Estimation of DOAs of Acoustic Sources in the Presence of Sensor with Uncertainties

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Estimation of DOAs of Acoustic Sources in the Presence of Sensors with Uncertainties

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By

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Declaration

I hereby declare that

1) The work presented in this paper is original and has been done by myself under the guidance of my supervisor.
2) The work has not been submitted to any other Institute for any degree or diploma.
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Anurag Patra

30th may 2014

Department of Electronics & Communication Engineering

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CERTIFICATE

This is to certify that the Thesis Report entitled "Estimation of DOAs of Acoustic Sources in the Presence of Sensors with Uncertainties" submitted by ANURAG PATRA bearing roll no. 212EC6193 in partial fulfilment of the requirements for the award of Master of Technology in Electronics and Communication Engineering with specialization in "Signal and Image Processing" during session 2011-2013 at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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Abstract

Direction of Arrival (DOA) estimation finds its practical importance in sophisticated video conferencing by audio visual means, locating underwater bodies, removing unwanted interferences from desired signals etc. Some efficient algorithms for DOA estimation are already developed by the researchers . The performance of these algorithms is limited by the fact that the receiving antenna array is affected by some uncertainties like mutual coupling, antenna gain and phase error etc. So considerable attention is there in recent research on this area.

In this research work the effect of mutual coupling and the effect of antenna gain and phase error in uniform linear array (ULA) on the direction finding of acoustic sources is studied. Also this effect for different source spacing is compared. For that, estimates of the directions of arrival of all uncorrelated acoustic signals in the presence of unknown mutual coupling has been found using conventional Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT). Also DOAs are computed after knowing the coupling coefficients so that we can compare the two results. Simulation results have shown the fact that the degradation in performance of the algorithm due to mutual coupling becomes more if the sources become closer to each other. Also we have estimated DOAs in the presence of unknown sensor gain and phase errors and we have compared this results with the results we got by considering ideal array. Finally in this case also the effect of gain and phase error as the source spacing varies has been tested. Simulation results verify that performance degradation is more if the sources become closer.

Keywords: gain error, phase error, ESPRIT algorithm, mutual coupling, acoustic signal uniform linear array (ULA), source spacing

TABLE OF CONTENTS

List of Figures

Abbreviations

DOA – Direction of Arrival

MUSIC - Multiple SIgnal Classification

ESPRIT- Estimation of SIgnal Parameters via Rotational Invariance Technique

EVD - Eigen Value Decomposition

DSB - Delay and Sum beamformer

GCC - generalized cross correlation

MCM - Mutual Coupling Matrix

PHAT - Phase Transform

TDE - Time Delay Estimate

SV- Steering Vector

MLM - Maximum Likelihood Method

CRLB - Cramer Rao Lower Bound

SNR- Signal to Noise Ratio

RMSE - Root Mean Square Error

ULA –Uniform Linear Array

URA - Uniform Rectangular Array

TM – Toeplitz Matrix

AWGN – Additive White Gaussian Noise

Chapter 1

Introduction

1.1 Motivation

Direction of arrival (DOA) estimation for Speech or acoustic signals with mouthpiece cluster has various applications now a days. Estimations of DOAs with the assistance of mouthpiece might be utilized to direct cameras to the speaker in video conferencing session or a long distance classroom [1]. The working of the camera might be took care of in one of three routes in present video conferencing frameworks or long separation feature classrooms,. Cameras that give distinctive settled perspectives of the room might be put at diverse areas in the meeting room to cover all the individuals. Furthermore the framework could comprise of one or two cameras worked through people. Finally the framework could comprise of switches those are operated manually for every client or gathering of clients that might control the camera toward them when actuated. Third class of frameworks is utilized regularly within long separation training that uses TV based classrooms. These frameworks end up being costly as far as additional equipment or labor needed to work them successfully and dependably. It might be attractive to have a few cameras that could be naturally controlled to turn to the speaker. Most gatherings and classrooms commonly have one man talking at once and all others tuning in. The speaker, also can move within the room. So it is needed to have a framework that successfully and dependably spots and tracks a solitary speaker. Single speaker confinement and following could be performed utilizing either visual or acoustic information. An exhaustive following framework utilizing video data was created by Wren et al. [2]. However, the algorithmic multifaceted nature and computational burden needed for such a framework infers, to the point that a completely devoted workstation be devoted to performing this task. Methods focused around acoustic information are commonly far easier as far as multifaceted nature and computational burden.

Another requisition of DOA estimation utilizing mouthpiece is a part of speech enhancement for human workstation interfaces that rely on upon speech inputs from human being [3]. Techniques utilized here, in the same way as superdirective beamforming, rely on upon faultless assessments of the DOA of the acoustic signals. The same is the situation in portable hearing assistants that utilize versatile beamforming to catch acoustic signals in the presence of noise.

In a radar framework DOA estimation is of incredible significance keeping in mind the end goal to spot the signal landing from a potential target. Electronic guiding might replace the mechanical directing for this situation of the radar antenna. Also in wireless communication environment determination of the coveted client at the base station might bring about minimizing the obstruction from different users. This may be carried out by heading the principle beam of the antenna towards the wanted client or regulating nulls towards the meddling clients.

1.2 Objective

The objective of this research is to study the effect of antenna array uncertainties like mutual coupling and antenna gain and phase error on the high resolution direction finding algorithms. Also effect of these uncertainties as the spacing between sources varies has been shown.

1.3 Outline

This thesis is divided into the following chapters in order to explain the works done and showing the results in a lucid manner.

Chapter 2 covers the basic antenna theory with a emphasis on sensor uncertainties between elements of an antenna array. After that the basic subspace based popular DOA estimation methods has been discussed.

Chapter 3 gives a glimpse of current research on this area by various scientists.

In Chapter 4 the modified ESPRIT algorithm in the presence of mutual coupling has been given and degradation in performance as the sources becomes closer has been studied.

In Chapter 5 the modified ESPRIT algorithm in the scenario of unknown antenna gain and phase errors has been given and the effect of these uncertainties on the estimation process for different source spacing has been given.

Chapter 6 , the last chapter of this thesis gives the conclusion, summary of the work done and some future scopes of this project.

Chapter 2

Fundamentals of DOA estimation

2.1 Basic theory of antenna array

The electric field variation with angle (θ,ϕ) (i.e radiation patterns) of solitary element antennas are broad normally., means directivity (gain) is relatively less. In the scenario of communication in long distance, radiators having high directivity are badly required. (This is true for receiving antennas too. For them it is required that they can only receive a particular signal from a particular direction to avoid picking up unwanted signal.) Such antennas are feasible to form by increasing the area of the radiating slot (size larger than λ). Multiple side lobes can appear however by adopting this technique. Besides, the size of the antenna becomes large and very problematic to manufacture. Another procedure to enhance the electrical size of a radiator is to form it as an Array of radiating elements in a efficient configuration –array antenna. Generally, the characteristics of array elements are same. But this is not indispensable always but it is practical and easier for manufacturing. The individual elements of the system may be of any shape like loops, wire dipoles, apertures, etc.

The total field of an array can be calculated by vector superposition of the fields radiated by a single antenna.(For receiving array same logic can be applied). For very directional pattern, it is must that the individual fields (emitted by the individuals) add up by constructive interference occur in the desired direction and destructive interference happens in the remaining space.

Five methods for shaping the beam pattern of antenna array:

a) the shape of the array (linear, rectangular, circular, spherical, etc.),

b) the position of the individuals with respesct to others,

c) the driving magnitude of individual elements,

d) the phase of the driving input of each member,

e) the radiation pattern of individuals

2.1.1 Two-elements antenna array

Let s give the electric fields in the far field of the antenna array in the following form

$$\boldsymbol{E_1} = M_1 E_{n1}(\theta_1, \phi_1) \frac{e^{-j(k_{r1} - \frac{\beta)}{2}}}{r_1} \widehat{p_1} \tag{2.1}$$

$$\boldsymbol{E_2} = M_2 E_{n2}(\theta_2, \phi_2) \frac{e^{-j(k_{r1} - \frac{\beta)}{2}}}{r_2} \widehat{p_2} \tag{2.2}$$

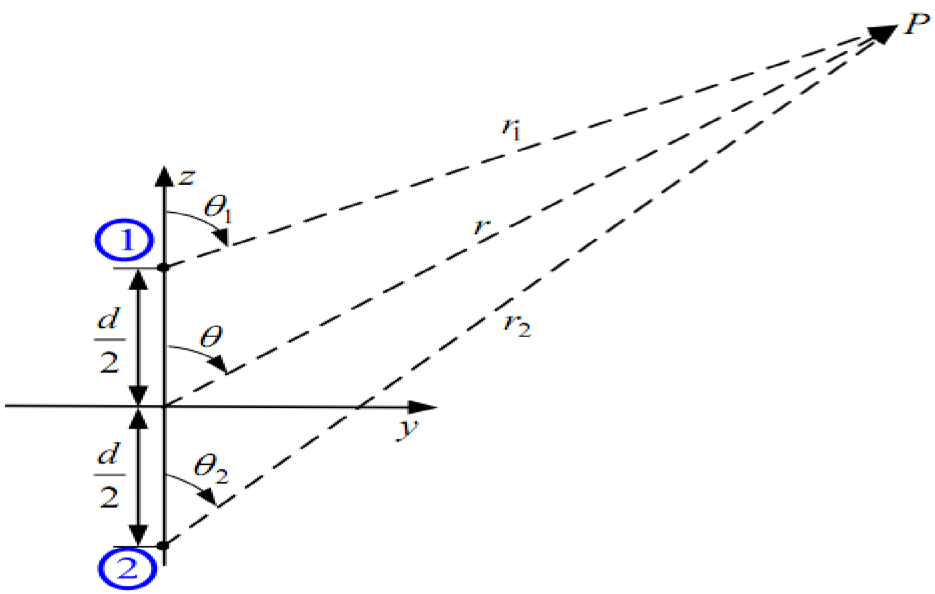

Fig 2.1: Two element antenna array

Here:

M_1 , M_2 : magnitudes of the electric fields(do not include the 1/r factor);

E_{n1} , E_{n2} : field patterns (normalized);

r_1 , r_2 : distances from the 1st and 2nd element to the observation point P;

β : difference in phase between the feed or excitation of the two array elements;

$\widehat{p_1}$, $\widehat{p_2}$: polarization vectors associated with the far-zone E fields.

The far field pattern for this two element antenna array can be given as

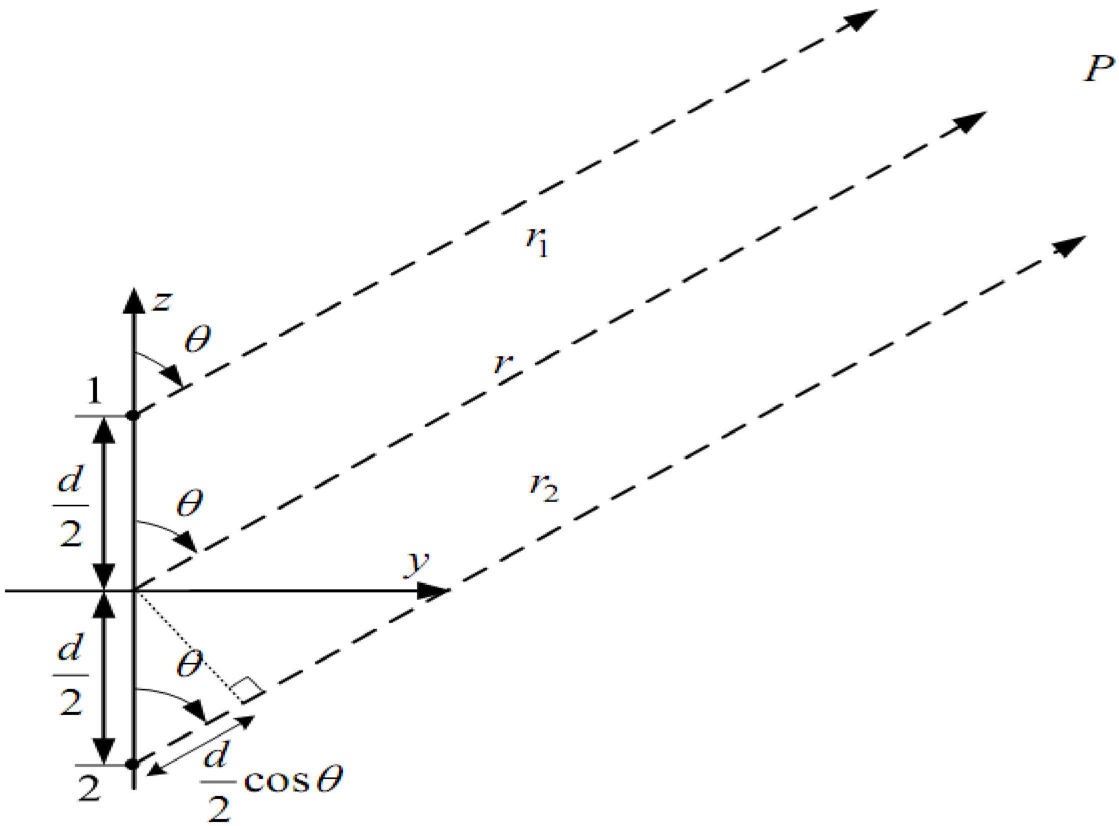

Fig 2.2: Far field approximation of two element array

Assumptions:

1) the array elements are identical so

$$E_{n1}(\theta_1, \phi_1) = E_{n2}(\theta_2, \phi_2) = E_n(\theta, \phi)$$

2) their polarization are same

$$\widehat{p_1} = \widehat{p_2} = \hat{p}$$

3) their excitation amplitudes are same so

$$M_1 = M_2 = \mathrm{M}$$

The total field is

$$\mathrm{E}=E_1 + E_2 \tag{2.3}$$

$$=\hat{p}M\,\frac{\boldsymbol{e^{-jkr}}}{\boldsymbol{r}}\,E_n(\theta,\phi) \times 2\cos(\frac{kdcos\,\theta+\beta}{2}) \tag{2.4}$$

So from the equation it can be inferred that the total field due to radiation from the array is the multiplication of individual field of an element and array factor(AF).

$$\mathrm{AF}=2\cos(\frac{kdcos\,\theta+\beta}{2}) \tag{2.5}$$

Using the normalized field pattern of a single element $E_n(\theta,\phi)$ and the normalized AF,

$$AF_n= \cos(\frac{kdcos\,\theta+\beta}{2}) \tag{2.6}$$

The normalized field variation of the whole antenna array is given as their product:

$$\boldsymbol{f}_{\boldsymbol{n}}(\theta,\phi) = E_n(\theta,\phi) \times AF_n(\theta,\phi) \tag{2.7}$$

This is the pattern multiplication rule which can be applied for antenna array with same type of elements. This formula can be extended for antenna array with N elements also.

2.1.2 *Steering matrix*

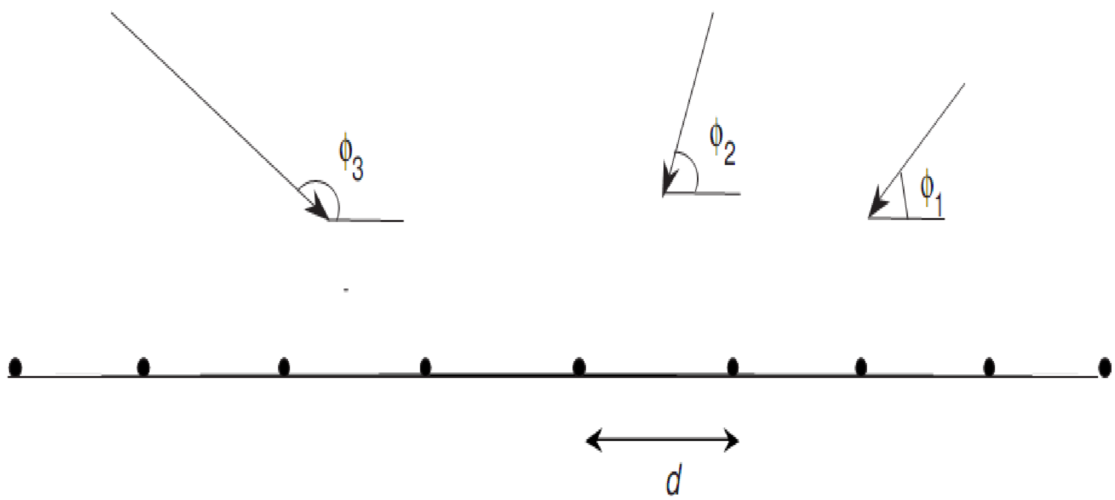

Fig 2.3: Set up for steering vector determination

Suppose this is the sensor array. For far field approximation the delay time between the first or leftmost individual and the next elements (2,3,4,5) are $\frac{dcos\phi}{c}, \frac{2dcos\phi}{c}, \frac{3dcos\phi}{c}$ $etc.$

Suppose the signal received by the 1st individual is $u_1(t) = s(t)$. So

$$u_2(t) = s(t)e^{-jw\frac{dcos\phi}{c}} = s(t)e^{-j2\pi\frac{dcos\phi}{c}} \tag{2.8}$$

In the same way

$$u_3(t) = s(t)e^{-jw\frac{2dcos\phi}{c}} = s(t)e^{-j2\pi\frac{2dcos\phi}{c}} \tag{2.9}$$

$$u_4(t) = s(t)e^{-jw\frac{3dcos\phi}{c}} = s(t)e^{-j2\pi\frac{d3cos\phi}{c}} \tag{2.10}$$

$$u_5(t) = s(t)e^{-jw\frac{4dcos\phi}{c}} = s(t)e^{-j2\pi\frac{d4cos\phi}{c}} \tag{2.11}$$

By combining these equations we can write

$$\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ u_5(t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j2\pi\frac{d cos\phi}{c}} \\ e^{-j2\pi\frac{d2cos\phi}{c}} \\ e^{-j2\pi\frac{d3cos\phi}{c}} \\ e^{-j2\pi\frac{d4cos\phi}{c}} \end{bmatrix} s(t) \tag{2.12}$$

$$u(t) = a(\phi)s(t) \tag{2.13}$$

Where $a(\phi) = \begin{bmatrix} 1 \\ e^{-j2\pi\frac{d cos\phi}{c}} \\ e^{-j2\pi\frac{d2cos\phi}{c}} \\ e^{-j2\pi\frac{d3cos\phi}{c}} \\ e^{-j2\pi\frac{d4cos\phi}{c}} \end{bmatrix}$

$a(\phi)$ is called steering vector or array manifold or steering vector.

2.1.3 Error or uncertainties in antenna array

Mutual coupling(M.C) is the unwanted interaction between the individual elements in an antenna array. This is actually the coupling of power between neighbouring elements. The effect of mutual coupling is severe if the inter-element distance is less. M.C has mainly three effects:

i) alter the array radiation pattern

ii) alter the array manifold

iii) alter the the input Impedances.

Eigen- structure based direction-estimating techniques such as MUSIC require exact information about the signals received by the sensor array from a source located at any angle. The performance of the eigenstructure based system depends strongly on the accuracy of this steering matrix or array manifold. Calibrating an antenna array system designed for two-dimensional (azimuth and elevation) direction finding with the precision required by these superresolution techniques have various practical problems. There is the problem of maintaining array Calibration also in addition to the problem of initial array calibration. Also due to numerous causes the response of the array changes with time.

1) Changes in the characteristics of the elements itself
2) Changes in the electronic components between the array and the outcome of the encoder (i.e temperature changes, aging of components.),
3) changes due to the ambience surrounding the sensor array
4) changes in the relative positions of the sensing elements (e.g., an antenna array located on the vibrating wing of an aircraft).

These factors greatly reduce the performance of the super resolution DOA Estimation techniques .Sometimes its efficiency is so poor that it gives worse result than the conventional DOA estimation methods.

2.2 Basic principles for DOA estimation

The basic theory of direction estimation using antenna or microphone arrays is to use the phase information that is present in signals received by sensors (microphones/antennas) that are separated in space. When the microphones/antennas are spatially separated, the signals or waves impinge on them at different time. For an known array geometry, the DOA of the signal itself defines these time delays or in other way it can be said that those delays in time are dependent on the signal DOAs. All methods that utilizes this logic to estimate the DOA can be classified under three heads[4].

The first category comprises of the steered beamformer employed methods. The Beam formers gather the waves from all the array-sensors in such a way that the array output emphasizes waves or signals from a certain desired direction or “look”-direction or the most probable direction. Thus if a signal is Arriving from the look-direction, the output power signal of the array output is high and if there is no signal coming from the look-direction the low array output power results. Hence, the help of the array can be taken for construction of beam formers that “look” in all probable angles and the estimate of the DOA is the angle that gives the maximum power. The simplest kind of beamformer that can be implemented is the delay and sum beam former (DSB) . The main pros of a steered beamformer employed procedure is that with one set of computations it is possible to estimate the desired angles of all the emitters that are emitting signals ,impinging on the array. So it is inherently very much applicable for detecting multiple emitters. From the theory of the eigen-values of the spatial correlation matrix, if we have N elements in an array, detection of more than N-1 independent sources is not feasible. Algorithms like complementary beamforming [6] have been overtured to Estimate DOAs when the number of sources is same to or greater than the number of elements in array. Problem is the computational complexity of a steered beamformer based methodology is bound to be very large. If a 3-dimensional Direction finding is needed we have to calculate the array output power using beam formers for all elevations (-90 to +90°) and for all azimuths (0 to 360°). This involves a searching at 64,979 search points if we take resolution of 1°.

The second classification consists of high- resolution(ability to locate closely spaced sources) subspace based methods. In these methods division of the cross-correlation matrix of the received array signals into signal and noises subspaces using eigen-value decomposition (EVD)is done to execute DOA calculation. These methods are also employed extensively for spectral estimation. Extensively used Multiple signal classification (MUSIC) is an paradigm of such method. These methods are used when it is required to differentiate multiple sources that are situated in close proximity to each other and their performance is much better than that of the steered beamformer based methods because the function that is calculated in MUSIC gives much sharper peaks or maximas at the true points. But the disadvantage lies in terms of computation. The algorithm employs an exhaustive search with very fine resolution around the set of possible source locations.

The final category of methods is a two-step process. In step 1 the delays in time are estimated for each pair of microphones or antennas in the array. Step 2 consists of merging these datas based on the known physical geometry of the array to get the best estimation of the angles. Various techniques are there those can be used to calculate pair-wise time lag, such as the generalized cross correlation (GCC) method [7] or filtering followed by phase difference calculation of sinusoid signals. The phase transform (PHAT) is the most frequently employed pre-filter for the GCC. For a pair of microphones/antennas the computed time-delay is assumed to be the delay that gives the extreme value of the GCC-PHAT function for that pair. Fusing of the pair-wise time delay estimates (TDE's) is usually executed by the least squares algorithm by solving a number of linear equations to minimize the least squared error. The easiness of the method and the fact that a closed form solution can be achieved (as opposite to searching) has made TDE based methods very popular.

We will discuss here the subspace based methods. The main logic behind DOA estimation using subspace based methods is one-to-one relationship between the direction of arrival of a signal and the associated received steering vector, means received steering vector is unique for a particular DOA. It is therefore feasible to invert the relationship and estimate the direction of a signal from the received signals.

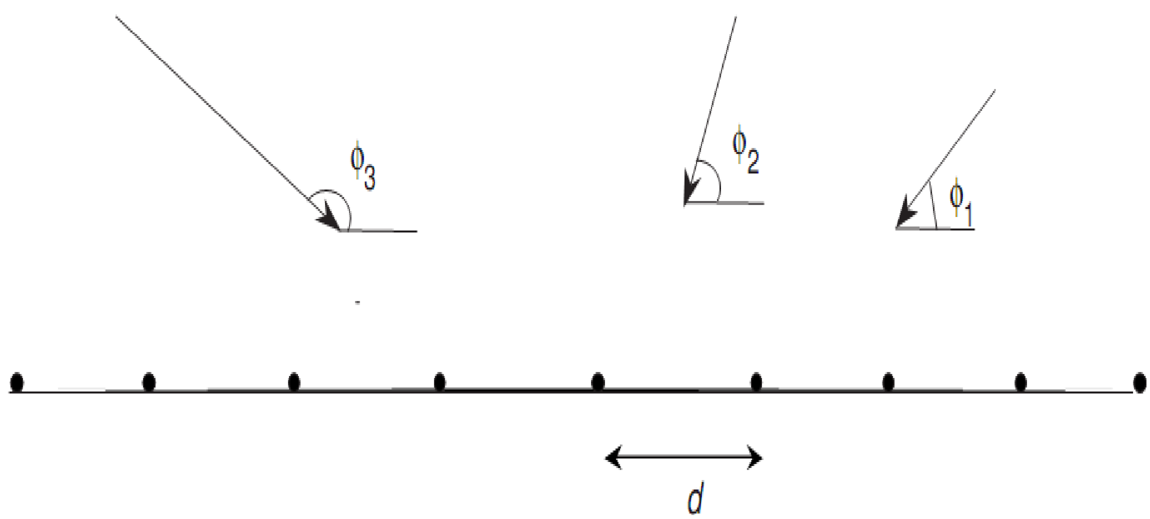

Fig 2.4: Set up for DOA estimation

The problem set up is introduced in Fig. 1. A Number of signls(M) hits a linear, equispaced array(Linear Uniform Array) having N elements, with signal directions φ_i. The motive of DOA estimation is to use the data received from the array to estimate φ_i, $i = 1, \ldots M$. Generally it is assumed that number of signals is less than no of array elements ($M < N$), though there exists some approaches (such as maximum likelihood estimation) that do not place this constraint.

Here we shall discuss some important methods to estimates the directions assuming the number of incoming signals is known to us. We will only discuss 4 techniques:

1. Correlation technique

2. Maximum Likelihood technique

3. MUltiple SIgnal Clasification(MUSIC)

4. EStimation of signal Parameters via Rotational Invariance Technique(ESPRIT)

2.2.1. DOA Estimation using Correlation

We now turn to actual calculations to determine the directions of arrival. The model is of M signals incident on the array, corrupted by noise, i.e.,

$$\mathrm{x} = \sum_{m=1}^{M} a_m s_{(\phi_m)+n} \tag{2.14}$$

The objective is to estimate $\varphi_m,\ m = 1, \ldots M$. The simplest approach to estimate the angles is through correlation. According to the Cauchy-Schwarz inequality, we can say that as a function of φ, $s^H(\varphi)s(\varphi_m)$ has a extreme at $\varphi = \varphi_m$. So actually the corresponding method plots $P_{corr(\varphi)}$ versus $\varphi,$ where

$$P_{corr(\varphi)} = s^H(\varphi)\mathrm{x}. \tag{2.15}$$

$P_{corr(\varphi)}$ is an estimate of the spectrum of the incoming information. The evaluated bearing of landings are the M largest peaks of this function.

2.2.2 DOA estimation using Maximum Likelihood Estimator

In this method DOA estimation is done of an incoming signal by maximizing the probability of coming of a signal from a particular direction. The information model we utilise is the same which is employed for correlation technique. The n vector is statistically colored and, generally can be given as, $E[nn^H] = R_n$.. The form of maximum likelihood estimator (MLE) is

$$\ddot{\Theta}, \tilde{\alpha} = \max[f_{X/\theta,\alpha}(x)], \tag{2.16}$$

Where $f_{X/\theta,\alpha}(x)$ is the probability distribution function of the information matrix x when parameters $\ddot{\Theta}$,$\tilde{\alpha}$ are known. If the noise vector is Gaussian we can write

$$f_{X/\theta,\alpha}(x) = \frac{1}{\pi^N det R_n} e^{-(x-\alpha s)^H R_n^{-1}(x-\alpha s)}, \tag{2.17}$$

i.e, the maximization in eqn is analogous to

$$\ddot{\Theta},\tilde{\alpha} = \min[(x - \alpha s)^H R_n^{-1}(x - \alpha s)] \tag{2.18}$$

$$= \min[x^H R_n^{-1} x - \alpha x^H R_n^{-1} s - \alpha^* s^H R_n^{-1} x + \alpha\alpha^* s^H R_n^{-1} s]$$

We must get the minimum value of this function. For that first we are differentiating the function w.r.t α^* for finding the value of α that will minimize the function.

$$\frac{d}{d\alpha^*} = s^H R_n^{-1}(x - \alpha s)$$

$$\widehat{\alpha} = \frac{s^H R_n^{-1} x}{s^H R_n^{-1} s} \tag{2.19}$$

Using this value of α, we can calculate $\widehat{\phi}$ as

$$\widehat{\phi} = \max[P_{MLE}(\phi)] = \max[\frac{\left|s^H R_n^{-1} x\right|^2}{s^H R_n^{-1} s}] \tag{2.20}$$

The function $P_{MLE(\phi)}$ is the maximum probable estimate of the spectrum of the incoming information. The extreme points of this function is the estimated DOAs.

An intriguing part of maximum probable estimator is that if there is single client and R_n = σ^2I, the only the diagonal elements of the correlation matrix is non-zero hence the MLE is reduced to the corresponding strategy of Section 3. This is normal on the ground that the correlation technique in that case is analogous to the matched filter, which is ideal in the solitary user scenario .

2.2.3. MUSIC: MUltiple SIgnal Classification

MUSIC is most popular eigen decomposition based DOA estimation technique. The received signal vector can be written as

$$x = A\alpha + n. \tag{2.21}$$

Where $A = [a(\varphi_1)\, a(\varphi_2) \ldots, a(\varphi_M)]$,

And $\alpha = [\alpha_1, \alpha_2 \ldots \alpha_M]T.$

The matrix A is a $N \times M$ matrix of the M steering or controlling vectors. For simplicity we are accepting that the different signals are uncorrelated. The correlation matrix of the received signal vector can be given as

$$R_x = E[xx^H]$$

$$=E[\mathrm{A}\alpha\alpha^{H}A^H] + E[nn^H]$$

$$=AaA^H+\sigma^2 I$$

$$=R_s+\sigma^2 I \tag{2.22}$$

Where

$$R_s=AaA^H \text{ and}$$

$$a=\begin{bmatrix} E[|\alpha_1|^2] & 0 & \ldots & 0 \\ 0 & E[|\alpha_2|^2] & \ldots & 0 \\ 0 & 0 & \ldots & E[|\alpha_M|^2] \end{bmatrix} \tag{2.23}$$

The signal covariance matrix,R_s, is obviously a $N \times N$ matrix with rank M. So there are $N - M$ eigenvectors that corresponds to the zero eigenvalue. Let q_m be such an eigenvector in relation to zero eigenvalue. Therefore it is feasible to write,

$$R_s q_m = \mathrm{Aa}A^H q_m = 0,$$

$$\Rightarrow q_m^H \mathrm{Aa}A^H q_m = 0,$$

$$\Rightarrow A^H q_m = 0 \tag{2.24}$$

This final equation is true as the matrix A is obviously positive definite. Equation implies that all $N - M$ eigenvectors (q_m) of R_S corresponding to the zero eigenvalues are orthogonal to all M signal steering vectors. This is the underlying logic for MUSIC. Q_n (noise subspace) is the $N \times (N - M)$ matrix of these eigenvectors.

MUSIC actually plots the pseudo-spectrum

$$P_{MU(\varphi)} = \frac{1}{|Q_n^H\, s(\phi)|^2} \tag{2.25}$$

As from the previous discussion it is conferred that the noise vector is orthogonal to signal steering vector so for any signal direction $|Q_n^H\, s(\phi)|^2$ becomes zero. Therefore, the estimated signal directions are the M biggest crests in the pseudo-spectrum. One trick is that in reality we will not be provided with the signal covariance matrix. We have to estimate that one. And from that estimate of R_s we have to calculate the noisy eigen vectors.

For any eigenvector $q_m \in Q$,

$$R_s q_m = \lambda q_m$$

$$\Rightarrow \mathrm{R} q_m = R_s q_m + \sigma^2 \mathrm{I} q_m = (\lambda_m + \sigma^2)\, q_m$$

It is clear that. any eigenvector of R_s is also an eigenvector of R with corresponding eigenvalue $\lambda + \sigma^2$. Letting $R_s = Q\Lambda Q^H$. We can write

$$\mathrm{R} = Q[\Lambda + \sigma^2 I]\, Q^H$$

$$= Q \begin{bmatrix} \lambda_1 + \sigma^2 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \lambda_2 + \sigma^2 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \lambda_M + \sigma^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \sigma^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & \sigma^2 \end{bmatrix} Q^H \tag{2.26}$$

By this eigen splitting we will get two different matrix. One is called signal space containing the eigen vectors of signal eigenvalues and the other is called noise space containing the vector of noisy eigenvalues. Q_S is the signal subspace, and Q_n is the noise subspace.

There are few paramount perceptions to be made:

• The smallest eigenvalues of R are the noise eigenvalues and are all same to σ^2,noise covariance i.e., one way of recognizing the signal and noise eigenvalues (equivalently the signal and noise subspaces) is to calculate the number of small eigenvalues that are equal.

• By orthogonality stated above it is logical to write for Q, $Q_S \perp Q_n$

Considering the last two observations, we see that all noise eigenvectors are orthogonal to the signal steering vectors. This is the logic behind MUSIC. Consider the following function of φ:

$P_{MUSIC(\varphi)}$ = 1/$S^H(\varphi)Q_n Q_n^H s(\varphi)$

If φ is equal to DOA of any one of the signals, $s(\varphi) \perp q_m$ and the denominator of the function is zero. MUSIC, therefore, gives the peaks of the function $P_{MUSIC(\varphi)}$,as the directions of arrival.

2.2.4 Estimation of Signal Parameters via Rotational Invariance Technique(ESPRIT)

ESPRIT is another very popular high resolution DOA estimation technique .It uses the fact that the signal at each member of the array is at constant phase shift from its earlier element.

$A\ ,the\quad N\ \times M$ matrix of steering vectors given by

$$A=\begin{bmatrix} 1 & 1 & \ldots & 1 \\ y_1 & y_2 & \cdots & y_M \\ \vdots & \vdots & \ddots & \vdots \\ y_1^{N-2} & y_2^{N-2} & \ldots & y_M^{N-2} \\ y_1^{N-1} & y_2^{N-1} & \cdots & y_M^{N-1} \end{bmatrix}$$

$y_m = e^{jkdcos\phi_m}$. Now we are incorporating two $(N\ -\ 1)\ \times M$ matrices, A_0 and A_1, where A_0 and A_1 comprises the 1st $N-1$ rows of A and last $N-1$ rows of A.

$$A_0 = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ y_1 & y_2 & \cdots & y_M \\ \vdots & \vdots & \ddots & \vdots \\ y_1^{N-2} & y_2^{N-2} & \ldots & y_M^{N-2} \end{bmatrix}$$

$$A_1 = \begin{bmatrix} y_1 & y_2 & \cdots & y_M \\ \vdots & \vdots & \ddots & \vdots \\ y_1^{N-2} & y_2^{N-2} & \ldots & y_M^{N-2} \\ y_1^{N-1} & y_2^{N-1} & \cdots & y_M^{N-1} \end{bmatrix}$$

Now we are defining another matrix ϕ with dimension $M \times M$

$$\Phi = \begin{bmatrix} y_1 & 0 & \dots & 0 \\ 0 & y_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & y_M \end{bmatrix}$$

It can be verified that $A_1 = A_0\, \Phi$ by simple mathematics.

Φ is a matrix of only diagonal elements, which correspond to the phase shift from one element to the next. From the above equation it is clear that if we can estimate Φ then we will get desired incoming directions.

If A_0 and A_1 were known, we could get Φ very easily. But we must resort an indirect method to obtain the desired result as they are not known .

From the concept of linear algebra it can be said that the same subspace is covered by the matrix A (steering matrix) and the matrix Q_s (matrix of signal eigenvectors). So there is existence of an unique invertible matrix T such that the following equation holds.

$$Q_s = AT \tag{2.27}$$

Now partition of Q_s has been done in the similar way of partitioning of A. The first matrix Q_0 comprises the first $(N - 1)$ rows of Q_s and Q_1 the last (N − 1) rows of Q_s.

$$Q_0 = A_0 T, \tag{2.28}$$

and

$$Q_1 = A_1 T = A_0 \Phi \mathrm{T} \tag{2.29}$$

Consider

$$Q_1 T^{-1}\phi^{-1}T = A_0\,\phi T T^{-1}\phi^{-1}T = A_0\,T = Q_0 \qquad (2.30)$$

Now, let

$$\Psi^{-1} = T^{-1}\phi^{-1}T$$

$$=> Q_1\Psi^{-1} = Q_0$$

$$=> Q_1 = Q_0\Psi \qquad (2.31)$$

where

$$\Psi = T^{-1}\,\phi T \qquad (2.32)$$

The above equation implies that the eigen decomposition of Ψ has been done and ϕ is the diagonal matrix containing the eigen values of Ψ. So if one can estimate Ψ,then by doing the eigen decomposition he will get the $\phi\ matrix$. If $k_1, k_2, \dots\dots, k_r$ be the r eigenvalues of Ψ then the DOAs can be estimated as

$$\theta_i = sin^{-1}\{\lambda \arg(k_i\ )/2\pi d\} \qquad (2.33)$$

One important point to be noted is the estimation of Ψ can be done using Total Least Square, which is an complex and improved version of Least square.

Chapter 3

Literature Review

Direction of Arrival (DOA) estimation finds its practical importance in sophisticated video conferencing by audio visual means, locating underwater bodies, removing unwanted interferences from desired signals etc. So considerable attention is there in recent research on this area and various algorithms are getting developed for this purpose by various researchers. It is found that the effective algorithms use sensor array processing in which a number of sensors are used to receive the signals from various directions. Popularly used high resolution methods for DOA estimation are MUltiple SIgnal Classification(MUSIC), root-MUSIC, Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT), and Maximum Likelihood (ML) algorithm which use array signal processing[8]-[12]. In [13] authors have estimated DOA of underwater target with acoustic array. For acoustic echo detection generally time delay based algorithm is frequently used. But they have introduced here adaptive phase difference estimator which works efficiently even with small array for determining the angular location of closely spaced sources. The phase difference is obtained by computing the adaptive weights of two parallel adaptive notch filters.

However, the above algorithms consider the ideal sensor array in which the imperfections such as mutual coupling between sensors, gain and phase errors which are obvious in practical scenario have not been taken into consideration . It has been found that performance in DOA estimation degrade due to those imperfections as it alters the ideal array manifold. Various techniques to combat those limitations for high frequency signals are available in existing literature. An iterative eigen structure based method for direction finding considering sensor mutual coupling , gain and phase uncertainties is given in[14]. Here along with DOAs mutual coupling coefficients, gain and phase errors have been calculated. 1st the directions have been found by taking the assumption that the gain, phase and mutual interaction parameters are in hand. After that using the results of DOAs they have minimized a certain cost function with respect to gain and phase error parameters. In the 3rd step they have again minimized that cost function to find the coupling coefficients, but this time with respect to both DOAs and gain, phase coefficients. The minimization process is repeated until they have got a sufficicent less value of the cost function. But they have not formulated sufficient conditions for convergence of the cost function. DOA estimation of multiple signals using uniform linear arrays with mutual coupling by setting the sensors at the boundary as auxiliary sensors is found in existing literature [15]. It has been shown in that the interaction between adjacent individuals with the same interspace is nearly equal, and the magnitude of the mutual coupling coefficient between two far apart elements is so small that it can be considered as null. Hence a banded symmetric Toeplitz matrix can be formed as model for the mutual interaction of ULA. With that the authors have also considered the case of coherent signal detection. They have compared two methods. 1st they have eliminated the sensors at the boundary, spatial smoothing has been done on the middle subarray. 2nd they have applied the spatial smoothing in the whole subarray. And they have concluded that the 2nd method gives better performance than the 1st one. Extension to uniform rectangular array(URA) of the same problem statement as [15] has been done in [16] for estimating both elevation and azimuthal angle of direction of arrival. They have proved that by setting the outermost sensors as auxiliary sensors and taking the rest sensors into consideration will not hamper the performance of 2-D MUSIC algorithm in a large way. Also they have used twice search method to reduce the computational burden. Not only that, after getting the DOA estimates they show how to get mutual coupling parameters utilizing those DOAs which is not done in [15].They have even calculated CRLB(Cramer Rao Lower Bound).

Spatial smoothing has been introduced in [17] . This is a technique to alleviate the problems faced when one trying to estimate the DOAs of fully correlated(coherent) signals resulting from multipath propagation. A Relatively different problem of DOA estimation for mixed signal (combination of correlated, uncorrelated and coherent signals) has been addressed in [18].They have developed a two stage process.1st they have estimated the DOAs of partially correlated and uncorrelated signals using any standard subspace approach. Then by oblique projection method they have eliminated the effect of lowly correlated and uncorrelated signals from the coherent signals. In the last step they DOAs of the coherent sources have been found by spatial smoothing technique. If number of signals is more than no of sensors then also their method works properly. But in this work they have taken ideal sensor array in which there was no uncertainties. Estimation of DOAs for mixed signls in the presence of mutual coupling has been studied in [19].Their procedure is partly same as of [18].Here also they have estimated the direction of falling of the non-coherent signls firstly. Using that information they have found the coupling coefficients by a lest square solution. In the final step for getting the DOAs of coherent signals they have taken the method of oblique projections to nullify the contribution of non coherent signals and also they have reduced the effect of mutual interaction. Iterative search is not needed at all in their method. With that they have shown the methodology to find the lost angles.

In [20] the authors have designed sensor imperfections as gain and phase errors. DOA estimation in the presence of these unknown gain and phase error of the sensors has been done using partly calibrated arrays. Partly calibrated array menas some of the array elements are ideal means gain is unity and no phase shift is there between falling and reflected signal. The conventional ESPRIT algorithm requires the fully calibrated array and the subarrays be oriented in the same way. Unfortunately, the arrays we deal with in practice may only be partly calibrated.As a result the ESPRIT algorithm cannot be directly applied . In this estimation of DOAs are executed by a ESPRIT based method for partly calibrated array along with the finding of unknown gain and phase coefficients by solving a optimization problem without spectral search. In addition they have derived the CRLB of the RMSE. By simulation results it is resoluted that the performance of this method is even better when the no of uncalibrated sensors is large. Author in [21] presented a maximum-likelihood calibration algorithm to compensate for the effect of mutual coupling, sensor gain and phase errors. But it requires a set of calibration sources at known locations. Though this method works efficiently with varying or changing uncertainties of array members but the implementation of this method is costly . Authors in [22] –[25] discussed receiving mutual impedance method for mutual coupling reduction.

Though in all these papers mutual coupling problem is addressed, that is done for very high frequency signals. With few exceptions mutual coupling problem has not been addressed in case of acoustic sensors due to their practical limitations and for less amount of coupling for low frequency signals. Though J.W Pierre and M. Kaveh presented the comparison between several DOA estimation algorithms using the University of Minnesota Ultrasonic Sensor array testbed in[26] means The efficiency of various DOA estimation procedure is investigated in hardware which was very rare in research paper. The array comprises eight ultrasonic transducers operating at 40 kHz. So automatically they have faced the nonideality of the practical arrays and as so they have also illustrated the calibration procedure. They have compared Capons MLM, MUSIC, Root MUSIC, Min Norm, ESPRIT and a weighted norm version of MUSIC .In simulation section they also provided the plot of bias and variance of RMSE versus SNR. But effect of mutual coupling as the sources becomes closer has not been addressed yet.

In this work our objective is to study the effect of coupling when spacing between the acoustic sources varies. For simplicity we consider that the signals are uncorrelated. This work has been motivated by the need to evaluate the performance of high resolution methods in the presence of mutual coupling or to see upto which degree of resolution their working is efficient. For that 1st we have estimated the DOAs using high resolution ESPRIT algorithm. Then by least square solution we have estimated the coupling coefficients. With these known coefficients we have got the more accurate DOA. Then we have compared the two results to get an insight about the degradation of performance due to coupling.

We have taken four sets of acoustic signal sources of different resolutions and shown the effect of coupling for different spacings.

Chapter 4

Effect of mutual coupling on DOA estimation

4.1 Formation of coupling matrix for DOA estimation

In this chapter the effect of mutual coupling on the DOA estimation performance has been studied.

We are taking a ULA consists of N alike acoustic sensors. The distance between neighbouring elements of the array is d. Suppose r narrowband uncorrelated acoustic signals $s_1(t)\, \dots \dots \dots, s_r\, (t)$ fall on the array with angles $\theta_1, \theta_2\, , \dots\ \theta_r$ at t[th] time index.

Suppose C denotes the mutual interaction matrix for the ULA. Then the output from the array sensors can be given as

$$x(t) = CAs(t) + n(t); \tag{4.1}$$

where $x(t)$, A, $s(t)$ and $n(t)$ are the received signal vector, the ideal array manifold, the signal 1-D matrix, and the noise 1-D matrix respectively.

$$x(t)=[x_1(t), x_2(t), \dots, x_N(t)]^T \tag{4.2}$$

$$A=[a(\theta_1), a(\theta_2), \dots \dots \dots, a(\theta_r)], \tag{4.3}$$

$$s(t) = [s_1(t), s_2(t), \dots \dots \dots, s_r(t)]^T, \tag{4.4}$$

$$n(t) = \ [n_1(t), n_2(t), \dots \dots \dots, n_N(t)]^T \tag{4.5}$$

The ideal steering vector is expressed as

$$a(\theta) = [1, \quad e^{j2\pi d sin\theta\lambda^{-1}}, \quad \dots, \quad e^{j2\pi(N-1)d sin\theta\lambda^{-1}}]^T \tag{4.6}$$

The form of coupling matrix for URA is given by [7].It is

$$C = \begin{bmatrix} C_1 & C_2 & 0 & \dots & 0 \\ C_2 & C_1 & C_2 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & C_2 & C_1 & C_2 \\ 0 & \dots & 0 & C_2 & C_1 \end{bmatrix}_{MN\times MN} \tag{4.7}$$

where M is no of sensor in a column ,N is no of sensor in a row.

C_1 and C_2 are sub-matrices of C with dimensions $N \times N$ and can be given by

$$C_1 = toeplitz\{1, c_x ,0,\dots\dots,0\} \tag{4.8a}$$

$$C_2 = toeplitz\{c_x , c_{xy}, 0, \dots\dots ,0\} \tag{4.8b}$$

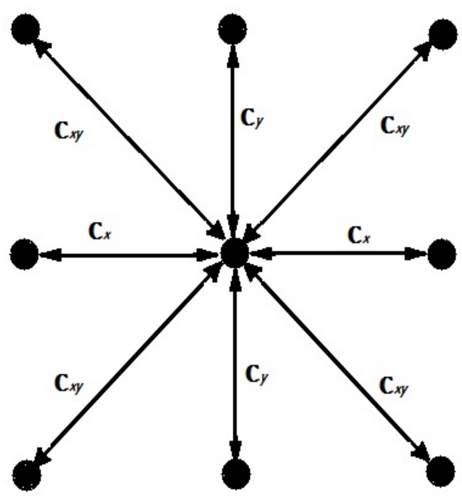

Fig. 4.1: Sketch map of mutual coupling for URA considering each sensor is affected by the coupling from the 8 neibhouring sensors.

In this work, we assume that one sensor is mutually coupled by only two nearest neighbours as mutual coupling reduces with distance . So for ULA the mutual coupling matrix is reduced to

$$C = \begin{bmatrix} 1 & c_1 & c_2 & 0 & \dots & \dots & 0 & 0 \\ c_1 & 1 & c_1 & c_2 & \dots & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & c_2 & c_1 & 1 & c_1 \\ 0 & 0 & \dots & \dots & 0 & c_2 & c_1 & 1 \end{bmatrix}_{NXN} \tag{4.9}$$

where c_1 is the coupling coefficient between one sensor and its nearest neighbour, c_2 is the coupling between one sensor and its second nearest neighbour.

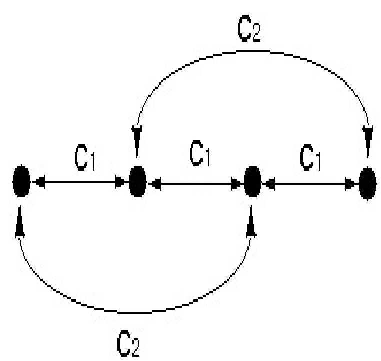

Fig. 4.2. Sketch map of mutual coupling for ULA considering each sensor is only affected by two nearest neighbour.

The covariance matrix of the received signal is

$$R_x = E\{x(t)x^H(t)\} = CAR_S A^H C^H + \sigma^2 I = A_m R_S A_m^H \tag{4.10}$$

in which R_s is the correlation matrix of the source, and I is an NXN identity matrix, and

$$A_m = CA \tag{4.11}$$

In standard subspace approach R_x can be given as

$$R_x = \sum_{i=1}^{r} w_i e_i e_i^H + \sum_{i=r+1}^{N} w_i e_i e_i^H = E_S \Lambda_S E_S^H + \sigma^2 E_n E_n^H \tag{4.12}$$

where $w_1 \geq w_2 \geq \ldots \geq w_r > w_{r+1} = \ldots = w_N$ are the eigenvalues of received vector and the respective eigenvectors are $e_1, e_2, \ldots \ldots, e_N$. E_S and E_n contains the eigenvectors associated with the r largest eigenvalues and $N - r$ smallest eigenvalues.

Now we are all prepared to estimate DOAs by ESPRIT using the mutually coupled steering matrix. After that we shall estimate the coupling coefficients .Using those known coefficients DOAs have been computed again. Then comparison of the results have been done for different source spacing to observe the effect of mutual coupling as the source spacing varies to see how much resolution can be achieved in practical application.

4.2 DOA estimation based on ESPRIT algorithm

We shall now estimate the DOAs using the mutually coupled steering vector. We can divide the ULA into two overlapping subarrays where the first subarray comprises of first (N-1) sensors and the second one consists of the last (N-1) sensors. Mutually coupled steering matrices of these two subarrays are denoted as A_{m1} and A_{m2} .

It can be verified that the relation between A_1 and A_2 is

$$A_2 = A_1 \phi \tag{4.13}$$

where A_1 *and* A_2 are ideal steering vector and ϕ is an rxr diagonal matrix , given by

$$\phi = diag\left\{e^{j2\Pi d sin\theta_1 \lambda^{-1}}, \ldots, e^{j2\Pi d sin\theta_r \lambda^{-1}}\right\} \tag{4.14}$$

As the span of signal subspace $\hat{E}_s$ and the span of the steering matrix A_m is same i.e, span{ $\hat{E}_s$}=span{ A_m },there exists an $r \times r$ nonsingular matrix T such that

$$\hat{E}_s = A_m T \tag{4.15}$$

Now we can partition the $\hat{E}_s$ as $\hat{E}_{s1}$ and $\hat{E}_{s2}$ .So we have

$$\hat{E}_{s1} = A_{m1} T \tag{4.16a}$$

$$\hat{E}_{s2} = A_{m2} T \tag{4.16b}$$

After further manipulation

$$\hat{E}_{s2}=\hat{E}_{s1}\Psi \tag{4.17}$$

where the $r \times r$ matrix Ψ is given by

$$\Psi = T^{-1}\,\phi T \tag{4.18}$$

If $k_1, k_2, \dots \dots, k_r$ be the r eigenvalues of Ψ then the DOAs can be estimated as

$$\theta_i = sin^{-1}\{\lambda \arg(k_i)/2\pi d\} \tag{4.19}$$

After getting the DOAs ,suppose the new steering vector is $a(\theta)$ which is a $N \times r$ matrix.So now we can find the mutual coupling coefficients by the method described in [8]and can form the MCM matrix C.

We can write $$Ca(\theta) = T_a(\theta)c \tag{4.20}$$

Where $T_a(\theta)$ is the sum of the following two matrices

$$[T_1]_{pq}=\begin{cases}[a(\theta)]_{p+q-1} & for\ p+q\leq N+1\\ 0 & otherwise\end{cases} \tag{4.21}$$

$$[T_2]_{pq}=\begin{cases}[a(\theta)]_{p-q+1} & for\ p\geq q\geq 2\\ 0 & otherwise\end{cases} \tag{4.22}$$

Now

$$\begin{bmatrix} U_n^H\ Ca(\theta_1) \\ \vdots \\ U_n^H\ Ca(\theta_r) \end{bmatrix} = \begin{bmatrix} U_n^H\ T_a(\theta_1) \\ \vdots \\ U_n^H T_a(\theta_r) \end{bmatrix} c \tag{4.23}$$

$$\triangleq (t_1 \quad t_2 \quad \dots \quad t_{P+1}) \begin{pmatrix} 1 \\ c_1 \end{pmatrix} \tag{4.24}$$

Where U_n^H is the noise subspace.

And we can get least square solution as

$$c_1 = -[t_2 \quad \dots \quad t_{P+1}]^{\#} \; t_1 \tag{4.25}$$

Once we know C we are free to calculate nearly ideal steering matrix from the equation

$$A_2 = C^{-1} A_m \tag{4.26}$$

Using this A_2 (the recalculated steering vector after the calculation of MCM)we can again compute the DOAs using ESPRIT algorithm which are nearer to the actual DOAs than the DOAs found in the previous iteration.

4.3 Simulation Results

In order to observe the effect of mutual coupling on the algorithm simulation of a ULA with N=15 sensors separated by d is done. We have taken four sets of three uncorrelated narrowband acoustic signals (spacing of 20°,10°,8° and 5°) with identical power.The background noise is assumed to be Additive White Gaussian Noise(AWGN). The power of the i^{th} signal is σ_i^2 ,noise power is σ^2 and the input SNR of the i^{th} signal is formulated as $10\log_{10}(\sigma_i^2 / \sigma^2)$ in dB.

In this experiment , we take the mutual coupling parameters as c_1= 0.107 - 0.1i, c_2=-0.013+0.02i. No. of snapshots taken in each experiment is 300.For each SNR level the same experiment is done 550 times. The RMSE of the estimated DOAs is calculated as

$$\text{RMSE}=\sqrt{\frac{\sum_{k=1}^{K}\sum_{n=1}^{r}(\theta_n - \ddot{\theta}_{n,k})^2}{(Kr)}}$$

where K is the number of Monte-carlo experiments, r is the signal source number and $\ddot{\theta}_{n,k}$ is the nth estimated DOA in the k-th Monte-carlo experiment.

The RMSE versus SNR curve are illustrated in fig 4.3 for DOAs 10°,30°,50°.It is shown that there is a significant improvement in the DOA estimates for the 2nd iteration in which nearly ideal steering vector has been calculated after estimating the mutual coupling coefficients.

Also RMSE versus SNR plots are illustrated in fig 4.4 for DOAs 10°,20°,30°.The same has been shown in fig 9 for DOAs 10°,18°,20°.Further decrease of the spacing of sources the system would not be able to compute DOAs of that sources due to large error. That case has been shown in fig 10 for DOAs 10°,15°,20° .

In fig 6 the difference of RMSE in two iteration is about 0.04° (at 10 dB) whereas the same is 0.1° and 0.15° in fig 4.5 and fig 4.6 respectively. So it is evident that the degradation in performance becomes more due to mutual coupling effect for more closely spaced sources.

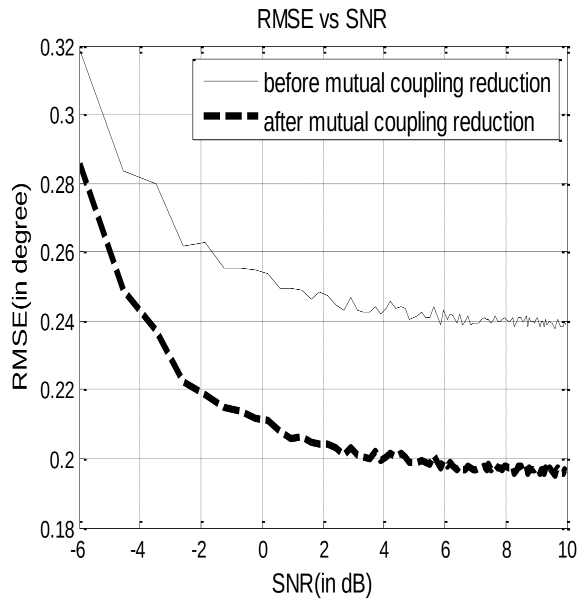

Fig 4.3. RMSE of the estimated directions versus SNR of the signals coming from $10^{\circ},30^{\circ},50^{\circ}$

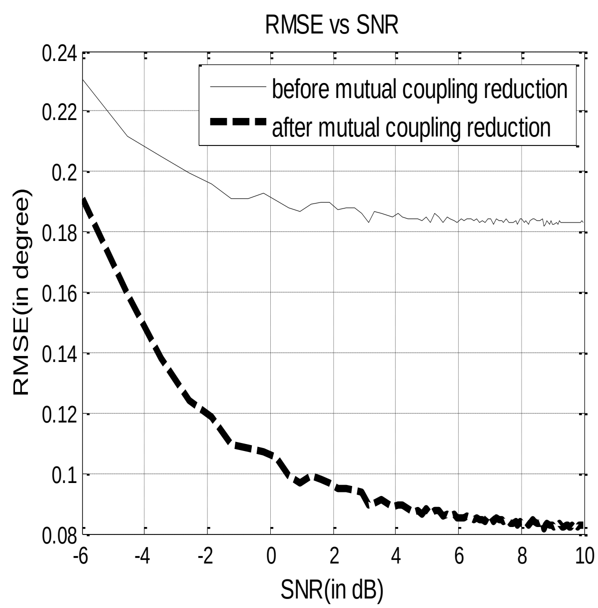

Fig 4.4. RMSE of the estimated directions versus SNR of the signals coming from $10^{\circ},20^{\circ},30^{\circ}$

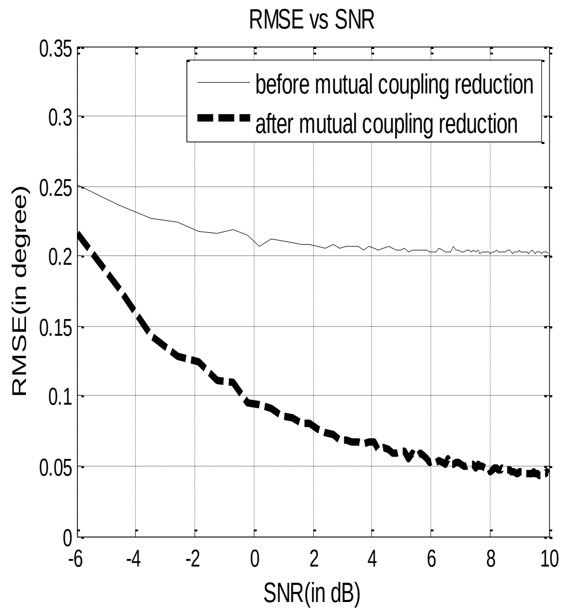

Fig 4.5. RMSE of the estimated directions versus SNR of the signals coming from $10^\circ\ 18^\circ, 26^\circ$

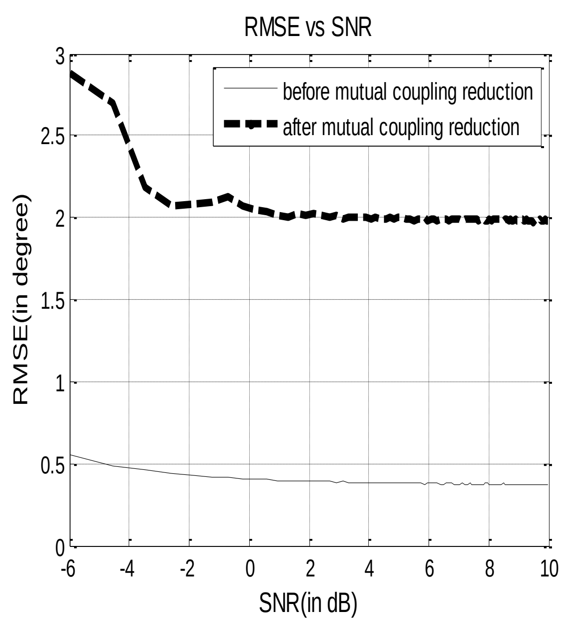

Fig 4.6. RMSE of the estimated directions versus SNR of the signals coming from $10^\circ\ 15^\circ, 20^\circ$

Chapter 5
Effect of antenna gain and phase error on DOA estimation

5.1 Formation of steering matrix for DOA estimation

In this chapter the effect of antenna gain and phase error on the performance of DOA estimation algorithm has been studied.

Now, we consider an ideal ULA with N isotropic sensors impinged by r uncorrelated narrowband source signals,$\{s_r(t)[r = 1, \dots, r]$,from far field of the receiver. At the t^{th} snapshot the observed array output consists of the outputs of the N isotropic sensors of the array and can be written as follows

$$x(t) = \sum_{r=1}^{r} a(\theta_r)s_r(t) + n(t) \tag{5.1}$$

where $a(\theta_r)$ is the array manifold vector for the rth signal, and A is the $M \times r$ steering matrix given by

$$A = [a(\theta_1), a(\theta_2), \dots \dots, a(\theta_r)] \tag{5.2}$$

Where A consists of all the steering matrix for all directions.

$s(t\ ) = [s_1(t), s_2(t), \dots \dots, s_r(t)]^T$ is the $r \times 1$ one dimensional matrix of the acoustic signal waveforms, and $n(t)$ is the perturbation matrix. The noise is additive white Gaussian noise (AWGN) with mean zero and covariance matrix $\sigma^2 I$. The steering vector for ideal ULAs, is

$$a(\theta) = [1, \quad e^{j2\pi d sin\theta\lambda^{-1}}, \quad \dots, \quad e^{j2\pi(N-1)d sin\theta\lambda^{-1}}]^T \tag{5.3}$$

In the above equation, λ , d and θ signifies the wavelength of signal, inter sensor distance , and DOA respectively.

From (1), the covariance matrix of the array output is

$$R = E\{x(t)x^H(t)\} = ASA^H + \sigma^2 I \qquad (5.4)$$

where $S = E\{s(t)s^H(t)\}$ is the coming signal covariance matrix

The model described above is for ideal cases where any antenna shortcoming has not been taken into account. So now we have to consider the practical situation where the sensor array is not errorless or not fully calibrated. For that we are taking the case where only part of the ULA is calibrated. In general cases, it is assumed that the some sensors from the starting of the array are calibrated, whereas the last sensors are uncalibrated which posess some errors. These errors or uncertainties of the ULA are modeled as unknown, direction-independent gains and phases as described in [20].

Let g and p represent the $N \times 1$ array gain and phase vectors, respectively. So

$$g = \left[1_{N_c}^T, g_1, \ldots, g_{N-N_c}\right]^T \qquad (5.5a)$$

$$\phi = \left[1_{N_c}^T, e^{j\phi_1}, \ldots, e^{j\phi_{N-N_c}}\right]^T \qquad (5.5b)$$

where $\mathbf{1}_{N_c}$ states an $N_c \times 1$ vector of ones (that signifies no uncertainties), and $g_1, g_2, \ldots, g_{N-N_c}$ and $\phi_1, \phi_2, \ldots, \phi_{N-N_c}$ are the uncertainties of the uncalibrated $N - N_c$ acoustic sensors, respectively. With these uncertainties the steering matrix can be formulated like this.

$$\overline{a(\theta)} = F(\gamma)a(\theta) = F(g^\circ \phi)a(\theta) \qquad (5.6)$$

Where ° denotes the Schur–Hadamard product between g and ϕ

Now we are introducing a new matrix,

$$\gamma = g^\circ \phi = \left[1_{N_c}^T, g_1 e^{j\phi_1}, \ldots, g_{N-N_c} e^{j\phi_{N-N_c}}\right]^{\mathrm{T}} \qquad (5.7)$$

and $F(\text{y}) = diag\{\text{y}_1, \text{y}_2, \ldots, \text{y}_N\}$ is an $N \times N$ diagonal matrix. Hence, the array covariance matrix becomes

$$R = \bar{A}\, S\bar{A}^H + \sigma^2 I = F(\text{y})ASA^H F^H(\text{y}) + \sigma^2 I \tag{5.8}$$

where $\bar{A} = F(\text{y})A$ is the array manifold of the partly calibrated or non ideal ULA. The subspace decomposition (EVD) of R is

$$R = E_S \Lambda_S E_S^H + E_N \Lambda_N E_N^H \tag{5.9}$$

where Λ_S is an $r \times r$ diagonal matrix. Its elements are r largest eigenvalues . Λ_N is an $(N - r) \times (N - r)$ diagonal matrix. And its elements are $(N - r)$ smallest eigenvalues. E_S is the $N \times r$ signal subspace matrix which carries the r eigenvectors associated with the r largest signal eigenvalues, while E_N is the $N \times (N - r)$ noise subspace matrix containing the eigenvectors related to the smallest noisy eigenvalues. For finite snapshots, the array covariance matrix is

$$\hat{R} = N^{-1} \sum_{t=1}^{N} x(t) x^H\,(t)$$

Which can be decomposed further as

$$= \widehat{E_S}\hat{\Lambda}\ \widehat{E_S^H} + \widehat{E_N}\hat{\Lambda}\widehat{E_N^H} \tag{5.10}$$

N is the number of time instant when the signal is sampled.

5.2 DOA estimation using non-ideal steering matrix

We are now prepared to estimate the DOAs using the partly calibrated ULA. As of conventional ESPRIT (with full calibrated array), we divide the nonideal ULA into two subarrays. The first subarray comprises of the first $N-1$sensors, and the second comprises of the last $N-1$ sensors. Also the steering matrix can be subdivided and the same for these subarrays is given as following

$$\overline{A_1} = F(\gamma_1)A_1 \tag{5.11a}$$

$$\overline{A_2} = F(\gamma_2)A_2 \tag{5.11b}$$

A_1 and A_2 are the ideal steering matrices of the 1st and 2nd subarrays respectively. $\gamma_1$and γ_2 are the $(N-1) \times 1$ error coefficient vectors of these two subarrays, and can be written in the following manner.

$$\gamma_1 = \left[1_{M_c}^T, g_1 e^{j\phi_1}, \dots, g_{M-M_c-1} e^{j\phi_{M-M_c-1}}\right]^{\mathrm{T}} \tag{5.12a}$$

$$\gamma_2 = \left[1_{M_{c-1}}^T, g_1 e^{j\phi_1}, \dots, g_{M-M_c} e^{j\phi_{M-M_c}}\right]^{\mathrm{T}} \tag{5.12b}$$

By simple mathematic it can be shown that A_1 and A_2 satisfy

$$A_2 = A_1\phi \tag{5.13}$$

$\phi = diag\left\{e^{j2\Pi d sin\theta_1 \lambda^{-1}}, \dots, e^{j2\Pi d sin\theta_r \lambda^{-1}}\right\}$ and it is a $r \times r$ matrix.

Since E_s spans the same subspace as the modified steering matrix $F(\gamma)A$,i.e.,span$\{E_s\}$ = span$\{F(\gamma)A\}$,there exists an unique nonsingular matrix T such that

$$E_s = F(\gamma)AT \tag{5.14}$$

Now we are dividing the signal suspace into 2 parts. Let first $(N-1)$ rows of E_s forms E_{s_1}, and the last $(M-1)$ rows of E_s forms E_{s2} .

So we have

$$E_{s_1} = F(\gamma_1)A_1T \tag{5.15a}$$

$$E_{s_2} = F(\gamma_2)A_2T \tag{5.15b}$$

Since the matrices $F(y_1)$, $F(y_2)$,and T are nonsingular, by substituting (12) into (15) we get

$$F(\overline{y})E_{s_2} = E_{s_1}\Psi \tag{5.16}$$

where the $r \times r$ matrix Ψ is given by

$$\Psi = T^{-1}\phi T \tag{5.17}$$

and $F(\overline{y}) = F(y_1)F^{-1}(y_2)$ with $\overline{y}$ is an $(N-1) \times 1$ vector as following

$$\overline{y} = [1_{N_c-1}^T, g_1^{-1}e^{-j\phi_1}, g_1 g_2^{-1}e^{-j(\phi_2-\phi_1)}, \dots\dots, g_{N-N_c-1}g_{N-N_c}^{-1}e^{-j(\phi_{N-N_c}-\phi_{N-N_{c-1}})}]^T \tag{5.18}$$

Here,we note that the as the first few elements are calibrated ,first $(N_c - 1)$ elements of $\overline{y}$ are equal to one,

i.e.,$\overline{y_i}$ =1,$for\ i = 1,2, \dots\dots, N_c - 1$.

By the concepts of linear algebra we can say that Ψ and ϕ are similar matrices. Therefore, the eigenvalues of Ψ must be equal to the diagonal elements of ϕ. If We denote $v_1, v_2, \dots\dots, v_r$be the eigenvalues of Ψ, then the DOAs can be estimated by the following equation,

$$\theta_l = sin^{-1}\{\tfrac{\lambda \arg(v_r)}{2\pi d}\} \tag{5.19}$$

These are the DOAs we have got by partly calibrated arrays. Also we have estimated the DOAs using conventional ESPRIT algorithm taking ideal steering matrix means considering the full calibrated array. We have compared the two results for partly calibrated array and full calibrated array to show the degradation in performance in presence of antenna error.

5.3 Simulation Results

For observing the effect of antenna gain and phase error on the algorithm, simulation of a ULA consists N=10 elements separated by half a wavelength is run. We have taken four sets of three uncorrelated narrowband acoustic signals(spacing of 12°,10°,8° and 5°) with identical power. If the power of the i^{th} signal is σ_i^2 and noise power is σ^2 then the SNR is $10\log_{10}(\sigma_i^2/\sigma^2)$ in dB.

In the following simulation ,we have considered that the first three sensors are calibrated and the rests are uncalibrated with unknown gain and phase errors given by $1.8e^{i*pi/3}$,$0.4e^{-i*pi/5}$,$0.8e^{i*pi/5}$, $1.25e^{-i*pi/3}$, $1.53e^{-i*pi/5}$, $0.75e^{i*pi/4}$,$1.36e^{-i*pi/10}$. No. of snapshots taken in each experiment is 300.For each SNR level the same experiment is done 550 times. The RMSE of the DOAs is calculated as

$$\text{RMSE}=\sqrt{\frac{\sum_{k=1}^{K}\sum_{n=1}^{r}(\theta_n - \ddot{\Theta}_{n,k})^2}{(Kr)}}$$

where K is the number of experiments, signal number is r and $\ddot{\Theta}_{n,k}$ denotes the n^{th} estimated DOA in the k-th experiment.

The RMSE versus SNR curve are illustrated in fig 5.1 for DOAs 20°,32°,44°.It is shown that there is a significant improvement in the DOA estimates for the 2^{nd} iteration in which ideal steering vector has been taken aS in the case of full calibrated array.

Also RMSE versus SNR plots are illustrated in fig 5.2 for DOAs 20°,30°,40°.The same has been shown in fig 5.3 for DOAs 20°,28°,36° and in fig 5.4 for DOAs 10°,15°,20° .

In fig 5.1 the difference of RMSE in two iteration is about 0.07° (at 10 dB) whereas the same is 0.1° and 0.1° in fig 5.2 and fig 5.3 respectively. And for 5° separation of sources it is 0.2° .So it is evident that the degradation in performance becomes more due to antenna gain and phase errors for more closely spaced sources.

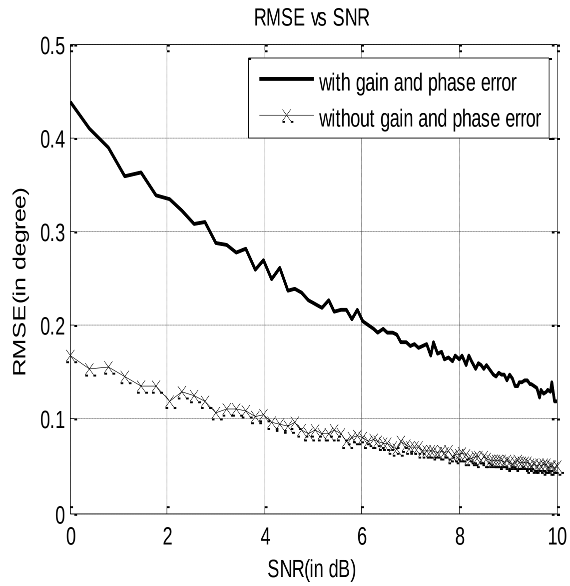

Fig 5.1 RMSE of the estimated directions versus SNR of the signals coming from 20°,32°,44°

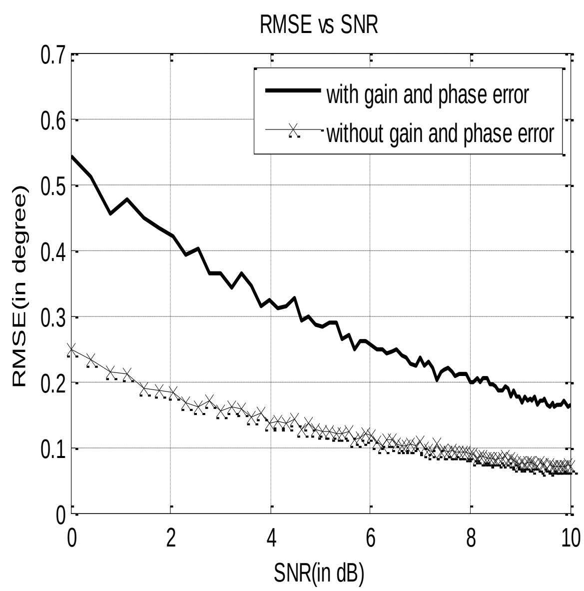

Fig 5.2 RMSE of the estimated directions versus SNR of the signals coming from 20°,30°,40°

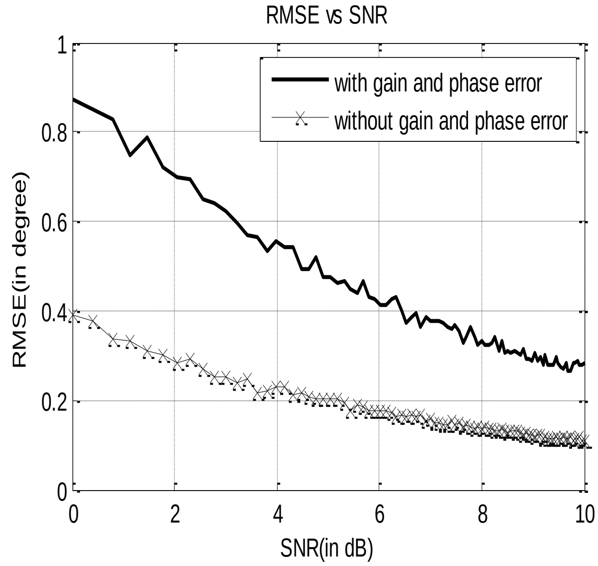

Fig 5.3 RMSE of the DOA estimates versus SNR of the signals coming from 20°,28°,36°

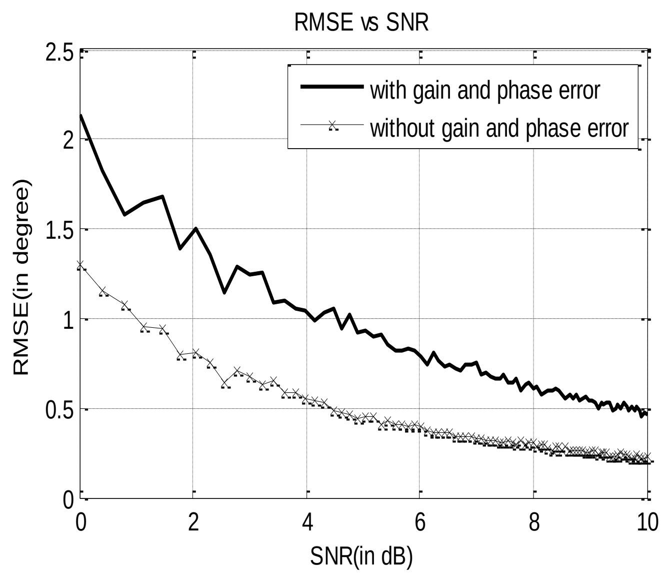

Fig 5.4 RMSE of the DOA estimates versus SNR of the signals coming from 20°,25°,30°

Chapter 6

Conclusion

6.1 Conclusion

By a very simple iterative algorithm based on ESPRIT, it is shown that unknown mutual coupling degrades the performance of DOA estimation algorithm. Moreover using the four simulation results it is shown that the error due to coupling effect increases as the source becomes closer and beyond a certain resolution the system fails to estimate the DOAs correctly. Also for antenna gain and phase error it is evident from the simulation results that the degradation in performance is more if the sources become closer. In this case also beyond a certain limit system fails to give correct DOAs.

6.2 Future work

1) The effect of changing the no of calibrated sensors, increasing the no of snapshots etc in the performance of the algorithms, can be studied.
2) Same work can be extended for URA and UCA also which are another two sensor array structure mostly used for DOA estimation.
3) Mathematical model can be given to estimate properly the amount of degradation as the sources becomes closer.

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