



# Modeling and Signal Processing of GPR System

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**Master of Technology**

in

*Communication & Networks*

by

**Varun Kumar**

Roll no-212EC5381

under the guidance of  
**Prof. Subrata Maiti**



Department of Electronics and Communication  
NIT Rourkela

# Declaration

I declare that this thesis is my own unaided work. It is being submitted for the degree of Master of Technology at the National Institute of Technology, Rourkela. It has not been submitted before for any degree or examination at any other university.

Signature of author.....

Department of Electronics & Communication Engineering  
National Institute of Technology, Rourkela  
May, 2014



National Institute of Technology  
Rourkela

# CERTIFICATE

This is to certify that the work in the thesis entitled “**Modeling and Signal Processing of GPR System**” by **Varun Kumar** is a record of an original research work done by him during 2013-2014 under my supervision and guidance in partial fulfillment of the requirement for the award of the degree of Master of Technology with the specialization of Communication & Networks in the Department of Electronics & Communication Engineering, National Institute of Technology, Rourkela. The result incorporated in the thesis has not been submitted for award of any degree elsewhere.

Place: Rourkela

Date:

S.Maiti  
Assistant Prof,ECE Department  
NIT Rourkela

# Abstract

Ground Penetrating Radar (GPR) is a geo-sensor, which is used for detecting, locating, identifying and 3D imaging of the buried target inside the shallow surface. It is very difficult to detect and locate the target when limited amount of information is delivered by geo-sensor. On the basis of scattering parameter we try to extract information (related to feature of surface and target) as many as possible. Geometrical modeling of ground and target, as well as mathematical modeling for transmitting and receiving signal are the key factor in the field of GPR research. Receiving signal is nothing but the scaled version of transmitting signal which is affected by noise. So for mathematical modeling of receiving signal scaling parameter and noise plays a vital role. Whereas in signal processing domain time and frequency are the observation parameter. In signal processing cross correlation, windowing technique and time domain gating are the excellent features, which help in delay measurement, for suppression of an unwanted frequency and for extracting the time domain information of GPR signal at any specified duration. SFCW technique is a modern technique which is more applicable in GPR comparison to conventional FMCW technique. This technique can be operated at low power and license free band (ISM band).

*Dedicated to Parents  
and My Family*

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Communication & Networks

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# Nomenclature

<b>AWGN-</b>	Additive White Gaussian Noise
<b>BER-</b>	Bit Error Rate
<b>CW-</b>	Continuous Wave
<b>DFT-</b>	Discrete Fourier Transform
<b>EM-</b>	Electro-magnetic
<b>EMI-</b>	Electromagnetic Interference
<b>FFT-</b>	Fast Fourier Transform
<b>FMCW-</b>	Frequency Modulated Continuous Wave
<b>GPR-</b>	Ground Penetrating Radar
<b>I-</b>	In-Phase
<b>IDFT-</b>	Inverse Discrete Fourier Transform
<b>IF-</b>	Intermediate Frequency
<b>IFFT-</b>	Inverse Fast Fourier Transform
<b>ISM-</b>	Industrial Scientific and Medical
<b>MW-</b>	Microwave
<b>Q-</b>	Quadrature
<b>RF-</b>	Radio- Frequency
<b>Rx-</b>	Receiver
<b>SFCW-</b>	Stepped Frequency Continuous Wave
<b>SNR-</b>	Signal to Noise Ratio
<b>Tx-</b>	Transmitter
<b>UWB-</b>	Ultra Wide-Band
<b>VNA-</b>	Vector Network Analyzer

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# Chapter 1

## Introduction

In the emerging world ground is an area of interest for researcher. These days lots of materialistic things like PVC pipe, cable e.t.c are being installed inside the ground or any surface which is not visible by naked eyes[1]. If any discrepancy arises then there should be proper detection mechanism. The nature of materialistic things may be metallic or non-metallic. Subsurface analysis has also an important role in soil mapping (agricultural sector)[2] and testing of strength of soil for road as well as railway track. In another side, due to enhancement of anti-social activity across the globe, landmine has become a major problem for human being[3]. Detection of any materialistic things becomes difficult if it has dielectric nature. If electrical parameter like permittivity  $\epsilon$ , Permeability  $\mu$ , Conductivity  $\sigma$  of target has similar value like environment then it becomes too difficult to detect the target. Shallow surface analysis has become an emerging field of an area of research because installed materialistic things, surface under the railway track, landmines are not placed at very high depth. To study the shallow surface and for locating, identifying and 3D imaging of [4],[5] the buried target, a technique came called as ground penetration radar (GPR). Apart from above application there are also another area, like tunnel detection, dead body detection e.t.c where GPR is being utilized. Basically in GPR EM wave strikes to the target and returned back to the receiver, on the basis of reflected signal we find the various parameter of ground and feature of target. There is another technique called as bore hole technique [6] where we find the ground parameter for higher depth but it is a too expansive method.

Many author proposed various type of algorithm for surface parameter estimation as well as target detection [7]. Andrew.D.Strange proposed a

novel method for true time determination of reflected echoes from the buried target [8]. Unambiguous range and range resolution are the primary observable parameter which deals the distance as well as separation between two target. O.Lopera proposed an algorithm for background (which appears due to presence of static reflector) removal in frequency domain for focusing GPR data[9]. Author has chosen some relevant area which has same working principle (incident, reflection, refraction) like a GPR as well as technical challenges regarding GPR.

**Related technology works on incident, refraction and reflection phenomenon:**

- Ultrasound
- Seismic wave observation
- Air-borne radar
- GPR
- Sonar
- Microphones
- Accelerometers
- Hydrophones

**Technical issue regarding GPR:**

**Scattering of EM wave:**

- Objects -shape, composition, orientation
- Surface Specular facets, Bragg resonance, Kirchhoff scattering, Small perturbation

**Materials:**

- Permittivity
- Permeability

- Conductivity

**Designing RF/Microwave Component:**

- Oscillators-stable reference
- Frequency Synthesizers
- Frequency Multipliers

**Numerical modeling, Simulation, Inversion:**

- Finite difference time domain (FDTD)
- Commercial CAD tools (HFSS,CST)

**Digital Circuitry:  
Timing and Control**

- Pulse repetition frequency
- Switch Control Signal
- Inter-pulse coding
- Waveform sequencing

**Data Acquisition**

- A/D converters
- Data buffering
- Real time processing
- Data storage

**Signal Processing**

- Fourier analysis
- Cross-correlation/cross-covariance
- FIR-filter/IIR- filter



**Matched Filters**

1. Pulse compression
2. Analog- track focusing
3. Phase coherence

**Math**

- System geometry (for mono-static, bi-static radar scenario)

**1.1 Problem Description:**

In mathematical modeling of transmitting signal proper nature of signal should be known i.e. pulse nature or continuous wave nature which has been discussed in next chapter.

Whereas for mathematical modeling of received signal, the received signal would be a convolution of the transmitting signals with the impulse response of antennas (beam width, gain, directivity and frequency), the response of subsurface medium (dispersion, attenuation and scattering losses), the impulse response of target and so on.

For convenient mathematically it can be expressed as follows:

$$v_{rx}(t) = v_{tx}(t - \tau) \odot g(t) \odot m(t) \odot T(t) \odot m(t) \odot g(t) + n(t) \quad (1.1)$$

Where  $v_{rx}(t)$  the received signal response,  $v_{tx}(t - \tau)$  delayed transmitted signal,  $m(t)$  response of antenna subsystem,  $g(t)$  impulse response of subsurface system,  $T(t)$  target response,  $n(t)$  be the noise and  $\tau$  are the two way time.

**1.2 Thesis Objective:**

The prime objective of this thesis is to give an overview of EM signal transmission and reception and also to give an information of signal behavior in time domain as well as in frequency domain. Time and frequency are the interdependent parameter for GPR signal. This thesis deals the impact of time and frequency for calculation of other dependent parameter of GPR signal. Using this time and frequency analysis we can find range resolution, unambiguous range and permittivity of target e.t.c.

## 1.3 Approach:

This thesis approaches the data modeling on the requirement to use computationally efficient algorithms to determine the parameters of the model. If pulse is taken for consideration then for the same pulse width shape of pulse causes difference in frequency domain. This thesis also describes a mathematical expression for the different types pulse. This thesis deals the mathematical expression of SFCW radar signal, and visualization of pulse repetition period and unambiguous range.

### 1.3.1 Signal Processing

Signal processing plays a vital role for extracting the features of target and behavior of surface when we work with limited amount of data sample. Data acquisition is the primary functional part where physical parameter is converted into electrical signals. A/D converter converts conditioned sensor signals to digital values.

Data processing unit process these sampled or digital data and make a suitable meaning. From signal processing points of view windowing technique and time domain gating has been used in this thesis. These techniques help for suppressing the inter-modulation distortion and data processing with finite number of sample.

## 1.4 Thesis Overview:

**Chapter 2** deals the deterministic approach to study the surface as well as target. It covers the various types of surface with an example. For our convenience we take the layered surface as the reference surface for analysis. We use ray tracing method to calculate the distance of target from the surface. Shape as well as orientation of the target is also a critical issue for detecting the target. We detect the target by the help of scattering parameter S11 or reflected power.

**Chapter 3** gives the information about the transmitting signal. Electromagnetic pulses as well as continuous wave show different characteristics when it penetrates to the ground media. When suitable pulse is sampled then the shape of pulse or distribution of discrete sample plays a vital role.

DFT operation gives the information of occupied band in frequency domain. Time domain shape of pulse or distribution of sample is suitable for doing mathematical operation. If analysis is performed in frequency domain then by doing IDFT operation we can observe the time domain behavior or time domain distribution of signal.

**Chapter 4** gives the information about receiving signal. It describes the mathematical model of receiving signal. When received data is processed then our prime intention is to suppress noise level. By making a theoretical model of noise and we process the received sample data with theoretical modeled data. We add some noise to nullify the effect of channel noise. The main aim of this project is to make suitable noise model which suppresses channel noise. Another aim is to do signal processing on noise free received signal for extracting the nature of ground surface and desired feature of target.

Finally in **Chapter 5** conclusions are drawn and future work has been discussed.

# Chapter 2

## Deterministic Approach to Study the Surface

### 2.1 Geometry of the surface and target:

Ground is a complex dielectric medium. We detect the target on the basis of its electrical parameter like permittivity, permeability, conductivity. We make a geometrical model of surface and target for this purpose surface modeling is used. Surface modeling is a branch of mathematics which deals the various types of surface and their mathematical interpretation. For convenient author has taken three modeling of surface we may assume the following type of surface

- **Mono layer surface** (like Fig.3.9)  
The best example of mono layered surface is desert land like *Thar, Sahara* desert or fully covered iced surface like *Antarctica, North pole of the earth*.
- **Layered surface** (like Fig.2.1/2.4).  
The best example of layered surface is the cricket pitch, surface beneath the railway track or surface under the high way which is made in USA, Canada or some European countries.
- **Complex surface**  
The best example of complex surface is any real world surface.

By making a synthetic model of ground we analyze the surface which helps to extract the dependent parameter of GPR signal theoretically. The

simplest way for studying the surface is a layered form structure, which is shown in Fig.2.1

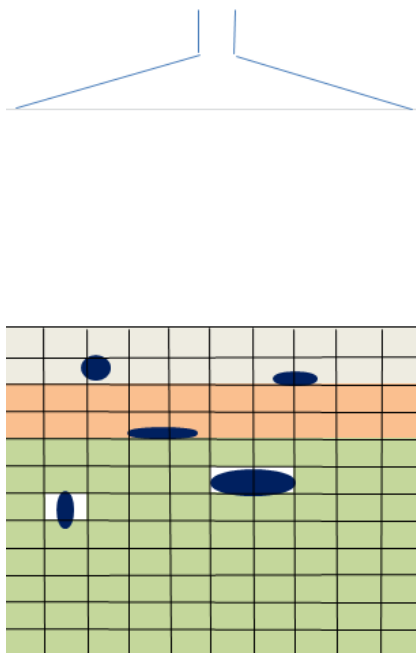


Figure 2.1: Layered surface having multiple targets

Three types of scanning mechanism is used for the detection of target called as

- A Scan: 1D scanning (Line scanning)
- B Scan: 2D scanning (Surface scanning)
- C Scan: 3D scanning (Volumetric scanning)

These scanning methods are the part of data acquisition section where we store the value of reflected voltage/power at different instant. On the basis of stored instantaneous value of voltage/power, we are capable to detect the range of target as well as another parameter of GPR. From Fig.2.2, brown line shows the ground surface and at different instant transmitter transmit EM wave, some part of EM wave strike to the target and some are returned back to the receiver. Below  $z_0$  distance, there exists a target which is shown in



## 2.2 Ray Tracing Mechanisms:

It is a basic mechanism to study the transmitting and receiving signal by optical ray. Ray tracing is a novel method for visualizing the incident, transmitted, reflected EM wave like light wave or like an optical signal. From Fig.2.3/2.4, ray tracing and its applicability is clearly shown. From Snails law when EM wave enters from one medium to another then either it bends towards the normal or away from the normal depending on the dielectric property of respective medium.

From Snail's law,

$$\frac{\sin(\theta_i)}{\sin(\theta_t)} = \frac{\mu_2}{\mu_1} \quad (2.6)$$

$$\theta_i = \theta_r \quad (2.7)$$

Where  $\mu_1$  and  $\mu_2$  are the reflective index of medium one and two. Light is also an EM wave, using this behavior we can find the deviation or scattering of EM wave. Scattering or Bragg resonance starts when surface is not perfectly polished which is the real scenario of ground penetrating radar. Deviation of EM wave as well as scattering provides the information about complexity of surface as well as composition of soil in case of GPR.

### 2.2.1 Reflected and Refracted Rays

When EM wave enters to a dielectric of certain thickness  $d$  and emit from another end then emitted wave and incident wave are parallel to each other and the separation between them is  $\Delta d$ , which is shown in Fig.2.3. Reflection and refraction starts from upper surface as well as the lower surface [10],[11]. By this way the intensity of secondary, tertiary or other multipath reflected and refracted wave decreases.

So lateral separation  $\Delta d$ , between incident ray and emitted ray can be expressed as

$$\Delta d = d \frac{\sin(\theta_i - \theta_t)}{\cos(\theta_t)} \quad (2.8)$$

Where  $\theta_i$  and  $\theta_t$  are angle of incidence and angle of refraction. Like above equation, we can also find the deviation of EM wave from the layered surface.

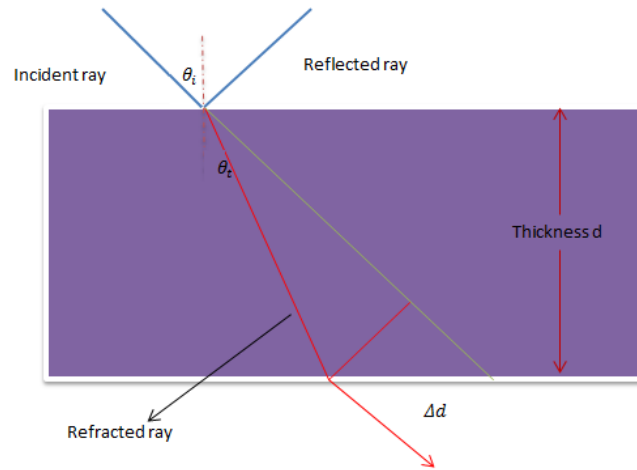


Figure 2.3: Ray tracing for transmission and reflection

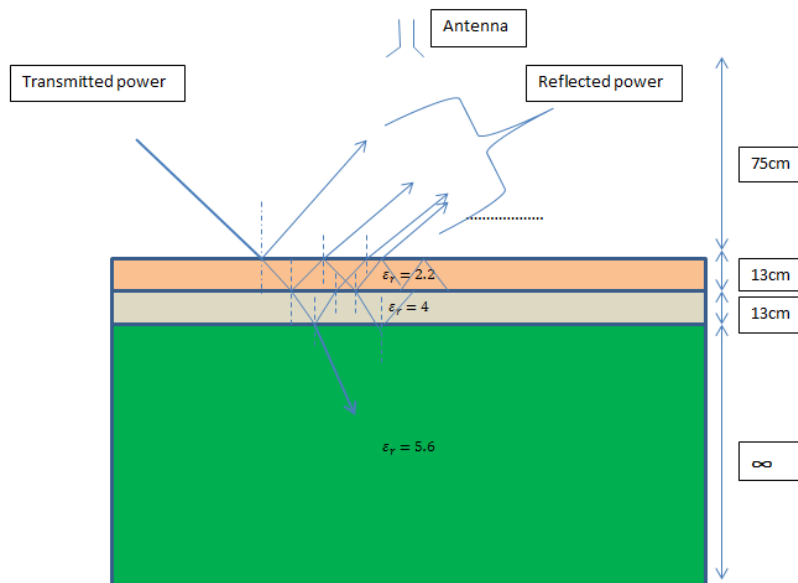


Figure 2.4: Layered surface showing multiple reflections



# Chapter 3

## Transmitted Signal Model

### 3.1 Pulse vs Continuous Wave Signal Model:

For making a mathematical model of transmitting signal is a challenging issue in case of GPR. So after making a surface model the next target is the selection of transmitting signal. Mainly two types of signal are taken into consideration i.e. either in pulse format or in continuous wave signal [12]. Both have some interesting feature which makes it relevant for different application.

- **Pulse form**

Pulse is a transient electromagnetic disturbance or it is a short burst of electromagnetic energy.

- **Continuous wave signal**

A continuous wave (CW) is a type of disturbance in which corresponding wave has constant amplitude and frequency, of infinite duration and it could be mathematically analyzed.

Following challenges has been observed and analyzed in this section which are as follows,

- What will be the effect of pulse and continuous wave signal in time domain as well as frequency domain?
- What should be the optimum operating bandwidth for GPR application?

- If pulse form of signal is being used then what should be the probable shape or probable distribution in discrete domain for pulse?
- What should be the mathematical expression for designing the probable shape of signal?
- If suitable pulse is generated in time domain then what is the operating bandwidth in frequency domain and vice versa as well as how much power contain in the desired band.
- If continuous wave (CW) signal is being used then whether that signal has single frequency component or multiple frequency components.
- What are the advantages and disadvantage of these two forms?

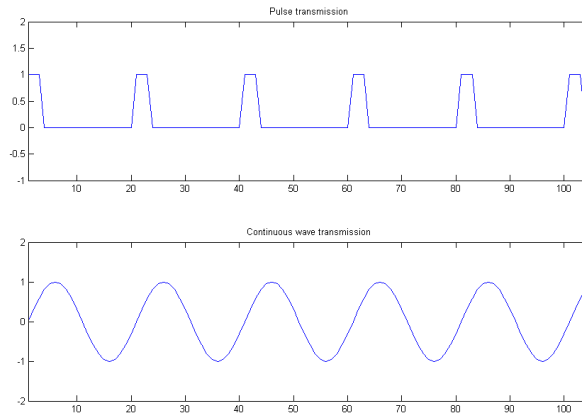


Figure 3.1: Pulse and CW signal

Fig.3.2 shows the variation of permittivity with respect to frequency. Mathematically effective complex permittivity of soil or surface can be expressed as

$$\epsilon_e = \epsilon' + j\epsilon'' \quad (3.1)$$

$$\epsilon_e = \epsilon' + j\left(\frac{\sigma}{\omega}\right) \quad (3.2)$$

From Fig.3.2 it is clear that the real and imaginary part of complex permittivity remains constant for frequency range few hundred MHz to few GHz.

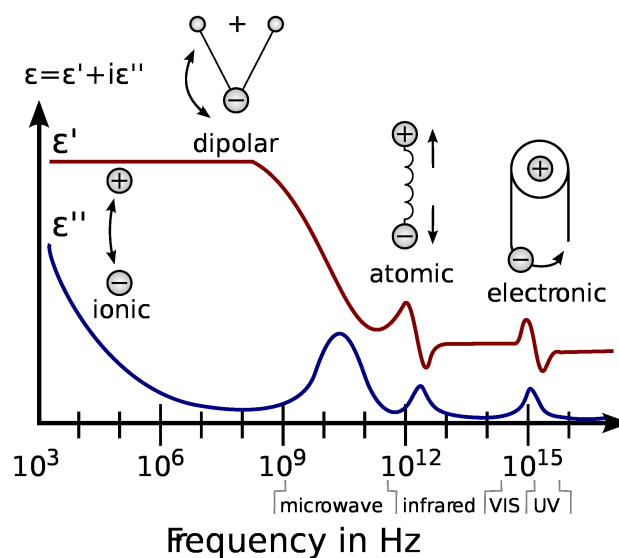


Figure 3.2: Permittivity vs Frequency

If operating frequency is too high then dielectric material shows non-linear characteristics with respect to frequency. Atomic as well as electronic polarization starts at very high frequency. 200MHz to 3GHz frequency range is suitable for GPR application. This is our design constraints where we have to generate suitable pulse, which fulfill our desired requirement.

On the basis of above design constraints we choose such type of signal which produces satisfactory desired bandwidth and power.

## Pulse Analysis

### Some probable shapes of pulse:

1. Gaussian pulse
2. Ricker wavelets
3. Sine wavelets
4. Sine wavelets with after ring
5. Sine wave mono pulse

## 6. Raised cosine pulse

Other probable shape or probable distribution may be considered for analysis points of view

Mathematical form of Gaussian signal

$$x(t) = e^{-at^2} \quad \frac{-\tau}{2} \leq t \leq \frac{\tau}{2} \quad (3.3)$$

Mathematical form of Ricker wavelets

$$x(t) = \frac{d(e^{-at^2})}{dt^2} \quad (3.4)$$

Mathematical form of Sine wavelets

$$x(t) = A \sin(2\pi ft) \sin(2\pi/3ft) \quad \tau_1 \leq t \leq \tau_2 \quad (3.5)$$

=0 otherwise

Mathematical form of Sine wavelets after ring

$$x(t) = A \sin(2\pi ft) \sin(2\pi/3ft) \quad \tau_1 \leq t \leq \tau_2 \quad (3.6)$$

=  $A \sin(2\pi ft)$   $t \geq \tau_2$

Mathematical form of Sine wave mono pulse

$$x(t) = A \sin(2\pi ft) \sin(2\pi/3ft) \quad \tau_1 \leq t \leq \tau_2 \quad (3.7)$$

Mathematical form of Raised cosine signal

$$x(t) = \text{sinc}\left(\frac{t}{T}\right) \frac{\cos\left(\frac{\pi\beta t}{T}\right)}{1 - \frac{4\beta^2 t^2}{T^2}} \quad (3.8)$$

Where  $\beta$  is the roll off factor and T the reciprocal of symbols rate.

Fig.3.3 is the time domain representation of above mathematical expression. Except raised cosine pulse more than 90% energy of pulse lies within 1ns. Above signal shows the different types of shape in confined duration. If it is analyzed in discrete domain then it is termed as distribution of data sample, like Gaussian distribution [13], sine wavelet distribution and so on. Shape of pulse plays a vital role in frequency domain.

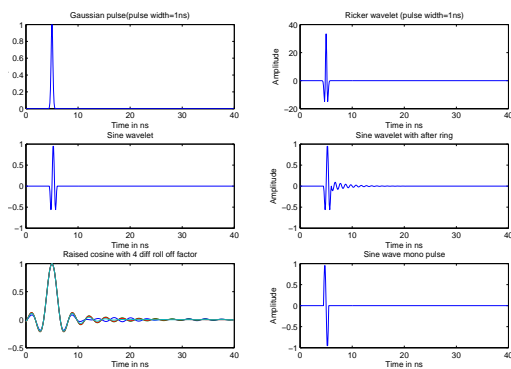


Figure 3.3: Plot of different types of pulses

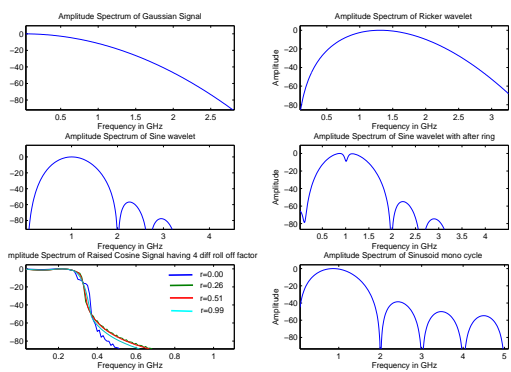


Figure 3.4: Plot of amplitude spectrum

Fig.3.4 shows the impact of shape of signal from time domain to frequency domain and its x-axis and y-axis shows the frequency in GHz and power in dB scale respectively. Table.1 summarizes the different shapes of signal and their power content in given band. From Table.1 it is clear that the 3dB bandwidth of Ricker wavelet is 720MHz, where the width of transmitting pulse is of 1ns. Peak power of Gaussian pulse as well as raised cosine pulse is confined at very low frequency which is not suitable for GPR. If this pulse is passed through band-limited channel then ringing effect arises which has been discussed in later section. Spectrum of Gaussian pulse decreases gradually so 3dB bandwidth is less. Sine wavelets and sine wave mono pulse shows its peak at fundamental frequency of sinusoid. It is our general convention that bandwidth is measured in terms reciprocal of pulse width. If this consideration is followed then Ricker wavelet gives 1GHz bandwidth at 6.16dB below the peak power whereas Gaussian pulse power is measured 11.84dB below from peak power. The power content for other pulses are mentioned in Table 1.

Table 3.1: Different types of signal, their operating band and power

S/N	Signal Type	3dB bandwidth		$B \approx .1/\tau$		Power in dB
		$f_l$	$f_h$	$f_l$	$f_h$	
1	Gaussian	few kHz	510MHz	few KHz	1.01GHz	-11.84
2	Ricker wavelet	970MHz	1.69GHz	840MHz	1.84GHz	-6.16
3	Sine wavelet	760MHz	1.26GHz	510MHz	1.51GHz	-12.2
4	Sine wavelet with ring	700MHz	1.3GHz	520MHz	1.53GHz	-11.24
5	Raised cosine	few KHz	300MHz	few KHz		
6	Sine wave mono pulse	570MHz	1.18GHz	370MHz	1.38GHz	-8.62

We can also analyze the signal in reverse order i.e. if signal information is in frequency domain then we can know the mathematical distribution scheme in its time domain counterpart.

## 3.2 Sinusoidal Signal Analysis on Layered Surface:

A novel characteristic of sinusoidal signal is its unique frequency. During filtering operation sinusoidal signal gives better response in comparison to other shape of signal (due to spreading in frequency domain and non-linear power distribution across all frequency). There is a practical difficulty with single frequency sinusoid to measure delay or unambiguous range when this signal penetrates to the ground. Due to multiple reflection as well as diffraction of signal, ambiguity arises during signal reception. We don't get the spikes in time domain which give information of reflected signal from the layered surface. For two layer surface there is a graphical signal model plotted in Fig.3.5, in which 1st, 2nd, 3rd, 4th subplots are transmitted signal, reflected signal from upper layered surface, reflected signal from bottom layer and composite signal at receiver end respectively. 1st and 2nd layer reflected signal superimpose in constructive manner. Now if we add complexity (taking  $n$  layered surface) then extraction of every layered reflected signal becomes difficult. This has also been discussed in later section.

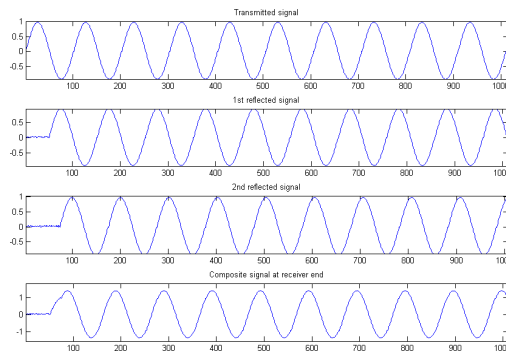


Figure 3.5: Sinusoidal signal analysis for layered surface

### 3.2.1 Short time Sinusoidal Signal analysis:

In this section we observe the behavior of short time sinusoidal signal in context to GPR (if transmitting signal like a short time sinusoid)[14]. Fig.3.6

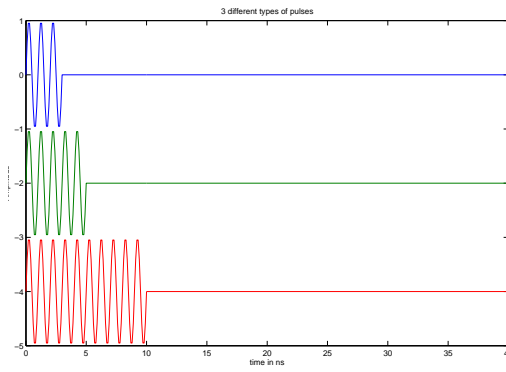


Figure 3.6: Plot of short time sinusoidal signal

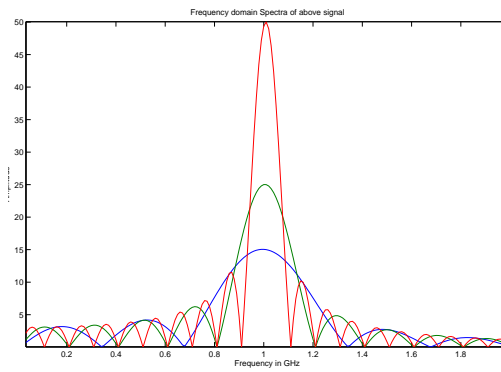


Figure 3.7: Plot of amplitude spectrum of short time sinusoid



Table 3.2: Analysis of pulse width and their 1st null bandwidth

S/N	Number of Cycles 1Cycle=1ns	Center Frequency	1st Null Bandwidth in GHz
1	3	1GHz	0.67GHz
2	5	1GHz	0.40GHz
3	10	1GHz	0.20 GHz

shows the short time sinusoidal signal having pulse width 3ns, 5ns and 10ns respectively, whereas pulse repetition period is 40ns for all the signals. All signals are resonating at 1GHz frequency which is shown in Fig.3.7 Narrow pulse width provides wider 3dB as well 1st null bandwidth and vice verse.

Table 2 summarizes the effect of short time domain sinusoid in frequency domain. From table 2 we can observe that

$$\tau B \approx 2 \quad (3.9)$$

$\tau$  and  $B$  are the pulse width and 1st null bandwidth respectively. But for getting 1<sup>st</sup> null bandwidth the power in dB scale is observed as below 80dB.

### Problem with Broader Pulse in GPR:

- If reflected pulse lies inside the transmitted pulse then ambiguity arises in delay measurement, which is shown in Fig.3.8
- Broader time domain pulse gives narrow frequency band therefore range resolution get affected since range resolution  $\Delta r$  is expressed as follows;

$$\Delta r = \frac{v\tau}{2} = \frac{v}{2B} \quad (3.10)$$

Where  $v$  is the velocity of EM wave in dielectric media,  $\tau$  is the pulse width and  $B$  is the occupied bandwidth( $B = \frac{1}{\tau}$ ).

These days a new technique is adopted for signal transmission called as SFCW, where we transmit CW multiple frequency signals instead of single frequency signal, which has been described in later section.

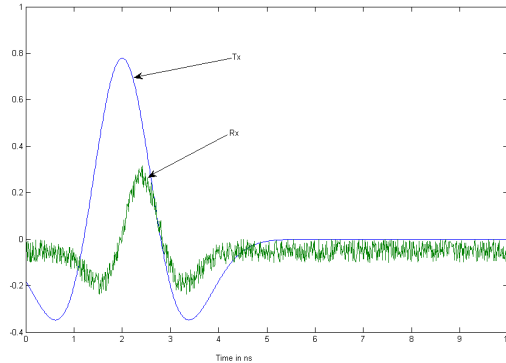


Figure 3.8: Plot of reflected pulse inside the transmitted one

### 3.3 SFCW Signal Analysis:

SFCW signal transmission is a modern approach in comparison to conventional FMCW signal transmission. We send the multiple frequencies in stepped manner. We choose suitable operating band for GPR which has been shown in Fig.3.2. We break this frequency band into  $n$  number of stepped frequency. Let  $B$ ,  $\Delta f$  and  $n$  be the suitable band of GPR, stepped frequency and total number of steps respectively then mathematically it can be expressed as follows [15],[16]

$$B = f_h - f_l = n\Delta f \quad (3.11)$$

Where  $f_h$  and  $f_l$  are the upper and lower cutoff frequency. If initial frequency is taken as  $f_0$  then for SFCW signal,  $k_{th}$  frequency component can be written as,

$$f_k = f_0 + (k - 1)\Delta f \quad (3.12)$$

We have several issues regarding frequency selection in case of GPR.

- What should be lower and upper cutoff frequency?
- What should be the step size?

For our convenient we choose lower cutoff and upper cutoff 400MHz and 4.845GHz respectively because in this frequency range real and imaginary part of permittivity remains constant which is shown in Fig.3.2 By taking

35MHz step frequency, we transmit 128 different frequencies. Unambiguous range and the range resolution depends on selection of frequency band and Frequency steps in case of SFCW radar[17],[18]. Unambiguous range and range resolution can be expressed as follows

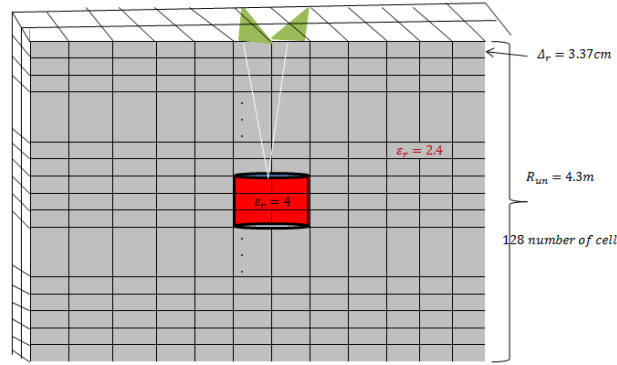


Figure 3.9: Target inside the mono dielectric media

$$R_{un} = \frac{v}{2\Delta f} \quad (3.13)$$

$$\Delta r = \frac{v}{2B} \quad (3.14)$$

where  $v$  is the velocity of propagation of EM wave in dielectric media. From above consideration unambiguous range and range resolution are 4.3m and 3.37cm respectively. Unambiguous range and range resolution are diagrammatically explained in Fig.3.9 where a target exists inside a mono dielectric media and small cell represent the range resolution (minimum separable distance between two targets). By ground sectioning, we can observe that 128 number of small cell equals to unambiguous range (beyond that range instrument cant observe the target).

Fig.3.10 shows the time domain SFCW signal which consist of 128 frequency components like the above consideration. All frequency components have same amount of power which has been plotted in Fig.3.11 Addition of multiple frequency components in time domain construct a series of pulses shown in Fig.3.10 Separation between two consecutive pulses is  $28.65\mu s$  i.e. nothing but reciprocal of step frequency

$$T = \frac{1}{\Delta f}, \quad 28.65\mu s = \frac{1}{35MHz}$$

Where T is the pulse repetition period of SFCW signal.

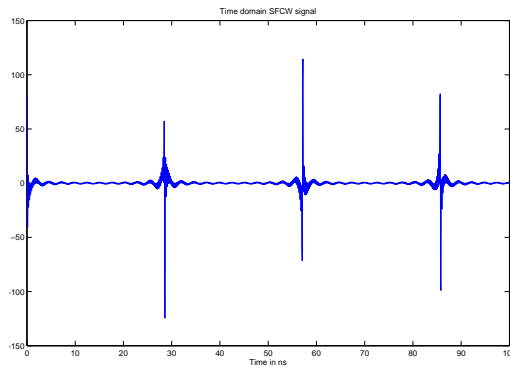


Figure 3.10: Time domain SFCW signal

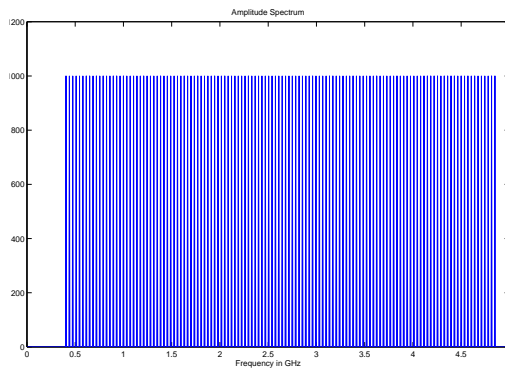


Figure 3.11: Plot of amplitude Spectrum of SFCW signal

### 3.4 Mathematical model of SFCW signal:

$$Y_t(n) = A \sum_{i=1}^{i=M} e^{j2\pi(\frac{f_i}{f_s})n} \quad (3.15)$$

Where  $Y_t$  is the sampled transmitting SFCW signal, since all frequency have same amount of power so here A is the amplitude of all frequency component,  $f_s$  is the sampling frequency and n is the sampling index n= 1 to N.

For processing points of view  $Y_t$  is the reference sampled signal of size  $1 \times N$ . We analyze the reflected signal using transmitted sample which has been discussed in later section.

# Chapter 4

## Received Signal Model

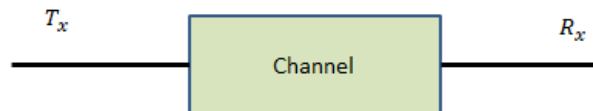


Figure 4.1: Block diagram of general communication system

In telecommunication engineering channel may be classified as

- Wired channel
- Wireless channel

In case of GPR the nature of channel is wireless, but there are lots of practical difficulty with wireless networks

- Wireless media is not a reliable media.
- It doesnt support very high bandwidth.
- Spreading loss and multipath fading comes into the picture.
- If multiple reflectors are present in the wireless medium then there is chance of destructive interference which causes deep fade, in that case the SNR of the signal becomes very low.

But in case of GPR our prime intention is to resolve proper information from the reflected signal. Both Tx and Rx are placed adjacently in GPR hardware. Air media and dielectric media make a wireless channel for GPR application. We make a synthetic model of ground for two different cases, and observe the theoretical power at the receiver end

### Case 1: Layered surface model where bottom surface is a metallic plate

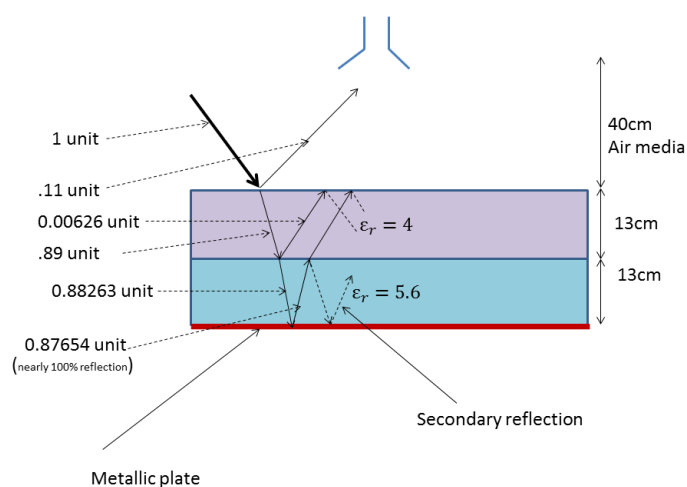


Figure 4.2: Ground model 1 and transient response analysis

### Case 2: Layered surface model where bottom surface has infinite distance with fix electrical parameter

A Gaussian pulse is taken as the reference transmitted pulse having pulse width 1ns which is shown in 1st subplot of Fig.4.3/4.5. When it passes through the air as well as dielectric media then effect of channel influences the behavior of signal which has been mentioned below;

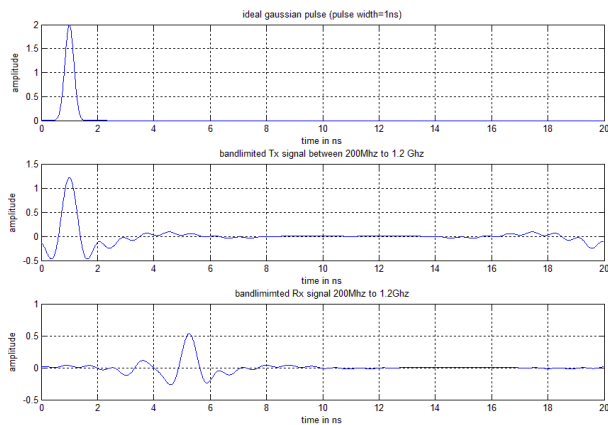


Figure 4.3: : Ideal Gaussian pulse (1ns pulse width), band limited Gaussian pulse and theoretically observed received power pattern for ground model 1

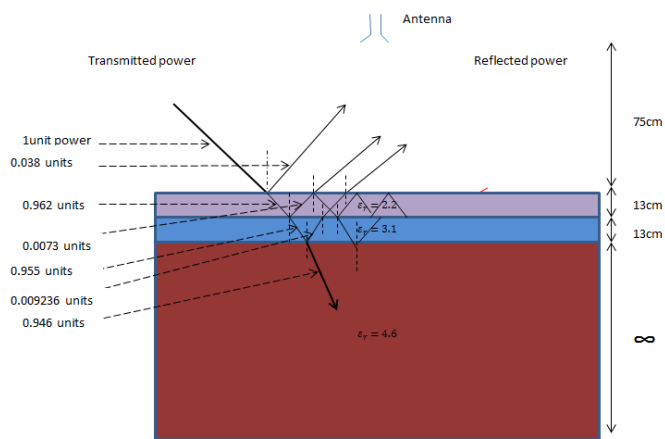


Figure 4.4: Ground model 2 and transient response analysis



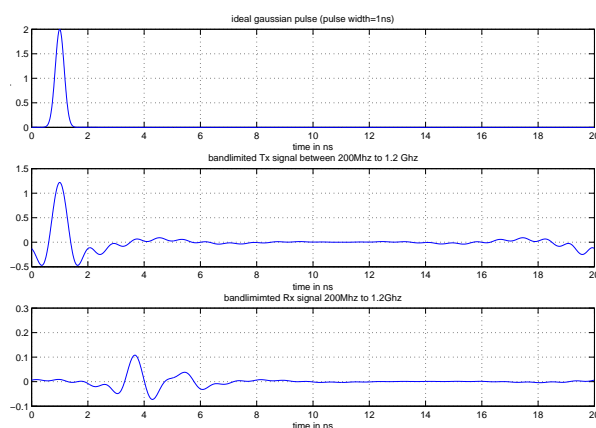


Figure 4.5: Ideal Gaussian pulse (1ns pulse width), band limited Gaussian pulse and theoretically observed received power pattern for ground model2

### Effect of channel:

- Wireless channel doesn't support very high bandwidth due to this behavior of channel it acts like a low pass filter or band pass filter up to certain extent. 2nd subplot of Fig.4.3/4.5 is the resultant signal when Gaussian passes through the band limited channel (200MHz to 1.2GHz). From 2nd subplot of Fig.4.3/4.5 we observe the ringing effect due to band limited operation.

### NOTE:

The effect of attenuation, distortion, damping, conductivity of ground and other environmental effect are ignored. Only the effect of channel and reflection from different layers are taken into consideration. Reflection from a layered surface has been described in later section.

### Observation

- Peak of the 3rd subplot of Fig.4.3/4.5 shows the strongest signal reaches at receiver end.
- Since Gaussian pulse stimulates at 1ns, so from Fig.4.3 1st peak and

2nd peak gives the information about reflected signal from the ground surface and metallic plate respectively whereas from Fig.4.5 1st peak and 2nd peak gives the information about reflected signal from the ground surface and bottom layer respectively .

- Due to time domain spreading of signal (ringing effect) if returned signal has low amplitude then sharp spikes cant be observed. From Fig.4.3/4.5 returned signal from interface of 1st layer and 2nd layer is not distinguishable.
- Amplitude of reflected signal from metallic plate is quite high so if dielectric target presents in a very high conductive environment then it becomes difficult to detect the dielectric target.

Still there exist an infinite set of geometry of surface and target which is really a challenging issue for an instrument designer.

We detect the returned signal on the basis of three fundamental quantities which are as follows

- Amplitude
- Frequency
- Phase

When EM signal propagate into a wireless media then EM signal influence by huge amount of noise. Lots of signal processing is required for proper reconstruction of transmitted signal at receiver end. Mathematically returned signal of GPR can be expressed as follows

$$Y_r(n) = s * Y_t(n) + R_n \quad (4.1)$$

where  $s$  and  $R_n$  are the scaling factor and the sampled noise respectively. Scaling parameter depends on attenuation constant, offered by air media as well as dielectric media or reflection coefficient. Both quantities depend on operating frequency, relative permittivity of media and conductivity of the medium. Reflection coefficient and attenuation are interdependent parameter. Mathematically global reflection and attenuation constant for layered surface can be expressed as [19]

$$\tilde{\Gamma}|_{i,i+1} = \Gamma|_{i,i+1} \prod_{k=0}^{i-1} (1 - \Gamma_{k,k+1}^2) \prod_{k=1}^i (e^{-2\alpha_k z_k}) \quad (4.2)$$

Where  $\Gamma_{i,i+1}$  is a local reflection coefficient and  $\Gamma_{k,k+1}$  is the  $k_{th}$  layer reflection coefficient,  $\alpha_k$  is the  $k_{th}$  layer attenuation constant and  $z_k$  is the thickness of  $k_{th}$  layer.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \quad (4.3)$$

where  $\eta$  is a wave impedance. From eqn.4.3 if  $\sigma \ll \omega\epsilon$  then

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \quad (4.4)$$

Relative permeability changes abruptly only in the case of fero-magnetic material like, Fe, Co, Ni and some artificial compound material otherwise permeability remains constant. So if we take this assumption then we can write as

$$\eta \propto \frac{1}{\sqrt{\epsilon}}$$

so local reflection coefficient can be expressed as follows,

$$\Gamma|_{i,i+1} = \frac{\eta_i - \eta_{i+1}}{\eta_i + \eta_{i+1}} = \frac{\sqrt{\epsilon_{i+1}} - \sqrt{\epsilon_i}}{\sqrt{\epsilon_{i+1}} + \sqrt{\epsilon_i}} \quad (4.5)$$

$$\alpha_k = \omega \sqrt{\frac{\mu_0 \mu_{rk} \epsilon_0 \epsilon_{rk}}{2} \left[ \sqrt{1 + \left(\frac{\sigma_k}{\omega \epsilon_0 \epsilon_{rk}}\right)^2} - 1 \right]} \quad (4.6)$$

Reflected power depends on many dependent parameters like transmitting power  $P_t$ , phase  $\theta$ , relative permeability  $\mu_r$ , relative permittivity  $\epsilon_r$ , conductivity  $\sigma$ , operating frequency  $f$ , distance  $z$  and so on. Mathematically we can express as;

$$P_r = g(P_t, \epsilon_r, \mu_r, \theta, f, z, \dots) \quad (4.7)$$

Another parameter may also affect the GPR signal characteristics like

- Temperature
- Pressure
- Humidity or moisture content
- Instrumental noise .....

## 4.1 Noise Model:

Modeling of noise is a way by which we make a deterministic approach for a random signal. Since Noise is a random signal which has no fix amplitude, frequency and phase so for theoretical modeling of channel noise is a challenging issue . Some probable distribution schemes for noise exist which is applied for different application. Like wireless application we take Rayleigh distribution or Rician distribution for calculating the fading. Poisson distribution has very wide domain not only in communication but also in another engineering application. For a highly robust system what is the BER that can be a practical example of Poisson distribution. Similarly for detecting the target in case of GPR what type noise added with a signal or in another sense what type of distribution scheme of noise. Like AWGN noise

A—Additive (it added with signal)

W— White (For all frequency Noise power remains constant)

G— Gaussian (Probability distribution)

N— Noise (randomness in nature)

### 4.1.1 Gaussian distribution:

The PDF of the Gaussian distribution can be expressed as

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (4.8)$$

where  $\sigma$  and  $\mu$  are the standard deviation and mean of random variable respectively.

### 4.1.2 Rayleigh distribution

The PDF of the Rayleigh distribution can be expressed as

$$f(x, \sigma) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \quad x \geq 0 \quad (4.9)$$

Where  $\sigma > 0$  is the scale parameter of the distribution. The CDF is

$$F(x) = 1 - e^{-\frac{x^2}{2\sigma^2}} \quad \text{for } x \in [0, \infty] \quad (4.10)$$

### 4.1.3 Rice distribution:

The PDF of the Rician distribution can be expressed as

$$f(x|v, \sigma) = \frac{x}{\sigma^2} e^{-\frac{(x^2+v^2)}{2\sigma^2}} I_0\left(\frac{xv}{\sigma^2}\right) \quad (4.11)$$

Where  $I_0(z)$  is the modified Bessel function of the first kind with order zero. The CDF is

$$F(x) = 1 - Q_1\left(\frac{v}{\sigma}, \frac{x}{\sigma}\right) \quad (4.12)$$

where  $Q_1$  is the Marcum Q function.

### 4.1.4 Poisson distribution:

A discrete random variable X is said to have a Poisson distribution with parameter  $\lambda > 0$ , if, for  $k = 0, 1, 2, \dots$ , the probability mass function of X is given by

$$f(k; \lambda) = P_r(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (4.13)$$

Where  $e$  is the Eulers number ( $e = 2.71828\dots$ )

The positive real number  $\lambda$  is equal to the expected value of X and also to its variance

$$\lambda = E(X) = Var(X) \quad (4.14)$$

The Poisson distribution can be applied when the chances of occurrence of an event is very rare. This distribution scheme may be applied in time frame or space frame or other type of real world problem. How many such events will occur during a fixed time interval? Under the right circumstances, this is a random number with a Poisson distribution.

### 4.1.5 Chi distribution:

The PDF is

$$f(x; k) = \frac{2^{1-k/2} x^{(k-1)/2} e^{-x^2/2}}{\Gamma(k/2)} \quad (4.15)$$

$\Gamma(z)$  is the Gamma function in above PDF.

### 4.1.6 Beta distribution:

The PDF of the beta distribution, for  $0 \leq x \leq 1$ , and shape parameters  $\alpha, \beta > 0$ , is a power function of the variable  $x$  and of its reflection  $(1-x)$  like follows:

$$\begin{aligned}
 f(x; \alpha; \beta) &= \text{constant} \cdot x^{(\alpha-1)}(1-x)^{(\beta-1)} & (4.16) \\
 &= \frac{x^{(\alpha-1)}(1-x)^{(\beta-1)}}{\int_0^1 u^{(\alpha-1)}(1-u)^{(\beta-1)} du} \\
 &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{(\alpha-1)}(1-x)^{(\beta-1)} \\
 &= \frac{1}{B(\alpha, \beta)} x^{(\alpha-1)}(1-x)^{(\beta-1)}
 \end{aligned}$$

where  $\Gamma(z)$  is the Gamma function in above PDF. Above distribution scheme gives the brief idea about probability or randomness which is helpful in prediction of noise content.

## 4.2 Deterministic approach for static observation:

For our convention we take surface as a layered form. If series of SFCW pulses like Fig.3.10 transmitted then between two consecutive pulses series of reflected echoes are observed which has been shown in Fig.4.6. If we transmit  $N$  number of pulses in a certain period of time then for analyzing the returned signal we just do the averaging. Since very fast switching is not an easy work. Let after  $k$  times switching action is performed then mathematically it can be expressed as

$$k = NT \quad \text{or} \quad T = \frac{k}{N}$$

Finally our observation is performed on a single transmitted pulse and its returned echoes. From Fig.4.6 four returned pulses has been shown which may represent reflected voltage from four layered surface or may exist the other possibility.

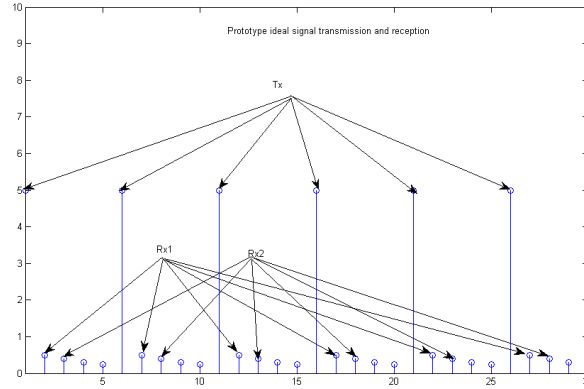


Figure 4.6: Prototype ideal pulse transmission and reception

### Experimental procedure for detection of target:

- First we have to set experimental setup in proper way.
- Horn antenna transmit EM wave and we get the returned signal information in terms of S11. Since single horn antenna is operated as a single port mode. The value of S11 is observed by vector network analyzer (VNA) which has been shown in Fig.4.7
- Since transmitting signal power is known and S11 is calculated by VNA [27] i.e. nothing but the voltage reflection coefficient. We can find the reflected voltage or reflected power, phase and frequency respectively from the VNA.

Mathematically reflection coefficient can be expressed as,

$$S11 = \Gamma = \frac{v^-}{v^+} \angle \theta \quad (4.17)$$

Where  $v^-$  is the reflected voltage and  $v^+$  is the incident or applied voltage and  $\theta$  is observed phase angle. Let  $\theta_1$  and  $\theta_2$  be the reference phase of transmitting signal (only for single frequency sinusoid) and the effective phase of reflecting signal then observed phase can be expressed as

$$\theta = \theta_2 - \theta_1 \quad (4.18)$$

- In case of SFCW radar signal we analyze the signal only for fix number of frequency. For our study we work on 128 frequency component (initial frequency 400MHz, step frequency 35MHz and final frequency 4845MHz).
- Based on these 128 frequency component, we find the amplitude as well as phase of the returned signal.
- From eqn.4.2 and eqn.4.4 we can observe that global reflection coefficient  $S_{11}$  and attenuation constant for different frequency. Effective permittivity also depends on frequency. Combining all information we can predict the target which is inside the ground. Still there is lots of signal processing required on these practical data for proper detection of target.



Figure 4.7: E5071C Vector Network Analyzer

### 4.3 Returned Signal Analysis:

In returned signal phase plays a vital role in the field of signal estimation. If two signals are added in, out of phase manner then resultant of the signal is to be very less where as in case of In phase addition resultant have higher value. There is no meaning of absolute phase of any signal. It is a relative quantity. The phase of any signal is measured with some reference signal. Let three sinusoidal signals are added in such a way that their operating



frequencies are same but amplitude and phases are different. Mathematically it can be expressed as

$$\begin{aligned} x(t) &= A_1 \cos(2\pi ft - \theta_1) + A_2 \cos(2\pi ft - \theta_2) + A_3 \cos(2\pi ft - \theta_3) \quad (4.19) \\ &= A_1 \cos(2\pi ft) \cos(\theta_1) + A_1 \sin(2\pi ft) \sin(\theta_1) + A_2 \cos(2\pi ft) \cos(\theta_2) + \\ &\quad A_2 \sin(2\pi ft) \sin(\theta_2) + A_3 \cos(2\pi ft) \cos(\theta_3) + A_3 \sin(2\pi ft) \sin(\theta_3) \end{aligned}$$

**In-phase component:**

$$x_I = A_1 \cos(\theta_1) + A_2 \cos(\theta_2) + A_3 \cos(\theta_3) \quad (4.20)$$

**Quadrature component:**

$$x_Q = A_1 \sin(\theta_1) + A_2 \sin(\theta_2) + A_3 \sin(\theta_3) \quad (4.21)$$

Since our reference signal  $x(t)$  is taken as cosine signal so cosine part will be our in-phase component and sine part will be our quadrature component. Similarly if our reference signal is a sine signal then sine part is the in-phase and cosine part is the quadrature component.

**Amplitude of resultant signal**

$$x = \sqrt{x_I^2 + x_Q^2} \quad (4.22)$$

**Phase of resultant signal:**

$$\theta = \tan^{-1} \left( \frac{x_Q}{x_I} \right) \quad (4.23)$$

**Representation of resultant signal:**

$$x_o = x \cos(2\pi ft + \theta) \quad (4.24)$$

Where,  $x_o$  is the observed resultant signal. In polar form above signal can be also expressed as follows

$$x_o = |x| \angle \theta \quad (4.25)$$

**Problem with resultant signal:**

1. In-phase as well as quadrature component depends on amplitude and phase of each individual sinusoid. From above analysis forward mapping of the signal is easy but it is difficult to make a mathematical model in reverse direction. From eqn.4.26 observed signal  $x_o$  depends on  $A_1, A_2, A_3, \theta_1, \theta_2, \theta_3$  and so on. Mathematically it can be expressed as

$$x_o = f(A_1, A_2, A_3, \theta_1, \theta_2, \theta_3) \quad (4.26)$$

So observed signal  $x_o$  can be estimated properly only; when we find the value of  $A_1, A_2, A_3, \theta_1, \theta_2, \theta_3$  properly. In case of reverse mapping we have not the sufficient number of equation to solve the problem. If we consider the linearly dependent equation then for finding the 6 variable we required 6 equations. Similarly for  $n$  number of variable we required  $n$  different equation. But we have only two equation in terms of amplitude and phase (eqn.4.22/4.23).

2. Another problem is that all equations are not linearly dependent for extracting the observed signals  $x_o$ .
3. There are different types of mathematical operation like linear algebraic, trigonometric, exponential, logarithmic e.t.c. If this type of operation comes into equation then there is no direct method to find the solution.
4. Above case is discussed in context to single frequency component. If multiple frequencies are taken into consideration then complexity arises. Like same manner we have thousands of unknown parameter and only limited numbers of mathematical solution exist.
5. SFCW signal carry finite number of frequency component so we work on those frequency component and try to estimate the phase frequency and amplitude of delayed signal.

#### Mathematical model of time domain phase of signal:

$$\phi(n) = \phi_0 + 2\pi\left(\frac{f}{f_s}\right)(n - 1) \quad (4.27)$$

Where  $\phi_0$  is the initial phase of the transmitted signal,  $f$  is the operating frequency of Tx signal and  $f_s$  is the sampling frequency and  $n$  is the sampling

index (only in case of single frequency). Where  $n=1, 2, 3, \dots, N$   
 According to Nyquist criteria we know that

$$f_s > 2f$$

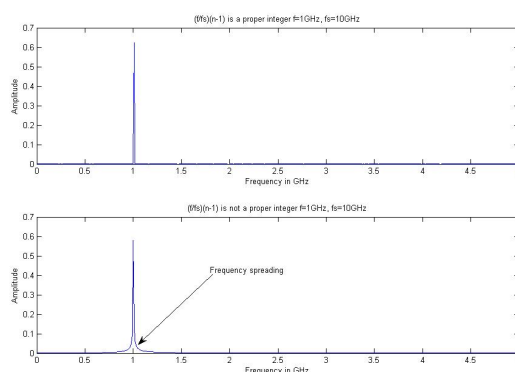


Figure 4.8: Frequency spreading due to selection of improper number of sample

## NOTE1

- In practical scenario, for proper reconstruction of transmitted analog signal, sampling frequency should be greater than 3 times to 4 times of message signal.
- If  $(f/f_s)(n-1)$  is not a proper integer then the resultant spectrum of signal spreaded which is shown in Fig.4.8.
- If  $N_1 > N_2$  then for  $N_1$  number of sample gives good amount of frequency resolution in comparison to  $N_2$  number of samples.

## NOTE2

### Merit and Demerit of Zero Padding:

- It increases the frequency resolution.
- Unwanted harmonics arises surrounded the center frequency.

# Chapter 5

## Delay Measurement and Frequency Domain Analysis

### 5.1 Correlation

Correlation shows the degree of similarity between two signals. It doesn't show the degree of exactness. There are two types of Correlation termed as

- **Auto-Correlation**

Auto-correlation shows the degree of similarity of a signal with itself. Degree of correlation is expressed in terms of correlation coefficient.

- **Cross-Correlation**

It shows the similarity between two different signals. In signal processing, cross-correlation measures the similarity between two signals as a function of a time-lag or in terms of space-lag applied to one of them. This is also known as a sliding dot product. It is commonly used for searching a long signal for a shorter, known feature.

The cross-correlation between a pair of signals  $x(n)$  and  $y(n)$  is given by

$$\gamma_{xy}(l) = \sum_{n=-\infty}^{+\infty} x(n)y(n-l) \quad l = 0, \pm 1, \pm 2, \dots \quad (5.1)$$

The index  $l$  is the shift lag parameter. The order of subscript  $xy$  indicates that  $x(n)$  is the reference sequence that remains not shifted in time whereas

the sequence  $y(n)$  is shifted  $l$  units in time with respect to  $x(n)$ . We can also write Eq.(5.1) as follows

$$\gamma_{xy}(l) = x(l) * y(-l) \tag{5.2}$$

Mathematically the normalized cross correlation sequence is as follows

$$\rho_{xy}(l) = \frac{\gamma_{xy}(l)}{\sqrt{\gamma_{xx}(0)\gamma_{yy}(0)}} \tag{5.3}$$

where  $\rho_{xy}(l)$  is a cross-correlation coefficient. Its value always lies between -1 to +1. A value 0 for cross correlation means no correlation.

### 5.1.1 Time Domain Delay Measurement:

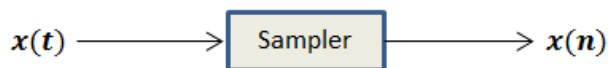


Figure 5.1: Simple block diagram of sampling

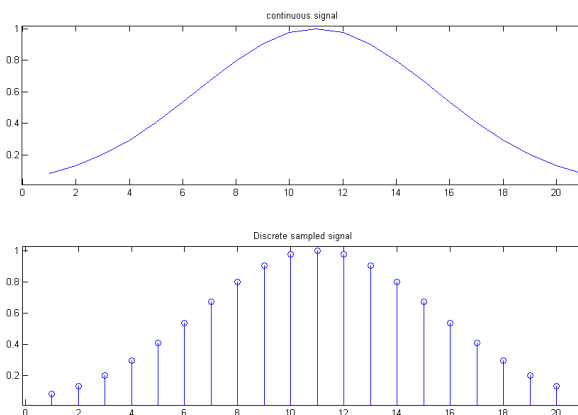


Figure 5.2: Continuous vs Discrete Signal

In case of GPR, correlation is performed between transmitted and received signal. We keep the track of transmitted sampled signal and try to

find the delay of received signal. If returned signal shows similarity with transmitted one than we say it is our genuine returned signal otherwise returned signal is noise. For performing this operation we use a system clock which is synchronized with transmitter as well as receiver. Correlation checks the pattern or shape of signal. If  $x(t)$  is the continuous time domain signal, which is sampled at the sampling rate  $f_s$  i.e.  $f_s = \frac{1}{T_s}$  then mathematically it can be expressed as

$$x(t) = \sum_{n=0}^N x(nT_s) \tag{5.4}$$

Where  $n$  is the sampling index which varies from 0 to  $N$  or in another sense  $N$  is the length of observation frame. Correlation is measured in terms of correlation coefficient where shift lag parameter  $l$  gives the information regarding delay. For better understanding we can visualize these things using below example

Ex- Let  $x=[1\ 2\ 3\ 4\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$  be the sampled transmitted sequence and  $y=[0\ 0\ 0.3\ 0.4\ 0.5\ 0.6\ 0\ 0\ 0\ 0\ 0]$  be the received sampled sequence, both sequence are highly correlated where  $x$  is the reference signal and  $y$  is the delayed version of  $x$ . Linear correlation sequence is as follows;

$$\begin{aligned} \gamma_{xy}(l) &= [0\ 0\ 0\ 0\ 0\ 0.6\ 1.7\ 3.2\ 5.0\ 3.8\ 2.5\ 1.2\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \\ \gamma_{xx}(0) &= 30 \\ \gamma_{yy}(0) &= 0.86 \end{aligned}$$

$$\rho_{xy}(-2) = \frac{5}{30 \times 0.86} = 0.9841 \approx 1$$

From above example if sampler samples the continuous signal at rate of 1MHz (sample duration= $1\mu s$ ) and if  $x$  is our transmitted sampled sequence and  $y$  is the received sampled sequence and at shift lag  $l=-2$  we get nearly 100% correlation then we conclude that after  $2\mu s$  reflected signal arrives at receiver end. Above example shows the degree of similarity not degree of exactness.

**NOTE:**

- Length of received sampled sequence should be higher in comparison to transmitted sampled sequence it is a general convention.

Cross- correlations are also useful for determining the spatial range in case of ground penetrating radar. If two targets exist inside the ground then reflected signal from upper target reaches early in comparison to lower one.

**Cross-correlations** are useful for determining the time delay between two signals, e.g. for determining time delays for the propagation of electromagnetic signals across a microphone array. After calculating the **cross-correlation** between the two signals, the maximum (or minimum if the signals are negatively correlated) of the cross-correlation function indicates the point in time where the signals are best aligned, i.e. the time delay between the two signals is determined by the argument of the maximum.

### 5.1.2 Practical difficulty in delay measurement:

- Noise is a key factor which influences the reflected signal and when we sample the (*signal + noise*) then delay may be different or irrespective to the true delay.

$$y_{received} = S + N \tag{5.5}$$

Ex-  $x = [1 \ 2 \ 3 \ 4 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$ ,  $S = [0 \ 0 \ 0.3 \ 0.4 \ 0.5 \ 0.6 \ 0 \ 0 \ 0 \ 0 \ 0]$  and  $N = [0.1119, 0.1032, 0.0111, -0.1143, 0.2442, -0.3001, 0.1009, -0.0012, 0.1041, 0.0351, 0.1123]$

$y_{received} = [0.1119 \ 0.1032 \ 0.3111 \ 0.2857 \ 0.7442 \ 0.3000 \ 0.1009 \ -0.0012 \ 0.1041 \ 0.0351 \ 0.1123]$

$\gamma_{xy_{received}}(l) = [0.1123 \ 0.2597 \ 0.5112 \ 0.7615 \ 0.5512 \ 0.9146 \ 1.6421 \ 3.0777 \ 4.3151 \ 4.5593 \ 2.3944 \ 1.7778 \ 0.7485 \ 0.4476 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$

$$\gamma_{xy}(-1) = \frac{4.5593}{(30 \times 0.8803)} = 0.8872$$

$$\gamma_{xy}(-2) = \frac{4.3151}{(30 \times 0.8803)} = 0.8396$$

Above correlation coefficient shows that after one unit delay we get maximum correlated output nearly 89% from the transmitted one whereas at 2 unit delay we get 84% correlated output, which create ambiguity. But in received sampled signal it shows 2 unit delays. This happens due random behavior of noise.

### 5.1.3 Solution for delay error

At this stage we should have proper information about distribution of sampled noise. Various type of distribution scheme as well as signal attenuation has been already discussed in above chapter. These distribution schemes are applicable for the modeling of noise. In case of GPR when signal get reflected then due to presence of noisy channel signal attenuated as well as distorted. To remove unwanted effect we should do some mathematical operation with signal in such a way that it counter balance or nullify the effect of noise.

$$\Delta N = N - N_0 \quad (5.6)$$

Where  $N$  is the observed noise and  $N_0$  is the modeled noise. Like from above equation we design a suitable  $N_0$  such that  $\Delta N$  became as minimum as possible. Still there are also lots of complexities at receiver end to extract multiple features from the GPR signal. For minimizing or nullifying the value of  $\Delta N$  different types of optimization techniques are used.

## 5.2 Windowing Technique

Windowing is a time domain operation which is used for suppressing the side lobe near the center frequency. There may be various reasons for generation of side lobes.

- Due to delay in signal which has also been discussed in next session.
- Improper number of selection of samples, which has also been already mentioned previously in Fig.4.8
- Effect of white noise, which is random in nature.

Windowing technique is a good substitute for making frequency response sharper which produces good visual impact. For different application different types of window function like Hamming, Hanning, Bartlett, Kaiser Window and so on are used in signal processing domain. In GPR application we process the returned signal with suitable window function. Hamming and Hanning window is the moderate window whereas for Kaiser Window we have to set suitable value of  $\beta$ . Mathematically, Hanning window function can be expressed as

$$\omega(n) = .5 + .5\cos\left(\frac{2\pi n}{N-1}\right) \quad (5.7)$$



Where  $n$  is the sampling index and  $N$  is the total number of sample.

Kaiser Window can be expressed as

$$\omega(n) = \frac{I_0[\beta \sqrt{1 - (\frac{2n}{N-1})^2}]}{I_0(\beta)} \quad (5.8)$$

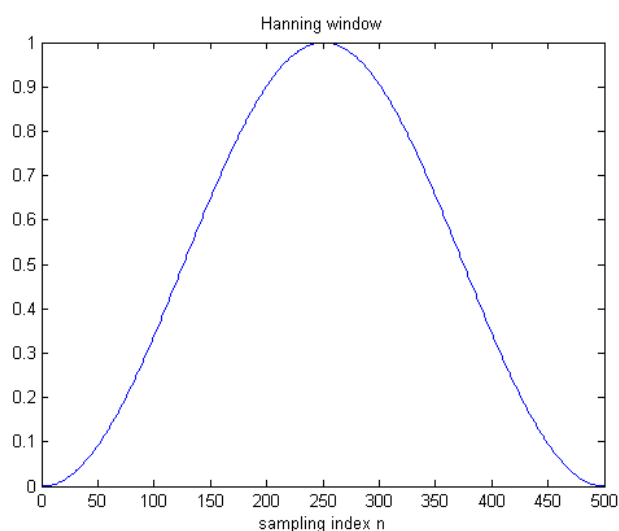


Figure 5.3: Hanning Window

Resultant time domain signal after window operation can be expressed as [28]

$$y(n) = y_r(n)\omega(n) \quad (5.9)$$

Windows are also used in the digital filters design, especially to convert an ideal impulse response of infinite duration, such as a sinc function, to a finite impulse response (FIR) filter design.

Current research is based on two types of observation

**Case 1: when single frequency is transmitted and it is received after some delay**

Fig.7.2 is a simple sinusoidal signal and its frequency response. In Fig.7.2 frequency response shows the delta function across the frequency of 1GHz

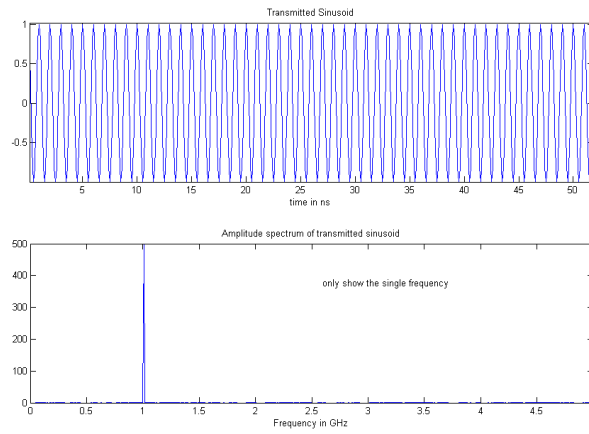


Figure 5.4: Sinusoidal signal and its spectrum

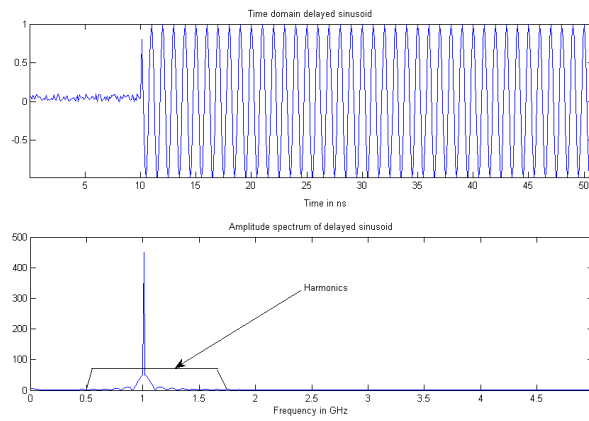


Figure 5.5: Delayed sinusoid and its spectrum

having no side lobes or in another term no spreading occurred in frequency domain.

Fig.7.3 shows the delayed sinusoid and its impact on frequency domain. We observe the frequency spreading near 1GHz frequency which is not like a transmitting frequency response.

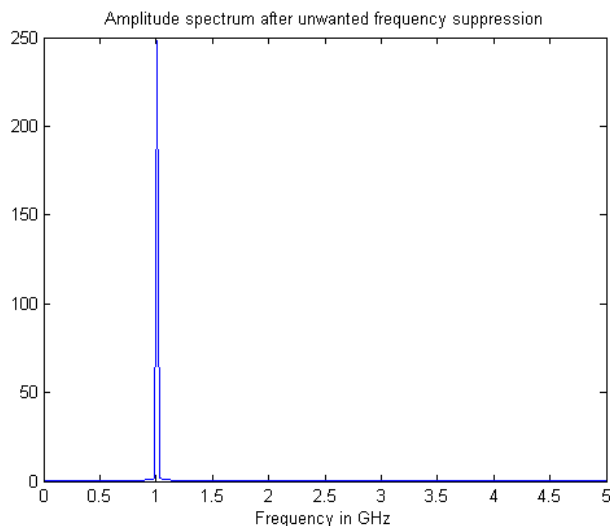


Figure 5.6: Unwanted frequency suppression

Windowing techniques suppress the unwanted frequency component. Fig.7.4 shows the frequency response of suppressed unwanted frequency.

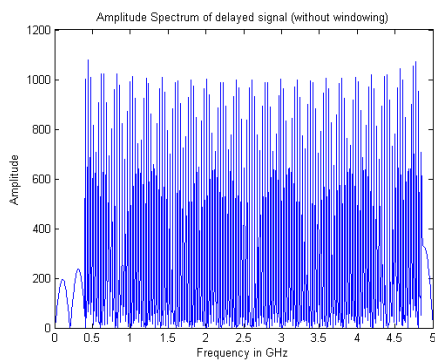


Figure 5.7: Spectrum of delayed SFCW radar signal

**Inter-modulation Distortion:**

When signal having multiple frequencies share the same transmission medium, then higher frequency component reaches early with respect to lower one at the receiver end which causes inter-modulation distortion.

SFCW signal consist of multiple frequencies which is applicable in for GPR signal transmission so inter-modulation distortion arises during SFCW signal reception.

**Case 2: When multiple frequencies (SFCW signal) are transmitted and received after some delay**

Fig.7.5 shows the delayed SFCW spectrum of previously taken example. Transmitted signal consist of 128 frequency in stepped manner (400MHz-35MHz-4845MHz). Due to effect of delay unwanted frequency added which can be observed in Fig.7.5.

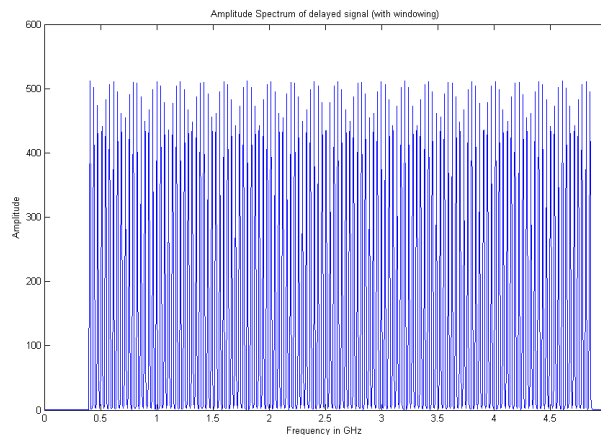
**Unwanted frequency suppression using windowing technique:**

Figure 5.8: Unwanted frequency suppression

**Pros and Cons of window function:**

Data truncation adds ringing to the time domain data and the resulting side lobes are high enough that they could obscure some responses of the device

under test (DUT). A windowing function can be applied which gradually reduces the frequency response and controls the side lobes created during the truncation process. However, the windowing function tends to reduce the sharpness of the response, spreading pulses, and stretching out slopes, thereby reducing the resolution of the transform and distorting the transitions of the frequency response. There is a trade-off between side lobe height and resolution when determining the windowing function.

### 5.3 Time domain gating:

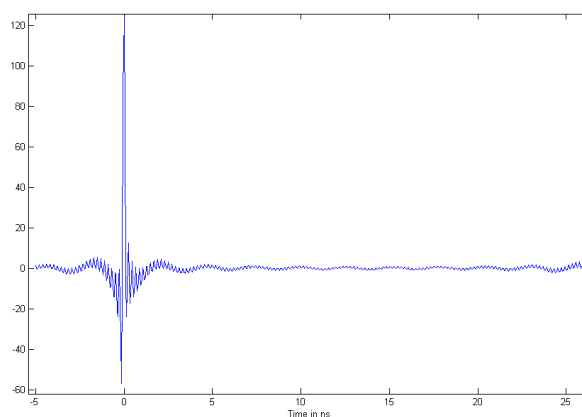


Figure 5.9: Time domain gated output

Fig.7.7 shows the time domain gated output of Fig.3.10 whereas Fig.7.8 is the amplitude spectrum of Fig.7.7. From spectrum of the gated output we observe fluctuation of power level across the corner of the extremities (towards 400MHz and 4845MHz). The original frequency domain data (magnitude and phase) are transformed using an Inverse Fourier Transform to give

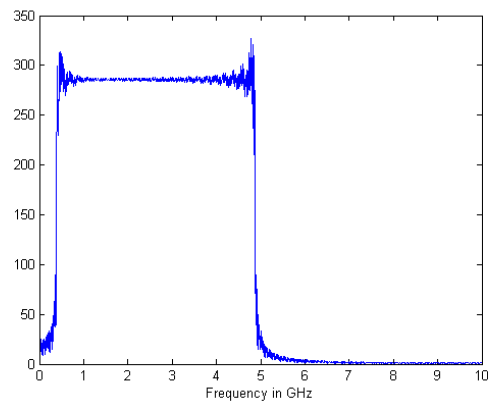


Figure 5.10: Amplitude spectrum of single pulse format SFCW signal

a time domain impulse response. The time domain impulse response is then gated using a modified Hanning window. The modified Hanning window has an initial rectangular window shape, but the late-time portion is a Hanning (cosine squared) taper. This allows the high frequency portion of the impulse response to be maintained, while smoothing the late-time effects of the gate truncation [27].

# Chapter 6

## Signature Analysis

### Signature Analysis

For same electrical specification of two targets, if it differs in their geometry or orientation then received powers are also different for both targets. From radar equation maximum detectable radar range can be expressed as

$$R^4 = \frac{P_t G_t G_r A_e \sigma F^4}{(4\pi)^2 P_r} \quad (6.1)$$

Where  $P_r$ =Received power

$G_t$ = Gain of transmitting antenna

$P_t$ = Transmitted power

$G_r$ = Gain of receiving antenna

$A_e$ = Effective aperture area of receiving antenna

$\sigma$ = RCS or scattering coefficient of the target

$F$ = Pattern propagation factor

$R$ = Distance from transceiver to target (since in case of GPR both transmitter and receiver are kept closely)

Modified radar equation, now becomes [20]

$$R^4 = \frac{P_t G A \rho_a \sigma n F^4 E_i(n) e^{-2\alpha R_{max}}}{(4\pi)^2 K T_0 F_n (B\tau) f_p (S/N) P_r L_s L_f} \quad (6.2)$$

Where  $\rho_a$ = Antenna aperture efficiency

$n$ = Number of pulse integrated

$k$ = Boltzmann constant

$F_n$ = Receiver noise figure

$B$ =Receiver bandwidth, Hz

$\tau$ =Pulse width, s

$f_p$ =Pulse repetition frequency, Hz

$L_f$ = Fluctuation loss

$L_s$ =System loss

From eq.6.1 and eq.6.2 received power of radar directly depends on radar cross section. If flare section of horn antenna is towards the tip of cone then the amount of reflected power is to be quite less in comparison to circular cross sectional part of cone.

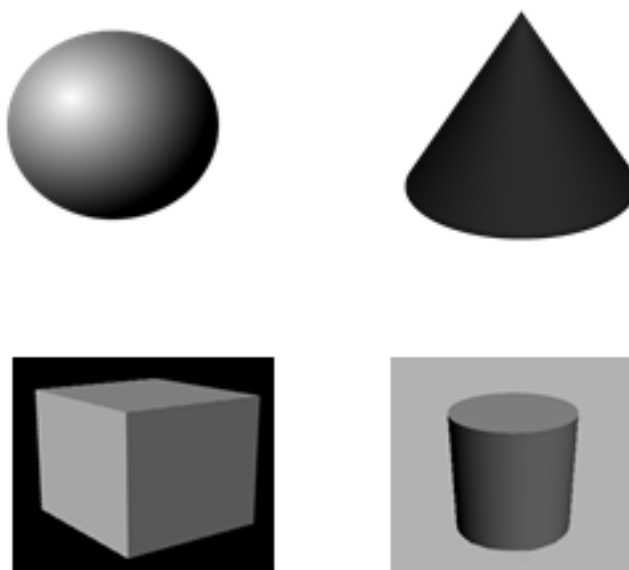


Figure 6.1: Some probable shape of target

Radar cross section is a virtual area of target observed by the receiver because inside ground determination of shape is a challenging issue. Radar cross section is a property of target reflectivity. For lab experiment we choose some suitable shapes of target like sphere, cylinder, cube, cuboid, cone e.t.c. We measure reflected power pattern and repeat same experiment several times. By averaging that received data we make a conclusion regarding the shape of particular material.

At a frequency of 3GHz (S band) a corner reflector (or flat plate) of physical area  $1m^2$  has a radar cross section of  $1000m^2$ . On the other hand a cone-sphere with  $1m^2$  projected area has a radar cross section at 3GHz equal to



$0.001m^2$ . (Based on the approximation  $\sigma \cong 1\lambda^2$  )

The RCS of a radar target is the hypothetical area required to intercept the transmitted power density at the target such that if the total intercepted power were re-radiated isotropically, the power density actually observed at the receiver is produced.

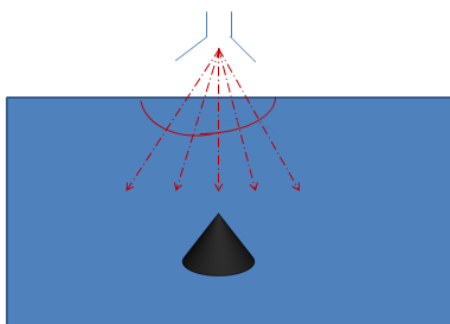


Figure 6.2: Target having conical shape

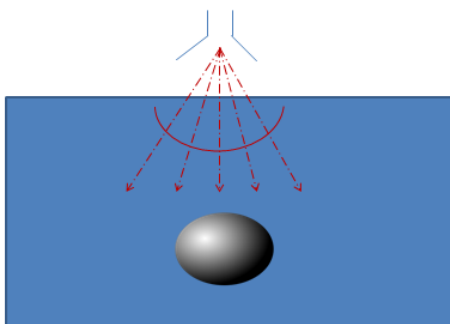


Figure 6.3: Target having spherical shape

Fig.(6.2-6.5) is a pictorial representation of different shape of target inside the mono dielectric media. Complexity arises when we work on real ground or surface.

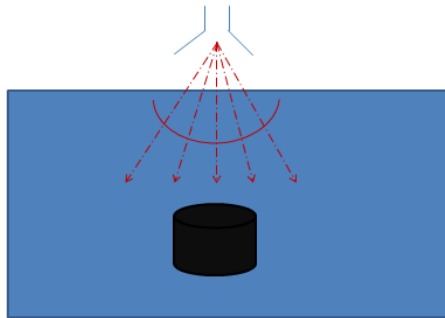


Figure 6.4: Target having cylindrical shape

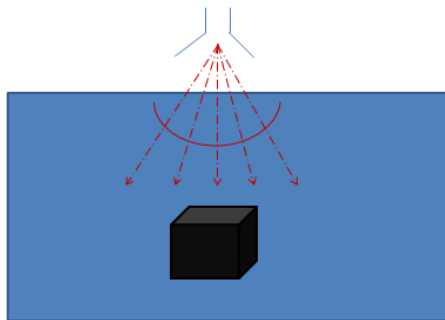


Figure 6.5: Target having cubical shape

# Chapter 7

## Conclusion and Future Work

### 7.1 Conclusion

For surface parameter extraction lots of variation observed in time and frequency domain . Both are essential for calculating delay, unambiguous range, range resolution etc. Still, there are lots of problem in GPR signal transmission and reception. There exist infinite set of geometry of surface as well as cross sectional area of target and also there is a trade of between System performance and system complexities. If we process the data by taking huge number of sample we observe better result but system complexities also increases.

In time domain delay measurement cross correlation can give better result if signal has pulsed nature. Due to delay, frequency of the receiving signal spreaded. So windowing technique resolve the problem of spreading of frequency. There is also a trade of between frequency and time. If we suppress the signal spreading in time domain then we require UWB.

### 7.2 Future Work

There are few areas where author believes the future work should be undertaken;

- Phase error
- Frequency error

- Error in power level
- Optimization of error
- Estimation of dependent variable of a signal
- To learn the hi-tech instrument made by Agilent, National instrument , GSSI and so on.

All observation deals the theoretical aspect of ground as well as signal. Author wants to work on practical data and learning of data acquisition and data processing with real time system.

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