Control Algorithms for a Two Tank Liquid Level System: An Experimental Study

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Control Algorithms for a Two Tank Liquid Level System: An Experimental Study

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CERTIFICATE

*This is to certify that the thesis titled "**Control Algorithms for a Two Tank Liquid Level System: An Experimental Study**", by **Mr. Soumya Ranjan Mahapatro** submitted to the National Institute of Technology Rourkela for the award of Master of Technology by Research in Electrical Engineering is a record of bona fide research work carried out by him in the Department of Electrical Engineering, under my supervision. We believe that this thesis fulfills part of the requirements for the award of the degree of Master of Technology by Research. The results embodied in this thesis have not been submitted for the award of any degree elsewhere.*

Place: Rourkela **Prof.Bidyadhar Subudhi**

Date:

Dedicated

To

My Loving Parents

...Soumya Ranjan Mahapatro

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Abstract

The liquid level control in the coupled tank system (CTS) is a classical benchmark control problem. The dynamics of CTS resembles with that of many real systems such as distillation column, boiler process, oil refineries in petrochemical industries and many more. It is a most challenging benchmark control problem owing to its non-linear and non-minimum phase characteristics. Furthermore, its physical constraints are also pose complexity in its control design.

The thesis provides the description of a CTS along with its hardware setup used for carrying out research work. Usually, system identification is a procedure to obtain the mathematical model of a physical system from the experimental input-output data of the system. The entire process of identifying a system from input and output data broadly consists of six steps. It begins with an experimental design followed by data collection and data preprocessing, next a suitable model structure is selected, then the parameters of the model are estimated and finally the model is validated using the experimental data. The present work is aimed at utilizing the existing as well as developing new tools of system identification for obtaining a suitable model for the studied coupled tank apparatus. Based on the identified model, control algorithms are developed in order to maintain constant liquid levels in the presence of disturbances which is arising due to sudden opening of the valve in the tanks. A lot of research works have been directed in the past several years to develop the control strategies for a CTS. But, few works have been reported for validating the developed control strategies through the experimental setup. Thus, there lies a good opportunity to develop some advanced controllers and to implement them in real-time on the experimental set-up of a CTS in the laboratory.

The objectives of the present work is to maintain the water level at the desired set point value and also simultaneously ensure robust performances when there is a load disturbance. Initially, for regulating desired liquid level in both the tanks, a LMI based PI controller has been designed and implemented in real-time on a CTS. Usually, in this approach PI controller design problem is formulated as a state feedback controller design problem, which is further solved by exploiting a convex optimization approach. But, it yields slower response. Hence, an adaptive fuzzy PI (AFPI) controller has been developed to obtain better liquid level performance compared to LMI based PI controller. This developed AFPI controller consists of two parallel connected PI controllers such as a primary and a secondary PI controller.In primary part, parameters of the PI controllers are fixed which is tuned by Ziegler-Nichols method and

in secondary part, parameters are altered implicitly by means a suitable choice of fuzzy rules in real-time.This developed AFPI controller provides precise liquid level owing to large range of operating conditions because the fuzzy logic controller (FLC) covers a wide range of operating conditions which is the main advantage of this controller. After implementing the developed AFPI in real-time, it has been observed from the experimental response that it gives good tracking response but it yields overshoot which is undesirable. Hence, in order to obtain good tracking as well as robust performance, a sliding mode controller has been designed. But from experimental as well as simulation results it is observed that, it suffers from chattering problem which possess a serious concern such as chance of damaging of the actuator of the setup. Therefore, in order to reduce the chattering problem, an adaptive fuzzy sliding mode controller (AFSMC) is developed and also it is implemented in real-time. From both the experimental results, i.e. both under load disturbance and without disturbance it is observed that the proposed AFSMC control gives robust control performance in order to maintain constant desired liquid level in both the tanks as compared to other presented controller.

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List of Abbreviations

Abbreviations	Description
CTS	Coupled Tank System
PSUPA	Power Supply and Power Amplifier
DAQ	Data Acquisition Card
PWM	Pulse Width Modulation
FOPDT	First Order Plus Dead Time
VSC	Variable Structure System
SMC	Sliding Mode Control
GA	Genetic Algorithm
IMC	Internal Model Based Control
MRAC	Model Reference Adaptive Control
RLS	Recursive Least Square Estimation
DMC	Dynamic Matrix Control
OE	Output Error Model
LS	Least square
MSE	Mean Square Error
LMI	Linear Matrix Inequality
PID	Proportional Integral Derivative
ISE	Integral Square Error
IAE	Integral Absolute Error
FLC	Fuzzy Logic Control
LQR	Linear Quadratic Regulator
FIS	Fuzzy Inference System
AFSMC	Adaptive Fuzzy Sliding Mode Control
ARE	Algebraic Riccati Equation
PI	Performance Index
DC	Direct Current

Chapter 1

Introduction

The International Federation of Automatic Control (IFAC) Committee in the year 1990 has defined a set of practical design problems that are helpful in differentiating new and present control methods and tools so that a significant comparisons of control performance can be made. The committee came up with a set of real-world control problems that were included as "benchmark control problems". Out of which, the level control problem in coupled tank system is featured as a benchmark problem in the category of nonlinear and unstable control systems. Process industries play a significant role in economic growth of a nation. Control of liquid level in tanks and fluid flow between tanks is a fundamental requirement in almost all process industries such as waste water treatment, chemical, petrochemical, pharmaceutical, food, beverages, etc. shown in Fig.1.1 Mostly, level and flow control in tanks are popular in all process control systems.

1.1 Description of the Coupled Tank System

Since last two decades, the control of coupled tank liquid level system has attracted attention of many researchers around the world. It is one of the most challenging benchmark control problems due to its nonlinear and non-minimum phase characteristics. The control objective in a coupled tank system is that a desired liquid level of the liquid in tank is to be maintained when there is an inflow and outflow of water out of the tank respectively. The coupled tank system is a multi-input multi output system (MIMO) with control voltage as input and water level as the output. Even though the coupled tank system is simple from construction point of view but there lies a lot of control challenge owing to following characteristics. Fig.1.2 depicts a basic representation of a typical liquid level system.

- Non-linear system
- Non-minimum phase system

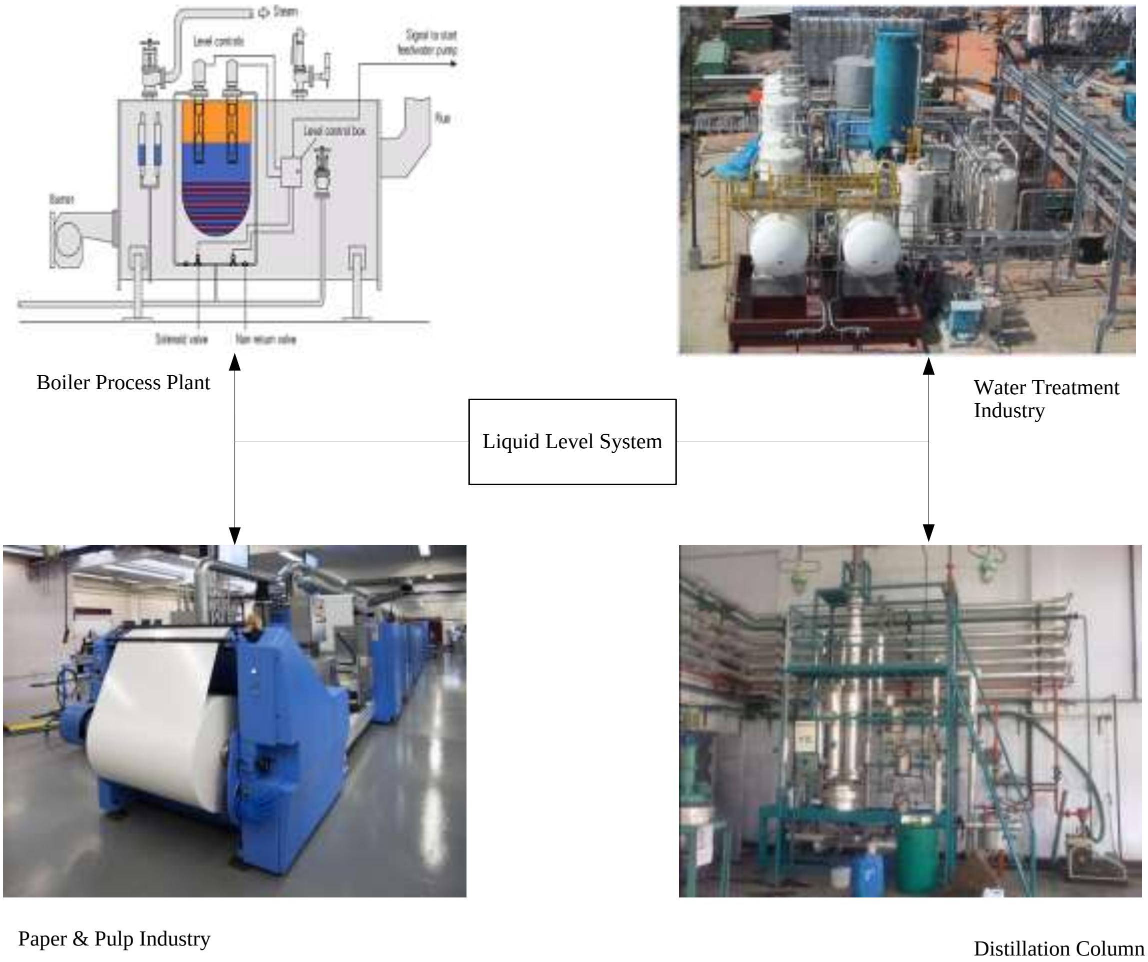

Fig.1.1 Coupled tank liquid level system examples

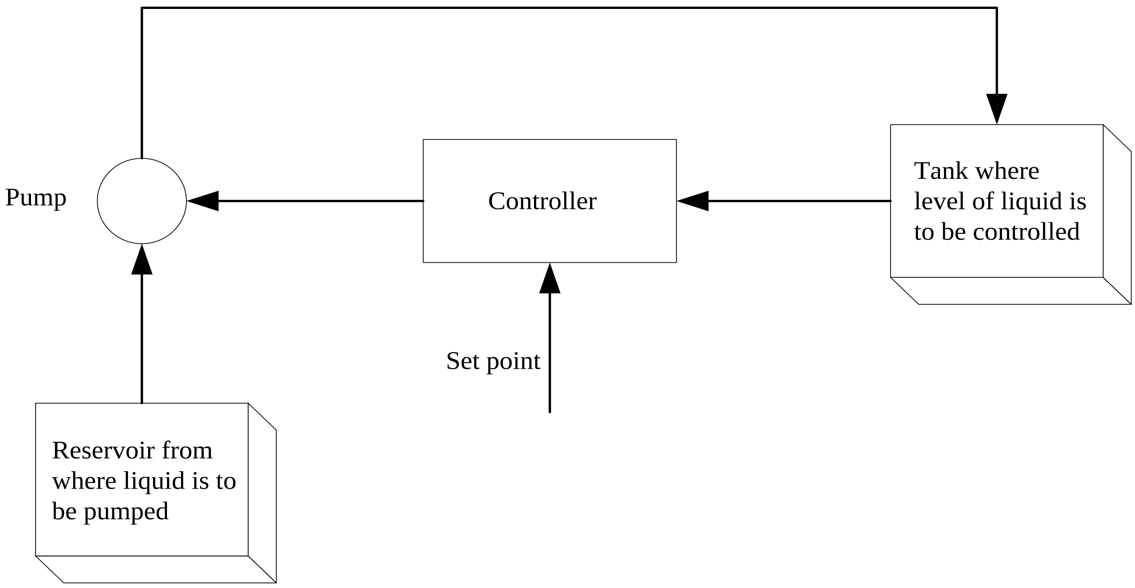

Fig.1.2 Representation of a typical liquid level system

Fig.1.3 illustrates the basic schematic representation of a coupled tank system. It consists four translucent tanks and each tank is fitted with an outlet pipe in order to transmit the over flow water to reservoir. In this process, bottom tank (fifth tank) is used for water storage purposes i.e. as a reservoir. A level sensor is also attached at the base of each tank in order to measure the water level of the corresponding tank [1]. The output of the level sensor is converted to 0-5 volt DC by the help of a signal conditioning circuit. There are two pumps installed in the reservoir in order to drive the water from bottom to the top of the tank. A scale is attached in front of all the individual tanks for the purpose of monitoring the water level. It works under two basic modes of operations i.e. local mode and remote mode. In local mode, two tanks are controlled by two separate potentiometers which are applied to two tanks to drive water to respective tanks.

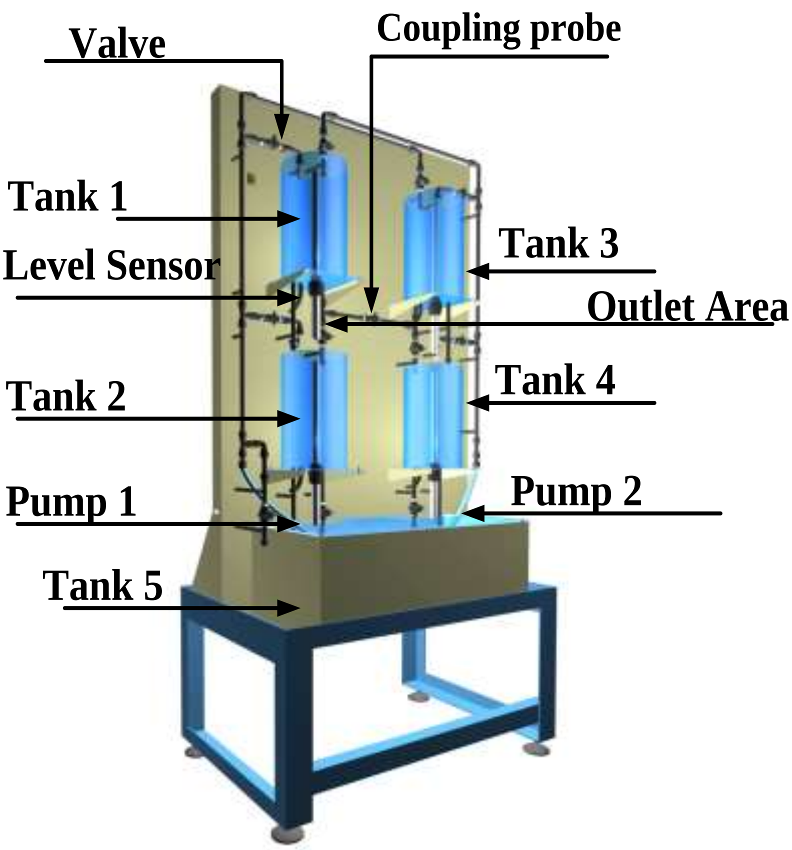

Fig.1.3 Schematic Diagram of a Coupled Tank Mechanical Unit

1.2 Description of Coupled Tank Experimental Setup

Apart from the mechanical parts of the coupled tank system it is also equipped with a power supply unit and a power amplifier (PSUPA) and a cable connector box which is shown in Fig.1.4. In this set-up, PC with Advantech card and MATLAB/SIMULINK environment serve as the main control unit. Basically, the PSUPA unit amplifies the water pressure-level signals and passes them as analogue signals to the PCI1711 DAQ card. Control signals to the pumps can be sent from the PC through the DAQ (PCI1711) card and PSUPA unit. The control signals, which are between 0V – 5V, are transferred to the PSUPA unit where they are transformed into 24V PWM signals in order to drive the pumps.

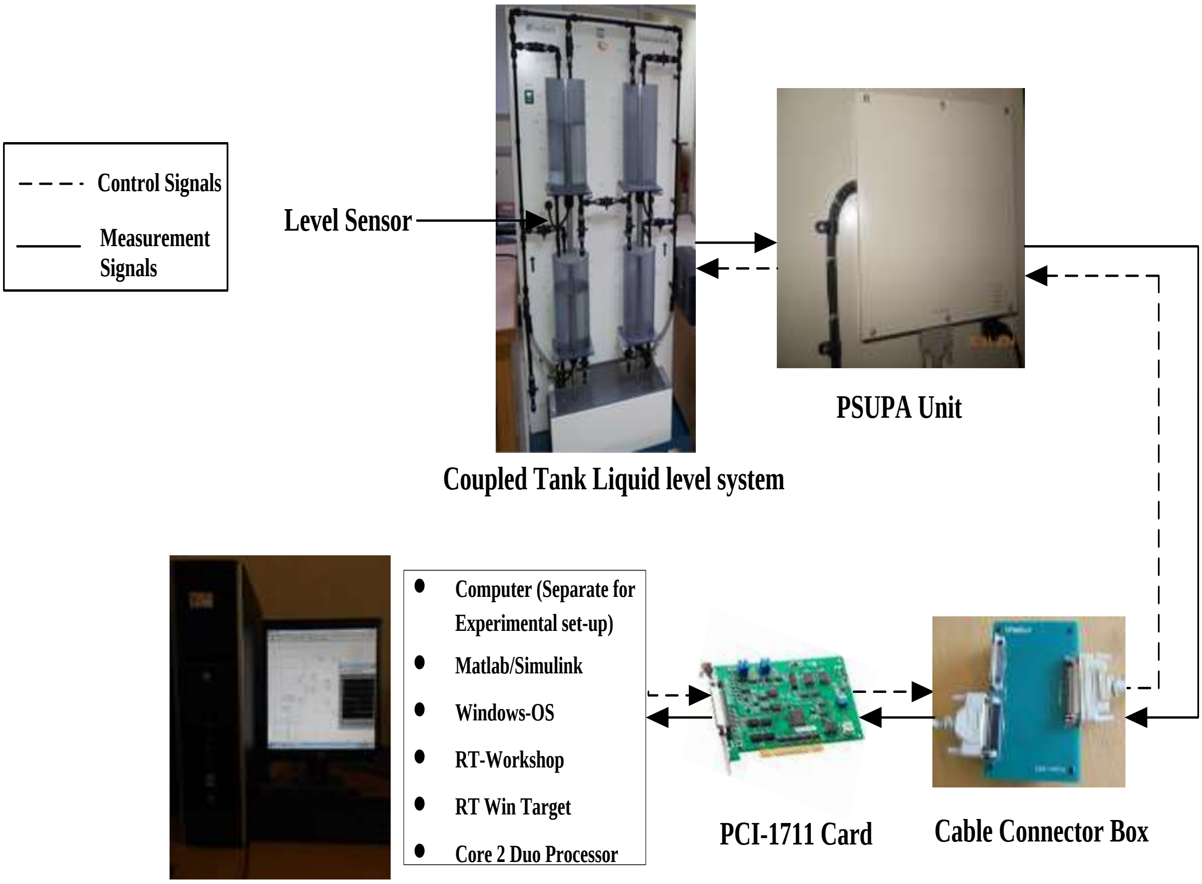

Fig.1.4 Schematic Representation of Experimental Set-up Showing Each Hardware

The next section explains how the SIMULINK and Real-Time Workshop are integrating with the hardware.

1.2.1 Real-Time Workshop

Usually, the Real-Time Workshop is an extension of SIMULINK which has rapid prototyping ability for real-time software applications [56]. It has the following features,

- Automatic code generation for several target platforms.
- A rapid and direct path from system design for implementation.
- Simple graphical user interface.
- Seamless incorporate with MATLAB/SIMULINK.

The toolbox has an automatic code to building up the process for real-time process. Fig. 1.5 explains the process diagrammatically.

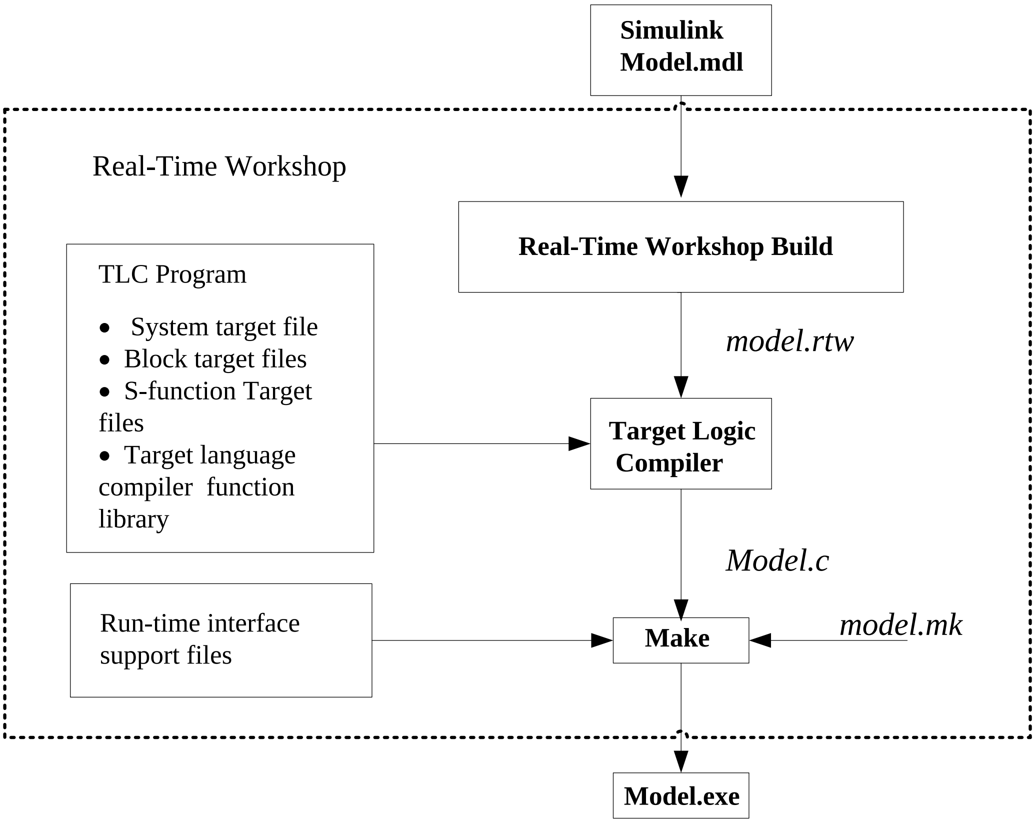

Fig.1.5 Schematic of the Real-Time Workshop code generation process [56]

The steps of real-time build process [56], are as follows

1. Real-Time Workshop analysis the block diagram and compiles it into an intermediate hierarchical depiction of the form ***model.rtw.***
2. Target language compiler (TLC) reads the ***model.rtw*** and converts it into C code which is placed in the build directory within the MATLAB working directory.
3. Further, the TLC constructs a make file from an appropriate target make file template and places in the build directory.
4. Then the system reads the make file to compile the source code and links object files and libraries and generates an executable file i.e. ***model.exe.***

1.3 Literature Survey on Control Strategies Applied To Coupled Tank System (CTS)

The last four decades have witnessed the development of several modeling and controller approaches for a coupled tank system. A coupled tank system is a challenging control problem, because it has right half-plane zeros (non-minimum phase system) which impose restrictions on the sensitivity function. An accurate model as well as an appropriate control strategy is highly essential in order to maintain desired level in tanks in face of uncertainty and disturbance. In [1-2], a brief description has been reported about a quadruple tank coupled system. A mathematical model for coupled tank by considering mass balance equation and Bernoulli's principle has been described in [2-3]. But this linear model fails to provide adequate performance, because during linearization by Taylor Series expansion, generally higher order terms are omitted and also some parameters of the coupled tank system are not known precisely. So in order to overcome this drawback some literature consider the system identification techniques [4-5]. In [4], subspace identification has been presented. A soft computing approach, i.e. ANFIS architecture based on TSK fuzzy modeling for liquid level control has been reported in [6] with a hybrid learning approach.

Although various control strategies have been successfully verified for the coupled tank system but the classical PID with some enhancement provides effective liquid level control performance. Also it is simple in view of its easy implementation and simple structure [7, 48, 60, 62]. An auto adjustable PI controller using Model Reference Adaptive Control (MRAC) technique was proposed in [3]. In this, the MRAC approach can adapt the controller parameters in response to changes in plant and disturbance occurring in real time by referring to the reference model that specifies properties of the desired control system. A characteristics ratio assignment (CRA) based PI controller method was proposed in [8]. A comparison of performances of PI controller with numerous tuning approaches has been reported in [9]. In [10], a two degree of freedom control for level control has been reported where instead of measuring the inlet flow rate, a load estimation scheme is proposed. The proposed control uses a feed-forward gain for load estimation and only a proportional control (P) for only feedback control. The proposed control scheme in [10] acts as only proportional control (P) in disturbance free condition and it works like a PI control under load disturbance. An auto tuning technique of PID controller has been reported in [11-12]. In [12], a comparison of responses has been analyzed between conventional PID and auto tuning PID. A comparison analysis has been explored between the conventional Proportional Integral (PI) based on Ziegler-Nichols tuning approach with Internal Model Control (IMC) based on Skogestad's setting [13].

A sampled-data level control for nonlinear coupled tanks was presented in [14]. Development of a web based laboratory control experiments has been reported in [15], with emphasis both on teaching and research. Further it has some attractive features such as the use of video conferencing for providing audio-visual feedback to the user and the provision for adjustment of the pan/tilt and the zoom of the camera capturing the real time video has been incorporated. In order to eliminate the draw backs of the standard PID controller, a robust PID controller design has been presented in [16-18], where two approaches such as edge theorem and Neimark's D-Partitions [16] have been considered in order to carry out the design. Digital control of a liquid level tank system has been reported in [19] where a digital state-feedback algorithm has been proposed for achieving level control. A comparison between conventional PID and fuzzy control has been reported in [20]. In order to tune the PID gains, an inverting decoupling technique has been proposed in [21] for the quadruple tank process. A comparisons of different controller such as Linear Quadratic Gaussian (LQG), H_{∞}, loop-shaping, feedback linearization and model predictive control (MPC) etc. has been presented in [22]. A different optimization technique also has been successfully applied to the coupled tank system for level control. In [23], cuckoo optimization has been considered in order to tuning the optimal fuzzy parameters for fuzzy logic controller which is used for liquid level control. Genetic algorithm has been reflected in [24], for on-line auto tuning of the PID parameters of a liquid level control system. A robust decentralized PID control for a quadruple tank system has been discussed in [25], where both minimum and non-minimum phase configuration of the quadruple coupled tank system are considered.

A model based control using internal model control (IMC) has been reported in [4] where two control techniques has been discussed such as IMC and DMC Initially IMC applied to the non-minimum phase process and later on, dynamic matrix control (DMC) is used to control the system and explicitly implement process constraints. In [26], the authors has proposed a distributed model predictive control where local measurements at the nodes are used to estimate the relevant plant state which is then used in the model predictive calculations.

Adaptive controllers and backstepping controllers also have been successfully implemented for coupled tank system [3, 27, 28]. In [3] real time implementation of model reference adaptive control (MRAC) has been explored based on MIT rule. A comparison between a direct model reference adaptive controllers (MRAC), an indirect MRAC with Lyapunov estimation and an indirect MRAC with a RLS parameter adaption estimation has been presented in [27].Two different backstepping control approaches have been designed in [28],namely model based backstepping controller and adaptive back stepping controller. Model based backstepping controller was initially designed in order to ensure the exponential tracking of level and then an adaptive backstepping controller compensating the uncertainties arising in the tanks

During last two decades significant interest on variable structure system and sliding mode control has been observed in the control research community worldwide. Apart from the above reported controller techniques such as PID controller, model based control, adaptive control and backstepping controller also fuzzy logic as well as sliding mode control have been successfully implemented for coupled tank liquid level system [29-43,57,61]. In general, the sliding mode control have numerous attractive features such as faster response, good transient performance, better disturbance rejection capabilities. Basically SMC laws are inherently more robust against the matched uncertainties [45-46]. Variable structure systems with sliding modes design and analysis of systems are surveyed in [29 and 32]. Basic concept behind the sliding mode controller has been analysed in [30], where a brief tutorial about the most fundamental issues in the field of VSC and SMC; also the most important novel trends and engineering applications have been reported in this field. A new approach to sliding mode controller design based on a first order plus dead time (FOPDT) model for a chemical process has been reported in [33].In [34], development of an Internal Model Sliding Mode Control has been presented, where a new control method, combining the IMC approach and the SMC concept for the process with a large dead time has been discussed. A continuous time sliding mode controller for a chemical process has been reported in [35]. A fuzzy sliding mode controller using nonlinear sliding surface for coupled tank systems has been discussed in [36], where in order to alleviate the chattering, a fuzzy logic controller was used in order to approximate the corrective control term. Development of a neuro-fuzzy-sliding mode controller with a nonlinear sliding surface for a coupled tank system has been proposed in [37], in this paper in order to reduce the chattering a fixed boundary layer around the switching surface was used. Also in order to smooth the switching signal, fuzzy logic control has been used and to compute the equivalent controller a feed-forward neural network has been considered. A feedforward-plus-sliding mode controller design for the coupled tank system has been reported in [38]. In this paper, a feedforward controller is used to achieve desired process output and the sliding mode controller is combined to ensure the robustness against different uncertainties and external disturbances. In [39], a static sliding mode control design has been proposed for a coupled tank system. Two different dynamic sliding mode control schemes were also proposed

in [39] in order to reduce the chattering problem. Combining feedback linearization with sliding mode control algorithm control scheme was presented quadruple tank system [40].
A second order sliding mode control algorithm has been discussed in [41].In order to realize level position control of a coupled tank system, a chattering free sliding mode controller has been proposed in [42]. To improve the tracking performance of a coupled tank system against various uncertainties a nonlinear sliding mode control with varying boundary layer has been presented in [43].Herein authors has made a comparative assessment between the sliding mode control with a varying boundary layer. In [44], a comparative study has been made on two controllers namely, fuzzyPI+fuzzyPD with the conventional PI for a real-time liquid level control experiment in real time has been discussed. A robust recursive method for parameter estimation of linear time invariant continuous systems has been proposed in [59]. In this paper basically the algorithm is developed to estimate the coefficient of Laguerre series expansions of the process signal, when the measurement contains outliers. In [7], a fuzzy type PID based on ANFIS model for a nonlinear liquid level system has been analysed. A neuro-fuzzy controller (NFCGA) based on the radial basic function neural network which is tuned automatically by using genetic algorithms (GA) has reported in [6], where a linear mapping method is used to encode the GA chromosome and effectiveness is demonstrated in a real time coupled tank liquid level set-up.

1.4 Motivation

The development of control algorithm for a coupled tank system is complex and more challenging because, the coupled tank system dynamics is nonlinear which exhibits non-minimum phase behaviour. It is the most popular form of coupled multivariable system. Level control in a coupled multivariable tank system is challenging due to the following issues.

- Nonlinear and Non-minimum phase system
- Commonly, multivariable nature causes interactions between the two tanks so water may flow in either two direction
- System Constraints
    - The capabilities of DC motors used to pump water is limited (0-5V) (Input Constraints)
    - The water level in the two tanks have to be maintained at a desired set point within a specific level (Output Constraints)

1.5 Thesis Objectives

The objectives of the thesis are as follows.

- To develop a suitable model for the coupled tank system by employing physical mathematical modeling as well as system identification techniques.
- Design and implement different advance controllers for the level control in order to maintain desired liquid level in the coupled tank liquid level system.
- To pursue a comparative study among all the developed controllers technique in order to choose the best controller based on tracking capability performance for the liquid level control.
- To validate all results from the simulation (using MATLAB) and then through real-time experimentation on a coupled tank liquid level setup

1.6 Thesis Organization

The thesis is organized as follows.

- In chapter 2 the dynamics modeling of the coupled tank system has been described by employing both mathematical modeling as well as system identification technique.
- Chapter 3, presents a Linear Matrix Inequality (LMI) based PI controller design and also a comparative study is pursued with the traditional approach PI controller design [7] where parameters are tuned employing Ziegler Nichols approach.
- It has been observed in Chapter 3 that a large control action is required to acquire desired liquid level. Hence in Chapter 4, an adaptive fuzzy based PI control approach is proposed for the coupled tank system.
- In view of overcoming the short comes of the PI controller that is unable to provide good robust performance against load disturbance, a sliding mode control algorithm is developed in Chapter 5
- Chapter 6 proposes development of an Adaptive Fuzzy Sliding Mode Controller algorithm for the CTS.
- Chapter 7 concludes the thesis and suggestions for future work are also discussed therein.

Chapter 2

Dynamics Modeling of a Coupled Tank System

2.1 Coupled Two Tank Dynamics

The simplest nonlinear model of the coupled tank system [1] can be obtained by considering the mass balance principle, which is relating the water level h_1, h_2 and applied voltage 'u' to the pump.

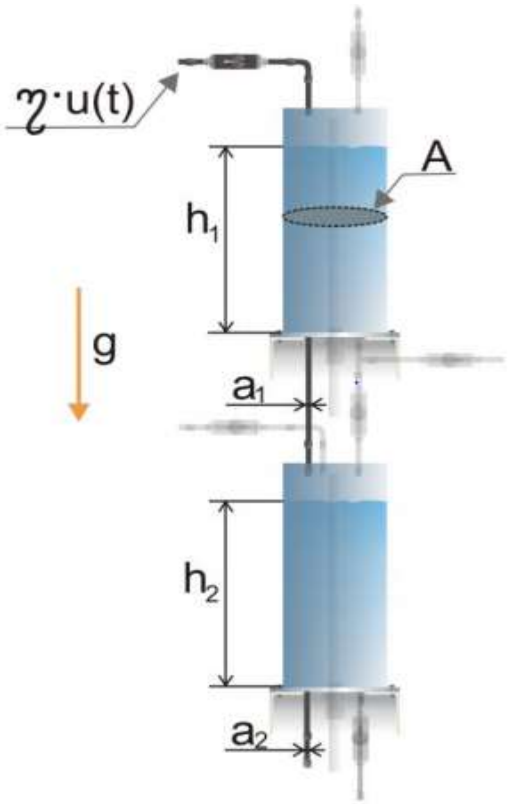

Fig.2.1 Representation of Coupled Two Tanks Model

$$\frac{dh_1(t)}{dt} = -\frac{a_1}{A}\sqrt{2gh_1(t)} + \eta u(t) \tag{2.1}$$

$$\frac{dh_2(t)}{dt} = \frac{a_1}{A}\sqrt{2gh_1(t)} - \frac{a_2}{A}\sqrt{2gh_2(t)} \tag{2.2}$$

where

h_1 = water level in tank 1

h_2= water level in tank 2

a_1= outlet area of tank 1

a_2= outlet area of tank 2

A = cross-sectional area of tanks

g = gravitational constant

η =constant relating to the control voltage

Eq (2.1) and eq (2.2) represent the dynamics of the coupled tank system. On performing Taylor's series expansion of eq (2.1) and eq (2.2) one can obtain linear mathematical model for the coupled tank system. At equilibrium point for constant water level the derivatives must be equal to zero, thus one obtains

$$0 = -\frac{a_1}{A}\sqrt{2gh_1(t)} + \eta u(t) \tag{2.3}$$

$$0 = \frac{a_1}{A}\sqrt{2gh_1(t)} - \frac{a_2}{A}\sqrt{2gh_2(t)} \tag{2.4}$$

$$\frac{a_1}{A}\sqrt{2gh_{10}} = \eta u_0 \tag{2.5}$$

$$a_1\sqrt{2gh_{10}} = a_2\sqrt{2gh_{20}} \tag{2.6}$$

Linearizing eq. (2.1) and (2.2), considering two operating point h_{10} and h_{20} as below

$$h_{10} = \frac{1}{2g}\left(\frac{\eta u_0 A}{a_1}\right)^2 \tag{2.7}$$

$$h_{20} = \left(\frac{a_1}{a_2}\right)^2 h_{10} \tag{2.8}$$

$$\Delta\dot{h}_1(t) = -\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}\,\Delta h_1(t) + \eta\Delta u(t) \tag{2.9}$$

$$\Delta\dot{h}_2(t) = \left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}\,\Delta h_1(t) - \left(\frac{a_2}{A}\right)^2 \frac{g}{\eta u_0}\,\Delta h_2(t) \tag{2.10}$$

Defining water levels of tank as state variables for equation (2.9) and (2.10) a state space model of the coupled tank system can be obtained as follows

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0} & 0 \\ -\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0} & -\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \eta & 0 \end{bmatrix} u \qquad (2.11)$$

Laplace transforming eq. (2.9) and (2.10), yields the following transfer function models of the coupled tank system as follows

Tank 1:-

$$\frac{\Delta H_1(\mathrm{s})}{\Delta U(\mathrm{s})} = \frac{\eta}{s + \left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}} \qquad (2.12)$$

Tank 2:-

$$\frac{\Delta H_2(\mathrm{s})}{\Delta H_1(\mathrm{s})} = \frac{\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}}{s + \left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}} \qquad (2.13)$$

Table.2.1 Parameters of the Coupled Tank System

Symbol	Description	Value	Unit
A	Cross sectional area of tanks	0.01389	cm^2
a_1	Tank1 outlet area	0.1245	cm^2
a_2	Tank-2 outlet area	0.1245	cm^2
g	Gravitational constant	9.8	m/sec
η	Constant relating the control voltage with the water flow from the pump	0.1194	

During linearization of eq.(2.1) and (2.2) by Taylor Series expansion, higher order terms are neglected because these are very small and also some parameters of the coupled tank system are not known perfectly. So there is an obvious need of an obtaining an accurate dynamics model of the system. Hence, the system identification technique is adopted in order to get a perfect model of the system.

2.2 System Identification to Obtain Model for Coupled Tank System

System identification is a procedure to obtain the mathematical model of a physical system from the input-output data [55]. System identification techniques can handle a wide range of system dynamics without any prior knowledge of the actual physical system. Thus, system identification technique is usually adopted in order to obtain flexible model of a physical system instead of its modeling formed by first principle method. It is of high significance for systems where the presence of large number of variables and nonlinear interactions among them hinders the determination of a model from the governing physical laws. Basically, system identification technique is broadly classified into two groups i.e. parametric approach and non-parametric. The entire process of identifying the system from input and output data is broadly consists of six stages. It starts with an experimental design followed by data collection and data processing; next a suitable model structure is selected then the parameters of the model are estimated and finally the model is validated with the experimental data.

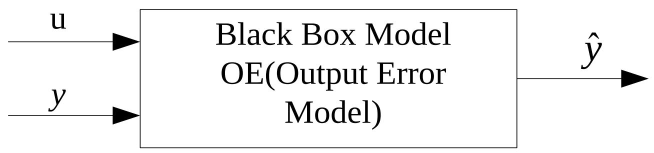

Fig.2.2 A Basic Representation of Black Box Model Identification

Generally different parametric model structures are selected while modeling of an unknown system. Parametric models commonly describe the true process behaviour exactly with finite number of parameters. The parametric model structure is also known as a black box model shown in Fig.2.2

The general used model description of a linear system is given by

$$y(k) = G(q)u(k) + H(q)e(k) \tag{2.14}$$

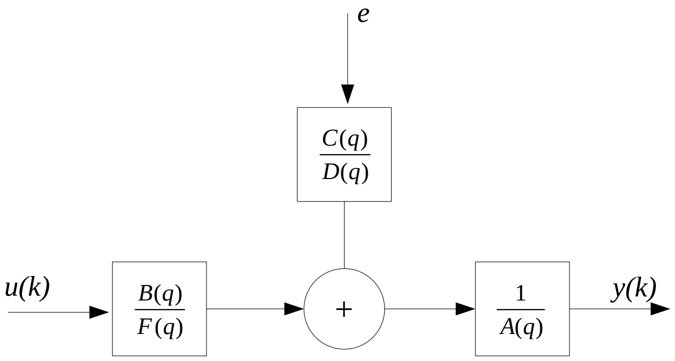

Fig.2.3 Representation of the General Model Structure

where

u(k)=input of the system

y(k)=output of the system

e(k)=zero-mean white noise or disturbance of the system

H(q)= transfer function of the stochastic part of the system

G(q)=Transfer function of the deterministic part of the system

This general model structure is usually divided into different structures which are discussed below.

Output Error (OE) Model

In the output error model (OE) structure the system dynamics is described separately. In this structure no parameters are used for modeling the disturbance characteristics. The model structure of output error model is depicted below

$$y(k) = \frac{B(q^{-1})}{F(q^{-1})} u(k) + e(k) \tag{2.15}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + ... + b_{n_b} q^{-nb}$$
$$F(q^{-1}) = 1 + f_1 q^{-1} + f_2 q^{-2} + ... + f_{n_f} q^{-nf}$$

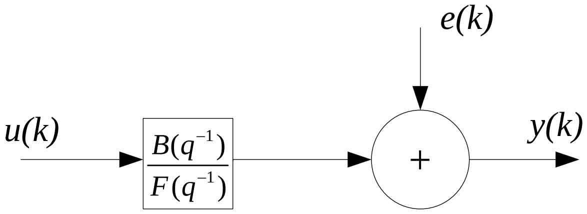

Fig.2.4 Block Diagram of the OE Model

Auto Regressive Exogenous (ARX) Model

In ARX (Auto Regressive Exogenous) model, auto regressive means that the current output has a relation to the previous values of output and exogenous signifies that the system relies not only the current input values but also history of output values. The estimation of the ARX model is the most efficient of the polynomial estimation method because it solves the linear regression equation in analytical form. The model structure of ARX model is similar to that of the output error model(OE) except that the model output in the ARX form is a basic function of past input and past process output while model output in OE model form is a function of past input and past model output.

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} u(k) + e(k) \qquad (2.16)$$

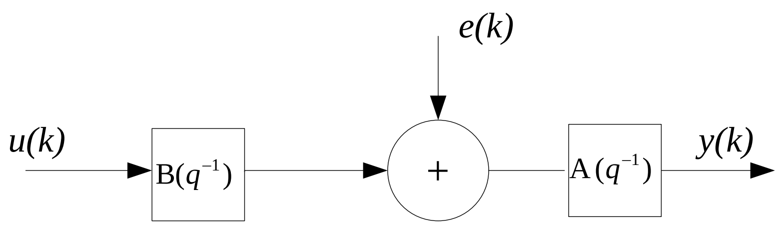

Fig.2.5 Block Diagram of the ARX Model

Auto Regressive Moving Average Exogenous (ARMAX) Model

The ARMAX model structure has more flexibility in handling disturbance, while compared to the ARX model structure. In ARMAX model structure, an extra moving average term is included as compared to the ARX model structure; except that part, both the ARX and ARMAX model structures are similar. Eq (2.17) gives the description of ARMAX model.

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} u(k) + \frac{C(q^{-1})}{A(q^{-1})} e(k) \qquad (2.17)$$

For describing OE, ARX and ARMAX, models the polynomial A, B and C can be defined as follows

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \dots\dots\dots + a_{n_a} q^{-n_a}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + \dots\dots\dots + b_{n_b} q^{-n_b}$$

$$C(q^{-1}) = c_0 + c_1 q^{-1} + c_2 q^{-2} + \dots\dots\dots + c_{n_c} q^{-n_c}$$

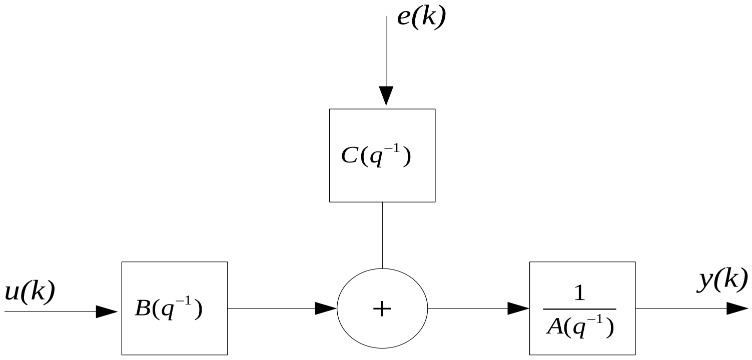

Fig.2.6 Block Diagram of the ARMAX Model

In this present work, a second order output error model is considered for model identification, to obtain good fit of the experimental data. In the simplest, form the OE model is represented as

$$y(k) = \frac{B(q^{-1})}{F(q^{-1})} u(k) \tag{2.18}$$

Least Square parameter estimation algorithm [55] is considered in order to estimate the parameters of the predicted model as that minimizes the error between the model and plant output in the sense of minimum square error. The parameter vector $\hat{\theta}_{Ls}$ can be obtained using the following normal equation.

$$\hat{\theta}_{Ls} = \left(\psi^T \psi\right)^{-1} \psi^T y \tag{2.19}$$

where $\hat{\theta}_{LS} = \begin{bmatrix} b_0 \\ b_1 \\ f_1 \\ f_2 \end{bmatrix}$

Obtained parameters for Tank 1 and Tank 2 are as follows,

Tank-1:-

$$\begin{aligned} B(q^{-1}) &= -0.004711\hat{q}^{-1} + 0.004877\hat{q}^{-2} \\ F(q^{-1}) &= 1 - 1.991\hat{q}^{-1} + 0.9911\hat{q}^{-2} \end{aligned} \tag{2.20}$$

Tank-2 :-

$$\begin{aligned} B(q^{-1}) &= 0.06481\hat{q}^{-1} - 0.062\hat{q}^{-2} \\ F(q^{-2}) &= 1 - 1.901\hat{q}^{-1} + 0.9017\hat{q}^{-2} \end{aligned} \tag{2.21}$$

Thus, the obtainable transfer function model of the CTS for both tanks using the system identification which are given below

$$T_1(s) = \frac{-0.04815s + 0.01668}{s^2 + 0.0892s + 0.002474}$$

$$T_2(s) = \frac{0.6672s + 0.2961}{s^2 + 1.035s + 0.04397} \tag{2.22}$$

These models i.e. $T_1(s)$ for tank-1 and $T_2(s)$ for tank-2, are to be used subsequently for designing controllers.

2.3 Results Obtained from System Identification

The coupled tank system is excited by a white noise for performing system which covers a broad range of frequencies for whole dynamics in the identification of parameters. The excited input signal is as shown in Fig.2.7. The outputs of the both tanks are as shown in Fig.2.8 and Fig.2.9.

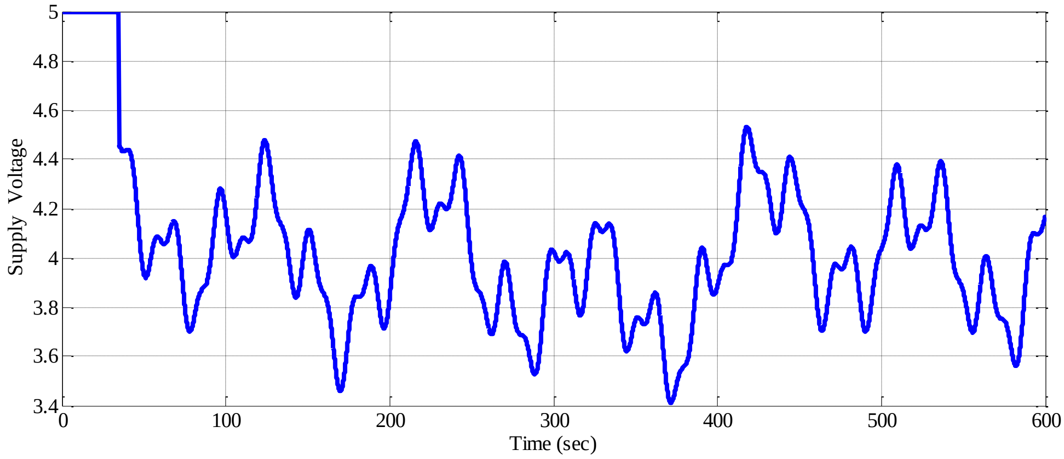

Fig.2.7 Experimental Input Data

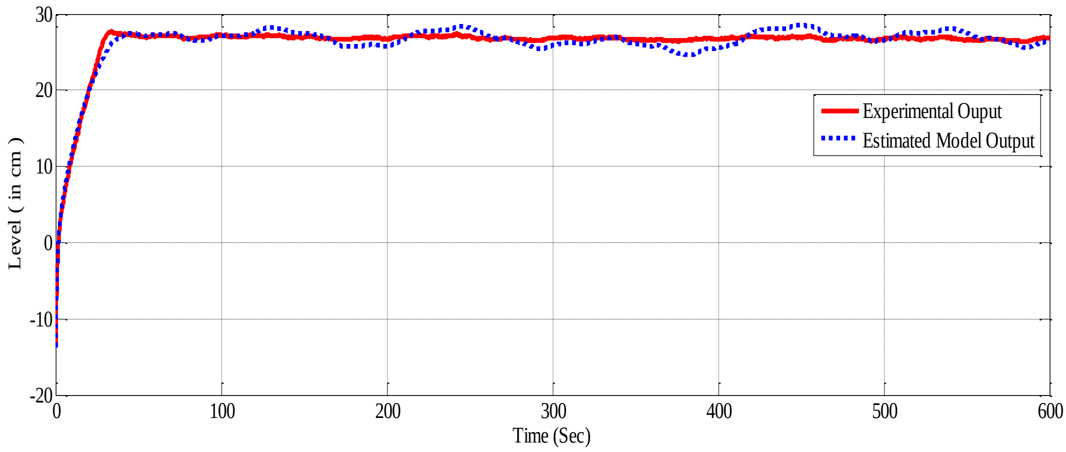

Fig.2.8 Experimental Output versus the Simulated Output of the Identified Model for Tank

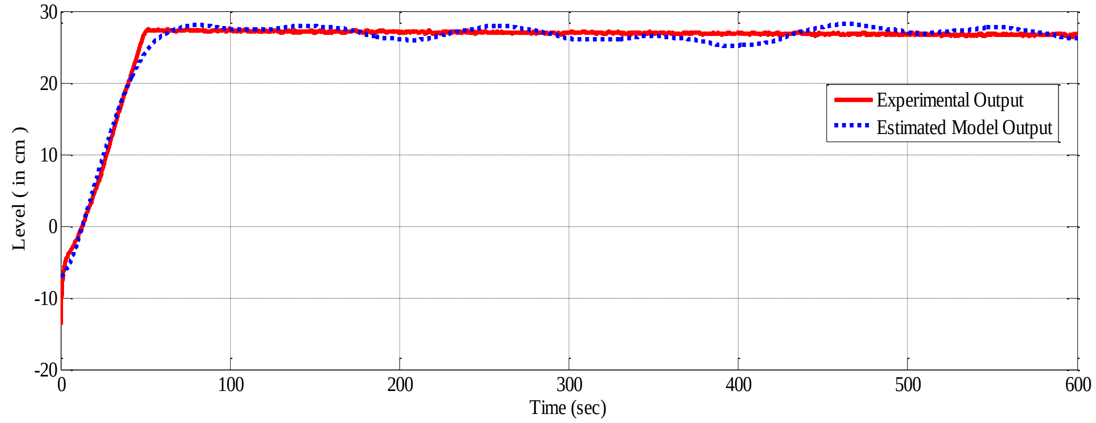

Fig.2.9 Experimental Output versus the Simulated Output of the Identified Model for Tank 2

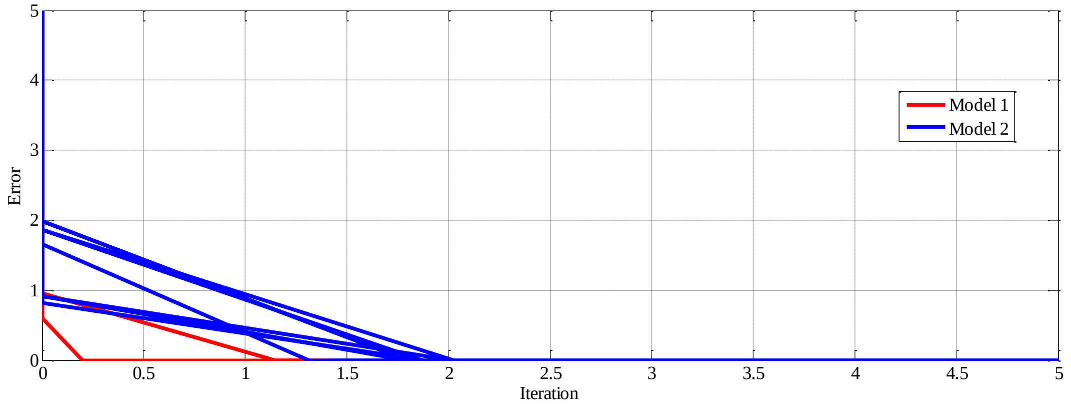

Fig.2.10 Response of Mean Square error plot (MSE)

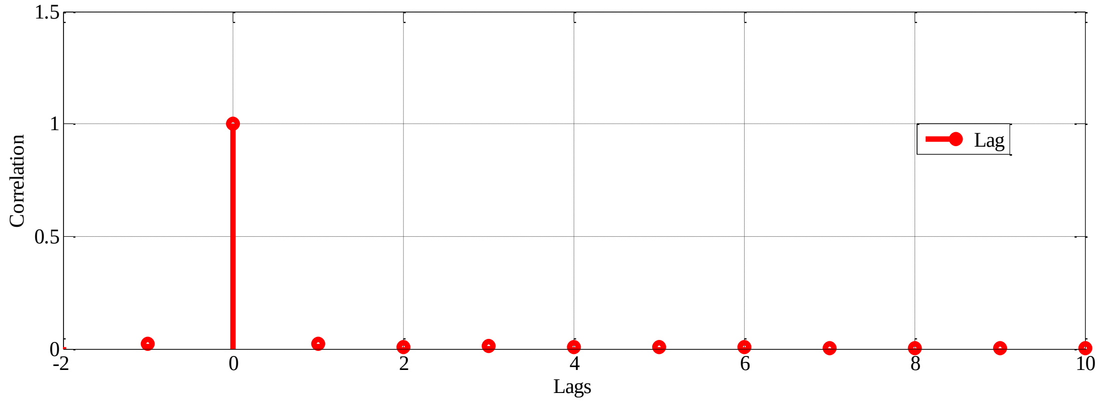

Fig.2.11 Model Validation Response by Using Auto-correlation Analysis

The experiment is performed for 600 secs with sampling time of 0.1 sec i.e. a record of 6000 experimental samples are considered. After getting the identified model, the model was validated by using Mean Square Error (MSE) approach which is shown in Fig.2.10 and nonparametric approach technique i.e. auto-correlation method.in Fig.2.11 From the auto-correlation analysis Fig.2.11 it is observed that all the lags (which is the time difference in samples between the signals at which the correlation is estimated) are lie inside the confidence interval. Hence, from both the obtained responses it is envisaged that the identified model is a good model which can be used to verify the performances different control algorithms developed for the coupled tank system.

2.4 Chapter Summary

In this chapter basically in order to obtain dynamic model of coupled tank system two approaches namely mathematical modeling and system identification has been presented. Furthermore in next chapter controller design have been carried out based on these obtained dynamic models.

Chapter 3

A LMI based PI Controller Design for the Coupled Tank System

The proportional-integral-derivative (PID) controller are extensively used in almost all the industries such as chemical, water treatment, and all process control industries etc. for last several decades. Ziegler and Nichols [7] have proposed first, the tuning of the PID controllers. The reason of popularity of PID controllers is their simplicity in design and parameter tuning [7-9] such as (Ziegler-Nichols approach). Usually PID controllers perform numerous important functions such as the steady state-state offset elimination and anticipation of deviation and adequate corrective signals generation through the derivative action. Along with combinational logic as well as sequential machines, these PID controllers are increasingly used in automation industries. However, there are many control problems where this simple PID control is inadequate such as nonlinear systems, systems with relative degree higher than two and also for non-minimum phase systems. The simple PID control structure does not provide satisfactory performance for time delay system and for the time varying system. Furthermore, in many real-world systems are time varying and uncertain systems. Therefore it is necessary that the controller should have good disturbance capability in face of the system uncertainties. Since a simple PID controller is not capable to handle this difficulties all together, in this chapter a LMI based convex optimization with simple PID controllers has been taken in order to overcome the above-mentioned problem.

This approach is based on the transformation of the PI controller design problem to a state feedback controller design problem, which is further solved using the convex constraint optimization approach [52-54].

3.1 Chapter Objectives

The objective of this chapter is that, to find an optimal state feedback gain *K*.so the cost function given in eq (3.7) is minimized which depends on the trajectory *x (t)*.So that the objective is to find out the worst possible of J for the worst case of *x (t)* i.e. to find out optimal cost $x_0^T P x_0$ .and by utilizing the obtained least optimal control effort the desired level will be maintain in both the tanks.

3.2 Linear Matrix Inequality (LMI): A Brief Introduction

Linear matrix inequalities in control system basically aim to describe how the convex optimization theory was established from the linear programming optimization tool to the interior-point approach and for analysis its significance in control systems [54]. Then after, the given control problem is transformed into a set of LMI constraints which will be further described by LQR optimal problem. In general the linear matrix inequality is represented in the following form

$$F(x) \triangleq F_0 + \sum_{i=1}^{m} x_i F_i > 0 \tag{3.1}$$

where $x \in R^m$ is a variable and $F_i = F_i^T \in R^{n*m}, i = 0 \ldots m$ are given. In eq (3.1), the inequality sign signifies that F(x) is positive definite, i.e. $F(x) > 0$ for all non-zero $x \in R^m$. A LMI is a set of n polynomial inequalities in x. Further multiple LMIs $F_1(x) > 0, \ldots. F_n(x) > 0$ can be stated as the simple LMI as follows

$$\begin{pmatrix} F_1(x) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & F_n(x) \end{pmatrix} \tag{3.2}$$

Usually, nonlinear inequalities are converted into LMI by utilizing Schur compliment which is described by the lemma 3.1

Lemma # 3.1.Schur Lemma

The LMI is given as follows

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0 \tag{3.3}$$

where $Q(x) = Q(x)^T, R(x) = R(x)^T$ and $S(x)$ depends affinely on x which is equivalent to

$$R(x) > 0, Q(x) - S(x)R(x)^{-1}S(x)^T > 0 \tag{3.4}$$

In other words, any matrix inequality of the form in equation (3.4) can be can also be represented as equation (3.3).

Basically, the LMI in equation (3.1) offers two kinds of questions such as

- The LMI feasibility problem amounts to testing whether there exists real variables $x_1 \ldots x_n$ such that equation (3.1) holds.
- The LMI optimization problem amounts to minimizing the cost function $c(x) = c_1 x_1 + \ldots\ldots + c_n x_n$ over all ($x_1 \ldots\ldots x_n$) that satisfies the constraints in Eq (3.1).

Usually, for control, most of the LMIs involve matrix variables rather than vector variables. That means most of the inequalities can be considered in the form as follows.

$$F(x) > 0 \tag{3.5}$$

3.3 A LQR-LMI framework based formulation for PI controller design

In this formulation mainly PI control design problem is converted into a state feedback control design problem which is solved using convex constraint approach. Generally, the convex optimization problem can be effectively solved by using the interior point LMI solver (MINCX)[52,53].It is a state space approach control design technique, where PI controller design is considered as a static state feedback control design problem and the static feedback gain vector 'K' contains all the parameters of the PI controller.

Consider a LTI system which is given as follows

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{3.6}$$

In the feedback control strategy, the static feedback gain k contains only proportional gain for inner and outer feedback loops. Hence in order to transform the system into a PI like frame work, an extra state variable needs to be included which is generally chosen as the integral of necessary output, for which zero steady state error is desired. If the augmented state is chosen as $\tau = -\int (r - x_1)$,where r is the desired trajectory for z_1 output, then the PI like state feedback control problem can be described using Fig (3.1).

The performance index for the above LTI system is given by

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt \tag{3.7}$$

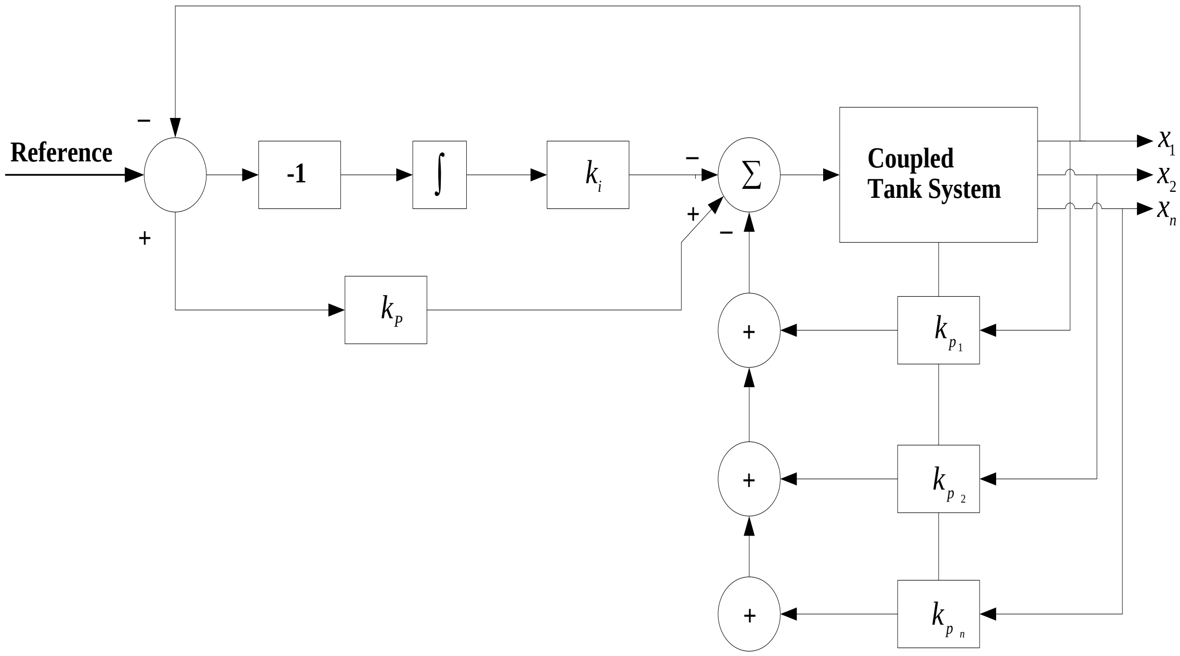

Fig. 3.1 Generalized structure of the PI like state feedback controller

The objective is to find an optimal state feedback gain *K*. The cost function given in eq (3.7) depends on the trajectory *x (t)*, so that the objective is to find out the worst possible of J for the worst case of *x (t)* i.e. to find out optimal cost $x_0^T P x_0$.

The control law is given by

$$u = -Kx = -R^{-1}B^T P \tag{3.8}$$

$$K = -R^{-1}B^T P \tag{3.9}$$

In above equation, 'P' is a positive definite solution of the Algebraic Riccati Equation (ARE)

$$A^T P + PA - PBR^{-1}B^T P = -Q \tag{3.10}$$

The minimum quadratic cost is given by

$$J_{min} = x_0^T P x_0 \tag{3.11}$$

The above LQR problem can be recasted as an optimization problem over $\hat{P}$ and y ,which is stated as follow

$J_{min} = x_0^T P x_0$

subject to

$$\begin{bmatrix} AP + A^T P + BY + Y^T B^T & \hat{P} & Y^T \\ \hat{P} & -Q^{-1} & * \\ Y & * & -R^{-1} \end{bmatrix} \le 0, \hat{P} \ge 0 \tag{3.12}$$

where $Y = -K\hat{P}$ and $\hat{P} = P^{-1}$

Proof:

From ARE we get

$$A^T P + PA - PBR^{-1}B^T P + Q \le 0 \tag{3.13}$$

$$A^T P + PA - K^T RK + Q \le 0 \tag{3.14}$$

where

$K^T = PBR^{-1}$ and $B^T P = RK$

$$A^T P + PA - K^T RK - K^T RK + Q + K^T RK \le 0 \tag{3.15}$$

By pre and post multiplying P^{-1} in the above equation we obtain as follows

$$A^T P^{-1} + AP^{-1} - BKP^{-1} - P^{-1}K^T B^T + P^{-1}QP^{-1} + P^{-1}K^T RKP^{-1} \le 0 \tag{3.16}$$

$$A^T \hat{P} + A\hat{P} - BK\hat{P} - \hat{P}K^T B^T + \hat{P}Q\hat{P} + \hat{P}K^T RK\hat{P} \le 0 \tag{3.17}$$

where in above equation (3.8) $P^{-1} = \hat{P}$ and replace $-K\hat{P} = Y$,one can obtain

$$A^T \hat{P} + A\hat{P} + BY + Y^T B^T + \hat{P}Q\hat{P} + Y^T RY \le 0 \tag{3.18}$$

By applying schur lemma in Eq (3.18) one can obtains

$$\begin{bmatrix} AP + A^T P + BY + Y^T B^T & \hat{P} & Y^T \\ \hat{P} & -Q^{-1} & * \\ Y & * & -R^{-1} \end{bmatrix} \le 0, \hat{P} \ge 0 \tag{3.19}$$

In equation (3.19) it is to be assumed that Q and R are invertible, hence once again by employing schur compliment the cost function can be rewritten as follows

$$x^T(0)\,\hat{\mathrm{P}}^{-1}\,x(0) \le \gamma \tag{3.20}$$

where γ be the specified upper bound. The inequality given in equation (3.20) also can be stated as LMI given by

$$\begin{bmatrix} \gamma & x^T(0) \\ x(o) & \hat{P} \end{bmatrix} \geq 0 \tag{3.21}$$

$$K = -Y^*(\mathrm{P})^{-1}, K = [\mathrm{K}_{p,}\ K_I] \tag{3.22}$$

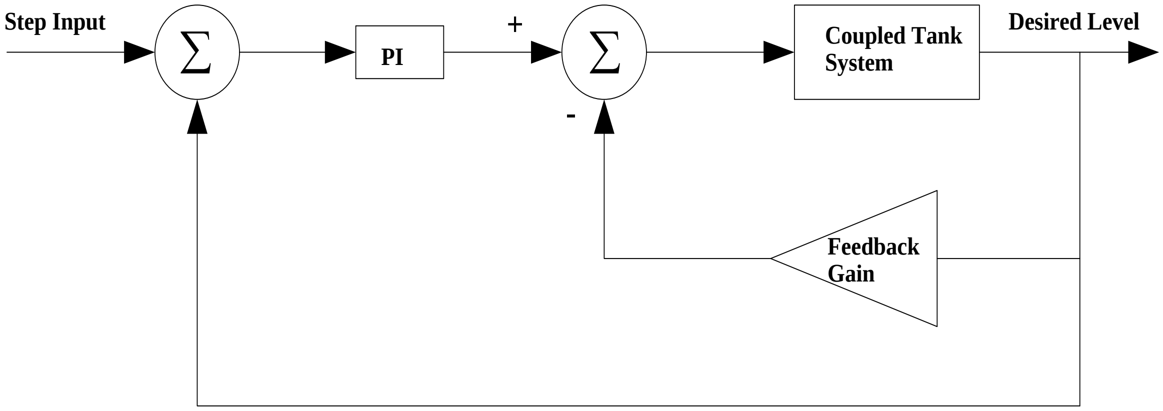

Fig.3.2 Block Diagram of the proposed LMI based PI Controller

3.4 Results and Discussions

For both the tanks Q, and R are chosen as follows.

For Tank 1:-

$$Q = 1.0e + 008 * \begin{bmatrix} 2.2555 & -2.4628 \\ -2.4628 & 7.5024 \end{bmatrix}$$

$$R = 18.647$$

$$Y = 1.0e + 008 * \begin{bmatrix} -5.9426 & -3.9373 \end{bmatrix}$$

For Tank 2:-

$$Q = 1.0e + 008 * \begin{bmatrix} 4.5781 & -3.3694 \\ -3.3694 & 7.1938 \end{bmatrix}$$

$$R = 16.371$$

$$Y = 1.0e + 008 * \begin{bmatrix} -1.3268 & -0.0115 \end{bmatrix}$$

From the simulation as well as experimental results it is observed that, the above chosen values of Q, R and Y offer satisfactory performance for level regulating in the both tanks. By utilizing LMI solver (MINCX) the gain parameters of the presented algorithm are obtained as follows i.e. gain parameters for tank 1 and tank 2 are obtained as $K = \begin{bmatrix} 0.2761 & 0.4542 \end{bmatrix}$, $K = \begin{bmatrix} 0.8265 & 0.6640 \end{bmatrix}$.

Simulation as well as experiment is performed using MATLAB in order to validate the performance of LMI based PI control law for regulating the level at a particular desired level in both the tanks. In order to evaluate the efficiency and feasibility of the LMI based PI controller, it has been compared with a traditional approach PI controller [7] and also implemented in real time. Fig.3.3, Fig.3.4 and Fig.3.5, Fig.3.6 show the simulation results obtained for tank 1 and tank 2 using both the controllers. Fig 3.7, Fig 3.8 and Fig.3.9, Fig.3.10 show the experimental results of both the LMI based and Ziegler-Nichols tuned PI controllers. From simulation as well as experimental results it is clearly observed that the proposed LMI based PI control exhibits better control performance for maintaining the desired liquid level as compared to the PI controller tuning with the traditional approach.

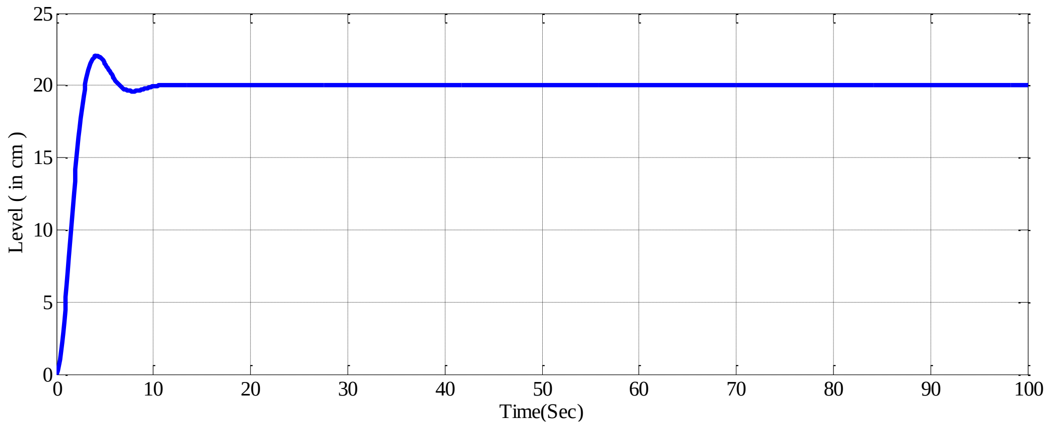

Fig. 3.3 Simulation Response of LMI based PI control for level control in Tank 1

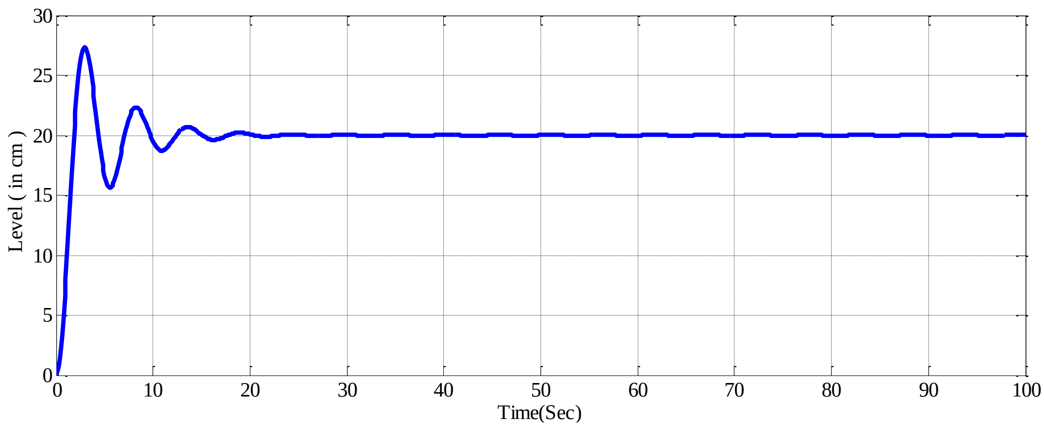

Fig. 3.4 Simulation Response of LMI based PI control for level control in Tank 2

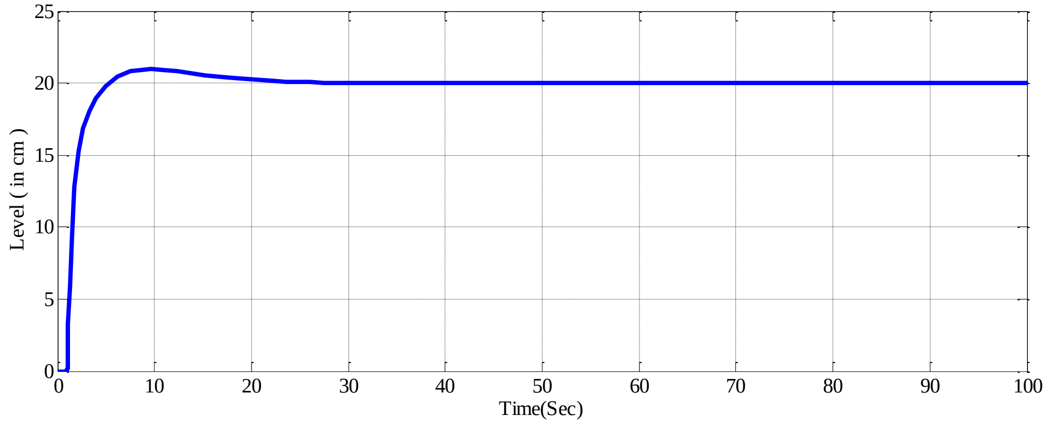

Fig. 3.5 Simulation Response of Ziegler Nichols tuned PI control for level control in Tank 1

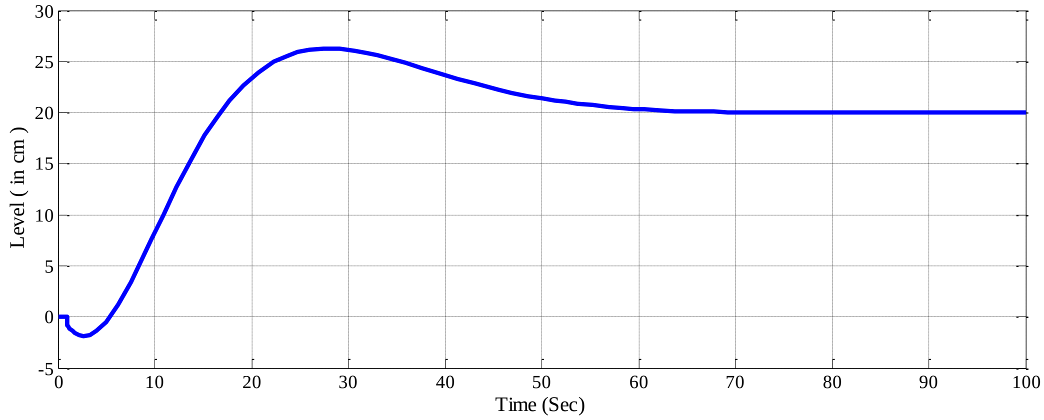

Fig. 3.6 Simulation Response of Ziegler Nichols tuned PI control for level control in Tank 2

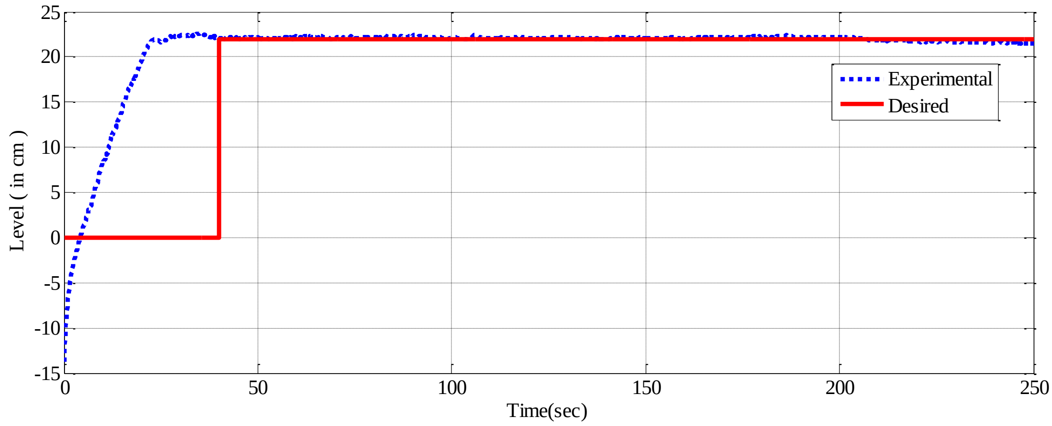

Fig. 3.7 Experimental Response of LMI based PI control for level control in Tank 1

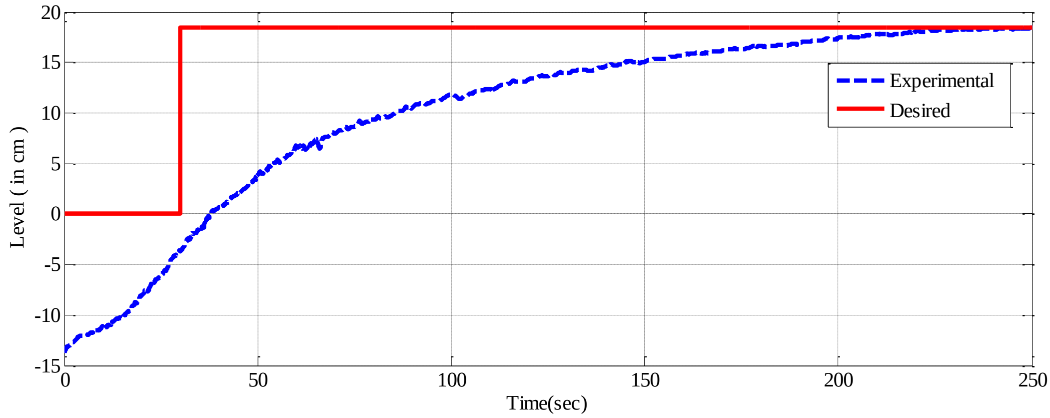

Fig. 3.8 Experimental Response of LMI based PI control for level control in Tank 2

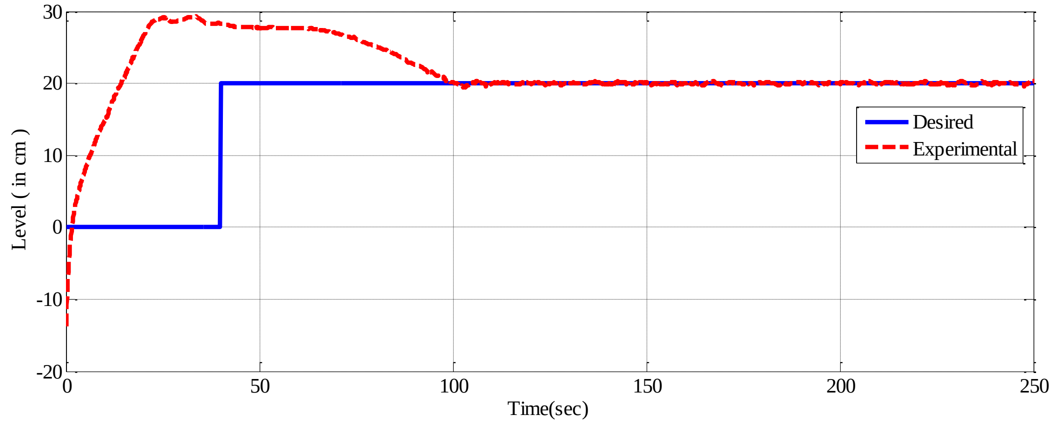

Fig. 3.9 Experimental Response of Ziegler Nichols based PI control for level control in Tank 1

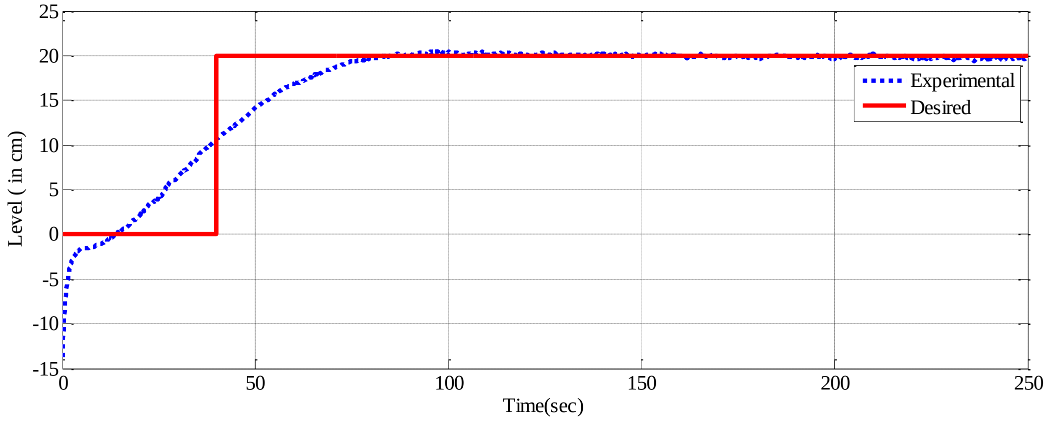

Fig. 3.10 Experimental Response of Ziegler Nichols based PI control for level control in Tank 2

It is clearly observed from simulation results as well as the experimental results Fig. 3.3, 3.4, 3.7 and 3.8 that by the LMI based PI control algorithm the system can reach the set-point in a short time with less overshoot and zero steady state error. On the contrary one can observe clearly, from the simulation as well as experimental results i.e.Fig.3.5, Fig.3.6 and Fig.3.9 and Fig.3.10 that by the Ziegler-Nichols based PI controller the system is settled at its desired level with taking large settling time and also while achieving desired level in tank 2 little steady state error is yielded as compared to the LMI based PI controller algorithm.

Table 3.1 Response Analysis from Time Domain Specifications and Performance Indices for Tank 1

Specification	**LMI Based PI**	**Traditional Approach PI tuned using Ziegler Nichols approach**
Rise Time (Sec)	2.9	3.12
Settling Time (Sec)	21.6	29.17
Peak Time (Sec)	10.5	8.65
Peak overshoot	2%	4%
ISE	27.81	43.55
IAE	62.82	64.33

Table 3.2 Response Analysis by Time Domain Specification and Performance Indices for Tank 2

Specification	**LMI Based PI**	**Traditional Approach PI tuned using Ziegler Nichols approach**
Rise Time (Sec)	1.78	16.46
Settling Time (Sec)	26.78	67.74
Peak Time (Sec)	20.93	27.48
Peak overshoot	9%	25%
ISE	17.81	27.33
IAE	43.221	39.54

3.5 Chapter Summary

In this chapter, a LMI tuned PI controller is presented for the regulation of liquid level in two tanks i.e. tank 1 and tank 2 of CTS. This chapter begins with the basic fundamental concept behind the LMI and furthermore PI control formulation based on LQR-LMI frame work has been described. Finally, this chapter concludes with a comparative assessment of the proposed controller and traditional approach tuned PI controller [7] using time domain specifications along with two different performance indices such as IAE and ISE. Also simulations as well as experimental results were pursued and the obtained result illustrates the efficacy of the proposed controller algorithm. However the proposed LMI based PI controller gives superior performance as compared to Ziegler Nichols tuned PI controller, but it yields sluggish response. Hence, in order to improve the response, in chapter 4 an Adaptive Fuzzy PI (AFPI) controller has been proposed.

Chapter 4

An Adaptive Fuzzy PI controller design for the Coupled Tank System

In chapter 3, we discussed LMI based PI controller, where the response is little sluggish. In order to get better control action an adaptive fuzzy PI controller is designed. The presented controller i.e. AFPI can provide precise liquid level owing to large range of operating conditions since, the fuzzy controllers cover a wide range of operating condition which is main advantage of this presented control algorithm. Generally, an adaptive fuzzy PI controller implies that the parameters of the PI controller are regulated by fuzzy rules in real-time. The input of the presented controller algorithm involve by two types of signals i.e. main and auxiliary signal, where the weighted system error is the main input signal and the secondary input signal is formed by FLC. In main input signal weights are the parameters of the PI controller where the parameters are fixed and tuned by Ziegler Nichols approach. As the output of FLC is variable, thus the controller gain is altered indirectly adapted. The next section describes the description of controller algorithm.

4.1 Design of an Adaptive Fuzzy PI Controller

Fig 4.1 depicts the schematic structure of an Adaptive Fuzzy PI (APFI) controller. In the presented controller, PI controller consists two parallel connected PI controller. In this controller, the total control input is as follows

$$u = u^0 + \Delta u$$

where

u^0= control input for main PI controller

Δu = control input for auxiliary PI controller

$$u(t) = [k_p(t)\phi_e(t)] + \int_0^t [k_i(\tau)\phi_e(\tau)]d\tau \qquad (4.1)$$

The control law given in equation (4.1) can be rewritten as follows

$$u(t) = [k_p^0 + \Delta k_p(t)]\phi_e(t) + \int_0^t [k_i^0 + \Delta k_i(\tau)]\phi_e(\tau)\, d\tau \qquad (4.2)$$

where

$$k_p(t) = k_p^0 + \Delta k_p(t)$$
$$k_i(t) = k_i^0 + \Delta k_i(t) \quad (4.3)$$

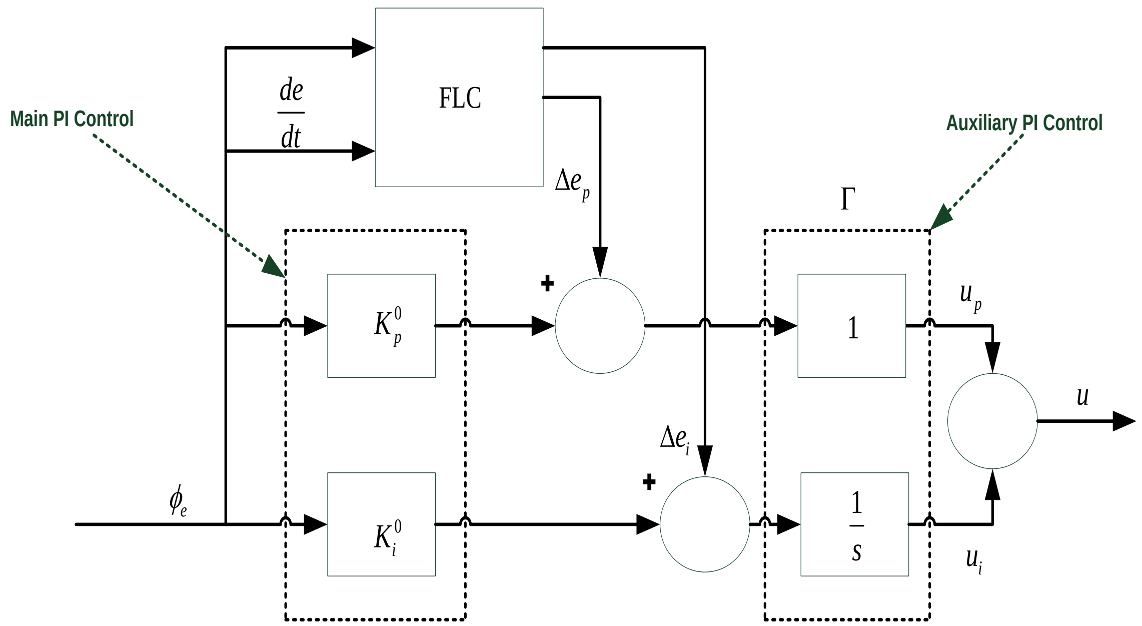

Fig. 4.1 Schematic Structure of Adaptive Fuzzy PI Controller

In equation (4.3), the parameters i.e.k_p^0 and k_i^0 are pre-tuned and are time invariant constants and $\Delta k_p\ (t)$ and $\Delta k_i\ (t)$ are time varying and the value of these parameters are adapted in real-time by using a FLC. In eq. (4.4), the parameters k_p^0 and k_i^0 are known as kernel of the parameters $k_p\ (t)$ and $k_i\ (t)$.Here, the adaptive perturbations of $\Delta k_p\ (t)$ and $\Delta k_i\ (t)$ make the parameters of $k_p\ (t)$ and $k_i\ (t)$ to move towards the kernel parameters k_p^0 and *ki. For* implicit adaptation of the values of $\Delta k_p\ (t)$ and $\Delta k_i\ (t)$, equation (4.3) can be represented as follows

$$u(t) = k_p^0 \phi_e(t) + k_i^0 \int_0^t \phi_e(\tau)\, d\tau + \Delta k_p(t)\phi_e(t) + \int_0^t \Delta k_i(\tau)\phi_e(\tau)\, d\tau \quad (4.4)$$

$$u(t) = u^0(t) + \Delta u(t) \quad (4.5)$$

where

$$u^0(t) = k_p^0 \phi_e(t) + k_i^0 \int_0^t \phi_e(\tau) d\tau$$
$$\Delta u(t) = \Delta k_p(t)\phi_e(t) + \int_0^t \Delta k_i(\tau)\phi_e(\tau) \quad (4.6)$$

In this proposed adaptive fuzzy PI control, the parameters of the main PI controller are determined by the kernel parameters i.e. k_p^0 and k_i^0 and the parameters of the auxiliary PI controller are defined by the $\Delta k_p\ (t)$ and $\Delta k_i\ (t)$. In this controller, the output of the main PI controller is $u^0\ (t)$ and the output of the secondary PI controller is $\Delta u\ (t)$. The output of the auxiliary PI controller i.e. equation (4.7) equally can be rewritten as

$$\Delta u(t) = \Delta\phi_{ep}(t) + \int_0^t \Delta\phi e_i(\tau)\, d\tau \qquad (4.7)$$

where

$$\begin{aligned} \Delta\phi_{ep}(t) &= \Delta k_p(t)\phi_e(t) \\ \Delta\phi_{ei}(t) &= \Delta k_i(t)\phi_e(t) \end{aligned} \qquad (4.8)$$

The parameters $\Delta k_p\ (t)$ and $\Delta k_i\ (t)$ in eq (4.8) are proportional to the error ϕ_e. It is ensured that whenever some changes occur in the values of $\Delta k_p\ (t)$ and $\Delta k_i\ (t)$ it may lead to change the $\Delta\phi_{ep}(t)$, $\Delta\phi_{ei}(t)$ and vice versa. As a whole, if the linking between the two signals and the error is redirected to be a generalized functions, the change of two the signals imply the adaption of $\Delta k_p\ (t)$ and $\Delta k_i\ (t)$. As defined, the adaption of $k_p\ (t)$ and $k_i\ (t)$, follow the adaption of $\Delta\phi_{ep}(t)$ and $\Delta\phi_{ei}(t)$. Hence, in order to provide the adaptation of signals $\Delta\phi_{ep}(t)$) and $\Delta\phi_{ei}(t)$, a fuzzy logic control is employed.

4.2 Design of Fuzzy Logic control (FLC)

Fig. 4.2 shows the schematic diagram of the fuzzy logic controller. Usually, fuzzy logic control system is a knowledge based or rule-based system. The basic idea behind the fuzzy logic control is that, it integrates the expert experience of a human operator in the design of the controller in controlling a process whose input-output relationship is described by a set of fuzzy control rules (IF-THEN Rules) by involving the linguistic variables. Fuzzifier transforms the crisp values of error and the change in error into corresponding fuzzy values and secondly, from knowledge or rule base, the fuzzy values of error and change in error determine which particular rules are to be fired through a fuzzy inference mechanism. A defuzzification mechanism transforms these fuzzy informed values into one crisp control value.

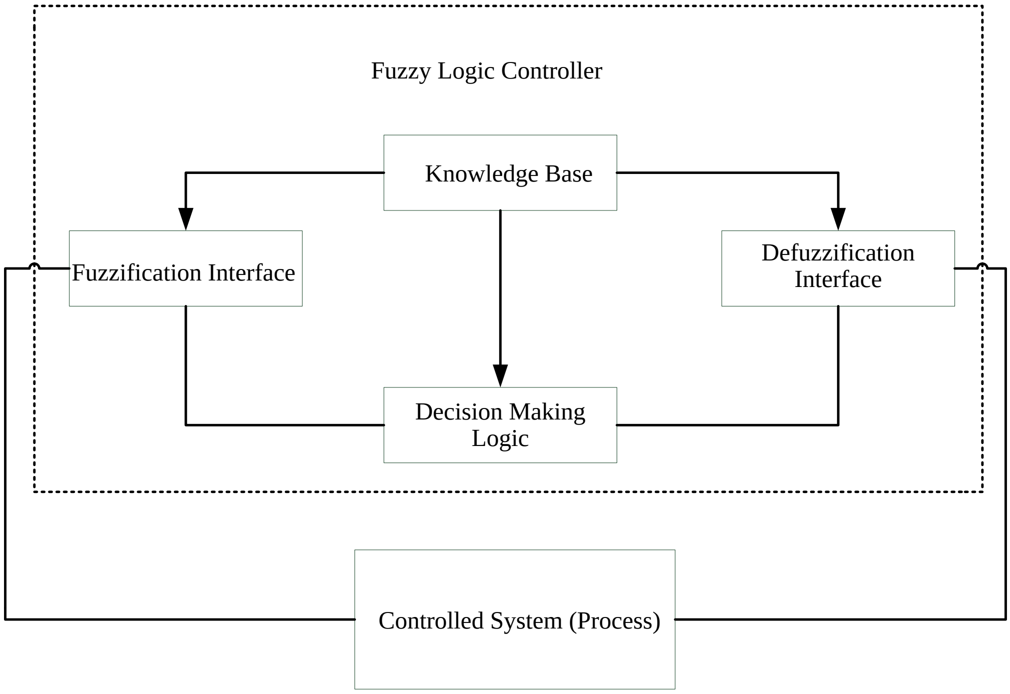

Fig.4.2 Schematic representation of a fuzzy logic control system

A two input two output fuzzy system is formulated by the MATLAB/SIMULINK using fuzzy logic toolbox. The inputs of the FIS are considered as, error [ϕ_e] and change in error [$\frac{d\phi_e}{dt}$].For both the inputs and outputs five membership functions have been defined namely,[NL,NS,ZE,PS and PL].

Table 4.1 Linguistic variables for input and output parameters

NL	Negative Large
NM	Negative Medium
ZE	Zero
PL	Positive Large
PS	Positive Small

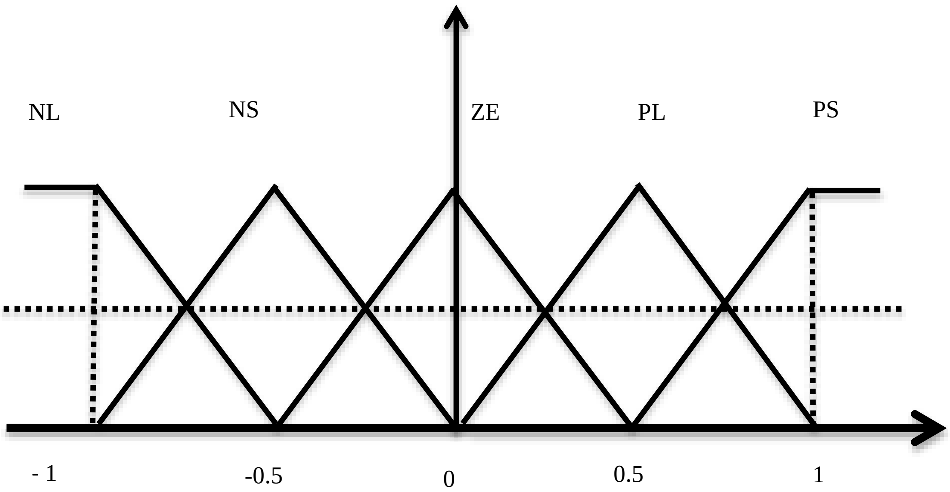

Fig.4.3 Fuzzy membership functions for input

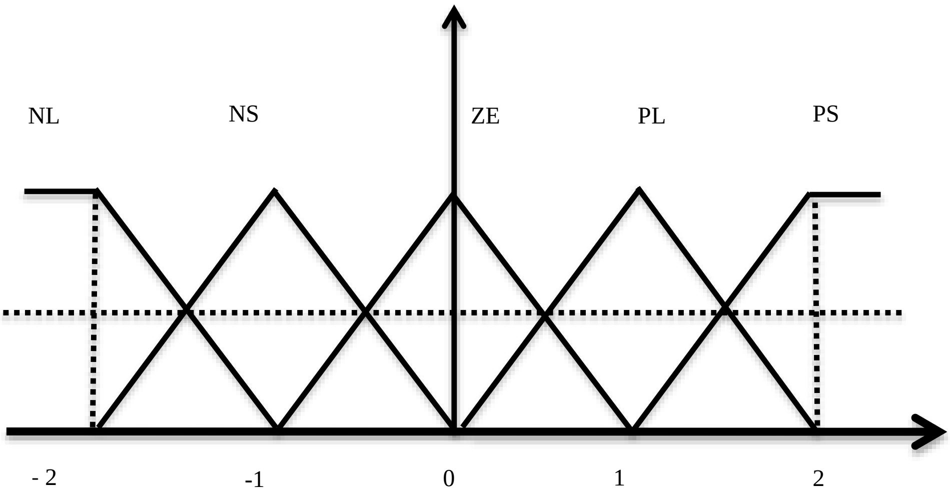

Fig. 4.4 Fuzzy membership functions for output

R[1]: *IF* change in error NL and error NL

THEN output is NL

R[2]: *IF* change in error NL and error NS

THEN output is ZE

R[3]: *IF* change in error NL and error ZE

THEN output is NL

R[4]: *IF* change in error NL and error PL

THEN output is NS

R[5]: *IF* change in error NL and error PS

THEN output is ZE

R^{6}: *IF* change in error NS and error NL
THEN output is NL

R^{7}: *IF* change in error NS and error NS
THEN output is NL

R^{8}: *IF* change in error NS and error ZE
THEN output is NS

R^{9}: *IF* change in error NS and error PL
THEN output is ZE

R^{10}: *IF* change in error NS and error PS
THEN output is PS

R^{11}: *IF* change in error ZE and error NL
THEN output is NL

R^{12}: *IF* change in error ZE and error NS
THEN output is NS

R^{13}: *IF* change in error ZE and error ZE
THEN output is ZE

R^{14}: *IF* change in error ZE and error PL
THEN output is PS

R^{15}: *IF* change in error ZE and error PS
THEN output is PL

R^{16}: *IF* change in error PS and error NL
THEN output is NS

R^{17}: *IF* change in error PS and error NS
THEN output is ZE

R^{18}: *IF* change in error PS and error ZE
THEN output is PS

R^{19}: *IF* change in error PS and error PL
THEN output is PL

R^{20}: *IF* change in error PS and error PS
THEN output is PL

R^{21}: *IF* change in error PL and error NL
THEN output is ZE

R^{22}: *IF* change in error PL and error NS
THEN output is PS

R^{23}: *IF* change in error PL and error ZE

THEN output is PL

R^{24}: *IF* change in error PL and error PL

THEN output is PL

R^{25}: *IF* change in error PL and error PS

THEN output is PL

Description of Fuzzy Rule-Base for the AFPI Algorithm [48]

For developing an adaptive fuzzy PI controller (AFPI) for the CTS in order to maintain liquid level at a desired level, tweenty five rules, along with five membership functions and Mamdani FIS system has been considered from the knowledge base and these are implemented using IF-THEN rules.The above mentioned rule base explains the relationship between input and output fuzzy variables which is defined as membership function.The shape of membership function has been decided based on the trade-off between reduced complexity and superior performance. The sole reason of chosen triangular shape membership function is due to its simple formula and computational efficiency. After using the fuzzy inference system, the output will be a fuzzy variable and it should be converted to crisp value in order to provide the plant. This method of conversion of fuzzy variable to crisp variable is known as the defuzzification mechanism. There are various defuzzification methods are existing such as centroid, bisector, middle of maximum (MOM), smallest of maximum (SOM) and largest of maximum (LOM). For conversion of fuzzy value to crisp value in this chapter, centroid defuzzification mechanism is used.

4.3 Results and Discussions

In this section, simulation results along with experimental results are provided in order to illustrate the efficacy of the proposed AFPI control algorithm. A variable step input is applied to both tanks in order to observe the tacking ability of the proposed AFPI control algorithm. From the simulation results given in Fig. 4.6 and 4.7, it is observed that, for both the tanks are taking less settling time in order to reach steady state for maintain desired liquid level.

Table 4.2 Values of the Kernel parameter

Parameter	K_p^0	K_I^0
Value	4.5 (Tank 1) 1.5 (Tank 2)	0.09(Tank 1) 0.06 (Tank 2)

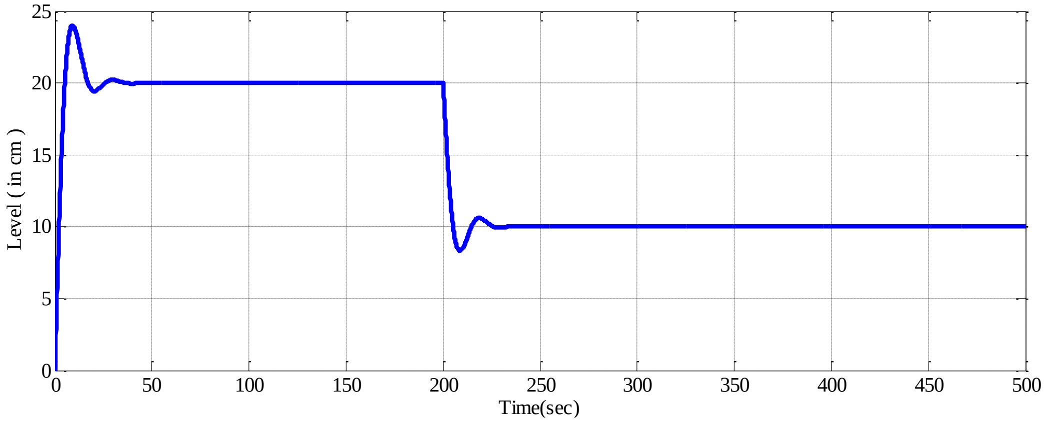

Fig. 4.5 Simulation Response of Adaptive Fuzzy PI (AFPI) for level control in Tank 1

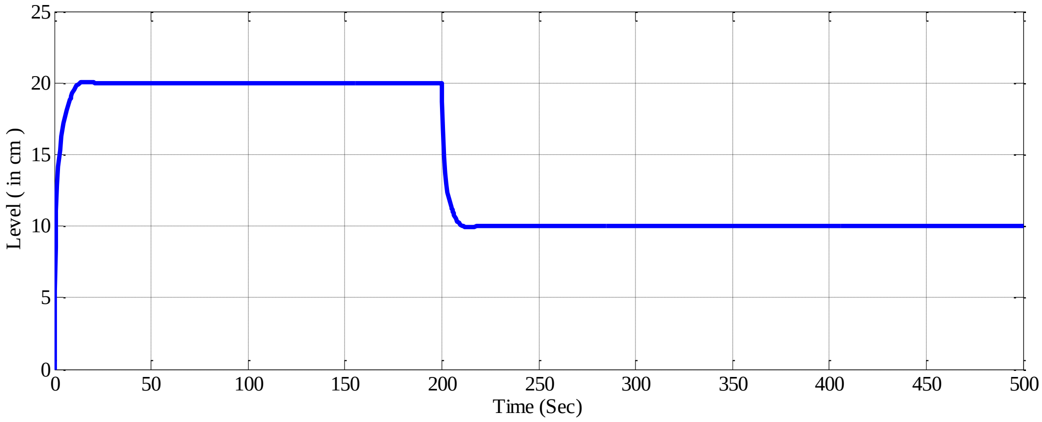

Fig. 4.6 Simulation Response of Adaptive Fuzzy PI (AFPI) for level control in Tank 2

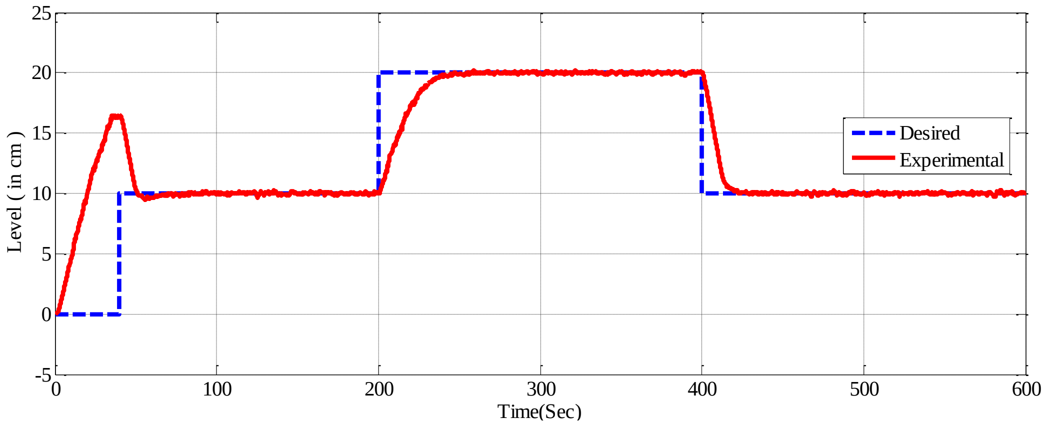

Fig. 4.7 Experimental Response of Adaptive Fuzzy PI (AFPI) for level control in Tank 1

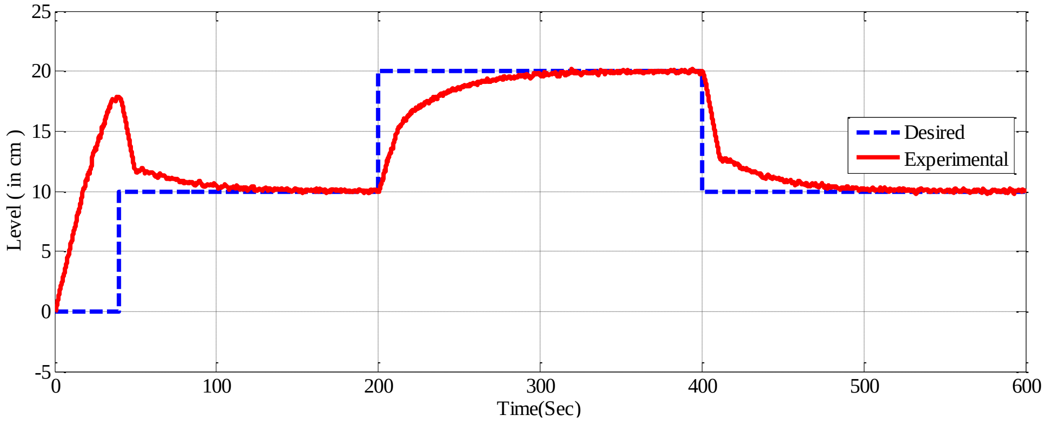

Fig. 4.8 Experimental Response of Adaptive Fuzzy PI (AFPI) for level control in Tank 2

From Fig. 4.5 and Fig. 4.6 it is observed that, during first step (i.e. 0 to 200 sec) in the tank1, initially a little overshoot arises thereafter it requires less time to settle at its desired steady state level and while achieving second desired level (i.e. 200 to 500 sec) it is immediately settled by utilizing very less settling time with less undershoot. On the other hand, it is observed that in tank 2, there is no overshoot occurs while regulating level during both the steps. Fig.4.7 and 4.8 shows the experimental response of both the tanks. The experiment was carried out with a sampling time of 0.1 sec for 600 seconds. From the obtained experimental results it is observed that while achieving desired level tank 2, it takes more time to settle at its desired level as compared to tank 1during each desired step. But on the other hand both the tanks maintain desired level without any steady state error and also without any overshoot.

Table 4.3 Performance assessment of AFPI controller for Tank 1

Performance	**Values**
Peak Overshoot (%)	0
Settling Time (Sec)	20.9
Rise Time (Sec)	4.195
ISE	4.356
IAE	2.087

Table 4.4 Performance assessment of AFPI controller for Tank 2

Performance	Values
Peak Overshoot (%)	0
Settling Time (Sec)	18.09
Rise Time (Sec)	13.98
ISE	13.017
IAE	23.13

4.4 Chapter Summary

In this chapter an adaptive fuzzy PI controller has been proposed for liquid level control in a coupled tank system. It is found, from the simulation as well as experimental results and also from performance assessment table 4.3 and 4.4 of the both tanks that, the presented control algorithm i.e. Adaptive Fuzzy PI (AFPI) controller provides better level control performance as compared to the LMI based PI controller.

Chapter 5

Design and Real-Time Implementation of a Sliding Mode Controller for the Coupled Tank System

As discussed, in chapter 3 and chapter 4 that using a PID controller accurate desired level cannot be maintained in presence of uncertainties in the model dynamics. Hence in order to overcome this drawback a sliding mode control has been developed and implemented for the CTS. The sliding mode control has been known as an effective robust controller [29-32]. The sliding mode control is a powerful approach for controlling nonlinear and uncertain systems in presence of model uncertainties and disturbances. The SMC is preferred due to its robustness against various kinds of uncertainties such as external disturbances and measurement error. In general, SMC consists of two steps namely design of a sliding surface so as to achieve the desired system behaviour, like stability of the origin, when restricted to the surface and the second step is to select suitable gain of the controller so that the closed loop system becomes stable on the sliding surface [45-46].

One of the most intriguing aspects of the sliding mode is the discontinuous nature of the control action whose primary function is to switch between two distinct structures such that a new type of system motion would occur which is called sliding mode that exists in a manifold. A sliding mode exists only if the vicinity of the switching surface of the system states are directed towards the sliding surface. The most crucial task in the sliding mode control design is achieving the precise switching control law which enforces the system towards the sliding surface $s(t)$. Once the state trajectories intersect the sliding surface they remain on it thereafter which is shown in Fig 5.1. Usually a sliding surface is chosen considering error. In this thesis, the sliding surface has been designed by taking the difference between the actual level and the desired level. In reality sampling noises, delays, discretization and hysteresis effects usually give rise to oscillations in the states of the system. In sliding control, the high frequency oscillation effect is commonly known as chattering which needs to be reduced.

This chapter is organized as follows. In section 5.2, the problem statement is given for controlling the liquid level in a coupled tank system. Section 5.3 describes the sliding mode controller algorithm. In order to verify the effectiveness of the proposed control algorithm, in section 5.4, simulation results as well as the experimental results are discussed. Finally the chapter summary is presented in section 5.5.

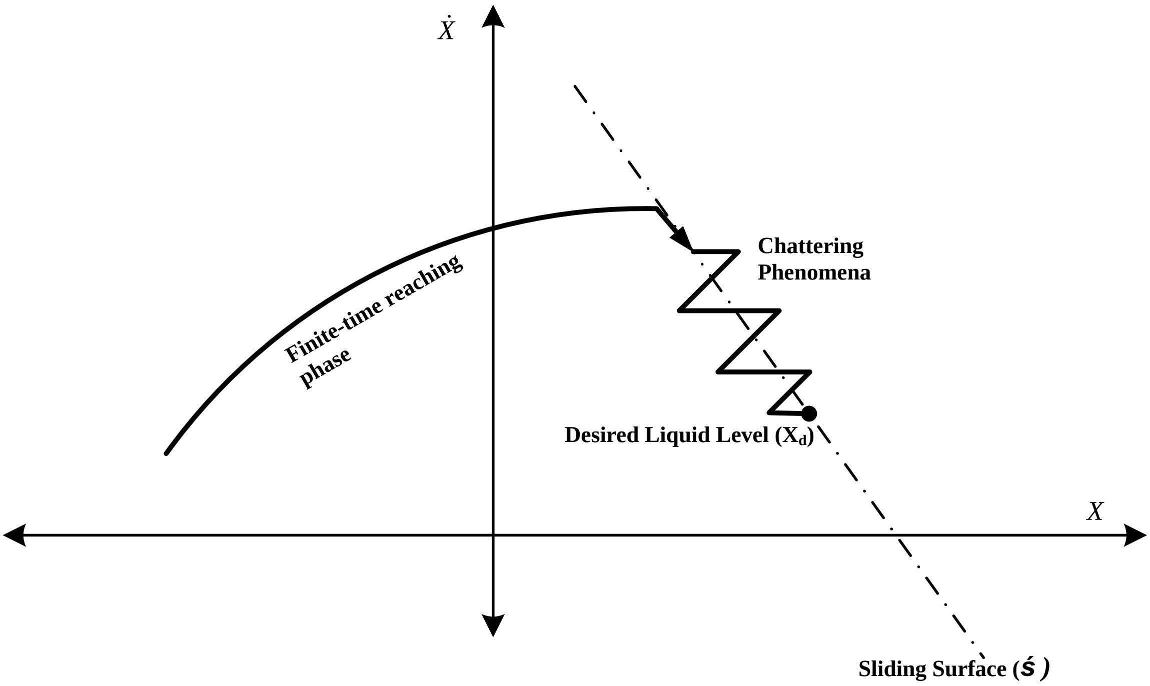

Fig. 5.1 Graphical Representation of the Sliding Surface

5.1 Problem Statement

In this chapter the objective of the controller development is that, to adjust the liquid level of both the tanks to the desired set point level i.e. h_{1d} and h_{2d} irrespective of the load disturbances in the system dynamics. For the control law development, we assumed two constraints i.e. (1) $q \geq 0$, (2) i.e. $h_1 \geq h_2$,where q is the inflow rate and h_1 and h_2 are the desired level of the tank 1 and tank 2 respectively.

5.2 Development of Sliding Mode Control Law

5.2.1 Control law for Tank -1

In a coupled tank system, the liquid flow into the tank cannot be negative, so the constraint on the inflow rate is given as follows

$$q \geq 0 \tag{5.1}$$

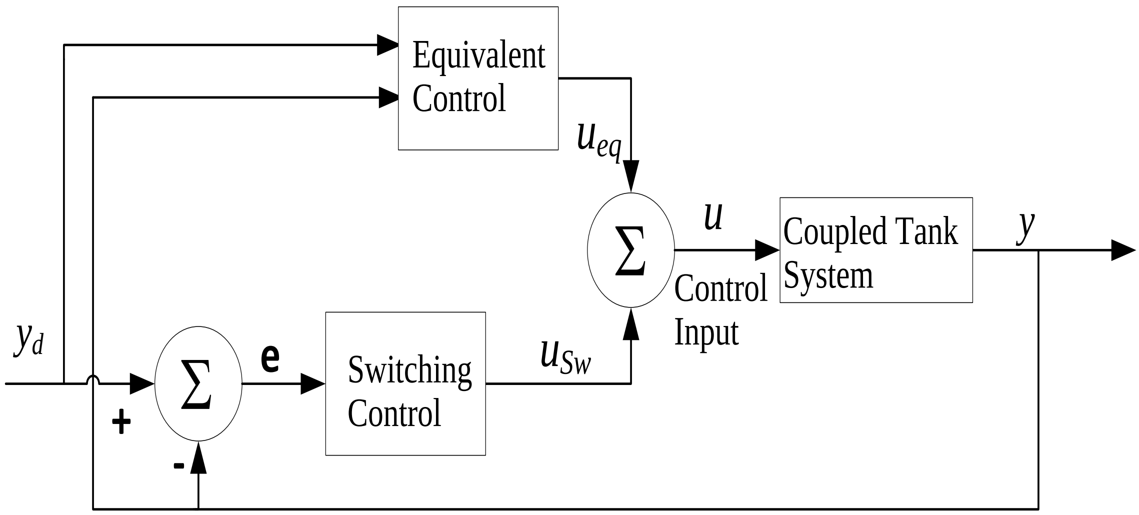

Fig 5.2 Schematic structure of sliding mode controller for level control in coupled tank system

Considering the dynamics of the coupled tank system is given in chapter 2, can obtain SMC control action as follows where u is the flow rate. In order to satisfy the constraints in eq (5.1) on the flow rate, following inequality must be satisfied

$$h_1 \geq h_2 \tag{5.2}$$

Thus, by considering eq. (5.2) and defining as follows

$$h_1 = z_1, h_2 = z_2$$

$$\frac{a_1\sqrt{2g}}{A} = k_1, \frac{a_2\sqrt{2g}}{A} = k_2 \tag{5.3}$$

$$k_1 = k_2 = k$$

The dynamic model of the coupled tank system as given in eq. (2.1-2.2) can be rewritten as follows

$$\begin{aligned} \dot{z}_1 &= -k\sqrt{z_1} + \eta u \\ \dot{z}_2 &= k\sqrt{z_1} - k\sqrt{z_2} \\ y &= z_1 \end{aligned} \tag{5.4}$$

The model of the coupled tank system dynamics is nonlinear. Therefore a transformation is defined so that the dynamics of coupled tank system in eq. (2.1-2.2) can be transformed into a seemly form for the control design.

Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and the transformation *x=T(z)*; such that

$$\begin{aligned} x_1 &= z_1 \\ x_2 &= \dot{z}_1 = -k_1\sqrt{z_1} + \eta u \end{aligned} \tag{5.5}$$

Now the dynamics of the coupled tank system can be rewritten is as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{-k}{2\sqrt{z_1}}.\dot{z}_1 \\ \dot{x}_2 &= \frac{-k}{2\sqrt{z_1}}(-k_1\sqrt{z_1} + \eta u) \end{aligned} \tag{5.6}$$

Hence the dynamics of the coupled tank system can be written in compact form as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f + \phi u \end{aligned} \tag{5.7}$$

where

$$\begin{aligned} f &= \frac{k^2}{2} \\ \phi &= \frac{k}{2\sqrt{z_1}}\eta \end{aligned} \tag{5.8}$$

Define a sliding surface s (t) as

$$s = \left(\frac{d}{dt} + \alpha_s\right)^{n-1} \tilde{\lambda} \tag{5.9}$$

where n is the order of the system to be controlled, α_s is a positive constant and λ is the error

$$\tilde{\lambda} = x_1 - H \tag{5.10}$$

$$s = \dot{x}_1 + \alpha_s(x_1 - H) \tag{5.11}$$

On differentiating both sides of equation (5.11) one gets

$$\dot{s} = \ddot{x}_1 + \alpha_s(\dot{x}_1) \tag{5.12}$$

$$\dot{s} = \dot{x}_2 + \alpha_s(x_2) \tag{5.13}$$

$$\dot{s} = \frac{k^2}{2} - \frac{k}{2\sqrt{z_1}}\eta u - \alpha_s k\sqrt{z_1} + \alpha_z \eta u \tag{5.14}$$

On solving the equation (5.14) yields the following control law

$$u = \frac{w_s \operatorname{sgn}(s) - \frac{k^2}{2} + \alpha_s k\sqrt{z_1}}{(\alpha_s - \frac{k}{2\sqrt{z_1}})\eta} \tag{5.15}$$

5.2.2 Control law for Tank -2

In the same manner the control law has been obtained for the level control in tank 2 considering the dynamics of the coupled tank system equation (2.1-2.2), which yields

$$h_1 = z_2, h_2 = z_1$$

$$\frac{a_1\sqrt{2g}}{A} = k_1, \frac{a_2\sqrt{2g}}{A} = k_2 \tag{5.16}$$

$$k_1 = k_2 = k$$

The dynamic model of the coupled tank system eq. (2.1-2.2) can be rewritten as follows

$$\begin{aligned} \dot{z}_2 &= -k\sqrt{z_1} + \eta u \\ \dot{z}_1 &= k\sqrt{z_2} - k\sqrt{z_1} \\ y &= z_1 \end{aligned} \tag{5.17}$$

Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and define the transformation *x=T (z)*, which yields

$$\begin{aligned} x_1 &= z_1 \\ x_2 &= \dot{z}_1 = k\sqrt{z_2} - k\sqrt{z_1} \end{aligned} \tag{5.18}$$

Now the dynamics coupled tank system can be rewritten by considering equation (5.18) as follows,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{-k}{2\sqrt{z_2}}.\dot{z}_2 - \frac{-k}{2\sqrt{z_1}}.\dot{z}_1 \qquad (5.19)$$
$$\dot{x}_2 = -\frac{k^2}{2}\sqrt{\frac{z_2}{z_1}} + \frac{k\eta}{2\sqrt{z_2}}u$$

The dynamic model of the for level control in tank 2 can be written in compact form as follows

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f + \phi u \qquad (5.20)$$

where

$$f = \frac{k^2}{2}\sqrt{\frac{z_2}{z_1}}$$
$$\phi = \frac{k\eta}{2\sqrt{z_2}} \qquad (5.21)$$

Define a sliding surface s (t) as

$$s = \left(\frac{d}{dt} + \alpha_s\right)^{n-1}\tilde{\lambda} \qquad (5.22)$$

where n is the order of the system to be controlled α_s is a positive constant

$$\tilde{\lambda} = x_1 - H \qquad (5.23)$$

$$s = \dot{x}_1 + \alpha_s(x_1 - H) \qquad (5.24)$$

By taking the derivatives of equation (5.24) one obtains

$$\dot{s} = \ddot{x}_1 + \alpha_s(\dot{x}_1) \qquad (5.25)$$

$$\dot{s} = \dot{x}_2 + \alpha_s(x_2) \qquad (5.26)$$

$$\dot{s} = \frac{k\eta}{2\sqrt{z_2}}u - \frac{k^2}{2}\sqrt{\frac{z_2}{z_1}} + \alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1}\right) \qquad (5.27)$$

Solving equation (5.27), yields the following control law

$$u=\frac{2\sqrt{z_2}}{k\eta}\left[\frac{k^2}{2}\frac{\sqrt{z_1}}{\sqrt{z_2}}-\alpha_s\left(k\sqrt{z_2}-k\sqrt{z_1}\right)-w_s\,\mathrm{sgn}\left(\dot{x}_1+\alpha_s\left(x_1-H\right)\right)\right] \tag{5.28}$$

Using the controller law given in equation. (5.15) and (5.28) in to equation (5.14) and (5.27), it follows that

$$\dot{s}=-w_s\,\mathrm{sgn(s)} \tag{5.29}$$

The state trajectories associated with this unforced discontinuous dynamics i.e. eq (5.29) exhibit a finite time reachability to zero from any value of initial condition subject to the value of w_S must be positive. Since system driven its states to zero in finite time, the desired level $y=z_1=h_2$ and $y=z_1=h_1$ in both tanks are regulated after a finite time.by first order dynamics $\dot{y}+\alpha_s(y-H)=0$. Hence, the output of both the tank will asymptotically converge to its desired value since α_S is positive.

5.3 Results and Discussions

The control law derived in the section 5.2, was simulated using MATLAB/SIMULINK. Fig. 5.3 and Fig 5.4 shows the simulation response of both tanks. It can be seen from Fig 5.3 and 5.4, it requires around 10 sec (Tank 1) and 45 sec (Tank 2) to converge the output to its desired value H of both tanks. The response of chattering effect of both tanks are represented in Fig 5.5 and 5.6. In this chapter a signum function has been considered in order to minimize the chattering effect of sliding mode control (SMC).

Table 5.1 Parameters of the Sliding Mode Controller

Symbol	**Value**
K	0.050
η	0.1194
w_s	10(Tank 1),0.1(Tank 2)
α_1	0.5(Tank 1),
α_2	0.8(Tank 2)

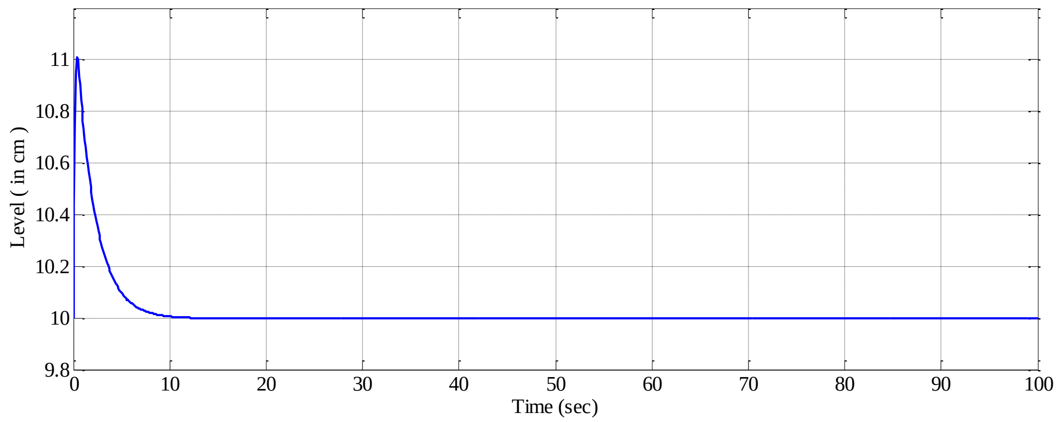

Fig. 5.3 Simulation Response of Sliding Mode Control while level control in Tank 1

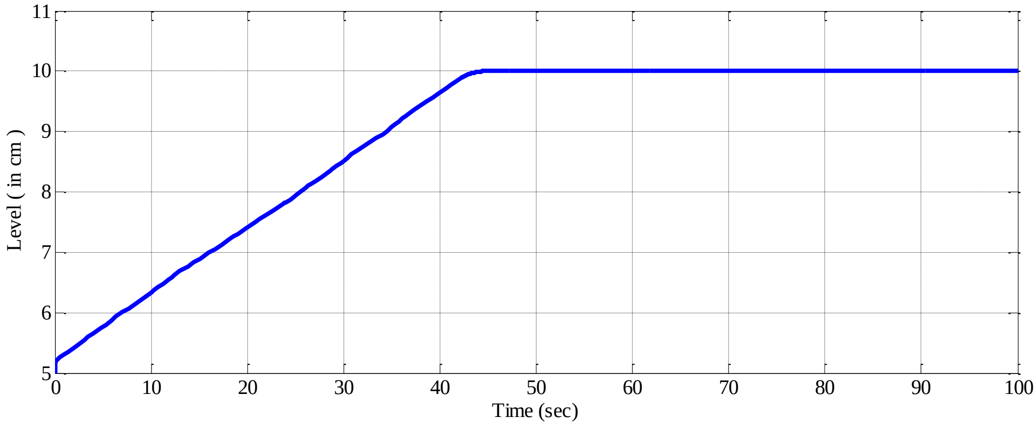

Fig. 5.4 Simulation Response of Sliding Mode Control while level control in Tank 2

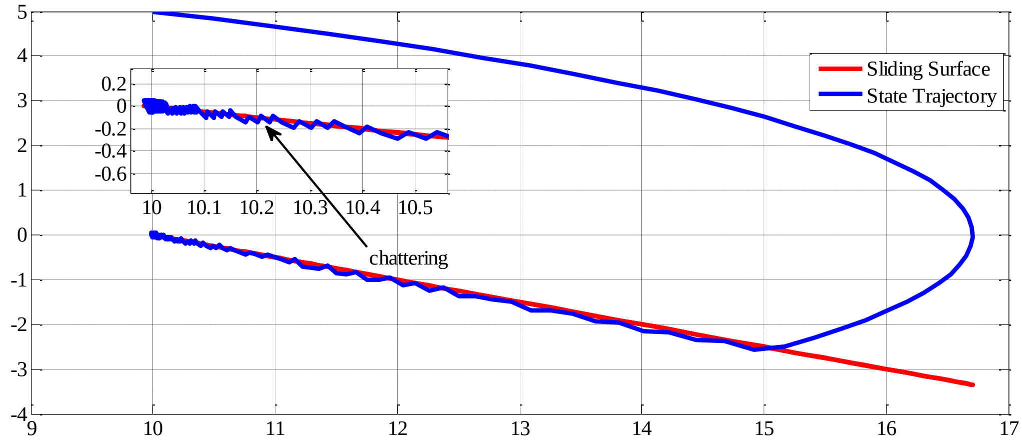

Fig. 5.5 Response of sliding surface while level control in tank 1

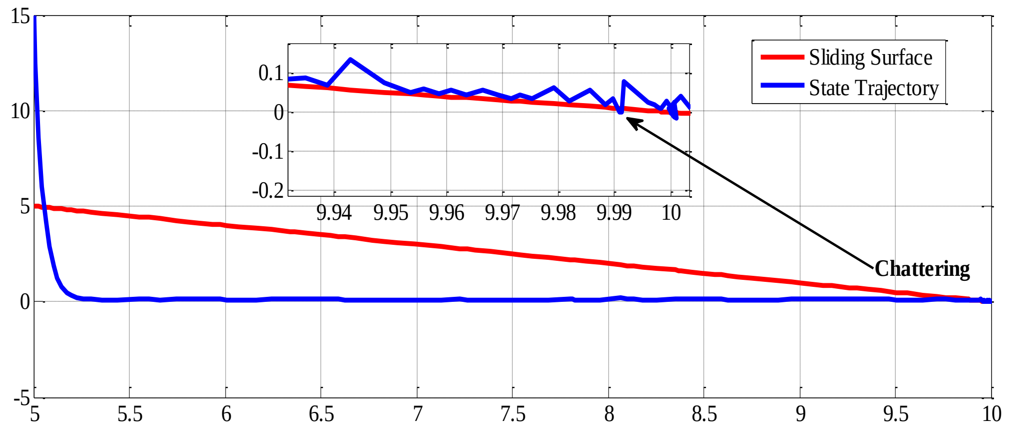

Fig. 5.6 Response of sliding surface while level control in tank 2

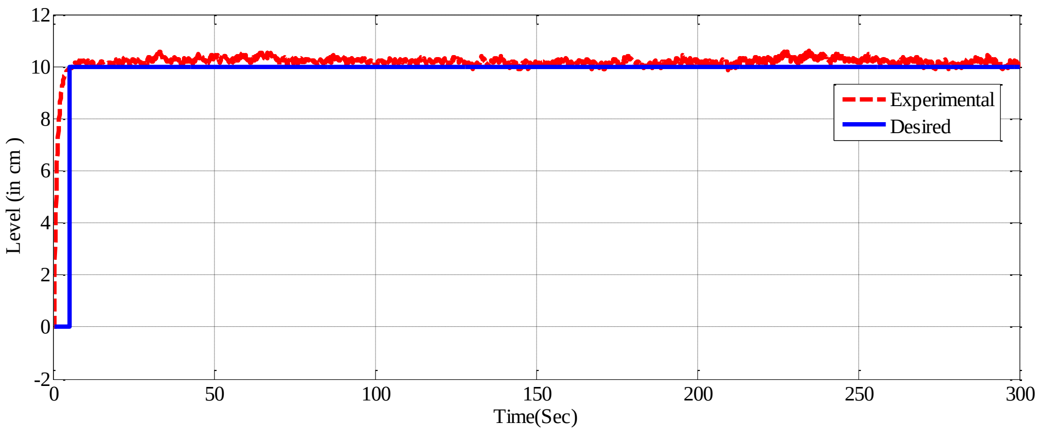

Fig. 5.7 Experimental Response of Sliding Mode Control while level control in Tank 1

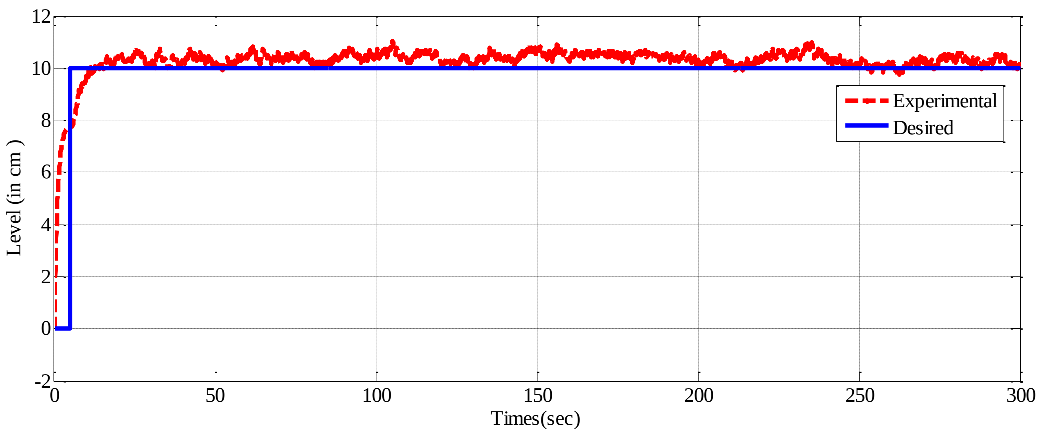

Fig. 5.8 Experimental Response of Sliding Mode Control while level control in Tank 2

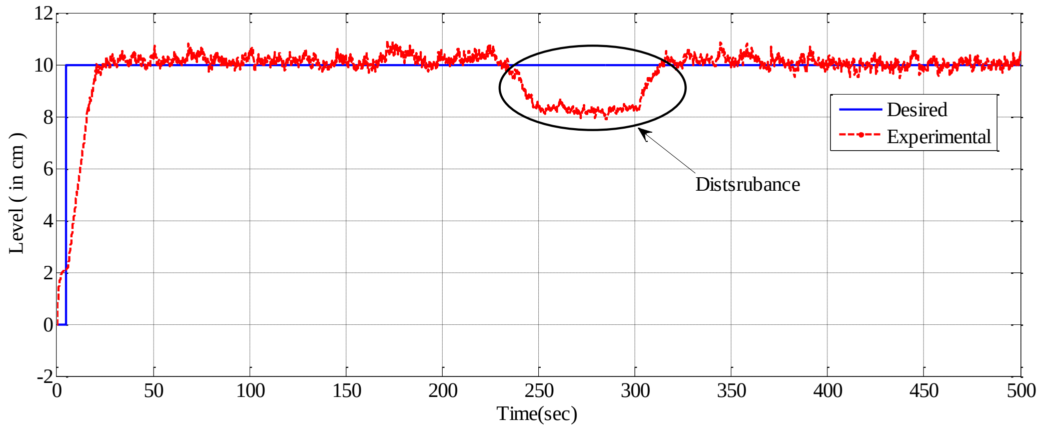

Fig. 5.9 Experimental Response of Sliding Mode Control under disturbance rejection mode while level control in Tank 1

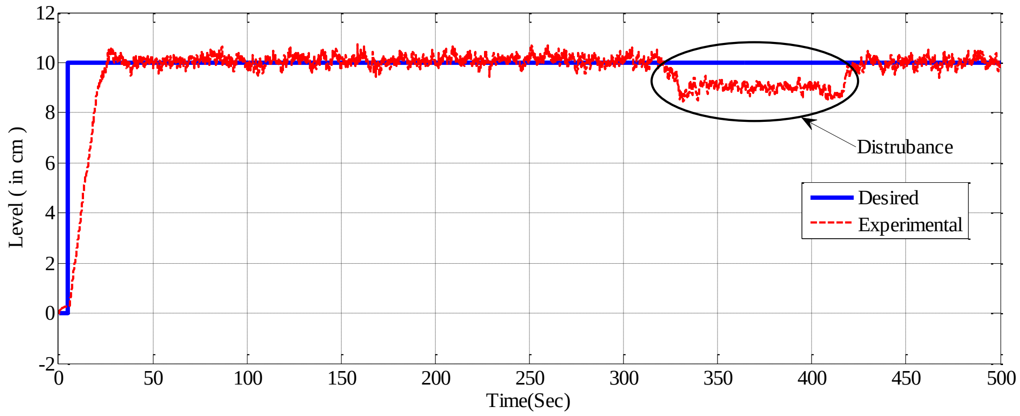

Fig. 5.10 Experimental Response of Sliding Mode Control under disturbance rejection mode while level control in Tank 2

In order to observe the effectiveness of sliding mode controller (SMC), the controller algorithm is verified in the real-time and also tested the disturbance rejection capability against uncertainties. The experiment is performed for 500 sec by taking sampling time 0.1 sec. Fig 5.7 and 5.8 exhibits the real time experimental results while SMC algorithm is applied for level control in the both tank 1 and tank 2. It has been seen from both Fig 5.7 and 5.8 that, less time requires in order to converge the output to its desired level H smoothly with small steady state error. Fig 5.9 and 5.10 represents the disturbance rejection capability of the SMC algorithm. The disturbances were applied during the steady state, where load is added into system by suddenly opening a valve. In both the tanks disturbances were applied for 80 sec in tank 1 and 100 sec in tank2. It clearly identified that, from the obtained Fig.5.9 and 5.10, the SMC delivers good control action because when the load disturbances are removed it has been seen that the output requires less time in order to converge to its desired liquid level.

Table 5.2 Performance assessment of the Sliding Mode Control (SMC) controller

Specification	**Simulation**	**Experimental**
Settling Time (Sec)	12.8(Tank1) 44.74(Tank2)	15.3(Tank1) 29.4(Tank 2)
Peak overshoot (%)	9.09(Tank1) 0(Tank2)	0 (For both Tanks)

5.4 Chapter Summary

In this chapter level control for the both tank of a coupled tank system has been carried out by employing the sliding mode control. Here the effect of partial opening of the valve while level maintaining in coupled tank is considered as a load disturbance. Generally, sliding mode control technique suffers from charting problem. In this chapter a signum function has been considered in order to minimize the chattering effect of sliding mode control (SMC). Simulation as well as experimental studies is carried out with taking model disturbance in order to validate the SMC algorithm.

Chapter 6

Development of an Adaptive Fuzzy Sliding Mode Controller Design for the Coupled tank System

In chapter 5, a sliding mode control algorithm is discussed which is generally suffers from the chattering problem. The chattering problem possesses a serious concern such as possibility of damage of actuators. In order to alleviate the chattering problem, in this chapter an Adaptive Fuzzy Sliding Mode Control (AFSMC) is proposed. In this presented algorithm, the sliding surface design involves a fuzzy variable for reducing the chattering. It is well known that an adaptive fuzzy control is robust in face of parametric uncertainties and disturbances [47]. Both adaptive fuzzy control and conventional adaptive control have some similarities as well as dissimilarities. Usually, they are similar in their basic configurations and principles are more or less the same and the mathematical tools used in the analysis and designs are similar. The differences are the fuzzy control has a special nonlinear structure that is universal for different plants, whereas the structure of a conventional adaptive controller changes from plant to plant and human knowledge about the plant dynamics and control strategies can be incorporated into adaptive fuzzy controllers, whereas such knowledge is not considered in conventional adaptive control system.

6.1 Chapter Objectives

The objectives of this chapter are as follows

- Development of an adaptive fuzzy sliding mode control algorithm in order to maintain the liquid level at a certain desired level.
- To introduce a fuzzy logic control in sliding surface design for alleviating chattering

6.2 Development of an Adaptive Fuzzy Sliding Mode Controller

This section presents the design of an Adaptive Fuzzy Sliding Mode Control (AFSMC) using the Lyapunov stability criteria together with a sliding condition. The adaptive fuzzy sliding mode control has several advantages over the conventional sliding mode control. Such as, it ensures zero steady state error with good set point tracking against parameter uncertainties and disturbance and it has robustness property.

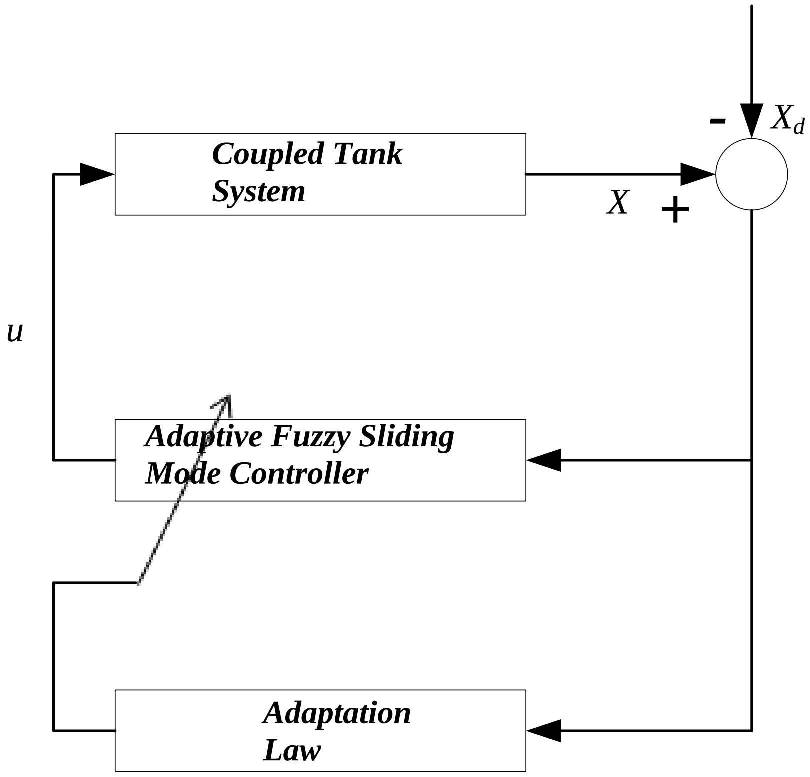

Fig.6.1 Schematic Control Structure of the Adaptive Fuzzy Sliding Mode Controller

Fig. 6.1 depicts the schematic control structure of the adaptive fuzzy sliding mode control, where controller adapts its parameters (*k*) by utilizing the adaption law Eq (6.12) and finally resultant control input is sent to the coupled tank system as its input control signal. The control law is derived for both the tank by using equation (5.6) and (5.16) given in chapter 5 which describes the dynamics of coupled tank system.

6.2.1 Development of Control law for Tank -1

By considering equation (5.6) in chapter 5, one can rewrite the model for tank1 in compact form as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f + \phi u \end{aligned} \tag{6.1}$$

Let

$$\begin{aligned} \theta &= f \\ \hat{\theta} &= \hat{f} \end{aligned} \tag{6.2}$$

Substituting θ, by replacing f in eq. (6.1) one can rewrite

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \hat{f} + bu \end{aligned} \tag{6.3}$$

Considering control law given in equation (5.15) in chapter 5 and equation (6.2) we have the final sliding control law u for the level control in tank 1 as follows

$$u = \frac{w_s \operatorname{sgn}(s) - \hat{\theta} + \alpha_s k \sqrt{z_1}}{\left(\alpha_s - \dfrac{k}{2\sqrt{z_1}}\right)\eta} \tag{6.4}$$

Consider a Lyapunov candidate function as

$$v(t) = \frac{1}{2}s^2 + \frac{1}{2}\gamma(\theta - \hat{\theta})^2 \tag{6.5}$$

where $\gamma \in R$ denotes a positive adaptive gain constant

By taking the time derivative of equation (6.5), yields

$$\dot{v}(t) = s\dot{s} + \gamma\tilde{\theta}\dot{\hat{\theta}} \tag{6.6}$$

where $\tilde{\theta} = \theta - \hat{\theta}$

By substituting the value of $\dot{s}$ from equation (5.14) in chapter 5, one obtains

$$\dot{v}(t) = s(\dot{x}_2 + \alpha_s x_2) - \gamma\tilde{\theta}\dot{\hat{\theta}} \tag{6.7}$$

$$\dot{v}(t) = s\left(\theta - bu + \alpha_s x_2\right) - \gamma\tilde{\theta}\dot{\hat{\theta}} \tag{6.8}$$

Using equation (6.4) in equation (6.8) one can get

$$\dot{v} = s(\theta + \frac{k}{2\sqrt{z_1}}\eta\left\{\frac{w_s\,\mathrm{sgn}(s) - \hat{\theta} + \alpha_s k\sqrt{z_1}}{\left(\alpha_s - \frac{k}{2\sqrt{z_1}}\right)\eta}\right\} + \alpha_s x_2) - \gamma\tilde{\theta}\dot{\hat{\theta}} \tag{6.9}$$

$$\dot{v} = \left[s\theta - \frac{sk\eta w_s\,\mathrm{sgn}(s)}{2\sqrt{z_1}\left(\alpha_s - \frac{k}{2\sqrt{z_1}}\right)\eta} - \frac{sk\eta\hat{\theta}}{2\sqrt{z_1}\left(\alpha_s - \frac{k}{2\sqrt{z_2}}\right)\eta} - \frac{s\alpha_s x_2 k\eta}{2\sqrt{z_1}\left(\alpha_s - \frac{k}{2\sqrt{z_2}}\right)\eta} - \gamma\tilde{\theta}\dot{\hat{\theta}} \right] \tag{6.10}$$

After eliminating the common factor from equation (6.10) yields

$$\dot{v} = s\left(\theta - \hat{\theta}\right) - \gamma\tilde{\theta}\dot{\hat{\theta}} - \frac{skw_s\,\mathrm{sgn}(s)}{2\sqrt{z_1}\left(\alpha_s - \frac{k}{2\sqrt{z_1}}\right)} - \frac{s\alpha_s x_2 k}{2\sqrt{z_1}\left(\alpha_s - \frac{k}{2\sqrt{z_1}}\right)} \tag{6.11}$$

Solving equation (6.11), yields the adaption law for $\hat{\theta}$ and w_s as follows

$$\dot{\hat{\theta}} = \frac{s}{\gamma} \tag{6.12}$$

$$w_s = \frac{\alpha_s x_2}{\mathrm{sgn}(s)} \tag{6.13}$$

The closed loop system can be proved to be stable if

$$\dot{v} \le 0 \tag{6.14}$$

$$\dot{v} = -sk\left[\frac{\left(\frac{\alpha_s x_2}{\mathrm{sgn}(s)}\mathrm{sgn}(s)\right)}{2\sqrt{z_1}\left(\alpha_s - \frac{k}{2\sqrt{z_1}}\right)}\right] - \frac{s\alpha_s x_2 k}{2\sqrt{z_1}\left(\alpha_s - \frac{k}{2\sqrt{z_1}}\right)} \tag{6.15}$$

$$\dot{v} = -sk\left(\frac{\alpha_s x_2}{\mathrm{sgn}(s)}\right)\mathrm{sgn}(s) - s\alpha_s x_2 k \tag{6.16}$$

$$\dot{v} = -\left[sk\,\mathrm{sgn}(s) + 2s\alpha_s x_2 k\right] \tag{6.17}$$

6.2.2 Development of Control law for Tank -2

By considering equation (5.19) in chapter 5, one can rewrite the model for tank 2 in compact form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f + \phi u \end{aligned} \tag{6.18}$$

Let

$$\begin{aligned} \theta &= f \\ \hat{\theta} &= \hat{f} \end{aligned} \tag{6.19}$$

Replacing *f* by substituting θ, eq. (6.18) can be altered as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \hat{f} + bu \end{aligned} \tag{6.20}$$

Considering control law given in equation (5.28) in chapter 5 and the equation (6.19), we have the final sliding control law u for the level control in tank 2 as follows

$$u = \frac{2\sqrt{z_2}}{k\eta}\left[-\hat{\theta} - \alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1}\right) - w_s \operatorname{sgn}\left(\dot{x}_1 + \alpha_s\left(x_1 - H\right)\right)\right] \tag{6.21}$$

Consider a Lyapunov candidate function as

$$v(t) = \frac{1}{2}s^2 + \frac{1}{2}\gamma(\theta - \hat{\theta})^2 \tag{6.22}$$

where

$\gamma \in R$ denotes a positive adaptive gain constant

By taking the time derivative of above equation (6.22), yields

$$\dot{v}(t) = s\dot{s} + \gamma\tilde{\theta}\dot{\hat{\theta}} \tag{6.23}$$

where

$\tilde{\theta} = \theta - \hat{\theta}$

By replacing the value of $\dot{s}$ from equation (5.26) in chapter 5, one can obtain

$$\dot{v}(t) = s(\dot{x}_2 + \alpha_s x_2) - \gamma\tilde{\theta}\dot{\hat{\theta}} \tag{6.24}$$

$$\dot{v}(t) = s\left(\theta + bu + \alpha_s x_2\right) - \gamma\tilde{\theta}\dot{\hat{\theta}} \tag{6.25}$$

Using equation (6.4) in equation (6.8) one can obtain

$$\dot{v} = s\left(\theta + \frac{2\sqrt{z_2}}{k\eta}\left[-\hat{\theta} - \alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1} \right) - w_s \,\mathrm{sgn(s)} \right] - \gamma\tilde{\theta}\dot{\hat{\theta}} \right) \tag{6.26}$$

$$\dot{v} = \left[s\theta - \frac{sk\eta 2\sqrt{z_2}\hat{\theta}}{2\sqrt{z_2}k\eta} - \frac{s2\sqrt{z_2}\alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1} \right)k\eta}{k\eta 2\sqrt{z_2}} - \frac{s\,2\sqrt{z_2}w_s \,\mathrm{sgn}(s)k\eta}{k\eta 2\sqrt{z_2}} - \gamma\tilde{\theta}\dot{\hat{\theta}} \right] \tag{6.27}$$

Eliminating the common factor of equation (6.27), yields

$$\dot{v} = s\tilde{\theta} - \gamma\tilde{\theta}\dot{\hat{\theta}} - s\alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1} \right) - sw_s \,\mathrm{sgn}(s) \tag{6.28}$$

Solving the above equation (6.28), which yield the adaption law for $\hat{\theta}$ and w_s as follows

$$\dot{\hat{\theta}} = \frac{s}{\gamma} \tag{6.29}$$

$$w_s = \frac{\alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1} \right)}{\mathrm{sgn}(s)} \tag{6.30}$$

The closed loop system can be proved to be stable if

$$\dot{v} \le 0 \tag{6.31}$$

$$\dot{v} = -\left[s\alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1} \right) + sw_s \,\mathrm{sgn}(s) \right] \tag{6.32}$$

$$\dot{v} = -\left[s\alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1} \right) + s.\frac{\alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1} \right)}{\mathrm{sgn}(s)}.\mathrm{sgn}(s) \right] \tag{6.33}$$

$$\dot{v} = -2s\alpha_s\left(k\sqrt{z_2} - k\sqrt{z_1} \right) \tag{6.34}$$

From equation (6.16) and (6.32), it is ensured that the derived control law for maintaining liquid level in tank 1 (eq. 6.4) and tank 2(eq. 6.21) guarantee the asymptotic stability of the closed loop system. The control law in equation 6.4 and 6.21 exhibits little oscillations, which is commonly well known as chattering which is undesirable. Because it can excite high frequency in dynamics of the system which owing to possibility of actuators in the system. So in order to decrease the chattering effect in this chapter we have considered a fuzzy control term in the sliding surface.

6.3 Design of Fuzzy Logic Control

For developing a fuzzy logic controller (FLC), error and change in error are considered as its input. Here, the difference between the actual water level and the desired water level is treated as error which is shown in Fig.6.2.Basically the fuzzy controller is a logical system, which is closer to human thinking and natural language than the traditional logic system [48-50]. In general, the fuzzy sliding controller design starts from extending the crisp sliding surface to fuzzy sliding surface defined by suitable linguistic expression. Here in order to carry out design seven membership function has been chosen based on the trade-off between reduced complexity and better performance.

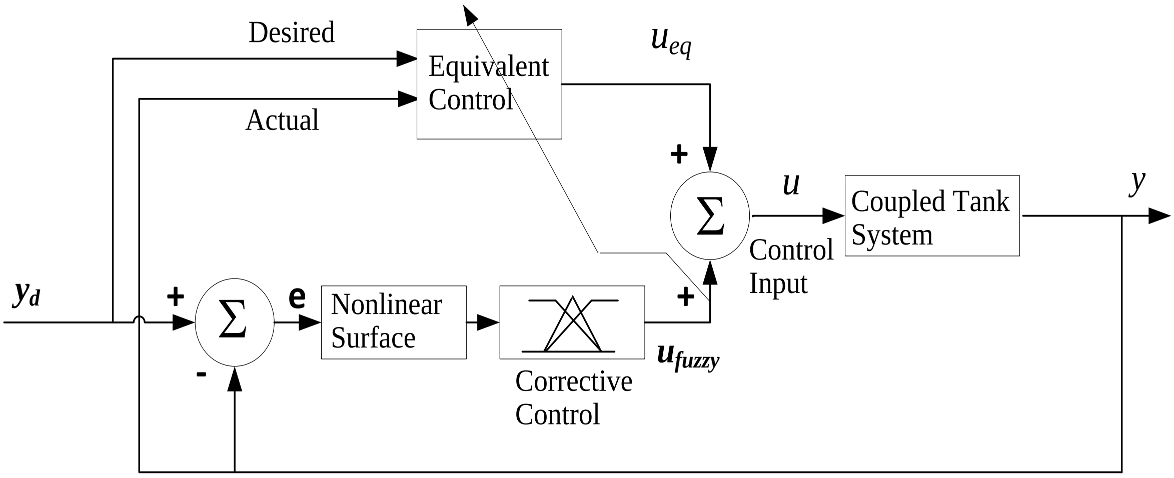

Fig. 6.2 Block diagram of adaptive fuzzy sliding mode control

Linguistic Variables

Commonly, if a variable can take words in natural languages as its value, referred as linguistic variable. There are seven I/O parameter used to develop the FLC, the linguistic used to describe the states of these parameters are defined in Table.6.1.

Membership Function

For defining the fuzzy variables, here the triangular membership functions are chosen. For each input and output variables the states are represented as linguistic variables and each variable is associated with the triangular function.

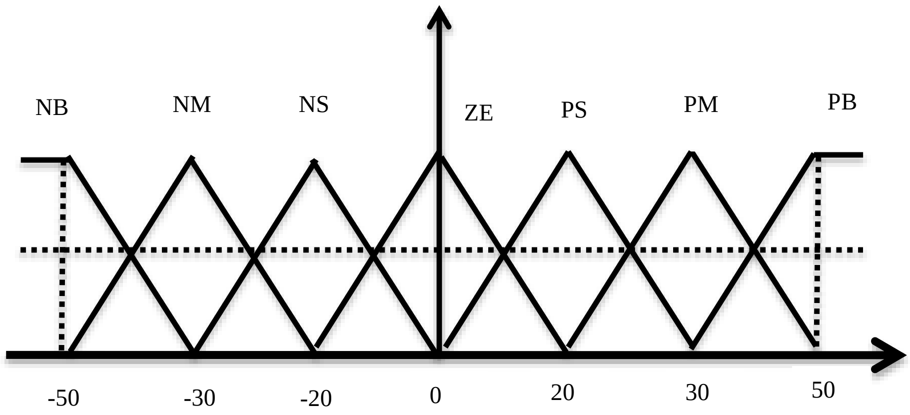

Fig..6.3 fuzzy membership function for error (Input 1)

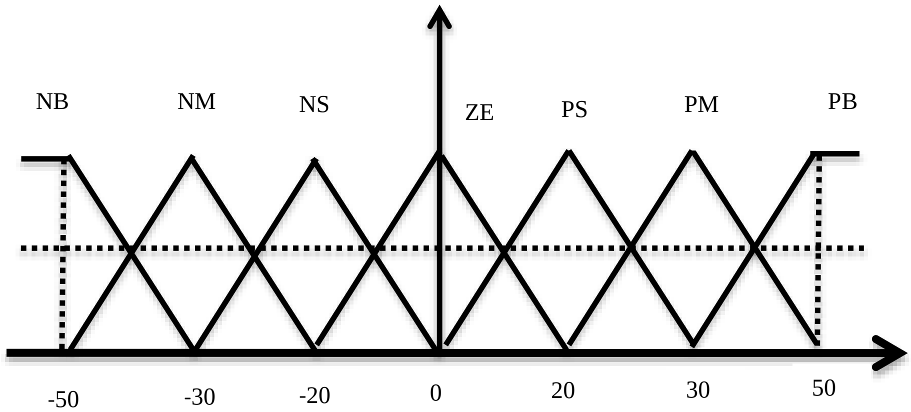

Fig.6.4 fuzzy membership function for change in error (Input 2)

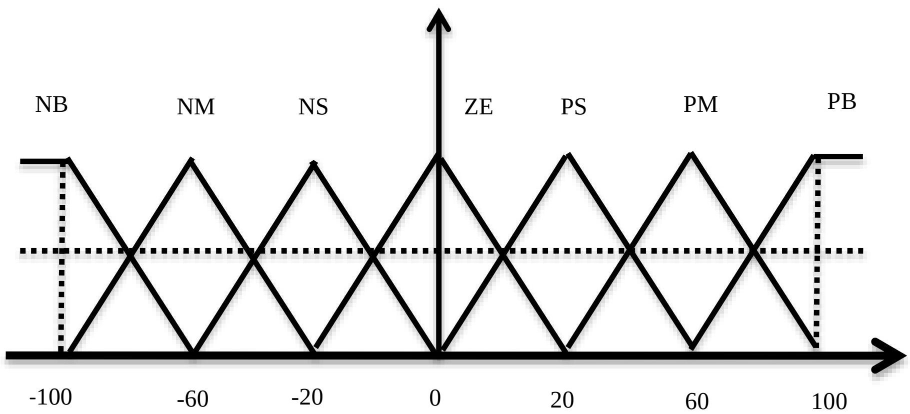

Fig.6.5 fuzzy membership function for output

Table 6.1 Linguistic variables for input and output parameters

NB	Negative Large
NM	Negative Medium
NS	Negative Small
ZE	Zero
PB	Positive Large
PM	Positive Medium
PS	Positive Small

v_e / $\Delta v_e/dt$	**NB**	**NM**	**NS**	**ZE**	**PS**	**PM**	**PB**
PB	ZE	NS	NS	NM	NM	NB	NB
PM	PS	ZE	NS	NS	NM	NM	NB
PS	PS	PS	ZE	NS	NS	Nm	NM
ZE	PM	PS	PS	ZE	NS	NS	NS
NS	PB	PM	PM	PS	PS	ZE	NS
NM	PB	PB	PM	PM	PS	PS	ZE

Description of fuzzy rule base while level control in tank 1 [49]

v_e / $\Delta v_e/dt$	**NB**	**NM**	**NS**	**ZE**	**PS**	**PM**	**PB**
PB	ZE	NS	NS	NM	NM	NB	NB
PM	PS	ZE	NS	NS	NM	NM	NB
PS	PS	PS	ZE	NS	NS	Nm	NM
ZE	PM	PS	PS	ZE	NS	NS	NS
NS	PB	PM	PM	PS	PS	ZE	NS
NM	PB	PB	PM	PM	PS	PS	ZE

Description of fuzzy rule base while level control in tank 2 [49]

For developeing the fuzzy controller for CTS for maintaing desired liquid level at particular level, two rule base has been designed for both the tank. These rule base are the knowledge base and these are implemented using IF-THEN rules. The above mentioned rule base which explains the relationship between input and output fuzzy variables which is defined as membership function. Here the membership function has been chosen based on the trade-off between reduced complexity and better performance. In this work, a Mamdani Fuzzy Inference system along with seven membership function defined as in table 6.1 have been considered for carry out design.

Defuzzification

After employing the fuzzy inference system the output will be a fuzzy and it should be converted to crisp value for giving to the plant. This method of conversion of fuzzy variable to crisp variable is called defuzzification process. There are various defuzzification methods are available such as centroid, bisector, middle of maximum (MOM), smallest of maximum (SOM) and largest of maximum (LOM). Here centroid defuzzification method was used to defuzzify the fuzzy sets into a crisp control signal. The reason for taking this centroid defuzzification method is only its intuitive plausibility [47] and also it provides most accurate signal.

6.4 Results and Discussions

Fig.6.6 and 6.7 presents the simulation results and Fig. 6.10 and 6.11 exhibits the experimental result for the coupled tank system. Fig.6.8 and 6.9 illustrates the chattering response of the both tanks. In order to improve the chattering response a fuzzy term has been considered. From Fig. 6.10 and 6.11, it is observed that the sliding variable converges to zero that means states remain on the sliding surface ($x = h, \dot{x} = 0$). It is clearly identified that from Fig.6.6 the output $y(t) = h_1(t)$ and Fig. 6.7 that the output $y(t) = h_2(t)$ converges to its desired level such as h_{1d} and h_{2d} in about 60 sec and 100 second.

Table 6.2 Parameters of the Adaptive Sliding Mode Controller

Symbol	**Value**
K	0.050
N	0.1194
α	0.1 (Tank 1),0.35 (Tank 2)
γ	500 (Tank 1),0.05 (Tank 2)

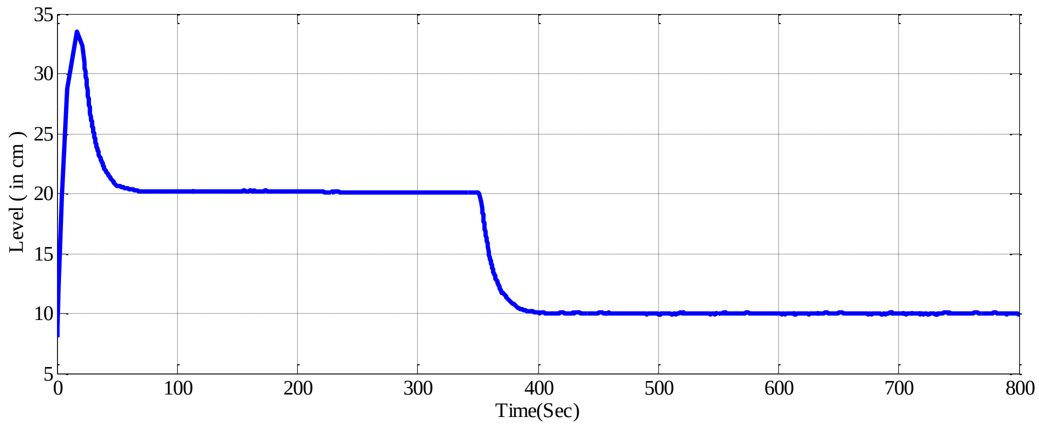

Fig. 6.6 Simulation Response of AFSMC while level control in Tank 1

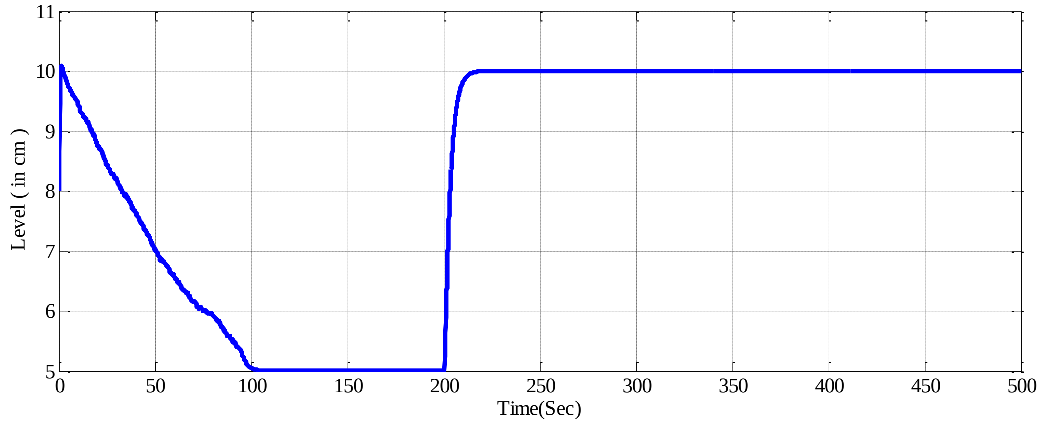

Fig. 6.7 Simulation Response of AFSMC while level control in Tank 2

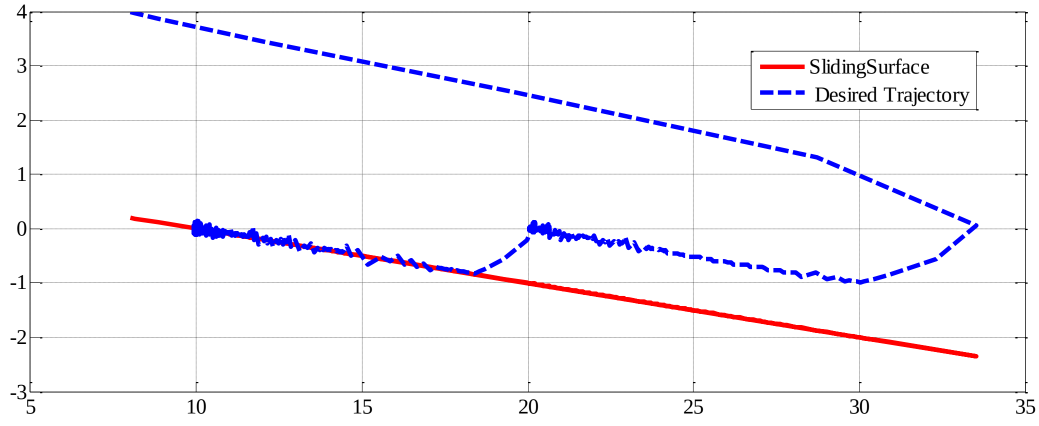

Fig. 6.8 Sliding Surface while level regulating in Tank 1

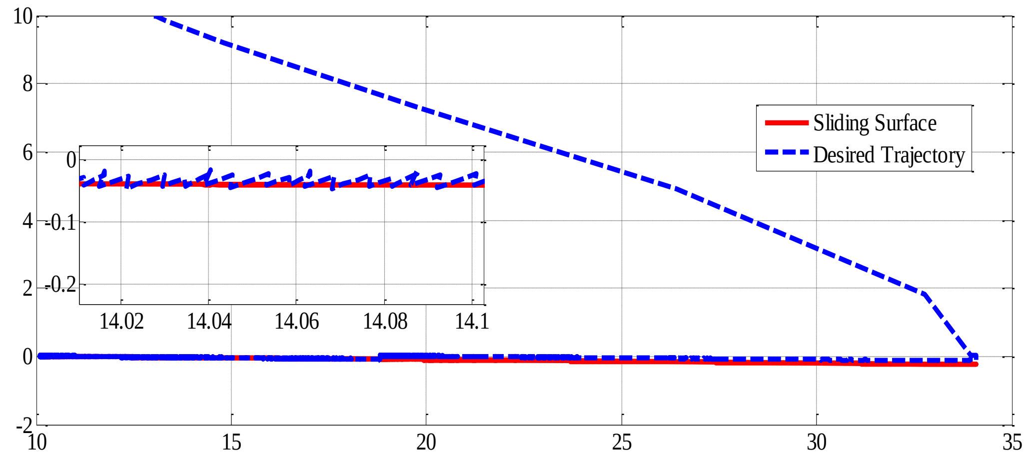

Fig. 6.9 Sliding Surface while level regulating in Tank 2

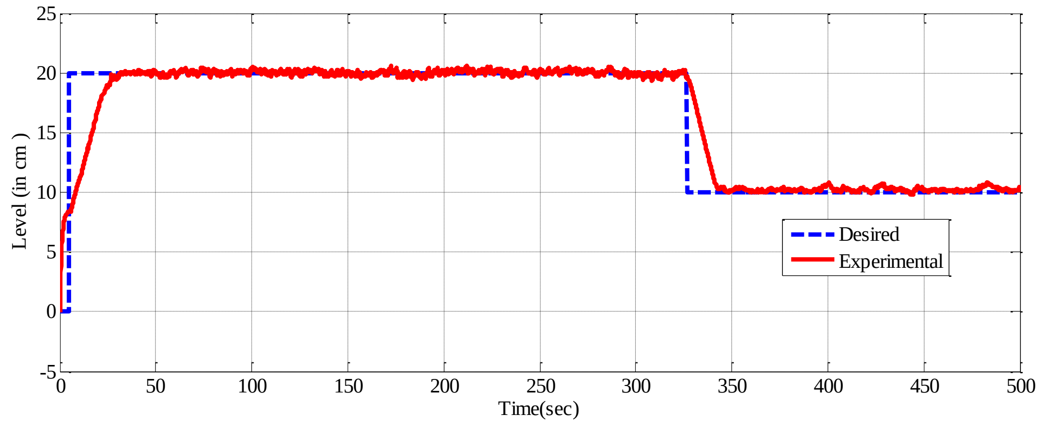

Fig. 6.10 Experimental Response of AFSMC while level control in Tank 1

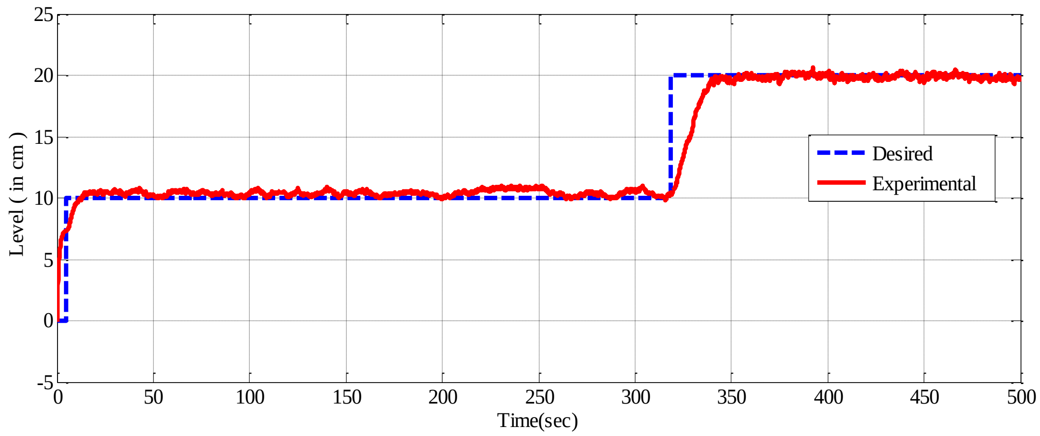

Fig. 6.11 Experimental Response of AFSMC while level control in Tank 2

From Fig. 6.10 and 6.11, it is observed that, with the proposed AFSMC control algorithm level of both tanks reach the desired level with taking less settling time. It also yields no overshoot and less steady state error. Here, water level of both tanks maintain its desired level in two desired step, where tank 1 maintain its first desired level at 20 cm for 0-320sec and second desired level at 10 cm for 330-500 sec and in tank 2 first desired level is regulated at 10 cm for 0-320 sec further the second desired level is at 20 cm for 330-500 sec. From Fig. 6.10 it is witnessed that, while level is regulating in tank 1 during the first desired step, level is smoothly settled around 30 second with zero steady state error but during second desired step it settles around 350 sec with little steady state error. Also from Fig.6.11 it is seen that, when level is regulating in tank 2 during first desired set point, level is settled around 10 sec with steady state error and in second desired step, level is settled around 340 sec with no steady state error.

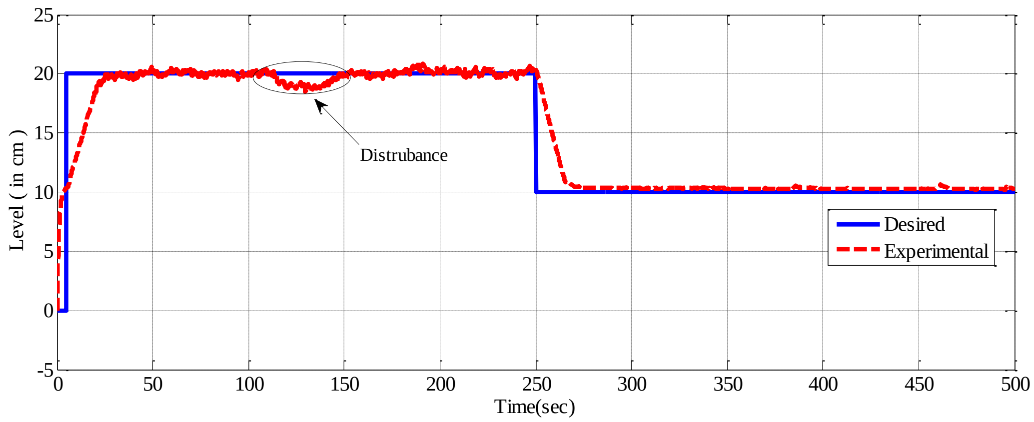

Fig. 6.12 Experimental Response of AFSMC under disturbance rejection mode while level control in Tank 1

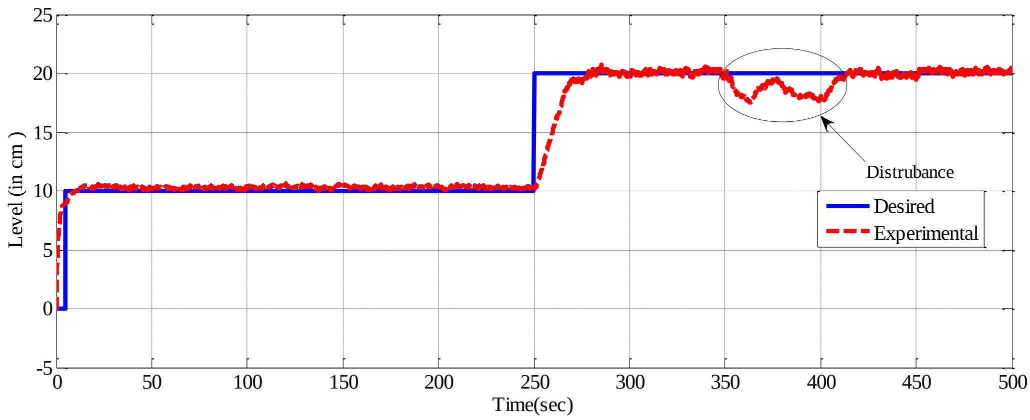

Fig. 6.13 Experimental Response of AFSMC under disturbance rejection mode while level control in Tank 2

Fig. 6.12 and 6.13 illustrates the disturbance rejection capabilities of the presented AFSMC control algorithm. Here the disturbances were applied to both the tanks during the steady state where load disturbance is added into the system by suddenly opening a valve for 30 sec in case of tank1 and 50 sec for tank 2.From Fig.6.12 and 6.13, it is observed that the proposed AFSMC algorithm brings the system response to the set point with less settling time and little steady state error after removal of the load disturbance.

Table 6.3 Performance assessment of AFSMC and SMC control algorithm

Comparison	Chattering Effect	Real time implementation issue	Reaching Time to the desired steady state level	Disturbance Rejection Capability
Adaptive Fuzzy Sliding Mode Control (AFSMC)	Smooth	Difficult	22 sec (Tank 1) 10 sec (Tank 2)	Better
Sliding Mode Control (SMC)	Less smooth as compared to AFSMC	quite easier than AFSMC	3 sec(Tank 1) 7 sec(Tank 2)	Less as compared to AFSMC

6.5 Chapter Summary

In this chapter an adaptive fuzzy sliding mode control law has been developed for maintaining desired liquid level in the both tank at a desired level. In this chapter, a fuzzy term has been included in the sliding surface in order to improve the chattering effect. It has been found that from both simulation and experimental results that the AFSMC controller exhibits best performance. It is also observed that the AFSMC control algorithm provides good robustness performances against disturbance rejection as well as tracking performance as compared to LMI based PI, Adaptive Fuzzy PI (AFPI), conventional sliding mode control (SMC).

Chapter 7

Conclusions and Suggestions for Future Work

7.1 Conclusions

This thesis presents a number of control strategies such as LMI based PI, Adaptive Fuzzy PI, Sliding Mode Control and Adaptive Fuzzy Sliding Mode controller. These control strategies have been fruitful in meeting with the control objectives i.e. maintaining of desired liquid level in both tanks of the coupled tank system as well as also satisfying the physical constraints in the control input.

The development of all the presented control strategies for the CTS have been successfully implemented using MATLAB/SIMULINK by considering vertical tanks coupling of the coupled tank system. In chapter 2 and chapter 3, a LMI based PI and Adaptive Fuzzy PI (AFPI) has been implemented in the real-time on a coupled tank liquid level system, which yields large overshoot and takes more time in order to maintain the desired level. Therefore for the improvement of response, in chapter 4 a sliding mode control designed in view of obtaining, as it is an effective approach for controlling nonlinear and uncertain system in presence of model uncertainties and disturbances. After implementation in the real-time, it is observed that, it suffers from the chattering problem which commonly possesses a serious concern to the possibility of damage of actuator. Hence in order to alleviate the chattering problem in chapter 5, an adaptive fuzzy sliding mode control has been developed, where the design of the sliding surface involves a fuzzy variable for the improvement of chattering problem. It is observed that the results obtained from AFSMC controller that, the developed control algorithm ensures best robust performance in face of system uncertainties as well as disturbance rejection and also it requires less time to settle at the desired steady level in both the tanks as compared to other controllers discussed in chapter 2, chapter 3 and chapter 4.

Table 7 1 Performances assessment of all controllers based on performance indices for Tank 1

Controller	IAE	ISE	Remarks
LMI based PI Controller	62.82	27.81	• Real-Time Implementation is easy but with this sluggish type of response is yielded. • Both ISE and IAE values are more as compared to AFPI, SMC and AFSMC.
Adaptive Fuzzy PI Controller (AFPI)	13.017	4.356	• Real-Time Implementation is quite difficult as compared to LMI based PI controller and also in this controller selection of range of membership function is time consuming. • Both Performance Indices are less as compared to LMI based PI controller.
Sliding Mode Controller (SMC)	3.296	10.86	• Real-Time Implementation is easier as compared AFPI. • Values of ISE and IAE are less compared to both LMI based PI and AFPI controller.
Adaptive Fuzzy Sliding Mode Controller (AFSMC)	12.32	9.695	• Real -Time Implementation is quite difficult as compared to SMC. It has better disturbance rejection capability as compared to SMC. • Both Performance Indices are less as compared to LMI based PI controller and AFPI controller and also the value of ISE is less as compared to the obtained values of ISE from SMC.

Table 7.2 Performances assessment of all controllers based on performances indices for Tank 2

Controller	IAE	ISE	Remarks
LMI based PI Controller	43.221	17.81	Values of ISE and IAE are higher as compared to other controller such as AFPI, SMC and AFSMC.
Adaptive Fuzzy PI Controller (AFPI)	23.13	13.017	The values of ISE and IAE are lesser as compared to LMI based PI Controller
Sliding Mode Controller (SMC)	10.843	18.97	Value of IAE is lesser as compared to LMI based PI and AFPI and ISE is less as compared to both AFSMC and LMI based PI controller.
Adaptive Fuzzy Sliding Mode Controller (AFSMC)	14.042	16.804	The value of IAE is less as compared to AFPI , LMI based PI and the values of ISE is less as compared to all presented controllers such as LMI based PI, AFPI,SMC and AFSMC.

7.2 Contributions of the thesis

The following are the contribution of the thesis

- PI controller based on LQR-LMI framework and an Adaptive Fuzzy PI (AFPI) is developed and implemented in real time for the regulation of level.
- In order to provide robust performance a sliding mode control is proposed. As usually the normal sliding mode control suffers from chattering problem, so in order to overcome this difficulty an adaptive sliding mode control is developed and also implemented in real time liquid level system.

7.3 Suggestions for the future work

- In the thesis, we have considered two tank systems in the dynamics equation and the controller design has been carried out accordingly. It can be further extended to four tanks with considering cross coupling and decoupling effect, which makes the problem more challenging.
- In chapter 4, in order to get robust response in face of model disturbance and also parametric uncertainties a sliding mode control has been designed and implemented in real-time. But usually the sliding mode controller suffers from the chattering problem. Hence due to that in chapter 5, an adaptive fuzzy sliding mode control has been developed for the improvements of chattering where a fuzzy variable is considered while designing the sliding surface. It can be further improved by utilizing higher order sliding mode (HOSM) and the super twisting algorithm.

Thesis Dissemination

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