DIFFERENTIAL EVOLUTION OPTIMIZATION TECHNIQUE TO DESIGN GEAR TRAIN SYSYTEM

A thesis submitted in partial fulfilment of the requirements

For the degree of

Bachelor of Technology
In
Mechanical Engineering

By

Tribed Kumar Mahanand
(111ME0309)
Under the guidance of

Prof. S.S. Mahapatra

Department of Mechanical Engineering National Institute of Technology Rourkela 769008

CERTIFICATE

This is to certify that the work in this thesis entitled "Differential Evolution Optimization Technique To Design Gear Train System" by ***Tribed Kumar Mahanand***, has been carried out under my supervision in partial fulfilment of the requirements for the degree of *Bachelor of Technology in Mechanical Engineering* during session 2014-2015 in the *Department of Mechanical Engineering, National Institute of Technology, Rourkela*.

To the best of my knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.

DATE- 7/5/2015
PLACE- NIT Rourkela

Prof. Siba Sankar Mahapatra

Head of the Department

Mechanical Engineering

National Institute of Technology Rourkela

ACKNOWLEDGEMENT

I am extremely fortunate to be involved in an exciting and challenging research project like "Differential Evolution Optimization Technique to Design Gear Train System". It has enriched my life, giving me an opportunity to work in a new environment of MATLAB. This project increased my thinking and understanding capability as I started the project from scratch. I would like to express my greatest gratitude and respect to my Supervisor P***rof. Siba Sankar Mahapatra***, for his excellent guidance, valuable suggestions and endless support. He has not only been a wonderful supervisor but also a genuine person. I consider myself extremely lucky to be able to work under the guidance of such a dynamic personality. Actually he is one of such genuine person for whom my words will not be enough to express.

Last, but not the least I extend my sincere thanks to all faculty members of Mechanical Engineering Department for making my project a successful one, for their valuable advice in every stage and also giving me absolute working environment where I unlashed my potential. I would like to thank all whose direct and indirect support helped me completing my thesis in time. I want to convey my heartiest gratitude to my parents for their unfathomable encouragement.

Tribed Kumar Mahanand

111ME0309

Bachelor of Technology, Mechanical Engineering

Contents

- Certificate …………………………………………………….…. 2
- Acknowledgement………………………………………….. 3
- Abstract…………………………………………….……. 6
- List of figures……………………………………………..7-8.
- List of tables…………………………………………………… 8
- Nomenclature…………………………………………………. 9
- Introduction…………………………………………….…10
    - Basic expectations from an optimization algorithm. ………………10
    - Basic features of differential evolution optimization algorithm……11
    - The Algorithm………………………………………………………11
    - Comparison between GA and DE…………………………………..12
    - Application of Differential Evolution………………………………12
- Literature review……………………………………………………13-16
- Differential evolution methodology……………………………...............
    - Problem definition……………………………........20
    - Variables…………………………......................20
    - Input parameters…………………………….............20
    - Constraint formation…………………………….......21
    - Experimental Data…………………………….........22
    - Input parameters…………………………….............23
    - Lewis equation for tooth bending stress……………..23

- ❖ Forming objective function…………………………25
- ❖ Penalty function…………………………..............26-27

- ➢ Input parameter values…………………………………………...........23

- ➢ Results…………………………………………….…………………28

- ➢ Discussions…………………………………………….……………39

- ➢ Conclusions…………………………………………….……………40

- ➢ References…………………………………………….……………41-42

Abstract

A high performance mechanical power transmission system needs least weight, minimum centre to centre distance and higher strength to maintain its performance. In the present paper the gear problem is solved by minimizing volume, centre to centre distance and maximizing gear strength of gear trains since they are crucial parameters of the gear design problem. Conventional optimisation techniques cannot be used to optimise multi- objective function with constraints easily. The expectation from a desired optimisation are it should find a true global minimum, convergence should be fast, have a minimum number of control parameters, simple and efficient to utilise.

Differential evolution optimisation, a simple and effective technique for global optimisation over incessant space, doesn’t need the function have to be continuous or differential as usually required by classical optimization. Some system parameters represented as vector are chosen, are decision variables. a multi objective function taking into consideration of module, width factor, number of teeth like its parametric vector or decision variable.

DE is a population based optimisation technique, tries to improve a candidate solution iteratively, accepts a solution vector and uses the formula in order to derive a new candidate solution from the existing candidates and find out the best function value from the existing functions by comparing. Penalty function is incorporated in order to handle constraints.

List of figures

figure no	description	Page no
1	Clear representation of whole phenomenon of differential evolution optimization algorithm.	17
2	Two-dimensional example of an objective function showing contour lines and the process for generating V in Scheme DE.	17
3	Flow chart showing diffrential evolution algorithm	18
4	gear tooth represented as cantilever beam	24
5	Plot of function value vs iterations for ordinary cut gears and decision variables for weightage (0.4, 0.6) considering only two objectives.	30
6	Plot of function value vs iterations for ordinary cut gears and decision for weightage(0.6,0.4) considering three objectives	30
7	Plot of function value vs iterations for ordinary cut gears and decision for weightage (0.5, 0.5) considering three objectives	31
8	Plot of function value vs iterations for ordinary cut gears and decision for weightage(0.4,0.35,0.25) considering three objectives	31
9	Plot of function value vs iterations for ordinary cut gears and decision for weightage (0.3, 0.3, 0.4) considering three objectives	32
10	Plot of function value vs iterations for ordinary cut gears and decision for weightage (0.35, 0.4, 0.25) considering three objectives	32
11	Plot of function value vs iterations and decision for weightage (0.35, 0.4, 0.25) considering three objectives for carefully cut gear	33
12	Plot of function value vs iterations and decision for weightage (0.3, 0.3, 0.4) considering three objectives for carefully cut gear	33
13	Plot of function value vs iterations and decision for weightage (0.4, 0.35, 0.25) considering three objectives for carefully cut gear	34
14	Plot of function value vs iterations and decision for weightage (0.3, 0.3, 0.4) considering three objectives for carefully cut & grounded metallic gear	34

15	Plot of function value vs iterations and decision for weightage (0.35, 0.4, 0.25) considering three objectives for Carefully cut & grounded metallic	35
16	Plot of function value vs iterations for ordinary cut gears and decision for weightage (0.4, 0.35, 0.25) considering three objectives for carefully cut & grounded metallic gear	35
17	Plot of function value vs iterations and decision for weightage (0.4, 0.35, 0.25) considering three objectives for Gear of Hardened steel, ground and lapped in precision	36
18	Plot of function value vs iterations and decision for weightage (0.35, 0.4, 0.25) considering three objectives for Gear of Hardened steel,ground and lapped in precision	36
19	Plot of function value vs iterations and decision for weightage (0.3, 0.3, 0.4) considering three objectives for Gear of Hardened steel,ground and lapped in precision	37
20	Plot of function value vs iterations and decision for weightage (0.3, 0.3, 0.4) considering three objectives for Gears whose teeth are finished by hobbing or shapping	37
21	Plot of function value vs iterations for weightage (0.4, 0.35, 0.25) considering three objectives for Gears whose teeth are finished by hobbing or shaping.	38
22	Plot of function value vs iterations decision for weightage (0.35, 0.4, 0.25) considering three objectives for Gears whose teeth are finished by hobbing or shaping	38

List of tables

Table no	description	Page no
1	description about the DE parameters	16
2	objective function value considering two objectives	28
3	comparing the objective function value at various strategies taking three objectives at weighted average (0.3, 0.3, 0.4)	28
4	objective function value at three various combination of weighted parameter taking three objectives	29

Nomenclature

1	kd	Dynamic Velocity Factor
2	Kfe	Form Factor
3	Kc	Stress Concentration Factor
4	Ft	Tangential Force
5	Ɛ	Overlap Ratio
6	b	Face Width
7	m	Module
8	σk	Ultimate Tensile Strength
9	Km	Material Factor
10	Kα	Flange Transverse Coefficient
11	KƐ	Tooth Overlap Factor
12	Kß	Tooth Slope Factor
13	Z1	No. of Teeth on pinion
14	Z2	No. of Teeth on Gear
15	Pall	Allowable Surface Pressure= 0.25 HB
16	HB	Brinell Hardness
17	K0	Working Factor
18	M	Bending moment
19	I	Moment of inertia
20	y	Lewis form factor

Introduction:

Gear plays a crucial role for transmission of powers in automobiles and various machineries with great accuracy still gear design is an on-going activity. It transfers power between parallel, perpendicular, angular shafts to have various transmissions. The system formed by interlocking of the teeth of gears on a frame and their resulting engagement is known as a gear train. The gear teeth are designed keeping in mind the pitch circle (imaginary circle designed on the gear axis) so as to ensure that no slipping occurs during the moment, while maintaining the gear engagement. This also aims to provide an efficient transfer of reactionary moments from one gear to the other. Weight, centre to centre distance and strength are crucial to be considered for gear design, a high performance gear train require a low weight, low centre to centre distance but a high strength. In case of the design of gear, bending strength, input power and rotational frequency can be easily calculated by using conventional design, but without an optimisation algorithm weight, centre to centre distance, strength can't be optimized. Involvement of empirical formulas, graphs, tables various constraints complicate the calculation and make it a time consuming process. We can take an example to say the design obtained with conventional technique may not be optimum. The variables considered follow one condition at a time, if module is found out depending on bending strength; same is used to calculate surface durability. The acceptance depends upon the constraint strength limit. Basically an optimization algorithm is adopted in order to fulfil the increasing demand for compact, efficient and reliable gears. Optimization performance depends heavily on the choice of the DE parameters like F, CS and NP. The primary objective of on-going research in the field is hence finding out the most optimal parameter to produce best results. Dynamic development of the variants applied in the DE algorithm ensures that optimization performance is enhanced. The most recent approach of research in the DE algorithm aims to generate more advanced DE variants so as to comply with the changing DE parameters during the optimization of problem.

Basic expectations from an optimization algorithm

Optimization of gear train acts at getting the minimum fitness function considering three objectives and three decision variables, module, face width, number of teeth. According to researcher R Storn and K Price differential evolution is a simple and efficient heuristic for global optimisation over continuous spaces. The three important expectations of user from a differential evolution optimization are

1) Regardless of parametric vectors value, true global minimum should be obtained.
2) Should converge fast.
3) Should be easy and efficient to use by keeping control variable minimum.

Basic features of differential evolution optimization algorithm are mentioned below:

- Used to optimize specifically minimize non-linear and non-differentiable continuous space function.
- Minimum number of control variables is required, robust in nature, easy to use; make itself suitable to parallel computation.
- System parameters or decision variables are considered as vectors.
- A multi objective function is developed including different objectives considering various constraints.
- New parameter vectors are generated using mutation and recombination and then decision is made to accept that new one or not.
- The parametric vector reduces the value of objective function, selected to replace the earlier one.
- Since the convergence rate is very fast and the technique is robust it avoids to get trapped in a local minimum, following its basic steps mutation, recombination and selection it reaches to its global minimum.

The Algorithm

The algorithm of differential evolution optimization technique is simple, robust and less time computing.it is mentioned below:

- Initialization of required DE parameters, include shape factor, cross over ratio etc.
- Parametric vectors are initialized randomly with in the boundary mentioned for decision variables.
- The various candidate solution derived from the population are evaluated. On dominated solution of the population are identified and kept in the non-dominated elitist archive (NEA).
- All the member of the population undergoes mutation and cross over operation.

For every population NP_I from NP number of population,

a) Excluding parent vector different other vectors are taken from the current population.
b) Using the mutation step, mutation vector is calculated.
c) Using the recombination step, cross over ratio or cross over probability is used to modify the mutated vector.
d) Upper and lower bound of decision variables mentioned earlier, restrict the variables with in it.

Every candidate solution of the population is evaluated. If the candidate is better than the parent in minimizing objective function then it takes the place of parent and vice versa also true. A temporary population (temppop) is there to which candidate is inserted.

- G the current generation is increased to G+1 and the two termination conditions are checked one the exceeding of maximum number of iteration and other of going less than value to reach.
- After mutation and recombination using CR selection procedure is followed to get the desired value of parametric vectors, minimizes objective function and gets the optimized fitness function.

Comparison between GA and DE

There are various points of comparison between Genetic Algorithms and Differential Evolution. While both the algorithms use the same variable transformation system to perform operations like mutation and crossover, there are various important differences in the approaches. GA performs mutation operations using small disturbances for the genes of a variable, while DE uses mathematical operators like addition, etc. to do the same task. Mutation is the most important function of DE, while GA lays more emphasis on crossovers. The iteration time of GA is much lower than DE, while the strength of DE lies in its accurate result producing algorithms and gives better output than GA or other evolutionary processes. Irrespective of hindrances like noise, multi-modals, multi-dimensional and multi-variables, DE gives sound results and is applicable in various real-world problems. Another perk of using DE algorithms is that fine-tuning of various parameters like CR and F is not required, which is a prime requisite of other evolutionary algorithms.

Application of Differential Evolution

Specified underneath is a list of various scientific and commercial applications of DE, accessible online through the listed URLs. As the field of DE is very dynamic and rapidly changing, the list is updated very frequently and is impossible to find any constant source for the same. The list is gathered by noting the prominent URls that have popped up during the search of the words “Differential Evolution” on Google.

1) Multiprocessor synthesis

2) Neural network learning

3) Crystallographic characterization

4) Synthesis of modulators

5) Optimization of an alkylation reaction

6) optimization for design of gear train

Literature review

Serial No	Name of title	Author	Journal	Year	Deliverables
1.	Optimization of multi-model discreet functions using Genetic Algorithms	Pham, D.T. & Yang, Y.		1993	The principle of Optimization of multi-model discreet functions using Genetic Algorithms is done. A few applications have been produced by the scientists utilizing diverse outline and estimation techniques. A gearbox was intended to deliver the wanted yield speed by utilizing GA.
2.	A new and generalised methodology to design multi-staged gear drives by integrating the dimensional and configuration design process	Chong, T.H., Bae, I., Park, G.J.	Mechanism and machine theory	2002	The target capacity expressed the quantity of teeth and number of shafts. The requirements utilized were most extreme transmission proportion, number of teeth of apparatus and greatest number of shafts. For computerizing preparatory outline of multi stage outfit a calculation was proposed.
3.	A solution method for optimal weight design problem of the gear using genetic algorithms	Yokota, T., Taguchi, T.,& Gen,M.	Computers and Industrial Engineering	1998	The calculation which comprised of four stages was run iteratively in order to get an alluring arrangement. The ventures in the calculation were directed physically, by arbitrary pursuit and create and test systems. A mimicked toughening calculation for minimizing geometrical volume of a gearbox by method for coordinating configurationally and dimensionally plan procedure was utilized. An ideal weight plan issue utilizing GA was considered for a rigging pair framework
4.	Tradeoff analysis in minimum volume design of multi stage spur gear	Thompson, D.F., Gupta, S., & Shukla, A.	Mechanisms and machine theory	2000	A summed up ideal outline plan to gear trains was exhibited considering the multi staged spur gear.

	reduction units				
5.	Genetic optimisation of gears	Marcelin, J.L.	International journal of advanced manufacturing technology	2001	The trade-off between least volume and surface weariness life utilizing multi-objective improvement was examined. A CAD way to deal with apparatuses was proposed for the genetic optimization of gear.
6.	Intelligent tutoring system for training in design and manufacturing	Abersek, B., & Popov, V.	Advances in engineering software	2004	to advance single stage rigging pair. GA was utilized for minimizing volume of rigging by lessening focus separation of apparatus sets and different parameters, for example, transmitting force, decrease proportion. An expert framework including a GA module was created in a study.
7.	A fast and elitist multi objective genetic algorithm: NSGA-II	Deb, K. Pratap, A., Agarwal, S., Meyarivan, Ti.	IEEE transactions on evolutionary computation	2002	Lately, numerous calculations have been presented for multi-objective enhancement. The vast majority of these exist in the field of Evolutionary Algorithms (EAs) – otherwise called Multi-objective Optimization EAs (MOEAs). Among these are NSGA (Non-ruled Sorting Genetic Algorithm-II) by Deb et
8.	Improving the strength pareto evolutionary algorithm	Zitzler, E., Laumanns, L.	Computer Engineering and networks laboratory	2001	To enhance the efficiency of the algorithm mentioned by pareto new algorithm is incorporated.
9.	Differential Evolution – a simple evolution strategy for fast optimisation	Price, K.V., Storn, R.	Dr. Dobb's general	1997	MOEA work by taking strong points of EAs and apply them to Multi-objective Optimization Problem (MOPs). An imperative EA utilized for multi-objective improvement will be Differential Evolution (DE).
10	A bibliography of differential evolution algorithms	Lampinen, J.			It has been successful in comprehending single-objective optimization issues not for more than one objectives

11	Multiobjective optimisation using a pareto differential evolution approach	Madavan , N.K.	Congress on evolutionary competition	2002	Accomplished great results by utilizing Pareto Differential Evolution Approach (PDEA1). PDEA is connected to DE to make new people. It joins both populaces and does the computation for non-dominated rank (with Pareto-based positioning task) and differing qualities rank (with the swarming separation metric) for all the people. Two variations of PDEA were discovered to be examined. The initially utilized a strategy to contrast every tyke and its parent. The tyke was found to supplant the guardian if had higher or same no dominated rank and a higher Diversity rank. Generally the calculation disposed of the kid. The variation didn't create likely results. Despite the fact that the differences was discovered to be great, yet the meeting was moderate.
12	Pareto-based multi objective differential evolution	Xue,F., Sanderson, A.C. , Graves, R.J.	Proceeding of the 2003 congress on evolutionary computation	2003	Presented a Multi-objective Differential Evolution (MODE). The calculation uses swarming separation metric and Pareto-based positioning task, however in a methodology that is unique in relation to PDEA (Pareto Differential Evolution Approach). Wellness is ascertained utilizing Pareto-based positioning and it is then decreased by people swarming separation esteem. This wellness worth is utilized to choose best people for the up and coming populace. It created preferable results over SPEA (Strength Pareto Evolutionary Algorithm) in five benchmark issues.
13	Module 2 – Gears: Spur Gear design	Prof. K. Gopinath & Prof. M. M. Mayuram	Machine Design II		Design of the gear considering Lewis equation to calculate bending strength.
14	A simple and efficient Heuristic for Global optimisation over continuous spaces	Storn, R. & Price, K.	Journal of Global Optimisation	1996	Another heuristic methodology for minimizing perhaps nonlinear and non-differentiable consistent space capacities is introduced. By method for a broad tested it is shown that the new strategy joins speedier and with more assurance than numerous other acclaimed worldwide streamlining strategies. The new technique obliges few control variables, is strong, simple to utilize, and loans itself extremely well to parallel calculation.

15	Genetic Algorithms and differential algorithms evolution algorithms applied to cyclic instability problems in intelligent environments with nomadic	Sosa, A., Zamiudio, V., Baltazar, R.	Workshop proceedings of the 9th international conference on intelligent environments	2013	In this paper the issue of cyclic instability in element situations is introduced. This cyclic instability is produced when parallel standard based roaming interface in complex ways, creating undesirable yields for the last client. Our technique is centred around minimizing this cyclic conduct, utilizing advancement calculations, specifically Genetic and Differential Evolution Algorithms.
16	Optimal weight design of a gear train using particle swarm optimization and simulated annealing	Savsani V., Rao R.V.,Vakharia D.P.	Deapartment of mechanical engineering	2009	In this paper the constraints are developed for the optimization of the gear design for weight minimization and optimized by particle swarm optimization and using simulated annealing

Differential evolution methodology

Differential evolution, a direct search method, consider NP parameter vectors as a population along with cross over ratio and shape factor as DE parameter for generation or iteration G. NP is a fix parameter which doesn't alternate while minimization is occurring. The starting assumption n is taken here by considering the upper and lower bound of the decision variable. A new parameter vector is obtained by by combining a third vector with the weighted difference of the other two candidates where all of them are distinct in nature. Objective function is calculated based upon these resulting vector which tries to minimize the fitness function. Parametric vector obtaining minimum value of objective function replaces the one with which its compared earlier. The main track of an optimization method is to always have the best population member which gives better means minimum value of objective function.

S.No	Name of the parameter	Description	value
1	Population Numbers(NP)	Population sizes determines the number of candidate solution vectors and the computing time	15
2	Mutation factor (F)	Determines the perturbation ratio, candidate solutions can achieve and the rate of convergence.	0.8
3	Cross over rate(CR)	Determines the probability of swapping in between trial and target vector.	0.8

Table 1: description about the DE parameters

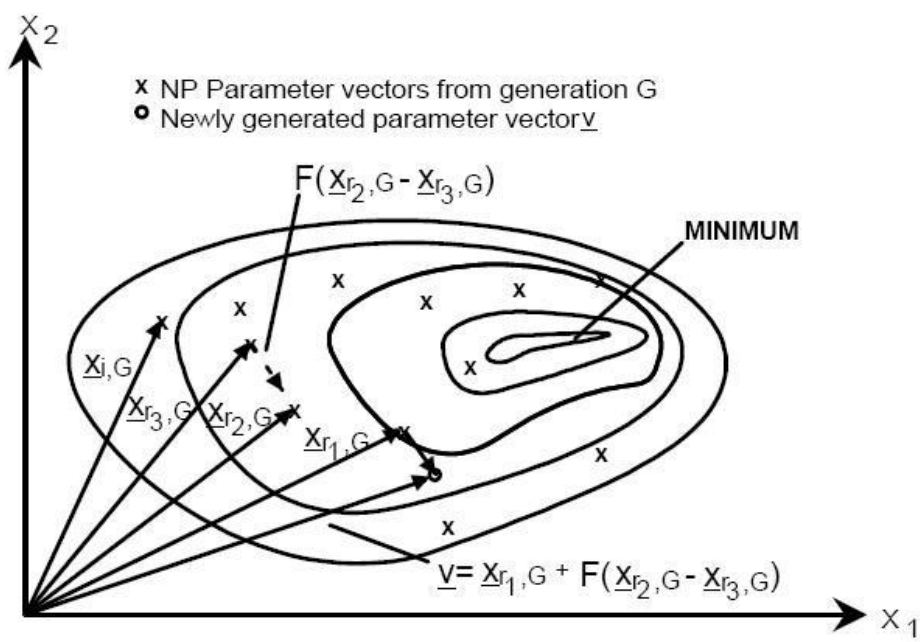

Fig. 1 Two-dimensional example of an objective function showing contour lines and the process for generating V in Scheme DE.

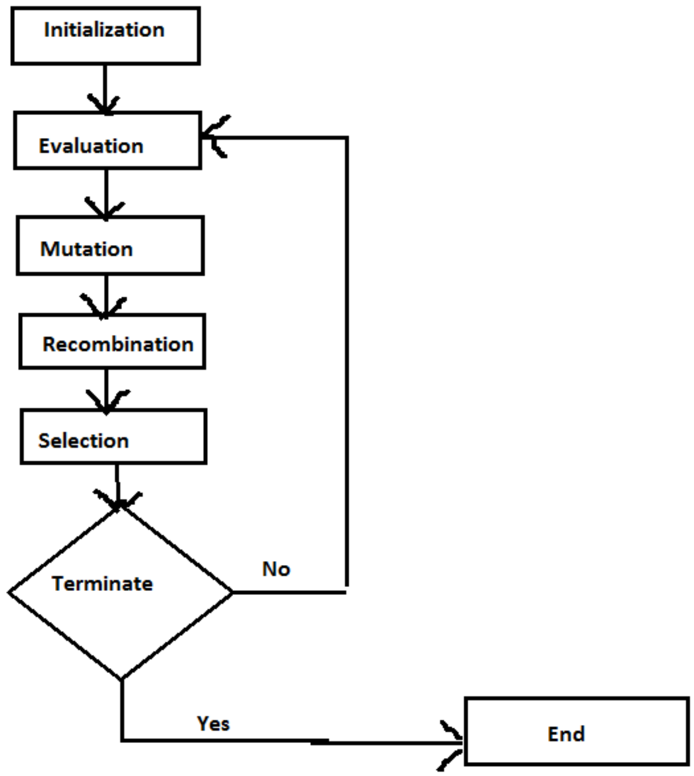

Fig. 2 Flow chart showing diffrential evolution algorithm.

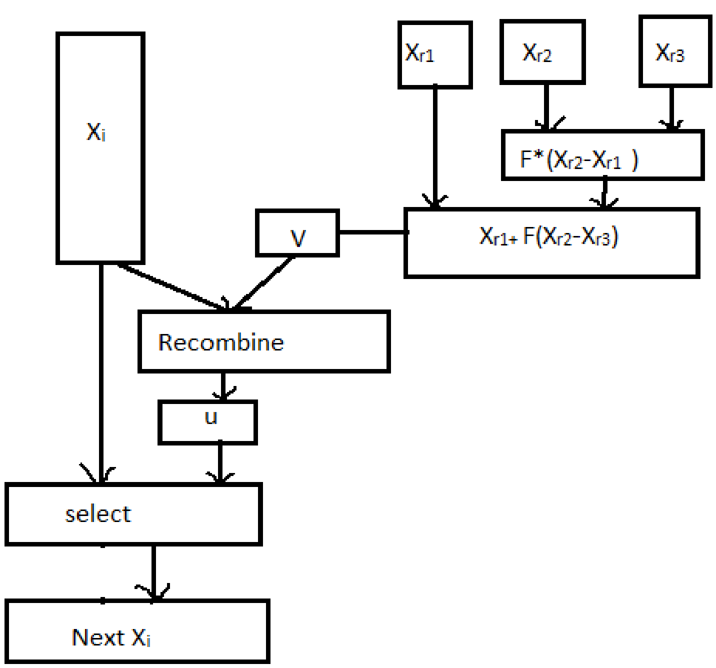

Fig. 3 Clear representation of whole phenomenon of differential evolution optimization algorithm.

Initialisation-

DE parameters number of population, mutation factor and cross over rate are initialized. Population of NP with D-dimensional decision variables or parametric vector where the candidate solution is encoded as $X_{i,G}=\{x^1_{1,G}\ldots\ldots\ldots x^D_{i,G}\}$where I varies from 1 to NP=15 and D varies from 1 to 3.

Evaluation-

In the generation G=0, the jth parameter at ith candidate is generated by

$X^j_{i,0} = x^j_{min}$ +rand(0,1*)*($x^j_{max} - x^j_{min}$) j=1,2,..,D

And rand (0, 1) represent a random variable with uniform distribution within the range 0 to 1.

Mutation-

There are nine strategy based upon which variants are chosen by calculating the difference of two randomly generated vector multiplied with F and adding with the third distinct vector to gt the mutated vector.the description of various strategies are shown below.

1)DE/rand/1

$V_{i,G} = x_{r1i,G} + F*(x_{r2i,G} - x_{r3i,G})$

2)DE/best/1

$V_{i,G} = x_{besti,G} + F*(x_{r1i,G} - x_{r2i,G})$

3)DE/rand to best/1

$V_{i,G} = x_{i,,G} + F*(x_{best,,G} - x_{i,,G}) + F*(x_{r1i,G} - x_{r2i,G})$

4)DE/best/2

$V_{i,G} = x_{best,,G} + F*(x_{best,,G} - x_{i,,G}) + F*(x_{r1i,G} - x_{r2i,G})$

5)DE/rand/2

$V_{i,G} = x_{r1,,G} + F*(x_{r2i,G} - x_{r3i,G}) + F*(x_{r4i,G} - x_{r5i,G})$

F is the positive control parameter helped to generate the variant.

For i=1 to NP

Generate a mutated vector $V_{i,G}$ for each target vector $x_{i,,G}$ using the above five strategy.

Crossover-

Swapping takes place between the donor vector and the target vector. Crossover leads to generate a trial vector

Generate a mutated vector $V_{i,G}$ for each target vector $x_{i,,G}$

$U^{j}_{i,G} =\{u^{1}_{i,G,\ldots\ldots,} u^{D}_{i,G})$

Binomial crossover

For i=1 to NP

J_{rand}=[rand(0,1)*D]

For j=1 to D

$u^{j}_{i,G}= v^{j}_{i,G}$, if (rand[0,1)<=CR) or ($j=j_{rand}$) otherwise

$u^{j}_{i,G} =x^{j}_{i,G}$

Selection-

Termination of crossover operation leads to approach towards fitness function and get it to continue for next generation.

For i=1 to NP

Evaluate the trial vector $U_{i,G}$

If $f(U_{I,G})<= f(X_{I,G})$, then $X_{I,G+1} = U_{I,G}$, $f(X_{I,G+1})=f(U_{I,G})$

If $f(U_{I,G})<=f(X_{best,G})$, then $X_{best,G} = U_{I,G}$, $f(X_{best,G})=f(U_{I,G})$

End

Problem definition

Optimization of weight, center to center distance and strength is done considering module, face width, number of teeth as decision variables. The numerical value of module is not taken high because of its less contribution towards objective function. s ame material for both gear and pinion is chosen and design is done taking consideration of the pinion. Gear ratio and number of teeth of pinion are there to predict the teeth of gear. Initially the range is mentioned of the module from 1 to 10, the face width from 20 to 40 and the number of teeth of pinion is taken from 17 to 24.

Variables

Three variables are taken for optimization which are the design variable vector

X1=module

X2=face width

X3=number of teeth

Input parameters

User specifies the input parameters by choosing the material of the gear thus obtaining its material properties. The main role of input parameters lie in predicting objective function value and various constraints. The input parameters are as follows.

1. Power transferred
2. Kind of material chosen
3. Input speed
4. Gear ratio
5. Brinell Hardness Number
6. Ultimate tensile strength
7. Working Factor
8. Overlap ratio
9. Helical angle
10. Stress concentration factor
11. Material factor
12. Flank transverse coefficient
13. Cross over probabilty

Constraint formation

In the design of gear train constraint formation is an important step considering various equations of contact stress, bending strength and face width.[16]

The fundamental equation for bending stress is given below:

σ_k= (F_t* K_d * K_{fe} * K_c)/(Ɛ*b*m*y)

The basicequation for bending stress is given below:

P_{alw}= {(K_d * K_m * K_α*K_ε*K_β* F_t*(Z_1 +Z_2)/ Z_1)/(b*m* Z_2)}$^{1/2}$

The constraints are derived from the above two fundamental equations. They are given as follows:[16]

K_d * K_{fe} * K_c*F_t – Ɛ*b*m(0.55*σ_k) <=0

K_d * (K_m * K_α*K_ε*K_β)^2*F_t*(Z_1 +Z_2)/ Z_1-b*m* Z_2*(P_{alw})^2<=0

20*m-b<=0

B - 40*m<=0

17 - Z<=0

Z-24<=0

Steps-

- Material of the gear was selected as cementite steel.
- Various material properties are considered as input they are transfer power, tooth overlap factor, stress concentration factor, material factor, flank transverse coefficient, gear ratio, BHN(brinell hardness number) ,ultimate tensile strength, helical angle, type of gear.
- Weightage value,three, to the objectives is allotted.
- Differential evolution optimization algorithm is run.
- Results get displayed after running the code
- Same procedure is being carried out for various kinds of gear,five type,with different weightages.
- The weightage of various kinds of gear giving the best value of a particular objective function is calculated

Experimental Data

Assumptions made are mentioned below for the design.

- Helical gear pair
- Pressure ang;e=20^0
- Full depth system
- Material=any material here mainly cementite steel
- Types of gear availability:
    a. ordinary cut gear
    b. carefully cut gear
    c. carefully cut & ground metallic gears
    d. hardened steel,ground and lapped in precision
    e. gears whose tooth are finished by hobbing or shapping
- module=1-10mm
- face width=20-40mm
- number of teeth=17-24
- shape factor=0.8
- cross over probability=0.8

Input parameters value:

Considering material of the gear as cementite steel following are the inputs to the design

14. Power transferred(KW)=7.5
15. Material: Cementite steel
16. Input speed (rpm)=1800
17. Gear ratio=i=6
18. Brinell Hardness Number=1460
19. Ultimate tensile strength, σ_k (N/mm^2)=1100
20. Working Factor, K_o= 1.25
21. Overlap ratio, Ɛ=1.6
22. Helical angle, ß =18^0
23. Stress concentration factor, * K_c= 1.5
24. Material factor, K_m(N/mm^2)=271.11
25. Flank transverse coefficient, K_α=1.76
26. Tooth overlap factor, K_ε=0.79

Lewis equation for tooth bending stress

Assumptions made for the derivation are

1. Full load is applied to the single tooth's tip at static load condition
2. Radial component is neglected due to its negligible contribution.
3. Load is distributed throughout the full face.
4. Force generating from tooth sliding friction are neglected.
5. Stress concentration of tooth fillet is neglected.

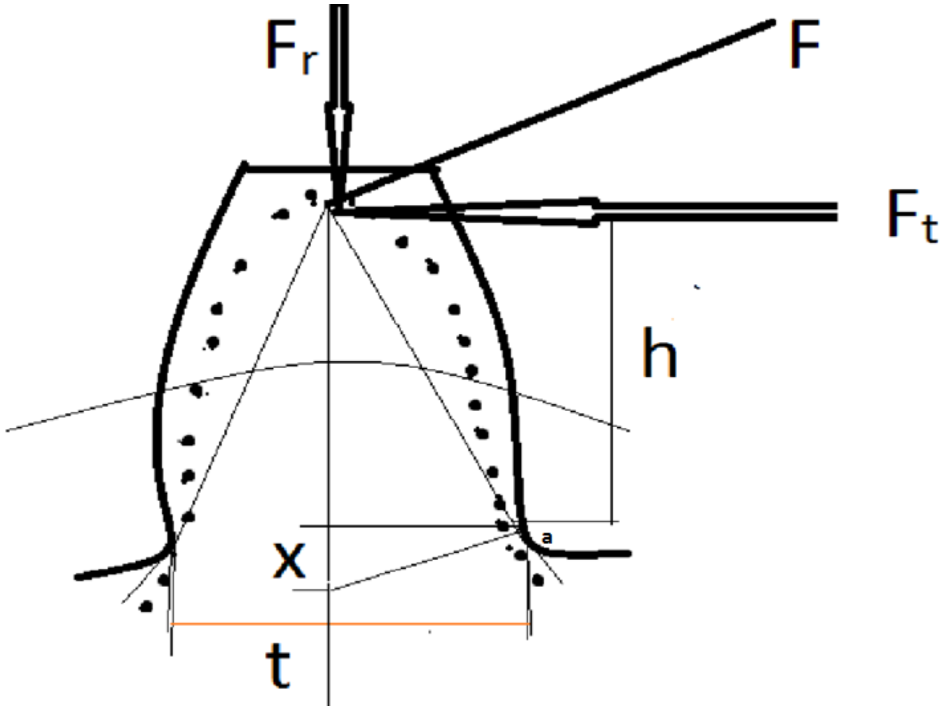

Fig. 4 gear tooth represented as cantilever beam

The equation for bending stress for beams is represented below:

$\sigma/y = M/I$ (1)

Bending stress at point 'a' is given by,

$\sigma=Mc/I=6F_t\,h/bt^2$ (2)

From the similarity of triangle we can obtained the equation as

$(t/2)/x=h/(t/2)$ (3)

Considering the above two equation the expression for bending stress is obtained as: $\sigma=6F_t/4bx$

y is the Lewis form factor which is given as

$y=2x/3p$ (4)

Substituting in the equation, we get

$\sigma = F_t / bpy$ (5)

$\sigma = F_t / bpy$

$F_t = \sigma bpy$ (6)

Where $p = \pi m$

$F_t = \pi \sigma bmy$ (7)

y is given as $y = (0.154 - (0.912/z))$

$F_t = \pi \sigma bm(0.154 - (0.912/z))$ (8)

As we have written in the form of x1,x2 and x3, the bending stress of gear tooth is one third of its ultimate tensile strength (1100N/mm^2)that is approximately 360N/mm^2.Since we are maximizing strength thus it should be considered as inverse of strength in the objective function. Considering all these the objective function for strength can be written as

1/π*360*(0.154-(0.912/x3))*x1*x2 (9)

Forming objective function

the quantity which is minimized or maximized under given constraints analysing under a search space is an objective function.in this paper three objective functions are taken, to minimize weight, centre to centre distance and to maximize strength of the gear. The objective function is given below considering weightage variable w1, w2 and w3 as

F_{obj}=w1*weight+w2*centre to centre distance+w3*1/(strength)

The sum of the weighted parameters w1, w2 and w3 is unity. Considering the above derived formula for strength of gear and taking directly the formula of other two objectives the objective function is given as

F_{obj}=w1*7.7005*x1^2*x2*x3^2*density*(1+gear ratio^2)/1000000000)+w2*0.5*x1*x3*(1+gear ratio)/1000+w3*1/π*360*(0.154-(0.912/x3))*x1*x2

Putting the numerical value of gear ratio (6) and density (3865.245 kg/m^3) in the equation we get the final objective function as below

F_{obj}=w1*0.00110127984*x1^2*x2*x3^2+w2*0.00175*x1*x3+w3*1/(π*360*(0.154-(0.912/x3))*x1*x2)

Various constraints are incorporated in the objective functions to be minimized.to calculates suitable solutions or content, they allow suitable design choices. Fatigue failure and tooth failure in gear design are the crucial failure seen in the gear power transmission system.so in design contact stress and bending strength are the crucial constraints to be considered. The other constraints like teeth constraint module constraint and face width constraints are based upon gear sizing.so the first constraint which is based on bending strength is derived from the fundamental bending strength equation and the second one is derived from the fundamental contact stress equation.

Penalty function

Penalty function is utilized for the optimization problem considering constraints to ensure not to violate them and give solution in the range. It is incorporated with the objective function. Whenever the constraints get violated a high positive value is computed to the objective function which is against our motive of optimization. The variation of left and right side of the constraint equation is measured from the positive value added is the penalty function which is calculated from the bending strength equation and Q is the penalty function generated from the contact shear stress equation where both face width and module which are our decision variables are taken into other side and used to check the violation of constraint.

In order to incorporate penalty function into the objective function, changes are nade to the objective function as shown below:

P= stress concentration factor * transferred power * 10000000*cos(20)*2/(input speed*2*3.14*overlap ratio*0.55*ultimate tensile strength)

Putting the value of stress concentration factor, transferred power overlap ratio and ultimate tensile strength we get P=115.934 and similarly another penalty function is

Q=cos(helical angle)*(material factor* flank transverse coefficient* tooth overlap ratio* tooth slope factor)^2transferred power*10000000*cos(20)*2*60/(2*3.14*input speed* gear ratio *(allowable surface pressure)^2)

After putting all the value of the above parameters in equation, we get Q=93086.06452.Both L&G are used

If (x1^2*x2*x3^2/dynamic velocity factor>=Q)

F_{obj}=w1*0.00110127984*x1^2*x2*x3^2+w2*0.00175*x1*x3+w3*1/(π*360*(0.154-(0.912/x3))*x1*x2);

Else F_{obj}=w1*0.00110127984*x1^2*x2*x3^2+w2*0.00175*x1*x3+w3*1/(π*360*(0.154-(0.912/x3))*x1*x2)+(Q-x1^2*x2*x3^2/dynamic velocity factor)

End

If (x1^2*x2*x3^2/dynamic velocity factor*(0.48356*x3-2.86368)<=P)

F_{obj} =F_{obj}+(P- x1^2*x2*x3^2/dynamic velocity factor) *(0.48356*x3-2.86368))

If (x2<20*x1)

F_{obj}= F_{obj}+(20*x1-x2)

If (x2>40*x1)

F_{obj}= F_{obj}+(x2-40*x1)

else

F_{obj}= F_{obj}

end

so in this way objective functions and penalty functions are developed for the given constraints and then using DE optimization algorithm it is optimized.

Results

We have mainly six selection criterias for gears and their corresponding dynamic velocity factor. They are

- Ordinary cut gears
  Dynamic velocity factor=3/(3+V)
- Carefully cut gears
  Dynamic velocity factor=4.5/(4.5+V)

- Carefully cut & ground metallic gears
  Dynamic velocity factor=6/(6+V)

- Hardened steel,ground and lapped in precision
  Dynamic velocity factor=5.6/(5.6+sqrt(V))

- Gears whose tooth are finished by hobbing or shapping
  Dynamic velocity factor=50/(50+sqrt(200*V))

Algorithm name	w_1	w_2	Function value	Module	Face width(mm)	Number of teeth
Differential evolution optimization	0.4	0.6	65.852	2.006	23.67	22.89
	0.6	0.4	45.02	2.006	21.67	22.89
	0.5	0.5	55.657	2.006	21.67	22.89

Table 2 : objective function value considering two objectives

Strategy number	Expressed as	Function value
1	DE/Best/1/exp	33.7475
2	DE/rand/1/exp	33.7475
3	DE/rand to Best/1/exp	33.7475
4	DE/Best/2/exp	33.7475
5	DE/rand/2/exp	33.7475
6	DE/best/1/bin	70.2297
7	DE/rand/1/bin	64.5369
8	DE/rand to Best/1/bin	67.423485
9	DE/best/2/bin	40.114485

Table 3: comparing the objective function value at various strategies taking three objectives at weighted average (0.3, 0.3, 0.4)

Algorithm	Type of gear	w_1	w_2	w_3	Function value	Module	Face width(mm)	Number of teeth
Standard Differential evolution	Ordinary cut gear	0.35	0.4	0.25	44.989	1.642	32.842	23.99
		0.4	0.35	0.25	39.379	1.642	32.842	23.99
		0.3	0.3	0.4	33.74	1.642	32.842	23.99
	Carefully cut gear	0.35	0.4	0.25	46.51929	1.7238	34.477	23.24
		0.4	0.35	0.25	52.0462	1.7238	34.477	23.24
		0.3	0.3	0.4	39.04001	1.7238	34.477	23.24
	Carefully cut & grounded metallic gear	0.35	0.4	0.25	73.426	1.77466	35.493	23.99
		0.4	0.35	0.25	49.696	1.77466	35.493	23.99
		0.3	0.3	0.4	42.5901	1.77466	35.493	23.99
	Hardened steel,ground and lapped in precision	0.35	0.4	0.25	31.9212	1.4644717	29.2894	23.91
		0.4	0.35	0.25	27.94276707	1.4644717	29.289434	24
		0.3	0.3	0.4	23.94548088	1.4644717	29.289434	23.99
	Gears whose teeth are finished by hobbing or shapping	0.35	0.4	0.25	24.9374	1.348657	26.973	23.9
		0.4	0.35	0.25	21.83098	1.348657	26.973	23.9
		0.3	0.3	0.4	18.70721	1.348657	26.973	23.9

Table 4: objective function value at three various combination of weighted parameter taking three objectives

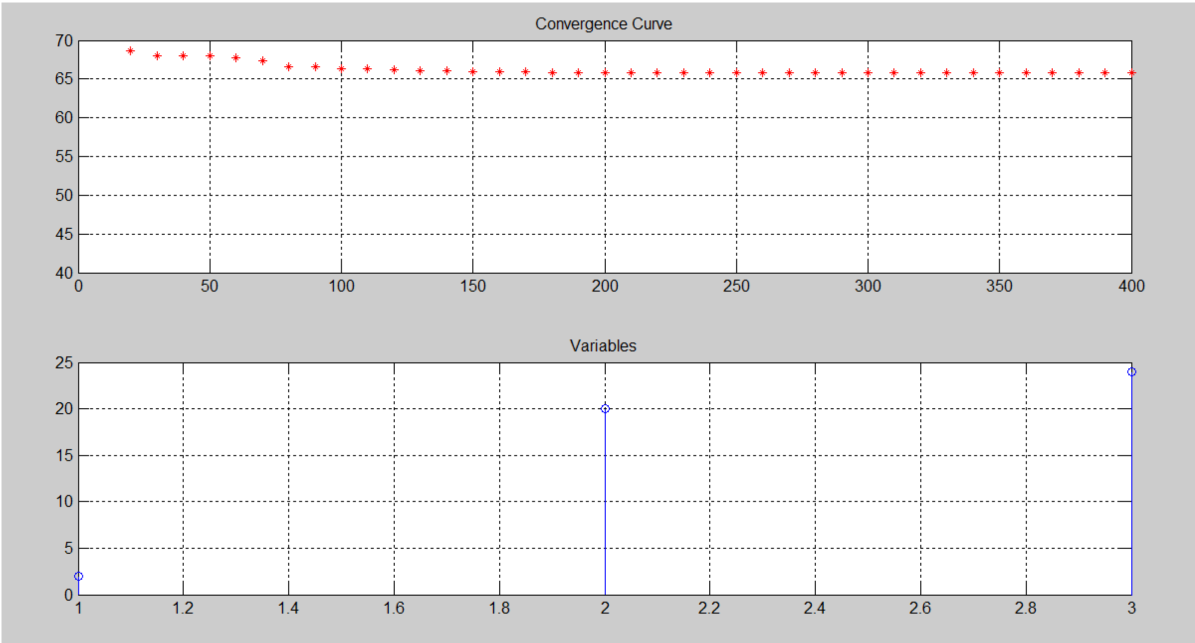

Fig. 5 Plot of function value vs iterations for ordinary cut gears and decision variables for weightage (0.4, 0.6) considering only two objectives.

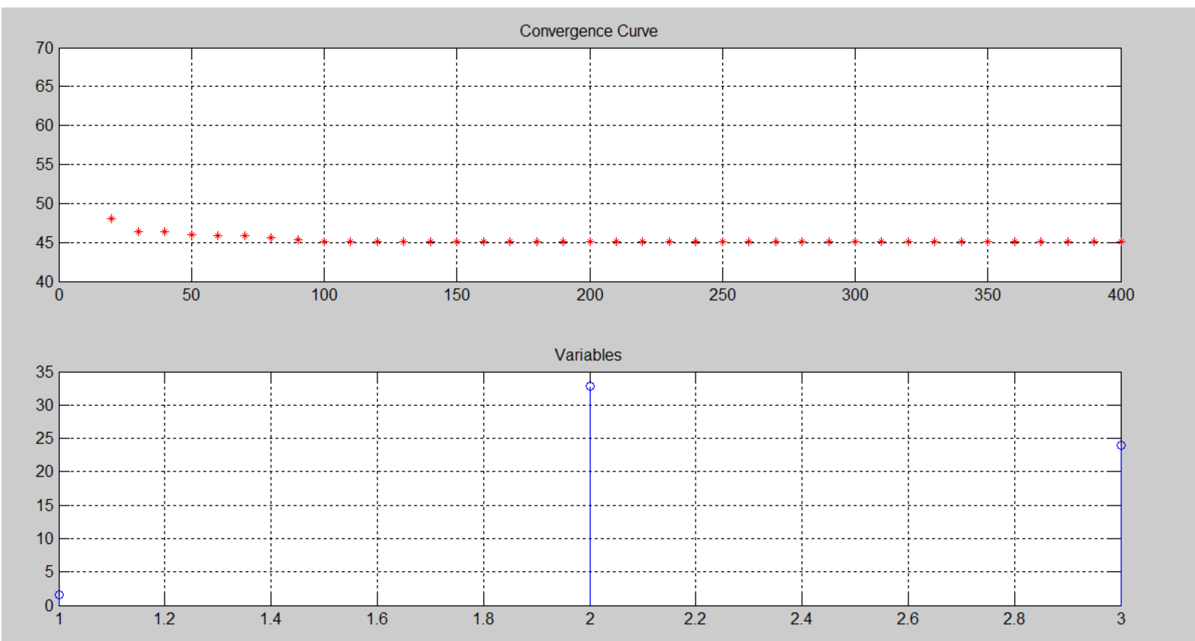

Fig. 6 Plot of function value vs iterations for ordinary cut gears and decision for weightage(0.6,0.4) considering three objectives

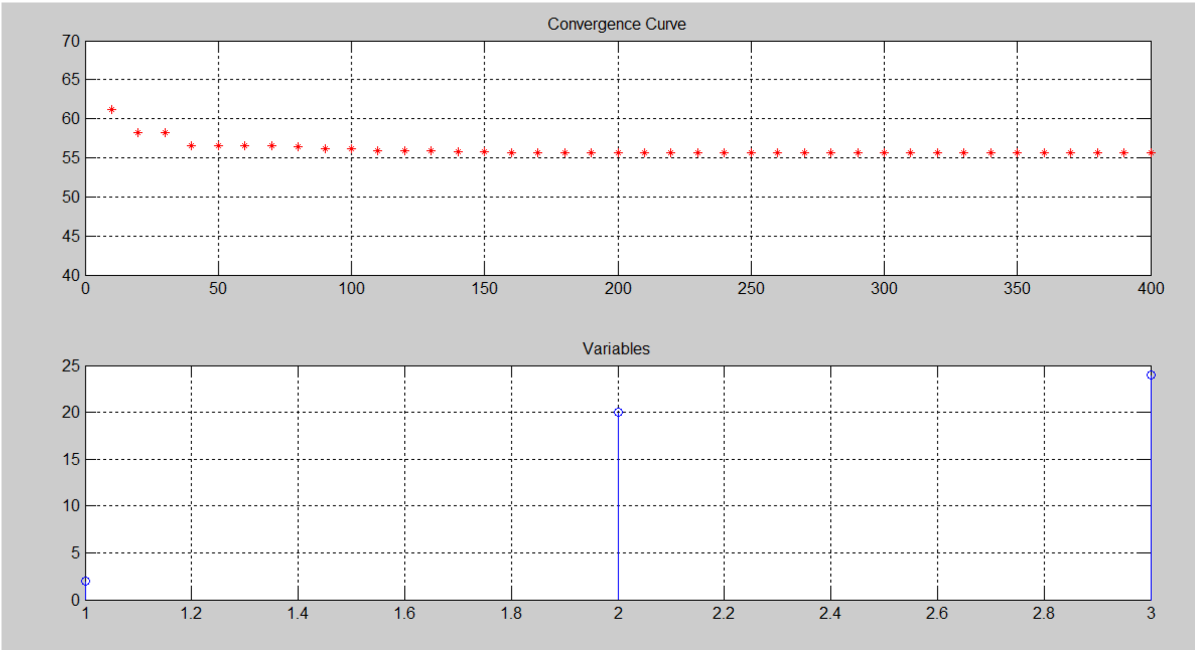

Fig. 7 Plot of function value vs iterations for ordinary cut gears and decision for weightage (0.5, 0.5) considering three objectives

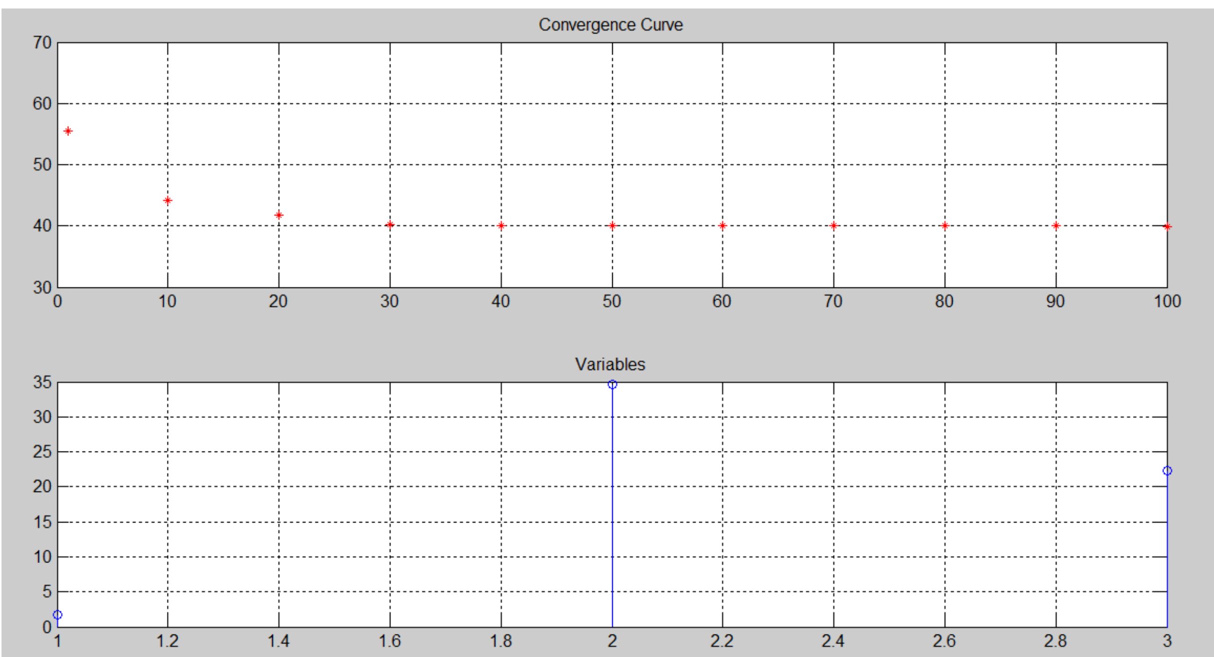

Fig. 8 Plot of function value vs iterations for ordinary cut gears and decision for weightage(0.4,0.35,0.25) considering three objectives

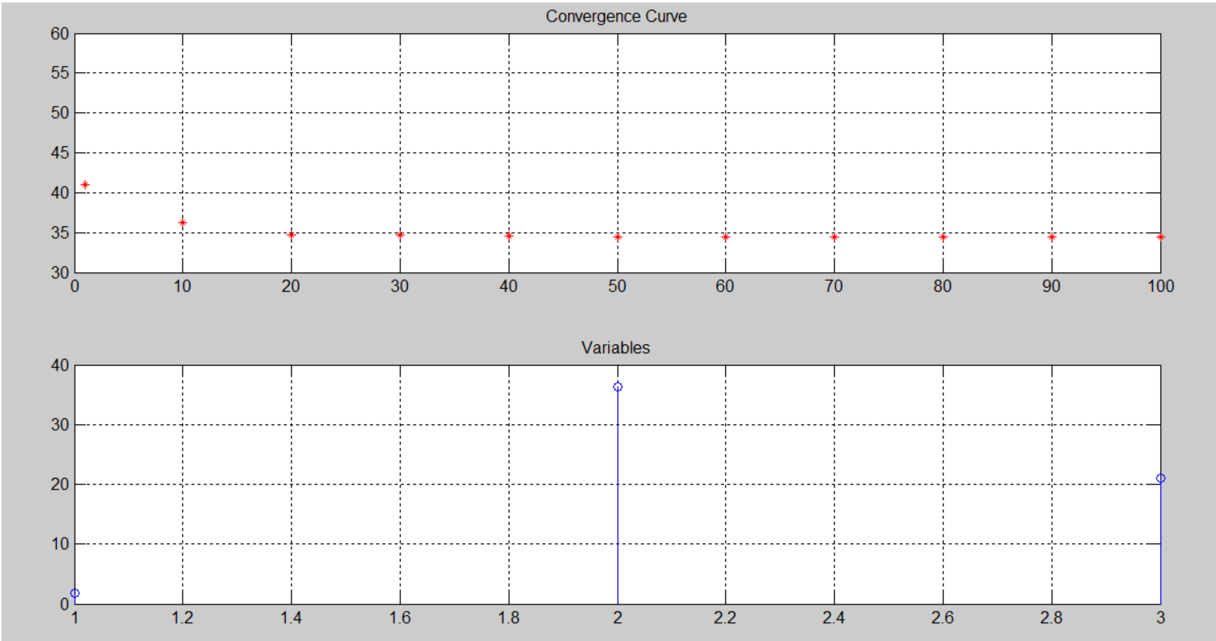

Fig. 9 Plot of function value vs iterations for ordinary cut gears and decision for weightage (0.3, 0.3, 0.4) considering three objectives

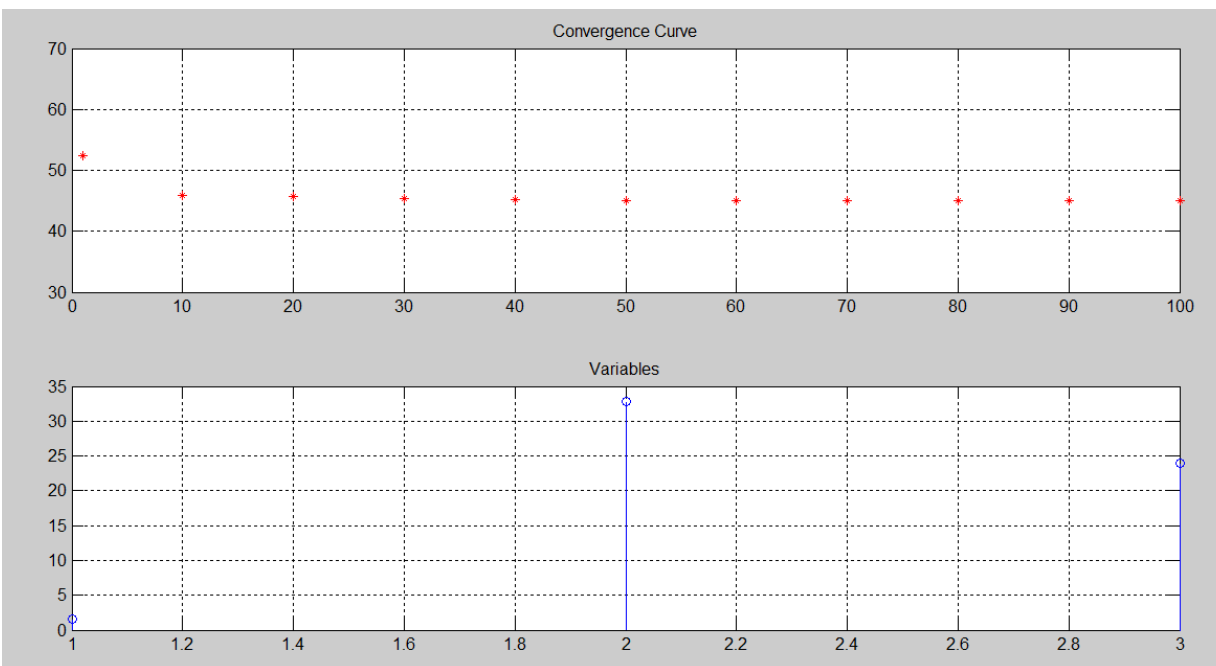

Fig. 10 Plot of function value vs iterations for ordinary cut gears and decision for weightage (0.35, 0.4, 0.25) considering three objectives

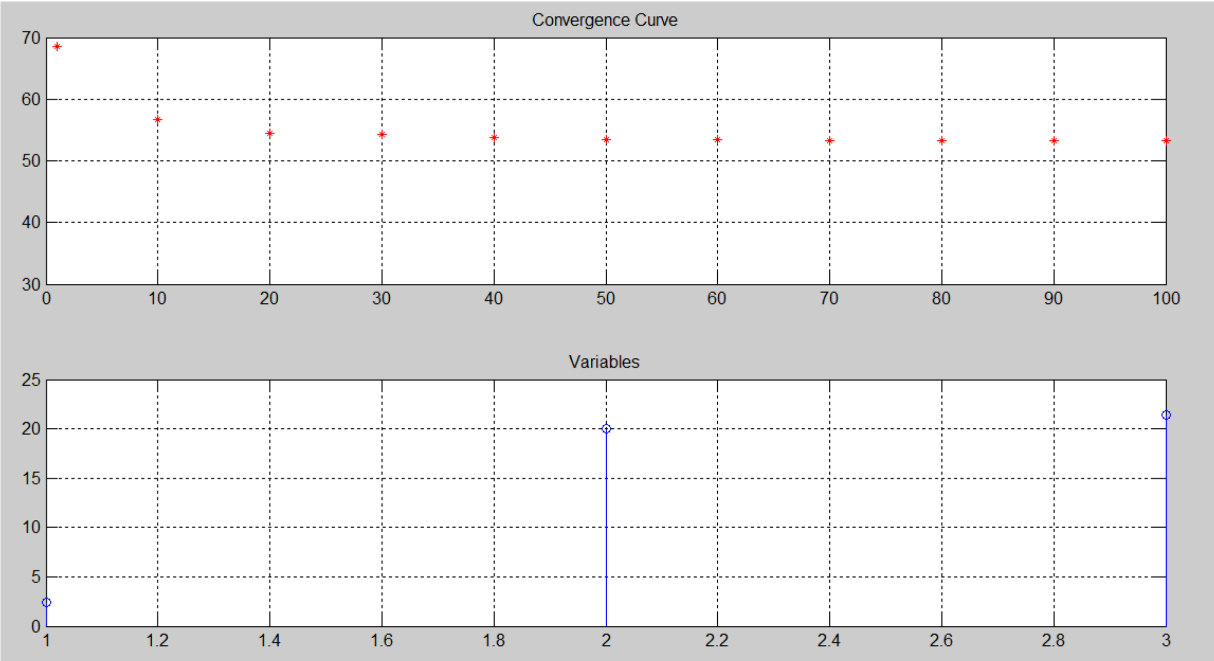

Fig. 11 Plot of function value vs iterations and decision for weightage (0.35, 0.4, 0.25) considering three objectives for carefully cut gear

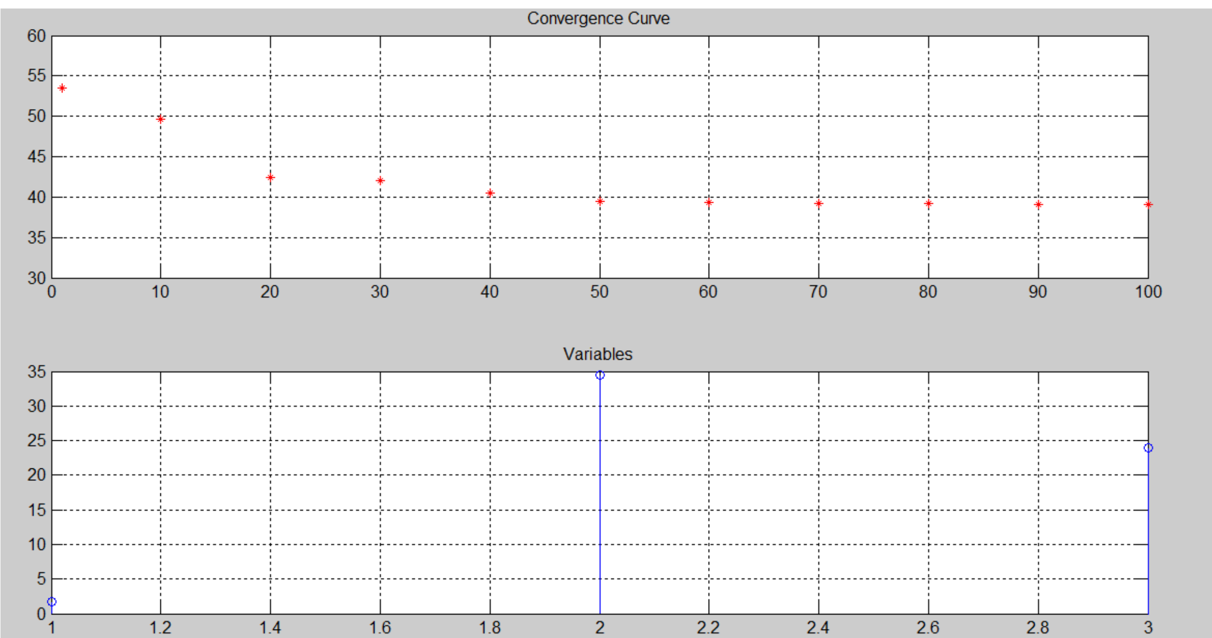

Fig. 12 Plot of function value vs iterations and decision for weightage (0.3, 0.3, 0.4) considering three objectives for carefully cut gear

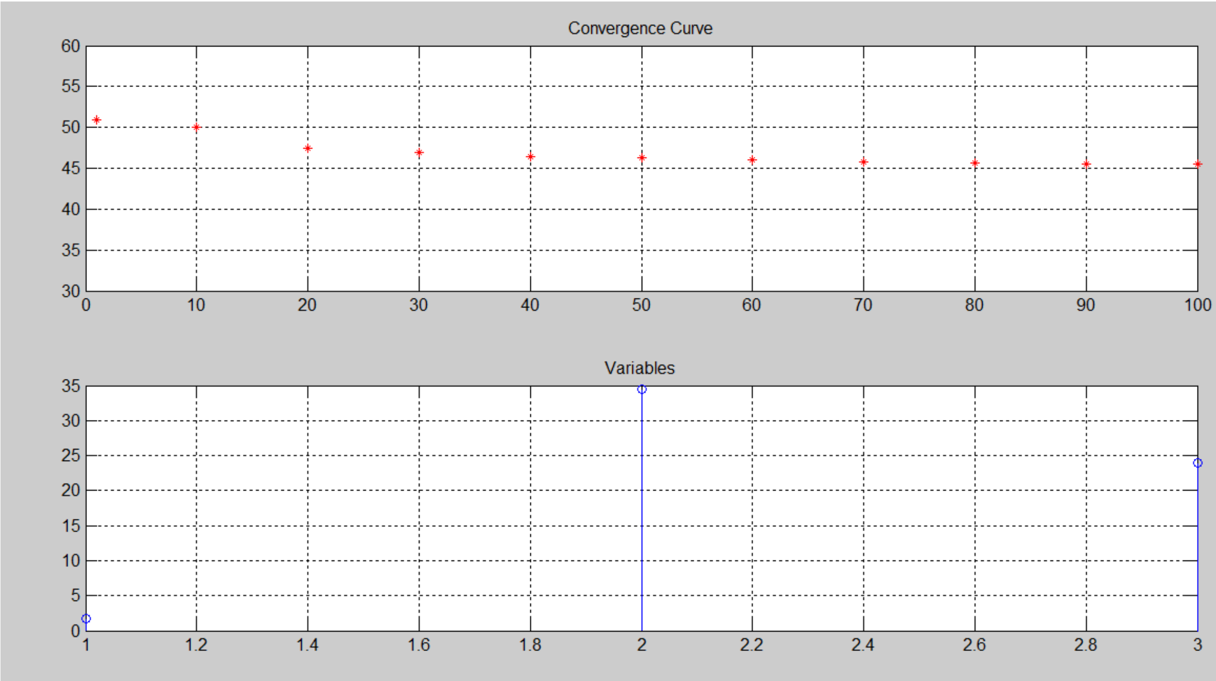

Fig. 13 Plot of function value vs iterations and decision for weightage (0.4, 0.35, 0.25) considering three objectives for carefully cut gear

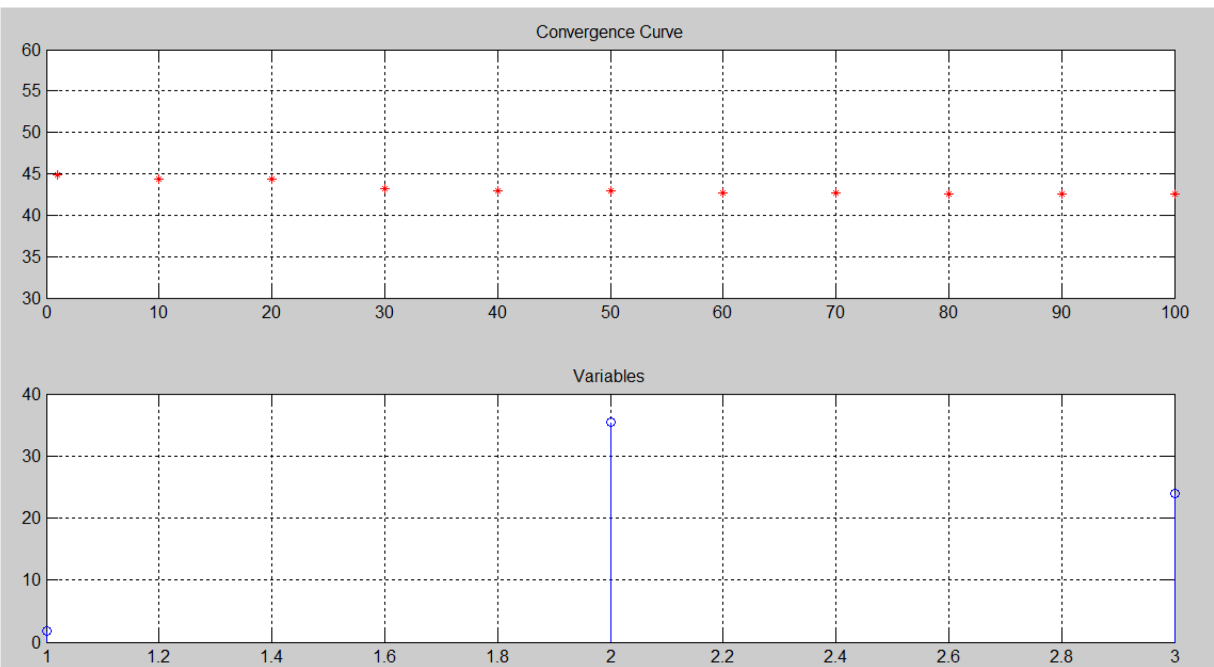

Fig. 14 Plot of function value vs iterations and decision for weightage (0.3, 0.3, 0.4) considering three objectives for carefully cut & grounded metallic gear

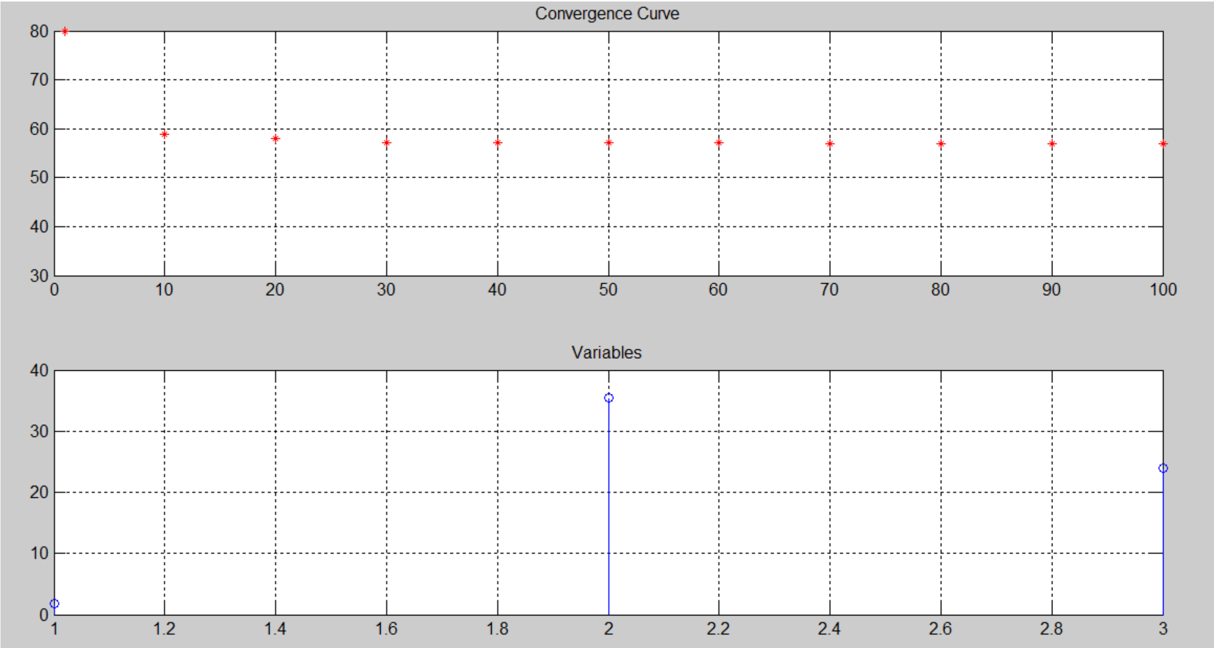

Fig. 15 Plot of function value vs iterations and decision for weightage (0.35, 0.4, 0.25) considering three objectives for Carefully cut & grounded metallic

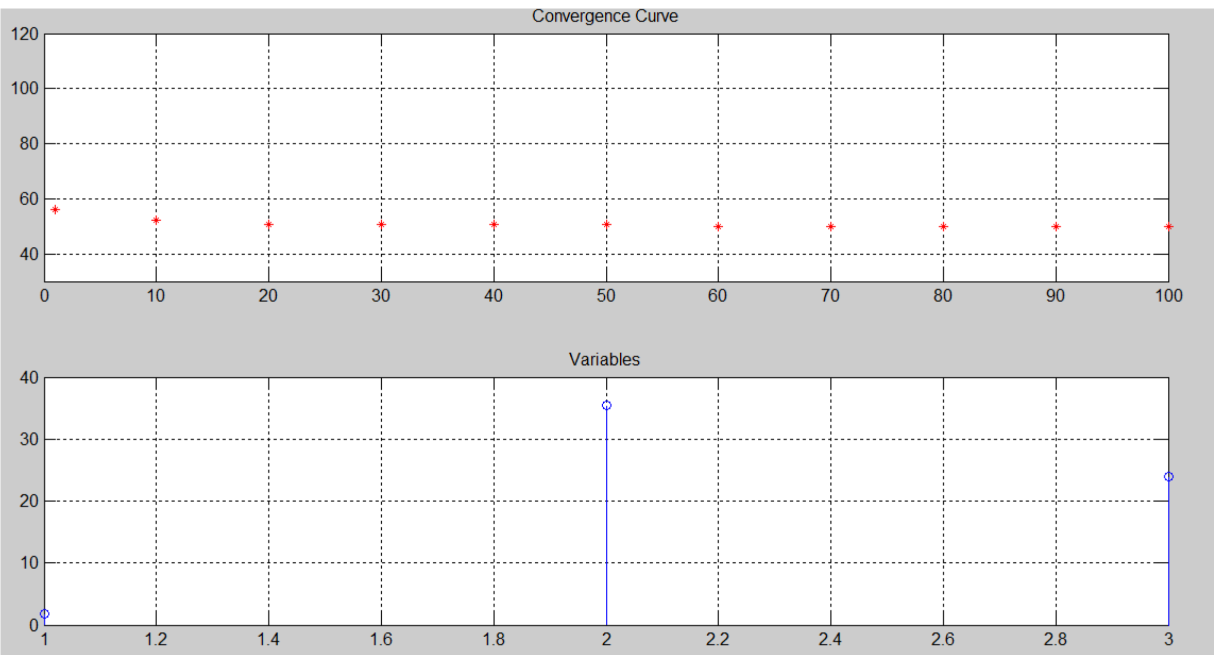

Fig. 16 Plot of function value vs iterations for ordinary cut gears and decision for weightage (0.4, 0.35, 0.25) considering three objectives for carefully cut & grounded metallic gear

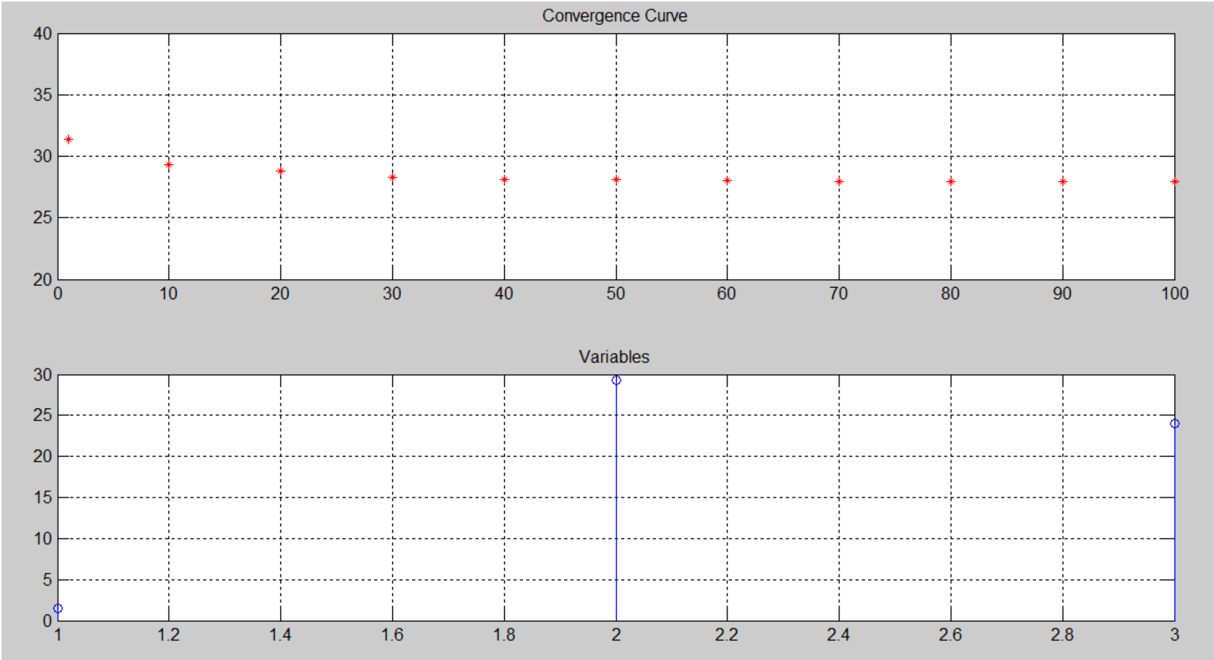

Fig. 17 Plot of function value vs iterations and decision for weightage (0.4, 0.35, 0.25) considering three objectives for Gear of Hardened steel, ground and lapped in precision

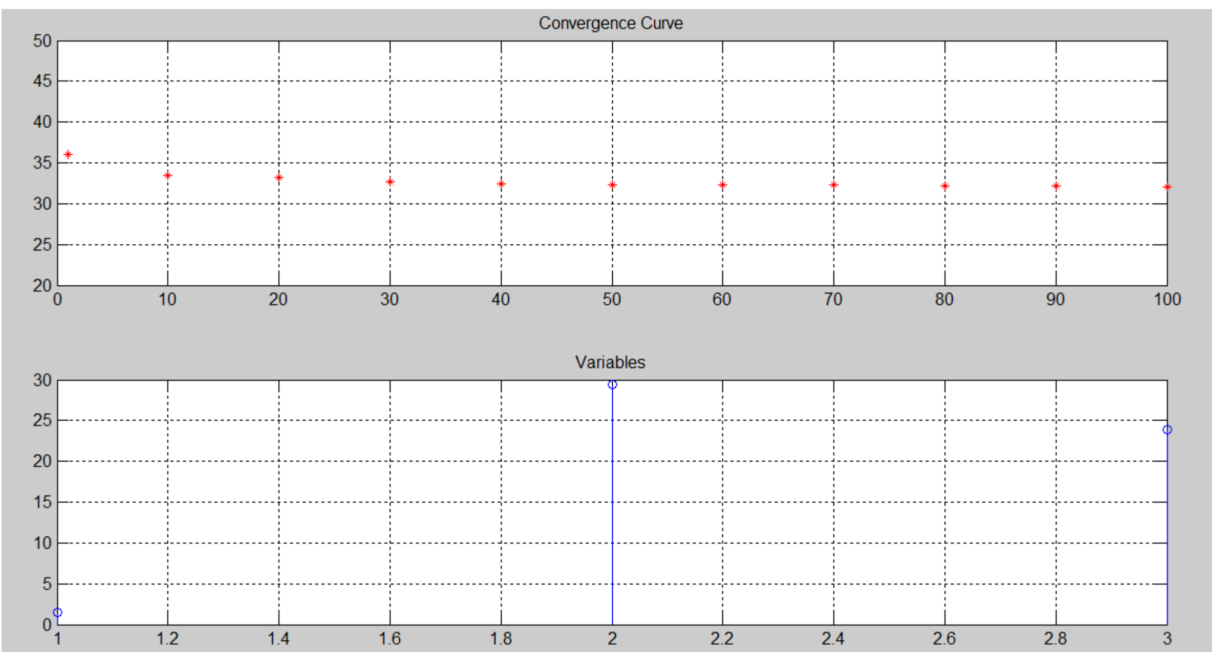

Fig. 18 Plot of function value vs iterations and decision for weightage (0.35, 0.4, 0.25) considering three objectives for Gear of Hardened steel,ground and lapped in precision

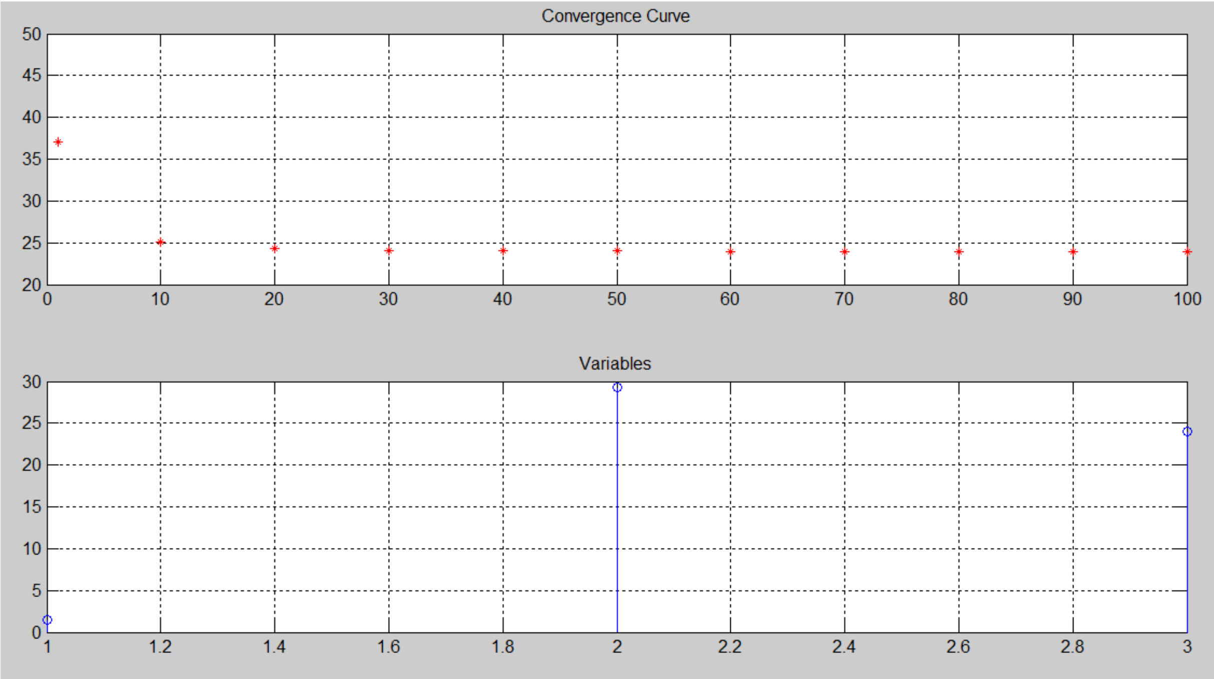

Fig. 19 Plot of function value vs iterations and decision for weightage (0.3, 0.3, 0.4) considering three objectives for Gear of Hardened steel,ground and lapped in precision

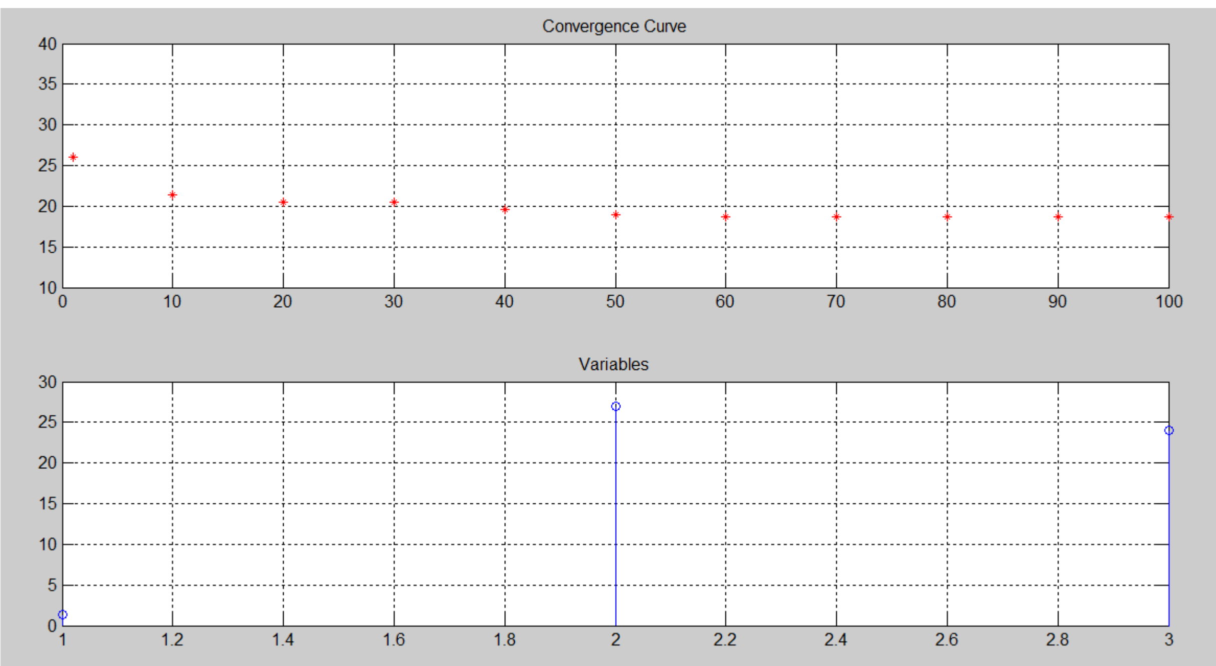

Fig. 20 Plot of function value vs iterations and decision for weightage (0.3, 0.3, 0.4) considering three objectives for Gears whose teeth are finished by hobbing or shapping

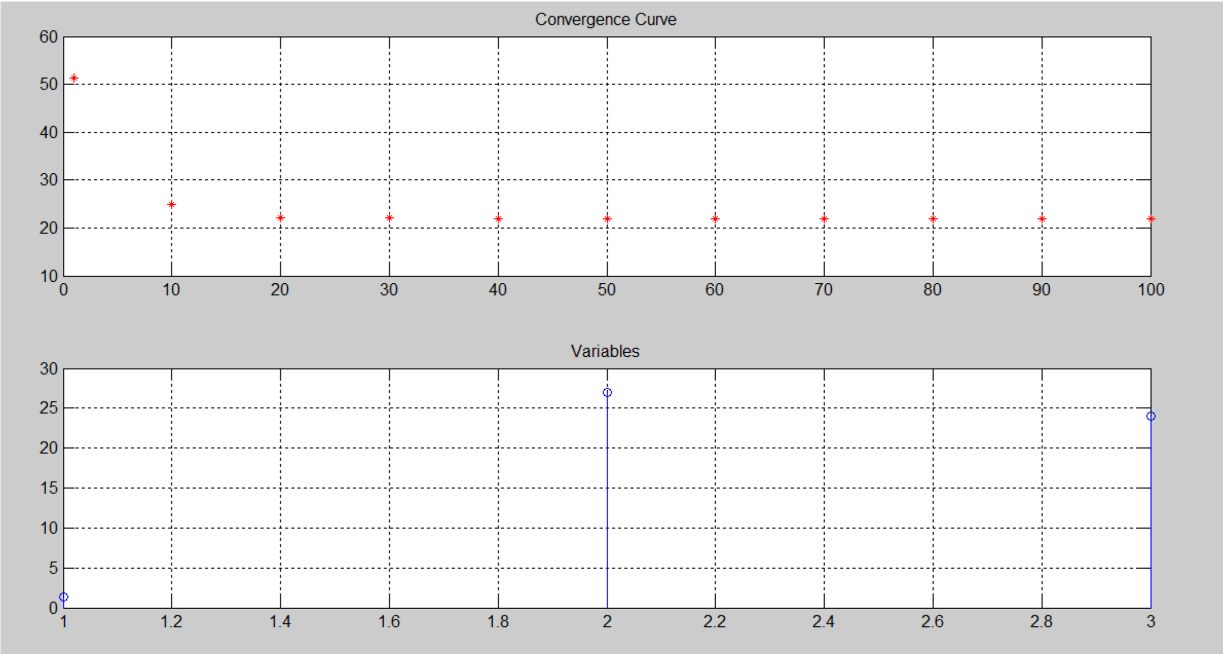

Fig. 21 Plot of function value vs iterations for weightage (0.4, 0.35, 0.25) considering three objectives for Gears whose teeth are finished by hobbing or shaping.

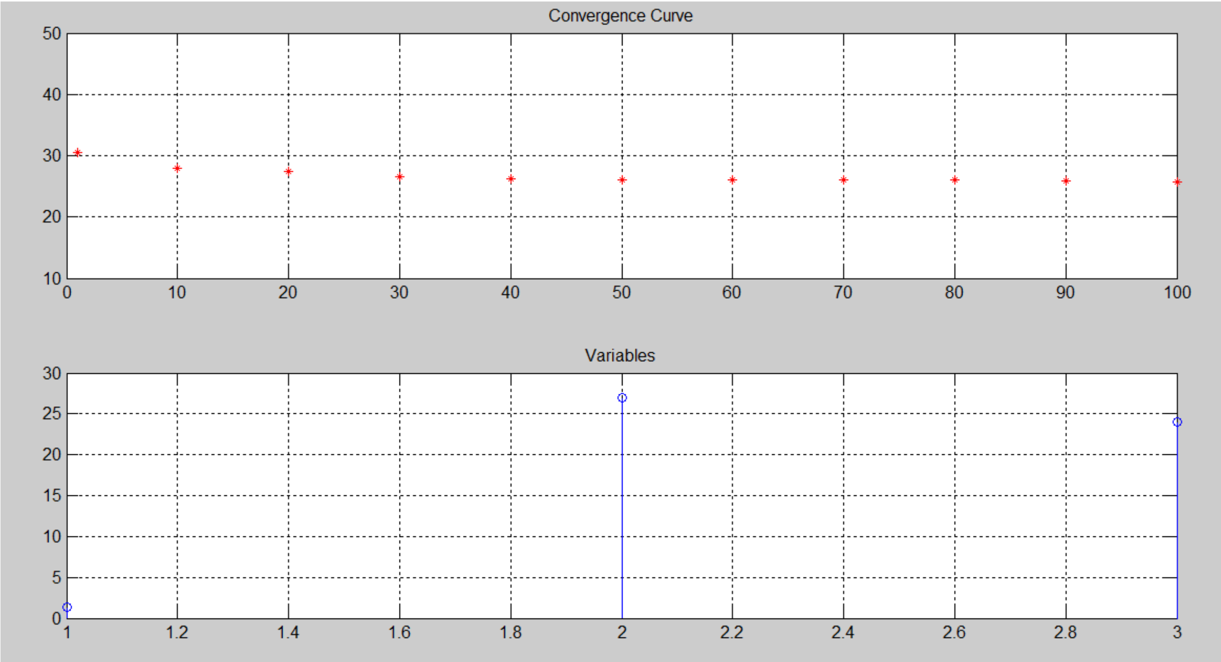

Fig. 22 Plot of function value vs iterations decision for weightage (0.35, 0.4, 0.25) considering three objectives for Gears whose teeth are finished by hobbing or shaping

Discussion

Considering the result obtained and graph obtained the following discussions are sated:

- Considering only two objective function weight and centre to centre distance considering the constraints possessed by the three decision variables at three different combination of weight ,(0.4,0.6) ,(0.6,0.4) ,(0.5,0.5) and minimum function value 45.02 is obtained at (0.6,0.4) weighted parameters obtaining 2.006,21.67,22.89 as the value of x1, x2, x3 respectively.
- For the same ordinary cut gear we took three objectives to minimize centre to centre distance, weight and maximize strength considering the same three decision variables a lower function value is obtained than earlier, 33.7475 at the weightage (0.3,0.3,0.4).For this weightage all the strategy are checked in order to get the minimum value and inferred that upto strategy 5 the value of objective function don't change from a constant minimum value but from strategy 6 to 9, objective function changes having a higher value.so strategy 1 is best suited for optimization due to its minimum corresponding objective function 33.7475.
- For carefully cut gear the minimum value of objective function attained is 39.04001 at the weightage (0.3, 0.3, 0.4) with module=1.7238, face width= 34.477, number of teeth=23.9 computing at the desired strategy=1.
- For carefully cut and ground metallic gear the minimum value of objective function attained is 42.5901 at the weightage (0.3,0.3,0.4) with module=1.77466, face width= 35.493243, number of teeth=23.99 computing at the desired strategy=1.
- For Hardened steel, ground and lapped in precision the minimum value of objective function attained is 23.94548088at the weightage (0.3,0.3,0.4) with module=1.4644717, face width= 29.289434, number of teeth=23.9 computing at the desired strategy=1.
- For Gears whose teeth are finished by hobbing or shapping the minimum value of objective function attained is 18.70721 at the weightage (0.3,0.3,0.4) with module=1.348657, face width= 26.973, number of teeth=23.99 computing at the desired strategy=1.
- Decrease in weightage value of weight and increase in weightage value of centre to centre distance lead to enhance the objective function value and increase in weightage value of weight and decrease in weightage value of centre to centre distance lead to give lower objective function value than earlier but keeping the weightagee of first two parameters fixed and increasing the third one leads to best and minimum objective function value in all the five different gears at the fixed strategy 1

Conclusion

Optimization of gear train of various kinds using DE leads to draw various conclusions as mentioned below:

- Decrease in weightage value of weight and increase in weightage value of centre to centre distance lead to enhance the objective function value.so it is required.
- Increase in weightage value of strength and keep a lower weightage value for both centre to centre distance and weight leads to a minimum value of the objective function. so it is required that weightage for strength should be more in case of this objective function considering centre to centre distance and weight minimization and strength maximization.
- Number of iterations in case of DE should be higher in order to avoid exploring and exploiting.
- Various kinds of gear can be taken into account based on availability by varying the decision variables.
- Module can be represented by discrete value also.
- Higher crossover value that is probability more than 0.5 decreases the number or times of crossovers.
- For future work regarding the design of gear above model can be utilised.

References

1. Pham, D.T., & Yang, Y. (1993). Optimization of multi-model discrete functions using genetic algorithms.

2. Chong T.H., Bae, I., Park, G. J. (2002). A new and generalized methodology to design multi stage gear drives by integrating the dimensional and configuration design process. Mechanism and Machine Theory, 37, 295-310.

3. Yokota, T., Taguchi, T., & Gen, M. (1998). A solution method for optimal weight design problem of the gear using genetic algorithms. Computers & Industrial Engineering, 35(3-4), 523-526.

4. Thompson, D.F., Gupta, S., & Shukla, A. (2000). Tradeoff analysis in minimum volume design of multi-stage spur gear reduction units. Mechanisms and Machine Theory, 35, 609-627.

5. Marcelin, J.L. (2001). Genetic optimization of gears. International journal of Advanced manufacturing Technology, 179, 910-915.

6. Abersek, B., & Popov, V. (2004). Intelligent tutoring system for training in design and manufacturing. Advances in Engineering Software, 35, 461-471.

7. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA–II. IEEE Transactions on Evolutionary Computation, 6, 182– 197.

8. Zitzler, E., Laumanns, M., Thiele, L. (2001). SPEA2: Improving the strength pareto evolutionary algorithm. Technical Report 103, Computer Engineering and Networks Laboratory (TIK), Swiss Federal Institute of Technology (ETH) Zurich, Gloriastrasse 35, CH-8092 Zurich, Switzerland.

9. Price, K.V., Storn, R. (1997). Differential evolution – a simple evolution strategy for fast optimization. Dr. Dobb's Journal, **22,** 18–24

10. Lampinen, J.: (A bibliography of differential evolution algorithm)

11. Madavan, N.K. (2002). Multiobjective optimization using a pareto differential evolution approach. In: Congress on Evolutionary Computation (CEC'2002). Volume 2, Piscataway, New Jersey, IEEE Service Center, 1145–1150

12. Xue, F., Sanderson, A.C., Graves, R.J. (2003). Pareto-based multi-objective differential evolution. In: Proceedings of the 2003 Congress on Evolutionary Computation (CEC'2003). Volume 2, Canberra, Australia, IEEE Press, 862–869.

13. Prof. K. Gopinath & Prof. M. M. Mayuram. Module 2 – Gears: Spur Gear design. Machine Design II.

14. Storn, R. & Price, K.(1996). A simple and efficient Heuristic for Global optimisation over continuous spaces. Journal of Global Optimisation.

15. Sosa, A., Zamiudio, V., Baltazar, R.(2013). Genetic Algorithms and differential algorithms evolution algorithms applied to cyclic instability problems in intelligent environments with nomadic agents. Workshop proceedings of the 9th international conference on intelligent environments

16. Savsani V., Rao R.V.,Vakharia D.P. (2009)Optimal weight design of a gear train using particle swarm optimization and simulated annealing. Deapartment of mechanical engineering