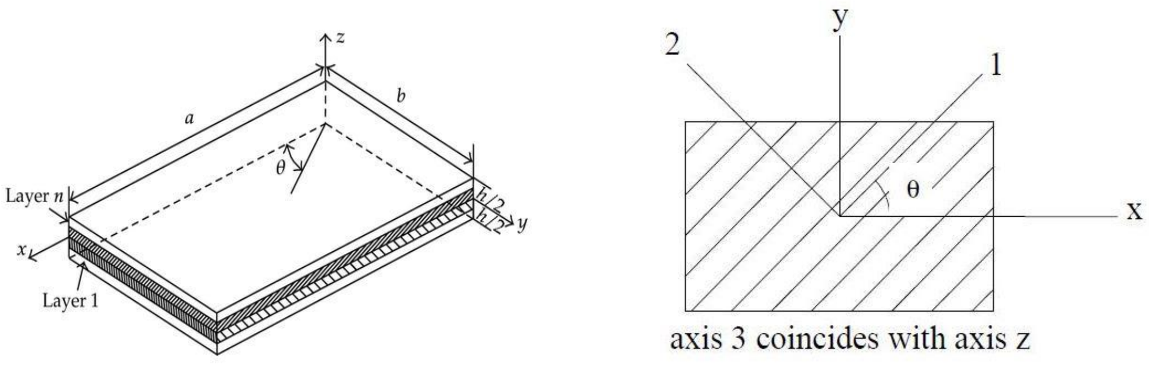


Fig.2: Laminated plate element showing principal axes and laminate directions

Where

$$Q_{11} = \frac{E_{11}}{(1 - v_{12}v_{21})}$$

$$Q_{12} = \frac{E_{11}v_{21}}{(1 - v_{12}v_{21})}$$

$$Q_{21} = \frac{E_{22}}{(1 - v_{12}v_{21})}$$

$$Q_{22} = \frac{E_{22}}{(1 - v_{12}v_{21})}$$

$$Q_{66} = G_{12}$$
$$Q_{44} = kG_{13}$$
$$Q_{55} = kG_{23} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

The on – axis elastic constant matrix corresponding to the fibre direction is given by

$$\left[\bar{Q}_{ij}\right] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{54} & \bar{Q}_{55} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

If the major and minor Poisson’s ratio are v_{12} and v_{21} then using reciprocal relation one obtains the following well known expression

$$\frac{v_{12}}{E_{11}} = \frac{v_{21}}{E_{22}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Standard coordinate transformation is required to obtain the elastic constant matrix for any arbitrary principle axes with which the material principal axes makes an angle.

Thus the off-axis elastic constant matrix is obtained from the on-axis elastic constant matrix as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{54} & \bar{Q}_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

$$\left[\bar{Q}_{ij}\right] = [T]^T \left[Q_{ij}\right][T] \qquad \text{……………………………………............ (7)}$$

Where ‘T’ is the transformation matrix. After transformation the elastic stiffness coefficients are.

$$\bar{Q}_{11} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4)$$

$$\bar{Q}_{22} = Q_{11}n^4 + 2(Q_{12} + Q_{66})m^2n^2 + Q_{22}m^4$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})mn^3$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n$$

$$\bar{Q}_{66} = (Q_{11} - Q_{12} - 2Q_{12} - 2Q_{66})m^2n^2 + Q_{66}(m^4 + n^4)$$

The elastic constant matrix corresponding to transverse shear deformation is

$$\bar{Q}_{44} = G_{13}m^2 + G_{23}n^2$$

$$\bar{Q}_{45} = (G_{13} - G_{23})mn$$

$\bar{Q}_{55} = G_{13}n^2 + G_{23}m^2$, Where m= $\cos\theta$ and n=$\sin\theta$

The stress strain relations are

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{54} & \bar{Q}_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

The forces and moment resultants are obtained by integration through the thickness h for stresses as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz$$

Where σx, σy are the normal stresses along X and Y direction, τxy, and τyz are shear stresses in xy , xz and yz planes respectively.

Considering only in-plane deformation, the constitutive relation for the initial plane stress analysis is

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{31} & A_{32} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \tau_{xy} \end{Bmatrix}$$

The constitutive relationships for bending transverse shear of a doubly curved shell becomes

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & 0 & 0 \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & 0 & 0 \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{54} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

This is also stated as

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ \kappa_j \\ \gamma_m \end{Bmatrix}$$

Or $\{F\} = [D]\{\varepsilon\}$ ………………………………………………………………… (9)

Where *Aij*, *Bij*, *Dij* and *Sij* are the extensional, bending-stretching coupling, bending and transverse shear stiffness.

They may be defined as:

$$A_{ij} = \sum_{k=1}^{n} (\bar{\mathrm{Q}}_{ij})_k (\mathrm{z}_k - \mathrm{z}_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (\bar{\mathrm{Q}}_{ij})_k (\mathrm{z}^2{}_k - \mathrm{z}^2{}_{k-1})$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} (\bar{\mathrm{Q}}_{ij})_k (\mathrm{z}^3{}_k - \mathrm{z}^3{}_{k-1}); i, j = 1, 2, 6$$

$$S_{ij} = k \sum_{k=1}^{n} (\bar{\mathrm{Q}}_{ij})_k (\mathrm{z}_k - \mathrm{z}_{k-1}); i, j = 4, 5$$

Where k is equal to transverse shear correction factor.

3.3 GOVERNING EQUATION FOR FREE VIBRATION ANALYSIS

The finite element formulation is established for the free vibration response of laminated composite plates with cut-out based on the first order shear deformation theory. An eight-nodded plate element is considered in the present work with six degrees of freedom i.e. u, v, w, θ_x, θ_y and θ_z at each node. The eigenvalue equation for the free vibration analysis of laminated composite plate can be expressed as

$$\left([K]-\omega^2[M]\right)\{\phi\}=\{0\} \quad \text{.................................} \quad (10)$$

Where [K] and [M] are the global stiffness and global mass matrices, ω is the natural frequency and ϕ is the corresponding eigenvectors i.e. mode shape.

CHAPTER- 4

MODELLING USING ANSYS 14.5

4.1 INTRODUCTION

ANSYS, a Finite Element Analysis (FEA) software is being generally used by engineer worldwide. ANSYS can be employed in virtually all the field of engineering such as structural, thermal and fluid mechanics etc. In this present work, ANSYS 14.5 is used to model the laminated composite plate to calculate the natural frequencies and deformed shapes of plates.

In this sub sections, the details of the ANSYS modelling are described. The terms related to ANSYS and the steps to be followed are discussed below.

4.2 TERMINOLOGY

Shell 8 node 281: Shell 281 is an element type for shell structures. This element has eight nodes. There are six degrees of freedom at each node i.e. both translations and rotations about the x, y, and z directions. The laminas are assumed to linear elastic and orthotropic. The geometry, nodes and co-ordinate system of a shell element is shown in Fig.3.

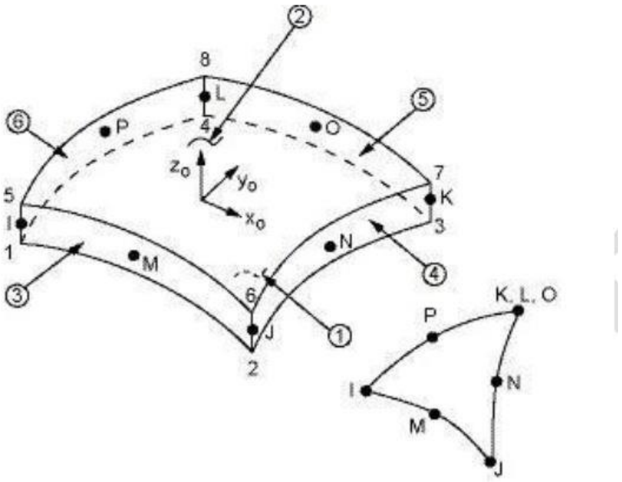


Fig.3: A SHELL 281 Element and triangular option

Modal Analysis: It is a linear analysis. There are various mode of extraction such as Block Lanczos, Super node, PCG Lanc zos, reduced, unsymmetric, damped, and QR damped are available. Block Lanczos mode of extraction is considered in the present modal analysis.

The following general guideline are used for ANSYS 14.5.

1. **Preprocessing:** It consists of defining element type, material properties, sectioning, modelling and meshing. In this study shell, Elastic 8 node 281 is selected as the element type. The laminated composite plate is modelled and meshed using ANSYS which are shown in Fig. 4 and Fig. 5 respectively.

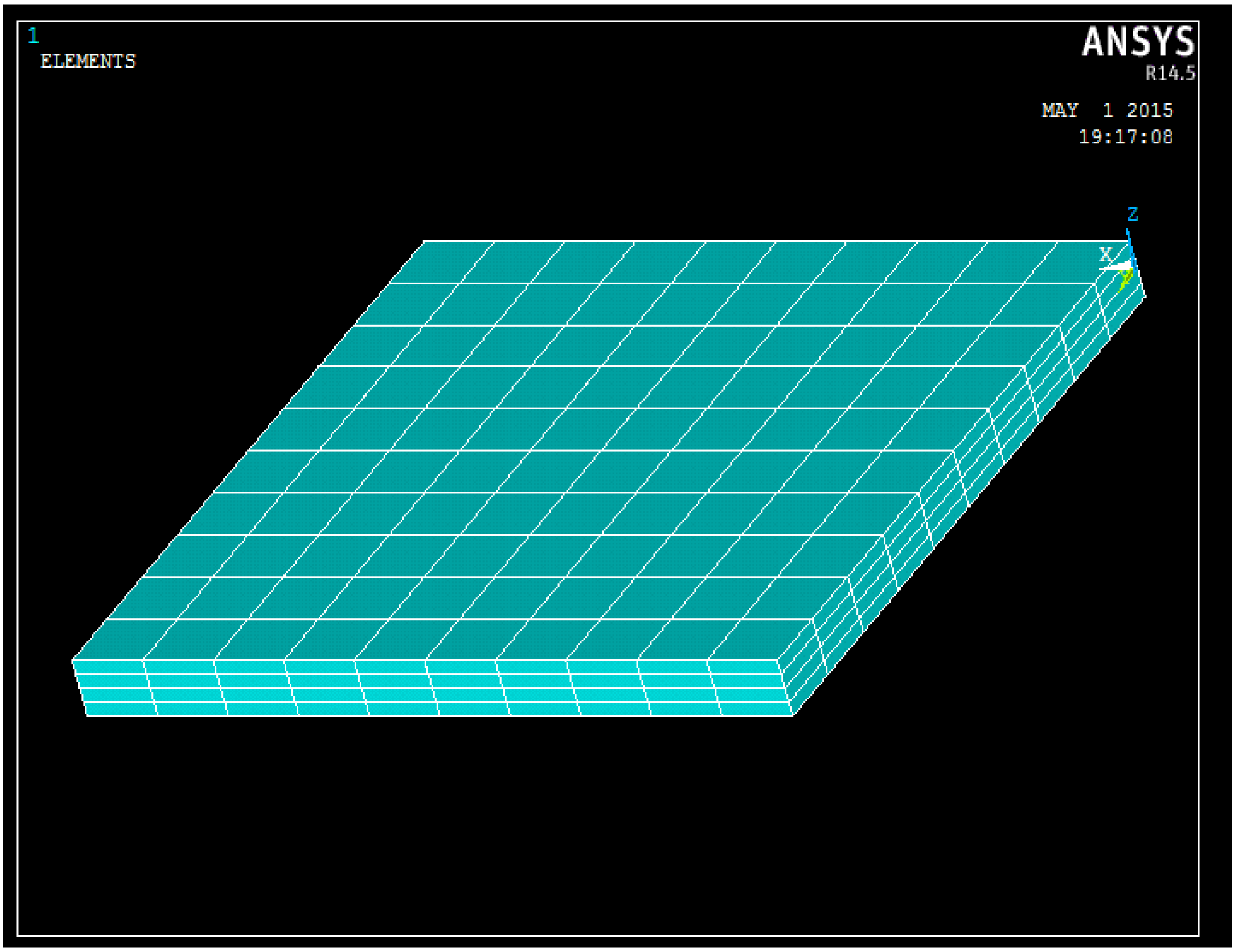


Fig.4: Laminated composite plate modelled in ANSYS

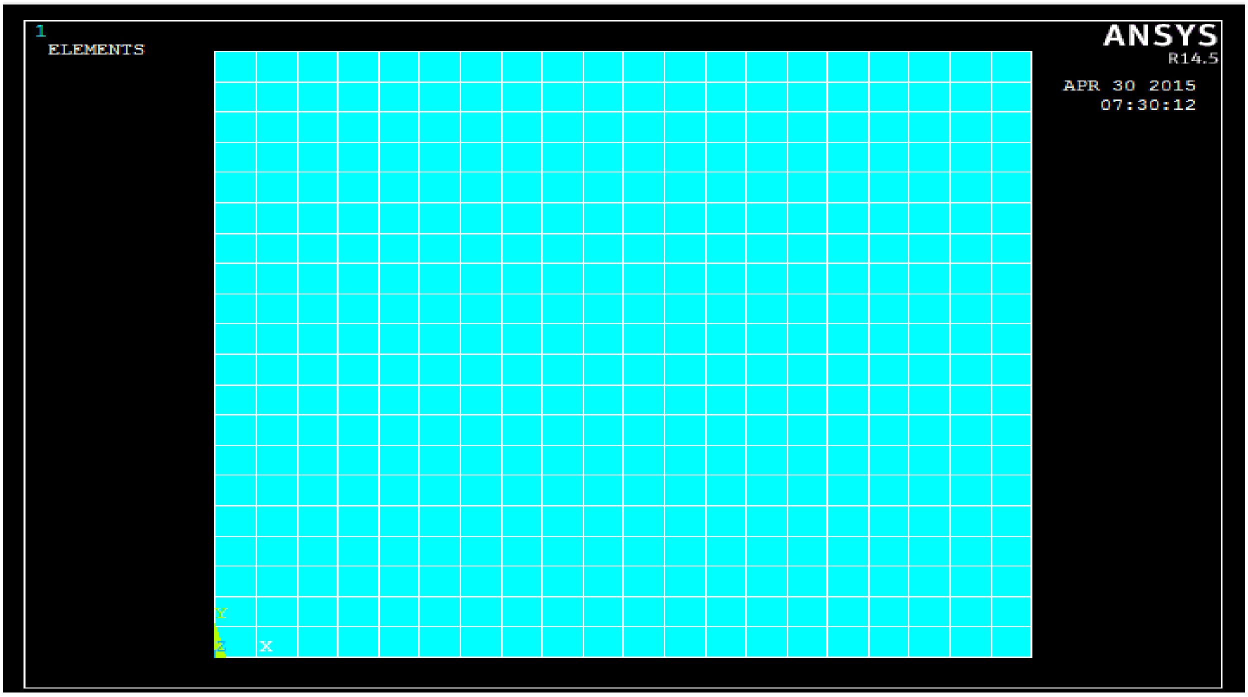


Fig.5: Meshing of plate

2. **Solution:** It includes assigning loads, applying boundary conditions and solving the modal analysis. The following table 1 indicates the constraints which are used in various boundary conditions. The constrains for different boundary conditions are presented in table 1. The loading and boundary condition of plate using ANSYS are shown in Fig.6.

Table 1. Constrains for different boundary conditions

Boundary condition	**UX**	**UY**	**UZ**	**ROTX**	**ROTY**	**ROTZ**
Simple supported	constrained	constrained	constrained	Not constrained	Not constrained	Not constrained
Cantilever supported	constrained	constrained	constrained	constrained	constrained	constrained
Fixed supported	constrained	constrained	constrained	constrained	constrained	constrained

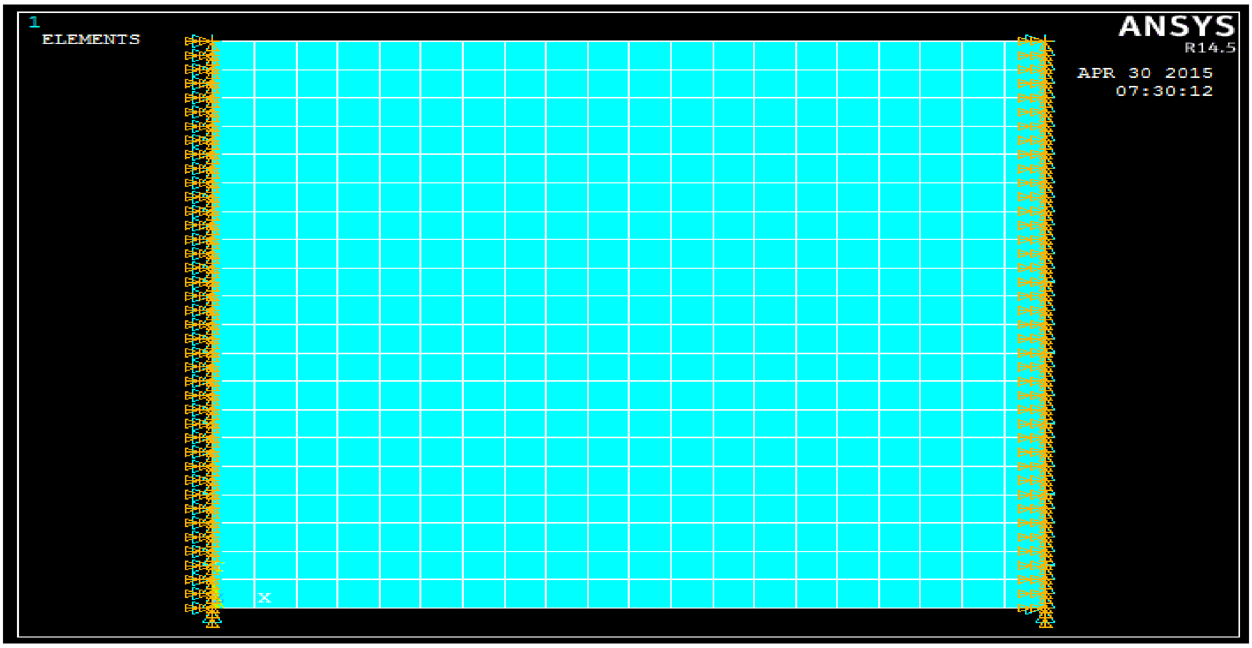


Fig.6: Loading and Boundary Condition of plate

3. Postprocessing: This step includes viewing of the results and plotting mode shapes. Also, the deformed shape and un-deformed shape of plate can be plotted. The mode shape of laminated composite plate with cutout under cfff boundary condition in ANSYS is shown in Fig.7.

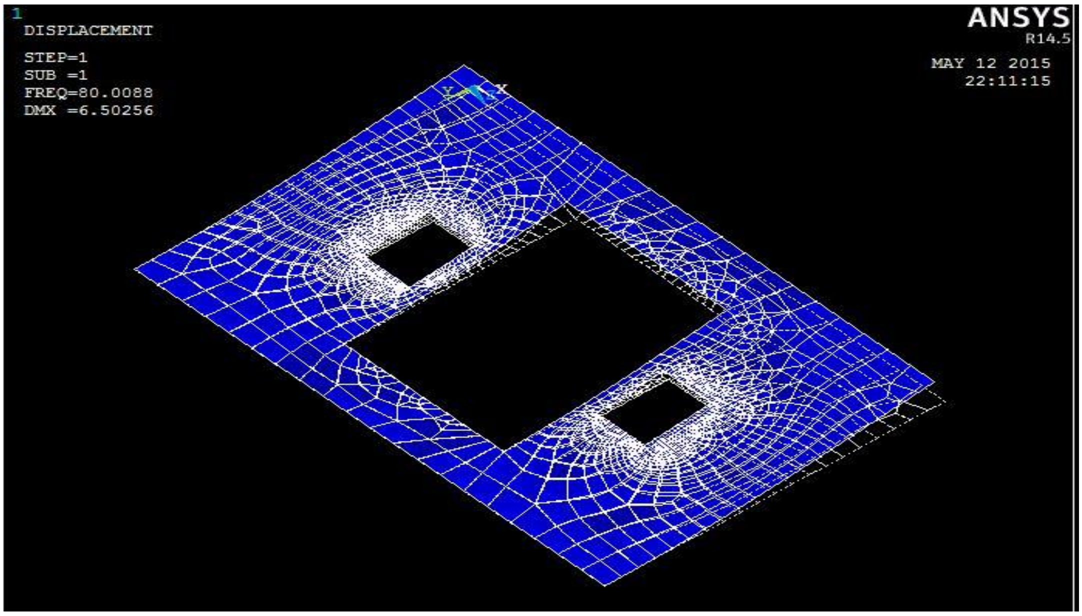


Fig.7: Mode shape of laminated composite plate with cutout under CFFF boundary condition.

CHAPTER- 5

EXPERIMENTAL PROGRAMME

5.1 INTRODUCTION

In this chapter the details of the experimental works are done on the vibration analysis of composite plate with cut-outs. The materials properties of the laminated composite plate are found out by tensile test as per ASTM D3039/ D3039M (2008) guidelines to characterise the laminated plate. The results obtained in experimental studies are compared with analytical predication.

5.2 MATERIALS

The following materials were used for preparation of laminated composite plate

- Woven glass fibre
- Epoxy
- Hardener
- Polyvinyl alcohol spray.

5.3 PRODUCTION OF LAMINTAED COMPOSITE SPECIMEN FOR TENSILE TEST

The percentage of fibre and matrix was taken in a proportion of 50:50 by weight. The hand lay-up process was used in this proposed works for fabrication of the laminated composite plate. This hand lay-up process is simplest method. In this process the resin is placed along with reinforcement against finished surface of an open mould. The glass fibre acts as reinforcement and resin acts as matrix. The glass fibres were cut into required shape and size for preparation of laminated plate. The matrix was prepared by using 8% of epoxy. A flat plywood platform was selected. Then, a plastic sheet was kept on the plywood and the releasing agent was sprayed on the plastic sheet by using spray gun. The resin was spread uniformly over the plastic sheet by using brush. Then the first layer of glass fibre was laid. Before laying resin on the first layer of glass fibre, a steel roller was used to remove air which may be entrapped. This process was continuation till all the eight layers are placed. Again a plastics sheet was covered the top of plate by applying releasing agent inside the sheet. Then a heavy flat metal rigid platform was kept on the top of the plate for compressing purposed.

The casting is cured under room temperature for 48 hrs. Then releasing sheet was removed from mould. The specimens were cut up to 250mmx25mm from the sheet of eight layers ply laminated as per ASTM D2344/D2344M (2006) specification by diamond cutter. The average thickness of laminated composite plate is found to be 3mm.

5.4 DETERMINATION OF MATERIAL CONSTANTS

Laminate composite plate are orthotropic in nature which are defined by material constant i.e. E_1, E_2, G_{12} and v_{12}. To perform the tensile test of specimens, an INSTRON 1195 UTM (as shown in fig.10) is used. The dimension of tensile test are given below in table 2. These specimens were tested which are shown in fig.9 for determination of mean values of material constant. The specimen was subjected to load in the INSTRON 1195UTM as described in ASTM standard D3039/D3039M (2008) at the rate of 0.2mm/minutes. The cracks were appeared in specimen during tensile test. This crack of specimen is shown in fig.11. Stress verses strain curve was plotted based on data obtained from testing and slope of this graph is young’s modulus of elasticity of materials. Poisson ’s ratio is assumed to be 0.25 for this proposed work.

Table 2: Size of the specimen for tensile testing

Length(mm)	Breadth(mm)	Thickness(mm)
250	25	3

The samples were cut from the casted plate as per requirement by tile cutter or diamond cutter as shown in Fig 8. In this study, three specimens were tested as shown in Fig 9 for mean values of material constants. According to Jones (1975), the given formula is used to determine the shear modulus as follows.

$$G_{12} = \frac{1}{\frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2\nu_{12}}{E_1}}$$

Where

E_1= elastic modulus in longitudinal direction ,E_2= elastic modulus in traverse direction

G_{12}= shear modulus in plane 1-2 and υ_{12} = Poisson’s ratio in plane 1-2

Fig.9: Laminated composite beam specimens for tensile testing.

Fig.8: Diamond cutter for cutting Specimen

Fig.10: Experimental set up for tensile testing

Fig.11: Specimen during tensile testing

5.5 PREPARATION OF LAMINATED COMPOSITE PLATE FOR VIBRATION TESTING

The preparation procedure for the composite plate for vibration analysis was same as that of plate preparation for tensile testing. Three different sizes of plate sample such as 235mmx235mm, 235mmx156.6mm and 235mmx117.5mm are prepared and cut into different cut-out ratio (d/D ratio) 0.0, 0.1, 0.2, 0.3 and 0.4 by using hacksaw for vibration test.

Fig.12: Fabricated mould

From Fig.13 to Fig.16 present the fabrication process of laminated composite plates.

Fig.13: Resin is applied on plastic sheet

Fig.14: Woven glass fibre is laid on gel coat

Fig.15: Steel roller is used to remove air

Fig.16: Curing process

The dimensions of laminated composite plate (Length x breadth x thickness), different orientations, different aspect ratio and different position of cut-out are given in the table 3 and table 4 respectively. The specimens were tested for three different end conditions. The following end conditions are considered such as

- CFFF-one edge is clamped
- SFSF-two opposite edges are simply supported
- CFCF-two opposite edges are clamped

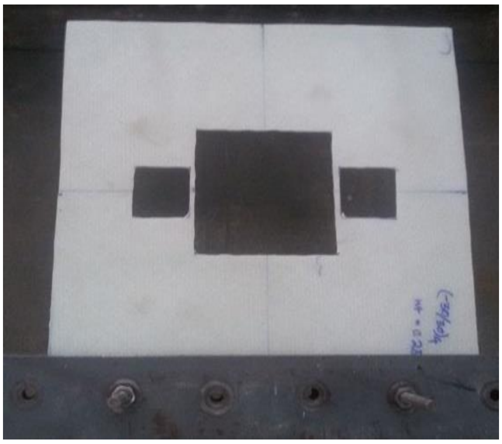


Fig.17:Frame for CFFF boundary Condition

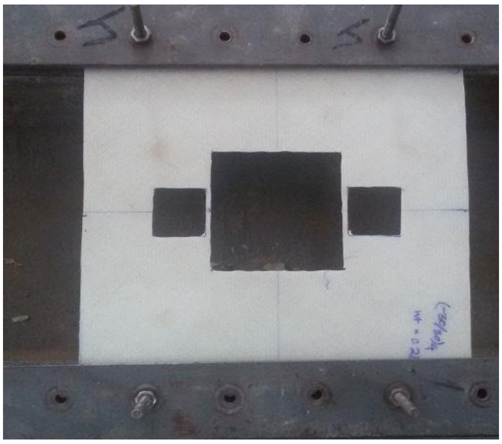


Fig.18: Frame for CFCF boundary Condition

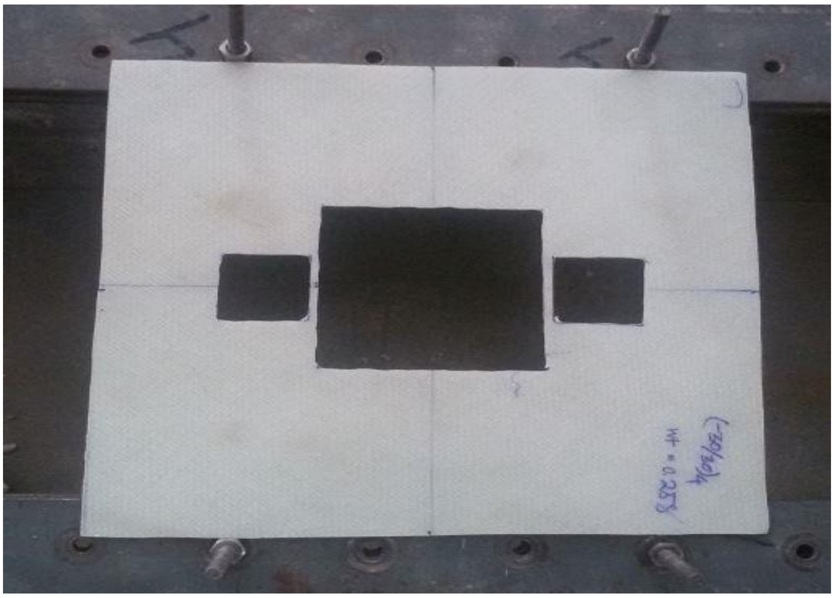


Fig.19: Frame for SFSF boundary condition.

Table 3:Dimensions of laminated composite plate with different cut-out ratio (d/D ratio).

Size of the specimen	No of layers	d/D ratio	Ply orientation	No of specimen
235x235x3	8	0	(0/90)4	1
235x235x3	8	0.1	(0/90)4	1
235x235x3	8	0.2	(0/90)4	1
235x235x3	8	0.3	(0/90)4	1
235x235x3	8	0.4	(0/90)4	1
235x235x3	8	0.4	(-30/30)4	1
235x235x3	8	0.4	(-45/45)4	1
235x235x3	4	0.4	(0/90)2	1
235x235x3	12	0.4	(0/90)6	1
235x156.6x3	8	0.4	(0/90)4	1
235x117.5x3	8	0	(0/90)4	1
235x117.5x3	8	0.1	(0/90)4	1
235x117.5x3	8	0.2	(0/90)4	1
235x117.5x3	8	0.3	(0/90)4	1
235x117.5x3	8	0.4	(0/90)4	1

Table 4: Dimensions of laminated composite specimens with different position of cutout.

Size of specimen	No of layers	Size of cut-out	Position of cut-out	Ply orientation	No of specimen
235x235x3	8	80x80	centre	(0/90)4	1
235x235x3	8	80x80	side	(0/90)4	1
235x235x3	8	80x80	corner	(0/90)4	1

5.6 EQUIPMENT FOR VIBRATION TEST

The following apparatus used in free vibration test

- Modal hammer
- Accelerometer
- FFT Analyzer
- PULSE software.

5.6.1 Modal hammer

Modal hammer is also known as impact hammer. It generates excitation on the structure. The accelerometer is attached on plate which receives the response generated by modal hammer. The specimen is excited by a small impact on a particular point with five times. The impact should be perpendicular to that specified point on the surface of the specimen.

Fig.20: Modal hammer Type 2302-5

5.6.2 Accelerometer

Accelerometer is used to sense the vibration from the specimen after excitation. Its main features are such as high sensitivity, durable titanium casing, light weight and use very low power etc. The accelerometer is attached to the composite plate by using bee wax before the start of the experiment.

Fig.21: Accelerometer Type 4507

5.6.3 FFT analyser

It receives time varying signal from accelerometer and convert into frequency based signal know as frequency response function. The FFT analyser is connected with computer where pulse lab shop is used. The outputs are demonstrated on analyser screen by using pulse lab shop.

Fig.22: Bruel & Kajer FFT analyzer Type 3560

5.6.4 Pulse lab shop

It is software used in computer to investigate the data obtained from the FFT analyser. It generates the frequency response. There is wide range of applications of pulse lab shop in static and dynamic study of structure.

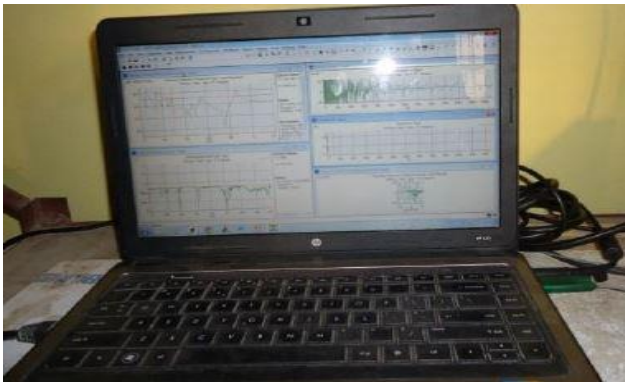


Fig.23: Display unit with pulse software

5.7 PROCEDURE FOR MODAL TESTING

As per required boundary condition, laminated composite plate was fitted. The FFT analyser, laptop, modal hammer and accelerometer were connected. The pulse lab shop version 10.0 was inserted to the laptop. An accelerometer was fixed to the plate by bee wax. The excitation is generated by means of modal hammer. The vibration generated by hammer was sensed by accelerometer. Then, the FFT analyser was processed this signal and obtained the frequency spectrum. The spectrum analyser was investigated the input and output signal and transmitted the resulting FRF to computer. The output signals are obtained on the analyser screen using pulse software. The natural frequencies of the plate were measured from the FRF graph.

CHAPTER- 6

RESULTS AND DISCUSSION

6.1 INTRODUCTION

Free vibration analysis of laminated composite plate with cut-outs is analytically studied by using ANSYS software. The effect of different parameters such as aspect ratio (a/b), position of cut-out, no of layers, orientation and cut-out ratio(d/D) under different boundary conditions. The experimental results on free vibration of laminated plate with cut -outs are verified with numerically using ANSYS. Vibration analysis of this study is presented as following

i. Comparison with previous results.

ii. Experimental and numerical results.

6.2 MODAL ANALYSIS

Cut-out may influence the dynamic behaviours of laminated composite structures. That's why in this present study, natural frequencies of composite plate with cut out are calculated both numerically and experimental programme. The effects of different parameters such as cut-out ratio (d/D ratio), no of layers, position of cut-out, aspect ratio (a/b ratio), different boundary condition and ply orientation on natural frequency of square cut-out plate were studied. The numerical results of modal analysis are compared with other existing literature.

6.2.1 COMPARISON WITH PREVIOUS RESULTS

Vibrational analysis of laminated composite plate with cut-out is computed numerically by finite element package ANSYS which are mentioned in previous chapter. The natural frequencies of laminated composite plates with two different end conditions such as SSSS (four edges of plate are simply supported) and CCCC (four edges of plate are clamped) are compared between the present numerical results with existing literature by Ju *et al.* (1995).

Table 5: Comparison of natural frequency (Hz) for glass epoxy laminated composite plates at different boundary condition

Material properties and geometry properties:

Ply orientation = (0/90/45/90/90/45/90/0), Density=1446.20 kg/m^3, E_{11} =132GPa, E_{22}= 5.35GPa, G_{12}=2.79GPa, $\upsilon_{12} = \upsilon_{13} = \upsilon_{23}$ =0.29.

Length=0.25m, width=0.25m, thickness=0.00212m

Boundary condition	**No of mode**	**Ju *et al.*(1995)**	**Present ANSYS**
CCCC	1st	346.59	342.52
	2nd	651.51	634.36
	3rd	781.06	766.52
	4th	1017.20	961.62
SSSS	1st	164.37	163.33
	2nd	404.38	400.12
	3rd	492.29	493.74
	4th	658.40	835.39

Similarly dynamics analysis of laminated composite plate with central cut-outs based on present ANSY modelling is compared with Sharma *et al* (2014).There are four different boundary condition such as FFFC, FCFC, CCCC and SSSS are considered to examine the influence of end conditions on natural frequencies of a square laminated composite plate with circular cut-out at centre.

Material properties and geometry properties of composite plate:

Length=1m, Width=1m, Cut-out ratio (diameter/side of square plate) = 0, 0.2, 0.4, 0.6, side to thickness ratio=100,

Ply orientation = (0/90)s, E_{11}=137.20GPa, E_{22}=E_{33}=14.48GPa,G_{12}=G_{13}=G_{23}=5.86GPa,

$\upsilon_{12} = \upsilon_{13} = \upsilon_{23}$ =0.21, Density=1500kg/m^3

Table 6: Natural Frequencies (Hz) of cut-out ratio=0.6 under different boundary conditions for square laminated composite plate with circular cut-out

Boundary condition	**No of mode**	**Cut-out ratio**	**Sharma *et al* (2014)**	**Present ANSYS**
FFFC	1st	0.6	7.21	6.99
	2nd		16.103	16.057
	3rd		45.34	45.277
	4th		60.251	59.793
	5th		64.197	64.973
FCFC	1st	0.6	68.24	65.066
	2nd		69.17	66.344
	3rd		171.02	156.30
	4th		171.54	158.44
	5th		176.67	167.58
CCCC	1st	0.6	263.64	232.97
	2nd		269.94	240.32
	3rd		271.34	244.68
	4th		281.46	253.27
	5th		385.14	345.35
SSSS	1st	0.6	54.293	52.976
	2nd		88.57	81.591
	3rd		89.481	81.798
	4th		145.29	133.09
	5th		191.81	162.95

Determination of material constants:

In this proposed work laminated composite specimens of eight layers are prepared to determine its mechanical properties. According to ASTM D2309/D2309M (2008) standard, tensile test on specimen was performed. The mechanical properties were found from the experiment which was given in Table 7.

Table 7: Material constants of laminated composite plate

Ply orientation	E_{11}(GPa)	E_{22}(GPa)	G_{12}(GPa)	υ_{12}	Density(kg/m^3)
(0/90)4	12.030	12.030	2.5	0.25	1530

E_{11}= elastic modulus in longitudinal direction

E_{22}= elastic modulus in traverse direction

G_{12}= shear modulus in plane 1-2

υ_{12} = Poisson's ratio in plane 1-2

6.2.2 EXPERIMENTAL AND NUMERICAL RESULTS

In this proposed work, three different layered laminated composite plates are prepared for modal analysis. The geometry properties of specimens are 235mmx235mm, 235mmx156.6mm, 235mmx117.5mm. The material properties are given in the table.7. The central cut-out with varying multiple holes laminated composite plate are considered which are shown in Fig.24. Also, this proposed work consider the effect of cut-out ratio (d/D ratio where 'D' is the side of centrally located square cut-out and 'd' is the side of multiple square cut-out located both side of central cut-out) ,no of layers, position of cut-out, aspect ratio(a/b ratio where a and b are length and width of composite plate respectively), different boundary condition and ply orientation on natural frequency of cut-out. The different cut-out ratio (d/D ratio) such as 0.1, 0.2, 0.3 and 0.4 of laminated composite plate are considered for the dynamics analysis of composite plate which are shown in Appendix. There is a comparison between natural frequency of central square cut-out of laminated composite plate and central square cut-out with multiple holes of laminated composite plate based on equal area of cutting.

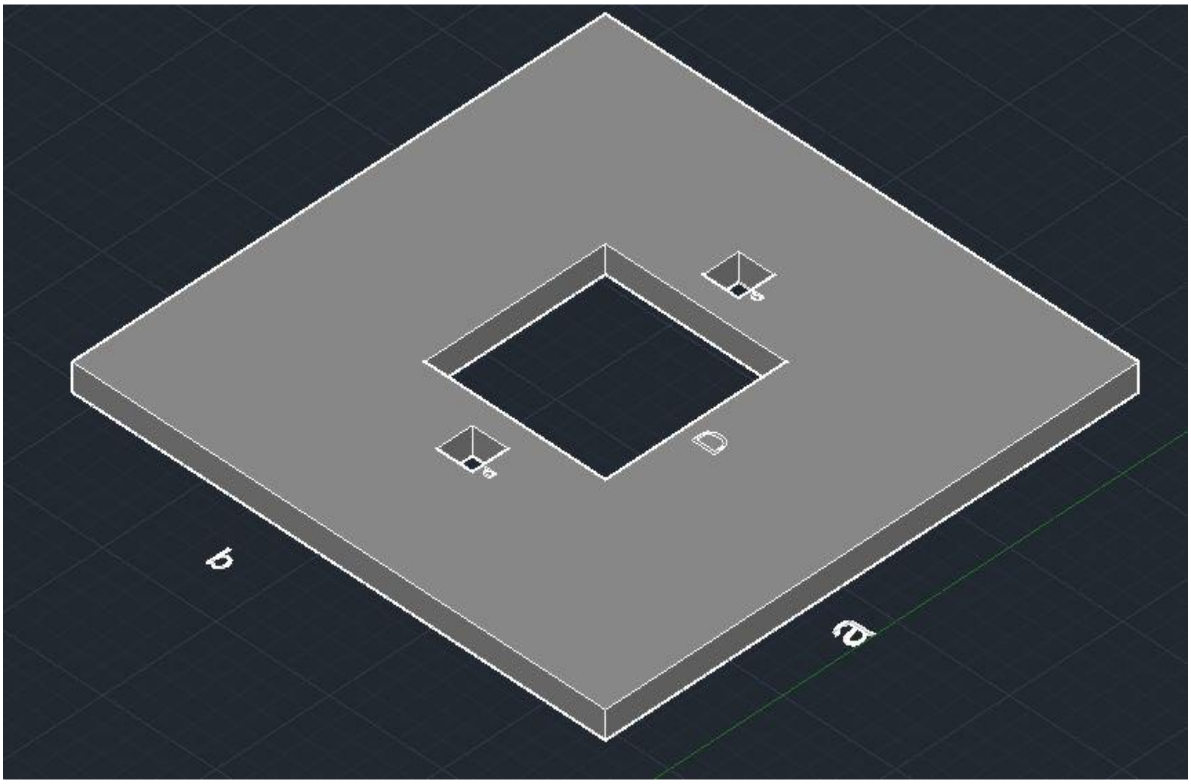
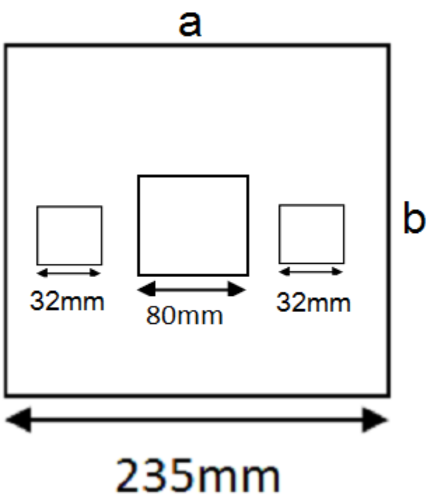


Fig.24: Composite plate with central square cut-out of side (D) with multiple square holes of side (d)

6.2.2.1 Effect of equal area of cutting

To study the variation of natural frequency between central cut-out with and without multiple holes based on equal area of cutting of the laminated composite plate are investigated. Two different samples having dimension 235mmx117.5mmx3mm are casted and material properties are given in table 7. The size of centrally located cut-out with multiple holes are 80mmx80mm at centre and 32mmx32mm at two side of the central cut-out (80x80+2x32x32=8448mm^2) which is also shown in Fig.25. The size of centrally located cut-out without multiple holes is 91.9mmx91.9mm (8448mm^2) which is shown in Fig.26. Two different end conditions such as CFFF and SFSF are considered. The end conditions are applied along the length of composite plate. The variation of natural frequencies between central cut-out with and without multiple holes based on equal area of cutting of the laminated composite plate under CFFF and SFSF boundary conditions are investigated in Fig:27 and Fig:28.



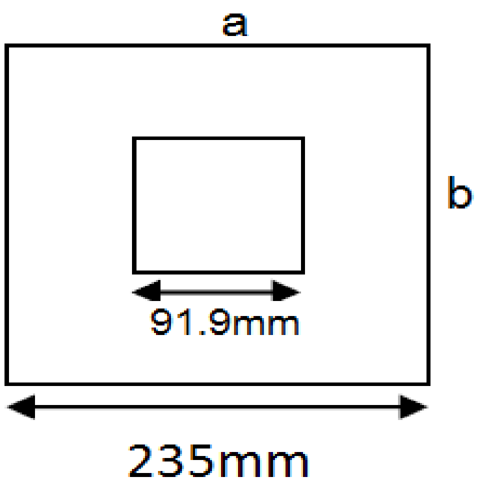


Fig.25:Central cut-out with multiple holes Fig.26:Central cut-out without multiple holes

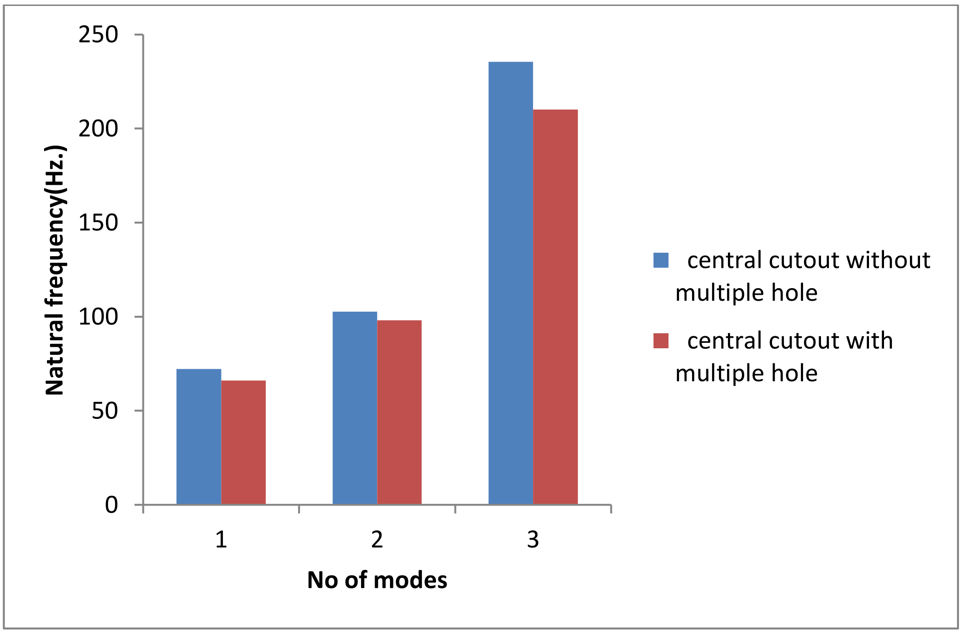


Fig.27: Variation of frequencies between central cut-out without holes and with multiple holes based on equal area of cutting under CFFF end condition

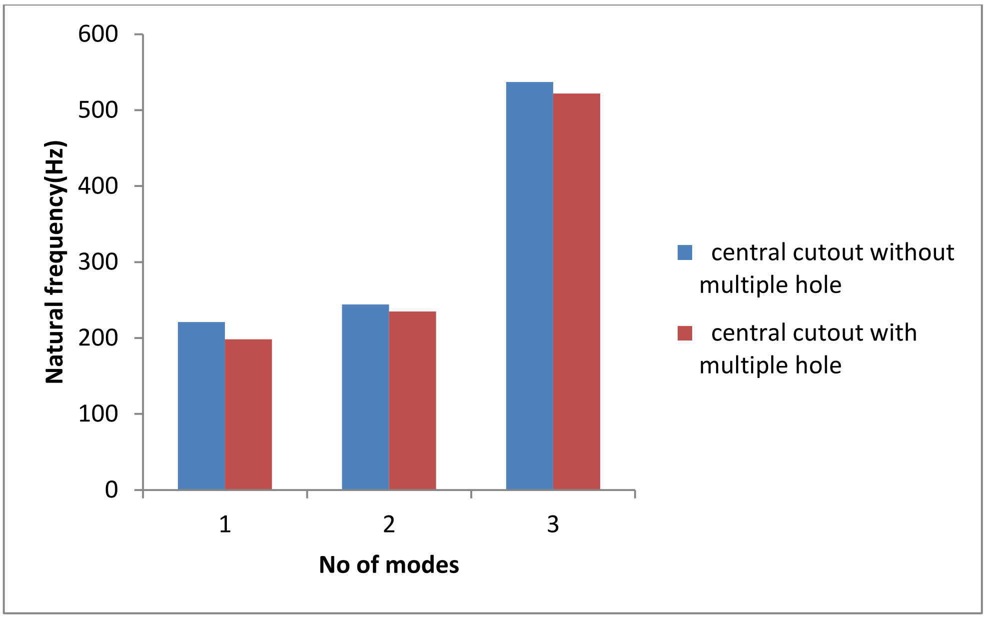


Fig.28: Variation of frequencies between central cut-out without holes and with multiple holes based on equal area of cutting under SFSF end condition

The investigation is related to the composite plate with SFSF (two opposite sides simple supported and others free) and CFFF (one side clamped and others free) restrained conditions, the results of which are presented in Fig: 27 and Fig: 28. The experimental natural frequencies of central cut-out with multiple holes are found to decrease by 10.21% from the experimental values of central cut-out without multiple holes for 1st mode and about 3.6% and 2.79% for 2nd and 3rd mode respectively for SFSF boundary condition and 8.4%, 4.5% and 10.81% for CFFF boundary condition respectively. This may be due to localisation of stress concentrations is reduced around central cut-out resulting declination in stiffness of laminated composite plate.

6.2.2.2 Effect of position of cut-out.

To study the influence of position of cut-out on dynamics characteristics of laminated composite plate, three different specimens are casted whose position of cut-out are at the centre, corner and side of the support. The geometry of specimens is 235mmx235mmx3mm, cut-out size of 80mmx80mm and material properties are given in the table 7. Both numerical and experimental results on natural frequency of composite plate for SFSF and CFCF boundary condition are presented. The natural frequencies for laminated composite plate having three different position of cut-out are presented in Fig: 29 and Fig: 30. .

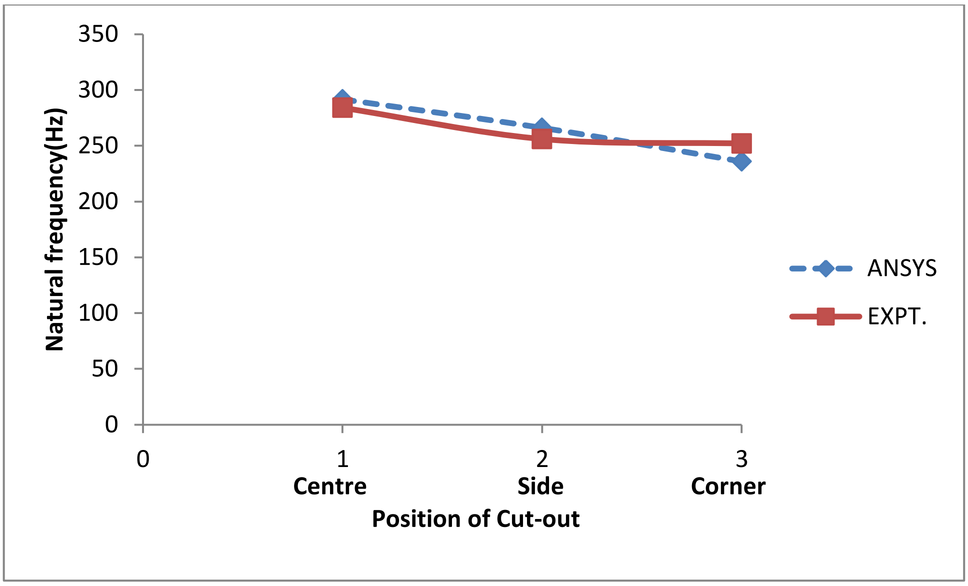


Fig.29: Variation of natural frequency laminated composite plate with different position for CFCF boundary condition

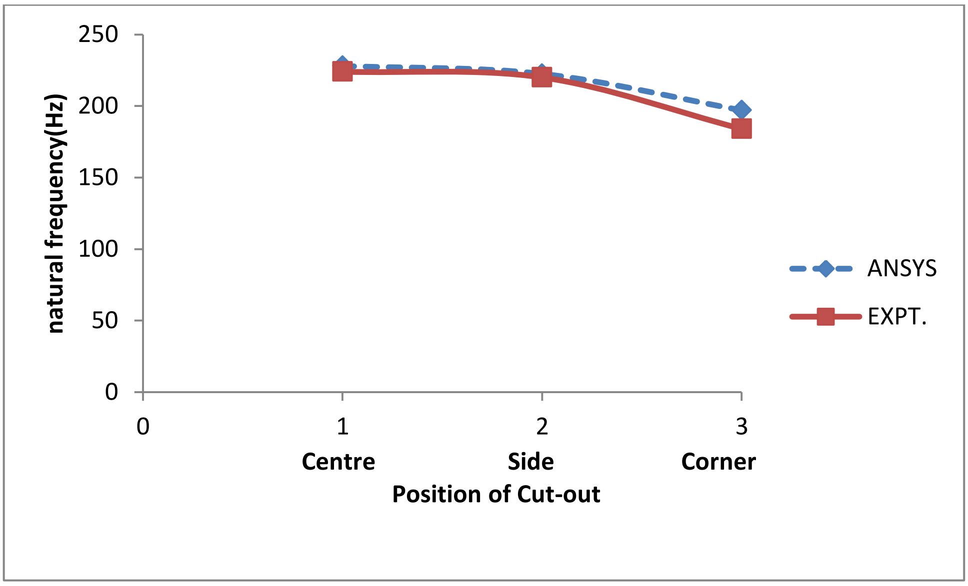


Fig.30: Variation of natural frequency laminated composite plate with different position for SFSF boundary condition

The relation between natural frequency and their relative position of the cut-out is plotted under CFCF and SFSF boundary condition. It is observed that in case of CFCF and SFSF end conditions maximum natural frequency occurs at centre position and minimum at corner position. From experimental values, it shows that natural frequency reduces about 9.85% from centre to side and about 11.20% from centre to corner position of cut-out in CFCF end condition. But in SFSF end condition natural frequency reduces about 1.78% from centre to side and about 17.8% from centre to corner position of cut-out respectively.

6.2.2.3 Effect of cut-out ratio (d/D ratio)

The influence of different cut-out ratio (d/D) on vibrational properties of composite plates are discussed for plate size of 235mmx117.5mmx3mm.(where D is the side of centrally located square cut-out and d is the side of multiple square cut-out located both side of central square cut-out).The central square cut-out of size 80mmx80mm and cut-out ratio (d/D) 0.0,0.1,0.2,0.3 and 0.4 are considered i.e. side of multiple square cut-out vary 0mm,8mm,16mm,24mm and 32mm. The end conditions are applied along the length of composite plate. The variation of natural frequency due to cut-out ratio under different boundary condition is examined in Fig: 31, Fig: 32 and Fig: 33.

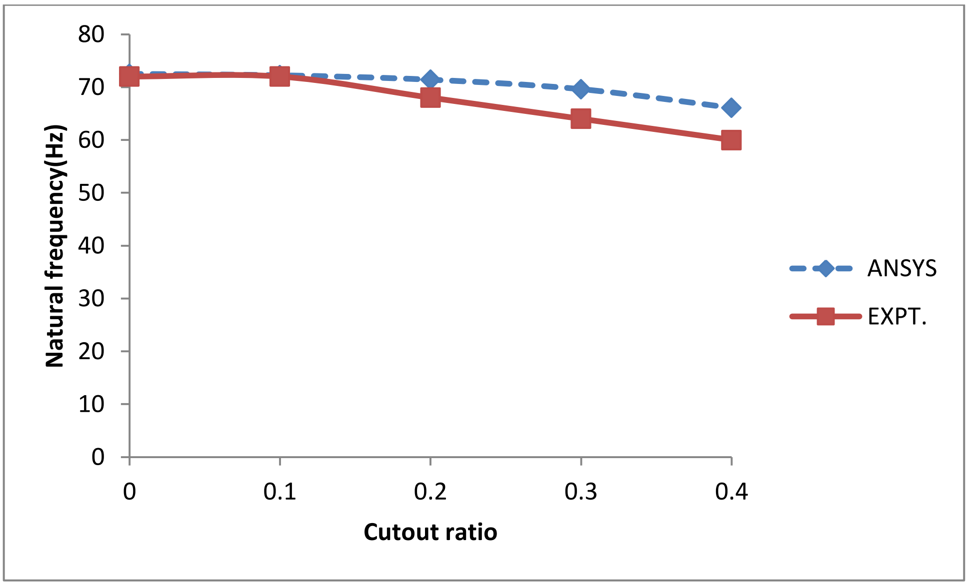


Fig.31: Variation of fundamental frequency for laminated composite plate with cut-out ratio (d/D ratio) under cantilever boundary condition

From this analysis, it is observed that at cut-out ratio=0.1, the frequency of specimen is least affected under CFFF boundary condition and also shows that the fundamental natural frequencies are considerable affects due to large cut-out ratio. It is also concluded from this investigation, the natural frequency decreases due to the increase in cut-out ratio. The experimental fundamental frequencies of cut-out ratio 0.1, 0.2, 0.3 and 0.4 are found to decrease by 0%, 5.55% ,11.11% and 16.6 % respectively as compared to fundamental frequency of cut-out ratio=0.0 under CFFF end condition .

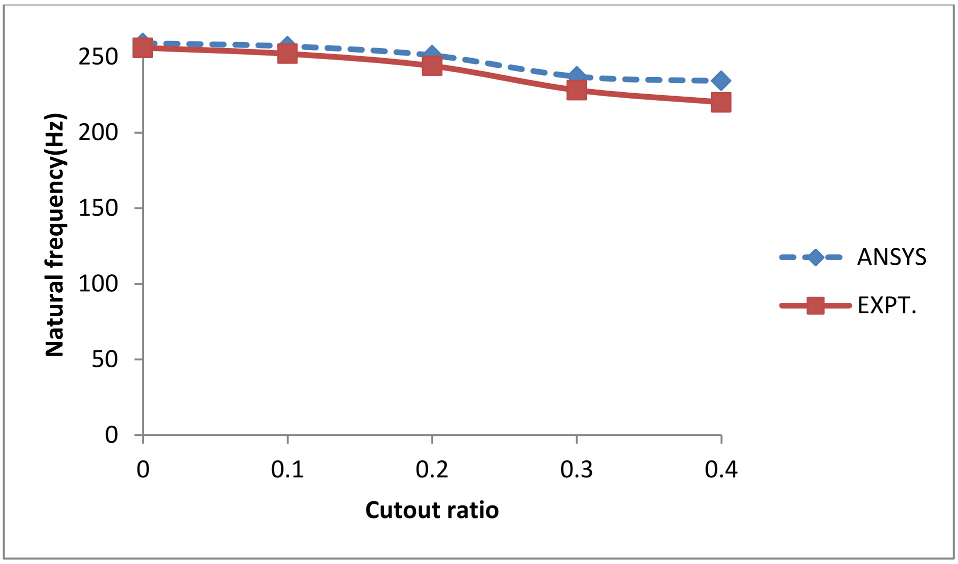


Fig.32: Variation of fundamental frequency for laminated composite plate with cut-out ratio (d/D ratio) under SFSF boundary condition

The same study has been extended to the composite plate under SFSF (two opposite sides are simple supported and others are free) condition, the results of which are plotted in Fig. 32. The experimental fundamental frequencies of cut-out ratio 0.1, 0.2, 0.3 and 0.4 are found to decrease by 1.56%, 4.68%,10.93% and 14.06 % respectively for SFSF boundary condition as compared to laminated plate with cut-out ratio=0.0.

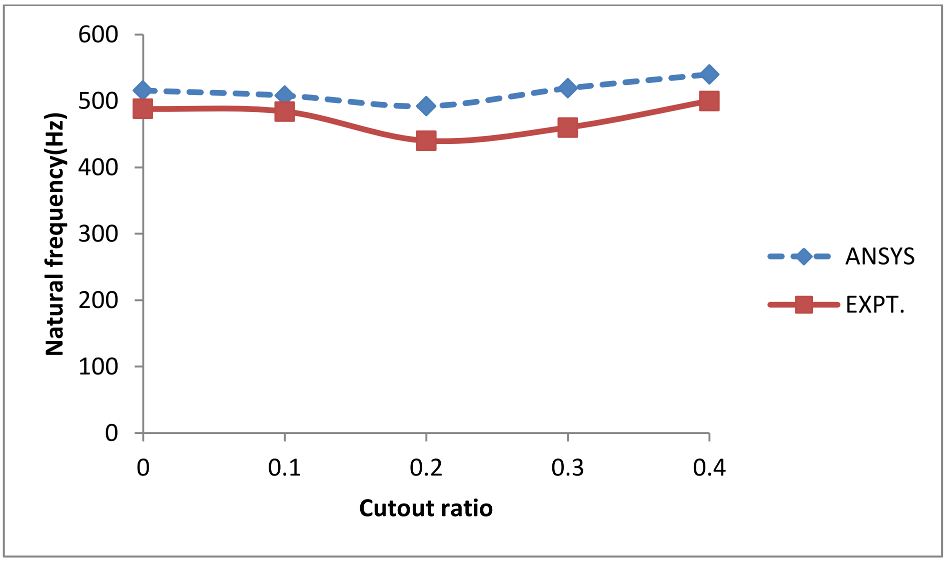


Fig.33: Variation of fundamental frequency for laminated composite plate with cut-out ratio (d/D ratio) under CFCF boundary condition

The mode shapes related with natural frequencies of (0/90)4s composite plate under CFCF end condition have been demonstrated in Fig. 34, Fig. 35, Fig.36, Fig.37 and Fig. 38 for cut-out ratio (d/D ratio) 0.0, 0.1, 0.2, 0.3 and 0.4 respectively. They are construed by using ANSYS package for fundamental frequencies of different cut-out ratio (d/D ratio) i.e. 0.0, 0.1, 0.2, 0.3 and 0.4 under CFCF restrained condition.

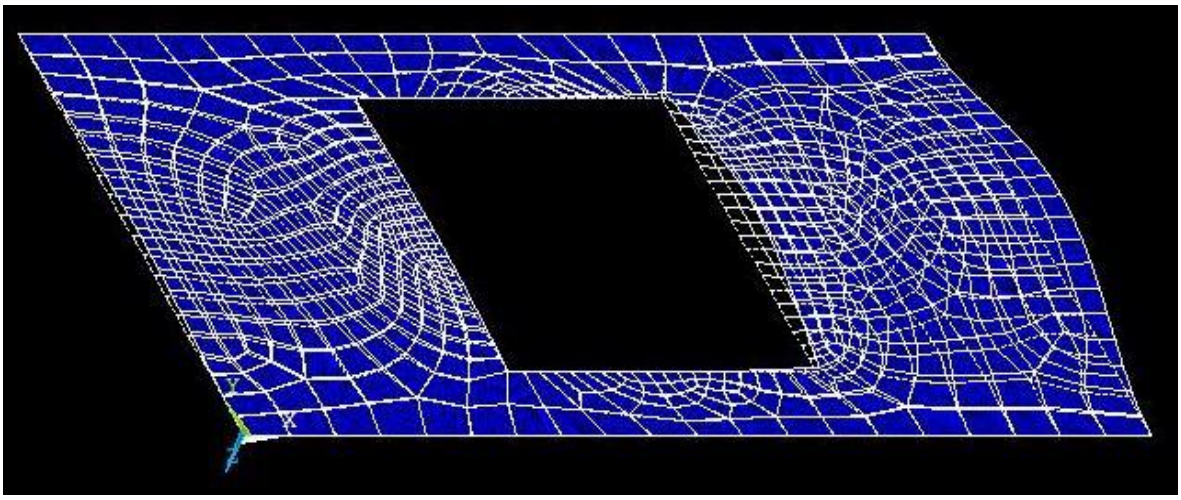


Fig.34: First mode shape (515.5 Hz) of laminated composite plate of size 235mmx117.5mmx3mm for cut-out ratio (d/D ratio) = 0.0

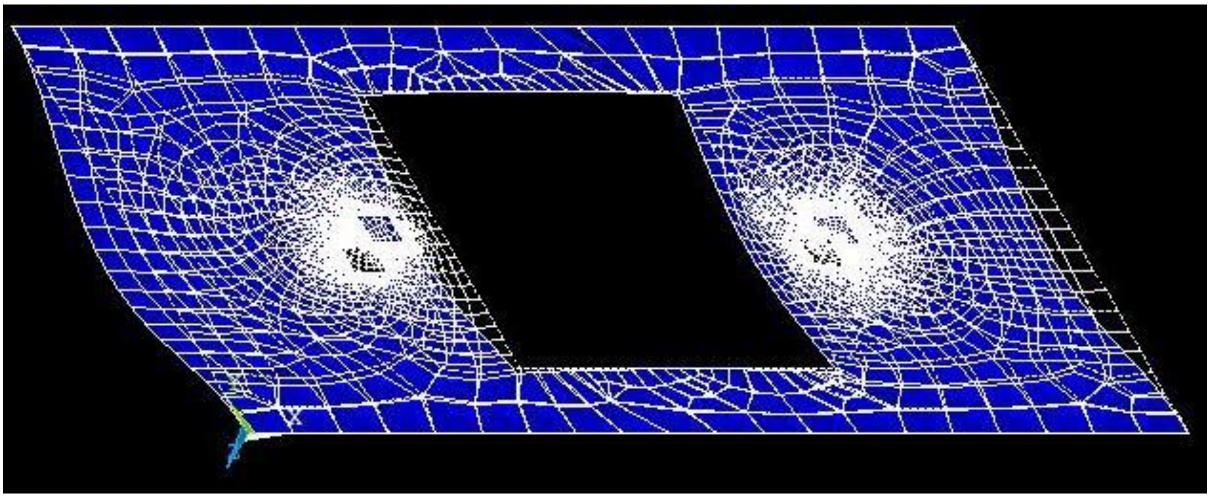


Fig.35: First mode shape (508.8Hz) of laminated composite plate of size 235mmx117.5mmx3mm for cut-out ratio (d/D ratio) = 0.1

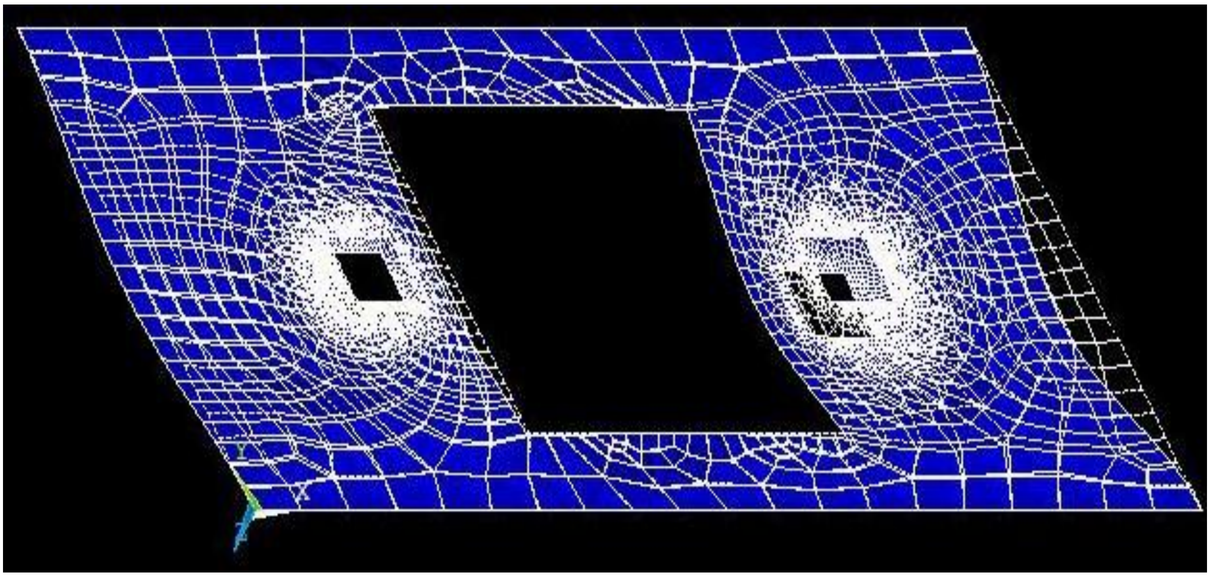


Fig.36: First mode shape (492.5 Hz.) of laminated composite plate of size 235mmx117.5mmx3mm for cut-out ratio (d/D ratio) = 0.2

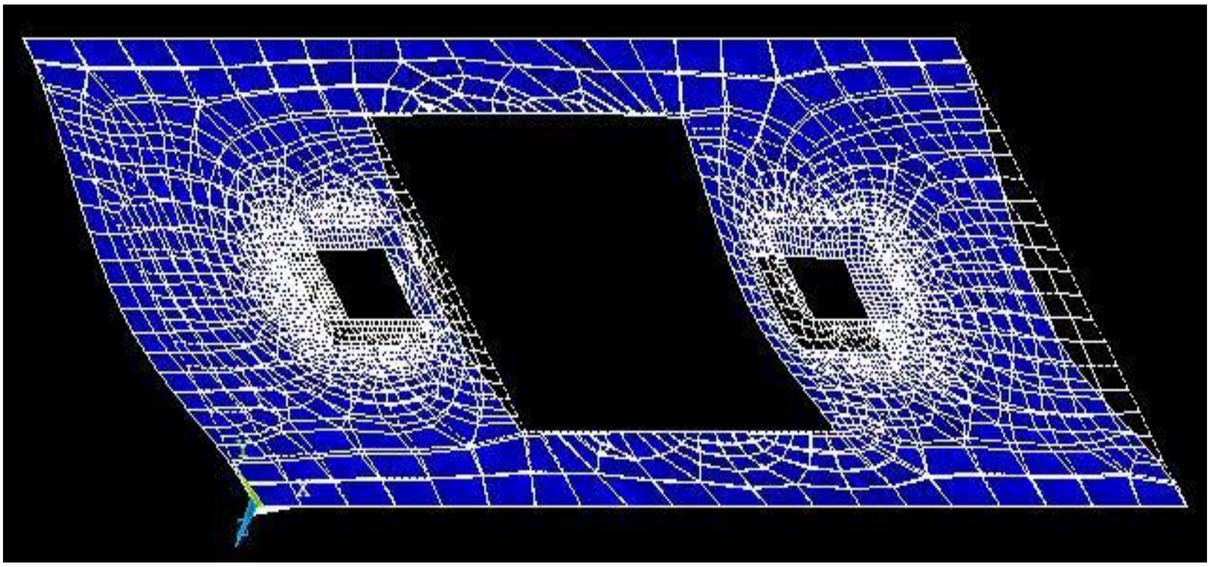


Fig.37: First mode shape (519 Hz) of laminated composite plate of size 235mmx117.5mmx3mm for cut-out ratio (d/D ratio) = 0.3

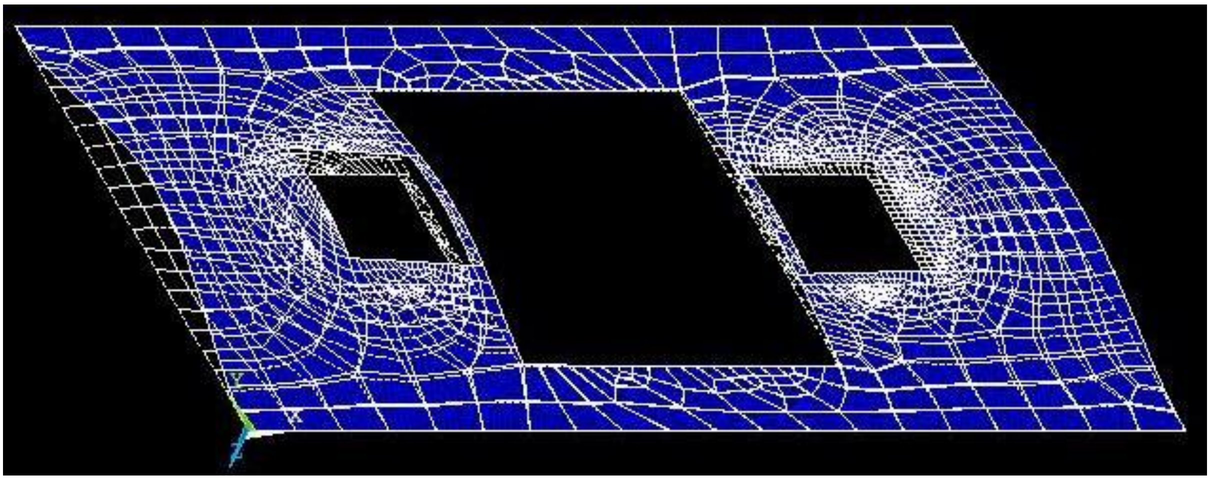


Fig.38: First mode shape (540 Hz) of laminated composite plate of size 235mmx117.5mmx3mm for cut-out ratio (d/D ratio) = 0.4

From the above graph, the fundamental frequencies of laminated composite plates decrease up to cut-out ratio (d/D ratio) equals to 0.2. Since the localisation of stress concentration is reduced by providing multiple square holes around the square central cut-out. Hence there is stiffness reduction of plate. Further natural frequency increases with increase in cut-out ratio (d/D ratio).

6.2.2.4 Effect of aspect ratio (a/b)

To examine the effect of aspect ratio on vibration behaviour of laminated composite plate of cut-out ratio (d/D ratio=0.4) are discussed. There are three different aspect ratio i.e. a/b=1,a/b=1.5 and a/b=2 are considered. The geometry properties of three different aspect ratio (a/b) are shown in table 8. The end conditions are applied along the length of composite plate. The dimensions of different aspect ratio are given in table. The variation of fundamental frequency of laminated plate with different aspect ratio (a/b ratio) for cut-out ratio=0.4 are demonstrated in Fig: 39 under CFCF boundary condition.

Table 8: The geometry properties of three different aspect ratio (a/b)

Aspect ratio(a/b)	Length(mm)	Width(mm)	Thickness(mm)
1	235	235	3
1.5	235	156.6	3
2	235	117.5	3

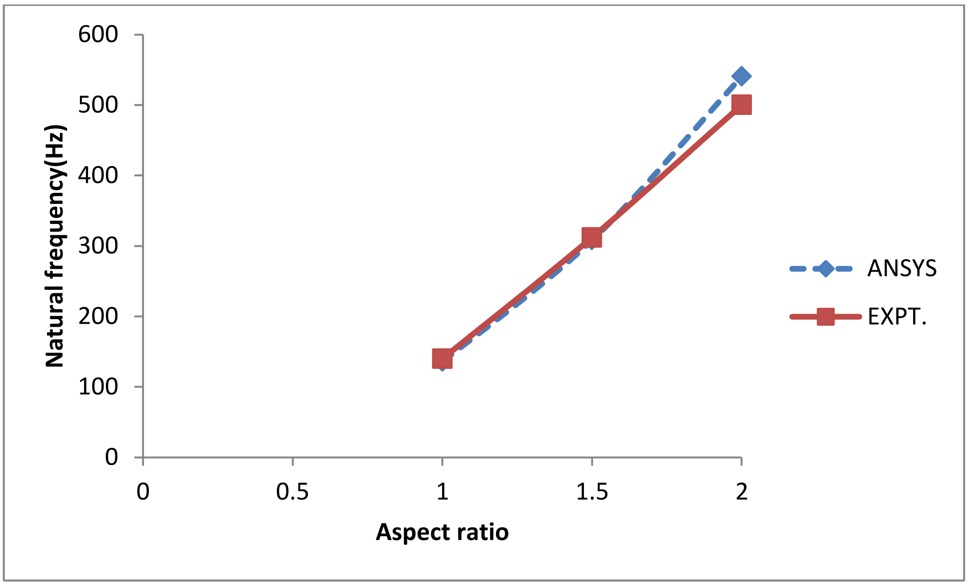


Fig.39: Variation of fundamental frequency of laminated plate with different aspect ratio (a/b ratio) for cut-out ratio=0.4 under CFCF end condition

It is found that the experimental fundamental natural frequency of composite plate with cut-out ratio=0.4 under CFCF boundary condition for aspect ratio 1.5 and 2.0 increases by 55.12% (from 140 Hz to 312 Hz) and 72% (140 Hz to 500 Hz) respectively as compared to aspect ratio 1.0. It also specifies that the fundamental frequencies of a laminated composite plate considerable increases with increase in aspect ratio (a/b).

6.2.2.5 Effect of no of layers of composite plate

Three different layers such as 4,8 and 12 layers are taken in this proposed work to investigate the influence of no of layers on fundamental frequencies of plate with cut-out ratio(d/D ratio=0.4). The geometry properties of plates are 235mmx235mmx1.5mm and 235mmx235mmx4.5mm for 4 and 12 layers respectively. The end conditions are applied along the length of composite plate. The densities of the specimens are calculated 1545 kg/m^3 and 1476 kg/m^3 for 4 and 12 layers respectively. The fundamental frequencies of laminated plate of cut-out ratio=0.4 w.r.t no of layers is presented in Fig: 40.

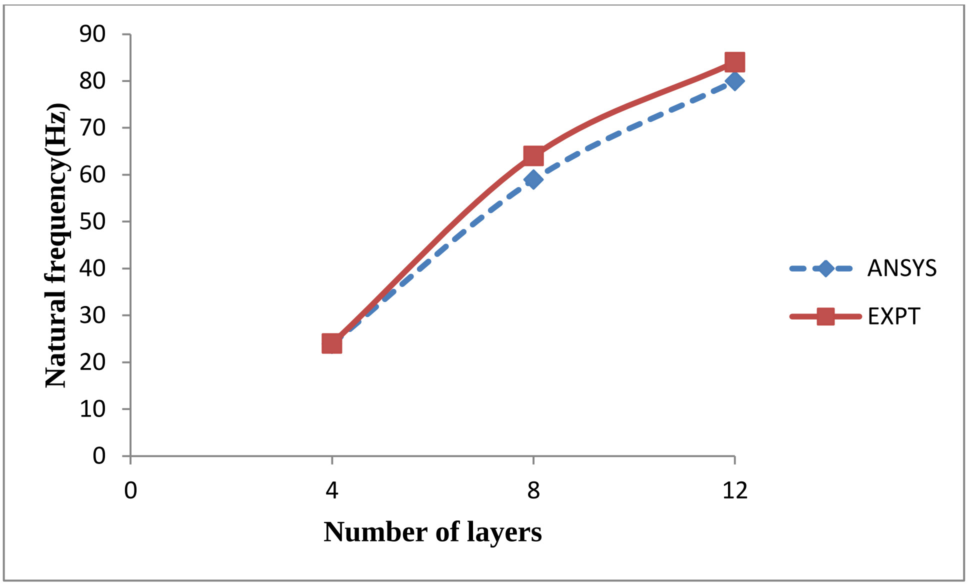


Fig.40: Variation of fundamental frequency of laminated plate of size 235mmx235mm with number of layers for cut-out ratio=0.4

From this above graph, it indicates that the fundamental frequencies of plate increases with increase in number of layer in composite plate. Also, it is observed that the fundamental frequencies of composite plate of cut-out ratio =0.4 for 8 layers and 12 layers is increased by 2.6 times and 3.5 times respectively w.r.t 4 layered laminated plate. This result shows that more no of layers has a positive effect on the stiffness of laminated composite plates.

6.2.2.6 Effect of ply orientation

Three different ply orientation such as $(0/90)_4$, $(45/-45)_4$ and $(30/-60)_4$ are considered in the present study to examine the effects of ply orientations on natural frequencies of laminated composite plate with cut-out ratio (d/D ratio=0.4). The geometry property of plate is 235mmx235mmx3mm.The young’s modulus of elasticity of composite plate corresponding to ply orientations $(30/-60)_4$ and $(45/-45)_4$ are 8.788 GPa and 7.574 GPa respectively. The end conditions are applied along the length of composite plate. The variation of fundamental frequency of laminated plate with respect to fibre orientation for cut-out ratio=0.4 is shown in Fig: 41.

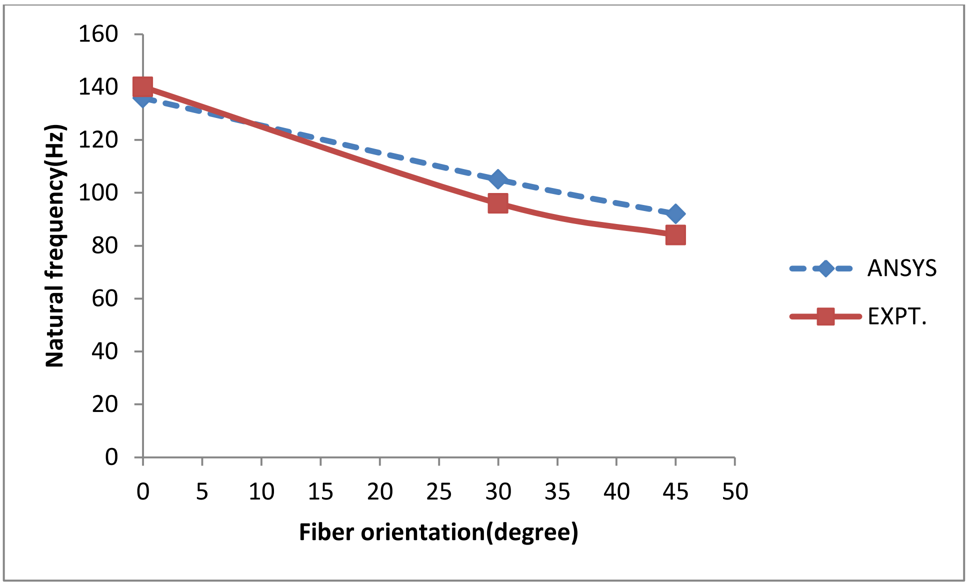


Fig.41: Variation of fundamental frequency of laminated plate with respect to fibre orientation for cut-out ratio=0.4

From this above graph, it finds that the experimental results show a good agreement with ANSYS results verifying that the ply orientation of fibre has been the effect on vibrational analysis of the laminated composite plates. The natural frequencies of plate decrease with increasing fibre angle. It is observed that increasing the orientation from 0^0 to 45^0 decreases the natural frequency by about 40% (from 140 Hz to 84Hz) for 1st mode. It is possible to verify the effect of fibre orientations on the free vibration of laminated plate. It is found that maximum natural frequency occurs at 0^0 and minimum at 45^0.

6.2.2.7 Effect of boundary condition

To investigate the effect of boundary condition on vibrational properties of eight layers laminated composite plate of size 235mmx235mmx3mm with cut-out ratio(d/D=0.4). The boundary conditions considered for present investigation are CFFF, SFSF and CFCF. The natural frequencies of glass epoxy composite plate with cut-out ratio (d/D=0.4) under three different boundary conditions are drawn in Fig.42.

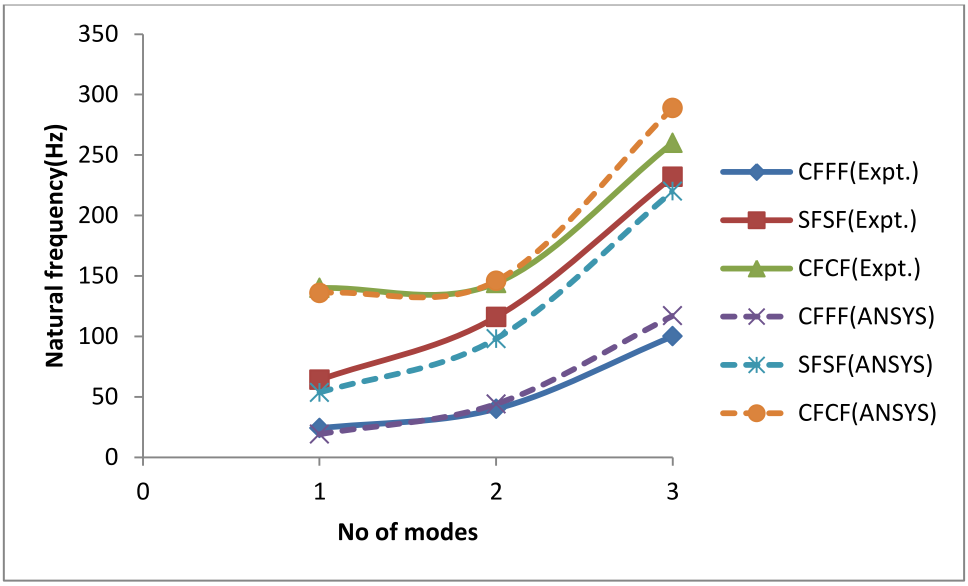


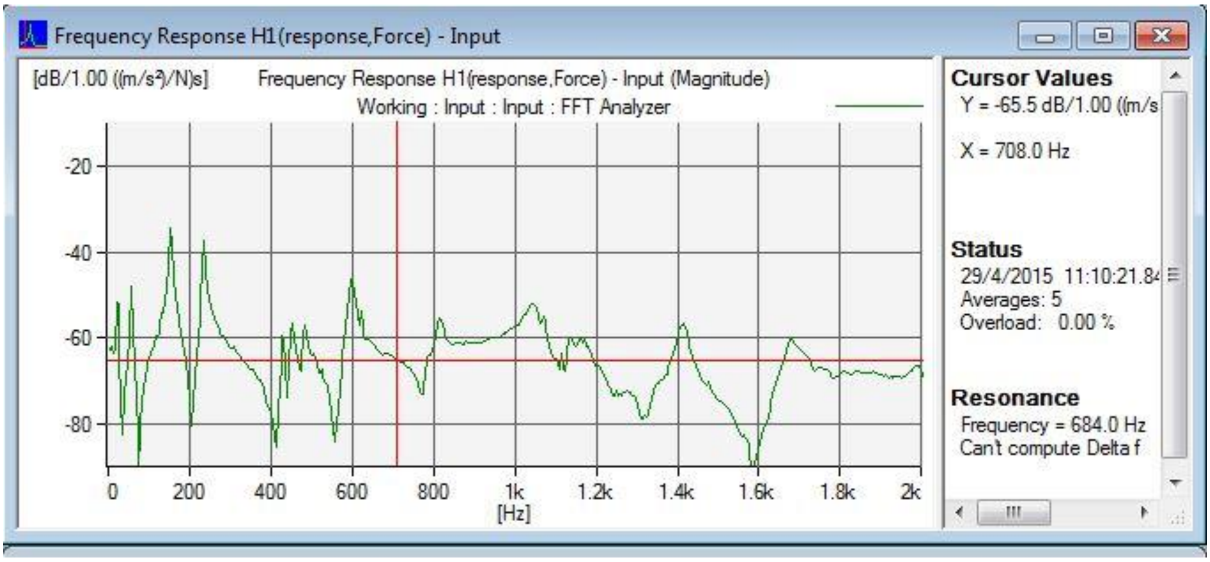
Fig.42: Variation of natural frequency of laminated plate of size 235mmx235mmx3mm with different boundary conditions for cut-out ratio=0.4

From this graph, It shows that the first, second and third mode of frequencies are least for CFFF (one side is clamped and other three sides are free) end condition and the maximum for CFCF boundary condition. The fundamental frequency for cut-out ratio=0.4 at CFFF and SFSF boundary conditions are decreased by 85.97% and 60.58% respectively with respect to CFCF boundary condition. This result indicates that the boundary conditions have large influence on the natural frequencies of laminated composite plate.

There is deviation of results between numerical values and the experimental values due to some possible error during measurement such as position of accelerometer, noise, mass of specimen, non-uniformity in properties of specimens (non-uniform surface finishing, voids and different thickness). These factors are not considered during numerical analysis, meanwhile the model of specimens are considered as homogeneous properties and fully perfect which is not possible in real life. Also, they do not considered irregular distribution of resin on the fibres. Models also do not consider damping effect which is large application effect on modal analysis of structure. Also, the finite element package ANSYS does not permit fibre interweaving present in composite plate.

6.2.3 Pulse report

The natural frequency of the laminated composite plate is recorded during free vibration test by inserting pulse software to computer. The pulse reports for laminated composite plate are shown in Fig: 43 and Fig: 44. The peaks of the FRF shown in Fig: 43 give the natural frequencies of free vibration. The coherence shows the accuracy of measurement during vibration test which is shown in fig: 44.

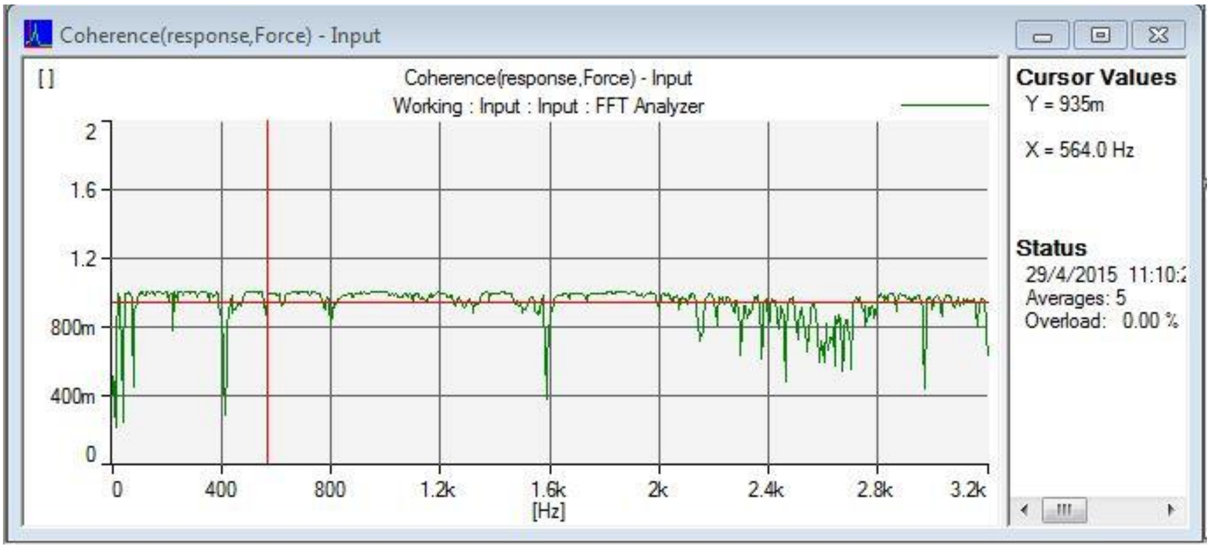


Cursor Values
Y = -65.5 dB/1.00 ((m/s
X = 708.0 Hz

Status
29/4/2015 11:10:21.8
Averages: 5
Overload: 0.00 %

Resonance
Frequency = 684.0 Hz
Can't compute Delta f

Fig 43: Frequency response function spectrum (X-axis indicates Frequency(Hz) and Y-axis indicates acceleration per force ((m/s2)/N))



Cursor Values
Y = 935m
X = 564.0 Hz

Status
29/4/2015 11:10:
Averages: 5
Overload: 0.00 %

Fig 44: Coherence (Response, Force)

CHAPTER-7

CONCLUSION

The present study is related with the effect of the free vibration behaviours on laminated composite plates with cut-outs. The finite element package ANSYS is used to investigate the laminated composite plate with cut-outs. The experimental results of free vibration behaviours of laminated composite plate with cut-outs are compared with numerical ones by considering the effect of various parameters such as cut-out ratio (d/D ratio), aspect ratio (a/b ratio), fibre orientation, no of layers , position of cut-out and different boundary conditions etc. From this analysis, we can be established the following remarks during the investigation of free vibration behaviours of laminated composite plate with cut-outs.

- The numerical results from ANSYS software showed in good agreement with experimental ones on vibration of laminated plates.
- The natural frequencies of plates increases with increase in aspect ratio (a/b) of plates by keeping the cut-out ratio (d/D ratio) constant.
- By keeping aspect ratio (a/b ratio) constant, as the cut-out ratio (d/D ratio) increases from 0.0 to 0.4 the fundamental frequency of composite plate decreases under CFFF and SFSF boundary conditions. But fundamental frequency decreases up to cut-out ratio (d/D ratio) equals to 0.2 under CFCF. Further fundamental frequency increases on increase of cut-out ratio d/D ratio. Since localisation of stress concentration is reduced by providing the multiple holes around the central cut-out
- Natural frequency of composite plate with square cut-out is maximum at centre and minimum at corner position irrespective of CFFF and SFSF boundary conditions.
- It is observed that natural frequency of composite plate with central cut-out without multiple holes is more than the natural frequency of plate with multiple holes.
- The fundamental frequencies of composite plate with fixed cut-out ratio, increase with increase in no of layers due to bending stretching coupling.
- By changing the ply orientation of laminates, it changes the dynamics characteristics of the plates, that is different frequencies for same geometry, mass and end

conditions. As the ply orientation increases, the natural frequency decreases and maximum natural frequency occurs at 0^0.

- The natural frequency of laminated plate with holes varies against different end conditions. It shows that the natural frequency under CFCF boundary condition is the maximum due to clamped at two opposite edges. However, the CFFF supported laminated composite plates shows the minimum frequency among three boundary conditions tested.

From the above conclusion, the vibration characteristics of laminated plates with cut-out is affected by the different geometry properties, boundary conditions, ply orientation, cut-out ratio and position of cut-outs. The presence of cut-out may be weakening the structure under the dynamic loading and causes resonance due to reduction in natural frequency. So cut-outs play a vital role on the vibration characteristics of the plate structures. So the designers have to be cautious while designing the structures with cut-outs. The vibration results of the laminated composite plates can be used as a tool for structural health monitoring and also helps in assessment of structural integrity of composite structures.

FUTURE SCOPE OF RESEARCH

In the proposed work, the natural frequency of laminated composite plate is computed both numerically and experimentally. The influences of various parameters such as cut-out ratio, fibre angle, aspect ratio, no of layers and different position of cut-out are analysed under different boundary conditions. The future scope of the present work can be expressed as follows.

- The present investigation is related to the square size cut-out. This can be extended for different shape of cut-out.
- Force vibration analysis of laminated composite plate with cut-outs can be extended.
- Dynamics analysis of stiffened plates with cut-out may be extended for future study.
- The present research work can be extended to free vibration analysis of shells with cut-out.
- The present investigation can be extended to vibration analysis of composite plates with delamination around cut-out.
- The present study deals plates with uniform thickness. The plate can be modified to non-uniformity in thickness.
- Free vibration of composite plate with cut-out can be extended in hygrothermal environment.
- Buckling analysis and dynamic stability of laminated composite plate with cut-out can be studied.

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APPENDIX

Modelling of Central Square cut-out with different multiple square holes of plate size 235mmx235mmx3mm.

Central square cut-out of size 80mmx80mm and cut-out ratio (d/D) 0.1,0.2,0.3 and 0.4 are considered i.e. side of multiple square cut-out vary 8mm,16mm,24mm and 32mm. (Where ‘D’ is the side of centrally located square cut-out and ‘d’ is the side of multiple square cut-out located both side of central square cut-out).The composite laminated plates with different cut-out ratio are presented in the Fig: 45 to Fig: 48.

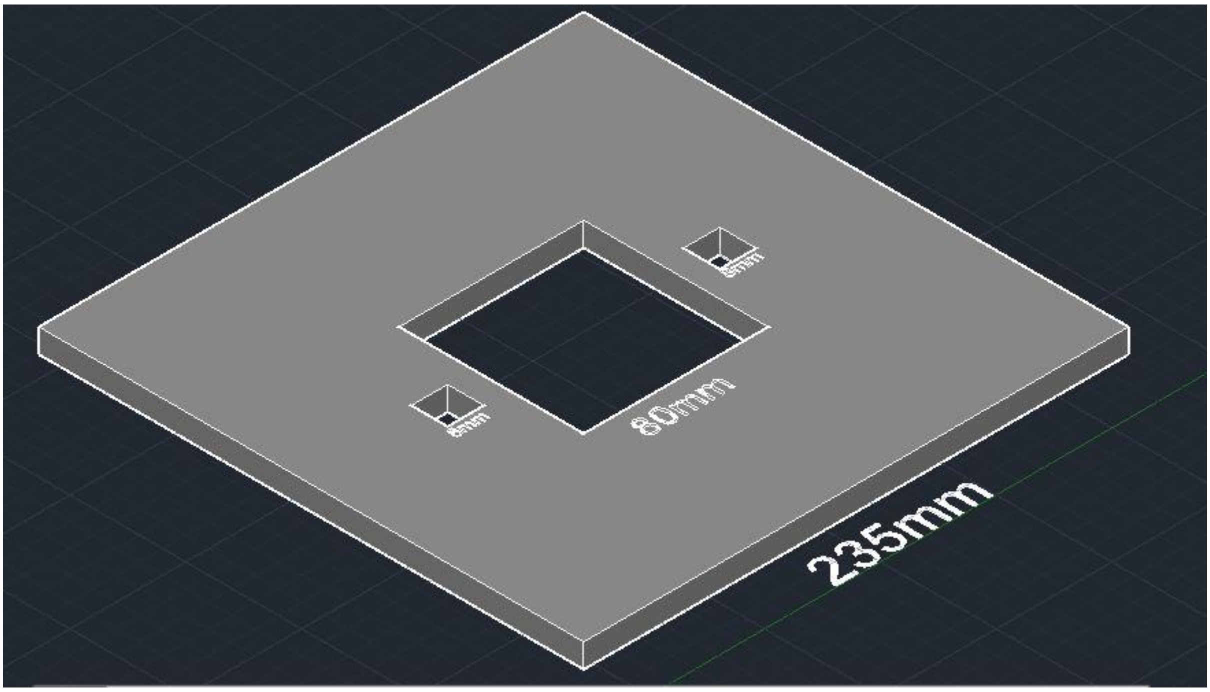

Fig.45: Plate with cut-out ratio (d/D ratio) = 0.1(where d=8mm and D=80mm)

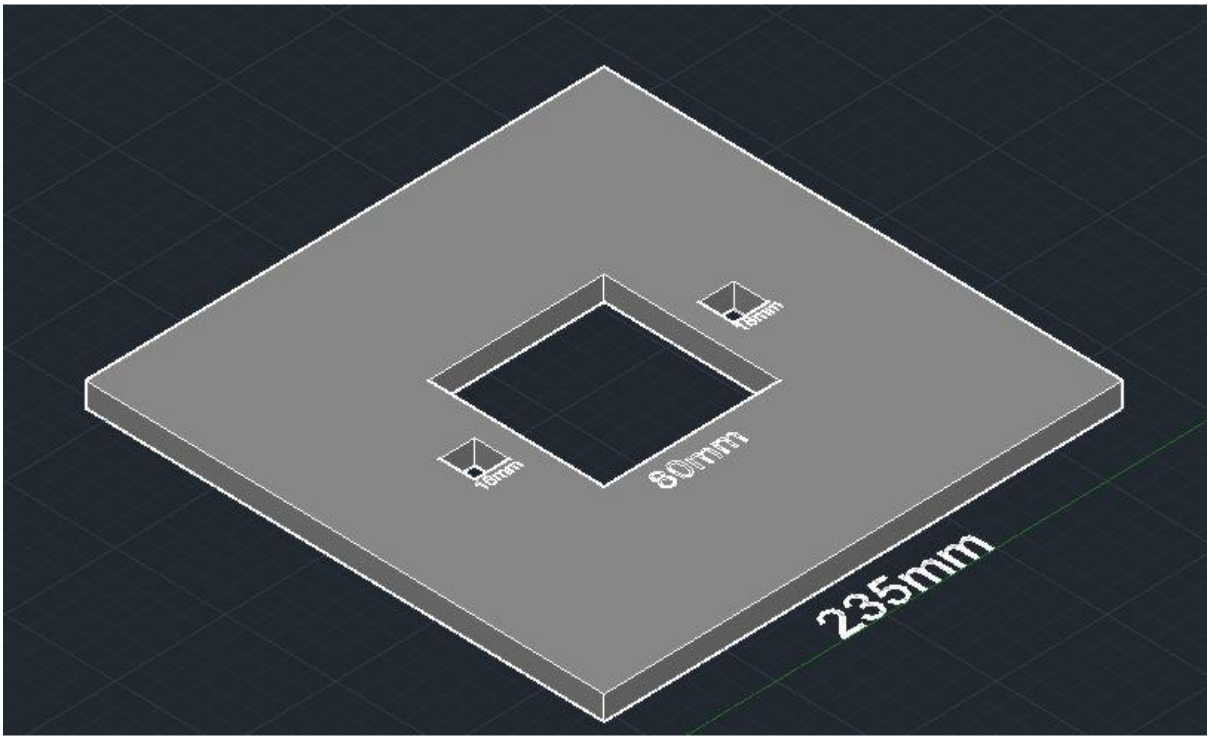

Fig.46: Plate with cut-out ratio (d/D ratio) = 0.2(where d=16mm and D=80mm)

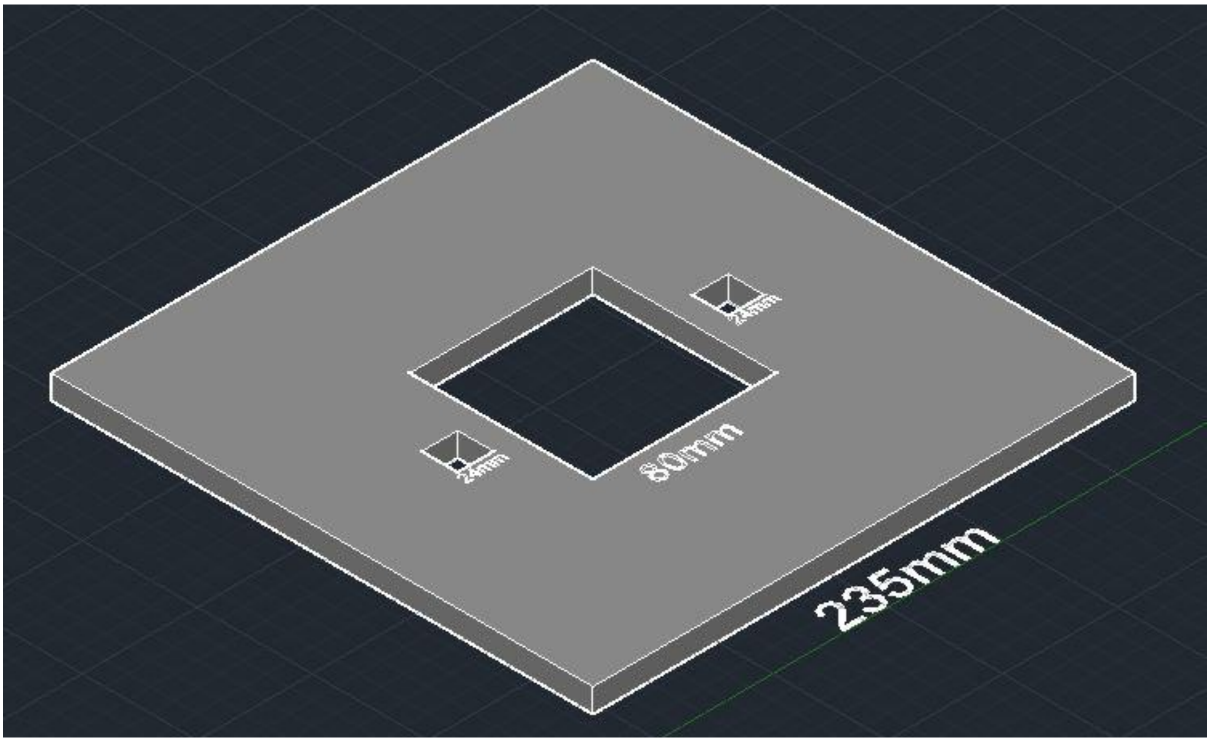

Fig.47: Plate with cut-out ratio (d/D ratio) = 0.3 (where d=24mm and D=80mm)

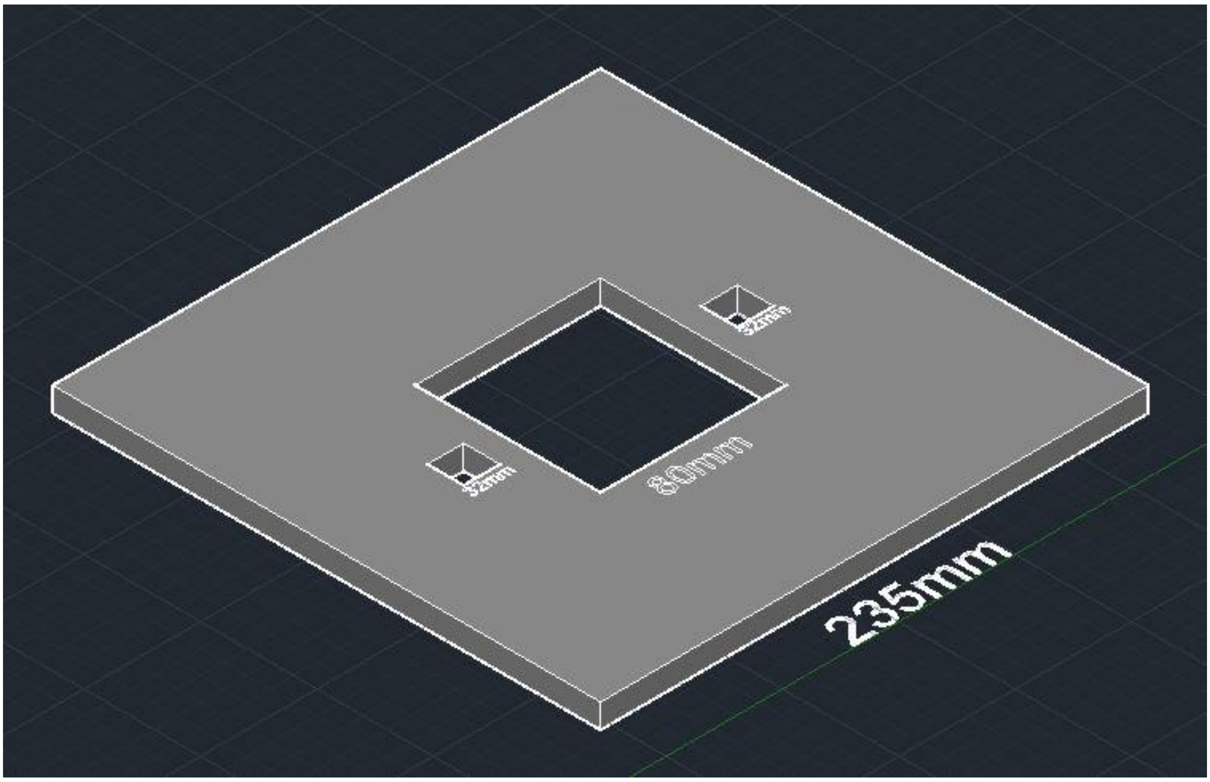

Fig.48: Plate with cut-out ratio (d/D ratio) = 0.4 (where d=8mm and D=80mm)