IMPLEMENTATION OF FIBER ELEMENT MODEL FOR NON-LINEAR ANALYSIS

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Implementation of Fiber element model for Non-linear analysis

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CERTIFICATE

This is to certify that the thesis entitled "Implementation of Fiber element model for Non-linear analysis" submitted by Shemin T John in partial fulfilment of the requirement for the award of Master of Technology degree in Civil Engineering with specialization in Structural Engineering to the National Institute of Technology, Rourkela is an authentic record of research work carried out by her under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Project Guide

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LIST OF SYMBOLS

f_c	Compressive Strength of Concrete
f_y	Yield Strength of Steel
E_c	Young's Modulus of Concrete
E_s	Elastic Modulus of steel
ϵ_{cc}	Strain corresponding to Compressive strength of Concrete
f_{co}	Compressive strength of Unconfined Concrete
ϵ_{co}	Strain corresponding to Unconfined Compressive strength
ϵ_{cu}	Ultimate strain of Confined Concrete

ABBREVIATIONS

K	Strength Enhancement factor
RC	Reinforced Concrete
IS	Indian Standard
DBM	Displacement Based Method
FBM	Force Based Method
MCS	Monte Carlo Simulation

ABSTRACT

Keywords: *Fiber element, Distributed Inelasticity, Numerical Integration, MCS, Confinement models for concrete*

RC frames undergo inelastic deformations in the event of an extreme earthquake. Nonlinear modelling of the concrete sections is very much necessary for the simulation of realistic behaviour of RC frames in earthquake loading. Concentrated/lumped plasticity and distributed plasticity are the two different approaches for nonlinear modelling of RC elements available in literature. The main objective of the present study is to implement a displacement based fiber element (stiffness) for nonlinear analysis of RC Sections. The present study focused on the element formulation of both stiffness and flexibility based fiber models, direct integration and numerical integration and incorporation of popular confinement models for stress-strain relationship for concrete. The present study is extended to a probabilistic analysis using the implemented model. It is found that fiber elements are appropriate tool for incorporating nonlinearity in the RC sections.

1
INTRODUCTION

CHAPTER 1

INTRODUCTION

1.1 GENERAL

RC frames behave inelastically under earthquake loading. Simulation of behaviour of RC frame in such earthquake loading requires nonlinear modelling and analysis techniques. There two different types of approaches for nonlinear modelling of RC elements, namely concentrated or lumped plasticity approach and distributed plasticity approach. The motivation of the present study is to simulate the nonlinear behaviour of RC sections using a fiber element model.

1.2 NONLINEAR MODELLING OF RC ELEMENTS

Material nonlinearity in a frame elements are primarily divided into two categories;

1.2.1 Distributed plasticity and Lumped plasticity

In the lumped plasticity model, elements consists of two zero-length nonlinear rotational spring elements with an elastic element between them as shown in the Fig. 1.1 The spring element accounts for nonlinear behaviour of a structure by having nonlinear moment-rotation relationships. The lumped plasticity model is popular since the computational cost of the analysis is high, e.g., in the case of nonlinear time-history analysis of a large structure.

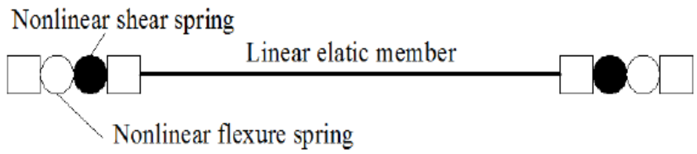

Fig 1.1: Zero length spring used in lumped plasticity model

The distributed plasticity model is used for more accurate estimation of the structural response. The distributed plasticity model is employed for the nonlinear frame element with the fiber section discretization. The distributed inelasticity members are modelled with the fiber approach, which consists of discretizing into integration sections and into several material fibers as shown in Fig. 1.2. The two main formulations are the displacement-based (DB) stiffness method and the force-based (FB) flexibility method. The DB formulation uses displacement shape functions, while the FB formulation uses internal force shape functions.

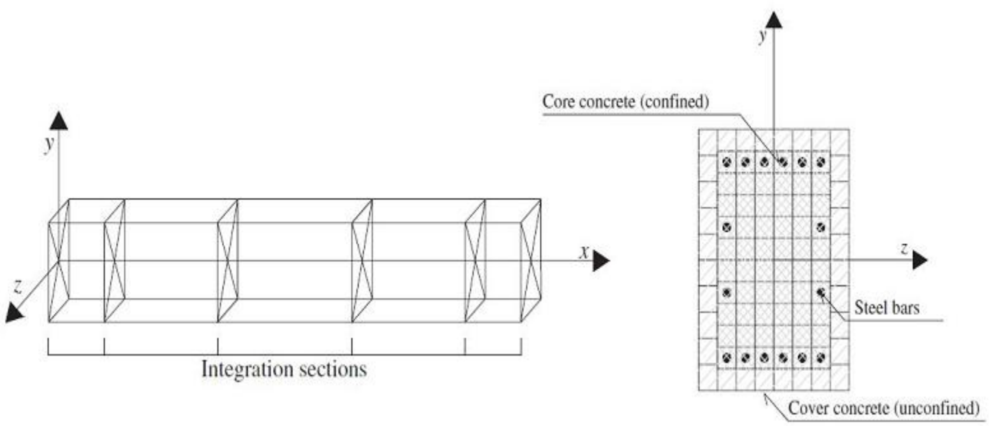

Fig 1.2: Fiber section discretization and sections

1.3 OBJECTIVES OF THE STUDY

Based on the preceding discussions, the main objectives of the current study has been quoted as follows

i. To implement the displacement based (stiffness) fiber element model for nonlinear analysis of RC Columns.

ii. To study the response of RC sections using various confinement models of concrete.

iii. To conduct probabilistic analysis of RC column considering uncertainties in the geometry and material properties.

1.4 METHODOLOGY

i. Conduct a literature review on various fiber element models to use in RC sections for non-linear static analysis.
ii. Identify a simple and easy to implement fiber element models
iii. Implement the fiber element model in MATLAB 2012b.
iv. Perform static non-linear analysis of RC section
v. Consider uncertainty of various random variables involved
vi. Conduct a probabilistic analysis to arrive at the uncertainty in the responses such as base shear and yield displacement. Analyse the results and arrive at conclusions.

1.5 SCOPE OF WORK

i. The present study is limited to only axial loading of RC sections.
ii. Only distributed plasticity formulations are considered for this study.
iii. Only material nonlinearity is considered in this study

1.6 ORGANISATION OF THE THESIS

Following this introductory chapter, the organisation of further Chapters is done as explained below.

i. A review of literature conducted on Element formulations of fiber element modelling, nonlinear solution and iterative strategies constitutive models for steel and concrete, and Monte Carlo simulation in Chapter 2.

ii. Element formulations of stiffness and flexibility fiber element model for the RC sections is explained in Chapter 3.
iii. Linear and Nonlinear analysis of RC sections using fiber element modelling and probabilistic studies such as Monte carlo simulations are explained in Chapter 4
iv. Finally in Chapter 5, discussion of results, limitations of the work and future scope of this study is dealt with.

2

REVIEW OF LITERATURE

CHAPTER-2

LITERATURE REVIEW

2.1 GENERAL

The literature review is divided into three parts. The first part deals with the distributed inelasticity models, numerical integration, solution strategies and iterative techniques for nonlinear analysis. The second part of this chapter discusses about the various confinement models for the concrete. Third part of the Chapter discuss the studies employing Monte Carlo simulation.

2.2 DISTRIBUTED INELASTICITY MODELS

The most accurate models for nonlinear analysis of reinforced concrete structures are fiber models. There are no predetermined lengths that lumps the inelastic behavior here. Thus inelasticity can develop anywhere in the structure. Distributed inelasticity models are modelled with fiber approach has got two methods

1. Displacement based Stiffness method
2. Force based Flexibility method

The following study shows the evolution of distributed inelasticity models along with flexibility and stiffness methods

2.2.1 Displacement based Stiffness method

Otani (1974) first introduced inelasticity spread of a member in one component model using two inelastic finite element length along with two additional nonlinear rotational springs. It was the first model to ascertain the importance of fixed end rotations for predicting seismic response.

Soleimani et *al*. (1979) considered a model with gradual spread of inelasticity along the member It consist of elastic and inelastic zones. The inelastic zones spreads from beam-column interface controlled by moment curvature relationship at member end section. Point hinges were also considered for fixed end rotations at beam column interface.

The proposal by Soleimani et *al*. (1979) was further extended by Filippou and Issa (1988) in a completely refined way. The member was subdivided into sub elements each accounting elastic behavior, inelastic behavior due to bending and fixed end rotations at beam column interface. Flexibility matrix and member end rotations are summed up from each sub element as they are all associated in series. The point hinge idealization was used in these models are based on bilinear moment rotation relationship with constant post yielding stiffness. This model was further improved by Filippou et *al* (1992) to include another sub element with shear distortions in inelastic zones. Constant axial force-bending moment interaction was included in the basic curve of the model.

Takanayagi and schnobrich (1979) have proposed another type of member model dividing elements into short sub elements (finite element springs) along the member with nonlinear moment rotation relationships. Axial force-bending moment interaction was included by limit surface for each spring. This model also encountered problem of unbalanced force in internal members which often resulted in numerical instability.

This model along with (Hellesland and Scordelis, 1981, Mari and Scordelis, 1984) were based on classical stiffness method using cubic hermitian polynomials to approximate displacements along the member. In these types, it encompasses 6 degree of freedom for the 3D elements.

In our present study element formulation presented by Lee and Mosalam (2004) is implemented which uses displacement interpolation functions from classical stiffness methods. The element stiffness matrix and nodal equivalent forces are obtained by integration of section stiffness and force distributions. The element formulation is straight forward and easy to implement.

2.2.1 Force based Flexibility method

Menegotto and Pinto (1977) proposed improved representation of internal deformations by combined approximation for both section deformation and flexibilities. Mahasuverachai (1982) proposed improvement of displacement interpolation functions. He introduced variable interpolation functions for piping and tubular structures.

Kaba and Mahin (1984) adapted this to RCC structures along with section layer discretization. Typically these functions were derived from from force interpolation polynomials. A mixed approach was used when both deformation and force interpolation functions are used. The model has inconsistencies led to numerical problems. State determination were such that equilibrium between applied and resisting section forces were not satisfied. The proposal was further improved by Zeris et *al.* (1986) and Kaba and Mahin (1988) improving element state determination.

The formulation of nonlinear flexibility based frame element cast into a unified and general theory by Taucer et *al.* (1991), Spacone et *al.* (1992) and Spacone (1994) derived from mixed finite element works of Zienkiewicks and Taylor (1989). Element state determination inserted classical stiffness based finite element which appear rather straightforward. The formulation was capable to carry moment curvature relationship or stress strain relationship at the fiber level. It requires a few control sections along the member. The force interpolation functions were used as they are exact regardless to the damaged state of the member. Element flexibility matrix were obtained by integration of flexibility distributions at control sections and an internal iterative scheme was proposed to find element resisting force for imposed displacements.

2.3 NUMERICAL INTEGRATION

Stiffness and Flexibility based formulations requires integration along the length of member. Conventional direct integrations are computationally not effective. These integrals can be evaluated by numerical quadrature. In our present study, we use gauss lobatto integration.

Location points x_i and weights w_i for i = 1,...,n. The domain of integration for such a rule is conventionally taken as [−1, 1], so the rule can be stated as:

$$\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i) \tag{2.1}$$

The Gauss-Lobatto rule with 'n' Integration points allows the exact integration of polynomials of degree up to 2n-3. The locations and the associated weights for this integration rule is shown in the Table 2.1. The points, x_i is the locations and w_i are the weights.

Table 2.1 Locations and the associated weights for Gauss Lobatto integration rule

Number of points	Points,XIP	Weights,W
3	-1.000	0.333
	0.000	1.333
	1.000	0.333
4	-1.000	0.167
	-0.447	0.833
	0.447	0.833
	1.000	0.167
5	-1.000	0.100
	-0.655	0.544
	0.000	0.711
	0.655	0.544
	1.000	0.100

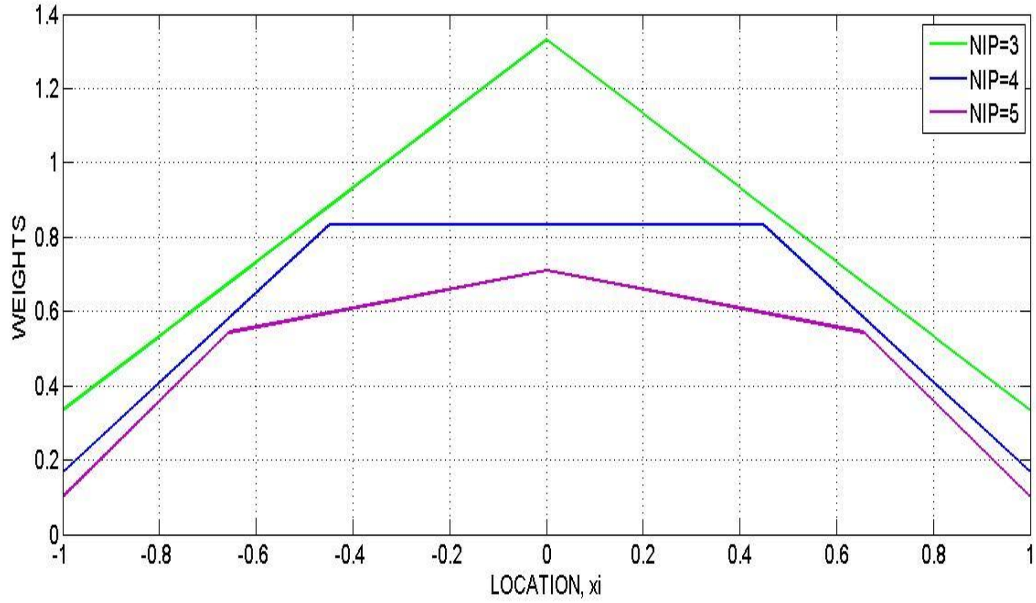

Fig 2.1: Locations and Weights of Gauss Lobatto quadrature rules

2.4 SOLUTION STRATEGIES FOR NONLINEAR ANALYSIS

2.4.1 Path Following Techniques

The objective of this techniques are to draw the equilibrium path of a nonlinear problem in the framework of a force-displacement relation. There are many techniques available and a Load control and Displacement control method is used in the present study.

2.4.1.1 Load Control

The total load is divided into small load increments. Displacement is calculated for each load level. This method gives equilibrium path up to failure point only as shown in the Fig.2.2.This method is not suitable at post critical yield regions and usually results in instability.

2.4.1.2 Displacement control

Displacement control method gives a good solution for nonlinear problems because it presents a great stability at the critical points.

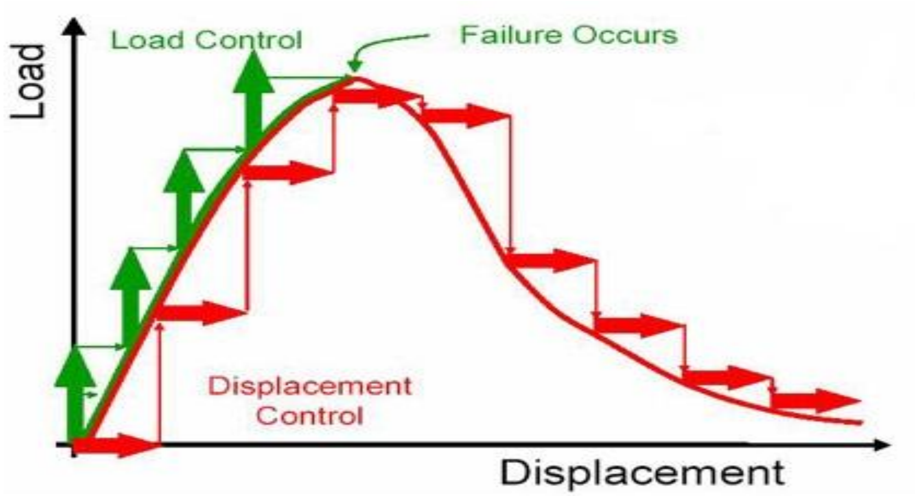

Fig 2.2: Load control and Displacement control method (www.theconcreteportal.com)

2.4.1.2.1 STEPS FOR DISPLACEMENT CONTROL METHOD

Step wise procedure for the displacement control method and a flow chart for the same is also shown in Fig. 3.17.

1. Let (d^0, λ^0) be the equilibrium initially at i=0 ($d^0$is the initial displacement vector and λ^0 is the load level)
2. qth component of d^0 is incremented by $\delta d(q)$. Alter the initial displacement vector d_0 such that $d^0(q) = d^0(q) + \delta d(q)$
3. Calculate the residual vector $r^i = q_i - \lambda q_e$

   Where q_i is the internal load vector

   λ is the load level parameter

   q_e is the external load vector
4. Find the displacement vectors $\delta \overline{d}^i$and δd_t^i.

   $\delta \overline{d}^i = K_t^{-1} r^i$ and $\delta d_t^i = K_t^{-1} q_e$
5. Calculate the incremental load level and incremental displacement $\delta \lambda^i and\ \delta d^i$

$\delta d^i = \delta \overline{d^i} + \delta\lambda\, \delta d_t^i$ and $\delta\lambda^i = \frac{\delta \overline{d^i}(q)}{\delta d_t^i(q)}$

6. Displacement vector and the load level are updated

$\mathrm{d}^{i+1} = \mathrm{d}^{\mathrm{i}} + \delta d^i$ and $\lambda^{i+1} = \lambda^i + \delta\lambda^i$

7. Repeat the steps until a desired accuracy or desired number of iterations are achieved.

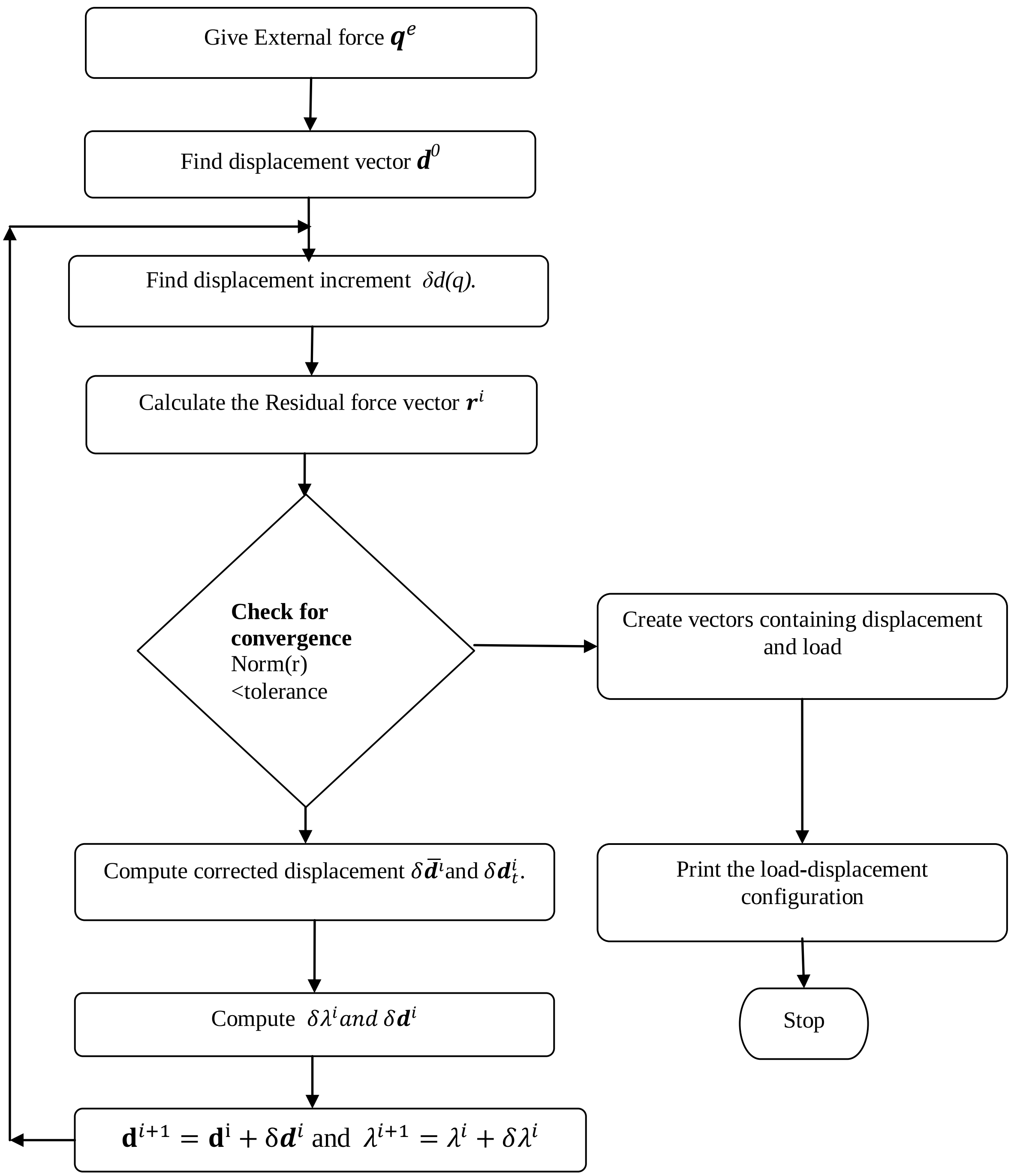

Fig 2.3: Flow chart for Displacement control method

2.5 ITERATIVE TECHNIQUES FOR NON-LINEAR ANALYSIS

2.5.1 Newton Raphson Method

Newton Raphson iterations are implemented to achieve equilibrium before going to the next load step. The incremental force is applied at the start of a step. Internal forces and external forces will not be in equilibrium so we have to use this techniques. Corrections are made to the displacement to achieve displacement. This is done by Newton-Raphson method by minimizing the residual,

r= $f_{int} - F_{ext.}$ Correction to the displacement is given by Δ d $^{j+1}$= $[K (d^{j})]^{-1} * r$

The new displacement d=d+ Δ d

Convergence is achieved by putting residual, 'r' to a tolerance say 10^{-2} .Tangent stiffness matrix is calculated at each load step. This method requires more computational time.

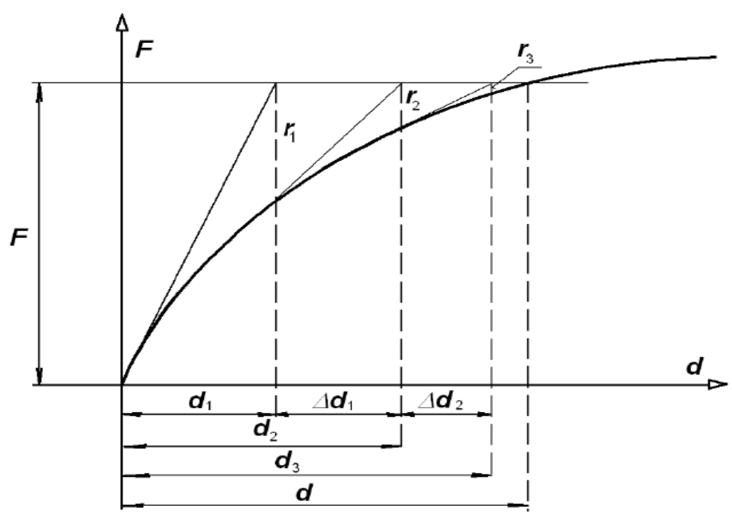

Fig 2.4: Newton Raphson iterative scheme

2.6 CONFINEMENT MODELS FOR CONCRETE

Capacity of RC sections can be significantly increased by confining the concrete with confining stirrups. Strength and strain capacity is increased due to the restrain of dilation of the concrete in compression. Confining action comes into play when the concrete is in compression and core expands against transverse reinforcement. In highly seismic region, the increase in strength and ductility is an important aspect for the design of RC structural elements. Confinement characteristics of concrete can be shown in stress stain curves. A review of confinement models are given below

2.6.1 MANDER et *al*. (1988) MODEL

This model first investigated different cross section columns to study the impact of transverse reinforcement. It was found that the performance over the complete stress-strain range was same if the peak strain and stress coordinates might be found (ε_{cc}, f_{cc}'). The Mander et *al*. (1988) model is popular and in this study we account material nonlinearity from this model.

The peak stress,

$$f'_{cc} = f'_{co}\left[1 + 3.7\left(\frac{0.5k_e\rho_s f_{yh}}{f'_{co}}\right)^{0.85}\right] \tag{2.1}$$

Where fco' is unconfined compressive strength equal to $0.75fck$, ke is the confinement effectiveness coefficient having a typical value of 0.95 for circular sections and 0.75 for rectangular sections, ρs = Volumetric ratio of confining steel, fyh= Grade of confining steel,

Strain corresponding to peak stress,

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + 5\left(\frac{f'_{cc}}{f'_{co}} - 1\right)\right] \tag{2.2}$$

The ultimate compressive strain,

$$\varepsilon_{cu} = 0.004 + \frac{0.6\rho_s f_{yh}\varepsilon_{sm}}{f'_{co}} \tag{2.3}$$

Where ε_{sm}= Steel strain at maximum tensile stress,

The stress at any strain,

$$f_c = \frac{f'_{cc} x r}{r-1+x^r} \tag{2.4}$$

$$x = \frac{\varepsilon_c}{\varepsilon_{cc}},\ r = \frac{E}{E_c - E_{sec}},\ E_c = 5000\sqrt{f'_{co}},\ E_{sec} = \frac{f'_{cc}}{\varepsilon_{cc}} \tag{2.5}$$

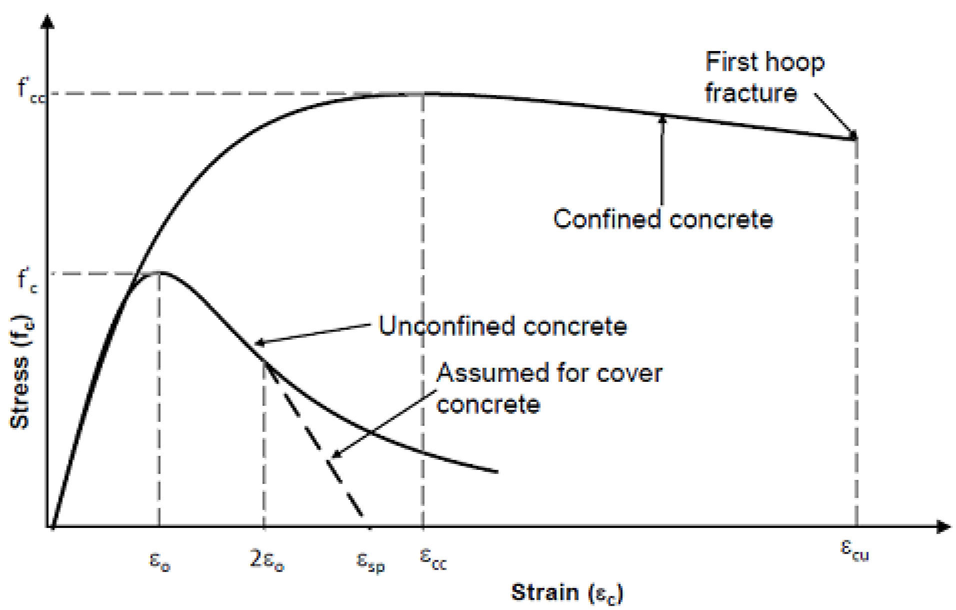

Fig 2.5: Mander et *al* 1988 model

2.6.2 MODIFIED KENT AND PARK MODEL (1982)

The strength enhancement factor, K was expressed in terms of volumetric ratio of confining reinforcement was introduced to existing Kent and park model (1971). This model of stress strain is also taken for the study.

For $e_c \leq 0.002K$

$$f_c = Kf_c'\left[\frac{2\varepsilon_c}{0.002K} - \left(\frac{\varepsilon_c}{0.002K}\right)^2\right] \tag{2.6}$$

For e≤0.002K

$$f_c = Kf_c'[1 - Z_m(\varepsilon_c - 0.002K)] \tag{2.7}$$

$$Z_m = \frac{0.5}{\frac{3 + 0.29f_c'}{145f_c' - 1000} + \frac{3}{4}\rho_s\sqrt{\frac{h''}{s_h}} - 0.002K} \tag{2.8}$$

ρ_s= ratio of volume of rectangular steel hoops to volume of concrete core measured to the outside of the peripheral hoop, h"= width of concrete core measured to the outside of the peripheral hoop and s_h= center to center spacing of hoop sets. In the above expressions the value of is obtained from the following expression:

$$K = 1 + \frac{\rho_s f_{yh}}{f_c'} \tag{2.9}$$

Where f_{yh} is the yield strength of the hoop reinforcement

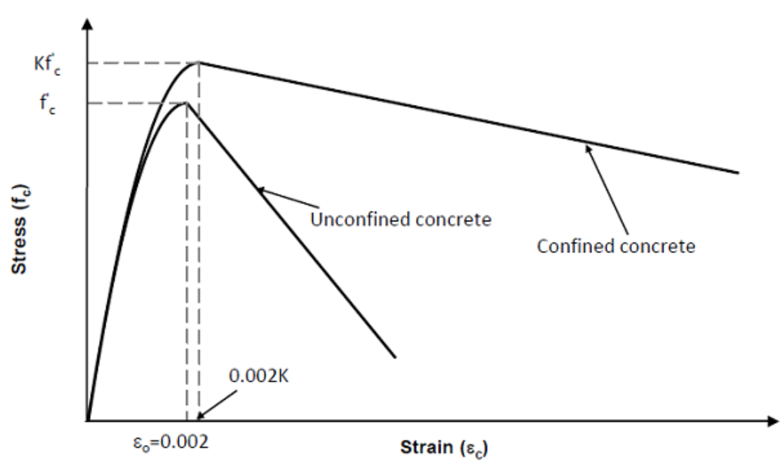

Fig 2.6: Modified Kent and Park model (1982)

2.6.3 IS 456 (2000) MODEL

IS 456 (2000) assumes the same ductility and strength for both confined and unconfined concrete .The maximum value of strain considered is 0.0035.IS 456 (2000) model underestimates the ductility and strength of the RC sections. The stress strain relationship is given by

$$\varepsilon_c \leq \varepsilon_{co} \qquad f_c = f'_{co}\left[\frac{2\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002}\right)^2\right] \tag{2.10}$$

$$\text{For } \varepsilon_{co} < \varepsilon_c < 0.0035 \qquad f_c = f'_{co} \tag{2.11}$$

where f_c is the stress in concrete corresponding to the strain ε_c and $f_{co}\,'$ is the strength concrete corresponding to the strain 0.002 (ε_{co}).

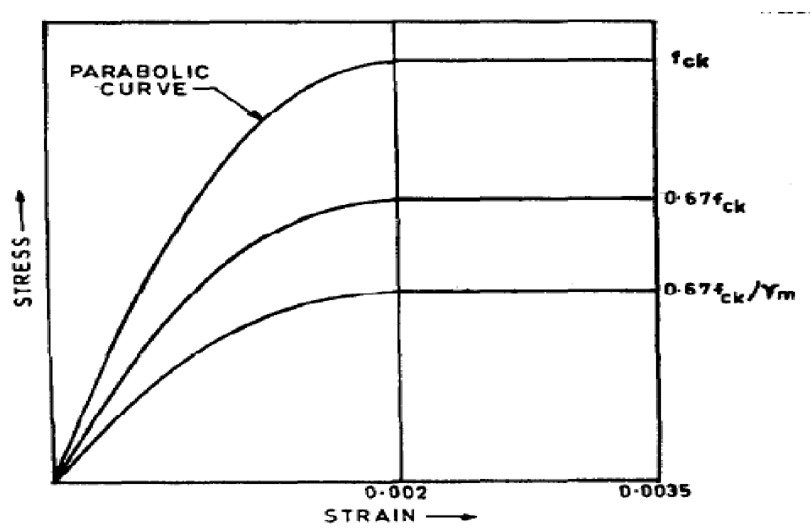

Fig 2.7: IS 456 model (2000)

2.6.4 NON LINEAR STEEL MODEL

The bilinear elastic-plastic portion followed by a strain hardening region shown in Lee and Mosalam,(2004) calculated by

$$f_s = f_u - (f_u - f_y)\left(\frac{\varepsilon_{su} - \varepsilon_s}{\varepsilon_{su} - \varepsilon_{sh}}\right)^2, \quad \varepsilon_{sh} < \varepsilon_s < \varepsilon_{su} \tag{2.12}$$

Where f_s is the steel stress corresponding to the steel strain e_s,

f_y is the yield stress, f_u is the ultimate stress, e_{sh} is the strain at the on-set of hardening, and e_{su} is the ultimate strain.The fig 2.6 shows the stress strain relationship for the steel

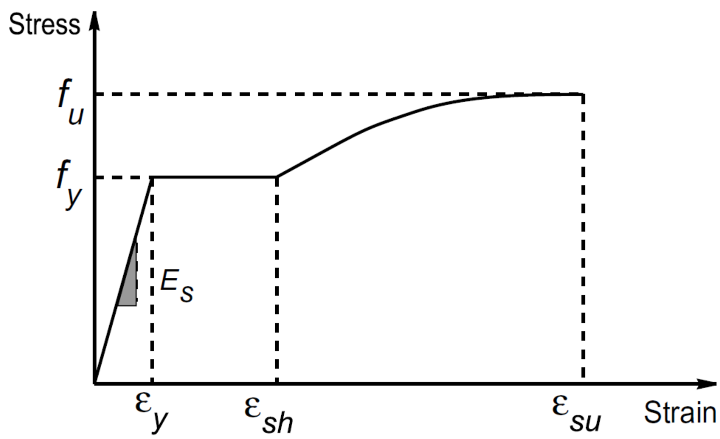

Fig.2.8: Reinforcing steel constitutive model (Lee and Mosalam, 2004)

2.7 MONTE CARLO SIMULATION

Shinozuka et *al.* (1972) reported that the importance of variability of the material properties for estimating the strength of RC structures. Monte Carlo simulation is an oldest computational approach used in several studies of RC sections such as beams and columns.

Reliability of a RC beams were studied by Knappe et *al.* (1975). Strength analyses of RC beam-column members by considering variability of material properties and dimensions were studied by Grant et *al*.(1978), Mirza and MacGregor (1975), and Frangopol et *al*. (1975). Probabilistic estimation of RC frames were done by Chryssanthopoulos et *al*. (1975), Dymiotis et *al*. (1975), Ghobarah and Aly (1975), and Singhal and Kiremidjian (1996) recently proposed systematic ways of evaluating RC framed structures by considering the uncertainty of ground motions and the material variability. Ghobarah and Aly (1998) accounted for uncertainties in member dimensions.

2.8 SUMMARY

This Chapter briefly describe previous studies on distributed inelasticity models, numerical integration, solution strategies and iterative techniques for nonlinear analysis. The present study uses various confinement models and hence a description of confinement models for the concrete such as Mander et al. (1988) etc are also discussed in detail. Third part of the Chapter discussed the studies employing Monte Carlo simulation. A simple and convenient model for nonlinear analysis of RC sections is required for the probabilistic analysis in this study. The classical displacement based - stiffness method used by Lee and Mosalam (2004) is found to be simple and easy to implement for the present study.

3

ELEMENT FORMULATIONS

CHAPTER-3

ELEMENT FORMULATIONS

3.1 GENERAL

Present study uses the distributed inelasticity approach based on the fiber element approach for nonlinear structural analysis. The fiber element can be used with two main formulations, namely, displacement-based (DB) stiffness method, which is the classical finite element formulation, and the force-based flexibility (FB) method. The DB formulation uses displacement shape functions, while the FB formulation uses internal force shape functions. This Chapter discuss the formulations and step wise procedure for the DB and FB method in detail.

3.2 DISPLACEMENT BASED-STIFFNESS METHOD

The displacement based-Stiffness method uses displacement interpolation function. It accounts for axial and transverse displacements of the elements. Linear Lagrangian shape function and cubic hermitian polynomial are the most used shape function for the beam-column elements. The element formulation of stiffness based models are comparatively easy when compared to flexibility based models.

The element force and deformation vectors are given by

$$p= [p_1, p_2, p_3, \ldots p_6]^T \tag{3.1}$$

$$u= [u_1, u_2, u_3, \ldots u_6]^T \tag{3.2}$$

The section force and deformation vector is given by

$$q(x)= [\ N(x), M(x)]^T \tag{3.3}$$

$$V_s(x)= [\ \varepsilon_0(x), \phi(x)\ ]T \tag{3.4}$$

Where N is the axial force, M is the bending moment, ε_0 is axial strain and ϕ is the curvature with respect to section position 'x'. Figure 3.2 depicts the element force and deformations

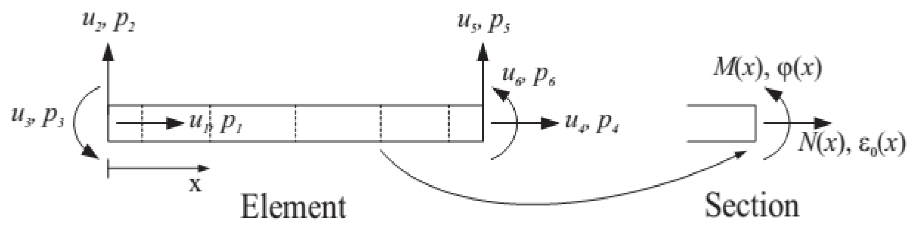

Fig 3.1: Element force and deformations

The strain increment in the 'i[th]' fiber is given by

$$d\varepsilon_i = a_s(y) \times dV_s(x) \qquad (3.5)$$

where $a_s(y) = [1\ -y_i]$ and $dV_s(x) = [\ d\varepsilon_0(x),\ d\phi(x)\ ]^T$

where y_i is the distance between the coordinate reference axis and i^{th} fiber

Section deformation are found from strain deformation relationship that is

$$Vs(x) = [\ B(x) + 0.5 \times G(x)] * u_{n+1} \qquad (3.6)$$

$u_{n+1} = u_n + \Delta u$ is the element deformation vector at the load step n+ 1,

$B(x)$, $G(x)$,C(x) is the strain-deformation transformation matrices

$$G(x)=\begin{bmatrix}1\\0\end{bmatrix}\{C(x)Un+1\}T \times\ C(x) \qquad (3.7)$$

The section stiffness matrix k(x) can be computed as

$$\mathbf{k_s}(x) = \int_{A(x)} \mathbf{a_s}^T(y) E_t(x,y) \mathbf{a_s}(y) dA \qquad (3.8)$$

Where $E_{(x,y)}$ is the tangent stiffness matrix

The section resisting force can also be determined by

$$\mathbf{r_s}(x) = \int_{A(x)} \mathbf{a_s}^T(y) E_t(x,y) dA \qquad (3.9)$$

The element stiffness matrix K_e

$$\mathbf{k_e} = \int_L \mathbf{T}^T(x)\mathbf{k_s}(x)\mathbf{T}(x)dx + \int_L \mathbf{C}^T(x)\mathbf{C}(x)N_s(x)dx \tag{3.10}$$

The element resisting vector re

$$\mathbf{r_e} = \int_L \mathbf{T}^T(x)\mathbf{r_s}(x)dx \tag{3.11}$$

Where

T(x) = B(x) + G(x), Transformation matrix

Ns(x) is a component of rs(x) representing the axial force resultant and L is the element length

For nonlinear analysis, we use $\Delta p = k_e \times \Delta u$

3.3 FLEXIBILITY BASED-FORCE METHOD

The Flexibility based-force method uses force interpolation function. In this formulation, element equilibrium is satisfied in strict sense. The implementation is quite challenging as existing finite element program generally uses stiffness formulations. The element formulation of flexibility based models are comparatively accurate when compared to flexibility based models. b(x) is the force interpolation function used for element state determination.

Element state determination

Step 1: Compute structural displacements and update

$$Ks \times \Delta p = Pext \tag{3.12}$$

$$p=p+\Delta p \tag{3.13}$$

Step 2: Compute Element deformations and update

$$\Delta q=Lele \times \Delta p \tag{3.14}$$

$$q=q+\Delta q \tag{3.15}$$

Lele=Transformation matrix

Step 3: Compute Element force and update

$$\Delta Q = K \times q \tag{3.16}$$

$$Q=Q+\Delta Q \tag{3.17}$$

Step 4: Compute section force and update

$$\Delta Dx=b(x) \times Q \tag{3.18}$$

$$Dx=Dx+\Delta Dx \tag{3.19}$$

Step 5: Compute section force and update

$$\Delta dx=f \times \Delta D(x) \times r(x) \tag{3.20}$$

$$dx=dx+\Delta dx \tag{3.21}$$

Step 6: Compute fiber stresses and tangent modulus from constitutive stress –strain curve

Step 7: Compute new section flexibility matrix

$$k(x)=\begin{bmatrix} \sum_i E_i A_i & -\sum_i E_i A_i y_i \\ -\sum_i E_i A_i y_i & \sum_i E_i A_i y_i^2 \end{bmatrix} \tag{3.22}$$

Where $f=[k(x)]^{-1}$

Step 8: Compute section resisting forces

$$D_R(x)=\begin{bmatrix} \sum \sigma_i A_i \\ -\sum \sigma_i A_i y_i \end{bmatrix} \tag{3.23}$$

Step 9: Compute Unbalanced force

$$D_u=D_x-DR(x) \tag{3.24}$$

Step 10: Compute residual section deformation

$$r(x)=f(x) \times Du \tag{3.25}$$

Step 11: Compute Flexibility matrix

$$K^i = \int_0^L B^T(x) k^i(x) B(x) \tag{3.26}$$

Where Flexibility,F=[K]$^{-1}$

Step 12: Check for convergence

Case 1: If Q_i=Q_j , K_i=K_j then the element converged

Case 2: If element not converged

$$\mathrm{s} = \int_0^L \mathrm{b}^T(x) \cdot \mathrm{r}(x)\,dx \tag{3.27}$$

and Δq=-s

Step 12: Compute structure stiffness and resisting forces

$$P_R = L_{ele}^T \times Q_{ele} \tag{3.28}$$

$$K_s = L_{ele}^T \times K_{ele} \times L_{ele}^T \tag{3.29}$$

3.1 SUMMARY

This Chapter presents the formulation and step wise procedure for the both the DB and FB method. This steps are implemented in the MATLAB 2012b for nonlinear analysis of RC sections and probabilistic analysis further.

4

IMPLEMENTATION OF FIBER ELEMENT FOR PROBABILISTIC ANALYSIS

CHAPTER-4

IMPLEMENTATION OF FIBER ELEMENT FOR PROBABILISTIC ANALYSIS

4.1 GENERAL

First part of this Chapter presents a flow chart that explains both the stiffness based method (DBM) and flexibility based methods (FBM) using fiber formulation for nonlinear analysis. Examples of a RC column section involving material nonlinearity is considered and analyses are conducted. Second part of this Chapter illustrate the results of these analyses and its comparison with exact results. Constitutive stress strain relations for concrete and steel reinforcement are incorporated in fiber formulation to obtain the nonlinear responses. Probabilistic analysis incorporating uncertainties in the material and geometric parameters properties of RC section, is carried out in the last part of this chapter.

4.2 METHODOLOGY

The flow chart of the different phases of the presented such as, comparison of DBM and FBM using linear static analysis, convergence study, discussion of numerical and direct integration method, nonlinear analysis using different confinement models, and a probabilistic analysis using the implemented model in MATLAB 2012b is displayed in Fig.4.1.

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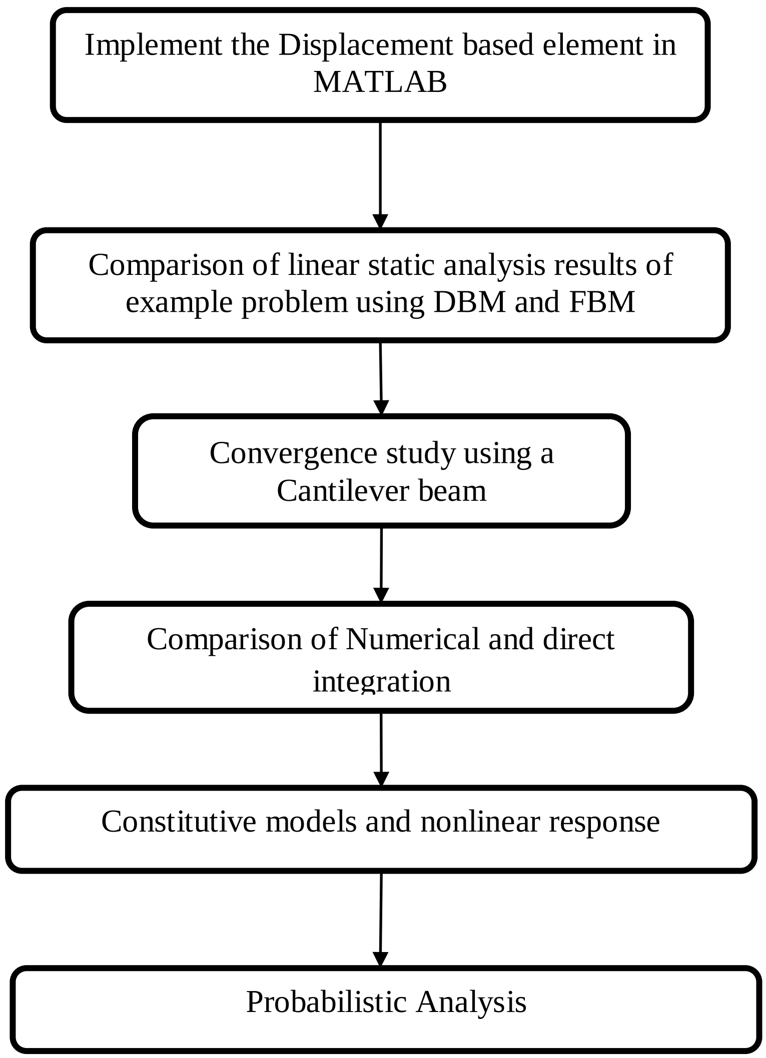


Fig. 4.1: Flow chart showing the present study

4.3 STEPWISE PROCEDURE FOR STIFFNESS AND FLEXIBILITY BASED METHOD The formulation of the stiffness and flexibility based fiber element is explained in Chapter 3. To have a more clarity in the steps involved in the implementation in MATLAB 2012b, a flow chart is provided for stiffness and flexibility based methods respectively in the Figs. 4.2a and Figs. 4.2b.

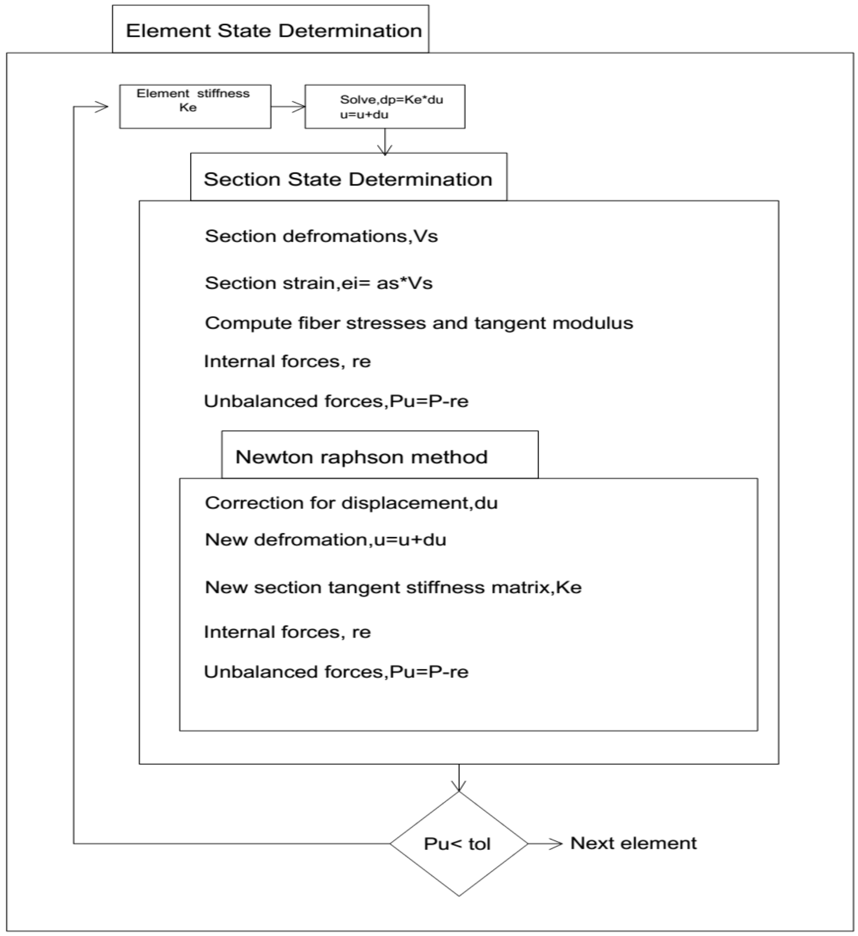


Fig 4.2a: Flow chart of state determination of Stiffness based method

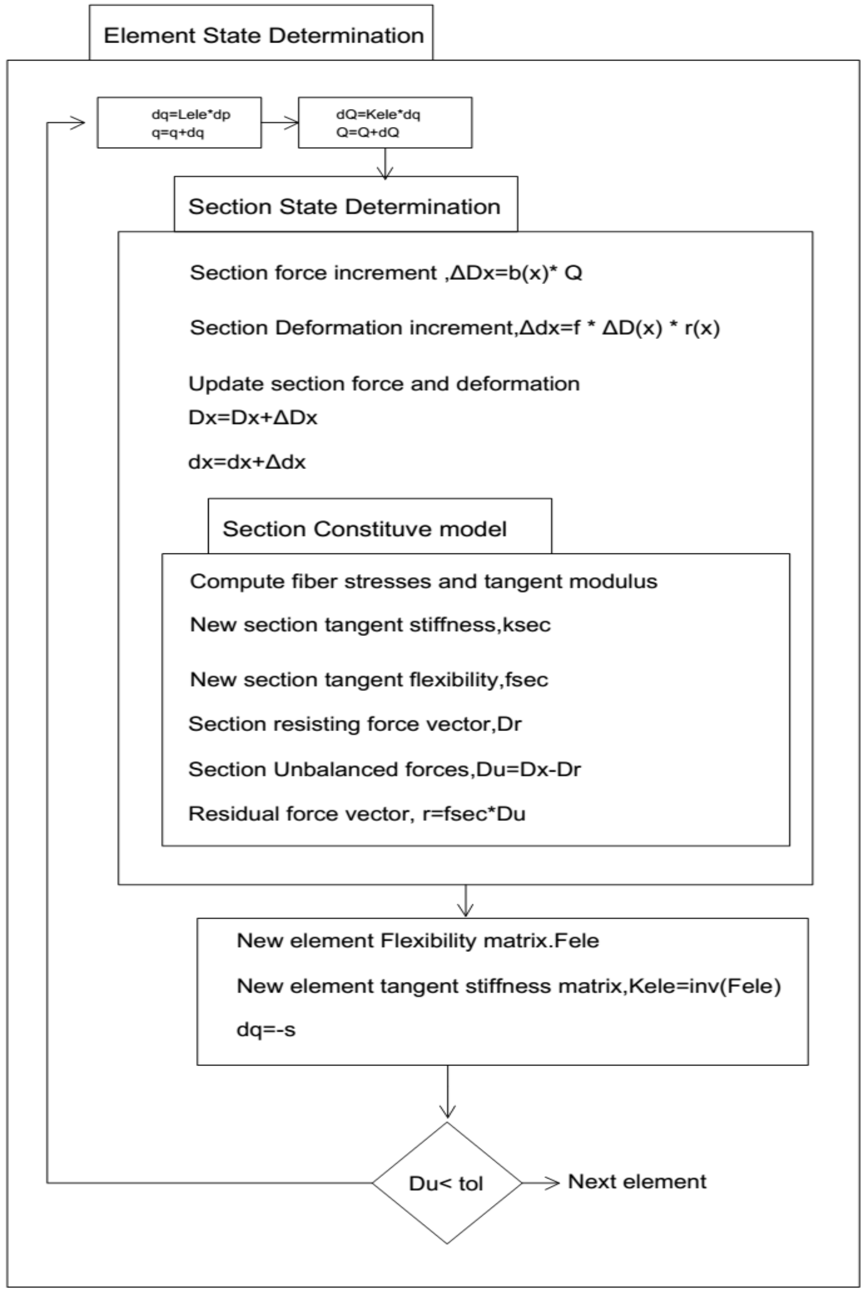


Fig 4.2b: Flow chart of state determination of Flexibility based method

4.4 COMPARISON OF LINEAR ANALYSIS USING DB AND FB FORMULATION

4.4.1 Elastic Column with Axial Force

An elastic column having 2m length and a cross section of 100mm x 300mm is chosen. Axial compressive load is considered at the free end. The details of the column, cross section and the constitutive linear relationship of the fiber element is provided in the Fig.4.3. The cross section is discretized as fiber element. The details of fiber discretization followed for both DBM and FBM are presented in the Table 4.1. Stiffness matrices at the section level and global level are computed as per both the DBM and FBM. The displacement is incrementally applied till a target displacement and the force and the displacement at the free end is monitored in each step for both the type of formulations. A comparison is done for both displacement based method and flexibility based method by using fiber element method. The force versus displacement responses from both DBM and FBM are illustrated in the Fig. 4.4. It can be seen the linear responses from both the approaches are matching. Hence this implemented model can be used for further parametric studies.

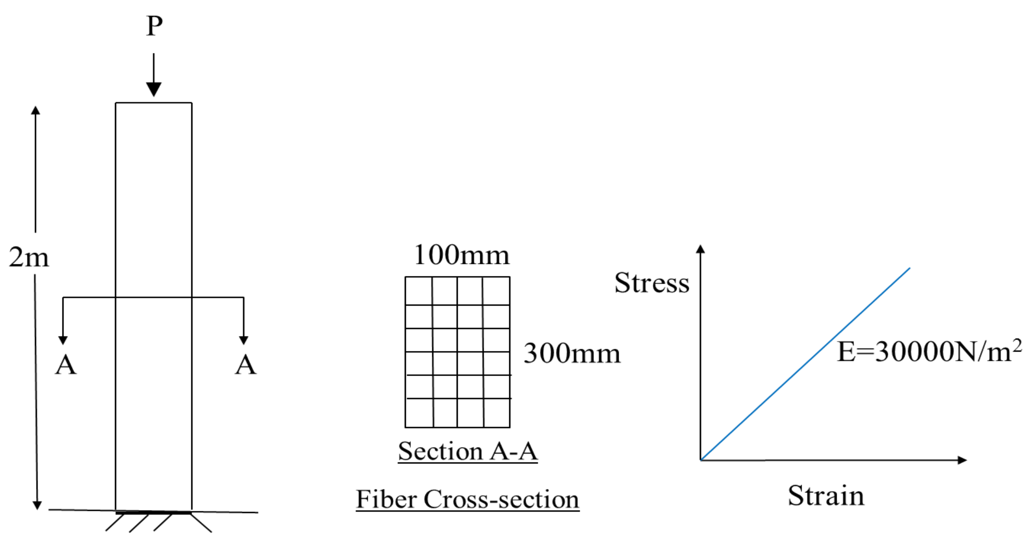


Fig 4.3: Homogenous column subjected to axial compression

Table 4.1: Parameters for number of fiber elements

No. of fiber in y direction	20
No. of fiber in z direction	20

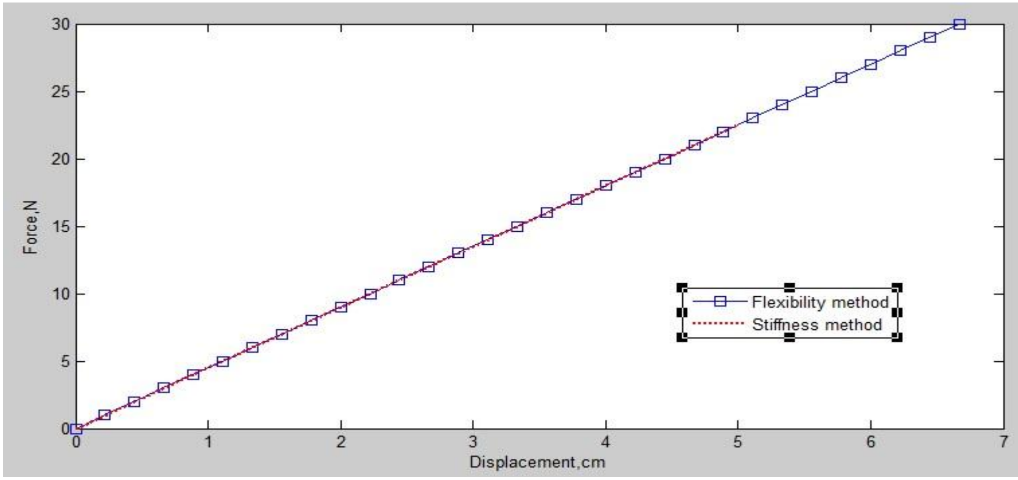


Fig 4.4: Force Displacement response for Elastic Column using DBM and FBM

4.5 CONVERGENCE STUDY - DIRECT INTERATION

The number of fiber elements in a section may influence the responses. In order to estimate the optimum number of fiber elements for reasonably accurate results, a convergence study is required to be conducted. A cantilever beam with 400 mm length with a cross section of 20mm x 20mm is chosen a load of 100N is applied at the free end. The linear constitutive relationship is considered for all the fiber elements as shown in the Fig. 4.4. The number of fibers (in both width and depth directions) and number of integration sections are treated as variables. The linear static analysis is conducted to find the displacement at free end for different number of fibers. The displacement at free end obtained for each case is tabulated in the Table 4.2. The deflection at free versus number of fibers is plotted in the Fig. 4.6. The percentage error versus number of fibers is expressed graphically in Fig. 4.7. It can be seen that the number of fibers in the cross section increases to 200 x 200 and the number of integration section sections to 400, and the deflection tends to converge to 6.40 mm. About 400 number of sections is required to have convergence in the case of direct integration method.

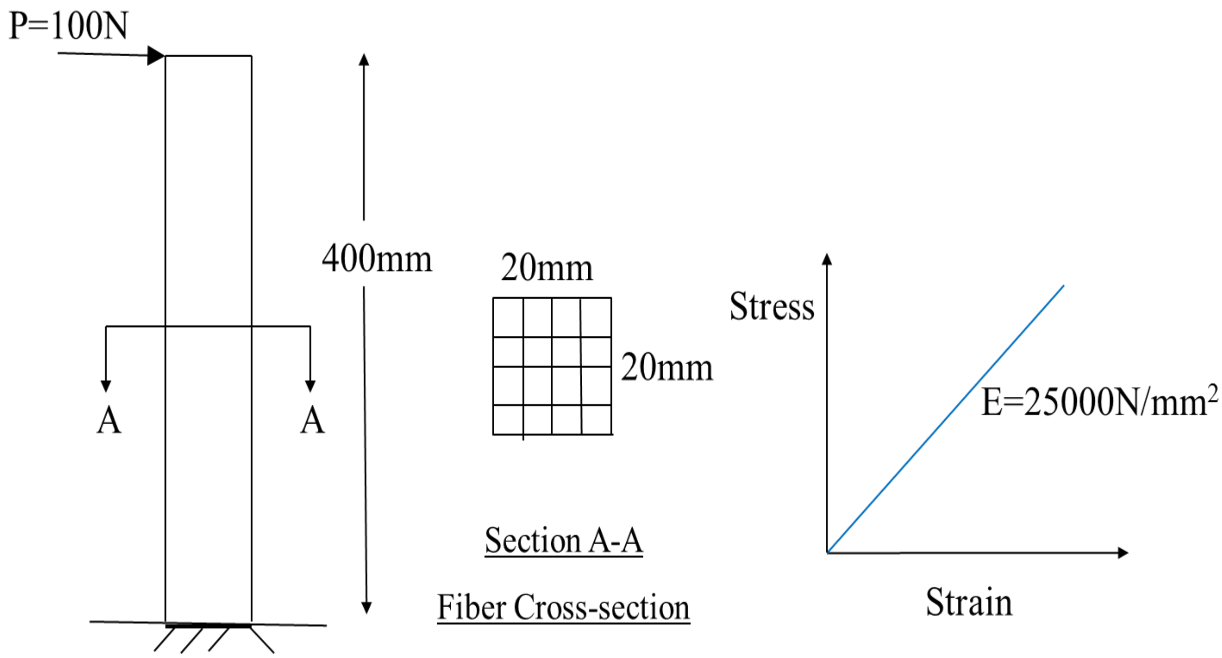


Fig 4.5: Cantilever beam with point load at the end

Table 4.2: Convergence study - Direct integration

Fibers	Number of Integration Section (Nos.)	Deflection using DBM (mm)	Exact deflection (mm)	Error (%)	Execution time (s)
5 x 5	25	6.77	6.40	5.75	16
10 x 10	50	6.56	6.40	2.50	16
50 x 50	100	6.50	6.40	1.56	16
100 x 100	200	6.45	6.40	0.78	17
200 x 200	400	6.40	6.40	0.05	28

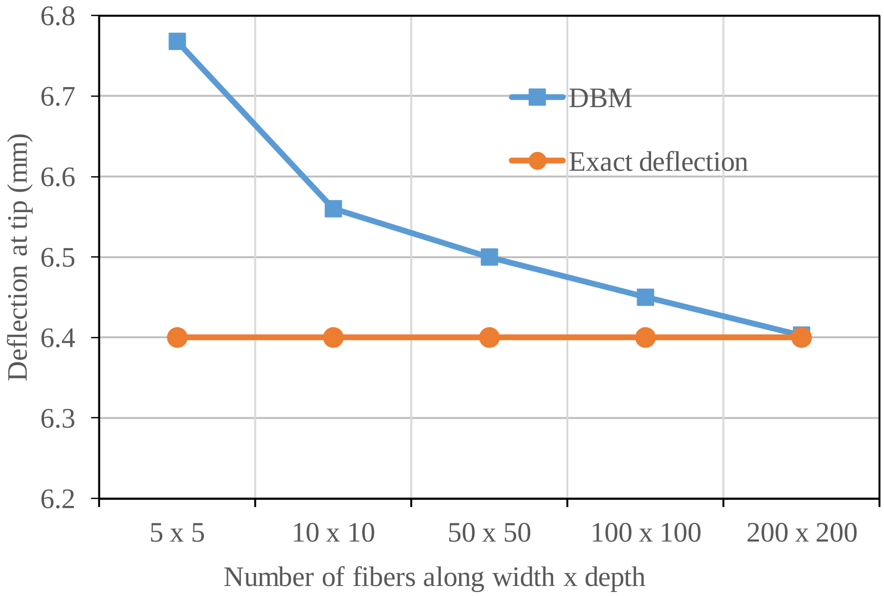


Fig 4.6: Deflection comparison for direct integration

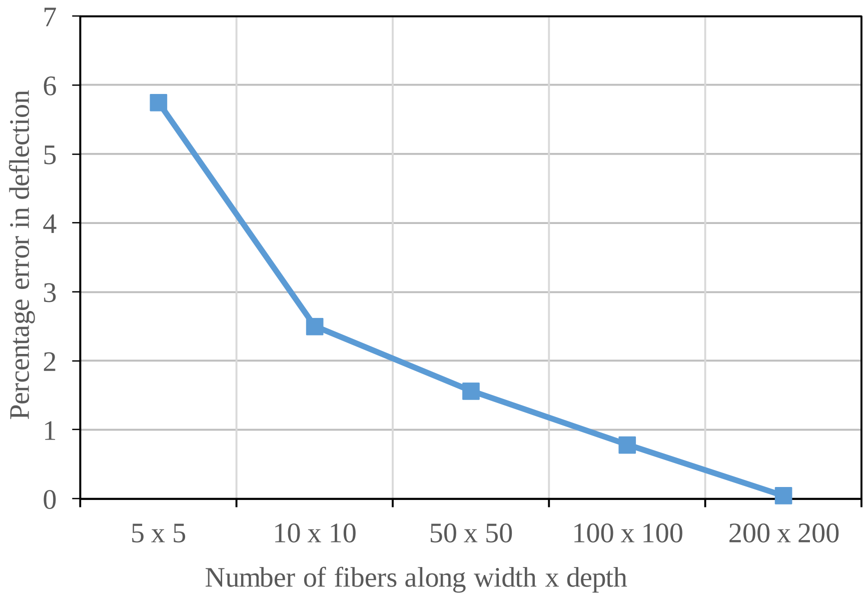


Fig 4.7: Percentage error comparison for direct integration

4.6 CONVERGENCE STUDY - NUMERICAL INTEGRATION

In order to estimate the optimum number of fiber elements for the numerical integration using Gauss Lobatto, the same cantilever beam problem as that of the previous convergence study is considered. The linear constitutive relationship considered is the same as shown in the Fig. 4.5. The number of fibers (in both width and depth directions) and number of integration points are treated as variables. The linear static analysis is conducted to find the displacement at free end for different number of fibers. The displacement at free end obtained for each case is tabulated in the Table 4.3. The deflection at free end versus number of fibers is plotted in the Fig. 4.8. The percentage error versus number of fibers is expressed graphically in Fig. 4.9. It can be seen that the number of fibers in the cross section increases to 50 x 50 and the number of integration section sections to 3, and the deflection tends to converge to 6.40 mm. It can be seen that only about 5 number of sections (instead of 400 in the case of direct integration) is required to have convergence in the case of direct integration method.

Table 4.3: Numerical integration: Fiber cross sections-Deflections and errors

Fibers	Number of Integration Section (Nos.)	Deflection using DBM (mm)	Exact deflection (mm)	Error (%)	Execution time (s)
5 x 5	3	6.66	6.40	4.16	2
10 x 10	3	6.46	6.40	1.00	2
50 x 50	3	6.41	6.40	0.04	2
100 x 100	5	6.40	6.40	0	2
200 x 200	5	6.40	6.40	0	2

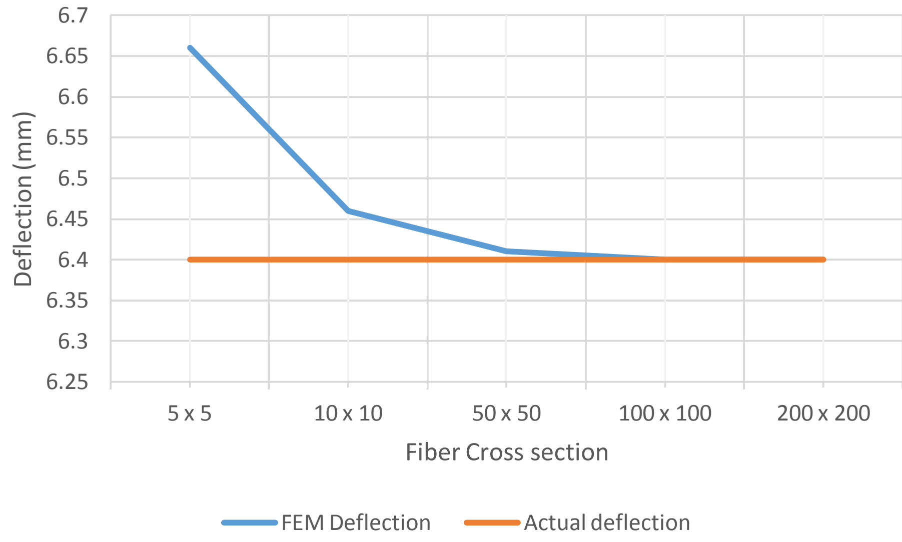


Fig4.8: Deflection comparison for Numerical Integration

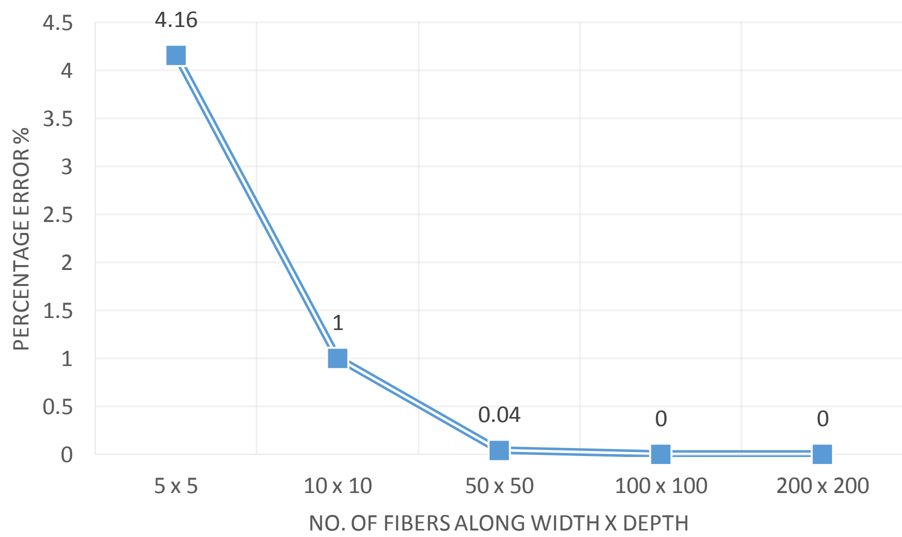


Fig 4.9: Error comparison for Numerical Integration

4.7 COMPARISON BETWEEN DIRECT AND NUMERICAL INTEGRATION

On order to have an understanding of the relative advantages of numerical integration over the direct integration the computational time and number of integration sections for both the methods are compared as shown in bar diagram in Fig. 4.10 and 4.11 respectively. It can be observed that the numerical integration is found to be more efficient due to computational efficiency.

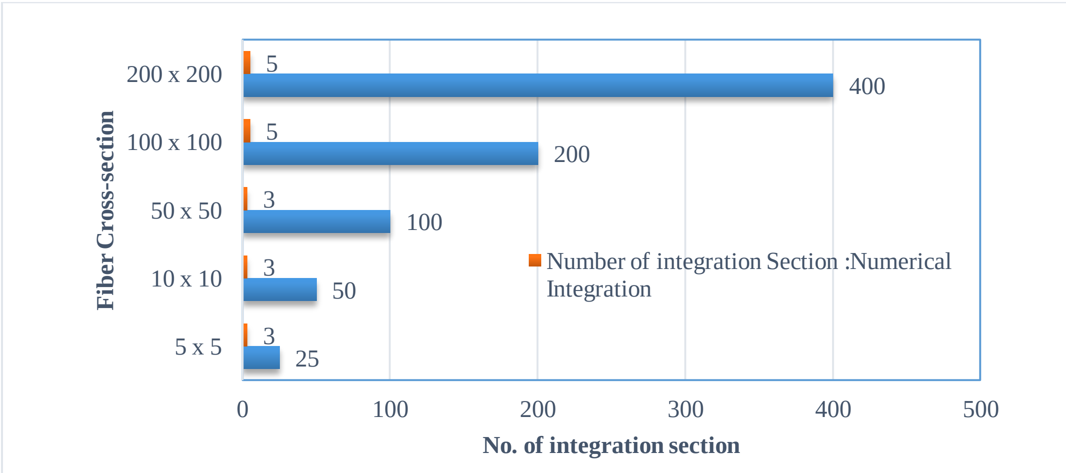


Fig 4.10: Number of fiber comparison for both integrations

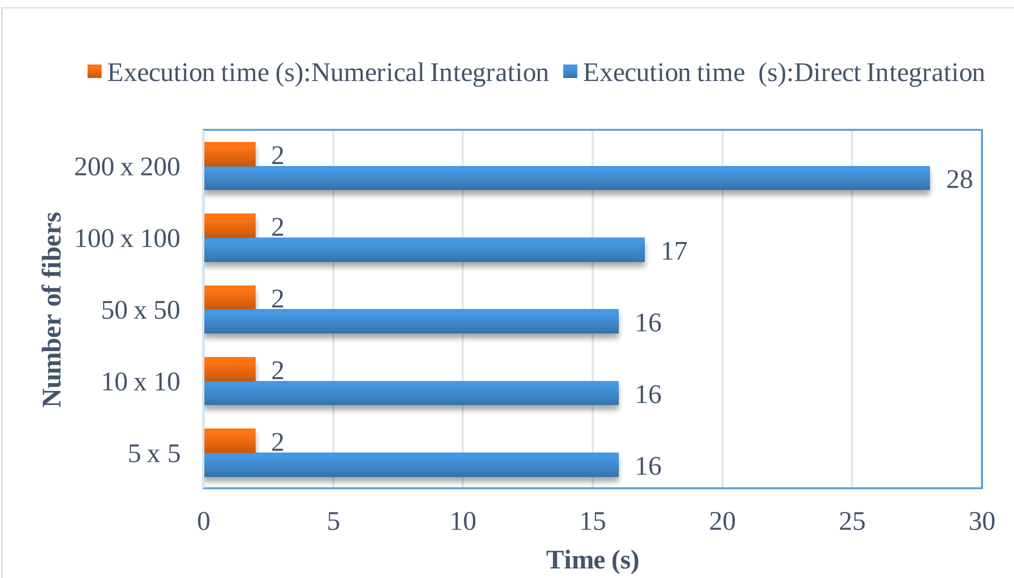


Fig 4.11: Execution time comparison for both integrations

4.8 CONFINEMENT MODELS IN NON-LINEAR RESPONSE

The effect of confinement in the concrete play a major role in the nonlinear response of frames. To study the effect of confinement models in the response, the fiber elements are considered to have constitutive relationship of confinement concrete in the core region (region over which the concrete is confined due to the transverse reinforcements) and unconfined region (region over which there is no confinement for the concrete) in the cover concrete.

4.9 RC COLUMN WITH AXIAL COMPRESSION

An RC column having dimensions 350 mm x 350mm with reinforcement detailing as shown in Fig. 4.15 is considered. To have the prominent effect of confinement, the transverse reinforcement in the column is assumed as high as 16mm dia @ 85 mm c/c. The confined and unconfined stress strain curves obtained for the above transverse reinforcement is calculated as per the expressions for various confinement models namely Mander et *al*. (1988), Modified Kent and Park model (1982), IS 456(2000) as given in the Chapter 2.

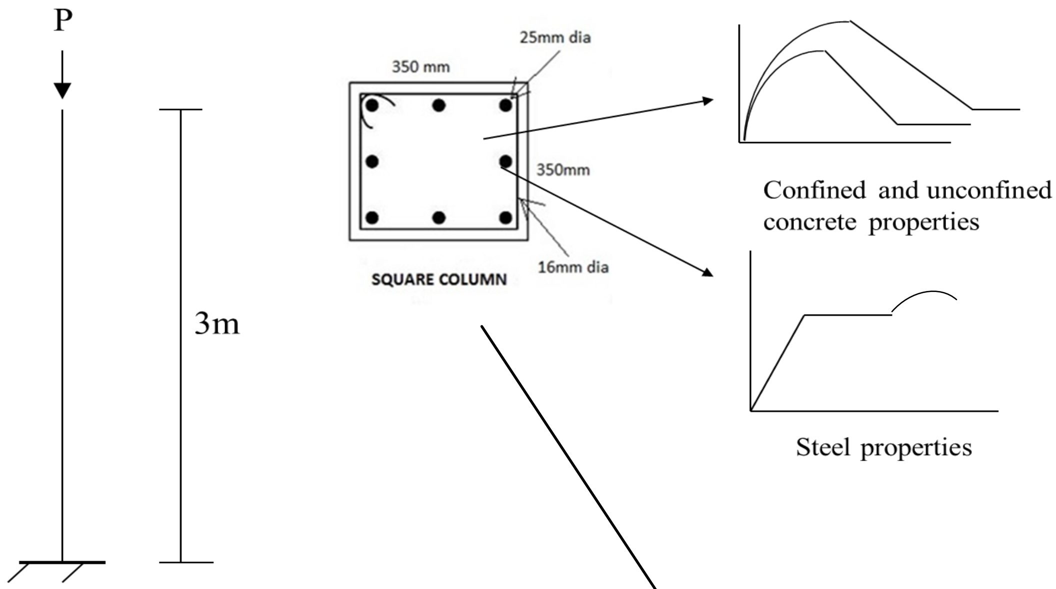
Nonlinear model of the cross section is developed using the implemented displacement based fiber element method. The RC column cross section is discretized to number of fibers, and for each fiber, depending on its location whether in core region or cover region, the corresponding confined and unconfined constitutive relations are used. The fibers at the locations of main reinforcement is modelled using the constitutive relation for the steel. Nonlinear analysis is conducted using displacement control method to obtain the force and displacement responses at the free end. The force –displacement curves obtained using Mander et *al.* (1988) is shown in Fig. 4.13(a). Fig. 4.16(b) and (c) shows the uniaxial constitute relations used for fibers at confined and unconfined regions respectively. The maximum axial force for this model is obtained as about 5000kN.

To study the effect of not considering confinement in core region, the above RC section is remodelled by assuming the unconfined stress strain curve for the core region. The corresponding axial force versus displacement is shown in the Fig. 4.14a. The stress strain relationship used for the concrete and steel is also shown in Fig. 4.14b and 4.14c.

IS 456 (2000) recommend stress strain curve of concrete to be considered for the limit state design of RC sections. Axial force versus displacement relationship is obtained as shown in the Fig. 4.15a. The stress strain relations for the steel and concrete are shown in Figs. 4.15b and 4.15c.

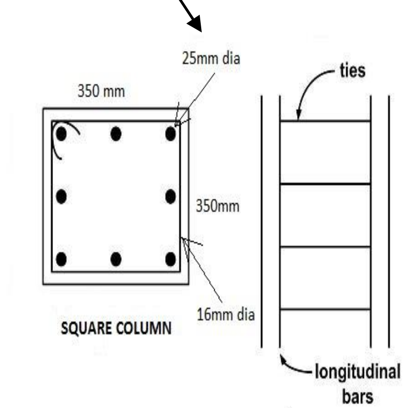
Force displacement relationship for the same RC cross section is obtained using the confinement model as per Kent and Park (1982). The force versus displacement responses and the corresponding stress strain curves used as shown in Fig. 4.16a, 4.16b and 4.16c respectively.

A comparison of axial force versus displacement curves for all the four case discussed in this section is shown in Fig. 4.17. It can be seen the Mander et al. (1988) predicts higher values for strength compared to other models. This is due to high value of confinement factor values. The maximum compressive strain by IS 456 (2000) is 0.0035. It can be seen that IS 456(2000) model has less ductility when compared to other models due the relative low value of maximum strain.



Cross section details of Column

width	350	mm
depth	350	mm
clear cover	40	mm
Main steel dia	25	mm
Stirrup dia	16	mm
Area of stirrups along X	402	mm^2
core width	270	mm
Area of stirrups along Y	402	mm^2
spacing	85	mm
Reinforcement ratio, X direction, ρsx	0.02	mm^2
Reinforcement ratio, Y direction, ρsy	0.02	mm^2
Reinforcement ratio ρs	0.04	mm^2
Characteristic_compressive strength,fck	25	MPa
f´co	18.8	MPa
Yield Strength, fy	415	MPa



No. of fiber in y direction	50
No. of fiber in z direction	50

Fig 4.12: RC Column subjected to different stress strain models

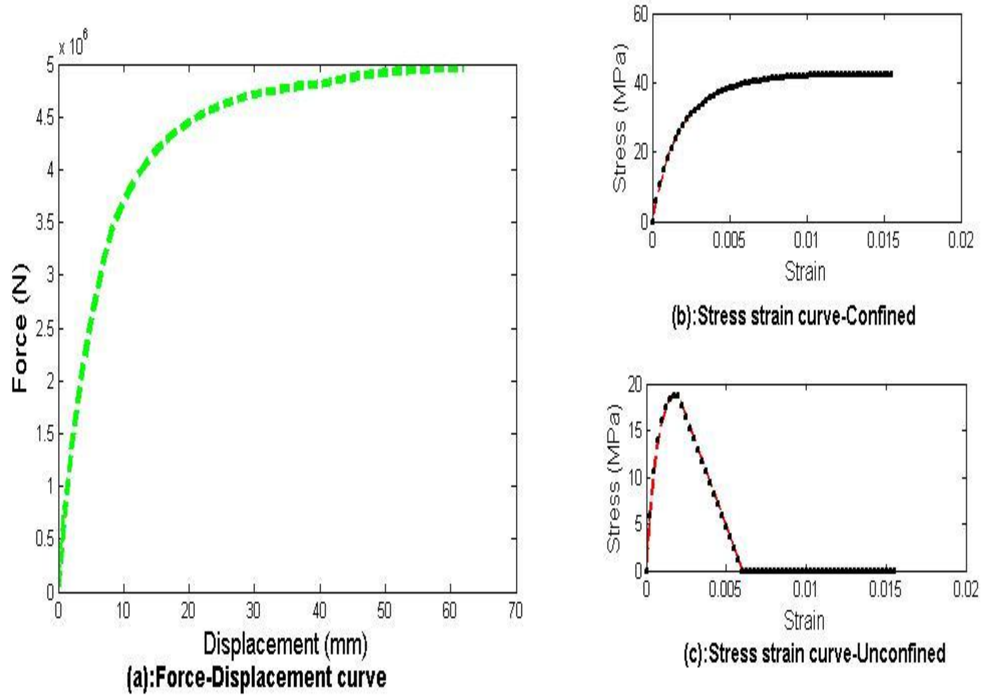


Fig 4.13: RCC column modelled with confined and unconfined sections using Mander et. *al,* (1988)

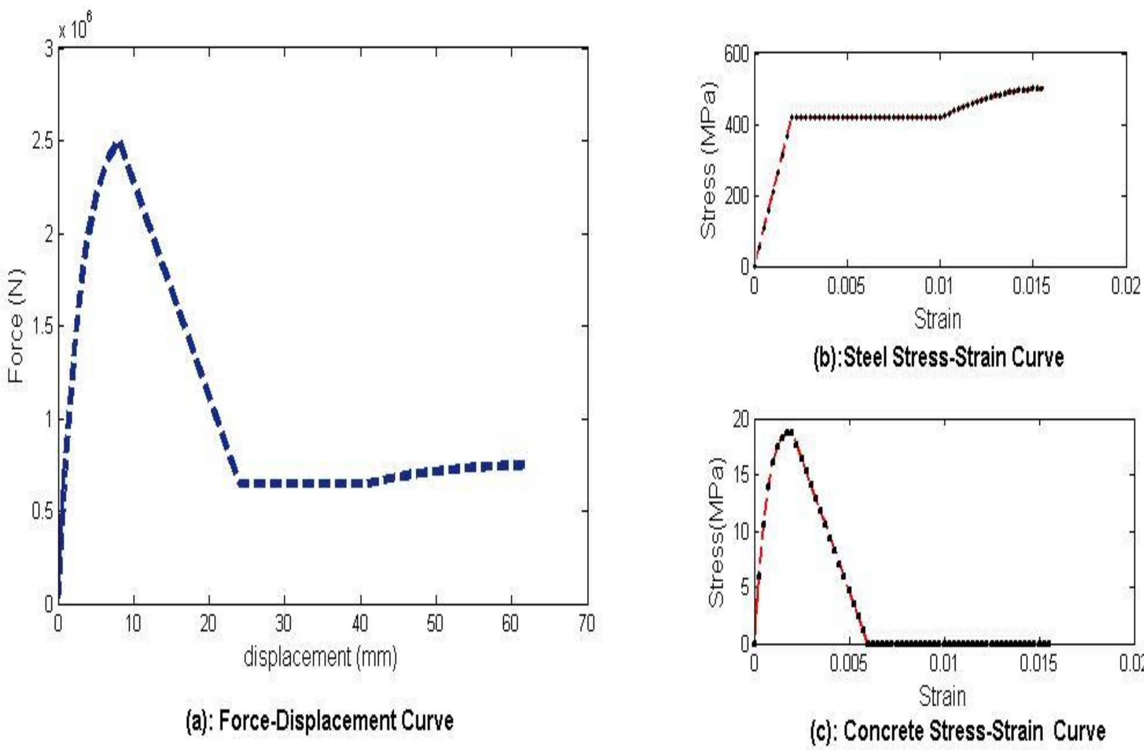


Fig 4.14: RCC column, entire section modelled as unconfined concrete as per Mander et *al.* (1988)

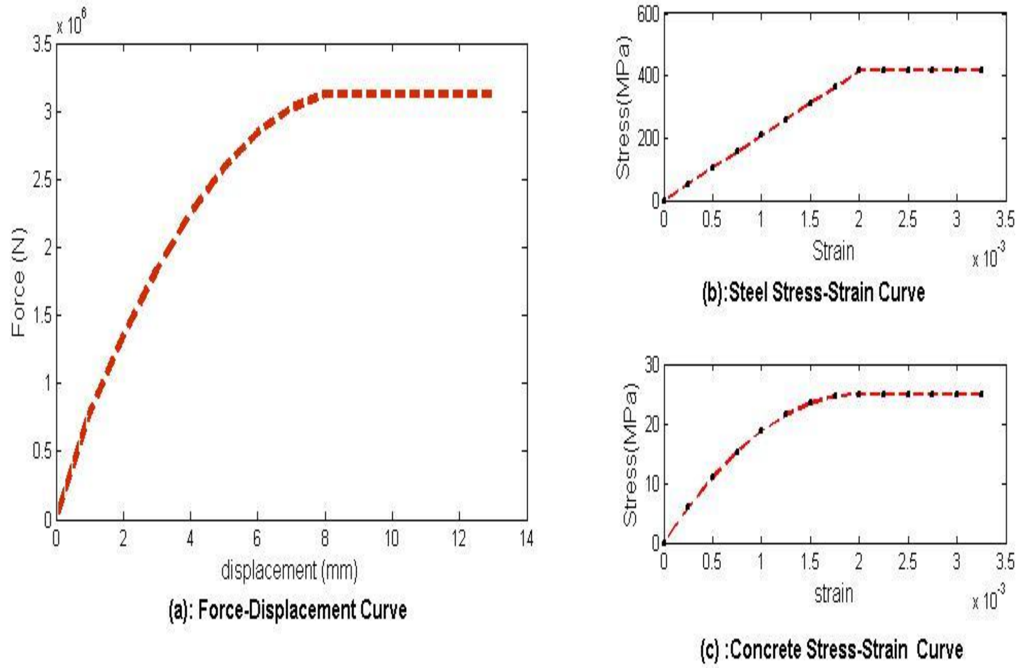


Fig 4.15: RCC column with concrete stress -strain using IS 456(2000) model

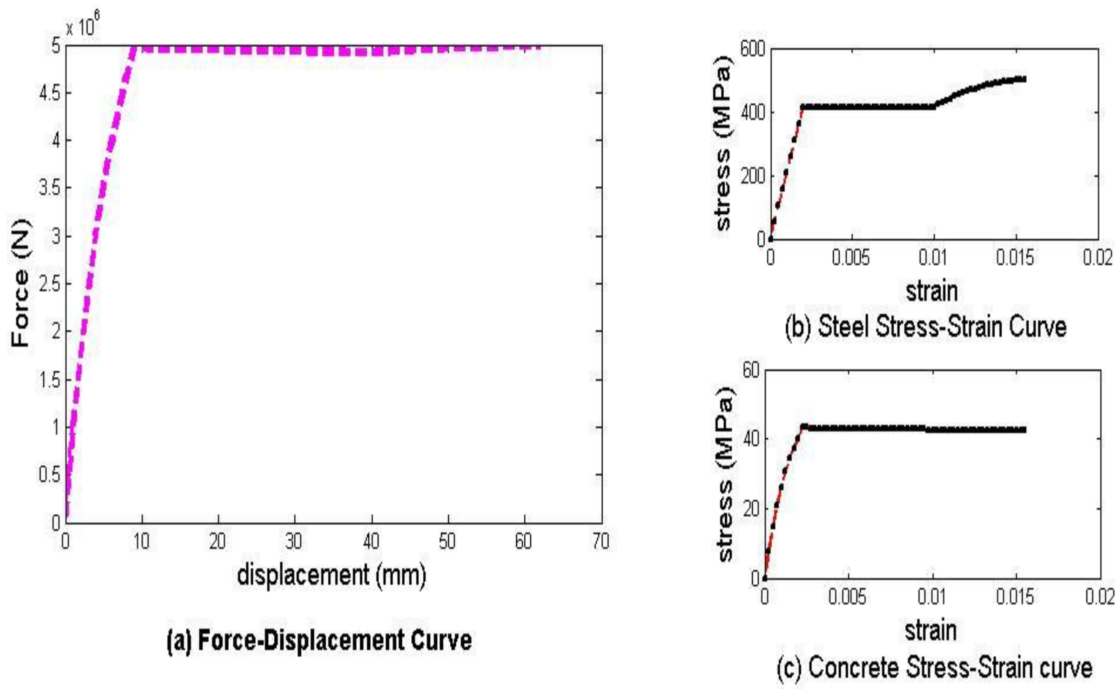


Fig 4.16: Nonlinear Response of RCC column with Modified Kent and Park (1982) model

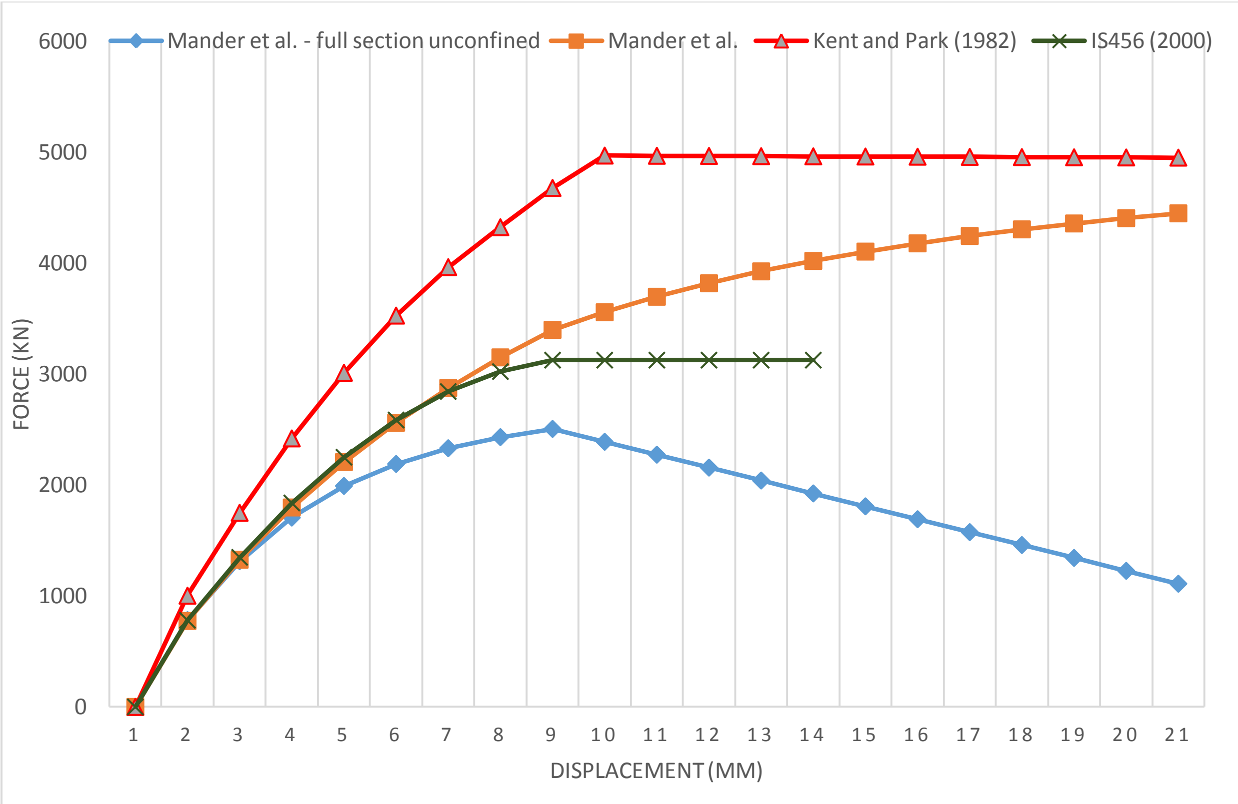


Fig.4.17: Comparison of force versus deformation of RC column with axial loading using different confinement models for concrete

4.10 PROBABILISTIC STUDY OF NONLINEAR RESPONSE

The methodology followed for the probabilistic analysis of RC sections is shown below. The present study consider the variables, compressive strength of concrete, yield strength of steel, initial tangent modulus of steel and concrete and dimensions of the member as random variables. The cross section and member shown in Fig.4.12 is considered for the probability analysis. Each random variable is treated as uncorrelated and 1000 samples have taken for the monte carlo simulation. The probability distributions of each variable considered are lognormal and the statistical details of the variables are given in the Table 4.5. The probability distributions of each random input variables that define the computational models is presented in the Figs. 4.20 to 4.26.

For each of the random samples of the above variables, displacement based finite element models are developed to conduct nonlinear analysis. The nonlinear response curves from probabilitistic analysis, histograms for peak axial strength and its probability distributions are found out. The Figs. 4.27, 4.28, 4.29 and 4.30 shows the probability distributions for peak axial strength for all the confinement conditions, namely, namely Mander et *al*. (1988), Modified Kent and Park model (1982), IS 456(2000). The mean and C.O.V. for the peak strength for each case is shown in the respective Figs. The coefficient of variations of all the input random variables and the peak axial strength response is summarized in a graphical form in Fig. 4.32. It can be seen that the C.O.V of the peak strength varies between 5.8 to 9.3%, when the C.O.V of input random variables is about 0.5 to 15%.

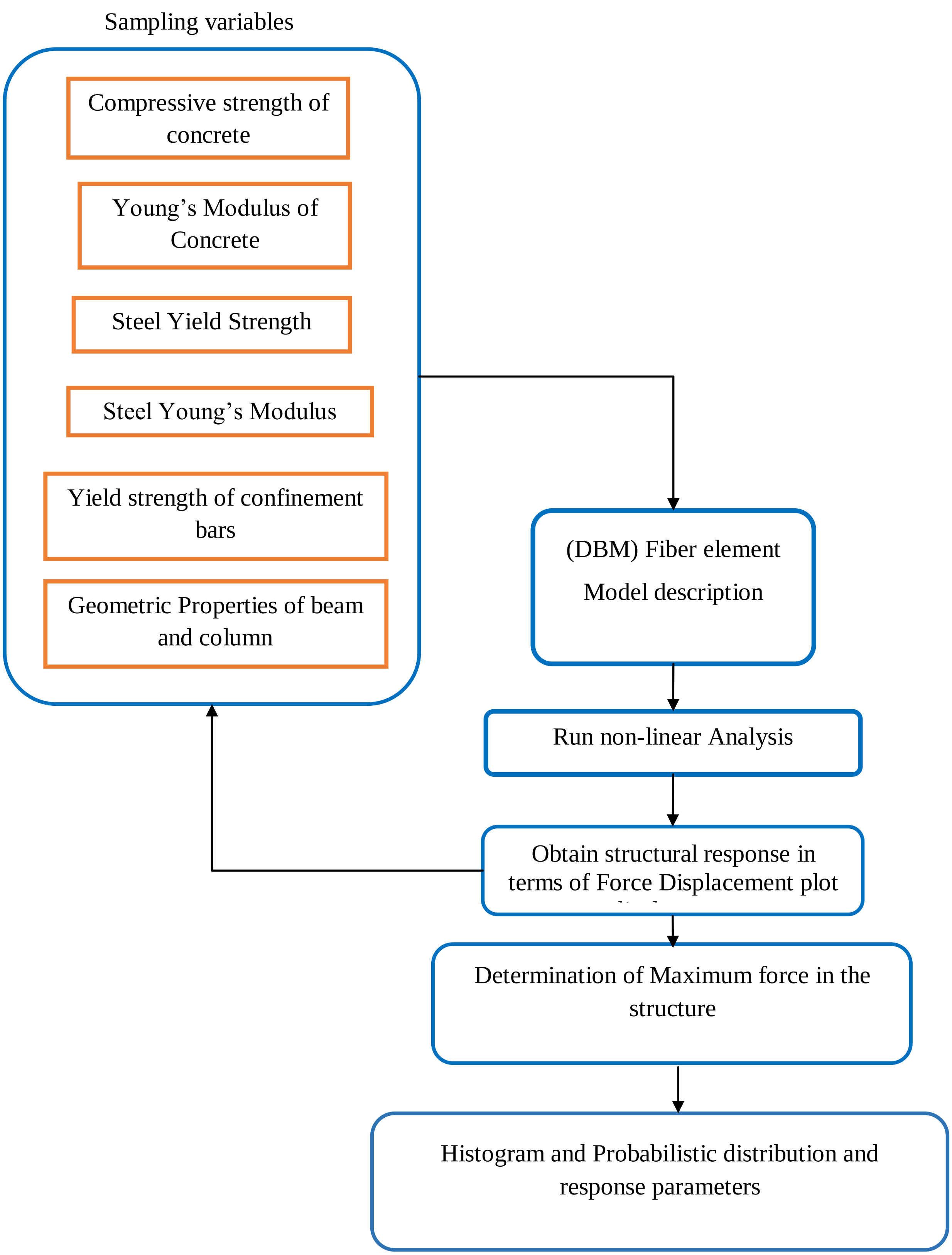


Fig 4.18: Flow chart of the probabilistic fiber element approach

Table 4.4: Parameters and distributions used for generation of random variables

Random variable	Parameters	Probability distribution function	Mean	C.O.V (%)	Reference
Characteristic compressive strength	f_{ck}	Lognormal	25MPa	15	Devandiran *et.al.*(2013)
Yield Strength of steel	f_y	Lognormal	415MPa	7.6	Devandiran et.al.(2013)
Column	Width	Lognormal	350mm	0.5	Assumed
	Depth		350mm	0.5	
	Length		4000mm	0.5	
Initial Tangent Modulus of Concrete	E_c	Lognormal	25000MPa	12	Lee and Mosalam (2004)
Initial Tangent Modulus of Steel	E_s		200000MPa	7.6	Devandiran et.al.(2013)

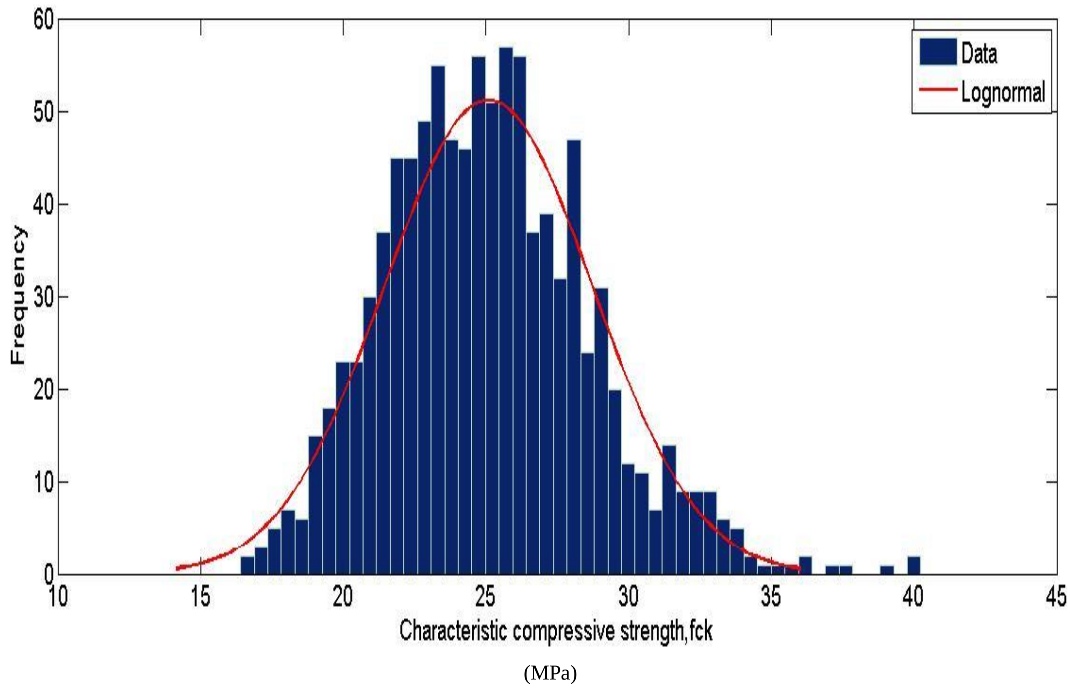
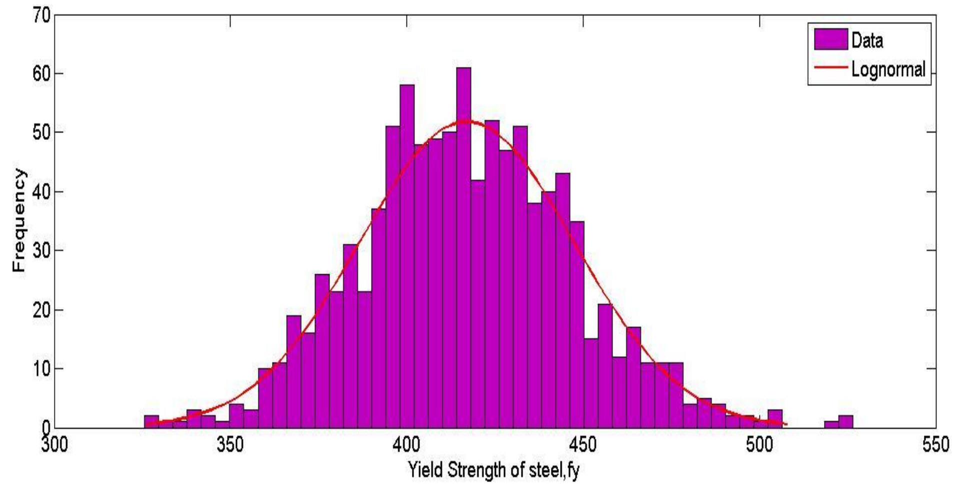
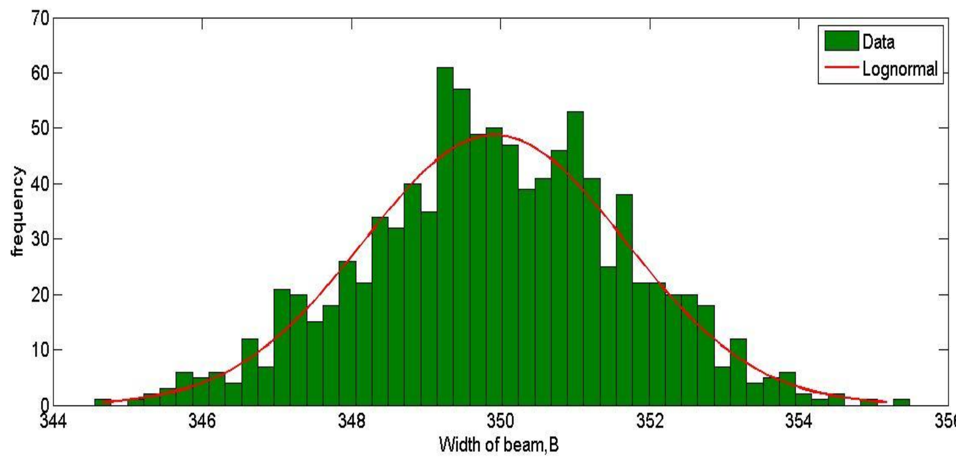


Fig 4.19: Probability distribution of Compressive strength of concrete.



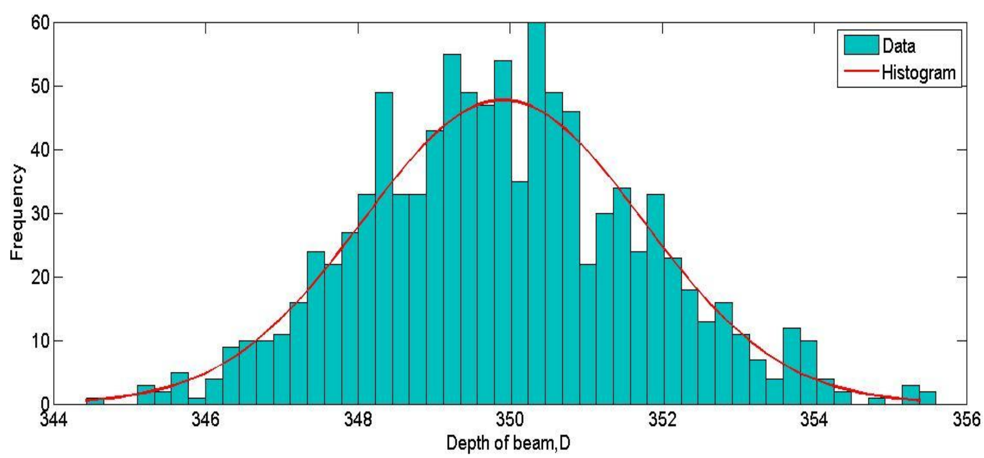
(MPa)

Fig 4.20 Probability distribution of yield strength of steel



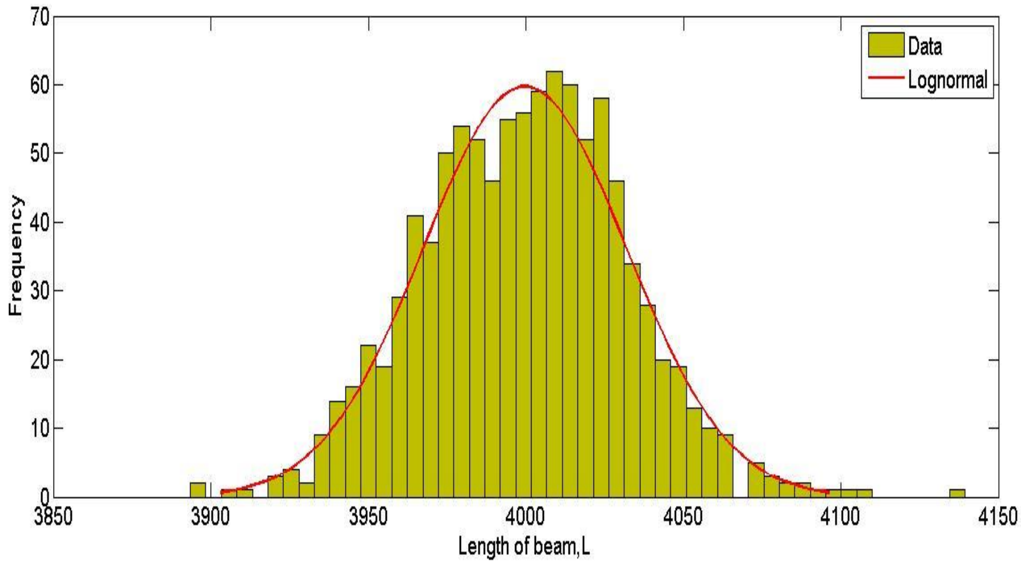
(mm)

Fig 4 21: Probability distribution of width of beam



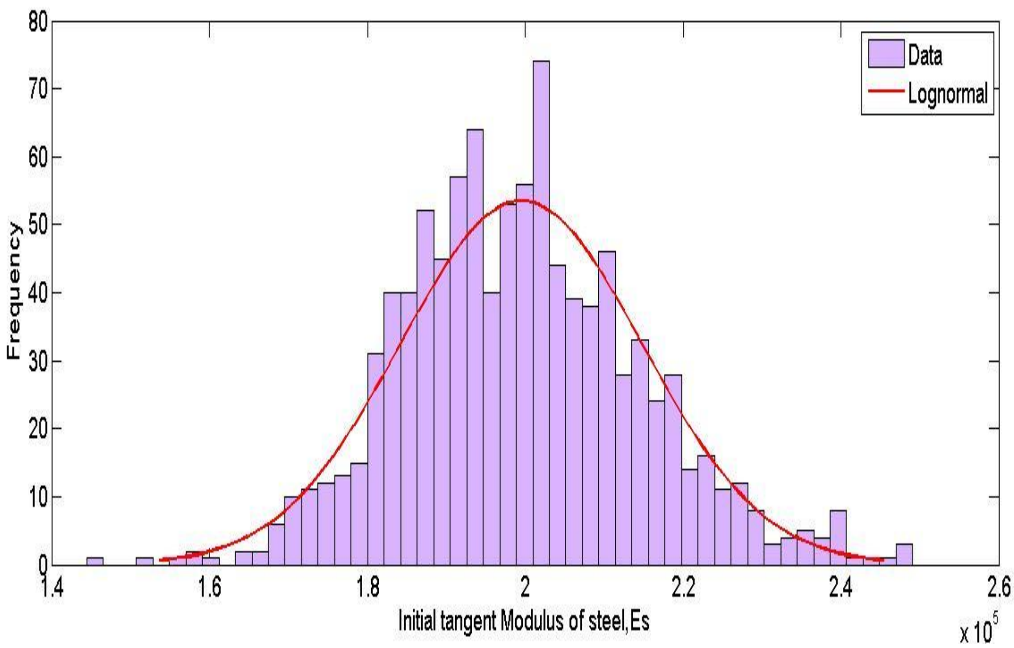
(mm)

Fig 4 22: Probability distribution of depth of beam



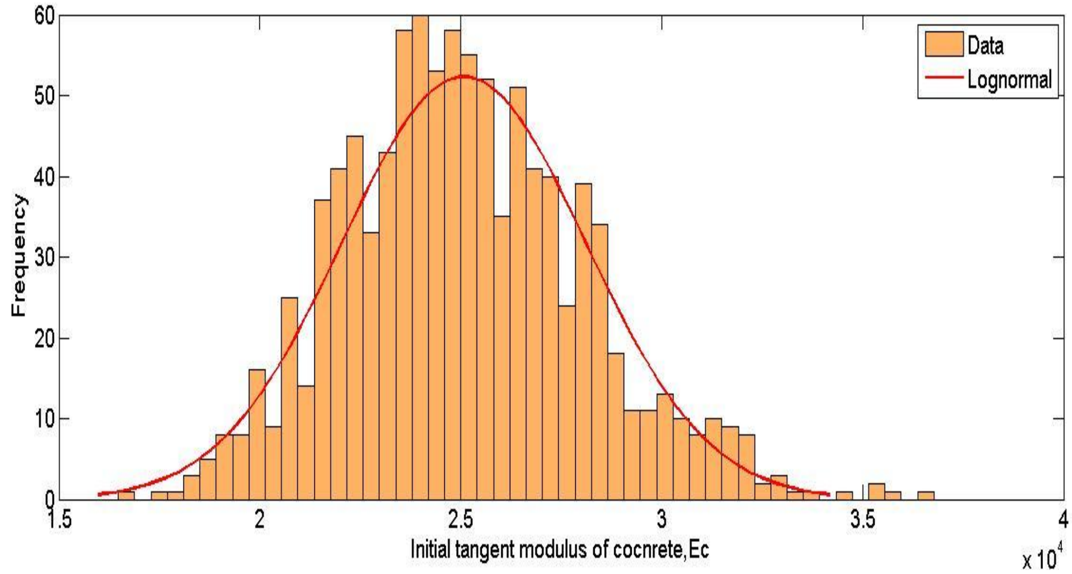
(mm)

Fig 4 23 Probability distribution of length of beam



(mm)

Fig 4 24: Probability distribution of Initial tangent modulus of concrete.



(MPa)

Fig 4 25: Probability distribution of initial tangent modulus of concrete.

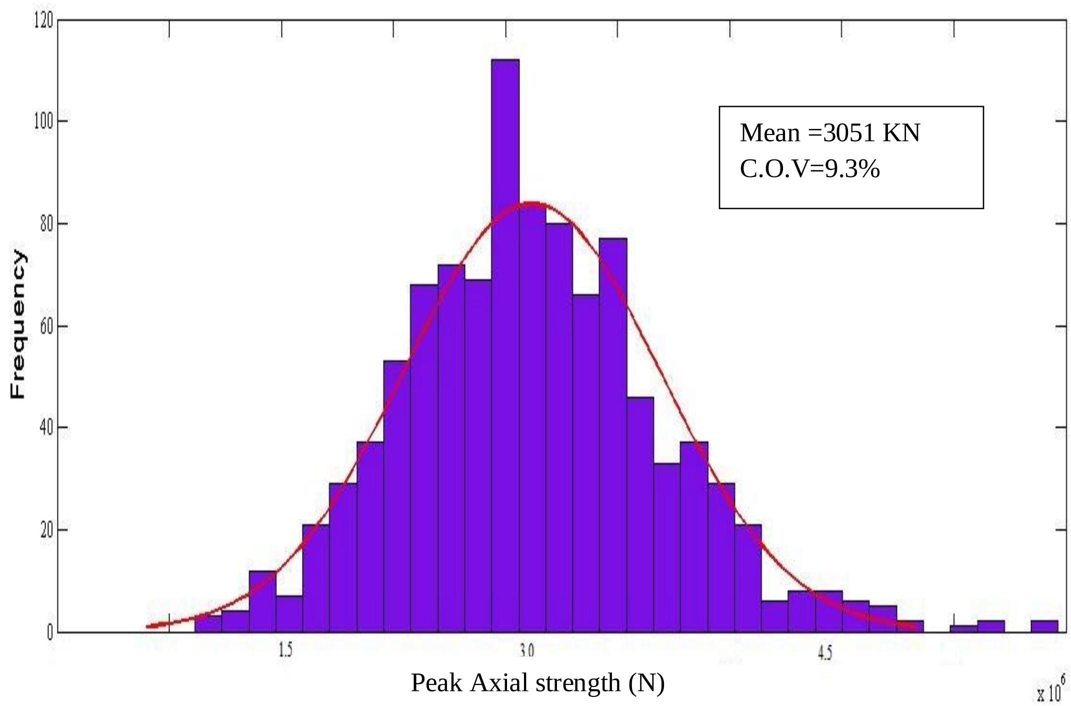


Fig 4.26: Histogram for peak axial strength using IS 456 (2000) model

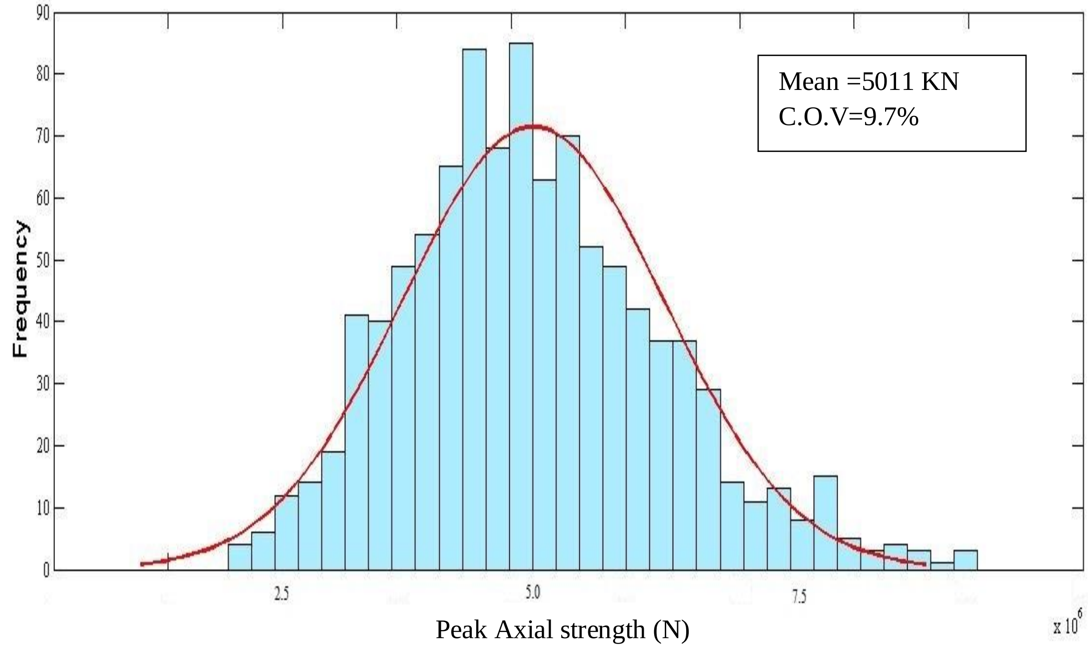


Fig 4.27: Histogram for peak axial strength for Modified Kent and Park (1982)

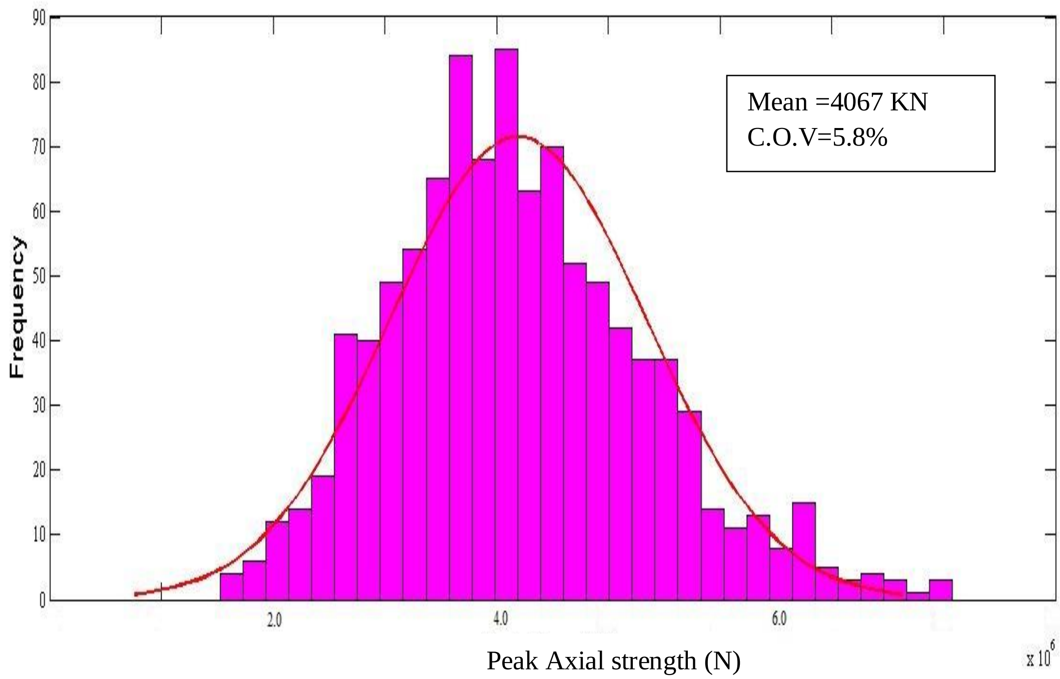


Fig 4.28: Histogram for peak axial strength for Mander et *al.* (1988)
–Modelled as Confined and Unconfined

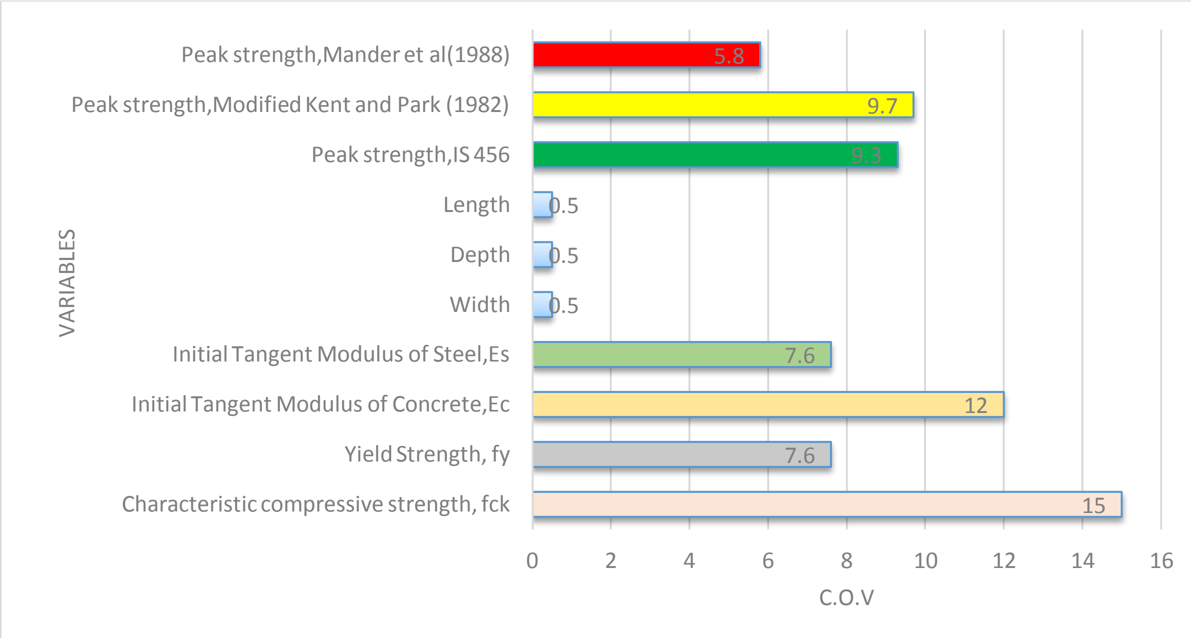


Fig 4.29: Coefficient of Variation (C.O.V) for all random input and response parameters using different confinement models

4.11 SUMMARY

This Chapter discuss about the comparison of responses of a column under axial force implemented using DBM and FBM method. The comparison for the selected problem is fairly matching. A convergence study for two integration types namely, direct and numerical, is conducted and discussed the advantages of numerical method. The confinement effects in RC sections can be modelled conveniently using fiber based element. The axial force responses of RC column is studied using various confinement models. A probabilistic analysis to incorporating nonlinearity is also carried out to study the uncertainty in the peak axial strength response considering the uncertainties in the sensitive input parameters.

5

SUMMARY AND CONCLUSION

CHAPTER 5

SUMMARY AND CONCLUSION

5.1 SUMMARY

The main objective of the present study was to implement the displacement based (stiffness) fiber element for nonlinear analysis of RC Sections. Element formulation of both stiffness and flexibility based fiber models were discussed. Global stiffness computation using both direct integration and numerical integration is discussed. Popular confinement models for stress-strain relationship for concrete were discussed and used as the constitutive relationship for fiber element to study the nonlinear response. The uncertainty exist in the constituent material properties of concrete needs a probabilistic analysis for realistic estimation of nonlinear responses. A probabilistic analysis is carried out further in the implemented model using a Monte-Carlo simulation. Major conclusions from the present study is presented in the following section.

5.2 CONCLUSIONS

1. The force displacement responses obtained from both the DBM and FBM are found to be same.
2. Direct integration method used for DBM required about number of sections as high as 400 compared to that of five in the case of numerical integration.

3. Confinement model as per Kent and Park et *al.* (1988) predicts higher values for strength compared to other models. This is due to high value of confinement factor values.
4. As the maximum compressive strain by recommended by IS 456 (2000) is as low as 0.0035, the displacement is reduced by 61.5% when compared to other confined models (Mander et *al*.(1988), Modified Kent and Park(1982))
5. Probabilistic analysis of the RC column under axial load shows that the C.O.V of the peak strength can vary between 5.8 to 9.7%, when the C.O.V of input random variables, f_{ck}, f_y, breadth, width, length, E_c, E_s are 15%,7.6%,0.5% , 0.5% , 0.5% ,12%,7.6% respectively..

5.3 LIMITATIONS AND FUTURE SCOPE OF WORK

- The RC columns in bending is not considered in the present study. It can be extended to RC frame sections in bending.
- Geometric nonlinearity can be incorporated with material nonlinearity for RC sections.
- It is learned that FBM is suitable for nonlinear problems of RC sections. The study can be extended further to implement this.

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