

# **DISTILLATION COLUMN CONTROL STRATEGIES; IMC & IMC BASED PID CONTROLLER**

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF  
THE REQUIREMENTS FOR THE DEGREE OF

**Bachelor of Technology in  
Electronics and Instrumentation Engineering**



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**National Institute of Technology**

**Rourkela**

**2011 - 2015**

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**Under the Guidance of**

***Prof. T. K. DAN***



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**CERTIFICATE**

This is to certify that the project report titled “**DISTILLATION COLUMN CONTROL STRATEGIES; IMC & IMC BASED PID CONTROLLER**” submitted by Biswa Bisruta Tripathy (111EI0450), Sandeep Kumar Khatua (111EI0246) & Labanya Behera (111EI0450) in the partial fulfilment of the requirements for the award of Bachelor of Technology in Electronics & Instrumentation engineering during the session 2011-2015 at National Institute of Technology, Rourkela is an authentic work carried out by them under my supervision.

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# **ABSTRACT**

## **DISTILLATION COLUMN CONTROL STRATEGIES**

Distillation column is a multi-input multi-output system, used especially in petrochemical industries. It is a multi-variable control, used to separate various components of a mixture. It is a highly interacting system. So the objective of this project is to control the compositions of top and bottom products.

The performance analysis of controlling different compositions has been found out using different control strategies i.e. PID controller as well as IMC controller. It is found out that the performance analysis of IMC controller is better than that of the PID controller.

The project emphasizes mainly on the tuning of the IMC controller. For that, different models of the process have been taken and the responses have been found out. Some empirical relationships have been derived between the tuning parameters and the process response characteristics. Based on this relationships, an empirical formula has been derived between the tuning parameter and the process parameters. That has been tested for an unknown process and verified in order to get the desired response characteristics.

## **IMC & IMC BASED PID CONTROLLER**

Internal Model Control (IMC) and the IMC based PID have widespread use in current control industries. Internal Model Control (IMC) is a commonly used mode to design and tune the various types of control transparently. Here, we analyse different concepts that are widely used in IMC design as well as IMC based PID for implementing a plant transfer function to show the benefits of using PID controller in IMC.

The IMC-PID controller are generally used over IMC for improved set-point tracking however poor disturbance occurs for the process that has a small time-delay, because in several areas that involve the use of process control techniques, set point tracking is not that important as disturbance rejection for an unstable process.

Hence, we have to choose for a better IMC filter so that we can design an IMC-PID controller to get improved set-point tracking in an unstable process. In order to obtain the requisite response, the controller functioned in a different manner for diverse set of values of the filter tuning parameters. Because the IMC method is based on cancellation of pole zero, techniques for designing an IMC gives improved set point responses. But the major demerit is that the IMC usually results in a large settling time for the load disturbances in lag dominant processes. This is a major disadvantage in control industries.

An approximation error generally occurs, for the reason that all the IMC-PID methods usually contain some type of model factorization techniques that is used to convert the IMC controller to the PID controller. This error is a major disadvantage for those processes that have time delay. Therefore it is important that we take some transfer functions that have significant time delay or they have some non-invertible parts (The transfer function contains RHP poles or the zeroes.)

The thesis also consists of the design of tuning for a generalized process. Both for the IMC and IMC based PID, we have designed an empirical formula between the tuning parameter and the process variable i.e. the process time constant. Thus, for a given desired value of settling time or rise time, we can easily find out the value of the tuning parameter. The equation developed is applicable for any process.



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# **Chapter 1**

## **A BRIEF DISCUSSION ON DISTILLATION COLUMN**

## 1.1 Distillation Column: Background

By and large, the main control objective in a distillation column operation is to maintain or keep the various process variables (i.e. controlled variables) at their desired set point, in the presence of various disturbances, by changing or manipulating the manipulated variables. The performance can be enhanced further through dead time compensation, better time response, and reduction in overshoot, improved set point tracking and improved disturbance rejection.

Distillation column is widely used in various industries such as:

- Used in petrochemical refineries and industries
- Coal tar processing
- Natural gas processing
- Liquor Production
- Liquefied air separation & Hydrocarbon Solvent Production
- Cryogenic distillation used in steel & metallurgical plant

The main idea behind designing this column is the separation of a mixture of two pure liquids that have different boiling points or in other words different volatility. The mixture is heated to a temperature in between the boiling points of the respective liquids, so that the more volatile of the two liquids boils first and get transformed into vapour which is then collected and condensed as the other liquid remains. For example, it is known that the boiling point of water is 100°C and that of ethanol is 83°C at atmospheric pressure. So if the mixture is heated to a temperature say 92°C, ethanol being the more volatile material will boil first and vaporize. So the differences in relative volatility of the two components is basic to a distillation column.

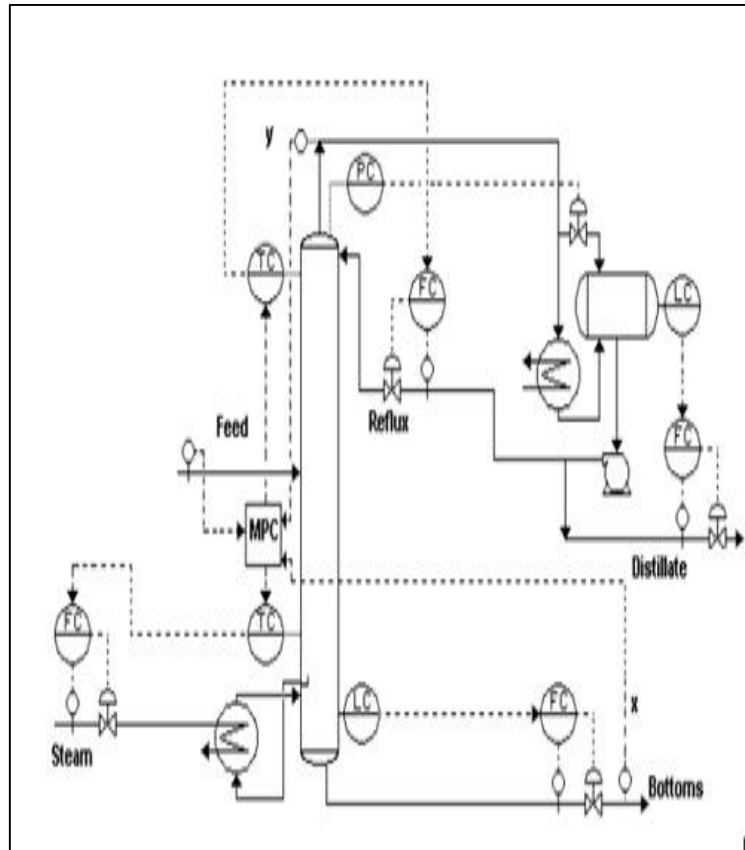


Fig 1.1:- Distillation Column

## 1.2 Description

In a general distillation column, there are a series or set of stacked plates in which a fluid feed (which is a combination of both the liquids) is allowed to enter into the column at one or more points. The feed flows over these plates, and the vapour bubbles up through the fluid through openings that are present in the plates. When this fluid flows down through the column, the vapour comes into contact with the fluid several times (because of the multiple plates) which is one of the most critical or important among all the processes that occurs in these columns. Both these phases, i.e., (liquid as well as vapour) are brought into direct contact so that one molecule having a greater boiling point (which in our case, is taken water) converts from its vapour phase to liquid phase by the liberation of energy, while at the same time another molecule that has a lower boiling point uses the free energy to convert from liquid phase to vapour phase.

Some of this liquid flows out of the base, out of which some amount is heated in the reboiler and sent back to the column and is known as boilup, labelled as  $V$ . Also the

left over fluid is called as the bottom product, labelled as *B*. Also some amount of vapour comes out from column's top end and is sent back to a liquid state in the condenser. Some amount of this liquid is sent back to the column as reflux *L*. The left over portion is called as the top product or distillate *D*. On a given plate, vapor and liquid phases approach pressure equilibrium, thermal equilibrium, and composition equilibrium which depends upon the efficiency of the plate.

Distillation columns are widely used in various types of separation processes particularly in chemical and other industries. Due to their large number of applications in various process and manufacturing industries and several other fields and also because their proper operation contributes significantly to improved product quality, manufacturing prices and various other capital costs, it is quite evident that their optimization as well as their control is of tremendous importance to an instrumentation engineer for these manifold reasons. However there is a major problem or difficulty associated with distillation control schemes because of the large number of different kinds of thermodynamic factors that arises from the separation process.

For example:

- Separations tend to deviate from linearity of the equations as purity of the product increases.
- When compositions are controlled, it may lead to/ result in Coupling of process variables.
- Feed and flow agitation can lead to disturbances
- Non-steady state behaviour may arise due to efficiency changes in trays.

Hence, in order to improve the desirability and the performance of distillation control one should identify these probable lacunas or challenges as well as realize their occurrence time as they are responsible for the dynamic behaviour of the column.

One of the most important aspects of control in this apparatus is the maintenance of both energy as well as material balances and also their various corollaries on the distillation column. The material balance formulas i.e.  $D/F = (z-x)/(y-x)$ , (where *z*, *x*,

and  $y$  denotes the feed, bottoms and distillate concentrations respectively), are employed. It was observed that as the distillate ( $D$ ) increases, its purity decreases and vice versa. So it gives us the conclusion that the purity level varies indirectly with the flow rate of that product. Energy input also plays a major role as it determines the vapor flow rate ( $V$ ) up the column which directly affects the  $L/D$  ratio (also called as reflux ratio) and therefore relates to a rise in the amount of separation taking place. Therefore, the amount of separation was determined primarily by the energy input, while the ratio of separation in the products was related by the material flow.

The different kind of disturbances that leads to deviation of the controlled variables from their respective set points are as follows:

1. Feed flow rate and Process loads

These include

- Feed composition ( $Z_f$ )
- Feed thermal condition
- Feed flow rate ( $F$ )

2. Changes in heating- and cooling- medium supply conditions

These include

- Steam supply pressure
- Cooling-water supply temperature
- Cooling-water header pressure
- Ambient temperature, such as those that are caused by rainstorms

3. Equipment Fouling

Heat exchanger fouls with extensive usage. However because its contribution is minimal it is not considered here.

The five controlled variables and their manipulated variables in the distillation column control strategy are as follows:

1. Controlled variables: Column pressure, Distillate Receiver level, Distillate composition ( $x_D$ ), and Bottoms composition ( $x_B$ ), Base Level,.

2. Manipulated variables: Condenser heavy duty, distillate flow rate, bottoms flow rate, reflux flow rate, and reboiler heavy duty.

### 1.3 Determination of $X_d$ and $X_b$

Our control objective here is to maintain  $X_d$  (the distillate composition) and/or  $X_b$  (the bottom composition) at the desired set point or specified value in spite of the presence of various disturbances.

#### Step 1:

The component material balance equation was written for each stage in the column.

Accumulation = Liquid entering  $i$ th stage + Vapor entering  $i$ th stage + Liquid leaving  $i$ th stage + Vapor leaving  $i$ th stage

Hence the component material balance for all stages, (except the feed tray, overhead condenser, and reboiler):

$$d(M_i X_i)/dt = L_{i-1} X_{i-1} + V_{i+1} Y_{i+1} - L_i X_i - V_i Y_i$$

*Assumption: For simplicity, accumulation in the each stage is constant;  $dM_i/dt=0$ .*

Now the simplified component material balance for each stage (only composition changes with time):

$$M_i dX_i/dt = L_{i-1} X_{i-1} + V_{i+1} Y_{i+1} - L_i X_i - V_i Y_i$$

These equations are used in the Excel Interactive ODE Distillation Column Model and are given so that the user can understand the working of the model.

The ODE employed here for solving the liquid composition leaving tray 2 (rectifying section):

$$dX_2/dt = [L_1 X_1 + V_3 Y_3 - L_2 X_2 - V_2 Y_2]/M_2$$

Now the ODE employed for the liquid composition leaving tray 5 (stripping section):



$$dX_5/dt = [L_4.X_4 + V_6.Y_6 - L_5X_5 - V_5Y_5]/M_5$$

Now for overhead condenser component balance:

$$dX_2/dt = V_1 (Y_1 - X_D)$$

Feed tray component balance:

$$dX_3/dt = [L_2.X_2 + V_4.Y_4 - L_3X_3 - V_3Y_3]/M_3:$$

Reboiler component balance:

$$dX_w/dt = [L_6.X_6 - WX_w - V_7Y_7]/M_w:$$

### **Step 2:**

The total material balances around the reboiler and condenser were written.

Condenser material balance:

Two conditions were taken.

Condition 1: Total condenser is taken constant.

Condition 2: Overhead accumulator liquid level remains constant.

$$D = [V_1 + LD]$$

Now we obtain the reboiler material balance:

$$W = F - D$$

We have to specify the following so that the equations are valid:

-reflux flow rate (mol/min)

-bottoms flow rate (mol/min).

### **Step 3:**

All flow rates were defined.

The following equations for various stages were obtained.

Vapor Leaving Feed Stage:

$$V_3 = V_4 + F(1 - q_f)$$

Liquid Leaving Feed Stage:

$$L_3 = L_2 + F(q_f)$$

Now for vapor flow rates in the stripping section:

Assumption: Equimolal overflow for vapor in the stripping section

$$V_4 = V_5 = V_6 = (V_7)$$

Now for vapor flow rates in rectifying section:

Assumption: Equimolal overflow for vapor in the rectifying section

$$V_1 = V_2 = (V_3)$$

Now for liquid flow rates in the rectifying section:

Assumption: Equi-molal overflow for liquid in rectifying section

$$L_1 = L_2 = (L_3)$$

Now for Liquid flow rates in stripping section:

Assumption: Equi-molal overflow for liquid in stripping section

$$L_6 = L_5 = L_4 = L_3$$

#### Step 4:

The equilibrium conditions were defined

The binary system considered for the Excel ODE model is a benzene-toluene system.

The equilibrium data for this binary system was put in the model and the relative volatilities were calculated for various equilibrium compositions.

Therefore, Relative Volatility (obtained from the equilibrium data):

$$\alpha = (Y_{\text{benzene}} \cdot X_{\text{toluene}}) / (X_{\text{benzene}} \cdot Y_{\text{toluene}})$$

Where  $\alpha$  is called as the relative volatility of the two components in the system.

The plot between relative volatilities versus temperature was obtained and the data was fit using linear regression.

Hence Relative volatility as a function of temperature gives:

$$\alpha = [-0.009T + 3.3157]$$

The equation shows how the separation changes on each tray depending on the temperature of the tray i.e. to express separation changes as a function of tray temperature, which decreases up the column.

Equilibrium Vapour Composition for each stage:

Assumption: The trays are considered to be completely efficient (i.e. vapour and liquid leaving any tray are in equilibrium)

$$Y_i = \frac{\alpha X_i}{1 + (\alpha - 1)}$$

Now we Replace alpha with the temperature dependent equation. This shows how the amount of benzene in the vapour leaving each tray is affected by the tray temperature.

### Step 5:

Finally the component energy balances for each stage was written.

In order that the dynamic model runs properly, the ODE energy balances are very important. The temperature changes from the top to the bottom of the column resulting in mass transfer within the column which allows the separation of the various components within the system.

The reboiler ODE is given as the first equation in the model. This is because the energy input into the column is added in the reboiler.

This is given in our model as:

$$dT_7/dt = [ \{L_6 X_6 - W.X_W\} \{T_6 - T_7\} ] / MW + q_r / MW_{CP}$$

Energy balances for each subsequent stage in the column are added. The stage which has a little different energy ODE is the feed stage.

This is given by:

$$dT_7/dt = [ [L_2 X_2][T_2 - T_3] + [V_4 Y_4][T_4 - T_3] - [L_3 X_3][T_2 - T_3] + [V_3 Y_3][T_4 - T_3] + [F.X_{feed}][T_{feed} - T_3] ] / M_3$$

Around the condenser we employ the last energy balance.

Assumption: Reflux return temperature is held fixed. (It is compensated by the changes in overhead condenser duty).

**Step 6:**

Inputs into the ODE model were determined.

After substituting all the equations into the model, all the remaining unknown variables must be placed in a section through which the user can specify these input values when running the model.

The users inputs for the Excel ODE distillation model include:

- 1) Feed flow rate
- 2) Mole fraction of light key in the feed
- 3) Reflux flow rate
- 4) Condenser, reboiler, and tray levels
- 5) Phase of the feed (q-value)
- 6) Feed temperature
- 7) Integration step size

To create the effects of disturbances, these input values may also be modified:

- Feed flow after 200 time steps
- Feed composition after 600 time steps

**Step 7:**

Euler's Method was employed to solve the ODE's.

Here Euler's method was used to integrate each ODE over each timestep in the interval so as to solve for the parameter value at the next time step. Making a graph of these values versus time allows one to see how variations in the input values affects the parameters like bottoms and distillate composition or flow rates.

There are several other Considerations that are employed for Dynamic Distillation Modelling. However for the purpose of simplicity they are not discussed or analyzed in this section.

## Glossary of Terms

$M_i$  = Molar holdup on tray  $i$

$L_{i-1}$  = Liquid molar flow rate into tray  $i$

$L_i$  = Liquid molar flow rate leaving tray  $i$

$V_{i+1}$  = Vapour molar flow rate entering tray  $i$

$V_i$  = Vapour molar flow rate leaving tray  $i$

$x_i$  = mole fraction of light component in the Liquid phase of Tray  $i$

$y_i$  = mole fraction of light component in the Gas phase of Tray  $i$

$B$  = Bottoms flow rate

$D$  = Distillate flow rate

$f$  = Feed flow rate

Alpha = Relative volatility of Benzene-Toluene system.

$q$  = Vapour Liquid composition value

## 1.4 Results

We assume the Steady state composition of different variables for the project to be:

Distillate Composition: $X_d$	0.99 mole fraction
Bottoms product: $X_b$	0.01 mole fraction
Reflux rate, $R$	2.706 Kmol/minute
Vapour Boilup Rate, $V$	3.206 Kmol/litre
Feed Flow Rate:	1 Kmol/min
Feed (more volatile) component mole fraction ( $Z_f$ ):	0.5
feed quality:	1

# **Chapter 2**

**MATHEMATICAL MODELLING**

**&**

**DECOUPLER DESIGN**

## 2.1 Introduction

In this section, we have to find the relationship between distillate column,  $X_d$  and bottom composition,  $X_b$  with reflux rate,  $L$  and vapour boil-up rate,  $V$ . Here, we also see the effect of multi-input on each multi-output in multi-input multi-output (MIMO) system and how to reduce this effect using decoupler method and Ziegler-Nicholas method.

## 2.2 Mathematical Modelling of Process

Here, the variables that were maintained at set point are distillate composition  $X_d(s)$  and Bottom composition  $X_b(s)$ , Disturbances are Feed Flow rate  $F(s)$  and Feed light component composition  $Z_f(s)$ . The manipulating variables used for manipulating/maintaining the controlled variables at their desired set point are Reflux rate  $L(s)$  and vapour boil-up rate  $V(s)$ .

This is a multi-input multi-output system where each output is affected by all the inputs or in other words it is an example of an interacting multivariable control system. We have to design it such that the output depends on only one synthetic input that is to make it non- interacting. This is possible by using decouplers.

$$\begin{bmatrix} X_d(S) \\ X_b(S) \end{bmatrix} = \begin{bmatrix} \frac{0.878}{(75S + 1)} & \frac{-0.864}{(75S + 1)} \\ \frac{1.082}{(75S + 1)} & \frac{-1.096}{(75S + 1)} \end{bmatrix} \begin{bmatrix} L(s) \\ V(s) \end{bmatrix} + \begin{bmatrix} \frac{0.394}{(75S + 1)} & \frac{0.881}{(75S + 1)} \\ \frac{0.586}{(75S + 1)} & \frac{1.119}{(75S + 1)} \end{bmatrix} \begin{bmatrix} F(s) \\ Z_f(s) \end{bmatrix}$$

For the time being we consider both  $F(S)$  and  $Z_f(S) = 0$ ;

So neglecting the disturbances for the time being we have,

$$\begin{bmatrix} Xd(S) \\ Xb(S) \end{bmatrix} = \begin{bmatrix} \frac{0.878}{(75S + 1)} & \frac{-0.864}{(75S + 1)} \\ \frac{1.082}{(75S + 1)} & \frac{-1.096}{(75S + 1)} \end{bmatrix} \begin{bmatrix} L(s) \\ V(s) \end{bmatrix}$$

So both the controlled variables,  $X_d(s)$  and  $X_b(s)$  depends both on  $L(S)$  and  $V(S)$ . Therefore it is a multivariable type process. The block diagram for this process is given in the following page.

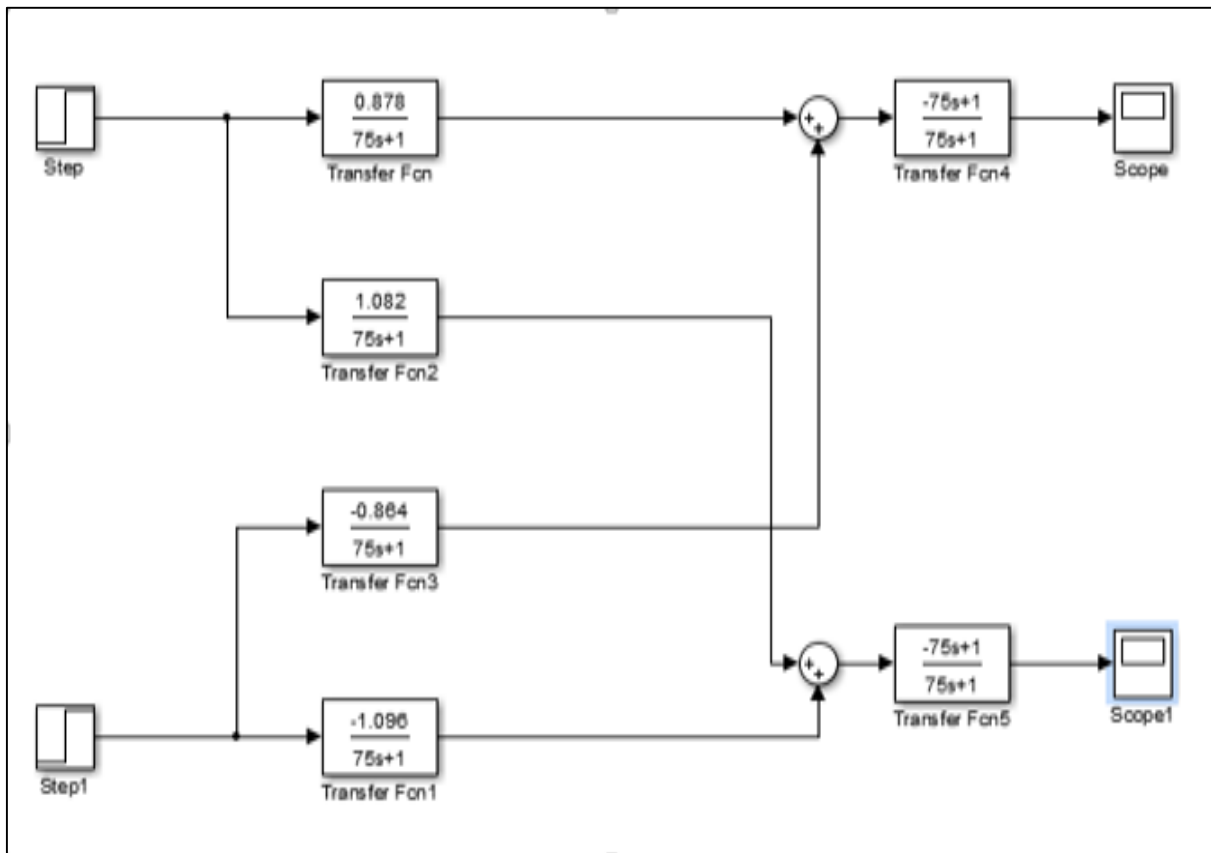


Fig 2.1:- Block Diagram for distillation column control (neglecting disturbances)

### 2.3 Decoupling

The controlled variables  $X_d(s)$  and  $X_b(s)$  depends on both  $L(s)$  and  $V(s)$ , or in other words they are interacting systems. To make it a non-interacting system where the outputs depend only on a single synthetic input we use decouplers. There are two types of decoupling techniques: ideal decoupling and simplified decoupling. The latter is generally used because of some inherent problems in the first one.



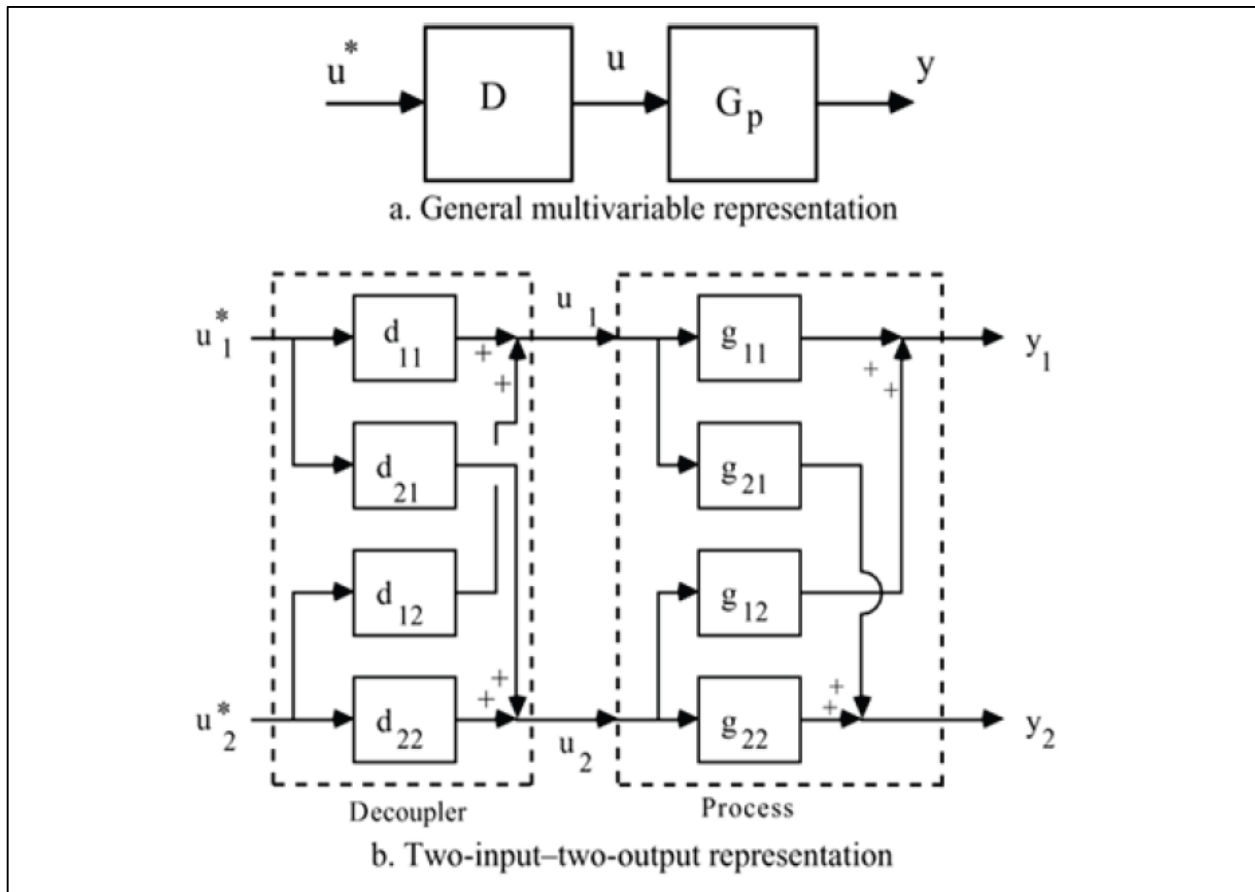


Fig 2.2:- Decoupling control strategy for two-input-two-output system

The synthetic input vector and process output vector are related to each other as:

$$Y(s) = G_p(s).D(s).U^*(s)$$

Now for a two input-two output process,

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = G_p(s).D(s) \begin{bmatrix} U_1^*(s) \\ U_2^*(s) \end{bmatrix}$$

Where,  $D(s)$  is the matrix for decoupler. There are several choices that are possible for the "target"  $G_p(s)D(s)$  matrix. Two popular methods are ideal decoupling and simplified decoupling.

### 2.3.1 Ideal Decoupling

In ideal decoupling we take,

$$G_p(s).D(s) = \begin{bmatrix} g_{11}(s) & 0 \\ 0 & g_{22}(s) \end{bmatrix}$$

Therefore,

$$D(s) = (Gp'(s))^{-1} \begin{bmatrix} g_{11}'(s) & 0 \\ 0 & g_{22}'(s) \end{bmatrix}$$

the (‘) notation denotes that the calculations are carried on a process model. The relationship between the synthetic inputs and process outputs is given by:

$$y(s) = GP(s)D(s)u^*(s),$$

which gives,

$$\begin{bmatrix} Y1(s) \\ Y2(s) \end{bmatrix} = \begin{bmatrix} g_{11}'(s) & 0 \\ 0 & g_{22}'(s) \end{bmatrix} \begin{bmatrix} U1^*(s) \\ U2^*(s) \end{bmatrix}$$

From here we get,

$$\begin{aligned} Y1(s) &= g_{11}'(s) U1^*(s) \\ Y2(s) &= g_{22}'(s) U2^*(s) \end{aligned}$$

For each control loop independent SISO tuning parameters are available. This is the major advantage. However major disadvantage is if there is any RHP transmission zeros, the decoupler may be unstable. Also it is extremely sensitive to model error.

### 2.3.2 Simplified Decoupling

Here, we specify a decoupled response and the de-coupler with the structure given in the matrix as:

$$D(s) = \begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix}$$

This is an alternate approach to ideal decoupling.

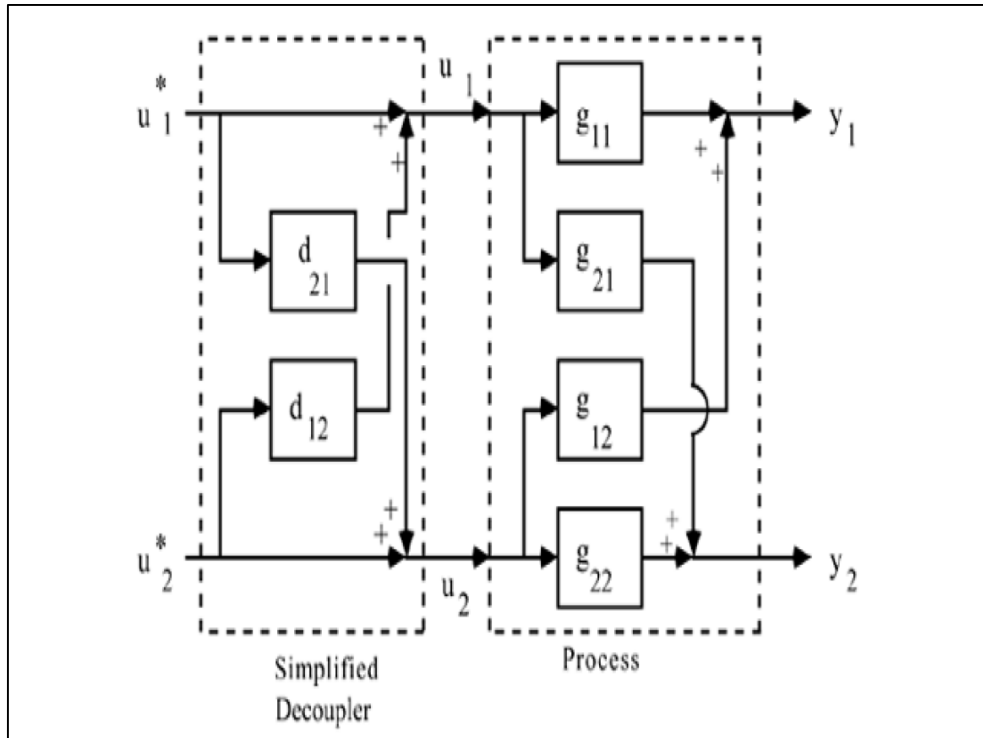


Fig 2.3:- Simplified Decoupling Control Strategy

Now we take,

$$Gp(s).D(s) = \begin{bmatrix} g_{11}^*(s) & 0 \\ 0 & g_{22}^*(s) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} * \begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix} = \begin{bmatrix} g_{11}^*(s) & 0 \\ 0 & g_{22}^*(s) \end{bmatrix}$$

We can find the four unknowns by solving the four equations:

$$d_{12}(s) = -\frac{g_{12}(s)}{g_{11}(s)}$$

$$d_{21}(s) = -\frac{g_{21}(s)}{g_{22}(s)}$$

$$g_{11}^*(s) = g_{11}(s) - \frac{g_{12}(s)*g_{21}(s)}{g_{22}(s)}$$

$$g_{22}^*(s) = g_{22}(s) - \frac{g_{12}(s)*g_{21}(s)}{g_{11}(s)}$$

For the process under consideration, we have:

$$g_{11}(s) = \frac{0.878}{(75s+1)}$$

$$g_{12}(s) = \frac{-0.864}{(75s+1)}$$

$$g_{21}(s) = \frac{1.082}{(75s+1)}$$

$$g_{22}(s) = \frac{-1.096}{(75s+1)}$$

From here we get after solving:

$$d_{12}(s) = -0.984$$

$$d_{21}(s) = -0.987$$

$$g_{11}^*(s) = 0.025/(75s+1)$$

$$g_{22}^*(s) = 0.03125/(75s+1)$$

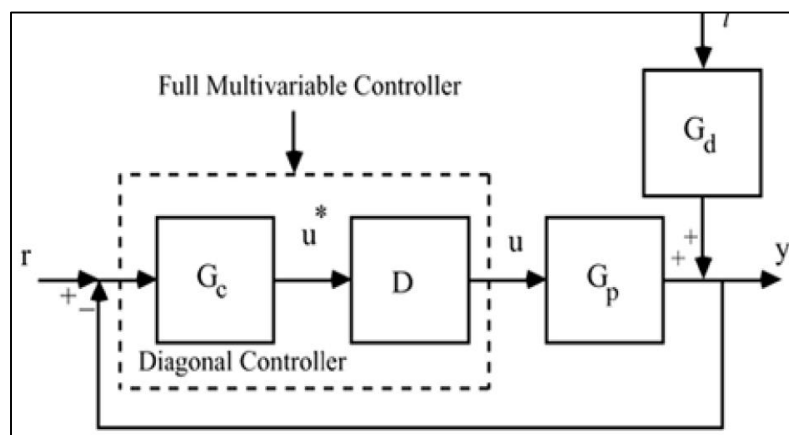


Fig 2.4:- Final (Simplified) Diagram.

# Chapter 3

## CONTROL STRATEGIES

### 3.1 Introduction

In this section, we have implemented different control strategies for set point tracking and to reduce the effects of disturbances entering into the system. Here, we have used 3 control strategies: PID (using Ziegler-Nichols method), Smith predictor and IMC and observed the effects of their parameters on the parameters of the process.

### 3.2 Control Strategies

#### 3.2.1 PID Controller

A PID controller has three tuning parameters:  $K_p$ ,  $T_i$  and  $T_d$ . If these are adjusted randomly, it will give unsatisfactory performance. Also, each observer will end up with a different set of tuning parameters. Therefore, Ziegler-Nichols closed-loop tuning technique is the best method to tune PID controllers. This method is not widely used because the closed-loop behaviour results in an oscillatory response and it's sensitive to uncertainty.

An ideal PID controller has the transfer function as:

$$C_{PID}(s) = k_p + \frac{k_i}{s} + k_d s = k_p \left( 1 + \frac{1}{T_i s} + T_d s \right)$$

Ziegler-Nichols PID Tuning Method 1 for First Order Systems:

A line was drawn tangent to the response curve through the *inflection point* of the curve.

The Time delay ( $L$ ) and Rise Time( $Tr$ ) were determined graphically as shown.

We obtained Using the requisite formulas for First Order Systems,

$$K_p=39.733$$

$$T_i=4.651$$

$$T_d=1.163$$

Therefore  $C_{PID}(S)$  was obtained as,  $C_{PID}(S) = 39.733 \times \left( 1 + \frac{1}{4.651S} + 1.163 S \right)$

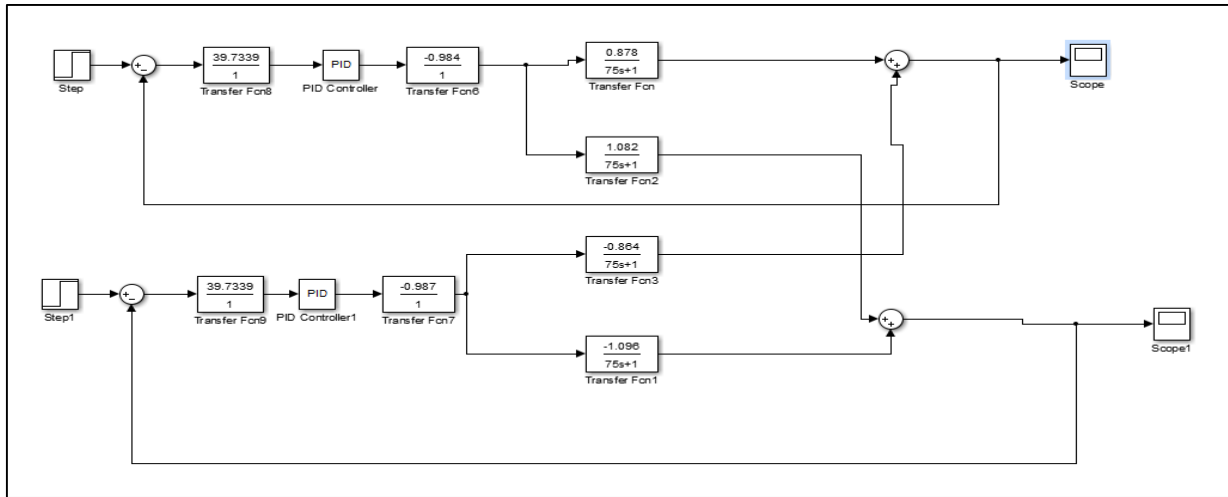


Fig 3.1:- Unity Feedback Control System with PID control:

### 3.2.2 Smith Predictor

It's a technique which employs a simple dynamic model in order to predict future outputs based on present information. Time-delay compensation methods & a traditional proportional-integral (PI) controller are applied in the control of the bottom & top compositions of a distillation column. To implement time-delay compensation, the control scheme is rearranged to a new configuration where a feedback loop has been implemented around the conventional controller.

#### Simulation:

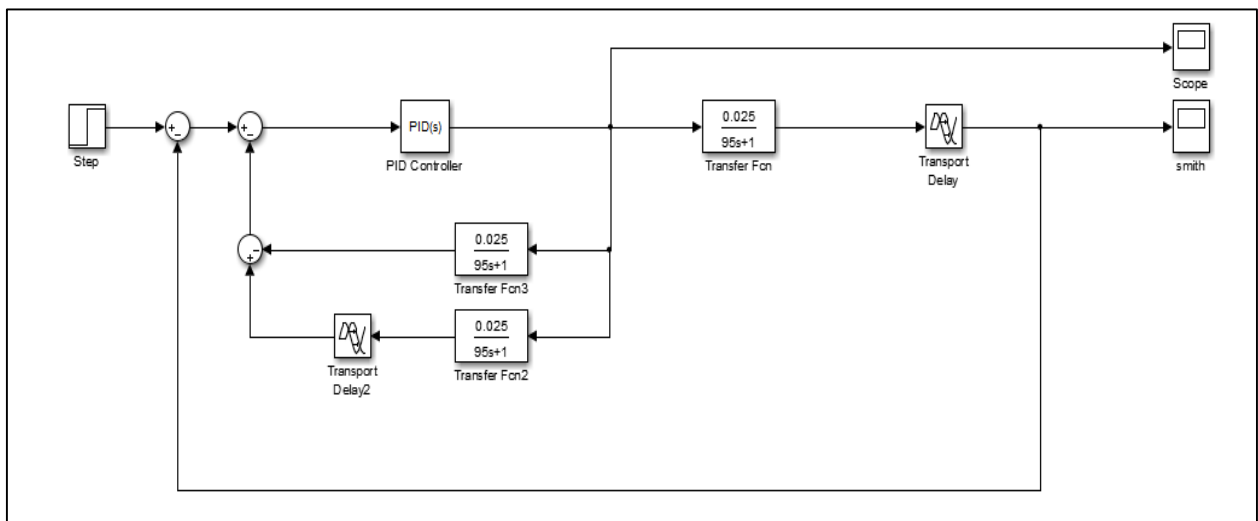
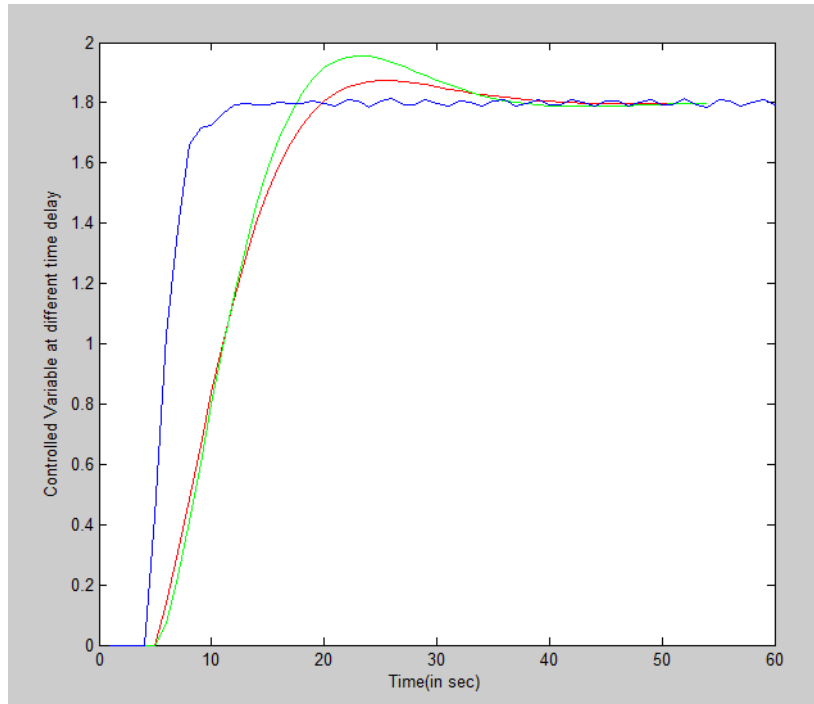


Fig 3.2:- Block Diagram of Smith Predictor



Sim 1:- Controlled Variable at different time delay

### 3.2.3 IMC Controller

In advanced process control applications, model-based control systems are often used to track set points as well as for reduction of the disturbances. The internal model control (IMC) design depends on the premise that any control system has different parameters which are to be controlled and as a result it is difficult to achieve perfect control.

#### Simulation:

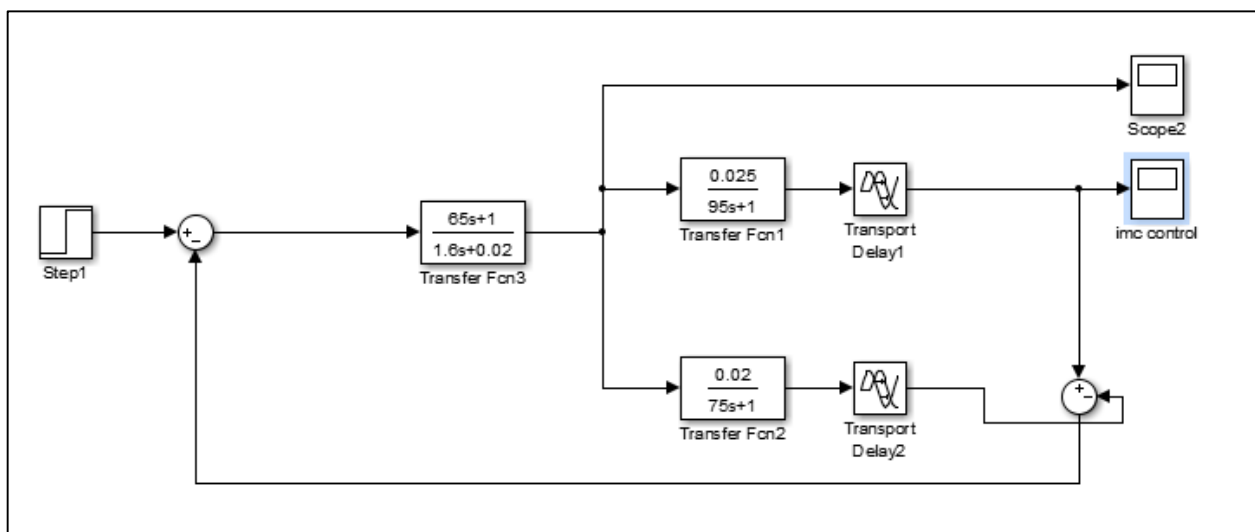
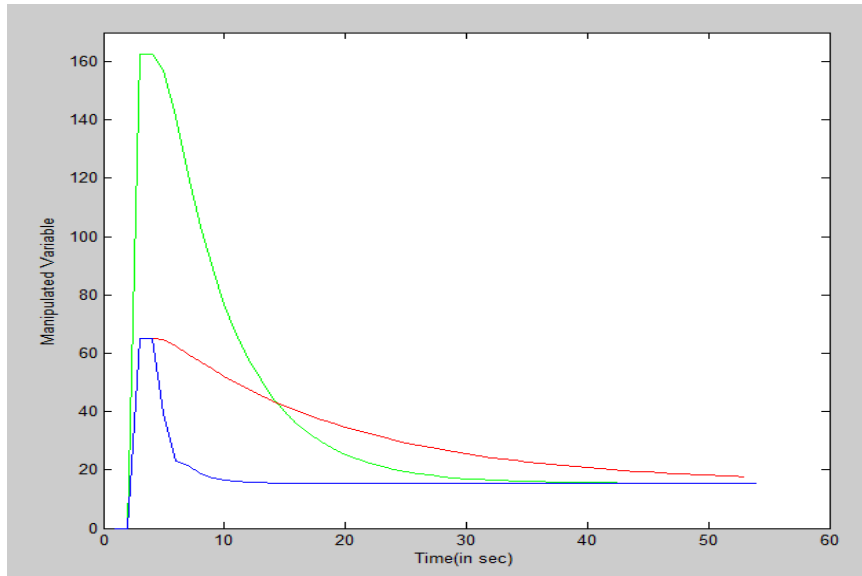
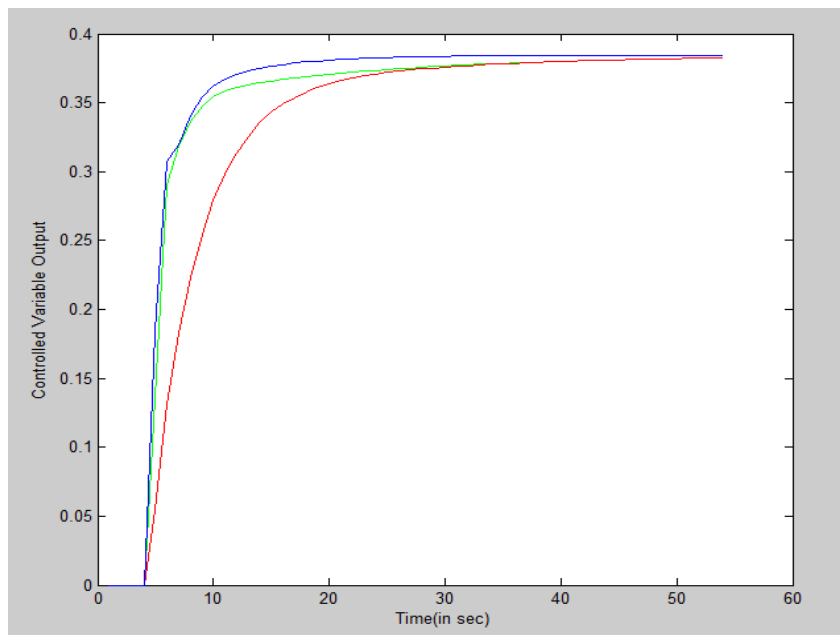


Fig 3.3:- Block Diagram of IMC Control





Sim 2:- Manipulated Variable at Different Tuning Parameter ( $\lambda=10, 30$  and  $50$ )



Sim 3:- Controlled Variable at Different Tuning Parameter ( $\lambda=10, 30$  and  $50$ )

# Chapter 4

## BRIEF INTRODUCTION OF INTERNAL MODEL CONTROL (IMC)

## 4.1 Background of IMC

In advanced process control applications, model-based control systems are often employed to track set points as well as for reduction of disturbances. The internal model control (IMC) design depends on the premise that any control system has various parameters that are to be controlled and as a result it is difficult to achieve perfect control. However, if a control scheme has been developed based upon the exact model of the process then an ideal control is theoretically achievable. There are a number of advantages to the IMC structure along with controller design procedure, compared with that of the classical feed-back control structure.

1. It becomes very clear how process characteristics such as time delays and RHP zeros affect the inherent controllability of the process.
2. IMCs are much easier to tune than other controllers in a standard feedback control structure.

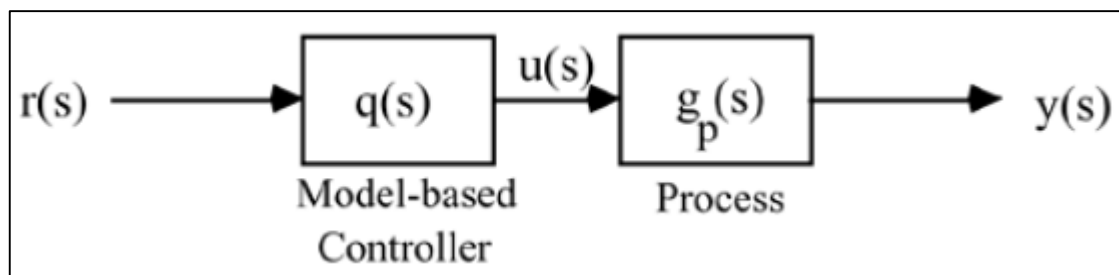


Fig 4.1:- Open loop control system

From the above block diagram:-

$$Y(s) = Q(s) * G_p(s) * r(s)$$

Where  $Q$  = model-based controller

$G_p$  = actual process

$r$  = set-point or input to the system

The above controller,  $q(s)$ , is used to control the process. It is given by:-

$$Q(s) = \text{inverse of } \check{G}_p(s)$$

Where  $\check{G}_p(s) =$  process model

But if  $\check{G}_p(s) = G_p(s)$ , i.e., if the model is exact as that of the process, it is seen that *for the above two conditions the output of the system,  $y(s)$ , will always be equal to the set point or input of the system,  $r(s)$ .*

As a result, if the different parameters of the process (as encapsulated in the process model) being controlled are known, we can have perfect control.

It shows that ideal control performance can be achieved without feedback which signifies that feedback control is necessary only when knowledge about the parameters of the process are uncertain.

Although, the designing procedure of IMC is identical to that of open loop control, the implementation of IMC results in a feedback system. Therefore, IMC tries to compensate for disturbances and model uncertainty, while, on the other hand, open loop control is not. As a disadvantage, IMC should be detuned to make sure of the stability if there is model uncertainty.

## 4.2 IMC basic structure

The important characteristic of IMC structure is the installation of the process model which is in parallel with the actual process or the plant.

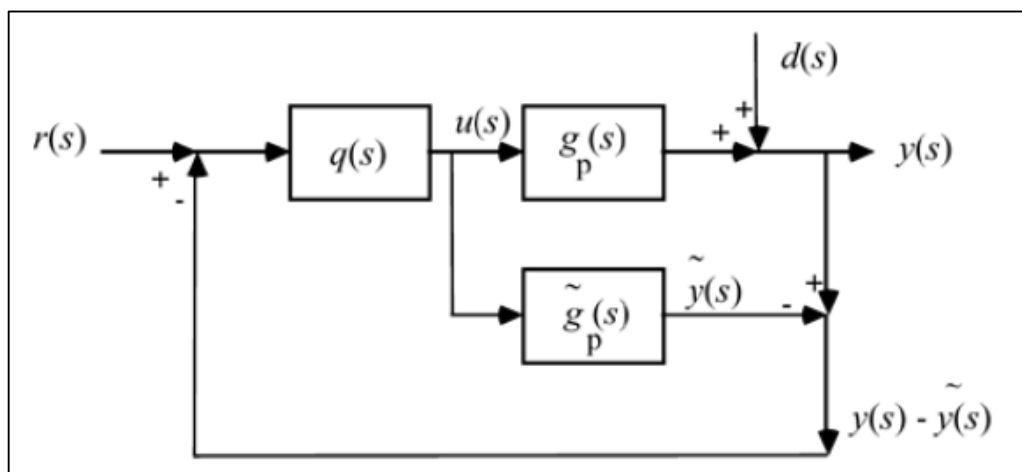


Fig 4.2:- IMC Basic Structure

### 4.3 IMC parameters

The various parameters used in the IMC basic structure shown above are as follows:

$Q_c$  = IMC controller

$G_p$  = actual process

$\check{G}_p$  = process model

$u$  = manipulated input (controller output)

$d$  = disturbance

$d^* = y - \hat{y}$  = estimated disturbance

$y$  = measured process output

$\hat{y}$  = process model output

Feedback signal:  $d^* = (G_p - \check{G}_p)u + d$

Now we consider a special case:-

#### **Perfect model without disturbance ( $d=0$ ):**

A model is said to be perfect if the process model is same as that of the process, i.e.,

$$G_p = \check{G}_p$$

Therefore, we get a relationship between  $r$  and  $y$  as

$$y = G_p * Q_c * r$$

The above relationship is similar to that of the open loop system. Thus, if the controller  $Q_c$  is stable and the process  $G_p$  is stable the closed loop system will be stable.

But in real cases, the disturbances and the uncertainties, always, do exist. Hence, actual process is always different from that of the process model.

### 4.4 IMC Strategy

As discussed above that the actual process always differs from the model of the process i.e. process model is not same as the process due to unknown disturbances entering into the system. Because of which the usual open loop control system is difficult to

implement, so we require a model-based control strategy by which we can achieve a perfect control. Thus the control strategy which we shall apply to achieve perfect control is known as INTERNAL MODEL CONTROL (IMC) strategy.

The error signal  $\check{r}(s)$  is because of the model difference and the disturbances which is send as modified set-point to the controller through the feedback loop and is given by

$$\check{r}(s) = r(s) - d^*(s)$$

And the output of the controller is  $u(s)$  which is given simultaneously to both the process and the model.

$$\begin{aligned} u(s) &= \check{r}(s) * Q_c(s) = [r(s) - d^*(s)] Q_c(s) \\ &= [r(s) - \{ [G_p(s) - \check{G}_p(s)].u(s) + d(s) \}] \cdot Q_c(s) \end{aligned}$$

$$u(s) = [ [r(s) - d(s)] * Q_c(s) ] / [ 1 + \{ G_p(s) - \check{G}_p(s) \} Q_c(s) ]$$

But,

$$y(s) = G_p(s) * u(s) + d(s)$$

Hence, closed loop transfer function for IMC is

$$y(s) = \{ Q_c(s) \cdot G_p(s) \cdot r(s) + [1 - Q_c(s) \cdot \check{G}_p(s)] \cdot d(s) \} / \{ 1 + [G_p(s) - \check{G}_p(s)] Q_c(s) \}$$

Also, to improve the robustness of the system mismatch of process and model should be minimum. Since, at higher frequencies mismatch of process and model occurs, a low pass filter  $f(s)$  is cascaded which can discard the higher frequencies and the problem can be avoided.

**Therefore, the internal model controller consists of the inverse of the process model and a low pass filter connected in cascade i.e**

$$Q(s) = Q_c(s) * f(s)$$

The order of the filter is selected so that the function becomes proper or at least semi proper (order of numerator is equal to the order of denominator). So, the above closed loop equation becomes

$$y(s) = \{ Q(s) \cdot G_p(s) \cdot r(s) + [1 - Q(s) \cdot \check{G}_p(s)] \cdot d(s) \} / \{ 1 + [G_p(s) - \check{G}_p(s)] Q(s) \}$$

# Chapter 5

## IMC DESIGN & ITS TUNING

## 5.1 Introduction

The methodology for designing IMC is exactly the same to that of the design of the open loop control technique. However unlike the case of an open loop control, the IMC structure is used for compensation of disturbances which are entering into the system and also model mismatch. The IMC filter tuning parameter “ $\lambda$ ” is used to prevent the effect of model mismatch. The general IMC design method is mainly centred on set-point tracking however better disturbance rejection can't be guaranteed, particularly those which are occurring at the process inputs. A change in the design method is made for maximization of the property of input disturbance rejection and also for making the controller internally stable at moderate higher frequencies.

## 5.2 IMC design procedure

We approximately take process model  $\check{G}_p(s)$  which is close to the process  $G_p(s)$ . The controller  $Q(s)$  helps in preventing the flow of the disturbances  $d(s)$  entering into the system. The various steps in the Internal Model Control (IMC) system design procedure are:

### 5.2.1 FACTORIZATION

This procedure includes factorizing the transfer function by dividing it into invertible and non-invertible parts. The factor which contain right hand zeroes and become the poles, when the process model is inverted leading to internal stability, is the non-invertible part which has to be removed from the transfer function. Mathematically, it is given as

$$\check{G}_p(s) = \check{G}_{p+}(s) \cdot \check{G}_{p-}(s)$$

Where,

$\check{G}_{p+}(s)$  is non-invertible part



$\check{G}_p(s)$  is invertible part

There are two methods used for factorization:

1. Simple
2. All pass

However, all pass factorization is used where the unstable RHP is compensated by a mirror image of it on the left hand side.

### 5.2.2 IDEAL IMC CONTROLLER

The characteristic of an ideal IMC is that the inverse of the process model is the invertible part. It is given as:-

$$Q_c^*(s) = \text{inv} [\check{G}_p(s)]$$

### 5.2.3 ADDITION OF FILTER

Now a filter is added to make the controller **proper** or at least **semi-proper** because the transfer function of the controller will be unstable if it is improper.

A transfer function is said to be as **proper** if the order of the denominator is greater than that of the numerator and for exactly of the same order the transfer function is said to be as **semi-proper**.

So to make the controller proper or semi-proper mathematically it is given as

$$Q(s) = Q_c^*(s) f(s) = \text{inv}[\check{G}_p(s)] f(s)$$

### 5.2.4 LOW PASS FILTER, f(s)

We have to reduce the instability at higher frequencies. So, a filter is added and the resulting controller, Q(s), is given as:

$$Q(s) = Q_c^*(s) .f(s) = \{\text{inv}[\check{G}_p(s)]\} f(s)$$

Where

$$f(s) = 1/(\lambda^* s + 1)^n$$

Where  $\lambda$  is the filter tuning parameter which varies the speed of the response of the closed loop system. When  $\lambda$  is smaller than the time constant of the first order process the response is faster.

The low pass filter is of two types:

- a) For input as set point change, the filter used is  $f(s) = 1/(\lambda s + 1)^n$ , where  $n$  is the order of the process.
- b) For good rejection of step input load disturbances the filter used is  $f(s) = (\gamma s + 1)/(\lambda s + 1)^n$  where  $\gamma$  is a constant.

### 5.3 IMC design implementation for 1st order system

Now applying the above IMC design procedure for a first order system:

**Given process and its model for 1st order system:**

$$\check{G}_p(s) = 0.025/[65s+1], K_p=0.025 \text{ and } T_p=75$$

$$\check{G}_p(s) = \check{G}_{p+}(s).\check{G}_{p-}(s) = 1.(0.02/[65s+1])$$

$$Q_c^*(s) = \text{inv}[\check{G}_{p-}(s)] = [65s+1] / 0.02$$

$$Q(s) = Q_c^*(s).f(s) = [65s+1] / [0.02(\lambda s + 1)] \quad f(s) = 1 / (\lambda s + 1)$$

$$y(s) = Q(s).G_p(s).r(s) = (0.02/[65s+1]).f(s).r(s)$$

$$\text{Output variable: } y(s) = r(s)/(\lambda*s + 1)$$

$$\text{Manipulated variable: } u(s) = Q(s).r(s) = [[65s+1].r(s)] / [0.025(\lambda s + 1)]$$

### 5.4 Empirical Formula between the process parameter and the tuning parameter

We generally need to tune the controller in such a way that we get a minimum value of percentage overshoot, rise time and settling time. But, in general we don't have a particular method to obtain the tuning parameter.

That is why we need to obtain a set of data bank by changing the process variables i.e. the process time constant and the process gain for different tuning parameter and finally get a relation between process time constant and tuning parameter.

And we need to find out the optimal value for the tuning parameter.

### 5.4.1 Basic Block Diagram

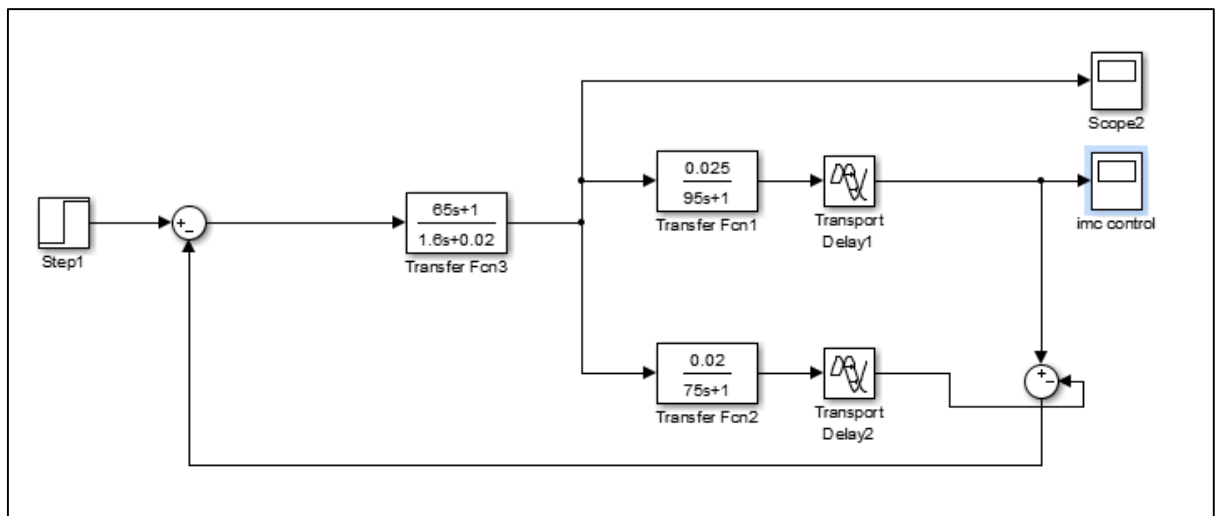


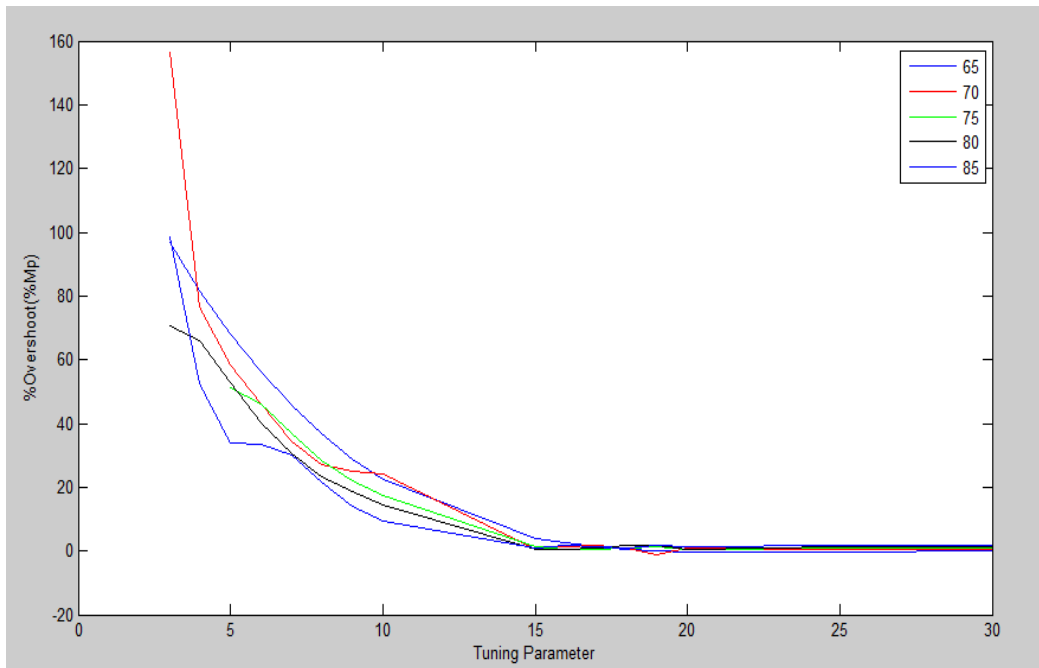
Fig 5.1:- IMC Block Diagram for the current process

### 5.4.2 Steps implemented for deriving the formula

- 1) A general first order process has been taken to derive the empirical formula between the process time constant and the tuning parameter. The process gain would not affect the response for the IMC controller. The formula can be then used to find out the tuning parameter for any given 1<sup>st</sup> order process with known time constant.
- 2) We took different process gains for the 1<sup>st</sup> order processes i.e.  $K_p=0.025, 0.035, 0.05, 0.1, 0.2$ .
- 3) For every value of  $K_p$  we took different process time constants i.e.  $T_p$  (in sec) = 65, 70, 75, 80, 85.
- 4) Now for each time constant, we varied the tuning parameter ( $\lambda$ ) and measured the % overshoot and settling time.
- 5) Now, we took  $K_p = 0.1$  for further analysis and to find out the required equation.
- 6) The values of the tuning parameter and the process time constant has been taken where we are getting the optimum values for the response characteristics.
- 7) Now, the empirical formula is formed between the tuning parameter and the time constant for minimum % overshoot and settling time.

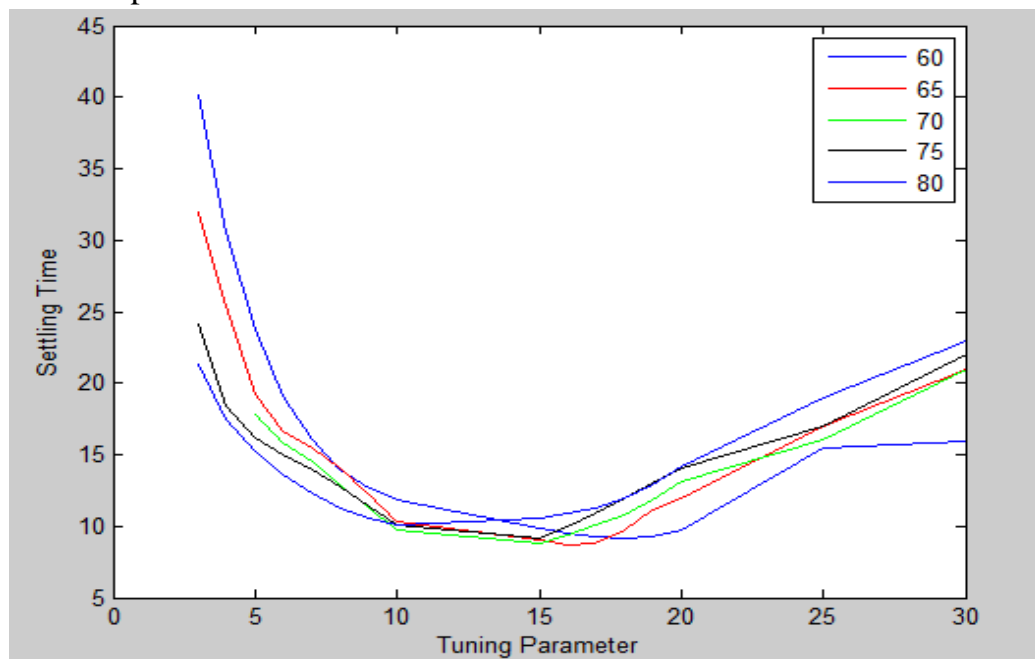
### 5.4.3 Simulation

- a) Graph between %Mp and Tuning Parameter at different values of  $T_p$  when  $K_p=0.1$



Sim 4:- Graph between %OS &  $\lambda$  at  $K_p=0.1$  at different  $T_p$

- b) Graph between Settling Time and Tuning Parameter at different values of  $T_p$  when  $K_p=0.1$



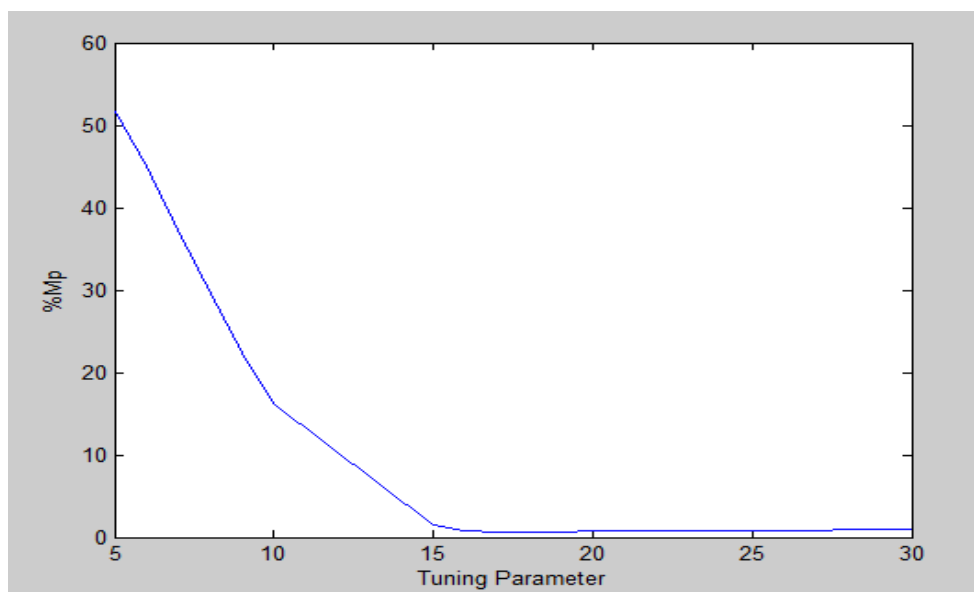
Sim 5:- Graph between TS &  $\lambda$  at  $K_p=0.1$  at different  $T_p$

c) For  $K_p=0.1$  and  $T_p=75$ , the values of % overshoot & settling time at different  $\lambda$

<b>Lambda(<math>\lambda</math>)</b>	<b>Overshoot(%Mp)</b>	<b>Settling Time(<math>T_s</math>)</b>
<b>5</b>	<b>50.87</b>	<b>18</b>
<b>6</b>	<b>47.37</b>	<b>15</b>
<b>7</b>	<b>35.27</b>	<b>16</b>
<b>8</b>	<b>29.09</b>	<b>12</b>
<b>9</b>	<b>22.80</b>	<b>11</b>
<b>10</b>	<b>16.76</b>	<b>10</b>
<b>15</b>	<b>1.15</b>	<b>9</b>
<b>16</b>	<b>0.74</b>	<b>9</b>
<b>17</b>	<b>0.71</b>	<b>10</b>
<b>18</b>	<b>0.71</b>	<b>11</b>
<b>19</b>	<b>0.73</b>	<b>12</b>
<b>20</b>	<b>0.74</b>	<b>13</b>
<b>25</b>	<b>0.82</b>	<b>16</b>
<b>30</b>	<b>0.89</b>	<b>21</b>

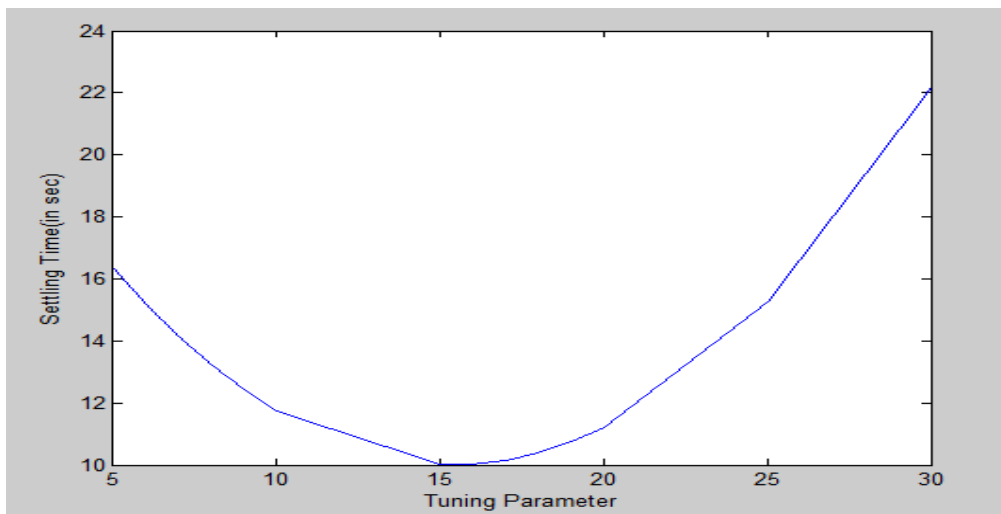
Table 1

d) Graph between %Mp and Tuning parameter for the above table



Sim 6:- Graph between %Mp &  $\lambda$  at  $K_p=0.1$

e) Graph between settling time and tuning parameter for table 1



Sim 7:- Graph between TS &  $\lambda$  at  $K_p=0.1$

f) Now taking tuning parameter for minimum settling time at different  $T_p$  when  $K_p=0.1$

Process Time Constant( $\tau_p$ )	Lambda( $\lambda$ )	Settling Time(TS)
65	17	8
70	16	8
75	16	9
80	14	8
85	13	8

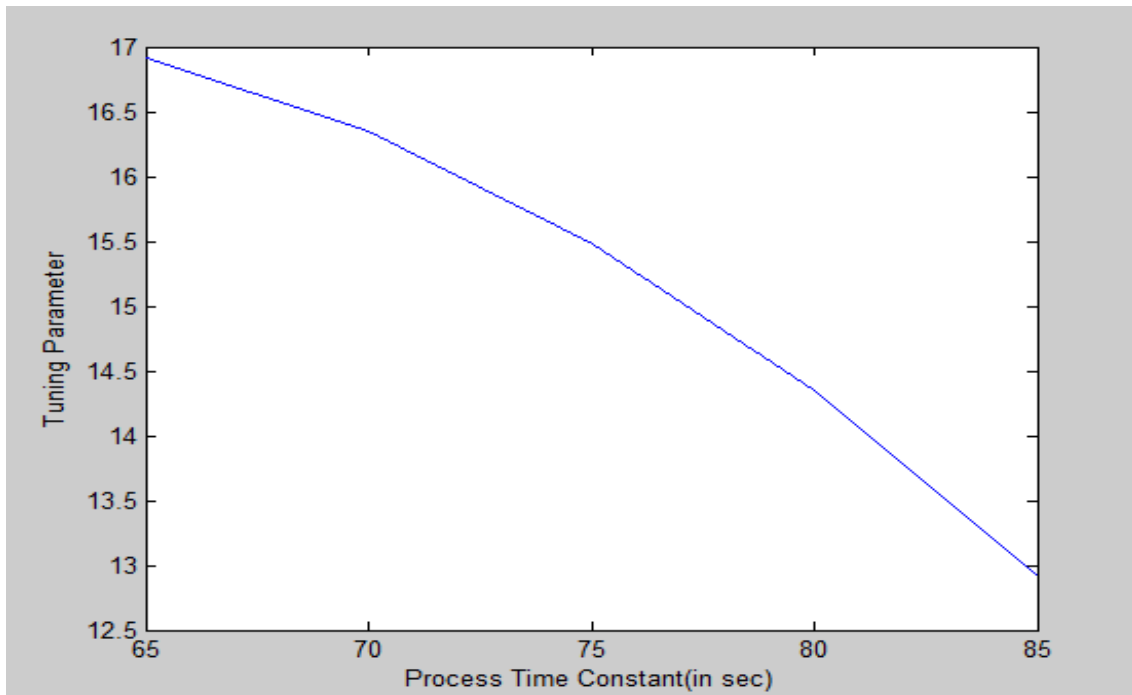
Table 2

g) The values of tuning parameter at different  $T_p$  for minimum % overshoot at  $K_p=0.1$

Process Time Constant( $\tau_p$ )	Lambda( $\lambda$ )	Overshoot(%Mp)
65	19	0
70	18	0.37
75	17	0.71
80	16	1.01
85	15	1.27

Table 3

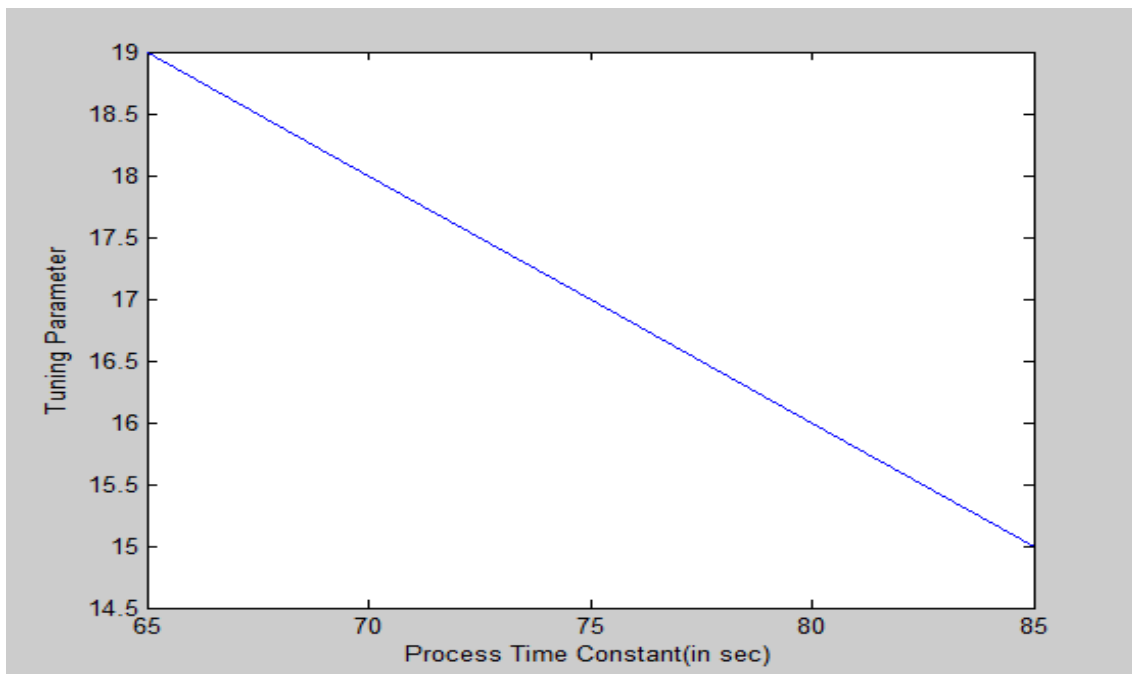
h) For minimum %overshoot, the graph between tuning parameter and  $T_p$



Sim 8:- Graph between  $\lambda$  &  $T_p$  for minimum %Mp

The normalized equation is,  $\lambda = -0.0057T_p^2 + 0.6571T_p - 1.6571$

i) For minimum settling time, the graph between tuning parameter and  $T_p$



Sim 9:- Graph between  $\lambda$  &  $T_p$  for minimum TS

The normalized equation is,  $\lambda = -0.0057T_p^2 + 0.6571T_p - 1.6571$

# Chapter 6

## IMC BASED PID & ITS TUNING



## 6.1 Introduction

In this section, the IMC structure is rearranged to get a standard feedback control system so that open loop unstable system can be handled. This is done because it improves the input disturbance rejection. Similarly to the IMC design, process model is also used in IMC based PID design. In the IMC design procedure, the IMC controller  $Q_c(s)$  is directly proportional to the inverse of the transfer function of the process model. The IMC depends on only one tuning parameter which is the low-pass filter tuning factor but the IMC based PID tuning parameters depends on this tuning factor. The selection of the filter parameter is directly based on the robustness. IMC based PID procedures uses an approximation for the dead time. And if the process has no time delays it gives the same performance as does the IMC.

## 6.2 IMC based PID structure

In ideal IMC structure, the model output is moved and connected to the summation of the input and the controller, as shown in the figure, to form a standard feedback controller which is known as IMC based PID controller.

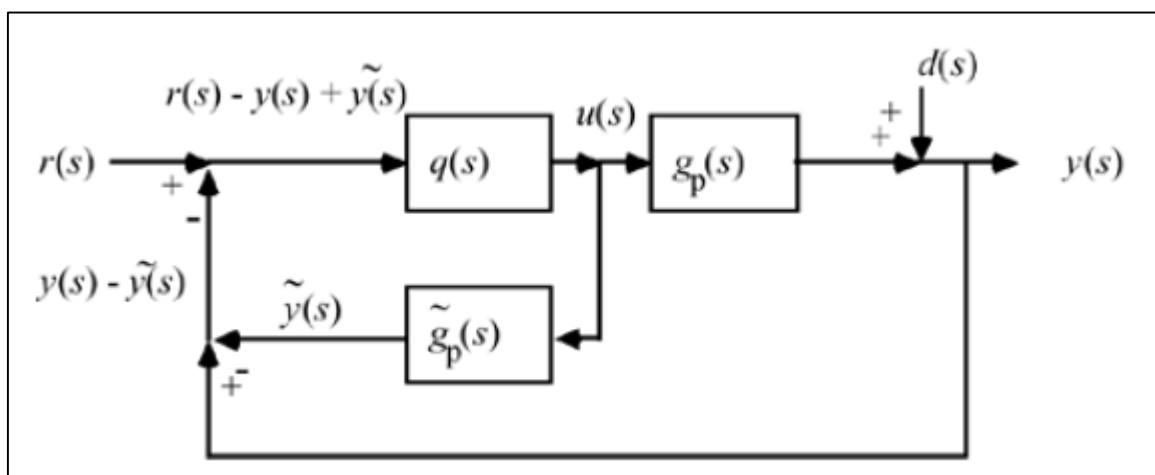


Fig 6.1:- Cosmetic change in the IMC structure

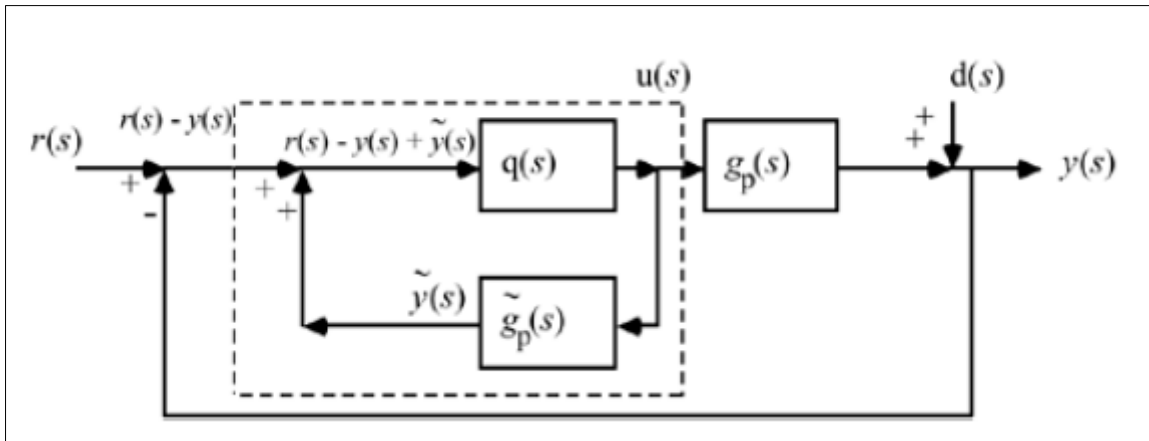


Fig 6.2:- Rearrangement of IMC structure

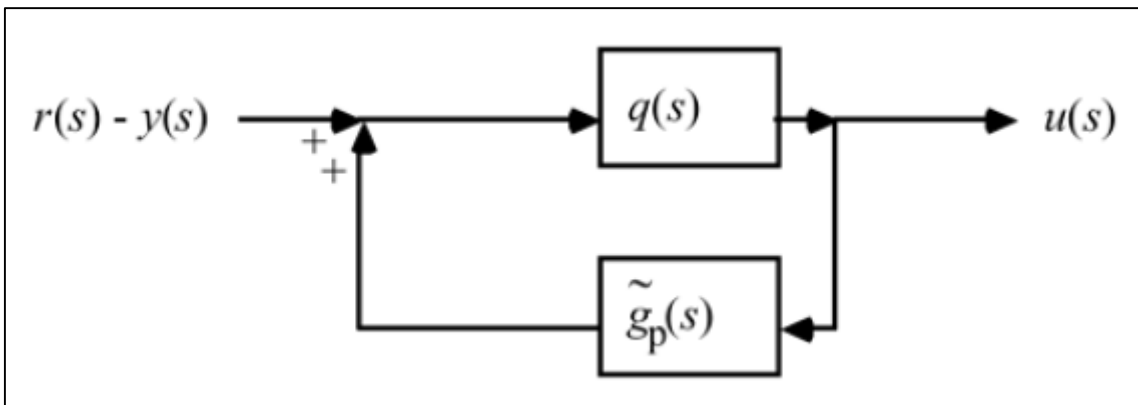


Fig 6.3:- Inner loop of figure 2

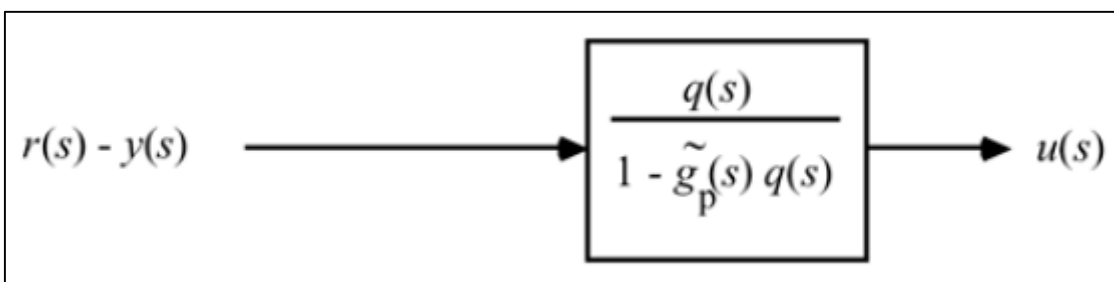


Fig 6.4- Equivalent block diagram of figure 3

### 6.3 IMC based PID design procedure

Let us consider a process model  $\check{G}_p(s)$  for an actual process  $G_p(s)$ . The controller  $Q(s)$  is used to prevent the flow of disturbance in the whole system and to reduce the effect of the mismatch of the process and the model. The IMC is designed as discussed in

chapter two and after that IMC based PID controller is designed.

### Equivalent feedback controller

By rearranging the IMC structure, we obtain the equivalent feedback controller using:

$$G_c = Q(s) / (1 - Q(s) \cdot \check{G}_p(s))$$

Thus, output  $y(s)$  is the cascade connection of  $G_c(s)$  and  $G_p(s)$  and the unity feedback system.

The manipulated variable now is;

$$u(s) = [r \cdot G_c] / [1 + G_c \cdot G_p]$$

Output is:-

$$y(s) = [r \cdot G_c \cdot G_p] / [1 + G_c \cdot G_p]$$

### Comparison of IMC with PID

Now we will compare the feedback controller,  $G_c(s)$ , with the PID transfer function to find out the tuning parameters of the PID controller.

#### 6.3.1 For First Order Process

**Given process model:**  $\check{G}_p(s) = K_p^* / [\check{T}_p(s) + 1]$

$$\check{G}_p(s) = \check{G}_{p+}(s) \cdot \check{G}_{p-}(s) = 1 \cdot K_p^* / [\check{T}_p(s) + 1]$$

$$Q_c^*(s) = \text{inv}[\check{G}_{p-}(s)] = [\check{T}_p(s) + 1] / K_p^*$$

$$Q(s) = Q_c^*(s) \cdot f(s) = [\check{T}_p(s) + 1] / [K_p^* \cdot (\lambda s + 1)]$$

$$f(s) = 1 / (\lambda s + 1)$$

Equivalent feedback controller using transformation

$$G_c(s) = Q(s) / (1 - Q(s) \cdot \check{G}_p(s)) = \{ [\check{T}_p(s) + 1] /$$

$$\{ K_p^* (\lambda s + 1) \} / \{ [1 - K_p^* / (\check{T}_p(s) + 1)] \cdot \{ \check{T}_p(s) + 1 \} / \{ K_p^* \cdot (\lambda s + 1) \} \}$$

$G_c(s) = \{ \check{T}_p(s) + 1 \} / K_p \cdot \lambda \cdot s$  (it is standard feedback controller for IMC )

$$G_c(s) = [K_c (T_i \cdot s + 1)] / (T_i \cdot s) \text{ (transfer function for PI controller)}$$

Comparing  $G_c(s)$  with PI transfer function, we get:

$$K_c = T_p / (K_p \cdot \lambda)$$

$$T_i = T_p$$

### 6.3.2 For 1<sup>st</sup> order process with delay

Here we use a first-order Padé approximation for dead time.

$$\text{Where, } e^{-\theta s} = (-0.5\theta s + 1) / (0.5\theta s + 1).$$

So, we approximate model transfer function as:

$$\begin{aligned} G_p^*(s) &= (K_p \cdot e^{-\theta s}) / (T_p \cdot s + 1) \\ &= (K_p \cdot (-0.5\theta s + 1)) / ((T_p \cdot s + 1)(0.5\theta s + 1)). \end{aligned}$$

Then we factored out the noninvertible elements:

$$G_p^{*-}(s) = K_p / ((T_p \cdot s + 1)(0.5\theta s + 1)).$$

So now  $q(s) = (G_p^{*-}(s))^{-1} \cdot f(s)$

$$= ((T_p \cdot s + 1)(0.5\theta s + 1)) / (K_p \cdot (\lambda s + 1))$$

Therefore  $G_c(s) = q(s) / (1 - G_p^*(s) \cdot q(s))$

$$= (0.5T_p \cdot s^2 + (T_p + 0.5\theta)S + 1) / (K_p \cdot (\lambda + 0.5\theta)S)$$

Where,  $T_i = T_p + 0.5\theta$ ,

$$T_d = T_p / (2T_p + \theta),$$

$$K_c = (T_p + 0.5\theta) / (K_p \cdot (\lambda + 0.5\theta))$$

### 6.3.3 For Second Order Process

**Given process model:**  $\check{G}_p(s) = K_p^* / [(\check{T}_p1(s) + 1) \cdot (\check{T}_p2(s) + 1)]$

$$\check{G}_p(s) = \check{G}_{p+}(s) \cdot \check{G}_{p-}(s) = 1 \cdot K_p^* / [\check{T}_p(s) + 1]$$

$$Q_c^*(s) = \text{inv}[\check{G}_{p-}(s)] = [\check{T}_p(s) + 1] / K_p^*$$

$$Q(s) = Q_c^*(s) \cdot f(s) = [\check{T}_p(s) + 1] / [K_p^* \cdot (\lambda s + 1)]$$

$$f(s) = 1 / (\lambda \cdot s + 1)$$

Equivalent feedback controller using

transformation,  $G_c(s) = Q(s) / (1 - Q(s) \cdot \check{G}_p(s))$

$$= [(Tp1 \cdot Tp2 \cdot s^2) + (Tp1 + Tp2)s + 1] / [Kp \cdot \lambda \cdot s]$$

(It is the transfer function for the equivalent standard feedback controller)

$G_c(s) = [K_c \{(T_i \cdot T_d \cdot s^2 + T_i \cdot s + 1)\}] / [T_i \cdot s]$  (transfer function for ideal PID controller for second order)

Comparing  $G_c(s)$  with PID transfer function, we get:

$$K_c = (Tp1 + Tp2) / (K_p \cdot \lambda)$$

$$T_i = Tp1 + Tp2$$

$$T_d = Tp1$$

## 6.4 Generalized Empirical formula for the Tuning Parameter

### 6.4.1 For First Order Process

#### Process model used

Now,  $T_p = 65$  &  $K_p = 0.02$ ;

So,  $K_c = T_p / K_p \cdot \lambda = 3250 / \lambda$

$T_i = T_p = 65$

#### Block Diagram

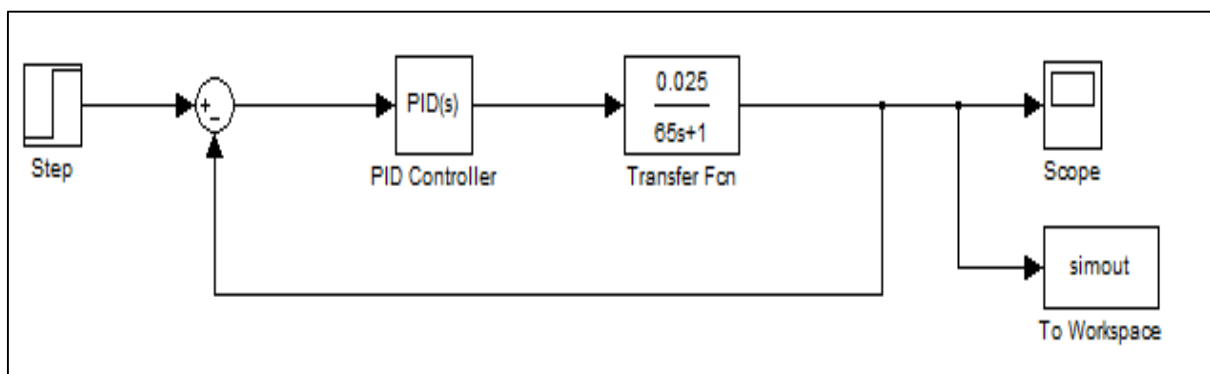


Fig 6.5:- Block diagram for 1<sup>st</sup> order IMC based PID

#### Simulation

a) For different values of  $T_p$ , the values of settling time & rise time at different values of tuning parameter

Tp	65		70		75		80		85	
$\lambda$	Tr	Ts	Tr	Ts	Tr	Ts	Tr	Ts	Ts	Ts
2	4.40	9.85	4.68	10.32	5.02	10.76	5.34	11.18	5.61	11.63
3	6.56	13.76	7.05	14.36	7.48	14.92	7.95	15.52	8.38	16.07
4	8.77	17.67	9.37	18.36	9.97	19.04	10.53	19.75	11.12	20.45
5	10.98	21.58	11.70	22.34	12.41	23.12	13.13	23.91	13.84	24.73
6	13.17	25.49	14.03	26.31	14.87	27.17	15.71	28.91	16.53	28.98
7	15.37	29.40	16.35	30.26	17.31	31.21	18.27	32.21	19.22	33.26
8	17.58	33.30	18.68	34.22	19.76	35.26	20.83	36.38	21.90	37.56
9	19.78	37.20	20.48	38.19	22.19	39.33	23.39	40.57	24.59	41.89
10	21.96	41.10	23.30	42.16	24.62	43.41	25.95	44.78	27.27	46.27
15	32.77	59.71	34.66	61.28	36.54	63.11	38.42	65.11	40.27	67.21

Table 4

b) Relationship between  $\lambda$ ; Tr & Ts for the above table

For Tp=65,  $Tr = -0.0022\lambda^2 + 2.2235\lambda - 0.0733$

$Ts = -0.0112\lambda^2 + 4.0342\lambda + 1.7493$

For Tp=70,  $Tr = -0.0037\lambda^2 + 2.369\lambda - 0.0421$

$Ts = -0.0096\lambda^2 + 4.0870\lambda + 2.1665$

For Tp=75,  $Tr = -0.004\lambda^2 + 2.5047\lambda + 0.0161$

$Ts = -0.0087\lambda^2 + 4.1747\lambda + 2.4560$

For Tp=80,  $Tr = -0.0056\lambda^2 + 2.6405\lambda + 0.0709$

$Ts = -0.0077\lambda^2 + 4.2758\lambda + 2.7120$

For Tp=85,  $Tr = -0.0073\lambda^2 + 2.7900\lambda + 0.0657$

$Ts = -0.0073\lambda^2 + 4.3975\lambda + 2.9060$

c) For different desired values of rise time

Tr=5,  $\lambda = -0.077Tp + 5.8394$

Tr=10,  $\lambda = -0.1378Tp + 10.97$

Tr=15,  $\lambda = -0.1855Tp + 15.6177$

$$Tr=20, \lambda=0.0010Tp^2-0.2451Tp+20.6843$$

$$Tr=25, \lambda=0.0013Tp^2-0.3095Tp+25.9691$$

d) Standard Equation:

$$\lambda=aTp^2+bTp+c$$

Values of the co-officiant 'a' at different Tr,

Tr	a
5	0
10	0
15	0
20	0.0010
25	0.0013

Table 5

So, a=0

Values of the co-officiant 'b' at different Tr,

Tr	b
5	-0.077
10	-0.1378
15	-0.1855
20	-0.2451
25	-0.3095

Table 6

So, b=-0.0098Tr-0.0288

Values of the co-officiant 'c' at different Tr,

Tr	c
5	5.8394
10	10.97
15	15.6177
20	20.6843
25	25.9691

Table 7

So,  $c=0.0021Tr^2+0.9371Tr+1.1876$

e) For desired value of settling time

$T_s=10, \lambda=-0.0563T_p+4.7423$

$T_s=25, \lambda=-0.0355T_p+8.2817$

$T_s=40, \lambda=-0.0384T_p+12.8406$

$T_s=55, \lambda=-0.0440T_p+17.6734$

$T_s=70, \lambda=-0.0647T_p+23.2151$

f) Standard Equation:

$\lambda=aT_p+b$

Values of the co-officiant 'a' at different  $T_s$ ,

$T_s$	a
10	-0.0563
25	-0.0355
40	-0.0384
55	-0.0440
70	-0.0647

Table 8

So,  $a=0.002T_s-0.0723$

Values of the co-officiant 'b' at different  $T_s$ ,

$T_s$	b
10	4.7423
25	8.2817
40	12.8406
55	17.6734
70	23.2151

Table 9

So,  $b=0.0014T_s^2+0.2003T_s+2.5560$



## 6.4.2 For First Order Process with Delay

### Process model used

For dead time  $\emptyset=2$ , first-order Padé approximation gives  $e^{-(2s)} = \frac{-s+1}{(s+1)}$ .

$$G_p^*(s) = \frac{0.025 * e^{-(2s)}}{(75s+1)}$$

$$= \frac{0.025 * (-s+1)}{(75s+1)(s+1)}$$

$$q(s) = \frac{(75s+1)(s+1)}{0.025 * (\lambda s+1)}$$

We obtain  $T_i=76$ ,

$$T_d=0.493$$

$$K_c=3040/(\lambda+1)$$

### Block Diagram

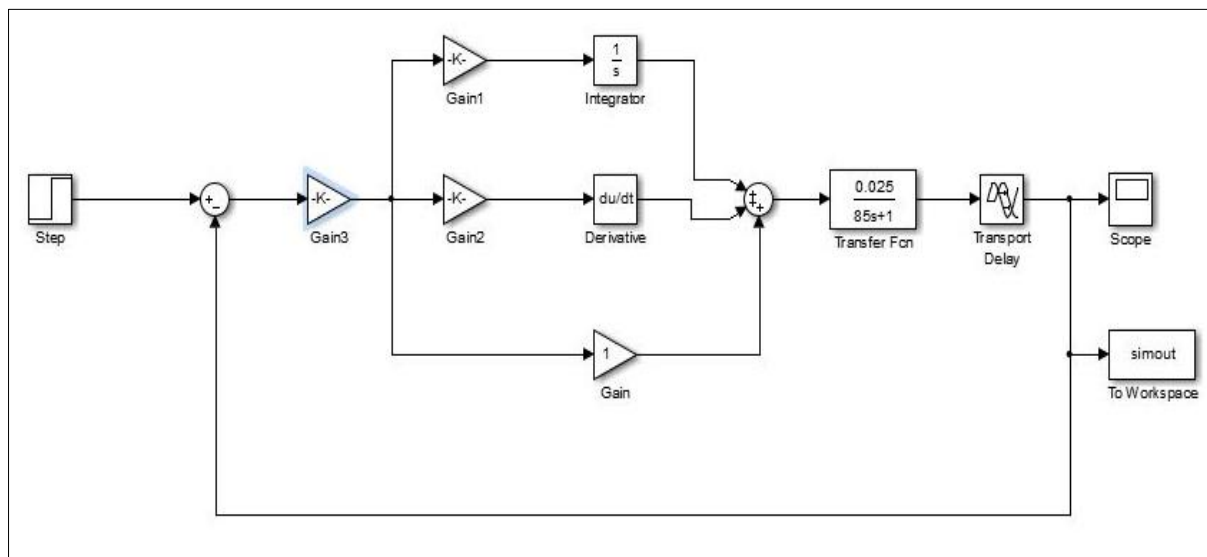


Fig 6.6:- Block diagram for 1<sup>st</sup> order IMC based PID with delay

### Simulation

a) For different values of  $T_p$ , the values of settling time & rise time at different values of tuning parameter

Tp	65		70		75		80		85	
$\lambda$	Tr	Ts	Tr	Ts	Tr	Ts	Tr	Ts	Tr	Ts
2	0.98	2.22	11.16	2.46	11.41	2.68	11.31	3.07	8.84	3.40
3	9.27	3.53	10.48	4.10	11.66	4.60	12.72	5.24	13.69	5.81
4	13.68	5.46	14.88	6.22	15.98	6.95	17.03	7.64	18.03	8.34
5	17.71	7.49	18.89	8.36	20.02	9.20	21.14	10.02	22.23	10.83
10	36.99	17.45	38.21	18.84	39.60	20.22	41.09	21.59	42.68	22.95
15	55.22	27.21	56.69	29.12	58.49	31.02	60.49	32.92	62.63	34.81
20	69.88	36.48	71.67	38.78	73.65	41.03	75.72	43.24	77.78	45.38
25	79.71	44.6	81.41	47.09	83.12	49.42	84.76	51.63	86.28	53.72
30	85.69	51.35	87.08	53.71	88.39	55.91	89.58	57.93	90.64	59.78

Table 10

b) Relationship between  $\lambda$ ; Tr & Ts for the above table

For Tp=65,  $T_s = -0.0407\lambda^2 + 4.1299\lambda + 0.6866$

$Tr = 0.0096\lambda^2 + 2.0892\lambda - 2.0942$

For Tp=70,  $T_s = -0.0385\lambda^2 + 4.035\lambda + 3.1412$

$Tr = -0.0131\lambda^2 + 2.2732\lambda - 2.1173$

For Tp=75,  $T_s = -0.0438\lambda^2 + 4.2309\lambda + 3.2889$

$Tr = -0.0169\lambda^2 + 2.4628\lambda - 2.1779$

For Tp=80,  $T_s = -0.055\lambda^2 + 4.6349\lambda + 2.0767$

$Tr = -0.0208\lambda^2 + 2.6488\lambda - 2.1761$

For Tp=85,  $T_s = -0.065\lambda^2 + 4.9942\lambda + 1.418$

$Tr = -0.0251\lambda^2 + 2.6488\lambda - 2.2114$

c) For different desired values of settling time

For Ts=10,  $\lambda = 0.0041Tp^2 - 0.6302Tp + 26.0626$

For Ts=20,  $\lambda = 0.0028Tp^2 - 0.4656Tp + 23.306$

For Ts=30,  $\lambda = 0.0024Tp^2 - 0.4349Tp + 25.6763$

For Ts=40,  $\lambda = 0.0012Tp^2 - 0.2797Tp + 23.6263$

For Ts=50,  $\lambda = 0.420Tp + 22.4674$

d) Standard Equation:

$$\lambda = a T_p^2 + b.T_p + c$$

Values of the co-officiant 'a' at different Ts,

A	Ts
0.0041	10
0.0028	20
0.0024	30
0.0012	40
0	50

Table 11

So, a = 0.0050

Values of the co-officiant 'b' at different Ts,

b	Ts
-0.6302	10
-0.4656	20
-0.4349	30
-0.2797	40
-0.420	50

Table 12

So, b = 0.0061Ts - 0.6280

Values of the co-officiant 'c' at different Ts,

c	Ts
-0.6302	10
-0.4656	20
-0.4349	30
-0.2797	40
-0.420	50

Table 13

So, c = -0.0687 Ts + 26.288

e) For different desired values of rise time

For  $T_r=10$ ,  $\lambda = -0.0614 T_p + 09.750$

For  $T_r=20$ ,  $\lambda = -0.0932 T_p + 16.492$

For  $T_r=30$ ,  $\lambda = -0.1032 T_p + 21.896$

For  $T_r=40$ ,  $\lambda = -0.0878 T_p + 25.721$

For  $T_r=50$ ,  $\lambda = 0.0366 T_p + 27.3270$

f) Standard Equation:

$$\lambda = a T_p^2 + b.T_p + c$$

Values of the co-officiant 'a' at different  $T_r$ ,  $a=0$

Values of the co-officiant 'b' at different  $T_r$ ,

b	Tr
-0.0614	10
-0.0932	20
-0.1032	30
-0.1355	40
-0.1732	50

Table 14

So,  $b = -0.0929$

Values of the co-officiant 'c' at different  $T_r$ ,

c	Tr
9.705	10
16.492	20
21.896	30
25.721	40
27.327	50

Table 15

So,  $c = -0.4447 T_s + 6.8863$

### 6.4.3 For Second Order Process

#### Process model used

$$G_p^*(s) = 1/(10s+1)(10s+1) \quad (\text{taking } T_{p1}=T_{p2}=T_p)$$

$$Q_c(s) = (10s+1)(10s+1)/(\lambda s+1)$$

$$G_c(s) = Q_c(s)/(1-Q_c(s)G_p^*(s)) \\ = (100s^2+20s+1)/(\lambda s)$$

Comparing with the standard PID controller

$$K_c = 20/\lambda$$

$$T_i = 20$$

$$T_d = 5$$

#### Block Diagram

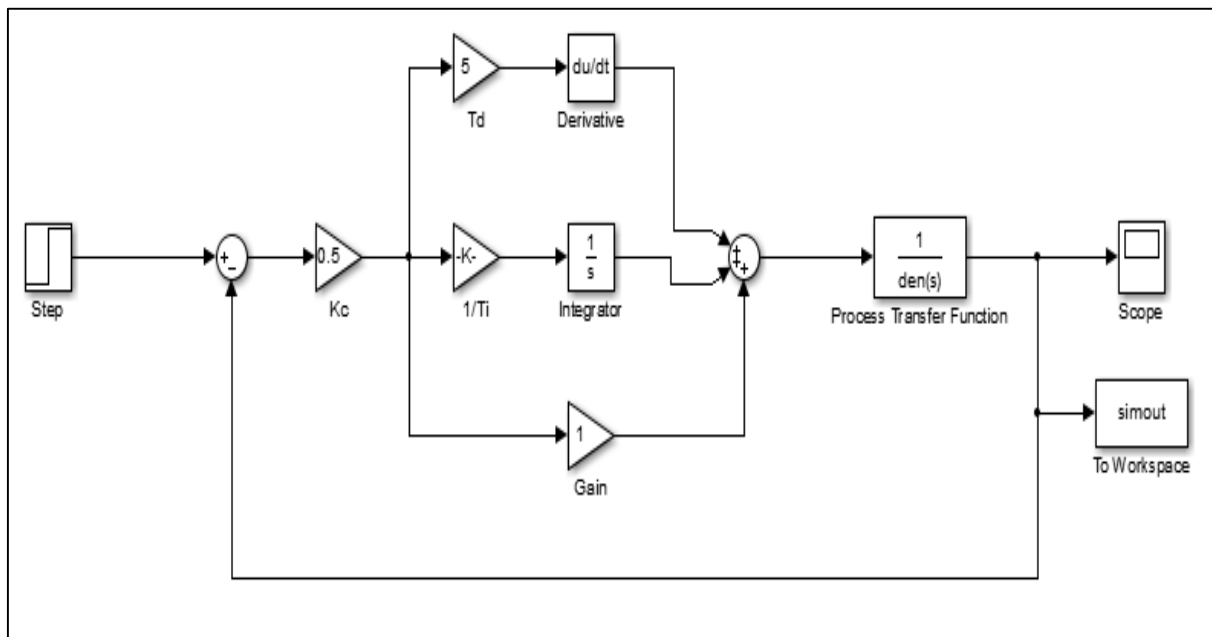


Fig 6.7:- Block diagram for 2<sup>st</sup> order IMC based PID

#### Simulation

a) For different values of  $T_p$ , the values of settling time & rise time at different values of tuning parameter

Tp	10		20		30		40		50	
$\lambda$	Tr	Ts	Tr	Ts	Tr	Ts	Tr	Ts	Tr	Ts
2	7.6	52.43	10.09	50.97	12.98	96.60	12.55	100.15	9.32	97.13
3	8.14	49	12.01	81.98	13.80	99.30	12.15	97.55	16.02	100.34
4	8.73	43.13	13.59	90.99	14.19	88.20	16.13	100.10	25.53	100.36
5	9.41	37.74	14.87	92.67	15.68	98.10	21.13	100.27	34.46	99.95
10	12.85	59.27	21.69	96.14	32.14	99.31	48.64	96.95	60.04	94.46
15	16.65	71.39	31.49	97.02	48.16	95.32	61.12	94.87	64.37	98.47
20	21.46	79.74	41.77	94.05	58.24	90.29	64.62	97.89	64.93	99.10
30	57.86	89.12	57.11	88.62	65.30	97.38	64.94	98.98	64.82	99.48
40	63.71	92.92	64.15	95.4	66.95	98.44	66.01	99.28	64.58	99.61

Table 16

b) Relationship between  $\lambda$ ; Tr & Ts for the above table

For Tp=10,  $Tr=0.0246 \lambda^2+0.5927 \lambda +5.6118$

$Ts=-0.0178 \lambda^2+2.0239 \lambda +42.452$

For Tp=20,  $Tr=-0.0131 \lambda^2+2.0269 \lambda +5.4287$

$Ts=-0.0591 \lambda^2+2.9381 \lambda +65.2912$

For Tp=30,  $Tr=-0.0511 \lambda^2+3.645 \lambda +3.0782$

$Ts=-0.0137 \lambda^2+0.7751 \lambda +87.4651$

For Tp=40,  $Tr=-0.0816 \lambda^2+4.7681 \lambda +2.8834$

$Ts=0.001 \lambda^2+0.0131 \lambda +97.1478$

For Tp=50,  $Tr=-0.0907 \lambda^2+4.9326 \lambda +7.4378$

$Ts=0.0045 \lambda^2-0.1653 \lambda +99.5601$

c) For different desired values of settling time

For Ts=60,  $\lambda= -0.061 Tp^2+6.03 Tp-44.75$

For Ts=70,  $\lambda= 0.4318 Tp^2-14.3685 Tp+116.32$

For Ts=80,  $\lambda= 0.3851 Tp^2-13.323 Tp+118.06$

For Ts=90,  $\lambda= 0.0761 Tp^2-4.5265 Tp+70.83$

For Ts=100,  $\lambda= -0.0733 Tp^2+5.792 Tp-67.2133$

d) Standard Equation:

$$\lambda = a T_p^2 + b.T_p + c$$

Values of the co-officiant 'a' at different Ts,

Ts	a
60	-0.061
70	0.4318
80	0.3851
90	0.0761
100	-0.0733

Table 17

So,  $a = -0.0011 T_s^2 + 0.173 T_s - 6.3937$

Values of the co-officiant 'b' at different Ts,

Ts	b
60	6.03
70	-14.3685
80	-13.323
90	-4.5265
100	5.792

Table 18

So,  $b = 0.0494 T_s^2 - 7.8132 T_s + 294.8187$

Values of the co-officiant 'c' at different Ts,

Ts	c
60	-44.75
70	116.32
80	118.06
90	70.83
100	-67.2133

Table 19

So,  $c = -0.4623 T_s^2 + 73.0612 T_s$

e) For different desired values of rise time

For  $Tr=10$ ,  $\lambda= 0.0037 Tr^2-0.3375 Tr+8.508$

For  $Tr=20$ ,  $\lambda= 0.0114 Tr^2-0.9482 Tr+22.882$

For  $Tr=30$ ,  $\lambda= 0.0138 Tr^2-1.208 Tr+32.192$

For  $Tr=40$ ,  $\lambda= 0.0138 Tr^2-1.2941 Tr+39.106$

For  $Tr=50$ ,  $\lambda= 0.0115 Tr^2-1.2239 Tr+44.162$

d) Standard Equation:

$$\lambda = a Tp^2 + b.Tp + c$$

Values of the co-officient 'a' at different Tr,

Tr	a
10	0.0037
20	0.0114
30	0.0138
40	0.0138
50	0.0115

Table 20

So,  $a=0.0011 Tr-0.0058$

Values of the co-officient 'b' at different Tr,

Tr	b
10	-0.3375
20	-0.9482
30	-1.208
40	-1.2941
50	-1.2239

Table 21

So,  $b=0.0011 Tr^2-0.087 Tr+0.401$



Values of the co-officiant 'b' at different Tr,

Tr	c
10n	8.508
20	22.882
30	32.192
40	39.106
50	44.162

Table 22

So,  $c = -0.015 \text{ Tr}^2 + 1.7767 \text{ Tr} - 7.4056$

# CONCLUSION

The Internal Model Control (IMC) is a powerful control strategy that can be used in various industrial and manufacturing processes for its robustness towards the uncertainties in various plant parameters and environments.

Also the IMC based PID controller provides a much simpler and robust way or technique to handle the various uncertainties and therefore is widely used in the design of control strategies in various industrial processes. IMC based PID has the added advantage of having only a single tuning parameter instead of the multiple tuning parameters used for control purposes in a simple PID based controller.

In addition to solving the problems that arise due to model uncertainty (i.e. by being robust to model inaccuracies) it is widely used in industrial procedures having large time delays that occurs when a process is made to operate in real-time environments. It also helps in reducing the effects of various kinds of discrepancies that somehow enter into the process through proper tuning of the process through the filter tuning parameter. The best performance for the PID is arrived through an optimum value of the tuning parameter that also determines how good the structure of the filter is.

It was also found out that an IMC can be restructured as a feedback controller based on PID control strategy using a single tuning parameter. It has the added advantage of improved set point tracking.

Without any time delay there is no significant difference between the performance characteristics of a simple IMC and IMC based PID Controller. Also IMC based PID control strategy helps in dealing with the problem of presence of RHP zero in the process that results in unstable closed loop response.

Hence IMC is used not just for its robustness to model inaccuracies, and disturbance compensation but also because of the above mentioned advantages. However it is important to detune the IMC particularly under the conditions of model uncertainty so that we guarantee for both stability and enhanced performance.

## REFERENCE

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