VIBRATION ANALYSIS OF FUNCTIONALLY GRADED NANOCOMPOSITE CONICAL SHELL STRUCTURES

A THESIS SUBMITTED

IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE

OF

Master of Technology

In

Mechanical Engineering

(Specialization: Machine Design & Analysis)

By

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Rourkela, Odisha (India)

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Rourkela-769008 (India)

May-2015

National Institute of Technology
Rourkela

CERTIFICATE

*This is to certify that the thesis entitled, **"Vibration Analysis of Functionally Graded Nanocomposite Conical Shell Structures"** submitted by **Mr. Mrityunjay Prasad Patel,** Roll No. **213ME1384** in partial fulfillment of the requirements for the award of **Master of Technology** Degree in **Mechanical Engineering** with specialization in **"Machine Design and Analysis"** at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other university/ institute for award of any Degree or Diploma.*

Date:

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A C K N O W L E D G E M E N T

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Machine Design and Analysis

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ABSTRACT

The present work deals with the free vibration analysis and buckling behavior of functionally graded nanocomposite conical shell structures reinforced with carbon nanotubes. The effective material properties of the functionally graded nanocomposite shell structures are obtained using the extended rule of mixture by using UD and some functionally graded distribution of single-walled carbon nanotubes (SWCNTs) in the thickness direction of the shell. In this study the nanocomposite is forming by mixing SWCNTs as reinforced phase with the polymer as matrix phase and makes a superior quality nanocomposite material at nanoscale level with some extraordinary material properties which provides advanced performance and service level. A suitable finite element model of functionally graded conical shell structure is developed using the ANSYS parametric design language (APDL) code in ANSYS environment. The model has been discretized using an eight noded shell element. The solution is obtained for fundamental natural frequencies and deformation of the composite shell. The effects of various geometric parameters, CNT volume fraction, boundary conditions and material properties are presented and discussed.

Keywords— Conical shell structures, CNTs based polymer nanocomposite, various CNT distributions, functionally graded material (FGM), free vibration, buckling analysis.

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INTRODUCTION

1.1 Composite Materials

Historical examples of composites are available in the various literatures. Significant examples include the use of reinforcing mud walls in houses with bamboo shoots, glued laminated wood by Egyptians (1500 B.C.), and laminated metals in forging swords (A.D. 1800). In the 20th century, high structural strength and glass fiber reinforced composites were developed in the early 1930s and the technology of modern composite materials has progressed significantly since then. Aircraft and boats were built out of these glass fiber composites which are commonly known as fiber-glass. The application of composite materials has extensively increased since the 1970s because of the development of new fibers like boron, carbon, and aramids and some other new composite structures with the matrices made from metals, polymer and ceramics.

In recent years the applications of the composite materials have increased extensively in different fields of engineering because of its great strength and light weight. Composite structures belong to the category of such engineering materials in which two or more materials mixed with each other at a macroscopic level with some significant changes in physical and chemical properties and forms a new material with better material properties than those of the individual components used alone. They are continuously used in various engineering fields because of its better properties than other conventional materials like higher strength, higher stiffness, low weight, design flexibility, wear resistance, corrosion resistance, good thermal properties and better fatigue life. Because of these properties there are a wide range of application of composite materials like in aircraft/military industry, construction industry, automobile/transportation sector, marine applications, chemical industry, electrical and electronics applications, nuclear industry etc.

Basically a composite material is a combination of two constituents one is called reinforcing phase (or embedded phase) and the other in which this reinforcing phase is embedded known as matrix. The matrix phase materials are generally continuous and the reinforcing phase are discontinuous or dispersed phase. The reinforcing phase serves to strengthen the composite materials, this phase may be in the form of particles, fibers or flakes.

Various varieties of reinforcing materials used in composite structures are glass fiber, graphite (carbon), asbestos, jute, sisal, boron kevlar 49 and whiskers, as well as chopped paper and synthetic fibers. There are three basic types of matrix materials polymers, metals or ceramics. The two chief purposes of the matrix are binding the reinforcement phases (embedded phase) in place and deforming to distribute the stresses between the constituent reinforcement materials under some applied forces. Examples of composite materials include concrete reinforced with steel and epoxy reinforced with graphite fibers, etc.

An increasing number of engineering structural designs, especially in the automobile, aerospace and civil engineering structures are extensively utilizing various types of fiber composite laminated structures such as beams, plates and shells. The laminated orthotropic shell structures belong to the category of composite shell. In recent years the use of laminated composite shell structures is increased for high performance structures such as wind turbine blades, fuselages of aircraft, ship and boat hulls etc. The main important factor in the analysis of the laminated shell structures is its individual layer properties, which may be made of orthotropic, isotropic or anisotropic materials. The primary function of a laminated shell is to transfer the loads from the edges of one layer to another. A thin shell is defined as a shell which has very small thickness as compared to its diameter, about 20 times smaller than its diameter. Laminated shells are widely used in various engineering fields because of its light weight and high strength like in structural engineering, power and chemical engineering, architecture and building, vehicle body structures, composite construction, armour, submarines etc.

1.2 Types of Composite Materials

1.2.1 Based on reinforcement materials

(i) Fibrous Composites

Fiber reinforced composite materials are composed by embedding the fiber materials in matrix material. Mainly fiber composites has two types first is random or short fiber reinforced composite and the other is continuous or long fiber reinforced composites. In random fiber reinforced composites the fiber materials is in the short discontinuous forms and are arranged in random order throughout the matrix material. In continuous or long fiber reinforced composites the long fiber is used in matrix material along its length or width according to the requirement.

(ii) Particulate composites

The particulate composites are composed by embedding or distributing the particle in the matrix. These particles may be in the form of flakes or powder in the matrix material.

(iii) Flake composites

Flake composites are consist of flat flakes as reinforcements which embedded in the matrix material. Some types of flake materials are glass, aluminum, mica and silver.

(iv) Filler composites

Filler composites are result from addition of filler materials in plastic matrices to replace a particular part of the matrix also to change or enhance the properties of the composite materials.

(v) Laminar composites

A laminar composite is a plane or curved layer of unidirectional fibers in the matrix material. Sandwich structures fall under this category.

1.2.2 Based on matrix materials

(i) Metal Matrix Composites (MMCs)

As the name indicates, Metal matrix composites (MMCs) have the main constituent is matrix of metals. In metal matrix composites, aluminum, magnesium, and titanium etc. are used as matrix and various types of reinforcement materials are used such as Fibers, whiskers, particulates, fillers or flakes which are capable of providing desired properties of composites. MMCs are mostly used to obtain desired material properties which are not achievable in monolithic metal alloys. The reinforcement materials such as ceramic particles, fibers or whiskers dispersed in the metal matrix by molten metal process or powder metallurgy methods to increase the strength, wear resistance, corrosion resistance, elastic stiffness and to control the temperature properties or control of coefficient of thermal expansion and thermal conductivity.

The thermal conductivity of reinforced matrix composites can be improved by the inclusion of fibers like silicon carbide. Because of these temperature and other properties of the metal matrix composites it has many potential applications like in aerospace, automobile and transportation, electronics fields and civil engineering.

(ii) Ceramic Matrix Composites (CMCs)

In Ceramic matrix composites (CMCs) the main constituent is ceramic which consist of ceramic fibers embedded in a ceramic material matrix and finally forms a ceramic fiber reinforced ceramic (CFRC) composite material. The ceramic fibers, used for ceramic matrix composites are carbon, alumina, silicon carbide and mullite fibers etc. For the matrix materials, usually the same materials are used which are used for reinforcing materials. The ceramics in ceramic matrix composites enhances the properties like hardness, strength, lightness, corrosion resistance, oxidation resistance, crack resistance and has very good thermal and electrical properties. The CMCs are processed and manufactured by slurry infiltration, liquid infiltration, hot pressing, hot isotactic pressing, pressure casting/squeeze casting, pultrusion, directed oxidation process and chemical vapor infiltration (CVI).

Ceramic matrix composites can be used in high temperature applications in which the polymer matrix composites (PMCs) and metal matrix composites (MMCs) cannot be used. Besides the high temperature applications, the some other potential applications includes armour of military vehicles, engine components, brake system components, space vehicles components, slid bearing components and components of gas turbine such as turbine blades, stator vanes and combustion chamber.

(iii) Polymer Matrix Composites (PMCs)

The polymer matrix composites (PMCs) are the most accepted advanced composites which consist of polymer (resin) matrix from thermoset (unsaturated polyester, epoxy, phenolic polyamide resins etc.) or thermoplastic (polycarbonate, PVC, polystyrene, acrylics, nylon, polyethylene, polypropylene etc.) in which the reinforcement phase embedded such as carbon fiber, glass fiber, steel or kelvar fiber etc. Thermosetting polymers are the most widely used polymers in PMCs. Polymer matrix composites (PMCs) are very common and famous due to their low material cost, simple methods of fabrication, high strength. Reinforcement of robust fibrous network allows fabrication of polymer matrix composites which are characterized by some enhanced material properties like high tensile strength, light weight, high fracture toughness, large stiffness, decent puncture resistance, abrasion and corrosion resistance etc.

The main drawback of polymer matrix composites (PMCs) includes light thermal resistance, large coefficients of moisture and thermal expansion and also in certain directions they exhibits low elastic properties. According to the reinforcement materials the most common group of PMCs are glass fiber reinforced polymer, carbon fiber reinforced composites and Kevlar (aramid) fiber reinforced polymers.

Polymer matrix composites are processed by spray up moulding, press moulding, hand lay-up, prepreg lay-up, autoclave forming, resin transfer moulding, filament winding etc. The potential applications of polymer matrix composites includes to make components of aircraft and space vehicles, marine transportation, automobile components and transportations, civil constructions, sporting equipments (golf clubs, tennis racquets, fishing rods, skis etc.), bullet-proof jackets, brake and clutch linings, radio controlled vehicles, boat bodies and medical devices.

1.3 Carbon Nanotubes (CNTs)

Carbon nanotubes (CNTS) are the allotropes of carbon which are in tubular cylindrical shape. Carbon nanotubes are cylindrical nanostructures which has very large length to diameter ratio significantly higher than the any other materials. The diameter of carbon nanotubes ranging from 1nm (nanometer) to 50nm and the lengths are millionth times than the diameters. The CNTs are long and hollow cylindrical structures with the walls developed by single atom thick sheet of carbon which is called graphene. These sheets are rolled at particular rolling angle and radius which decides the properties of nanotubes. Carbon nanotubes are playing very important role in various engineering field because of its extraordinary mechanical, thermal, electrical, chemical, optical properties, large yield strength and Young's modulus and also valuable for electronic, nanotechnology and some other fields of materials science and technology. CNTs have the strongest tensile strength and have very good flexibility of any other material known and about 20 times higher than the steel alloys. It has also very high current carrying capacity and very high electrical current density about 1000 times than copper and because of extraordinary thermal properties these are efficient conductor of heat. The nanotubes are composed similar to those of graphite which has the chemical bonding of entirely sp^2 bonds.

1.3.1 *Types of carbon nanotubes*

Basically there are two main types of carbon nanotubes, single walled carbon nanotubes (SWCNTs) and multi walled carbon nanotubes (MWCNTs). Single walled carbon nanotubes have the diameters about 1 nanometer (nm), with a very large length to diameter ratio that can be many millions of times longer. SWCNTs consist of a one atom thick layer of grapheme into a seamless cylinder. MWCNTs are consisting of multiple rolled layers of grapheme which is in concentric tubular form. For multi walled carbon nanotubes, the diameter ranges from 5nm to 50nm. Besides these two basic types of carbon nanotubes there are some other types of CNTs on the basis of special shapes and size such as torus, nanobud, nitrogen doped carbon nanotubes (N-CNTs), graphenated carbon nanotubes (g-CNTs), peapod carbon nanotubes, cup stacked carbon nanotubes, extreme carbon nanotubes etc.

1.3.2 *Applications of carbon nanotubes*

Because of the extraordinary properties of the carbon nanotubes, CNTs are promising to revolutionize and controls the other nanostructures in various fields of engineering such as aerospace, structural engineering, automobile and transportation, defence sector, marine engineering, composite construction, energy sector, medicine and healthcare, nanotechnology, material science, chemical industry, electrical and electronics applications etc.

By using the carbon nanotubes base composite materials we can obtain the structure of superior material properties as per the requirements by varying the composition and distribution of CNTs in composite which gives directional properties and control parameters.

1.4 Disadvantages of Composite Materials

Though laminated composites have superior advantages and properties over the conventional materials but their major drawback is however represented by the effects of different loads and pressures impact load, crippling loads repeated cycle stress etc. on the different layers of composite, which causes to weak or separate the two adjacent layers, this phenomena is known as delamination. This failure of one or more layers of composite may lead to start the failure of the whole structure. Composite material repairing is also a major downside of it which is complex process as compared to that for metals. If a particular part or a layer of composite material is not working then the repairing for that part will be time consuming and costly process. These problems can be reduced by introducing such materials which gives particular properties and functions to prevent the sudden change or failure of the composite parts.

1.5 Functionally Graded Materials (FGMs)

Functionally graded materials (FGMs) are a new variety of composite materials with properties that vary spatially according to a certain non-uniform distribution of the reinforcement phase. Much work has been done on functionally graded materials in a wide range of fields since the concept of FGMs was originated in Japan around 1984 during the project of space plane. The unique properties can overcome the defects of the conventional laminated composites. Recently, the FGMs have been employed to build various types of shell structures in different fields of engineering applications. By using the carbon nanotubes base functionally graded composite materials we can obtain the desired material properties as per the requirements by varying the composition and distribution of CNTs in composite which gives directional properties and control parameters.

1.6 Applications of functionally graded materials (FGMs)

A wide variety of applications exist for functionally graded material structures.

1. Aerospace
   - Aerospace skins
   - Engine components
   - Vibration control
   - Adaptive structures
   - Fuselages of aeroplane
2. Engineering
   - Cutting tools
   - Shafts
   - Engine components
   - Turbine blades
3. Optics
   - Optical fiber
   - Lens

4. Electronics

- Semiconductor
- Substrate
- Sensor
- Actuator
- Integrated chips

5. Chemical plants

- Heat exchanger
- Heat pipe
- Reaction vessel
- Substrate

6. Energy conversion

- Thermoelectric generator
- Thermo-ionic converter
- Solar cells, fuel cells

7. Biomaterials

- Implants
- Artificial skin
- Drug transport system
- Prosthetics
- Artificial tissues

8. Commodities

- Building material
- Sports goods
- Car body
- Casing of different materials

1.7 Importance of the present work

The present work deals with free vibration analysis and buckling behavior of functionally graded nanocomposite conical shell structures in which carbon nanotube as reinforcing phase and polymer as matrix and forms a very robust composite structure which perform a superior and extraordinary service. Very few research works has been carried out considering FG nanocomposite conical shell which gives immense scope of research in this area. In this present work the effect of vibration on functionally graded nanocomposite conical shell structures are analyzed under the different distribution of CNT volume fraction like UD, FG-X and FG-V etc. Finally the developed model is analyzed using the ANSYS parametric design language (APDL) code in ANSYS environment to investigate the effect of various parameters on vibration behavior of the structure.

1.8 Aim and objective of present thesis

i. The aim of this thesis is to evaluate the material properties of FG-CNTRC conical and doubly curved (catenoidal) shell structures using the extended rule of mixture by using UD and some other types of distributions of CNT volume fraction.
ii. Develop a mathematical model for functionally graded carbon nanotubes based composite shell structures under the effects of compressive loads using the ANSYS parametric design language (APDL) code to develop the model in the environment of ANSYS software.
iii. To evaluate the effects of buckling load and free vibration on the functionally graded carbon nanotubes shell structure is based on UD, FG-V, FG-X and FG-Λ with the help of geometrical parameters of the different layers of functionally graded carbon nanotube composite shell such as thickness, volume fraction of CNT, Young's modulus, shear modulus, Poisson’s ratio and boundary conditions are discussed.

1.9 Outline of the present work

The present work can be categorized into following six chapters:

Chapter 1: Presents an introduction about the composite materials, carbon nanotubes (CNTs), functionally graded materials (FGMs) and the applications of these and presents the importance and objective of the thesis.

Chapter 2: This chapter includes the literature reviews related to the present work and gives and detailed knowledge about the carbon nanotubes, functionally graded materials, conical shells and nanocomposite to analyze the vibration of FGM based nanocomposite shell structures.

Chapter 3: This chapter includes mathematical formulation to analyze the vibrational behavior of FG-CNTRC shell structures. First part of this chapter includes the evaluation of material properties of carbon nanotubes and polymer. Second part includes the modeling and analysis of shell structures under the some boundary conditions by using the ANSYS parametric design language (APDL) code in ANSYS software.

Chapter 4: This chapter presents the results and discussions of the present work and gives a detailed analysis about the effects of vibration on shell structures under the different distributions of the carbon nanotubes in the composite.

Chapter 5: presents the conclusion for the present work and contains the future work research for this work.

LITERATURE SURVEY

A composite material of FGM based shell structure performs a superior service which can be used in high temperature applications and high loading conditions. In recent years, functionally graded materials (FGMs) have received a great attention for the applications in high loading and high temperature conditions. Various literatures, studies and journals are available for the study of functionally graded material based nanocomposite shell structures.

Lei et al. [6] investigated the buckling behavior of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) plates under the effects of different mechanical loadings by using element-free kp-Ritz method. The buckling analysis is carried out for functionally graded single-walled carbon nanotubes (SWCNTs) reinforced plates by using first-order shear deformation plate theory and mesh free method. The effective material properties for single-walled carbon nanotubes reinforced plate materials is based on a micromechanical model, either the extended rule of mixture or the Eshelby Mori Tanaka method. Chakraborty et al. [7] modeled a new beam element to analyze the thermoelastic behavior of FGMs based beam structures. The element considered first order shear deformation theory and the thermal and elastic properties are varying in the thickness direction. Wave propagation, static and free vibration problems are considered to examine the variation in thermoelastic behavior of functionally graded material (FGM) beam with pure ceramic beams or pure metal. Azadi [8] presents finite element method (FEM) based forced and free vibration analysis of functionally graded material based beams by considering the temperature dependent material properties. In this work, material properties were graded in the thickness direction of beams by considering simple power law distribution of constituent’s volume fractions. Finally the dynamic analysis has been done for the damped and undamped systems. Yas and Heshmati [9] presented the vibrational study of functionally graded nanocomposite beams under the effect of moving loads which is reinforced by randomly oriented straight single walled carbon nanotubes (SWCNTs) by considering the Timoshenko and Euler-Bernoulli beam theories. Dastjerdi et al. [10] investigated dynamic analysis of functionally graded materials based nanocomposite cylindrical structures reinforced by single walled carbon nanotubes subjected to an impact load by a mesh free method. Stress wave propagation study and free vibration analysis of single-walled carbon nanotube reinforced composite (SWCTNRC) cylinders are studied in this work. Woo et al. [11] presented an analytical solution for the postbuckling behavior of functionally graded shallow cylindrical shells and plates under the effect of compressive loads and thermomechanical effects. The material properties for functionally graded (FG) shell structures are assumed to vary in the thickness direction of the shell by applying power law distribution of the constituent's volume fraction. This work shows the effects of thermomechanical coupling and different boundary conditions on the response of the FG shells and plates under the action of compressive loads. Shen [12] developed thermal buckling and postbuckling analysis of functionally graded nanocomposite cylindrical shell structures reinforced by single-walled carbon nanotubes (SWCNTs) under the action of temperature effects. Functionally graded and uniformly distributed reinforcements types of carbon nanotube (CNT) reinforced composite shells are considered. The governing equation is based on higher order shear deformation theory (HSDT) and von Karman equation to analyze the buckling behavior of functionally graded carbon nanotube reinforced composite (FG-CNTRC) cylindrical shell structures. Xiang [13] developed a computational model for the forced and free vibration analysis of functionally graded materials (FGMs) based laminated beam of variable thickness under the action of temperature field by considering the Timoshenko beam theory. Wali et al [14] investigated free vibration analysis of functionally graded material (FGM) based shell structures by using 3d shell model based on a discrete double directors shell elements. The material properties of the shell structures are assumed to vary in the direction of shell thickness according to the power law distributions of the constituents and the fundamental frequencies are derived from the virtual work principle.

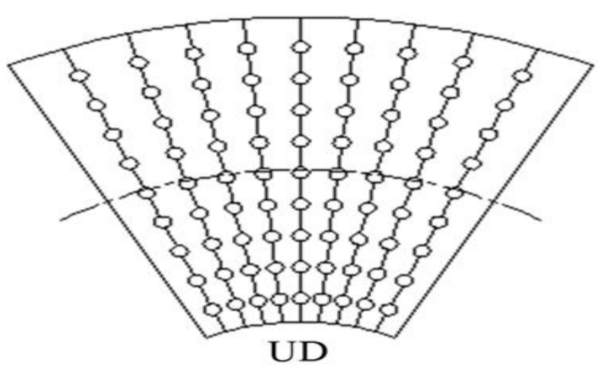
Bhangale et al. [15] presented the effect of vibration behavior and thermal buckling of FGM based truncated conical shells in the environment of large temperature condition. The analysis for the free vibration and the thermal buckling has been done by considering the temperature dependent material properties. Haddadpour et al. [16] presented the analysis of free vibration for simply supported functionally graded cylindrical shell structures under the effect of temperature. The equations of motions are based on von Karman Donnell and shell theory of Love's equations. Kadoli and Ganesan [17] investigated the free vibration analysis and the effect of buckling load on the functionally graded cylindrical shell structures with the clamped-clamped boundary condition under the effect of temperature. For the modeling of functionally graded material (FGM) based shell structure, first order shear deformation theory (FSDT) and

Fourier series expansion of the displacement variables are used. Heydarpour et al. [18] presented the effects of coriolis and centrifugal forces on the free vibration behavior of rotating functionally graded carbon nanotubes reinforced composite (FG-CNTRC) truncated conical shell structures. The governing equations are based on the FSDT of shell structures by considering the Hamilton's principle. The differential quadrature method (DQM) is used to solve the equations of motion and applied boundary conditions. Jam et al. [19] presented the free vibration analysis of carbon nanotube reinforced functionally graded cylindrical panels under the simply supported boundary conditions. The material properties of FG carbon nanotube reinforced nanocomposite cylindrical panels are obtained by using the extended rule of mixture which varies in the radial direction. Viola and Tornabene [20] presented free vibration analysis of homogeneous and isotropic conical shell structures using the numerical technique method known as Generalized Differential Quadrature (GDQ) method. Tornabene [21] investigated the dynamic behavior and free vibration analysis of thick functionally graded material based cylindrical shells, conical shells and annular plates with a four parameter power law distribution based on the first-order shear deformation theory (FSDT). Tornabene et al. [22] presented the dynamic behavior of functionally graded conical shells, annular plates and cylindrical shells of moderately thick structural elements by using the 2-D differential quadrature solution based on first order shear deformation theory (FSDT). Shu [23] studied free vibration analysis of homogeneous and isotropic conical shell structures by using the generalized differential quadrature (GDQ) method. Shakouri and Kouchakzadeh [24] presented the mode shapes and natural frequencies of two joined isotropic conical shell structures by incorporating the Donnell and Hamilton's principle. Zhu Su et al. [25] investigated a three dimensional vibration analysis of thick functionally graded cylindrical shells, conical shells and annular plates with arbitrary elastic restraints. The effective material properties of FG structures are obtained by using Voigt's rule of mixture which varies in the direction of shell thickness. The solution has been done for the different shell structures by using modified Fourier series equations. Tornabene et al. [26] developed the free vibration analysis of panels and shells of revolution with a free form meridian by using the Generalized Differential Quadrature (GDQ) method based on the first order shear deformation theory (FSDT) and considering the general anisotropic doubly curved shell theory. to show the differences and the accuracy of the shell structures, four different types of anisotropic shell theories are considered which are general first order shear deformation theory by Toorani–Lakis (GFSDTTL), general first order shear deformation theory by Qatu (GFSDTQ), classical Reissner–Mindlin theory (CRMT) and general first order shear deformation theory by Reissner–Mindlin (GFSDTRM). Yas et al. [27] presented free vibration analysis of FGMs based nanocomposite cylindrical panels by using single walled carbon nanotubes (SWCNTs) as reinforced phase and based on the three-dimensional (3-D) theory of elasticity. The material properties are obtained by using the extended rule of mixture and the variation of the carbon nanotube is in the radial direction of the functionally graded nanocomposite shell structures. Lei et al. [28] analysed free vibration analysis of CNT reinforced functionally graded material based rotating cylindrical panels by using element-free kernel particle Ritz method.

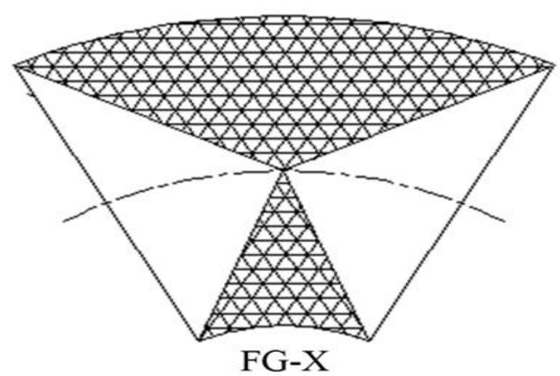
MATHEMATICAL MODELING

3.1 Material Modeling

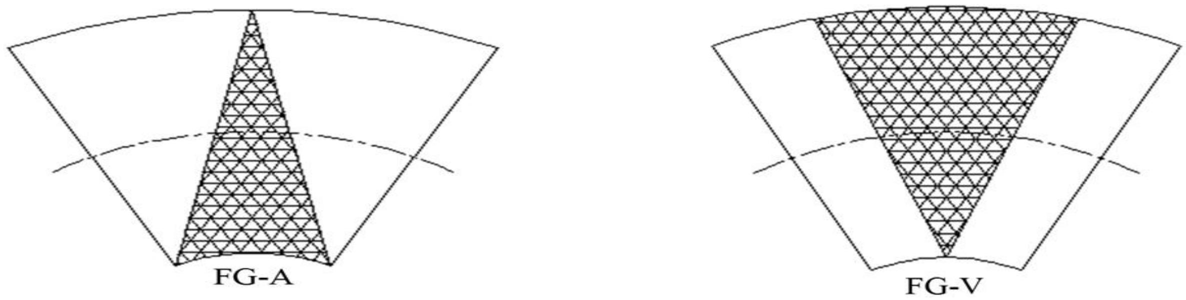
Consider a truncated conical shell made of carbon nanotube reinforced composites (CNTRCs) in which the distribution of carbon nanotubes (CNTs) is graded along the thickness direction of FG-CNTRCs. Four different types of distributions of CNTs are considered along the shell thickness directions which are shown in Figure 1. In the first case of distribution, the CNTs has a uniformly distribution through the direction of shell thickness, which is referred to UD type as shown in Figure 1(a). In the second case of distribution, the distribution of CNTs have a mid-plane symmetry and both the inner and the outer surfaces are rich CNTs, which is referred to FG-X type as shown in Figure 1(b). In the third type of distribution, the outer surfaces have lean CNTs and has rich matrix, whereas the inner surface is CNTs-rich. This type of distribution is known as $FG-\Lambda$ type distribution as shown in Figure 1(c). In the last case of distribution, the distribution of CNTs are opposite of third type, in which the inner surface is matrix-rich and the outer surface is CNTs-rich and known as FG-V type of distribution as shown in Figure 1(d).



(a) UD



(b) FG-X



(c) $FG-\Lambda$ **(d)** FG-V

Figure 1. Different types of distribution of CNTs [10].

In order to model the effects of the carbon nanotubes on the overall properties of the nanocomposite conical shell structure, the extended rule of mixture is used as a convenient and simple micromechanics model. According to this rule, the effective Young's modulus and shear modulus of FG-CNTRC shell structures can be expressed as [18]:

$$E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_M E^M \tag{1}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_M}{E^M} \tag{2}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_M}{G^M} \tag{3}$$

Where,

E_{11}^{CNT}, E_{22}^{CNT} = Young's modulus of CNTs,

G_{12}^{CNT} = shear modulus of CNTs,

E^M = Young's modulus of matrix,

G^M = shear modulus of matrix,

V_{CNT} = volume fraction of CNTs,

V_M = volume fraction of matrix,

$\eta_1, \eta_2, \eta_3 =$ efficiency parameters of CNTs.

In addition, the relationship between the volume fraction of CNT (V_{CNT}) and volume fraction of matrix (V_M) is given by:

$$V_{CNT} + V_M = 1 \tag{4}$$

The material properties of the FG-CNTRC conical shells vary smoothly and continuously in the direction of the shell thickness. Similarly, in order to evaluate the various CNT distributions effects on the free vibration characteristics of a FG-CNTRC conical shell, different types of material profiles through the shell thickness are considered. In this present work, we are assuming only linear distribution of the CNT volume fraction for the various types of the FG-CNTRC conical shell that can easily be achieved in practice and given by [18]:

$$\text{UD:} \quad V_{CNT} = V_{CNT}^{*} \tag{5}$$

$$\text{FG-V:} \quad V_{CNT} = V_{CNT}^{*}\left(1 - \frac{2z}{h}\right) \tag{6}$$

$$FG - \Lambda: \quad V_{CNT} = V_{CNT}^{*}\left(1 + \frac{2z}{h}\right) \tag{7}$$

$$\text{FG-X} \quad V_{CNT} = V_{CNT}^{*}\left(\frac{4|z|}{h}\right) \tag{8}$$

Similarly, Poisson's ratio υ and mass density ρ can be determined by:

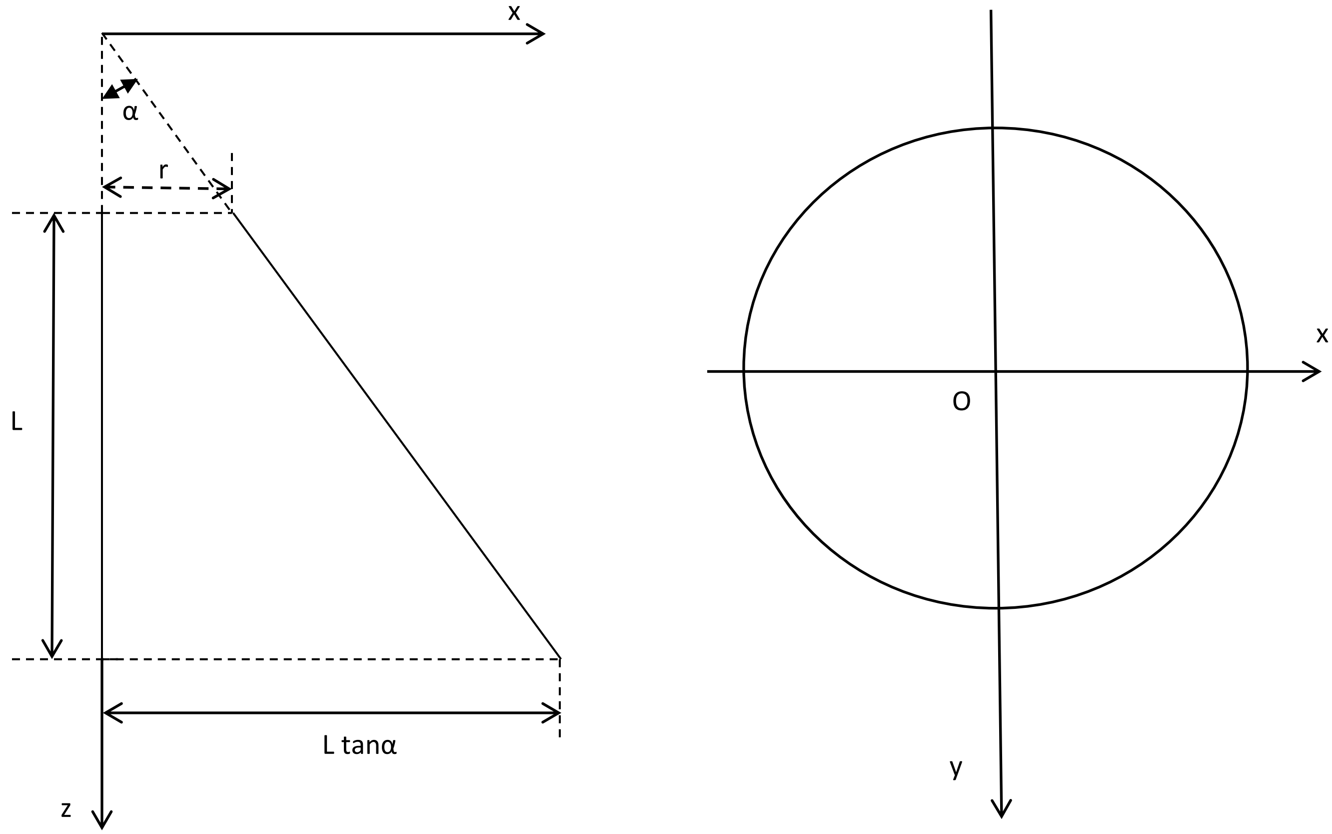
$$\nu_{12} = V_{CNT}\nu^{CNT} + V_M \nu^{M} \tag{9}$$

$$\rho = V_{CNT}\rho^{CNT} + V_M \rho^{M} \tag{10}$$

Where, ν^{M} and ν^{CNT} are Poisson's ratios of matrix and carbon nanotube, respectively.

3.2 Geometric modeling

To investigate the vibration analysis and buckling behaviour of functionally graded carbon nanotubes reinforced nanocomposite shell structures, the present work considers the two structures, one is conical shell structure and the other one is doubly curved (catenoidal) shell structure. The geometry for both the structures are describes as follows:

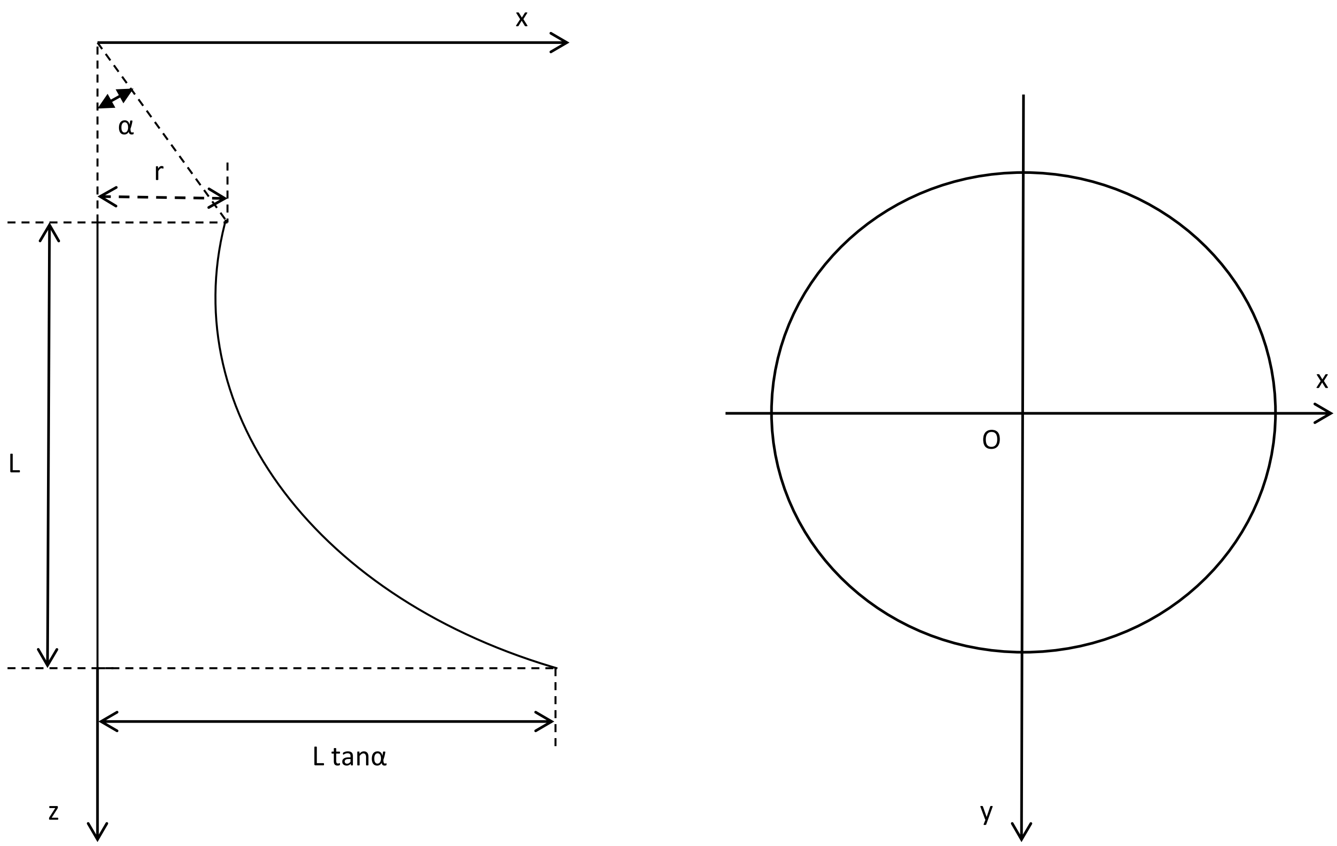


(a) meridian section (b) parallel section

Figure 2. Geometry of cone [22].

In the above conical shell geometry first part shows the meridian section and second part shows the parallel section of the conical shell in which α is the apex angle of the cone, r is the top or small radius of the cone, R shows the bottom or larger radius of the cone and the length of cone is given by L. The bottom or larger radius of cone is equal to the Ltanα of the cone.

The geometry for the doubly curved shell structure (catenoidal shell) is given in above figure. It has the similar dimensions of conical shell as apex angle α, top radius is r, bottom radius is Ltanα and the length is L and it has a curved shape on its circumference as shown in figure above. To solve the present work and analyse both the structures we use ANSYS parametric design language (APDL) code in ANSYS software. For the modelling purpose, a SHELL 281 element is selected from the ANSYS element library which is shown is the figure 4. This element is eight noded linear shell element and has six degree of freedom on every node (three degree of freedoms are translations along the x, y and z axis and the remaining three degree of freedoms are rotations about the x, y and z axis per node). The SHELL 281 element is suitable for analysing thin, moderate and thick shell structures. This element can be used for large rotation, linear and large strain nonlinear applications. The formulation of this element is based on the logarithmic strain and true stress measures.



(a) meridian section (b) parallel section

Figure 3. Geometry of doubly curved (catenoidal shell) [22].

The mid plane kinematics of carbon nanotubes based nanocomposite materials has been taken as the FSDT using the inbuilt steps in ANSYS and conceded as follows:

$$\begin{aligned} u(x,y,z) &= u^0(x,y) + z\theta_x(x,y) \\ v(x,y,z) &= v^0(x,y) + z\theta_y(x,y) \\ w(x,y,z) &= w^0(x,y) + z\theta_z(x,y) \end{aligned} \tag{11}$$

Where, u, v and w represents the displacements of any point along the (x, y, z) coordinates. u_0, v_0 are the in-plane and w_0 is the transverse displacements of the mid-plane and θ_x, θ_y are the rotations of the normal to the mid plane about y and x axes respectively and θ_z is the higher order terms in Taylor's series expansion.

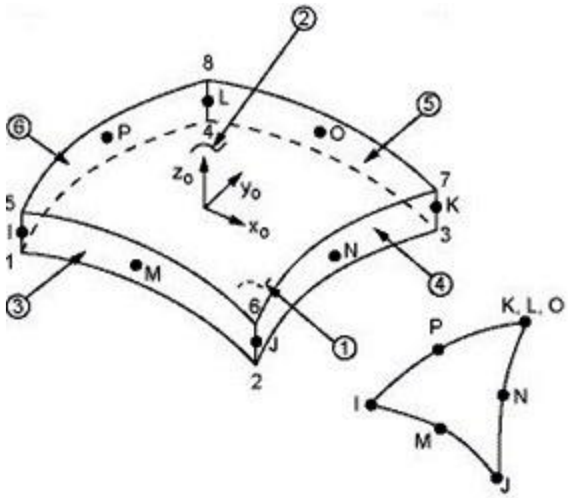


Figure 4. SHELL 281 geometry [29].

The displacements u, v and w can be expressed in terms of shape functions (N_i) as:

$$\delta = \sum_{i=1}^{j} N_i \delta_i \tag{12}$$

Where, $\{\delta\} = \begin{bmatrix} u_{oi} & v_{oi} & w_{oi} & \theta_{xi} & \theta_{yi} & \theta_{zi} \end{bmatrix}^T$ (13)

The shape functions for the 8 noded (j=8) shell elements (SHELL 281) are given by:

$$\begin{aligned}
N_1 &= \frac{1}{4}(1-\xi)(1-\eta)(-1-\xi-\eta) \\
N_2 &= \frac{1}{4}(1+\xi)(1-\eta)(-1+\xi-\eta) \\
N_3 &= \frac{1}{4}(1+\xi)(1+\eta)(-1+\xi+\eta) \\
N_4 &= \frac{1}{4}(1-\xi)(1+\eta)(-1-\xi+\eta) \\
N_5 &= \frac{1}{2}(1-\xi^2)(1-\eta) \\
N_6 &= \frac{1}{2}(1+\xi)(1-\eta^2) \\
N_7 &= \frac{1}{2}(1-\xi^2)(1+\eta) \\
N_8 &= \frac{1}{2}(1-\xi)(1-\eta^2)
\end{aligned} \tag{14}$$

Strains are obtained by derivation of displacements as:

$$\{\varepsilon\} = \begin{bmatrix} u,_x & v,_y & w,_z & u,_y + v,_x & v,_z + w,_y & w,_x + u,_z \end{bmatrix}^T \tag{15}$$

where, $\{\varepsilon\} = \begin{bmatrix} \varepsilon_{xx}^0 & \varepsilon_{yy}^0 & \gamma_{xy}^0 & k_{xx} & k_{yy} & k_{xy} \end{bmatrix}^T$ is the strain matrix containing normal and shear strain components of the mid-plane in in-plane and out of plane direction.

The strain components are rearranged by using the following steps by in plane and out of plane sets.

The in-plane strain vector:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{16}$$

The transverse strain vector:

$$\begin{Bmatrix} \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{z0} \\ \gamma_{yz0} \\ \gamma_{xz0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} \tag{17}$$

Where, the deformation components are given as:

$$\begin{Bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \gamma_{xy_0} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}; \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \theta_x}{\partial x} \\ \dfrac{\partial \theta_y}{\partial y} \\ \dfrac{\partial \theta_x}{\partial y} + \dfrac{\partial \theta_y}{\partial x} \end{Bmatrix} \tag{18}$$

$$\begin{Bmatrix} \varepsilon_{z_0} \\ \gamma_{yz_0} \\ \gamma_{xz_0} \end{Bmatrix} = \begin{Bmatrix} \theta_z \\ \dfrac{\partial w_0}{\partial y} + \theta_y \\ \dfrac{\partial w_0}{\partial x} + \theta_x \end{Bmatrix}; \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \dfrac{\partial \theta_z}{\partial y} \\ \dfrac{\partial \theta_z}{\partial x} \end{Bmatrix} \tag{19}$$

The strain vector expression in term of nodal displacement vector is given as:

$$\{\varepsilon\} = [B]\{\delta\} \tag{20}$$

where, [B] = strain displacement matrix

$\{\delta\}$ = nodal displacement vector.

The generalized stress strain relationship with respect to its reference plane may be expressed as:

$$\{\sigma\} = [D]\{\varepsilon\} \tag{21}$$

Where, {σ} and {ε} is the linear stress and linear strain vector correspondingly and [D] is the rigidity matrix.

The element stiffness matrix [K] and mass matrix [M] can expressed as:

$$\left[K^e\right] = \int_{-1}^{1}\int_{-1}^{1} [B]^T [D][B] \left|J\right| d\xi d\eta \tag{22}$$

$$\left[M^e\right] = \int_{-1}^{1}\int_{-1}^{1} [N]^T [m][n] \left|J\right| d\xi d\eta \tag{23}$$

where, |J| is the determinant of Jacobian matrix, [N] is the shape function matrix and [m] is the inertia matrix. The integration has been carried out using the Gaussian quadrature method.

The static analysis determines the deflections as:

$$[K]\{\delta\} = \{P\} \tag{24}$$

Where, {P} is the static load vector acting at the nodes.

In the form of shape functions, the nodal displacements are given as follows:

$$u^0 = \sum_{i=1}^{8} N_i u_i^0 \,,\; v^0 = \sum_{i=1}^{8} N_i v_i^0 \,,\; w^0 = \sum_{i=1}^{8} N_i w_i^0 \,,$$

$$\theta_x = \sum_{i=1}^{8} N_i \theta_{xi}^0 \,,\; \theta_y = \sum_{i=1}^{8} N_i \theta_{yi}^0 \,,\; \theta_z = \sum_{i=1}^{8} N_i \theta_{zi}^0 \tag{25}$$

The above equation can be arranged in the form of i^{th} nodal displacement as follows:

$$\left\{\delta_i^*\right\} = \left\{u_i^0 \quad v_i^0 \quad w_i^0 \quad \theta_{xi}^0 \quad \theta_{yi}^0 \quad \theta_{wi}^0\right\} \tag{26}$$

$$\left\{\delta_i^*\right\} = \left[N_i\right]\left\{\delta_i\right\} \tag{27}$$

Where, $$\left[N_i\right] = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & 0 & N_i \end{bmatrix} \tag{28}$$

The expression for the strain energy can be given by substituting the value of nodal displacement in strain vector equation as follows:

$$\left\{\varepsilon\right\} = \left[B_i\right]\left\{\delta_i\right\} \tag{29}$$

$$U = \frac{1}{2}\int \left[B_i\right]^T \left\{\delta_i\right\}^T \left[D\right]\left[B_i\right]\left\{\delta_i\right\} dA - \left\{F\right\}_{mi} \tag{30}$$

Where, $[B_i]$ = strain-displacement matrix,

$\{F\}_m$= mechanical force.

The final equation can be expressed by minimizing the total potential energy (TPE) as follows:

$$\partial \prod = 0$$

Where, $\prod$ = total potential energy.

The free vibration analysis is used to determined natural frequencies by given equation:

$$([K] - \omega_n^2 [M]) = 0 \tag{31}$$

Where, [K] = global stiffness matrix,

[M] = global mass matrix.

The buckling equation for laminated shells can be given as follows:

$$[K]\{\delta\} = \{F\}_m \tag{32}$$

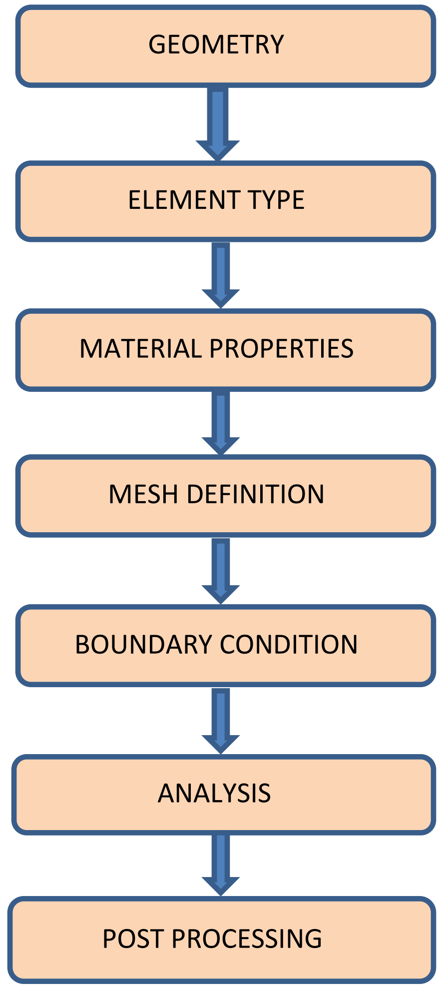
The eigenvalue type of above buckling equation can be given as in the following step by neglecting the force vector and conceded to

$$\{[K] + \lambda_{cr}[K_G]\}\{\delta\} = 0 \tag{33}$$

where, $[K_G]$ = the geometric stiffness matrix,

λ_{cr} = critical mechanical load at which the shell structure buckles.

3.3 Solution technique and steps:



RESULT AND DISCUSSION

4.1 Material Modeling

Based on the above formulation the free vibration analysis and buckling behavior of functionally graded nanocomposite conical shell is carried out by using ANSYS (APDL code) and some of the results are presented here. The material properties of matrix material are ν^{M} = 0.34, ρ^{M} = 1150 kg/m^3 and E^{M} = 2.5Gpa at environment temperature (300^{o}K). The material properties of single walled carbon nanotube (SWCNTs) are E_{11}^{CNT} = 5.6466 TPa E_{22}^{CNT} = 7.0800 TPa, G_{12}^{CNT} = 1.9445 TPa, ρ^{CNT} = 1440 kg/m^3 and ν^{CNT} =.175 mixes with the polymer at 12%, 17% and 28% distributions of CNTs.

Table 1: CNT efficiency parameters for different CNT volume fraction

V_{cnt}	CNT efficiency parameters		
	η_1	η_2	η_3
0.12	0.137	1.022	0.715
0.17	0.142	1.626	1.138
0.28	0.141	1.585	1.109

To analyze the vibration and buckling effects on the FG nanocomposite conical shell structures, two types of boundary conditions are considered, namely simply supported boundary condition and clamped- clamped boundary condition. The below equation describes the boundary condition for both the cases [22]:

Simply supported boundary condition:

u = v = w = θ_y = 0, M_x = 0 at x=0,

u = v = w = θ_x = 0, M_y = 0 at y=0,

Clamped-clamped boundary condition:

u = v = w = θ_x = θ_y = 0, at x=0,

u = v = w = θ_x = θ_y = 0, at y=0,

4.2 Numerical illustrations

The numerical results for the current work by considering the above material properties and boundary conditions under the effect of compression load are given below. From the above solution we get the effects of buckling parameter on the functionally graded nanocomposite conical shell at different distributions of the CNTs (V_{CNT}^{*} = 12%, 17% and 28%) under the simply supported and clamped-clamped boundary conditions. It is observed from the results obtained from the table that with increase in the CNTs volume fraction the buckling load parameter also increases. From the table it is note that the frequency of vibration is found maximum in case of $FG-\Lambda$ type CNTRC conical shell and minimum for the case FG-V type CNTRC conical shell.

Table 2 and 4 shows the variation of buckling parameter for different CNT distribution and volume fraction for conical and catenoidal shell. Both the Tables shows that there is a prominent effect of FG-V and $FG-\Lambda$ type CNT distribution on the buckling parameter. Table 3 and 5 shows variation of first 10 fundamental frequencies of conical and catenoidal shell respectively and it clear from both Tables that as the CNT volume fraction increases the fundamental frequency also increases which results in increase in the stiffness of the structure.

Table 2: Buckling load parameter under compression load for conical shell.

V_{cnt}	Type of FG	Buckling load (N)	
		Simply supported	Clamped-Clamped
12%	UD	10.1778	10.0189
	FG-X	10.1784	10.1243
	FG-V	10.0437	10.0112
	$FG-\Lambda$	10.4437	10.3678
17%	UD	10.5281	10.2884
	FG-X	10.6914	10.4619
	FG-V	10.0494	10.0336
	$FG-\Lambda$	11.7903	11.9244
28%	UD	11.9393	11.9234
	FG-X	12.5034	12.3584
	FG-V	10.2157	10.1052
	$FG-\Lambda$	14.6948	14.8058

Table 3: The first 10 fundamental frequencies for conical shell.

Frequencies (in Hz)	
f_1	82.7332
f_2	82.7332
f_3	85.7238
f_4	85.7285
f_5	86.2338
f_6	86.2338
f_7	86.7088
f_8	86.7118
f_9	89.5396
f_{10}	89.5396

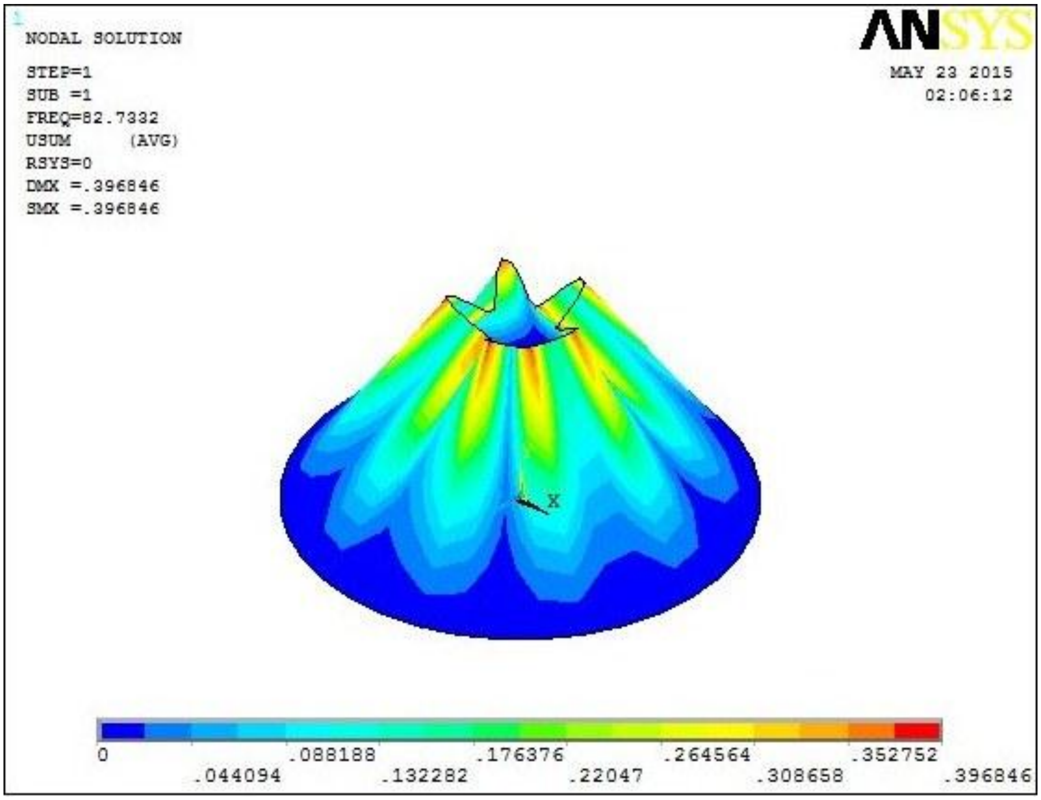
Table 4: Buckling load parameter under compression load for doubly curved (catenoidal) shell.

V_{cnt}	Type of FG	Buckling load (N)	
		Simply supported	Clamped-Clamped
12%	UD	10.1709	10.1882
	FG-X	10.2017	10.2007
	FG-V	10.0833	10.1137
	$FG-\Lambda$	10.2785	10.2677
17%	UD	10.2506	10.1474
	FG-X	10.2687	10.1920
	FG-V	10.0963	10.1084
	$FG-\Lambda$	10.4721	10.3352
28%	UD	10.3339	10.4250
	FG-X	10.9989	10.9940
	FG-V	10.1565	10.1080
	$FG-\Lambda$	11.0660	11.0684

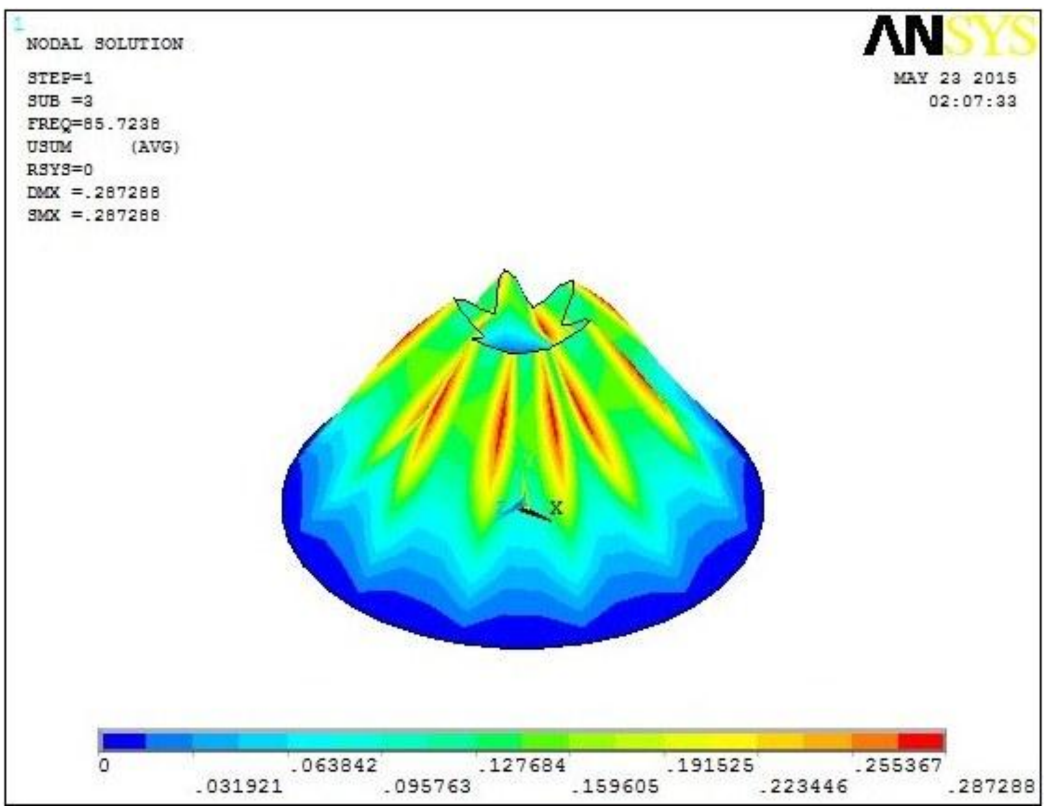
Table 5: The first 10 fundamental frequencies for doubly curved (catenoidal) shell.

Frequencies (in Hz)	
f_1	154.656
f_2	154.656
f_3	157.434
f_4	157.613
f_5	157.954
f_6	157.954
f_7	158.460
f_8	159.000
f_9	161.226
f_{10}	161.488

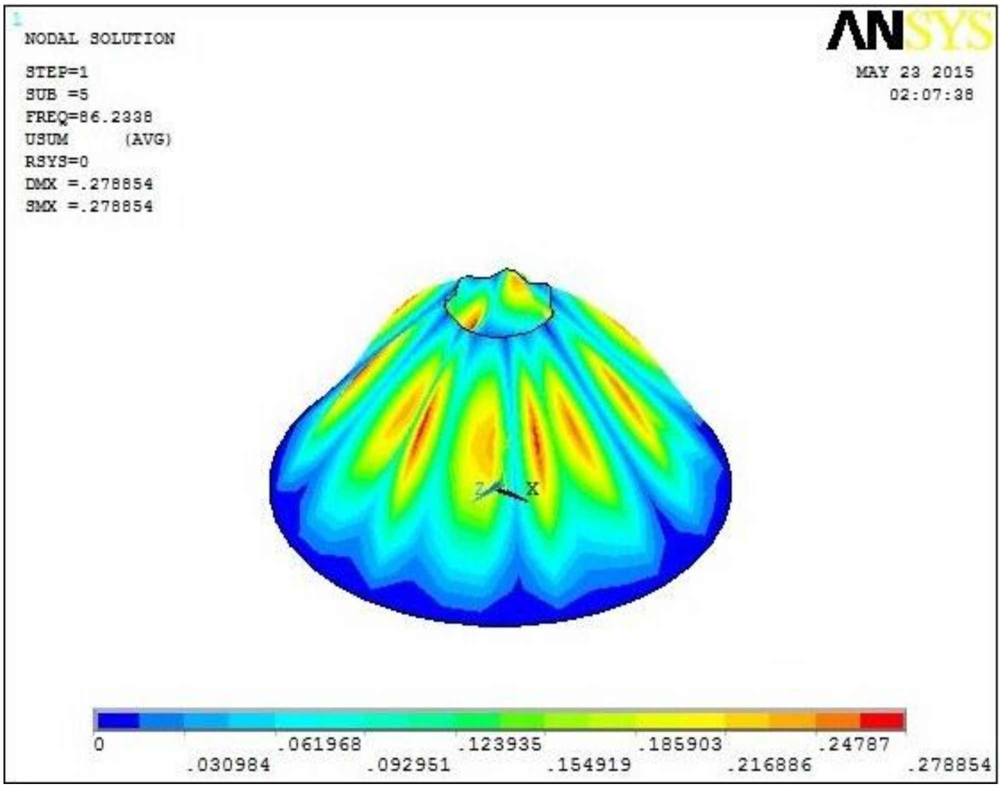
Figure 5 and 6 shows the mode shapes for the conical shell structure and for doubly curved (catenoidal) shell structure respectively. From the table 3 and 5 it is noticed that the fundamental frequencies are coming almost same for pair of two frequencies which are shown in the Figure 5 and 6.



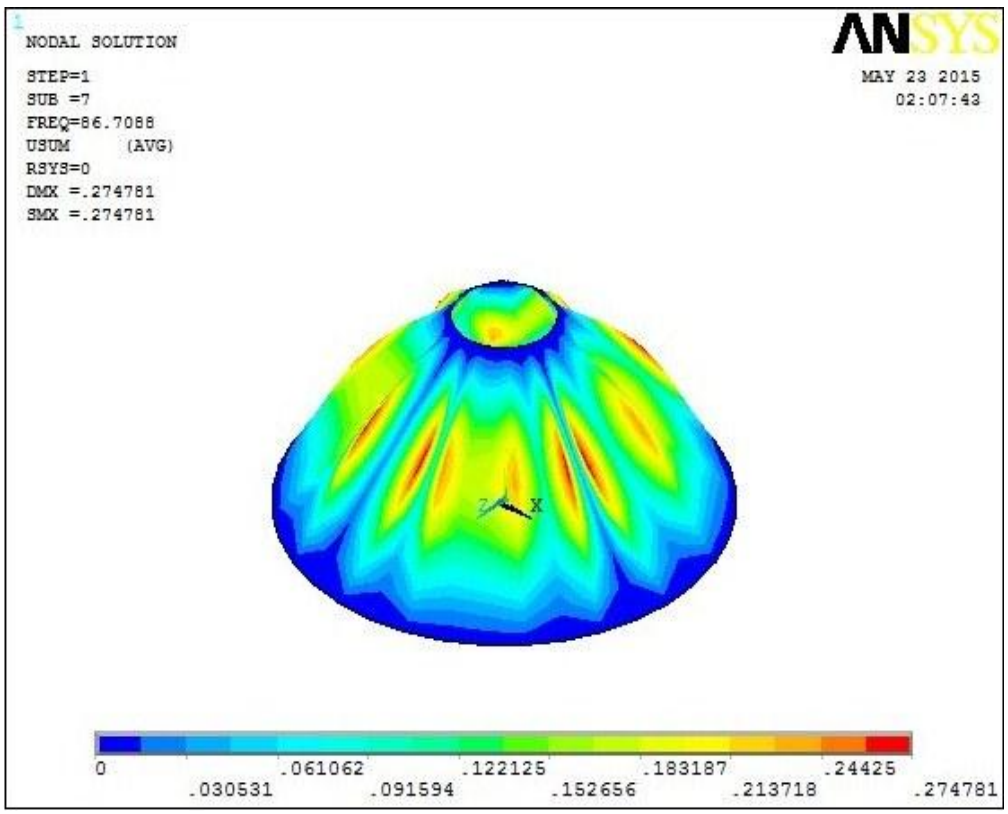
(a) mode shape 1-2



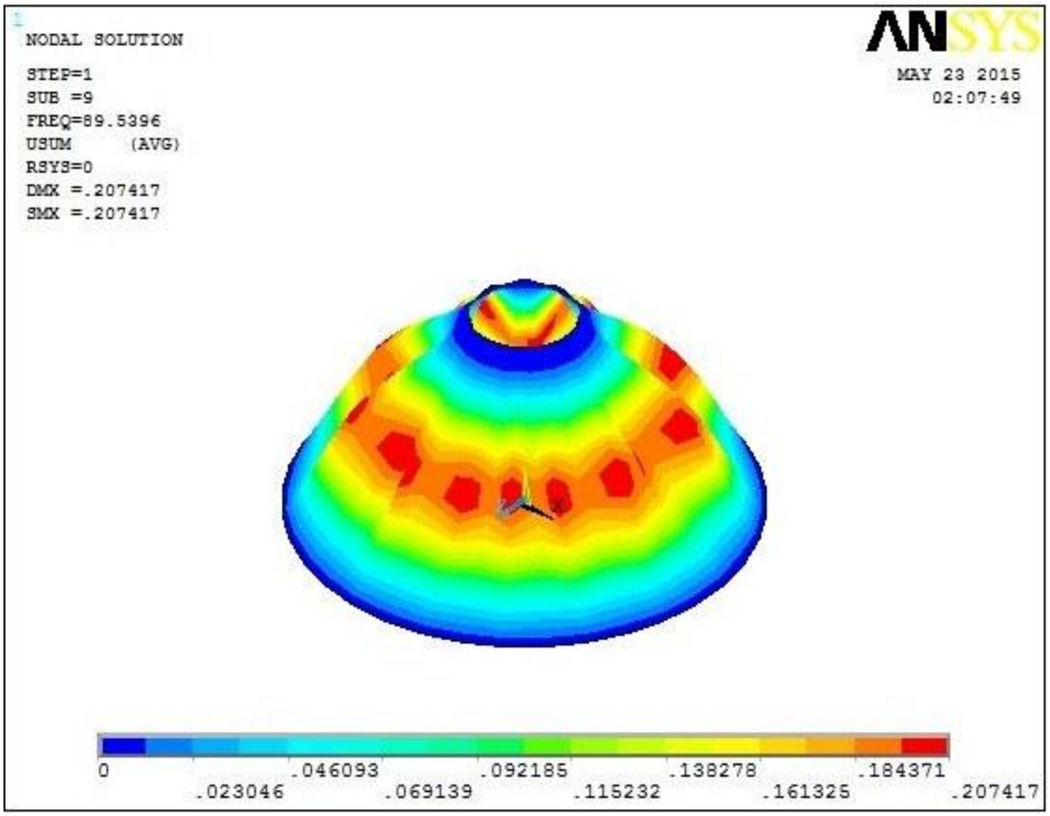
(b) mode shape 3-4



(c) mode shape 5-6

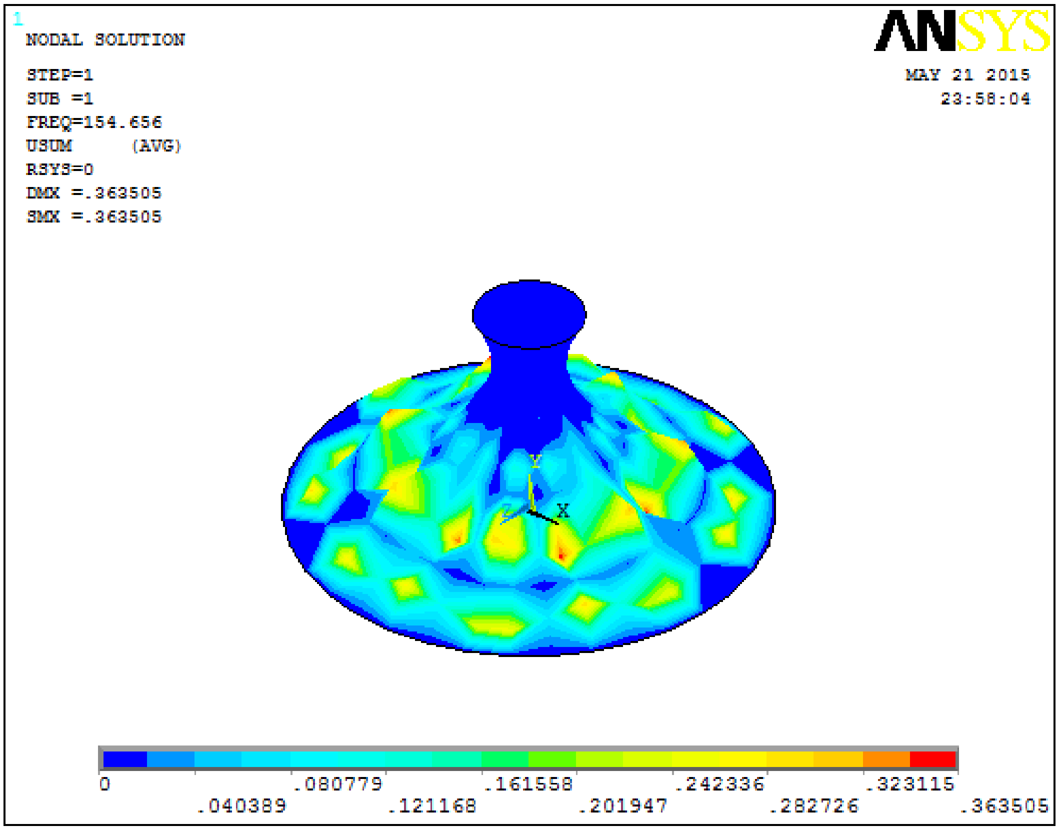


(d) mode shape 7-8

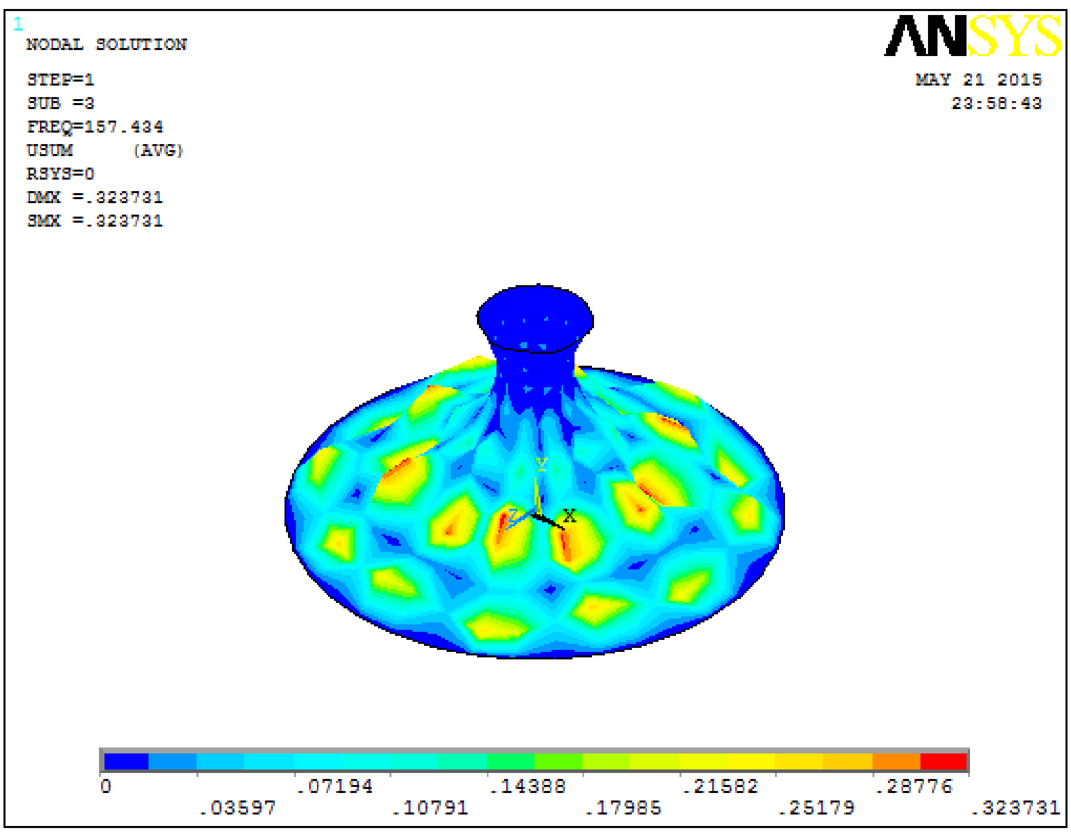


(e) mode shape 9-10

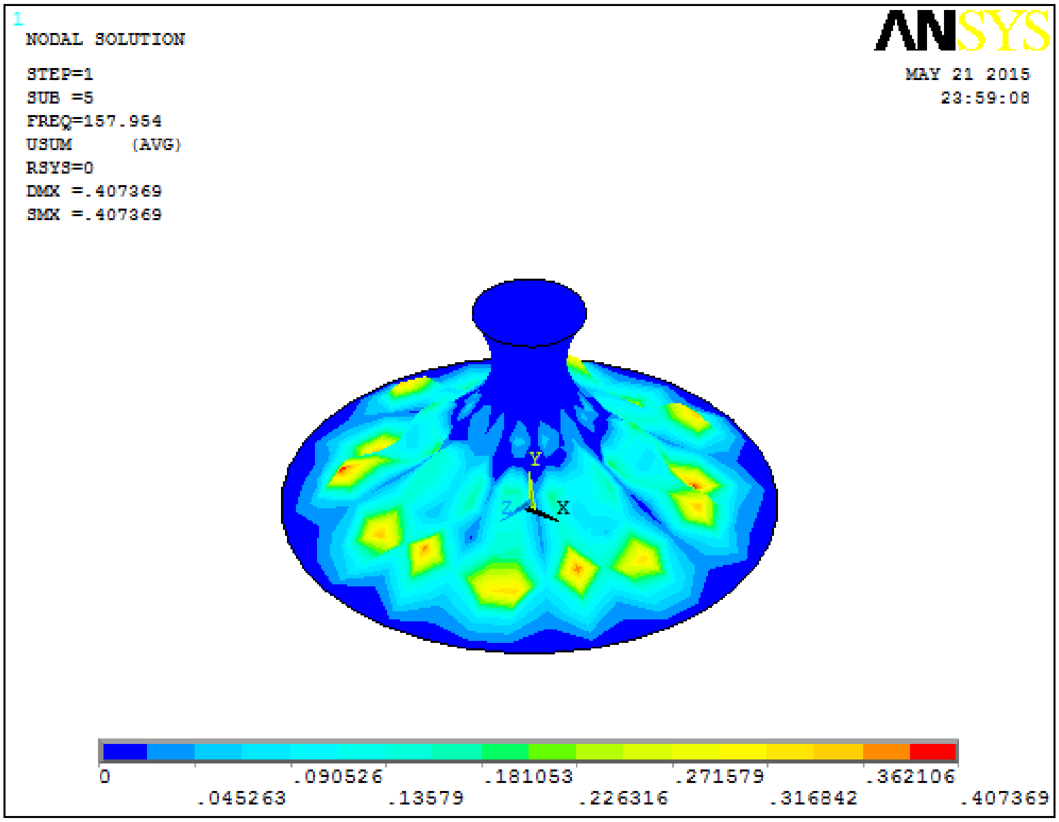
Figure 5. Mode shapes for conical shell.



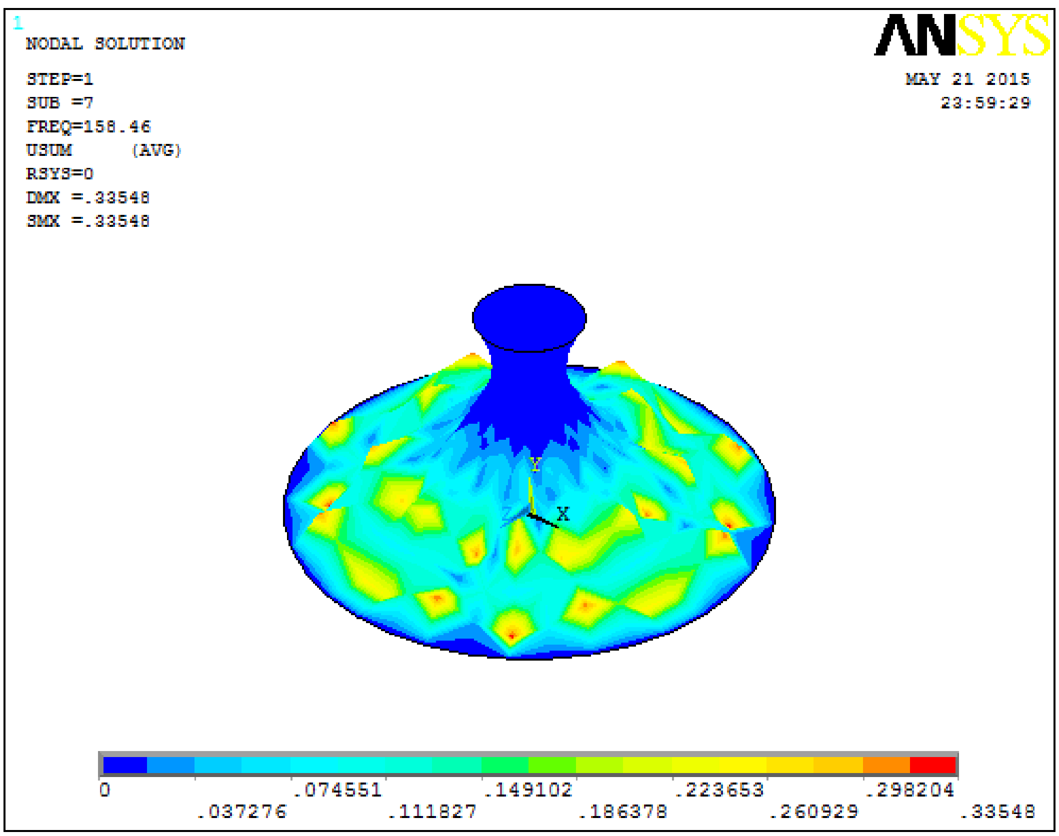
(a) mode shape 1-2



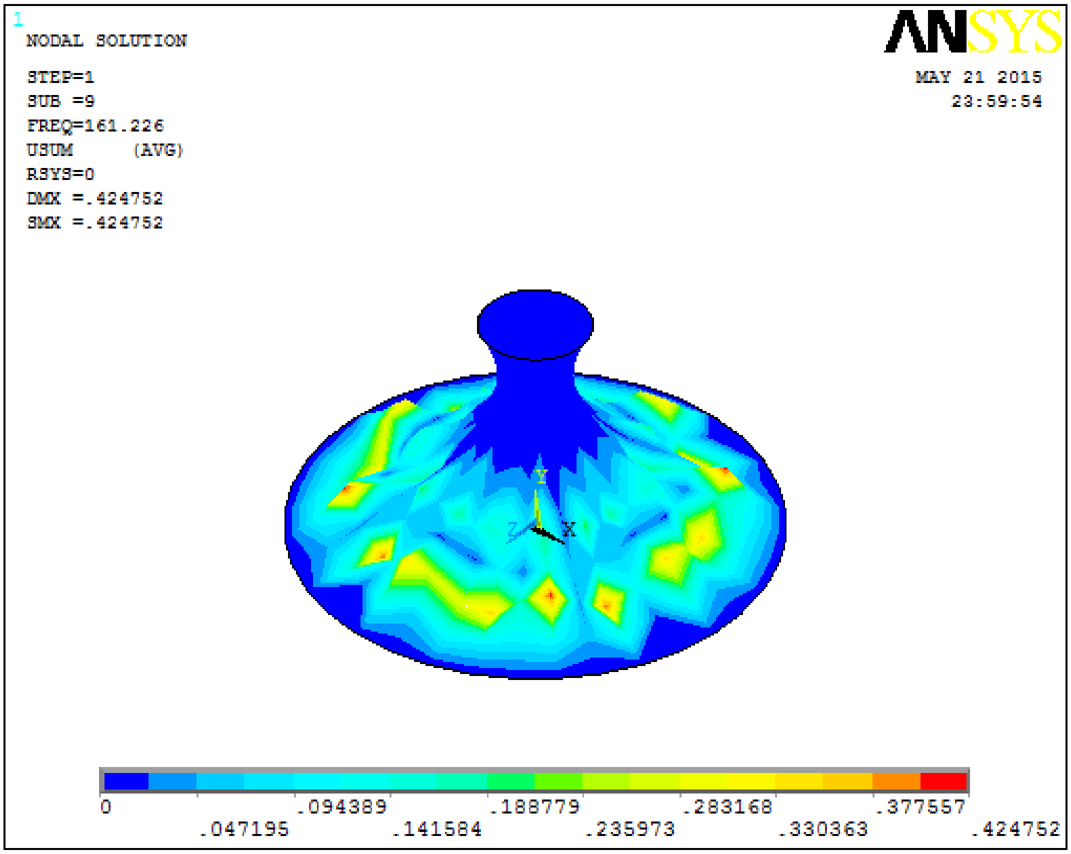
(b) mode shape 3-4



(c) mode shape 5-6



(d) mode shape 7-8



(e) mode shape 9-10

Figure 6. Mode shapes for doubly curved (catenoidal) shell.

CONCLUSION AND SCOPE OF FUTURE WORK

5.1 CONCLUSION

From the above solutions, the present work has been investigated. The vibration behavior and buckling analysis for the functionally graded carbon nanotube reinforced conical shell and for doubly curved (catenoidal) shell has been carried out by using the extended rule of mixture for the FG-CNTRC shell structures and considering the different types of distributions of CNTs. From the results, it is found that the buckling load parameter is maximum for the $FG-\Lambda$ and minimum in the case of FG-V type of CNT distribution. The buckling behavior and fundamental frequencies has been obtained by using the ANSYS parametric design language (APDL) code in the ANSYS software. The fundamental frequencies have been carried out by using the Block-Lanco’s method in ANSYS environment.

It is noticed that the fundamental frequencies and buckling load parameter increases with the increase of carbon nanotube volume fraction which implies that there is increase of stiffness of the FG-CNTRC shell structures with decrease in deflection.

5.2 SCOPE OF FUTURE WORK

The present work is based on the analysis of free vibration and buckling behavior of functionally graded material based carbon nanotube reinforced nanocomposite shell structures under the effect of compression load. Moreover, on the basis of some other vital parameters the present work can be extended for further research as follows:

i. To analyze the vibrational behavior and buckling analysis, the present work can be extended on the basis of temperature dependent material properties and thermomechanical loading conditions.

ii. The present work can be extended to evaluate the forced vibration for nanocomposite functionally graded carbon nanotube reinforced shell structures.

iii. The present work has been done by using a single walled carbon nanotube (SWCNT) which can be further extended on the basis of multi-walled carbon nanotube (MWCNT) and other different types of CNT to analyze the vibrational behavior.

iv. On the basis of this work, we can investigate the vibrational behavior for different geometric shell structures such as plate, spherical, hyperboloid, cylinder etc.

References:

[1] Ajayan P. M., Schadler L. S., Braun P. V. Nanocomposite Science and Technology. Weinheim, Wiley-Ych Verlag GmbH Co. KGaA, 2003.

[2] Iwahori Y., Ishiwata S., Sumizawa T., Ishikawa T. Mechanical properties improvements in two-phase and three-phase composites using carbon nano-fiber dispersed resin. Composites: Part A 36 (2005): pp. 1430–1439.

[3] Zhao P., Wang P.J., Zhang Z., Ren M.J., Liu D.S. First-principles study of the electronic transport properties of the carbon nanobuds. Physica B 405 (2010): pp. 2097–2101.

[4] Zhong G., Wang H., Yu H., Peng F. Nitrogen doped carbon nanotubes with encapsulated ferric carbide as excellent electrocatalyst for oxygen reduction reaction in acid and alkaline media. Journal of Power Sources 286 (2015): pp. 495-503.

[5] Bailey S.W.D., Lambert C.J. The electronic transport properties of N@C_{60}@(n,m) carbon nanotube peapods. Physica E 40 (2007): pp. 99–102.

[6] Lei Z.X., Liew K.M., Yu J.L. Buckling analysis of functionally graded carbon nanotube-reinforced composite plates using the element-free *kp*-Ritz method. Composite Structures 98 (2013): pp. 160–168.

[7] Chakraborty A., Gopalakrishnan S., Reddy J.N. A new beam finite element for the analysis of functionally graded materials. International Journal of Mechanical Sciences 45 (2003): pp. 519–539.

[8] Azadi M. Free and forced vibration analysis of FG beam considering temperature dependency of material properties. Journal of Mechanical Science and Technology 25 (1) (2011): pp. 69-80.

[9] Yas M.H., Heshmati M. Dynamic analysis of functionally graded nanocomposite beams reinforced by randomly oriented carbon nanotube under the action of moving load. Applied Mathematical Modelling 36 (2012): pp. 1371–1394.

[10] Dastjerdi R.M., Foroutan M., Pourasghar A. Dynamic analysis of functionally graded nanocomposite cylinders reinforced by carbon nanotube by a mesh-free method. Materials and Design 44 (2013): pp. 256–266.

[11] Woo J., Meguid S. A. and Liew K. M. Thermomechanical postbuckling analysis of functionally graded plates and shallow cylindrical shells. Acta Mechanica 165, (2003): pp. 99–115.

[12] Shen H.S. Thermal buckling and postbuckling behavior of functionally graded carbon nanotube-reinforced composite cylindrical shells. Composites: Part B 43 (2012): pp. 1030–1038.

[13] Xiang H.J., Yang J. Free and forced vibration of a laminated FGM Timoshenko beam of variable thickness under heat conduction. Composites: Part B 39 (2008): pp. 292–303.

[14] Wali M., Hentati T., Jarraya A., Dammak F. Free vibration analysis of FGM shell structures with a discrete double directors shell element. Composite Structures 125 (2015): pp. 295–303.

[15] Bhangale R.K., Ganesan N., Padmanabhan C. Linear thermoelastic buckling and free vibration behavior of functionally graded truncated conical shells. Journal of Sound and Vibration 292 (2006): pp. 341–371.

[16] Haddadpour H., Mahmoudkhani S., Navazi H.M. Free vibration analysis of functionally graded cylindrical shells including thermal effects. Thin-Walled Structures 45 (2007): pp. 591–599.

[17] Kadoli R., Ganesan N. Buckling and free vibration analysis of functionally graded cylindrical shells subjected to a temperature-specified boundary condition. Journal of Sound and Vibration 289 (2006): pp. 450–480.

[18] Heydarpour Y., Aghdam M.M., Malekzadeh P. Free vibration analysis of rotating functionally graded carbon nanotube-reinforced composite truncated conical shells. Composite Structures (2014).

[19] Jam J. E., Pourasghar A., Kamarian S., Namdaran N. Free vibration analysis of functionally graded nanocomposite cylindrical panel reinforced by carbon nanotube. Metall. Mater. Eng. Vol 19 (3) (2013): pp. 203-216.

[20] Viola E. and Tornabene F. Vibration analysis of conical shell structures using GDQ method. Far East J. Appl. Math. 25(1) (2006): pp. 23-39.

[21] Tornabene F. Free vibration analysis of functionally graded conical, cylindrical shell and annular plate structures with a four-parameter power-law distribution. Comput. Methods Appl. Mech. Engrg. 198 (2009): pp. 2911–2935.

[22] Tornabene F., Viola E., Inman D.J. 2-D differential quadrature solution for vibration analysis of functionally graded conical, cylindrical shell and annular plate structures. Journal of Sound and Vibration 328 (2009): pp. 259–290.

[23] Shu. C. An Efficient Approach For Free Vibration Analysis Of Conical Shells. Int. J. Mech. Sci. Vol. 38, Nos 8-9, (1996): pp. 935-949.

[24] Shakouri M., Kouchakzadeh M.A. Free vibration analysis of joined conical shells: Analytical and experimental study. Thin-Walled Structures 85 (2014): pp. 350–358.

[25] Su Z., Jin G., Ye T. Three-dimensional vibration analysis of thick functionally graded conical, cylindrical shell and annular plate structures with arbitrary elastic restraints. Composite Structures 118 (2014): pp. 432–447.

[26] Tornabene F., Liverani A., Caligiana G. General anisotropic doubly-curved shell theory: A differential quadrature solution for free vibrations of shells and panels of revolution with a free-form meridian. Journal of Sound and Vibration 331 (2012): pp. 4848–4869.

[27] Yas M.H., Pourasghar A., Kamarian S., Heshmati M. Three-dimensional free vibration analysis offunctionally graded nanocomposite cylindrical panels reinforced by carbon nanotube. Materials and Design 49(2013): pp. 583–590.

[28] Lei Z.X., Zhang L.W., Liew K.M. Vibration analysis of CNT-reinforced functionally graded rotating cylindrical panels using the element-free kp-Ritz method. Composites Part B 77 (2015): pp. 291-303.

[29] Pietraszkiewicz W. and Gorski J., Shell Structures: Theory and Applications: Volume 3. Leiden, CRC Press, 2013.