A preliminary study of wavelet method for solving ordinary differential equations

A thesis Submitted in partial fulfillment of the requirements for the award of the degree of

MASTER OF SCIENCE
IN
MATHEMATICS

By

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Under the Supervision of

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May, 2015

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DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"A preliminary study of wavelet method for solving ordinary differential equations"** in partial fulfillment of the requirement for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology Rourkela is an authentic record of my own work carried out under the supervision of **Prof. S. Chakraverty**. The matter embodied in this has not been submitted by me for the award of any other degree.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Date: Prof. S. Chakraverty

Place: (Supervisor)

ACKNOWLEDGEMENT

I want to specific my deepest sense of feeling to my supervisor **Prof. S. Chakraverty**, HOD, Department of Mathematics, National Institute of Technology Rourkela for his valuable steerage, help and time to time inspiration throughout my project. I am significantly grateful to **Prof. S. Sarangi**, Director, National Institute of Technology, Rourkela for providing wonderful facilities within the institute for effecting analysis.

I would prefer to offer sincere thanks to **Mr. Sukanta Nayak** for his inspirative support throughout my project work.

Finally all credits goes to my family and friends too for their continued support.

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ABSTRACT

Wavelet method is the backbone of various wavelet residue methods. In this context, Wavelet Galerkin Method is becoming a powerful tool to solve various type of differential equations. In this method, discrete orthogonal wavelets (family of functions with compact support) are used as shape functions which are easier to compute. These discrete orthogonal wavelets form a basis on a bounded domain. The connecting coefficients obtained by using Daubechies wavelet are presented to calculate the coefficient matrix.

Initially we have considered an example problem and the general solutions of the same has been discussed by using wavelet Galerkin method. Then Haar wavelet has been studied in detail. Finally using wavelet method various example problems have been investigated. The obtained results are found to be in good agreement.

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Chapter 1

Introduction

Wavelet method is very well-known to handle differential equations. Further, wavelet residue method viz. wavelet Galerkin method has been widely used by various researchers for solving differential equations. In this context, Chen et al. (1996) used Daubechies compact orthogonal wavelets as shape functions. These shape functions are used in Galerkin method. Then the wavelet Galerkin method is being used to obtain the solution of various differential equations. Further Lepik (2007) used wavelet method for solving nonlinear integral equations.

In view of the above literature, we have seen that the wavelet method has advantage over the other numerical techniques due to the involved scaling and shrink functions. The above idea is used in this thesis. The details and solution of various ordinary differential equations have been discussed here.

Chen (1997) and Hwang (1996) have used Haar wavelets as a tool to find the solution of different science and engineering problems. The main goal of this thesis is to find the solution of ordinary differential equation using wavelet method.

1.1 Wavelet

Wavelet analysis was developed within the mathematical literature in 1980. Wavelet is nothing but a wave in the form of function. There are many types of wavelet. Here we have discussed two types of wavelets viz. **Haar** and **Daubechies** wavelet.

1.2 Wavelet Function

Wavelets are well localized, oscillatory functions which provide a basis of $L^2(R)$ and can be modified to a basis of $L^2[a,b]$ where $[a,b]$ is a bounded domain. Where L is the filter coefficient and it must be even. The fundamental wavelet function is given by

$$\Psi(x) = \sum_{l=0}^{L-1} (-1)^l p_{1-l} \phi(2x - l) \tag{1.1}$$

where $\phi(x)$ is the scaling function, L is filter coefficient and $\Psi(x)$ is the wavelet function.

There has been an increasing interest within the wavelet-based approach in recent years. Any ordinary differential equation may be solved using wavelet method. Here wavelet function has been used to investigate various ordinary differential equations. In this context, various researchers have worked. Few important and related literatures have been presented.

1.3 Scaling function

Wavelets are outlined by the wavelet function $\Psi(x)$ and scaling function $\phi(x)$ within the bounded domain. Scaling functions are discrete type of functions which may produce the Daubechies function. We have defined a function to understand more about the scaling functions.
Scaling function is defined as

$$\Psi_{j,k}(t) = 2^{j/2} \phi(2^j t - k) \tag{1.2}$$

1.4 Haar Wavelet Function

Haar wavelet function is defined as

$$\phi(x) = \begin{cases} 1 \; if \, 0 \leq x \leq 1 \\ 0 \text{ otherwise} \end{cases} \tag{1.3}$$

within the range $0 \leq x \leq 1$.

Chapter 2

Developed Model

2.1 Galerkin Method

Let us consider a second order differential equation,

$$\frac{d^4\tilde{u}}{dx^4} + p(x) = 0, 0 \leq x \leq L \tag{2.1}$$

$$y(0) = 0$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}(1) = 1$$

Natural boundary conditions are given below,

$$\frac{d^2y}{dx^2}(L) = M$$

$$\frac{d^3y}{dx^3}(L) = -v$$

Let us consider the approximate solution of given equation as

$$\tilde{y}(x) = \sum_{i=1}^{N} c_i\Omega_i(x) \tag{2.2}$$

where Ω is trial function. Now we will find weighted residual equation (Galerkin method).

$$\int_0^L \left(\frac{d^4\tilde{u}}{dx^4} - p(x)\right)\Omega_i(x)dx = 0 \tag{2.3}$$

Let us integrate twice Eq.(2.3) and substituting approximate solution, we will have,

$$\int_0^L \sum_{j=1}^{N} C_j \frac{d^2\Omega_j}{dx^2}\frac{d^2\Omega_i}{dx^2}dx = \int_0^L p(x)\Omega_i(x)dx - \frac{d^3\tilde{y}}{dx^3}\Omega_i|_0^L + \frac{d^2\tilde{y}}{dx^2}\frac{\Omega_i}{dx}|_0^L, i = 1, ..., N \tag{2.4}$$

The matrix form can be written as

$$[K]_{n\times n}\{C\}_{n\times 1} = \{F\}_{n\times 1} \tag{2.5}$$

where

$$K_{ij} = \int_0^L \frac{d^2\Omega_j}{dx^2}\frac{d^2\Omega_i}{dx^2}dx \tag{2.6}$$

$$F_i = \int_0^L p(x)\Omega_i(x)\,dx - \frac{d^3y}{dx^3}\Omega_i\Big|_0^L + \frac{d^2y}{dx^2}\frac{d\Omega_i}{dx}\Big|_0^L \tag{2.7}$$

2.2 Wavelet Galerkin Method

According to the Galerkin method, we consider trial solutions and scaling functions as

$$u(x) = 2^{\frac{J}{2}}\sum_{k=2-L}^{2^J-1} u_k\phi_{J,k}(x) \tag{2.8}$$

or

$$u(x) = 2^{\frac{J}{2}}\sum_{k=2-L}^{2^J-1} u_k\phi(2^Jx-k) \tag{2.9}$$

$J > 0$

$$\phi_{J,k}(x) = \phi(2^Jx-k) \tag{2.10}$$

we will write

$$E(x) = 2^{\frac{J}{2}}\sum_{j=2-L}^{2^J-1} E_j\phi(2^Jx-j) \tag{2.11}$$

where the coefficients E_j are calculated using the inner product,

$$E_j = \int_0^1 E(x)\,\phi_{J,j}(x)dx \tag{2.12}$$

The Galerkin residual R is found by substituting Eqs.(2.9) and (2.11) in Eq.(2.12),

$$R = \sum_{j=2-L}^{2^J-1}\sum_{k=2-L}^{2^J-1} E_ju_k(\phi^{(0)}_{J,j}(x)\phi^{(2)}_{J,k}(x) + \phi^{(1)}_{J,j}(x)\phi^{(1)}_{J,k}(x)) + \beta\sum_{k=2-L}^{2^J-1} u_k\phi^{(0)}_{J,k}(x) \tag{2.13}$$

The nth derivative of the scaling function $\phi^{(n)}_{J,j}(x)$ is

$$\phi^{(n)}_{J,j}(x) = 2^{nJ+\frac{J}{2}}\phi^{(n)}_{J,j}(2^Jx-j), \quad n = 0, 1, ..., \frac{L}{2}-1 \tag{2.14}$$

According to the Galerkin method, scaling functions of level J are selected as the weighting functions. The inner product of the residual and the weighting functions is equal to zero.

$$\int_0^1 R\phi_{J,l}(x)dx = 0 \ for \ \ i = 2-L, 3-L, ..., 2^J - 1 \tag{2.15}$$

which using Eq. (2.13),

$$\sum_{j=2-L}^{2^J-1} \sum_{k=2-L}^{2^J-1} E_j u_k (a_{l,j,k} + b_{l,j,k}) + \beta \sum_{k=2-L}^{2^J-1} u_k c_{l,k} = 0 \tag{2.16}$$

for $l = 2-L, 3-L, ..., 2^J - 1$,
where

$$a_{l,j,k} = \int_0^1 \phi_{J,l}(x)\phi_{J,j}^{(0)}(x)\phi_{J,k}^{(2)}(x)\,dx \tag{2.17}$$

$$b_{l,j,k} = \int_0^1 \phi_{J,l}(x)\phi_{J,j}^{(1)}(x)\phi_{J,k}^{(1)}(x)\,dx \tag{2.18}$$

$$c_{l,j,k} = \int_0^1 \phi_{J,l}(x)\phi_{J,k}^{(0)}(x)\,dx \tag{2.19}$$

Hwang (1996) referred to integrals of form Eqs.(2.17) and (2.19) as three term connection coefficients, and Eq.(2.19) as two term connection coefficients. Zhao (2010) suggested improved algorithms for exact calculation of these connection coefficients. The standard notation for the two-term and three term connection coefficients are as follows

$$\Gamma_k^n(x) = \int_0^x \phi(y)\phi^{(n)}(y-k)dy \tag{2.20}$$

$$\Omega_{j,k}^{m,n}(x) = \int_0^x \phi(y)\phi^{(n)}(y-j)\phi^{(n)}(y-k)dy \tag{2.21}$$

Now the value of $\Gamma_k^n(x)$ and $\Omega_{j,k}^{m,n}(x)$ can be computed from in Hwang (1996).

2.3 Integration of Haar Wavelet Function

Haar wavelet system and integrals of first four Haar wavelet function can be expressed as Zhao (2010).
The Haar wavelet function is

$$\phi(x) = \begin{cases} 1 \ if 0 \leq x \leq 1 \\ 0 \text{ otherwise} \end{cases} \tag{2.22}$$

Now let us denote the Haar wavelet as $\phi(x) = h(x)$ and divide this into first four intervals to compute the Haar integral function in the range $[0,1]$, Lepik (2007)

$$\int_0^t h_0(t)dt = \{t, \quad 0 \le t \le 1 \tag{2.23}$$

$$\int_0^t h_1(t)dt = \begin{cases} t, & 0 \le t \le \frac{1}{2} \\ 1-t, & \frac{1}{2} \le t \le 1 \end{cases} \tag{2.24}$$

$$\int_0^t h_2(t)dt = \begin{cases} t, & 0 \le t \le \frac{1}{4} \\ \frac{1}{2}-t, & \frac{1}{4} \le t \le \frac{1}{2} \end{cases} \tag{2.25}$$

and

$$\int_0^t h_3(t)dt = \begin{cases} t-\frac{1}{2}, & \frac{1}{2} \le t \le \frac{3}{4} \\ 1-t, & \frac{3}{4} \le t \le 1 \end{cases} \tag{2.26}$$

Suppose the integration of first four Haar wavelet function P_0, P_1, P_2 and P_3 are defined on the same interval as the interval of Haar wavelet function Lepik (2007)

$$\int_0^t h_0(t)dt = P_0(t) \tag{2.27}$$

$$\int_0^t h_1(t)dt = P_1(t) \tag{2.28}$$

$$\int_0^t h_2(t)dt = P_2(t) \tag{2.29}$$

and

$$\int_0^t h_3(t)dt = P_3(t) \tag{2.30}$$

Let us again consider the function and integrate it function in the range $[0,1]$. Hence obtained as, Lepik (2007)

$$\int_0^t P_0(t)dt = \begin{cases} \frac{t^2}{2}, & 0 \le t \le 1 \end{cases} \tag{2.31}$$

$$\int_0^t P_1(t)dt = \begin{cases} 0, & t \in [0,0) \\ \frac{t^2}{2}, & t \in [0,\frac{1}{2}) \\ \frac{1}{4}-\frac{1}{2}(1-t)^2, & t \in [\frac{1}{2},1) \\ \frac{1}{4}, & t \in [1,1) \end{cases} \tag{2.32}$$

$$\int_0^t P_2(t)dt = \begin{cases} 0, & t \in [0,0) \\ \frac{t^2}{2}, & t \in [0,\frac{1}{4}) \\ \frac{1}{16}-\frac{1}{2}(1-t)^2, & t \in [\frac{1}{4},\frac{1}{2}) \\ \frac{1}{16}, & t \in [\frac{1}{2},1) \end{cases} \tag{2.33}$$

and

$$\int_0^t P_3(t)dt = \begin{cases} 0, & t \in [0, \frac{1}{2}) \\ \frac{1}{2}\Big(1 - \frac{t}{2}\Big)^2, & t \in [\frac{1}{2}, \frac{3}{4}) \\ \frac{1}{16} - \frac{1}{2}(1-t)^2, & t \in [\frac{3}{4}, 1) \\ \frac{1}{32}, & t \in [1, 1) \end{cases} \tag{2.34}$$

Suppose the second integration of Haar wavelet function Q_0, Q_1, Q_2 and Q_3 which are defined on the same interval as the interval of Haar wavelet function,

$$Q_0(t) = \int_0^t P_0(t)dt \tag{2.35}$$

$$Q_1(t) = \int_0^t P_1(t)dt \tag{2.36}$$

$$Q_2(t) = \int_0^t P_2(t)dt \tag{2.37}$$

and

$$Q_3(t) = \int_0^t P_3(t)dt \tag{2.38}$$

and so on.

CHAPTER 3

Example Problems

3.1 Example of Galerkin method

Example 1

Let us consider fourth order ordinary differential equation

$$\frac{d^4y}{dx^4} + 5x = 0, \quad 0 \leq x \leq 1 \tag{3.1}$$

subject to the boundary conditions

$$y(0) = 0$$

$$y'(0) = 0$$

$$y''(1) = 2$$

$$y'''(1) = -1$$

Two trial functions will satisfy the boundary conditions

$$\Omega_1(x) = x^2, \Omega_1'(x) = 2x, \Omega_1''(x) = 2 \tag{3.2}$$

$$\Omega_2(x) = x^3, \Omega_2'(x) = 3x^2, \Omega_2''(x) = 6x \tag{3.3}$$

An approximate solution has been considered which satisfies the essential boundary conditions. Substituting the values of scaling function in approximation solution of the differential equation, we will get

$$\tilde{y}(x) = \sum_{i=1}^{2} c_i\Omega_i(x) = c_1x^2 + c_2x^3 \tag{3.4}$$

From Eq.(2.1)

$$p(x) = -5x \tag{3.5}$$

the coefficient matrix can be mentioned as

$$K_{11} = \int_0^1 \left(\Omega_1''\right)^2 dx \ = 4 \tag{3.6}$$

$$K_{12} = K_{21} = \int_0^1 \left(\Omega_1'\Omega_2'\right) dx \ = 6 \tag{3.7}$$

$$K_{22} = \int_0^1 \left(\Omega''{}_2\right)^2 dx = 12 \tag{3.8}$$

Putting the value of K_{11}, K_{12}, K_{21} and K_{22} in a matrix form

$$[K] \ = \begin{pmatrix} 4 & 6 \\ 6 & 12 \end{pmatrix} \tag{3.9}$$

from Eq.(2.7), we will get

$$F_1 = \int_0^1 (-5x)\, x^2 dx + 1 + 4 = \frac{15}{4} \tag{3.10}$$

$$F_2 = \int_0^1 (-5x)\, x^3 dx + 1 + 6 = 6 \tag{3.11}$$

We may find

$$\{c\} = [K]^{-1}\{F\} \tag{3.12}$$

So we have the approximated solution

$$\tilde{y}(x) \ = \frac{3}{4}x^2 - \frac{1}{8}x^3 \tag{3.13}$$

The exact solution of Eq.(3.1) may be written as

$$y(x) \ = \frac{-1}{25}x^5 \ + \frac{1}{4}x^3 + \frac{2}{3}x^2 \tag{3.14}$$

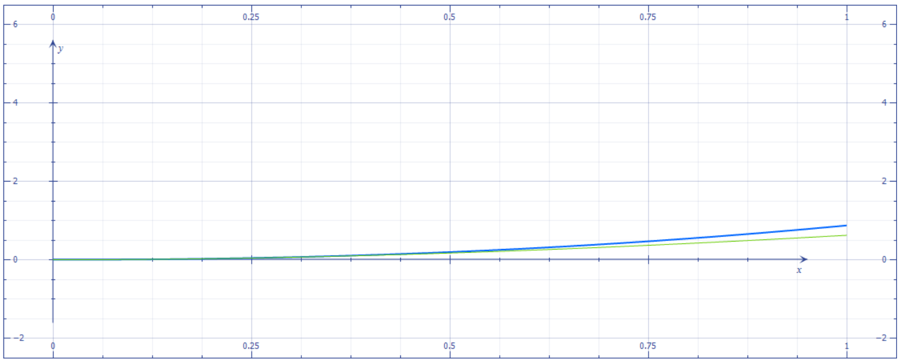
The comparison of exact and obtained solutions for values of x viz. $x = 0.1, 0.2, 0.3$ and 0.4 are given in Table 1.

Table 1. comparison of exact and Haar wavelet solutions of example 1.

x	$y(x)$	$\tilde{y}(x)$
0.1	0.00738	0.00692
0.2	0.02900	0.02865
0.3	0.06413	0.0665
0.4	0.11200	0.12226

The solutions of Eqs(3.13) and (3.14) have been plotted to show the comparison between exact and approximated solutions.

Figure 3.1: **Comparison between exact and approximated solution in the interval [0,1] (where light green line for approximate solution and blue line for exact solution)**



3.2 Example of wavelet Galerkin method

Let us consider the second order ordinary differential equation as

$$\frac{d^2u(x)}{dx^2} + \beta^2 u(x) = 0 \tag{3.15}$$

Suppose the solution of Eq (3.15)

$$u(x) = 2^{\frac{J}{2}} \sum_{k=2-L}^{2^J-1} u_k \phi(2^J x - k) \tag{3.16}$$

where

$$\phi_{J,k}(x) = 2^{\frac{J}{2}} \phi(2^J x - k) \tag{3.17}$$

and then we have

$$u(x) = 2^{\frac{J}{2}} \sum_{k=2-L}^{2^J-1} u_k \phi_{J,k}(x) \tag{3.18}$$

The Galerkin residual is found by substituting Eq.(3.18) into Eq.(3.15)
After putting $L = 6$,$J = 0$ Simon (2012), we will get

$$\sum_{k=-4}^{0} u_k \phi_{0,k}^2(x) + \beta^2 \sum_{k=-4}^{0} u_k \phi_{0,k}(x) = 0 \tag{3.19}$$

Using Galerkin Method, scaling functions of level J are selected as the weighing function .The inner product of residual and the weighing functions is set to be zero, Simon (2012).

$$\int_0^1 R\, \phi_{0,l}(x)\, dx \ = 0 \tag{3.20}$$

Assume the integral (3.20) to solve Eq.(3.15)

$$\Omega_{j,k}^{m,n}(x) = \int_0^x \phi(y)\, \phi^{(m)}(y-j)\, \phi^{(n)}(y-k)\, dy \tag{3.21}$$

Consider Eqs (3.18),(3.19) and integral (3.21) at $\beta = 1$
Eq.(3.19) may be represented in omega integral as, Simon (2012)

$$u(1) = \sum_{k=-4}^{0} \Omega_{0,k}^{0,2}(1) \ + \sum_{k=-4}^{0} \Omega_{0,k}^{0,0}(1) \tag{3.22}$$

3.3 Examples of Haar wavelet method to solve differntial equation

3.3.1 Example 2

Let us consider another second order differential equation as

$$\frac{d^2y}{dx^2} + 1 = 0, 0 \leq x \leq 1 \tag{3.23}$$

subject to

$$y(0) = 0$$
$$y'(1) = 1$$

The exact solution of Eq.(3.23) may be obtained as

$$y(x) = 1.85082 \sin(x) \tag{3.24}$$

Now we will solve this differential equation by Haar wavelet function method to get the approximate solution.
So, Let us consider

$$\frac{d^2y}{dx^2} = \sum_{i=0}^{3} a_i h_i(x) \tag{3.25}$$

Where function $h_i(x)$ is Haar wavelet function. Let us integrate the both side of Eq.(3.25) with respect to independent variable x, then we will get,

$$\frac{dy}{dx} = \sum_{i=0}^{3} a_i p_i(x) + k_1 \tag{3.26}$$

Where function $p_i(x)$ is the first integration of the Haar wavelet function and k_1 is the constant of integration in Eq.(3.26). Now again Let us integrate the both side of Eq.(3.26) with respect to independent variable x, then we will get,

$$y(x) = \sum_{i=0}^{3} a_i q_i(x) + k_1 x + k_2 \tag{3.27}$$

where function $q_i(x)$ is the second integration of Haar wavelet function and k_2 is the constant of integration in Eq.(3.27).
Now substituting the value of Eq.(3.25) in the differential Eq.(3.23) and we will get,

$$\sum_{i=0}^{3} a_i h_i(x) + 1 = 0 \tag{3.28}$$

where $i = 0, 1, 2$ and 3
Substituting the value of $i = 0, 1, 2$ and 3 in the Eq.(3.28)

$$a_0 h_0(x) + a_1 h_1(x) + a_2 h_2(x) + a_3 h_3(x) + 1 = 0 \tag{3.29}$$

and we will get

$$a_0 + a_1 + a_2 + a_3 = -1 \tag{3.30}$$

$$y(x) = a_0 q_0(x) + a_1 q_1(x) + a_2 q_2(x) + a_3 q_3(x) + k_1 x + k_2 \tag{3.31}$$

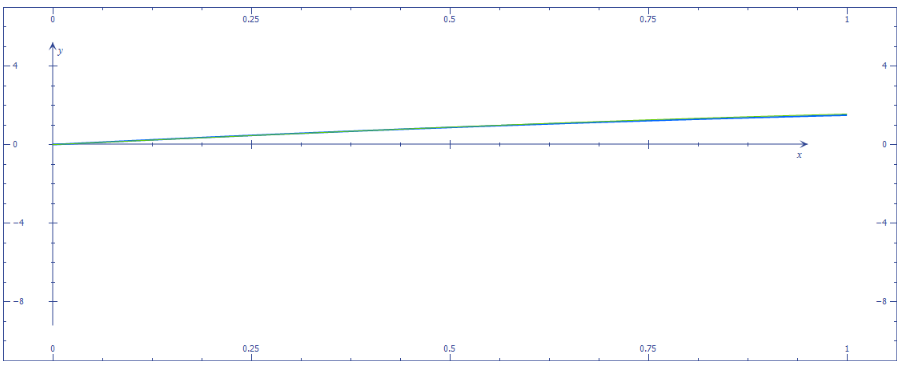
$$y(x) = (a_0 + a_1 + a_2 + a_3)\frac{x^2}{2} + k_1 x + k_2 \tag{3.32}$$

Hence we will get approximated solution as

$$\tilde{y}(x) = -\frac{x^2}{2} + 2x \tag{3.33}$$

we have graph of solution Eqs(3.24) and (3.33) to show the comparison between exact and approximated solutions.

Figure 3.2: **Comparison between exact and approximated solution in the interval [0,1] (where light green line for exact solution and blue line for approximated solution)**



3.3.2 Example 3

Let us consider the second order differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0, 0 \leq x \leq 1 \tag{3.34}$$

subject to

$$y(0) = 2$$
$$y'(0) = 2$$

The exact solution of Eq.(3.34) may be obtained as

$$y(x) = e^{-\frac{x}{2}} \left[2\cos\frac{\sqrt{3}}{2}x + \frac{2}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}x \right] \tag{3.35}$$

Now we will solve the above differential equation by Haar wavelet function method to get the approximate solution.
So, let us consider

$$\frac{d^2y}{dx^2} = \sum_{i=0}^{1} a_i h_i(x) \tag{3.36}$$

Where function $h_i(x)$ is the Haar wavelet function. Integrating both sides of Eq.(3.36) with respect to independent variable x, we will get,

$$\frac{dy}{dx} = \sum_{i=0}^{1} a_i p_i(x) + k_1 \tag{3.37}$$

Where function $p_i(x)$ is the first integration of the Haar wavelet function and k_1 is the constant of integration in Eq.(3.37). Similarly integrating Eq.(3.37), we will get,

$$y(x) = \sum_{i=0}^{1} a_i q_i(x) + k_1 x + k_2 \tag{3.38}$$

Where function $q_i(x)$ is the second integration of the Haar wavelet function and k_2 is the constant of integration in Eq.(3.38).

Now substituting the values of Eqs.(3.36),(3.37) and (3.38) in Eq.(3.34).
We will get,

$$\sum_{i=0}^{1} a_i h_i(x) + \sum_{i=0}^{1} a_i p_i(x) + k_1 + \sum_{i=0}^{1} a_i q_i(x) + k_1 x + k_2 = 0 \tag{3.39}$$

where $i = 0, 1$
Substitute the value of $i = 0, 1$ in the Eq.(3.39)

$$a_0 h_0(x) + a_1 h_1(x) + a_0 p_0(x) + a_1 p_1(x) + k_1 + a_0 q_0(x) + a_1 q_1(x) + k_1 x + k_2 = 0 \tag{3.40}$$

we will get

$$a_0 + a_1 = -\left(\frac{k_1 + k_2 + k_1 x}{1 + x + \frac{x^2}{2}}\right) \tag{3.41}$$

$$y(x) = a_0 q_0(x) + a_1 q_1(x) + k_1 x + k_2 \tag{3.42}$$

and

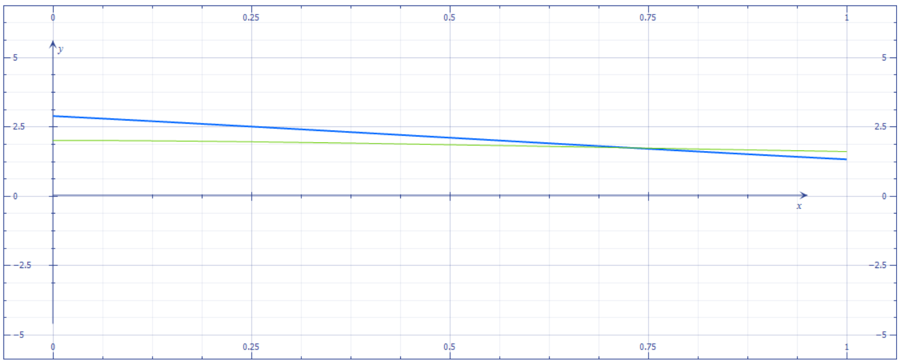
$$y(x) = (a_0 + a_1)\frac{x^2}{2} + k_1 x + k_2 \tag{3.43}$$

Hence the approximated solution can be obtained as

$$\tilde{y}(x) = \frac{4 + 4x}{2 + 2x + x^2} \tag{3.44}$$

The comparison between exact and approximated solutions of Eqs(3.35) and (3.44) have been shown in the Figure 3.3 .

Figure 3.3: **Comparison between exact and approximated solution in the interval [0,1] (where light green line for approximate solution and blue line for exact solution)**



3.4 Conclusion

In recent years wavelet method has gained an important role and become popular among researchers. The scaling and shrink functions are main tools of wavelet method which distinguish it from other numerical methods. In view of this we have considered wavelet and wavelet Galerkin method to investigate various ordinary differential equations. As such, Haar and Daubechies wavelets have been used for the investigation. Further, some example problems are solved and obtained results are compared with exact solutions. Finally we have seen that the obtained solutions are found to be in good agreement.

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List of Publication

1. Agrawal D[1]., Nayak S., and Chakraverty S., *Wavelet Galerkin method for solving differential equations,* 42nd annual conference of Orissa Mathematical Society, Vyasanagar Autonomous College Jajpur Road, Odisha, Feb 7-8,2015.

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