Numerical and Experimental Investigations on Vibration of Delaminated Composite Shell

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CERTIFICATE

*This is to certify that the thesis entitled, **"NUMERICAL AND EXPERIMENTAL INVESTIGATIONS ON VIBRATION OF DELAMINATED COMPOSITE SHELL"** submitted by **JANGYA NARAYAN GOUDA** in partial fulfilment of the requirements for the award of **Master of Technology** degree in Civil Engineering with specialization in "**Structural Engineering**" during 2012-2014 session at the National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.*

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institution for the award of any Degree or Diploma.

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CONTENTS

ABSTRACT

The laminated composite shell structures are acting as most desired structure in various field of modern engineering. These are frequently used in various industries such as aerospace, construction, nuclear field, automotive, petrochemical industries, ship & rocket building due to high specific strength and stiffness of composite materials. Because of their ability to meet the design requirements of strength & stiffness, composite materials have greater importance in recent years. But during the manufacturing and service life, there will be delamination damage in composite laminate which is not visible due to embedded within the composite structures. However it reduces the strength and stiffness of the laminated structures and also effects on the vibration characteristics of the structures.

The present work deals with the numerical and experimental investigation on free vibration characteristics of delaminated composite cylindrical shell. Experimental setup and procedure of the modal testing were described. Fabrication procedure of the shell was also described. Natural frequencies were determined using a computer programme in MATLAB environment based on finite element method (FEM). The results using the present formulation were compared with other existing literature. Also comparison was made between the experimental values and numerical predictions using FEM. The effects of percentage delamination on natural frequency of the composite shell were studied. The influence of various parameters i.e. curvature, delamination area, ply lay-up and aspect ratio on vibration response were discussed for different boundary conditions.

KEYWORD: Free vibration, woven fiber, composite shell, delamination, natural frequency.

LIST OF SYMBOLS

a, b	Dimensions of shell in X and Y axis respectively
h	Thickness of shell
[B]	Strain displacement matrix
[D]	Flexural rigidity or elasticity matrix of the shell
$[E_{11}]$, $[E_{22}]$	Elasticity modulii of lamina in both 1 & 2 direction respectively
$[G_{12}]$	Shear modulus of rigidity
J	Jacobian
[K]	Global elastic stiffness matrix
κ_x, κ_y, κ_{xy}	Curvature of the shell
[M]	Global mass matrix
$[N_x]$, $\lfloor N_y \rfloor$, $\lfloor N_{xy} \rfloor$	In plane internal stress resultants of the shell
$[M_x]$, $\lfloor M_y \rfloor$, $\lfloor M_{xy} \rfloor$	Moment resultants of the shell
n	Number of layer of the laminated panel
$[Q_x]$, $\lfloor Q_y \rfloor$	Transverse shearing forces
T	Transformation matrix
u, v, w	Displacements in X, Y, Z direction respectively
u^0, v^0, w^0	Mid plane displacements in X, Y, Z direction respectively
X, Y, Z	Global coordinate axis system
σ_{xx}, σ_{yy}, σ_{xy}	Stresses at a point

ε_{xx}, ε_{yy}, χ_{xy}	Bending strains
υ	Poisson’s ratio
θ_x, θ_y	Slopes with respect to Y and X axes
ω	Natural frequency

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CHAPTER-1

INTRODUCTION

Composite materials are frequently used in various applications such as aerospace field, construction of dome roof structures in airport, nuclear field, automotive industries, petrochemical industries, ship & rocket building due to high specific strength and stiffness, thermal characteristics etc. Shell panels have more priority in modern engineering applications because of their ability to meet the design criteria of strength, stiffness and aesthetic requirements. Due to distinct structural advantages, laminated curved panels have been extensively used in aerospace industries such as fuselage, aircraft wings etc. where structures are mostly. Shell members have often been used in modern structural design applications. Comprehensive understanding of the mechanical behaviour of composite shell is important to assure the unification of these structures during service life.

But there will be delamination in composite laminate during manufacturing (e.g. incomplete wetting, air entrapment) or during service life (e.g. low velocity impact, bird strikes). They may not be visible or barely visible on the surface, since they are embedded within the composite structures. However the presence of delamination may significantly reduce the strength and stiffness of the structures and also may affect some design parameters such as the vibration characteristics of the structure (e.g. natural frequency and mode shapes).

This is because the desired performance can be achieved by controlling the ply orientation and the stacking sequence can alter their structural properties leading to an optimal design. Since the safety and durability of the structures are greatly affected by the delamination presence in laminated curved panel, a comprehensive understanding of the delamination behaviour is more vital in the assessment of structural performance of laminated composites. Shell members with delamination may result in significant changes to their dynamic characteristics. So it plays a critical role in the structural element from the point of design aspect. Therefore it is essential to study the effects of delamination on the composite shell.

CHAPTER-2

REVIEW OF LITERATURE

INTRODUCTION

The strength and stiffness of the structures are greatly affected due to the delamination present in the composite specimens and subsequently it influence on the vibration characteristics of the composite structures. Therefore this delamination behaviour is an importance aspect during the structural design of laminated composite structures. There are so many researcher investigated on laminated composite shell for free vibration analysis. But the available literature on free vibration of the delaminated composite shell is very limited. The studies in this chapter are grouped into two parts as follows.

- Review on laminated composite shells.
- Review on composite shells with delamination.

2.1 REVIEWS OF LAMINATED SHELLS

Plenty of studies are available on the vibration analysis of laminated composite shells. The earlier investigations on dynamics analysis of composite shells is reviewed by Qatu, Sullivan and Wang (2010). In an earlier study, free vibration analysis of laminated cylindrical shells based on Sander's first approximation theory was studied by Bert and Kumar (1982).The theoretical formulation on free vibration analysis of cross ply laminated cylindrical panels was developed by Soldatos (1984). He also compared the numerical results of vibration with most of thin shell theories. Librescu *et al*. (1989) deals with the evidence of a shear deformable theory of cross-ply laminated composite shallow shells and also investigated the vibration analysis and static buckling problems of doubly curved shallow panels for a variety of boundary conditions. Narita and Ohta (1993) studied the free vibration analysis of laminated cylindrical shell using finite element approach. He investigated the vibration characteristics by varying the composite material constants and cross ply stacking sequences with typical edge conditions. Lam and Loy (1995) presented the analysis on natural frequencies of the forward and backward modes of thin rotating laminated cylindrical shells. He determined the vibration characteristics of cylindrical shells by using four thin shell theories, namely Donnell's, Flugge's, Love's and Sanders'.Dasgupta and Huang (1997) used the finite element method for a layer wise analysis for free vibration of thick composite spherical panels. A similar layer wise laminated shell theory was developed for doubly curved thick composite panels subjected to different combination of three dimension boundary conditions. Piece-wise continuous, quadratic interpolation functions through the thickness, were combined with beam function expansions in the two in plane directions of the laminate, to model the dynamic behaviour of laminated spherical panels. Singh (1999) studied the free vibration analysis of doubly curved deep sandwich panels. Natural frequencies are obtained by using Rayleigh-Ritz method. The formulation was developed for the analysis of open panels circumscribed by four curvilinear edges. Numerical results were obtained for circular, cylindrical and spherical sandwich panels with one edge clamped and other remaining free and also two opposite edges clamped and two others free.

Lam and Qian (2000) analysed the free vibration of thick symmetric angle ply laminated composite cylindrical shells using first order shear deformation theory. The frequency characteristics for thick symmetric angle ply laminated composite cylindrical shells with different h/R ratio and L/R ratios are studied in comparison with those of symmetric cross ply laminates. Also the author investigated the influence of lamination angle and number of lamination layers on frequency with simply supported boundary condition. Sahu and Datta (2001) studied the vibration and parametric resonance characteristics of laminated composite doubly curved shells subjected to various in-plane static and periodic loadings using finite element analysis based on first order shear deformation theory. The effects of number of layers, static load factor, side to thickness ratio, shallowness ratio, boundary conditions, degree of orthotropic, ply orientations and various load parameters on the principal instability regions of doubly curved panels were studied in this paper**.** Bhattacharya et al.(2002) developed a shear deformable shell element based on Reissner's hypothesis for the analysis of smart laminated composite shells. The electric field was defined in the curvilinear co-ordinate system. Lame's parameters and the radii of curvature are generated within the model. He studied the active vibration control of laminated spherical shells by considering radius of curvature and different ply lay-up.. Sahu and Datta (2002) studied the frequencies of vibration and instability behaviour of curved panels with cut outs subjected to in-plane static and periodic compressive edge loadings using finite element method. Sairam and Sreedharbabu (2002) investigated on natural frequency of composite spherical shell cap with and without a cut-out using the finite element method based on a higher-order shear deformation theory. The author observed that, the fundamental frequency of composite spherical panels generally increases with the increase in cut-out size except in the case of simply supported orthotropic spherical shell cap with fibres along meridional direction in which the fundamental frequency decreases. The dynamic analysis of laminated cross-ply composite non-circular thick cylindrical shells subjected to thermal/mechanical load is carried out based on higher-order theory was studied by Ganapathi *et al*. (2002).

Zhao *et al* (2004) analysed the free vibration of laminated two side simply supported cylindrical panels by the mesh free kp-Ritz method. This study also examined the effects of different curved edge boundary conditions on the frequency characteristics of cylindrical panels. Latifa and Sinha (2005) improved the finite element analysis of multi-layered, doubly curved composite shells by using Koiter's shell theory and Mindlin's hypotheses. He presented the free vibration analysis of doubly curved, laminated composite shells. The nonlinear free vibration behaviour of laminated composite shells subjected to hygrothermal environments using the finite element method based on first-order shear deformation theory is investigated by Swamy and Sinha (2007). The analysis is carried out by varying the curvature ratios and side to thickness ratios of composite cylindrical shell, spherical shell and hyperbolic paraboloid shell panels with simply supported boundary conditions. Nguyen-van *et al*.(2008) analysed laminated plate/shell structures based on FSDT with a quadrilateral element. The author presented free vibration analysis of laminated composite plate/shell structures of various shapes, span to thickness ratio, boundary conditions and lay-up sequences. Oktem and Chaudhuri (2009) used higher order theory for the vibration behaviour of cross ply doubly curved panels. The author determined the natural frequency by considering the influence of curvature, lamination, material properties, thickness as well as their interactions. He used all the edges simply supported boundary conditions. Kurpa *et al* (2010) studied on the vibration behavior of laminated composite shells by using R-function theory and variational methods based on first order shear deformation theory. They investigated on the dynamic behavior of the shallow shell by considering the influence of different parameters such as curvature, geometry, ply orientation and boundary conditions. Alijani *et al* (2011) investigated on the nonlinear vibration analysis of doubly curved shell due to thermal effects. Nonlinear HSDT was considered for the geometric nonlinear analysis of functionally graded shells. They discussed the effects of FGM law index, thickness ratio and temperature variations on the frequency amplitude nonlinear response. Mochida *et al* (2012) studied the dynamic behavior of the double curved shallow shell by using Superposition-Galerkin Method (SGM). Also studied the natural frequency of thin shells by considering the different curvature ratio, aspect ratio and boundary conditions. The freee vibration analysis of laminated conical shells was investigated by Civalek (2013) through discrete singular convolution method based on the shear deformation theory. Also examined the influence of geometry and material properties on the dynamic behhavior of conical shell. Kouchakzadeh and Shakouri (2014) determined the dynamic characterestics of two joined cross ply laminated conical shells using Donnel's shallow shell theory. Also studied the influence of semi-vertex angle, meridional lengths and shell thicknesses on the natural frequencies of conical shells.

2.2 REVIEWS OF SHELLS WITH DELAMINATION

The literatures containing vibrations of composite shells with delamination are scarce in literature. The free vibration and transient response of multiple delaminated doubly curved composite panels subjected to hygrothermal environment was analyzed by Parhi et al. (2001) using finite element formulation based on first order shear deformation theory. The author observed that the frequency decreases with increase in moisture content and temperature for any delamination area. It also observed that, the vibration characteristic of shells are more influenced by curvature. Liu and Yu (2003) studied the finite element modelling of delamination by layer wise shell element allowing for interlaminar displacements. In his paper, he gave a brief review of the layer wise shell element. He considered geometrically non-linear analysis involving finite rotation and finite strain. According to his results he suggested that central delamination is more damaging than the edge and corner delamination for it causes a greater reduction in the strength and integrity of the plate, in particular in the early stage of shear mode delamination. Kim and Cho (2003) derived an higher order shell theory for composite shell with several delamination. It studied the effects of number, shape, size and locations of the delamination. Yang and Fu (2006) studied the composite curved panel for delamination growth. They also developed a equation for buckling analysis of single curved laminated panel based on the vibrational principle of moving boundary. The author analysed on growth of delamination by considering the effects of delamination size, stacking sequence, material properties and boundary conditions. Jinhua and Yiming (2007) analysed the dynamic stability for composite single curved panel with delamination. It investigate the influence of the amplitude of external excitation, material properties, the delamination size and the location of delamination and the principal dynamic instability region of the delaminated cylindrical shells. Acharyya *et al.* (2009) analysed the bending behaviour of laminated single curved panel with delamination by the finite element method. In this paper, analysis was done by considering the influence of delamination area, curvature with different boundary condition.

Park and Lee (2009) investigated on delaminated spherical shell by considering in-plane pulsating forces for dynamic stability analysis. In this study, a finite element formulation was developed based on higher order shell theory. They presented the dynamic analysis due to the effects of the radius to length ratio, size and location of delamination, fiber lay-up etc. It also investigated the effects of the amount of the periodic in-plane load on the instability region. Lee and Chung (2010) developed a formulation for vibration analysis of curved panels with delamination around a central cut-out using finite element analysis. In this literature, it mainly shows that the effect of the curvature to length ratio, delamination and cut out size, the number of layer and the location of delamination in the layer direction. Nanda and Sahu (2012) studied the vibration behaviour of the delaminated composite shell with delamination by using finite element method based on first order shear deformation theory. In this paper, the author determined the natural frequencies of the delaminated cylindrical, spherical and hyperbolic paraboloid shells due to the effect of delamination size and number of layers.

2.3 OBJECTIVE AND SCOPE OF PRESENT INVESTIGATION

Various analytical and numerical methods have been used in studying the dynamic behaviour of delaminated composite shells. Experimental studies on vibration of delaminated shells are not available in open literature. The present work deals with an experimental investigations on free vibration analysis of composite cylindrical shell by considering the effects of curvature, percentage of delamination area, aspect ratio, number of layer and different boundary condition. The experimental results are compared with prediction based on finite element methods.

CHAPTER-3

MATHEMATICAL FORMULATION

In this chapter the finite element mathematical formulation for vibration of the shell structures of various geometries with delamination is carried out. For analysis, here a double curved panel of composite laminate is considered. The boundary conditions are incorporated in the most general manner.

3.1 FINITE ELEMENT ANALYSIS

A composite doubly curved panel as shown in Fig: 1 consists of 'n' number of laminate is considered for analysis.. The sides of panel in x and y directions are represented as a and b respectively. The ply details of curved panel are shown below in Fig: 2.

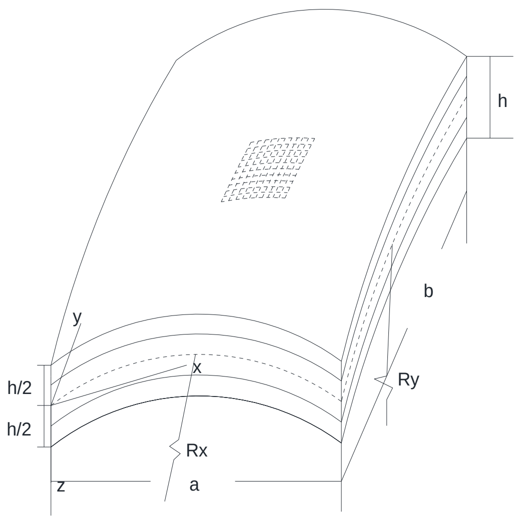


Fig 1: Doubly curved panel with delamination

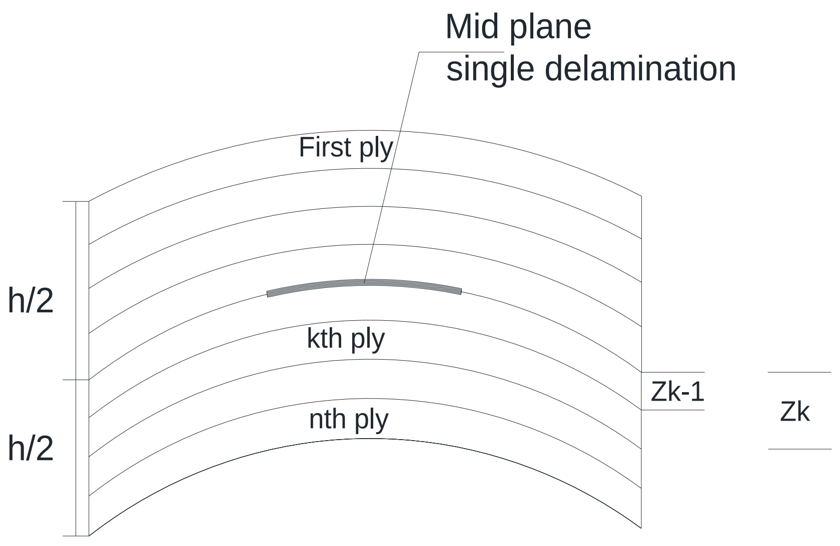


Fig 2: Layer details of curved panel.

Let the shell thickness h has mid surface at a distance h/2 from each lateral surface. For the analysis purpose, X-Y plane is located in mid surface, therefore z=0 identifies the mid surface. Let u, v, w be the displacements at any point (x, y, z) and θ_x, θ_y are the rotations of the cross section perpendicular to the y and x axis respectively.

The displacement field is given by

$$u(x,y,z,t) = u^0(x,y,z,t) + z\theta_x(x,y,t),$$

$$v(x,y,z,t) = v^0(x,y,z,t) + z\theta_y(x,y,t) \qquad (1)$$

$$w(x,y,z,t) = w^0(x,y,z,t)$$

Where, u^0, v^0 and w^0 are mid plane values. Using Sander's first theory for thin shells, the mid-plane strains and curvatures are represented as

$$\{\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^T = \{\varepsilon_{xx}^0 \quad \varepsilon_{yy}^0 \quad \gamma_{xy}^0 \quad \gamma_{xz}^0 \quad \gamma_{yz}^0\}^T + \{\kappa_{xx} \quad \kappa_{yy} \quad \kappa_{xy} \quad \kappa_{xz} \quad \kappa_{yz}\}^T \text{...(2)}$$

Where,

$$\begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \\ \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u^0}{\partial x} + \dfrac{w}{R_x} \\ \dfrac{\partial v^0}{\partial y} + \dfrac{w}{R_y} \\ \dfrac{\partial u^0}{\partial y} + \dfrac{\partial v^0}{\partial x} + \dfrac{2w}{R_{xy}} \\ \theta_x + \dfrac{\partial w}{\partial x} \\ \theta_y + \dfrac{\partial w}{\partial y} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \\ \kappa_{xz} \\ \kappa_{yz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \theta_x}{\partial x} \\ \dfrac{\partial \theta_y}{\partial y} \\ \dfrac{\partial \theta_x}{\partial x} + \dfrac{\partial \theta_y}{\partial y} \\ 0 \\ 0 \end{Bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

R_x , R_y and R_{xy} are the three radii of curvature of the shell element. Equation of equilibrium reduce to that for cylindrical shell, when the values of R_x and R_{xy} becomes ∞. Let u^0, v^0, w and θ_x, θ_y are the degree of freedoms considered for this formulation at nodes of shell element. By taking the eight-nodded element shape functions, the element displacements are expressed as

$$u^0 = \sum_{i=1}^{8} N_i u_i^0 , \qquad v^0 = \sum_{i=1}^{8} N_i v_i^0 , \qquad w^0 = \sum_{i=1}^{8} N_i w_i^0 , \qquad (4)$$

$$\theta_x = \sum_{i=1}^{8} N_i \theta_{xi} , \qquad \theta_y = \sum_{i=1}^{8} N_i \theta_{yi}$$

Where N_i s are the shape functions used to interpolate the generalized displacements u_i^0, v_i^0 , w_i , θ_{xi} and θ_{yi} at node i within an element.

The stress resultants are related to the mid-plane strains and curvatures for a general laminated shell element as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_y \\ Q_x \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \\ \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \quad ....(5)$$

Where, N_x, N_y and N_{xy} are in-plane stress resultants, M_x, M_y and M_{xy} are moment resultants and Q_y, Q_x are transverse shear stress resultants. The extensional, bending-stretching and bending stiffness's of the laminate are expressed in the usual form as

$$\left(A_{ij,} B_{ij} D_{ij}\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\overline{Q}_{ij}\right)_k \ (1, z, z^2)\ dz, \qquad ......(6)$$

$$i, j = 1, 2, 6.$$

Similarly, the shear stiffness is expressed as

$$\left(S_{ij}\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \alpha \left(\overline{Q}_{ij}\right)_k \ ds, \qquad i, j=4, 5. \qquad ......(7)$$

α is the shear correction factor which is derived from the Timoshenko beam concept by applying the energy principle is assumed as 5/6. It accounts for the non-uniform distribution of transverse shear strain across the thickness of the laminate.

$\left(\overline{Q}_{ij}\right)_k$ in equations (6) and (7) are the off-axis stiffness values defined as

$$\left(\overline{Q}_{ij}\right)_k = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2-n^2 \end{bmatrix} \quad \left(Q_{ij}\right)_k = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2-n^2 \end{bmatrix} \quad i, j = 1, 2, 6.$$

$$(8)$$

$$\left(\overline{Q}_{ij}\right)_k = \begin{bmatrix} m & -n \\ n & m \end{bmatrix} \qquad \left(Q_{ij}\right)_k = \begin{bmatrix} m & n \\ -n & m \end{bmatrix}, \qquad i, j=4, 5.$$

Where, m= $\cos\theta$, n= $\sin\theta$

and $(Q_{ij})_k$ are the on axis stiffness coefficients in the material direction given by

$$(Q_{ij})_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}, \qquad \text{i, j= 1, 2, 6}$$

And

$$(Q_{ij})_k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \qquad \text{i, j= 4, 5}$$

$(Q_{ij})_k$ are calculated by taking young's modulus, shear modulus and Poisson's ratio values of the material

$$Q_{11} = E_1/(1-\upsilon_{12}\upsilon_{21}), \qquad Q_{12} = \upsilon_{12}E_2/(1-\upsilon_{12}\upsilon_{21}), \qquad Q_{21} = \upsilon_{12}E_2/(1-\upsilon_{12}\upsilon_{21}),$$
$$Q_{22} = E_2/(1-\upsilon_{12}\upsilon_{21}), \quad Q_{66} = G_{12}, \; Q_{44} = G_{13}, \quad Q_{55} = G_{23}. \qquad (9)$$

E_1, E_2 = Young's moduli of a lamina along and across the fibres, respectively

G_{12}, G_{13}, G_{23} = Shear moduli of a lamina with respect to 1,2 and 3 axes.

$\upsilon_{12}, \upsilon_{21}$ = Poisson's ratios.

The strain can be described in term of displacements as

$$\{\varepsilon\} = [B]\{d_e\}$$

Where $\{d_e\} = [u_1 v_1 w_1 \theta_{x1} \theta_{y1} \ldots\ldots\ldots\ldots u_8 v_8 w_8 \theta_{x8} \theta_{y8}]^T$

$$[B] = [[B_1]\ldots\ldots\ldots\ldots[B_7][B_8]]$$

$$[B_i] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \\ 0 & 0 & \frac{\partial N_i}{\partial x} & N_i & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & N_i \end{bmatrix}_{i=1 to 8}$$

The potential energy of the deformation for the element is given by

$$U_e = \frac{1}{2} \iiint \{\varepsilon\}^T [\sigma] dV$$

$$\{\varepsilon\} = \{\varepsilon^0{}_{xx} \varepsilon^0{}_{yy} \ \gamma^0{}_{xy} \kappa_{xx} \kappa_{yy} \kappa_{xy} \gamma^0{}_{yz} \gamma^0{}_{xz}\}$$

Where $\{\varepsilon\} = [B]\{d_e\}$

Then $U_e = \frac{1}{2} \iint \{d_e\}^T [B]^T [D][B]^T \{d_e\} dxdy$

$$= \frac{1}{2} \iint \{d_e\}^T [K_e]\{d_e\}$$

Where the element stiffness matrix, $[K_e]$ is given by

$$[K_e] = \int_{-1}^{1} \int_{-1}^{1} [B]^T [D][B] J| d\zeta d\eta ....(10)$$

Where [B] is stain-displacement matrix, [D] is the elasticity matrix and J is the Jacobean matrix

Similarly the element mass matrix $\left[M_e\right]$ is expressed as

$$\left[M_e\right]=\int_{-1}^{1}\int_{-1}^{1}\left[N\right]^T\left[\rho\right]\left[N\right]Jd\zeta d\eta \text{..........................(11)}$$

With [N], the shape function matrix and $\left[\rho\right]$, the inertia matrix.

The shape functions N_i are defined as

$$N_i=\left(1+\xi\xi_i\right)\left(1+\eta\eta_i\right)\left(\xi\xi_i+\eta\eta_i-1\right)/4 \qquad \text{i=1 to 4}$$

$$N_i=\left(1-\zeta^2\right)\left(1+\eta\eta_i\right)/2 \qquad \text{i=5, 7}$$

$$N_i=\left(1+\zeta\zeta_i\right)\left(1-\eta^2\right)/2 \qquad \text{i=6, 8}$$

Where ζ and η are the local natural coordinates of the element and ζ_i & η_i are the values at the i^{th} mode. The derivatives of the shape function N_i with respect to x, y are expressed in term of their derivatives with respect to ζ and η by the following relationship.

$$\begin{bmatrix} N_{i,x} \\ N_{i,y} \end{bmatrix}=\left[J\right]^{-1}\begin{bmatrix} N_{i,\zeta} \\ N_{i,\eta} \end{bmatrix}$$

Where $$\left[J\right]=\begin{bmatrix} \dfrac{\partial x}{\partial \zeta} & \dfrac{\partial y}{\partial \zeta} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} \end{bmatrix}$$

Multiple delamination modelling:

A typical composite laminate having p number of delamination is considered as shown in Figure 3. The displacement field within any sub laminate t is denoted by

$$u_t=u_t^0+\left(z-z_t^0\right)\theta_{x,} \qquad v_t=v_t^0+\left(z-z_t^0\right)\theta_{y,} \qquad (12)$$

Where, u_t^0, v_t^0 are the mid-plane displacement of the t_{th} sub laminate and z_t^0 , the distance between the mid-plane of the original laminate and the mid-plane of the t_{th} sub laminate.

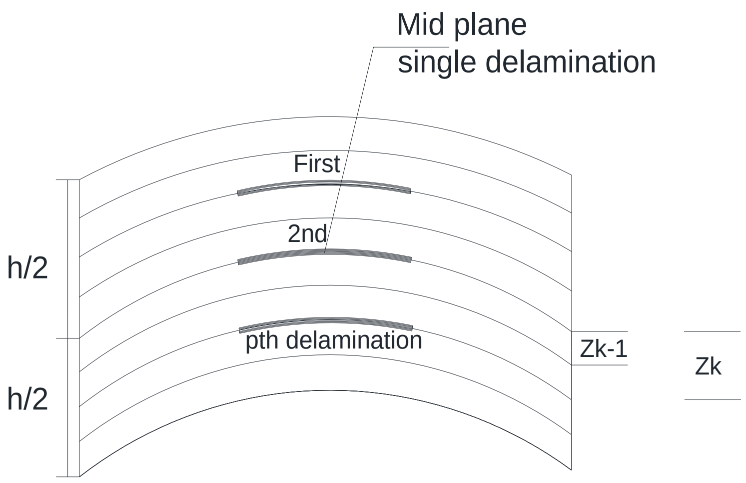


Fig 3: Multiple delamination model

The strain component of any layer of a sub laminate are found from equation (12) in the form of

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}_s = \begin{Bmatrix} \varepsilon^0{}_{xx} \\ \varepsilon^0{}_{yy} \\ \varepsilon^0{}_{xy} \end{Bmatrix}_s + \left(Z - Z_t^0\right) \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \qquad (13)$$

For compatibility condition at the periphery of delamination, the transverse displacements and rotations at a common node for all sub laminates is assumed to be constant. So the sub laminate displacements at mid-point can be expressed as

$$u_t = u^0 + Z_t^0 \theta_{x,}, \quad v_t = v^0 + Z_t^0 \theta_{y,} \qquad (14)$$

The mid plane strain components of the t_{th} sub laminate are derived from it as

$$\begin{Bmatrix} \varepsilon^0{}_{xx} \\ \varepsilon^0{}_{yy} \\ \varepsilon^0{}_{xy} \end{Bmatrix}_s = \begin{Bmatrix} \varepsilon^0{}_{xx} \\ \varepsilon^0{}_{yy} \\ \varepsilon^0{}_{xy} \end{Bmatrix} + Z_t^0 \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \qquad (15)$$

By substituting the equation (15) into equation (13), then we get strain components at any layer within a sub laminate.

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}_s = \begin{Bmatrix} \varepsilon^0{}_{xx} \\ \varepsilon^0{}_{yy} \\ \varepsilon^0{}_{xy} \end{Bmatrix} + Z_t^0 \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} + \left(Z - Z_t^0\right) \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}$$

For any lamina of the t_{th} sub laminate, the in-plane and shear stresses are found from the relations

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} + \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}_t \text{.......................(16)}$$

$$\begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{44} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}_t \text{....................................(17)}$$

Where σ_{xx} and σ_{yy} are the normal stresses along X and directions respectively and τ_{xz}, τ_{yz} are shear stresses in XZ , YZ planes respectively.

Integrating these stresses over the thickness of the sub laminate, the stress and moment resultants of the sub laminate are desired which lend to the elasticity matrix of the t_{th} sub laminate is the form

$$[D]_t = \begin{bmatrix} A_{ij} & Z_t^0 A_{ij} + B_{ij} & 0 \\ B_{ij} & Z_t^0 B_{ij} + D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \text{..(18)}$$

Where $$\left[A_{ij}\right]_t = \int_{\frac{-h_t}{2}+Z_t^0}^{\frac{h_t}{2}+Z_t^0} \left[\overline{Q}_{ij}\right] dz$$

$$\left[B_{ij}\right]_t = \int_{\frac{-h_t}{2}+Z_t^0}^{\frac{h_t}{2}+Z_t^0} \left[\overline{Q}_{ij}\right]\left(Z - Z_t^0\right) dz$$

$$\left[D_{ij}\right]_t = \int_{\frac{-h_t}{2}+Z_t^0}^{\frac{h_t}{2}+Z_t^0} \left[\overline{Q}_{ij}\right]\left(Z - Z_t^0\right)^2 dz\,, \qquad \text{imp=1,2,6}$$

$$\left[S_{ij}\right]_t = \int_{\frac{-h_t}{2}+Z_t^0}^{\frac{h_t}{2}+Z_t^0} \left[\overline{Q}_{ij}\right] dz\,, \qquad \text{i, j=4,5}$$

Here h_t is the thickness of the t_{th} sub laminate.

The in-plane stress and moment resultants for the t_{th} sub laminate can be expressed in a generalised manner as

$$[D]_t = \begin{bmatrix} A_{11} & A_{12} & A_{16} & z^0{}_t A_{11} + B_{11} & z^0{}_t A_{12} + B_{12} & z^0{}_t A_{16} + B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & z^0{}_t A_{12} + B_{12} & z^0{}_t A_{22} + B_{22} & z^0{}_t A_{26} + B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & z^0{}_t A_{16} + B_{16} & z^0{}_t A_{26} + B_{26} & z^0{}_t A_{66} + B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & z^0{}_t B_{11} + D_{11} & z^0{}_t B_{12} + D_{12} & z^0{}_t B_{16} + D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & z^0{}_t B_{12} + D_{12} & z^0{}_t B_{22} + D_{22} & z^0{}_t B_{26} + D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & z^0{}_t B_{16} + D_{16} & z^0{}_t B_{26} + D_{26} & z^0{}_t B_{66} + D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix}$$

3.2 GOVERNING EQUATION FOR FREE VIBRATION ANALYSIS

The finite element formulation is developed for the dynamic analysis of laminated composite shells with delamination using the first order shear deformation theory. An eight-nodded continuous isoparametric shell element is employed in the present analysis with five degrees of freedom viz. u, v, w, θ_x, and θ_y at each node.

The eigenvalue equation for the free vibration analysis of delaminated composite shell can be expressed as

$$\left([K] - \omega^2 [M]\right)\{\Phi\} = \{0\}\text{..........................(19)}$$

Where [K] and [M] are the global stiffness and global mass matrices, ω is the natural frequency and Φ is the corresponding eigenvectors i.e. mode shape.

3.3 COMPUTER PROGRAMME

A computer programme is developed by using MATLAB environment to perform all the necessary computations. The element stiffness matrix and element mass matrix are derived using above formulation. Numerical integration technique by Gaussian quadrature is adopted for element matrices. The global matrices are obtained by assembling the corresponding element matrices. The boundary conditions are imposed restraining the generalised displacements in different nodes of the discretized structures. Different delamination such as 6.25%, 25% and 56.25% of area of shell are considered for the vibration analysis of composite cylindrical shell as shown in Appendix.

CHAPTER-4

EXPERIMENTAL PROGRAMME

4.1 INTRODUCTION

In this chapter, the details of the experimental works are conducted on the vibration analysis of woven roving composite cylindrical shells with delamination. Therefore composite shells are fabricated for the aforementioned experimental work and the material properties are found out by tensile test as per ASTM D3039/D3039M (2008) guidelines to characterise the delaminated composite shells. The experimental results are compared with the analytical or numerical predictions.

4.2 MATERIALS

For fabrication of laminated composite specimens which are required for tensile as well as free vibration testing, following materials were used as

- Woven roving glass fiber
- Epoxy
- Hardener
- Polyvinyl alcohol spray
- Teflon film for artificial delamination

4.3 FABRICATION OF SPECIMENS FOR TENSILE TESTING

In the present investigation, the glass: epoxy laminate was fabricated in a proportion of 50:50 by weight fractions of fibre: matrix. Lapox L12 is a liquid, unmodified epoxy resin of medium viscosity and hardener K6 with low viscosity were used with woven roving glass fiber to fabricate the laminated composite specimen. Woven roving glass fibers were cut into required shape and size for fabrication. Epoxy resin matrix was prepared by using 8% hardeners. A flat rigid platform was selected as a contact mould in order to get the flat samples for tensile testing. Hand lay-up method was used for combine the glass fiber with epoxy resin of required sequence. The mould releasing sheet was held on the flat plywood and a polyvinyl alcohol spray was used as releasing agent on plastic sheet. A gel coat was applied over the plastic sheet by brush to get the smooth surface and also protect the fiber from direct contact of environment. Then a layer of woven roving glass fiber was laid on gel coat over which steel roller was used as rolling purpose to remove air entrapment. Subsequent plies were placed one upon another with the matrix in each layer to get eight stacking plies. The laminate consisted of 8 layers of identically $0-90^{\circ}$ oriented woven fibres as per ASTM D2344/ D2344M (2006) specifications. A releasing sheet with polyvinyl alcohol thin film was placed over the lay-up and mould to prevent the bonding between layup and mould surface. Then the prepared specimens were cured under normal temperature for 48hrs. The releasing sheets were removed from the mould after the proper curing of specimen. From the laminates, the specimens were cut into 250 x 25mm (Length x Breadth) size as per ASTM D2344/ D2344 specification by diamond cutter and the thickness was taken as per the actual measurement. The average thickness of specimens for tensile test is found 2.62mm.

4.4 DETEMINATION OF MATERIAL CONSTANTS

Laminated composite plates behave like orthotropic lamina, the characteristics of which can be defined by material constants i.e. E_1, E_2, G_{12} and υ_{12}. Laminate having eight layers was casted to estimate its properties. An instrument INSTRON 1195 UTM was used to perform the tensile test on samples cut along longitudinal, transverse and $45°$ to the longitudinal direction as described in ASTM standard D3039/D3039M (2008). The tensile test specimens are having a constant rectangular cross section in all the cases. The dimensions of the specimen are mentioned below in Table 1.

Table 1: Size of the specimen for tensile testing

Length(mm)	Width(mm)	Thickness(mm)
250	**25**	**2.62**

The samples were cut from the casted plate as per requirement by tile cutter or diamond cutter as shown in Fig 4. In this study, six specimens were tested as shown in Fig 5 for mean values of material constants.

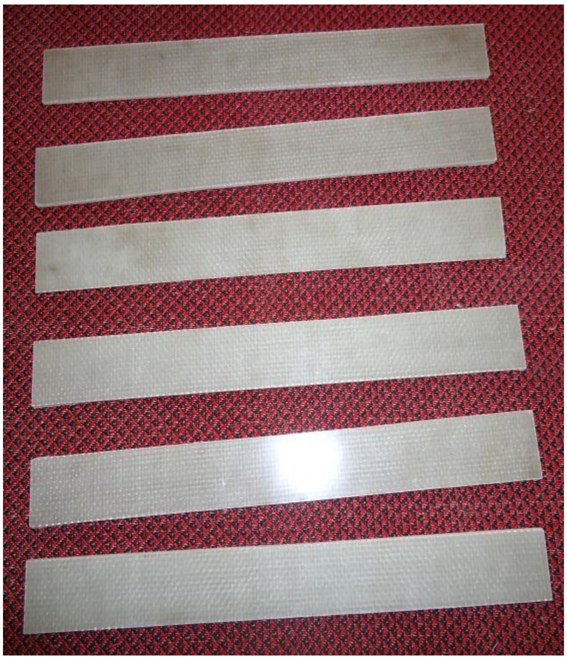


Fig 4: Diamond cutter for cutting specimens **Fig 5: Specimens for tensile testing**

For measuring the Young's modulus, the specimen was loaded in INSTRON 1195 UTM (as shown in Figure 6) with a rate of loading of 0.2 mm/minute. Specimens were fixed in the upper jaw first and then gripped in the movable jaw (lower jaw). Gripping of the specimen should be as much as possible to prevent the slippage. Here, it was taken as 50mm in each side for gripping. Initially strain was kept at zero. The load, as well as the extension, was recorded digitally with the help of a load cell and an extensometer respectively. Crack pattern of specimen during testing is shown in Figure 7. From these data obtained, stress vs strain curve was plotted and the initial slope of this curve gives the young’s modulus of the materials. By using bi-axial strain gauge, longitudinal and lateral strains are recorded and then Poisson’s ratio was determined. Poisson’s ratio of the composite is considered as 0.25 for this study. As per Jones (1975), the shear modulus was calculated from the formula as follows.

$$G_{12} = \frac{1}{\frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2\upsilon_{12}}{E_1}}$$

Where,

E_1 = Modulus of elasticity along longitudinal direction

E_2 = Modulus of elasticity along transverse direction

E_{45} = Modulus of elasticity along $45°$ direction

υ_{12} = Poisson's ratio

Fig 6: Tensile test set up **Fig 7: WR glass/epoxy specimen during tensile testing**

4.5 FABRICATION OF SPECIMENS FOR FREE VIBRATION TESTING

The fabrication procedure for preparation of the shell sample in case of vibration study was same as that of plate only difference is that instead of flat ply, fabricated moulds (as shown in Fig.8) of different curvature were selected for preparing the specimen. Hand lay-up method was used for combine the glass fiber with epoxy resin of eight layers of 0-90^0 oriented. A Teflon tape was introduced at mid plane 6.25%, 25%, 56.25% central delamination of specimen during casting. After completion of all eight layers of fiber lay-up with resin matrix, again a releasing sheet with polyvinyl alcohol agent were placed over the lay-up and mould to prevent the bonding of epoxy from ply and mould surface. One cover mould of specified curvature with filled sand was kept over the casted mould and left for a 48hrs for curing purpose under normal temperature. Then samples are being transported and cut into required size for vibration testing. A size of 235mm x 235mm shell samples are prepared for conducting the vibration test as shown in Table 2.

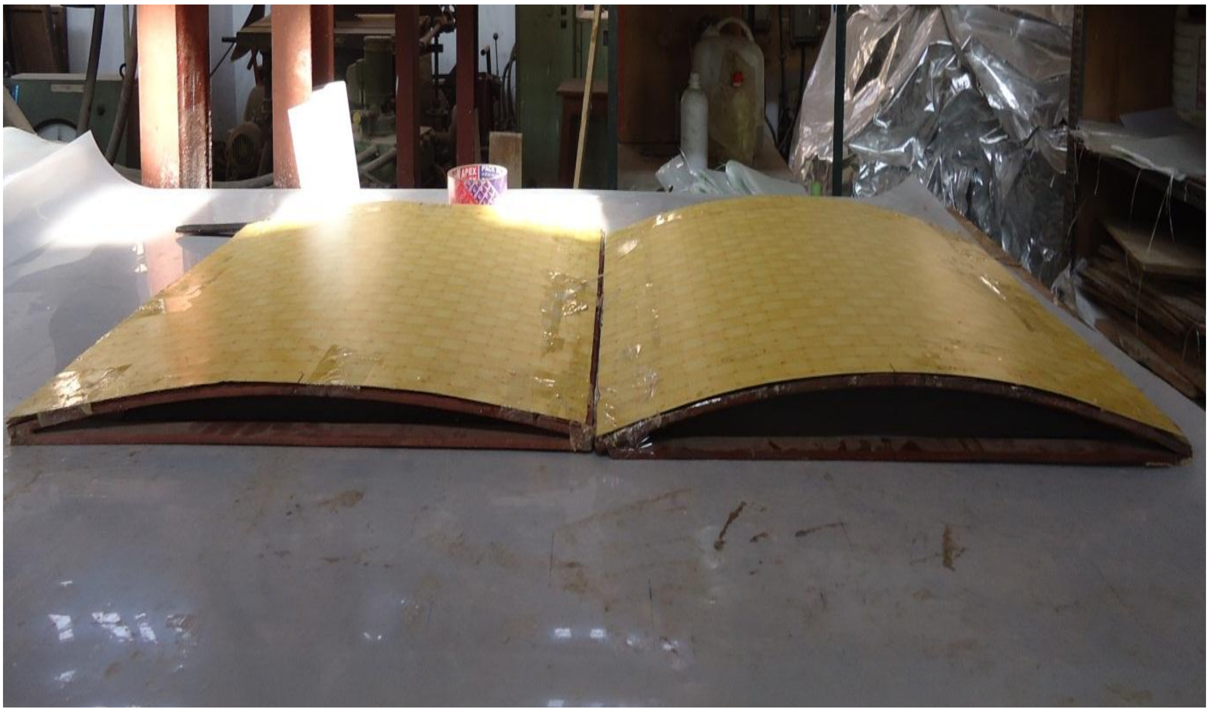


Fig 8: Fabricated moulds

Fig.9 to Fig.12 shows the process of casting for laminated composite cylindrical shells. Fig.13 & Fig.14 shows the introduction of artificial delamination during casting of composite cylindrical shells.

Fig.9

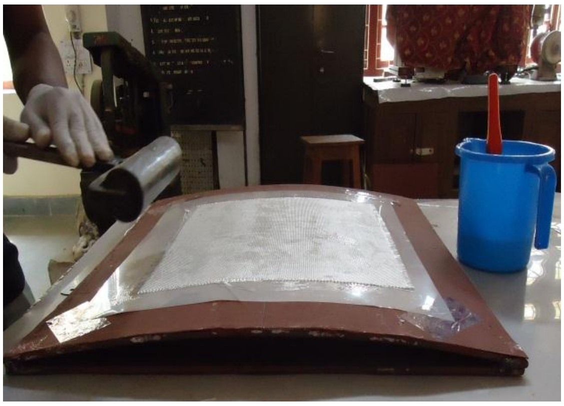


Fig.10

Fig.(9) Use of gel coat on plastic sheet, Fig (10) Placing of woven roving glass fibre on gel coat & removal of air entrapment using steel roller

Fig.11

Fig.12

Fig (11) Placing of releasing sheet, Fig (12) Set-up for fabrication of delaminated composite shell.

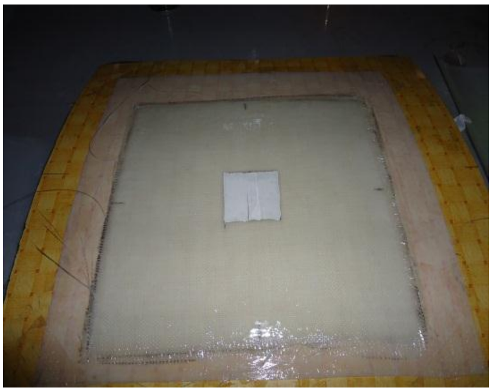


Fig 13: 6.25% Delamination

Fig 14: 56.25% Delamination

The geometrical dimensions (i.e. length, breadth, and thickness), ply orientations and percentage of delamination of the fabricated shells are shown in Table 2. All the specimens described in Table 3 were tested for its vibration characteristics. To examine the influence of boundary condition on the natural frequency of delaminated shells, the samples were tested for three different boundary conditions (B.C) i.e. for four sides free, two opposite sides simply supported and four sides clamped. For different boundary conditions, one iron frame was used. Some of the test specimens with different boundary conditions i,e. F-F-F-F, S-F-S-F and C-C-C-C are shown in Fig 15 to Fig 17.

Fig 15: Frame for F-F-F-F boundary condition.

Fig 16: Frame for C-C-C-C boundary condition.

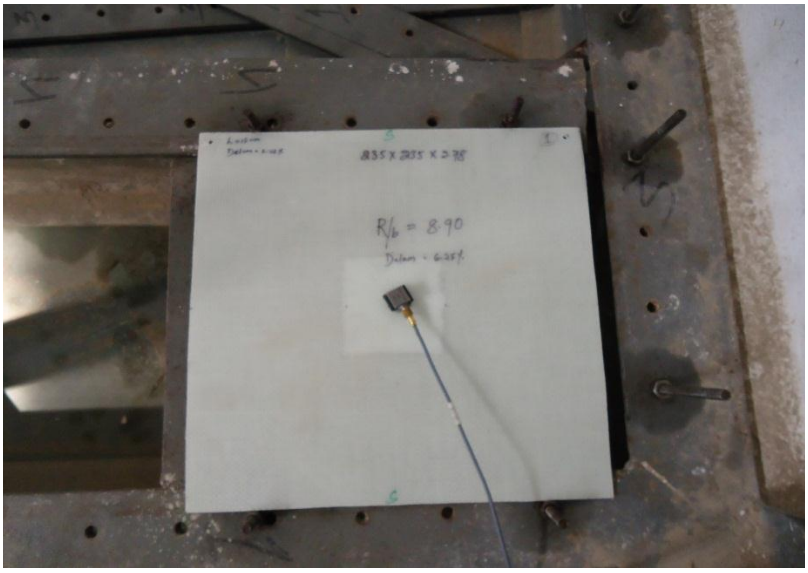


Fig 17: Frame for S-F-S-F boundary condition

Table 2: Dimensions of composite specimens with and without delamination

Size of Specimen(mm)	Curvature	No. of layers	% of Delamination	Ply orientation	No. of delamination	No of specimens
$235 \times 235 \times 2.32$	$R_x = \infty$ $R_y = \infty$	8	0	$[0/90]_4$	0	1
$235 \times 235 \times 2.84$		8	6.25	$[0/90]_4$	1	1
$235 \times 235 \times 2.89$		8	25	$[0/90]_4$	1	1
$235 \times 235 \times 3.17$		8	56.25	$[0/90]_4$	1	1
$235 \times 235 \times 2.31$	$R_x = \infty$ $R_y = 0.91$	8	0	$[0/90]_4$	0	1
$235 \times 235 \times 2.69$		8	6.25	$[0/90]_4$	1	1
$235 \times 235 \times 2.57$		8	25	$[0/90]_4$	1	1
$235 \times 235 \times 2.55$		8	56.25	$[0/90]_4$	1	1
$235 \times 235 \times 2.61$	$R_x = \infty$ $R_y = 1.2625$	8	0	$[0/90]_4$	0	1
$235 \times 235 \times 2.72$		8	6.25	$[0/90]_4$	1	1
$235 \times 235 \times 2.76$		8	25	$[0/90]_4$	1	1
$235 \times 235 \times 2.88$		8	56.25	$[0/90]_4$	1	1
$235 \times 235 \times 2.52$	$R_x = \infty$ $R_y = 2.09$	8	0	$[0/90]_4$	0	1
$235 \times 235 \times 2.78$		8	6.25	$[0/90]_4$	1	1
$235 \times 235 \times 2.72$		8	25	$[0/90]_4$	1	1
$235 \times 235 \times 2.85$		8	56.25	$[0/90]_4$	1	1

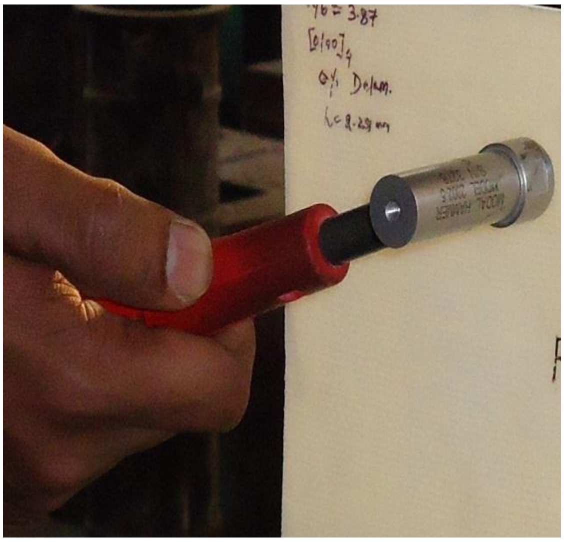
Size of Specimen(mm)	Curvature	No. of layers	% of Delamination	Ply orientation	No. of delamination	No of specimens
$235 \times 235 \times 1.50$	$R_x = \infty$ $R_y = 1.2625$	4	25	$[0/90]_2$	1	1
$235 \times 235 \times 2.76$		8	25	$[0/90]_4$	1	1
$235 \times 235 \times 3.97$		12	25	$[0/90]_6$	1	1
$237 \times 158 \times 2.81$		8	25	$[0/90]_4$	1	1
$236 \times 118 \times 2.82$		8	25	$[0/90]_4$	1	1
$235 \times 235 \times 2.76$		8	6.25	$[0/90]_4$	3	1
$235 \times 235 \times 2.76$		8	25	$[0/90]_4$	3	1
$235 \times 235 \times 2.76$		8	56.25	$[0/90]_4$	3	1
$235 \times 235 \times 2.78$		8	25	$[30/-30]_4$	1	1
$235 \times 235 \times 2.79$		8	25	$[45/-45]_4$	1	1

4.6 EQUIPMENT FOR VIBRATION TEST

The apparatus which are used in free vibration test are

- Modal hammer (type 2302-5)
- Accelerometer (type 4507)
- FFT Analyzer (Bruel & Kajer FFT analyzer type -3560)
- PULSE software.

The apparatus which are used in the vibration test are shown in Figure 18 to Figure 21.



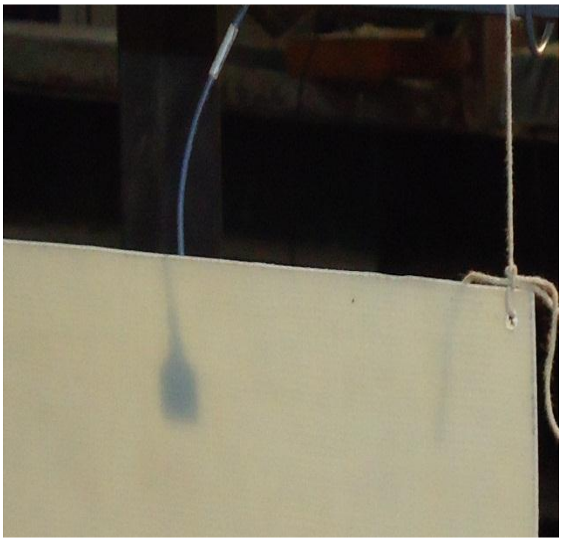


Fig 18: Modal Impact Hammer (type 2302-5) **Fig 19: Accelerometer (4507)**

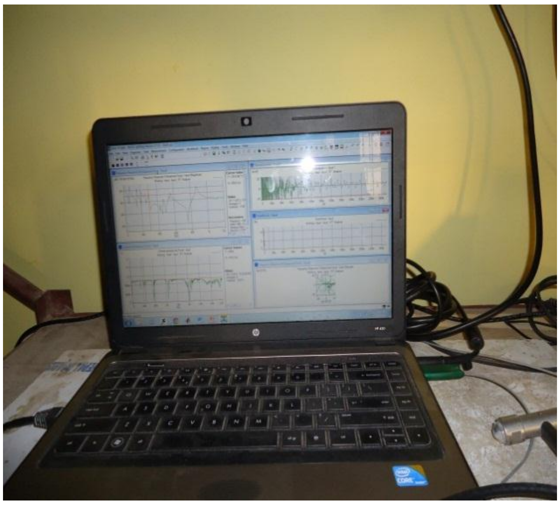


Fig 20: Bruel & Kajer FFT analyser **Fig 21: Display unit**

4.7 PROCEDURE FOR FREE VIBRATION TEST

The set up and the procedure for the free vibration test are described sequentially as given below. An iron frame was kept for creating the different boundary condition. For all sides free boundary condition, the test specimens were fitted with the string attached to the iron frame as shown in Fig 15. Similarly for C-C-C-C and S-F-S-F boundary conditions, special arrangements have been made by cast iron frame/plate attached to the main iron frame as shown in Fig 16 and Fig 17 respectively. The connection of FFT analyser, laptop, transducer, modal hammer and accelerometer with laptop were done. The pulse lab shop version-10.0 software key was inserted to the laptop port. Accelerometer (B&K, 4507) was attached to the sample by means of bee wax and then fitted to the specified frame for different boundary condition. Now with modal hammer (2302-5), a specimen was excited by means of small impact on particular points with 5 times and care should be taken the impact should be perpendicular to the point of contact on the specimen surface. The input signals were captured by the force transducer fixed on the modal hammer. Then the accelerometer sensed the resulting vibration. The signal was then processed by the FFT analyser and the frequency spectrum was also obtained. Both input and output signals are investigated by spectrum analyser and resulting FRF are transmitted to the system. By using pulse software, the output from the analyser was displayed on the analyser screen. Then the natural frequencies of the fitted specimens are measured by moving the cursor to peak of the FRF graph.

CHAPTER-5

RESULTS AND DISCUSSION

5.1 INTRODUCTION

The free vibration results i.e. natural frequency of composite shell structures with delamination are numerically studied by using the above FEM formulation in this chapter. The effect of different constraints i.e. curvature radius to side ratio, area delamination, numbers of layers, aspect ratio on different boundary conditions on the vibration of delaminated composite shells are presented using numerical method. The experimental results on free vibration of composite shells are also used to support the numerical predictions. The study is made for vibration analysis as following objectives.

i. Comparison with previous studies.
ii. Numerical and experimental results.

5.2 VIBRATION ANALYSIS

The dynamic behaviours of the composite structures are greatly affected by the delamination. Therefore in the present investigation, natural frequencies of delaminated composite cylindrical shells were determined both finite element analysis and experimental programme. The effects of different constraints like delamination area, curvature to side ratio, boundary condition, aspect ratio, and number of layers were studied critically. For free vibration analysis, numerical results are compared with other existing literature. Experimental results are also presented on natural frequency of composite cylindrical shells with delamination.

5.2.1 COMPARISION WITH PREVIOUS STUDIES

In order to compute the vibration analysis of composite shell numerically, MATLAB programs are developed based on finite element formulation and delamination modelling mentioned in earlier chapter. For free vibration of laminated composite plate with different boundary conditions, comparison was made between the present numerical results with the other existing literature by Ju *et al.* (1995) using FEM to validate the program as shown in Table 3.

Table 3: Comparison of natural frequency (Hz) for laminated composite plates at different boundary condition.

Ply orientation=(0/90/45/90/90/45/90/0), a=b=0.25m, h=0.00212m, $\upsilon_{12} = \upsilon_{13} = 0.291$, $\upsilon_{23} = 0.3$, ρ =1446.20 kg/m^3, $E_{11} = 132GPa$, $E_{22} = 5.35GPa$, $G_{12} = 2.79GPa$.

Boundary Condition	**mode**	**Ju *et al.* (1995)**	**Present FEM**
Four sides Free	1st	73.309	72.534
	2nd	202.59	201.398
	3rd	243.37	243.549
	4th	264.90	263.260
Four sides simply supported	1st	164.37	163.71
	2nd	404.38	401.25
	3rd	492.29	494.57
	4th	658.40	650.93
Four sides clamped	1st	346.59	342.86
	2nd	651.51	637.34
	3rd	781.06	768.41
	4th	1017.2	969.73

Similarly, comparison was made for the fundamental frequencies of a composite cylindrical shells with mid plane single delamination based on present finite element formulation and delamination modelling through analytical results of Parhi *et al*. (2001) as shown in Table 4.

Material and geometry properties:

a=b=0.5m, a/h=100, υ_{12}=0.25, ρ =1600 kg/m^3,

E_{11}=172.5GPa, E_{22}=6.9GPa, $G_{12} = G_{13}$=3.45GPa, $G_{23} = 1.38$GPa.

Table 4: Comparisons of natural frequency (Hz) for simply supported composite cylindrical shells (R/b=10) with various percentage delamination.

% delamination	Stacking sequence	Results of Parhi *et al* (2002)	Present FEM Results
0	$(0/90)_2$	103.03	103.0197
	$(30/-30)_2$	148.95	148.9423
	$(45/-45)_2$	193.38	193.3716
	$(60/-60)_2$	242.97	242.9612
6.25	$(0/90)_2$	88.17	88.1633
	$(30/-30)_2$	136.33	136.3336
	$(45/-45)_2$	181.48	181.4845
	$(60/-60)_2$	230.27	230.2714
25	$(0/90)_2$	69.60	69.5945
	$(30/-30)_2$	121.51	121.5228
	$(45/-45)_2$	168.20	168.0395
	$(60/-60)_2$	216.66	216.6927
56.25	$(0/90)_2$	59.88	59.9258
	$(30/-30)_2$	109.20	109.2110
	$(45/-45)_2$	153.51	153.5305
	$(60/-60)_2$	169.86	169.9099

Determination of material constants:

In this investigation, composite specimens of eight layers are prepared to estimate its properties. Tensile test on samples are performed following the procedure described in ASTM D2309/D2309M (2008) standard. Material constants are found from the experiment as given in Table 5.

Table 5: Material properties of the composite specimen

Lay-up	No. of layers	$E_1(GPa)$	$E_2(GPa)$	$E_{45}(GPa)$	$G_{12}(GPa)$	υ_{12}	$\rho(Kg/m^3)$
WR	8	10.846	10.846	7.253	2.42	0.25	1631

E_1 = Longitudinal Elastic modulus

E_2 = Transverse Elastic modulus

E_{45} = Elastic modulus obtained in 45^o tensile test

ν_{12} =Poisson's ratio

G_{12} =In plane shear modulus

ρ = density of specimen

5.2.2 NUMERICAL AND EXPERIMENTAL RESULTS

In the present work, an eight layered cross ply composite specimens are fabricated for numerical and experimental analysis of vibration. The specimen sizes i.e. length and width are taken as 235mm both and thickness is 2.62mm. The materials properties of composite specimens are given in Table 2. Mid plane single square size delamination of composite cylindrical shells are considered as shown in Fig 22. Also it investigate the influence of percentage delamination, number of layers, aspect ratio (i.e. a/b), curvature to side ratio (R/b) and different boundary conditions on natural frequency of delaminated cylindrical shells.

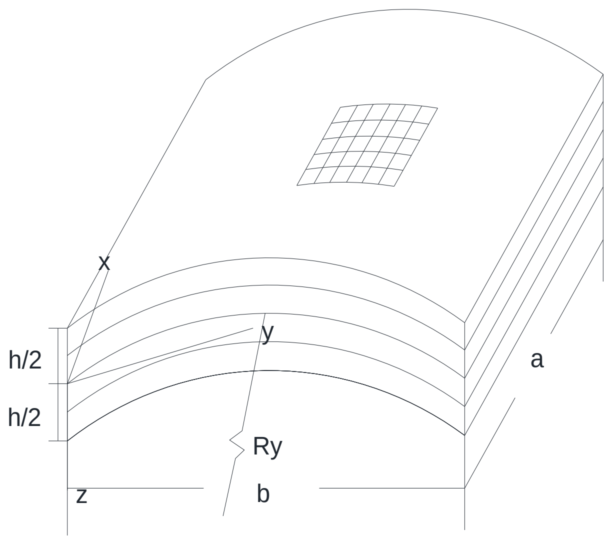


Fig: 22 Laminated composite shell with square size mid plane delamination

5.2.2.1 Effects of Curvature:

To study the effects of curvature on vibration characteristics of laminated composite shell, three different types of specimens are casted whose radius of curvature are R1=0.91m, R2=1.2625m and R3=2.09m. The geometry and material properties are same as given in earlier. Both the numerical and experimental natural frequency of laminated composite plate at F-F-F-F boundary condition is shown in Fig 23. Similarly, the natural frequencies for shells having three different radius of curvatures i.e 0.91m, 1.2625m and 2.09m are presented in Fig 24, Fig 25 and Fig 26 respectively.

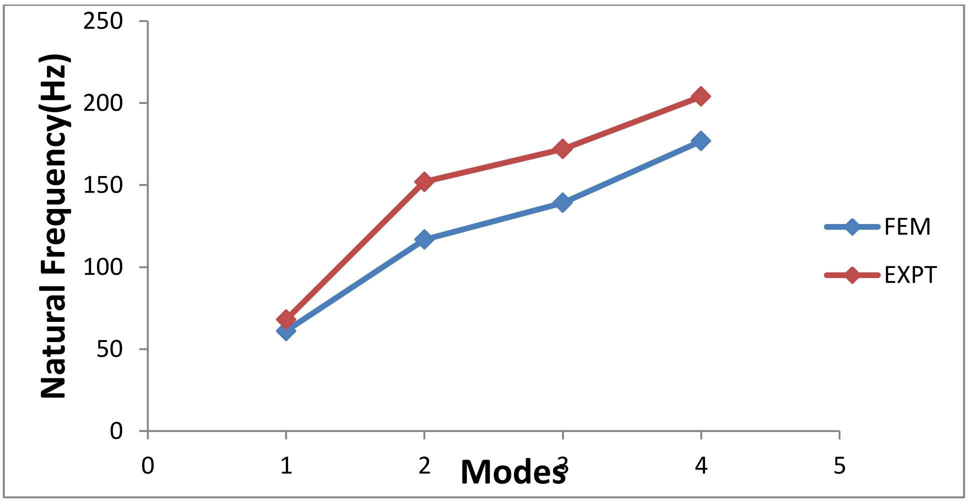


Fig 23: Variation of natural frequencies of laminated composite plate with modes.

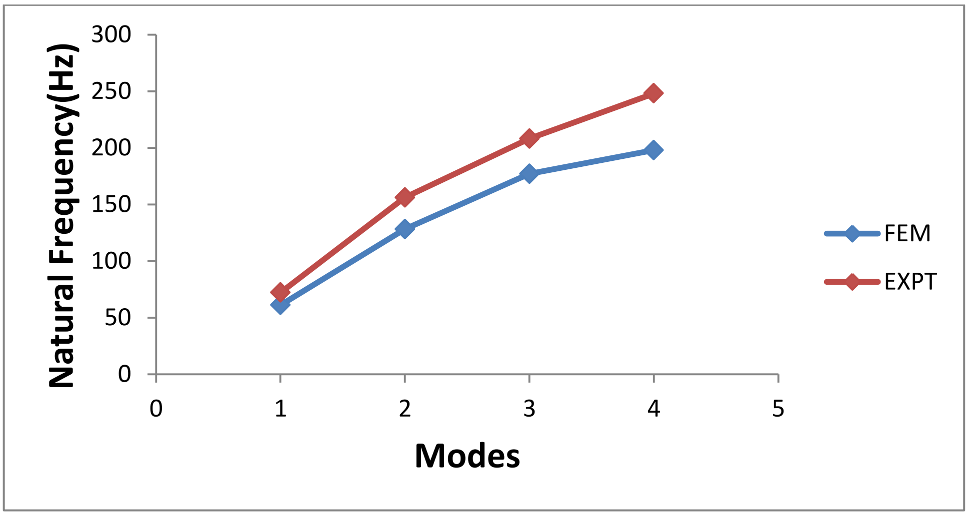


Fig 24: Variation of natural frequencies of laminated composite shell (R=0.91m) with modes.

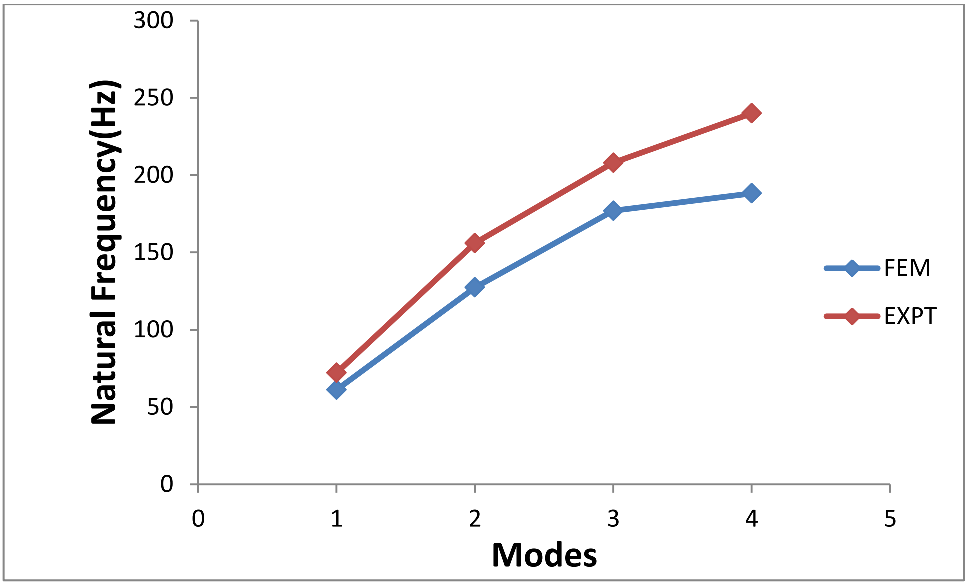


Fig 25: Variation of natural frequencies of laminated composite shell (R=1.2625m) with modes.

From graph, it is observed that there is a good agreement between numerical and experimental results for fundamental natural frequency. The natural frequency on laminated plate and shells for different curvature radius to side ratio (R/b) are given in Table 6. It represents that at higher modes, there is significant decrease in natural frequency with increase in curvature to side ratio.

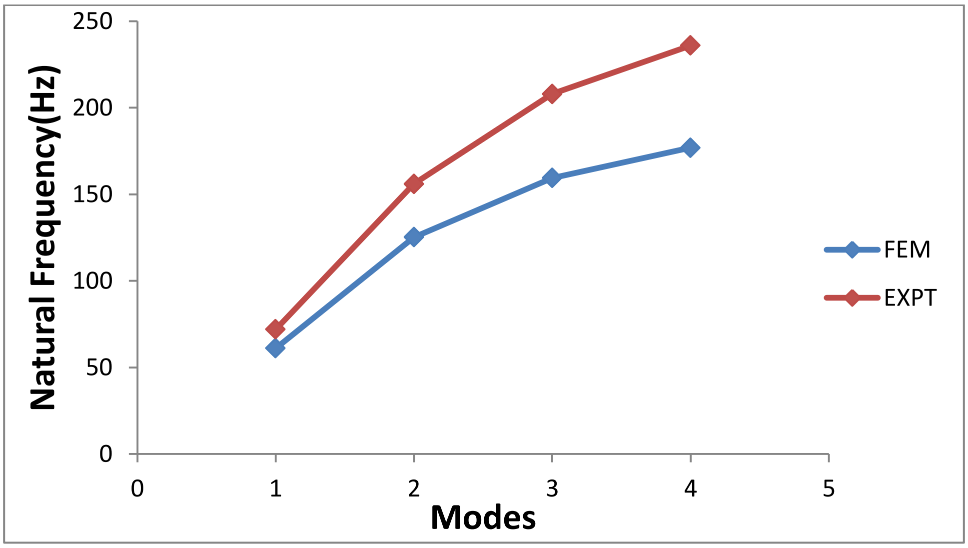


Fig 26: Variation of natural frequencies of laminated composite shell (R=2.09m) with modes.

Table.6: natural frequency (Hz) of laminated composite specimens with different R/b ratio.

R/b=3.87	R/b=5.37	R/b=8.90	R/b=∞
61.039	61.031	61.023	61.017
128.007	127.363	125.249	116.789
176.965	176.937	159.465	139.145
197.799	188.169	176.883	176.826
222.187	192.153	181.088	176.826
355.363	333.357	318.392	311.348
360.685	355.345	355.326	355.313
388.264	384.754	376.615	355.313
438.793	407.063	380.982	378.114
441.316	430.452	423.652	420.168

From Table 6, it is observed that the natural frequency is reduced significantly in higher modes as increasing the curvature radius to side ratio value.

5.2.2.2 Effects of delamination area:

The effects of percentage delamination on vibration characteristics of an eight layered composite shell were studied by introducing central mid-plane delamination at 6.25%, 25% and 56.25% of specimen area. The least natural frequency of the delaminated cross ply composite shells with four sides free boundary condition is illustrated in Figure: 27 as a function of delamination area. The experimental fundamental frequencies of 6.25%, 25% and 56.25% delaminated shells are found to decrease by 5%, 10% and 15 % respectively as compared to the laminated shell. This may due to the reduction in stiffness of the laminates.

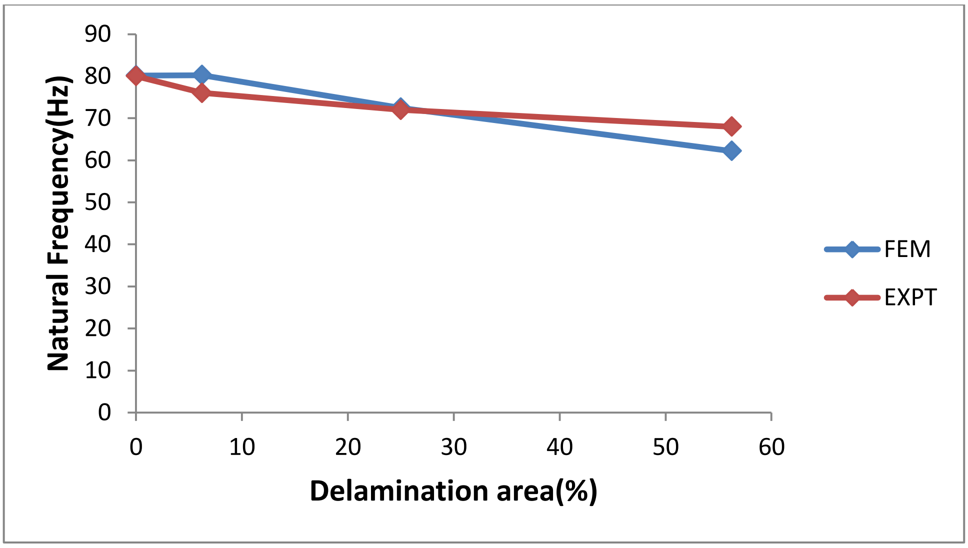


Fig 27: Variation of fundamental natural frequency with delamination area of cylindrical shell at F-F-F-F boundary condition.

The same investigation was extended to the composite shells with S-F-S-F (two opposite sides simple supported and others free), C-C-C-C boundary conditions, the results of which are presented in Fig: 28 and Fig: 29. The experimental fundamental frequencies of 6.25%, 25% and 56.25% delaminated shells are found to decrease by 7%, 14% and 28 % respectively for S-F-S-F boundary condition and 2%, 7.29% and 14.58% for C-C-C-C boundary condition as compared to laminated shell. This may be due to reduction in stiffness of the laminates.

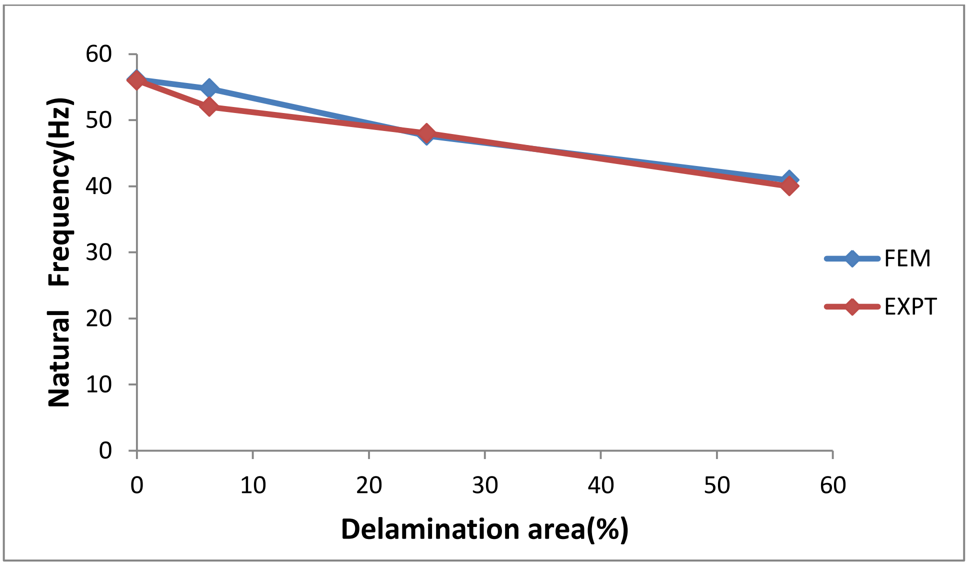


Fig 28: Variation of fundamental natural frequency with delamination area of cylindrical shell at S-F-S-F boundary condition.

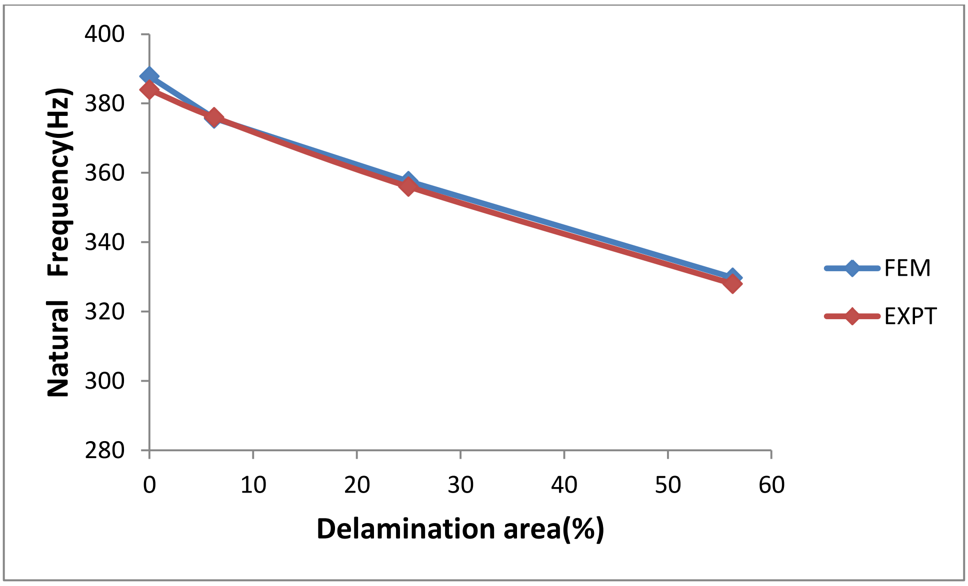


Fig 29: Variation of fundamental natural frequency with delamination area of cylindrical shell at C-C-C-C boundary condition.

From this investigation, it is observed that at 6.25% delamination area, the frequency of specimen is least affected at C-C-C-C boundary condition as compared to other two boundary conditions and also shows that the fundamental natural frequencies are considerable affects due to large delamination area at all three boundary conditions. It is also concluded from this investigation, the natural frequency decreases due to the increase in delamination area.

5.2.2.3 Effects of number of layers of laminate:

To study the influence of number of layers on vibration behaviour of cross ply composite cylindrical shell with 25% area delamination, samples are prepared having 4 , 8, and 12 plies. The specimen densities are calculated as 1650 kg/m3 and 1627 kg/m3 for 4 layers and 12 layers respectively. Both the numerical and experimental values of natural frequency with number of layers are depicted in Figure: 30. From this graph, it shows that due to increase in number of layer, the frequency of shell also increases. And also observe that the fundamental frequency of composite shell of 25% delaminated shell for 8 layers and 12 layers is increased by 2 times and 2.8 times respectively with respect to 4 layered laminates. This result indicates that relatively more number of layers have considerable positive effect on the stiffness of delaminated composite shell.

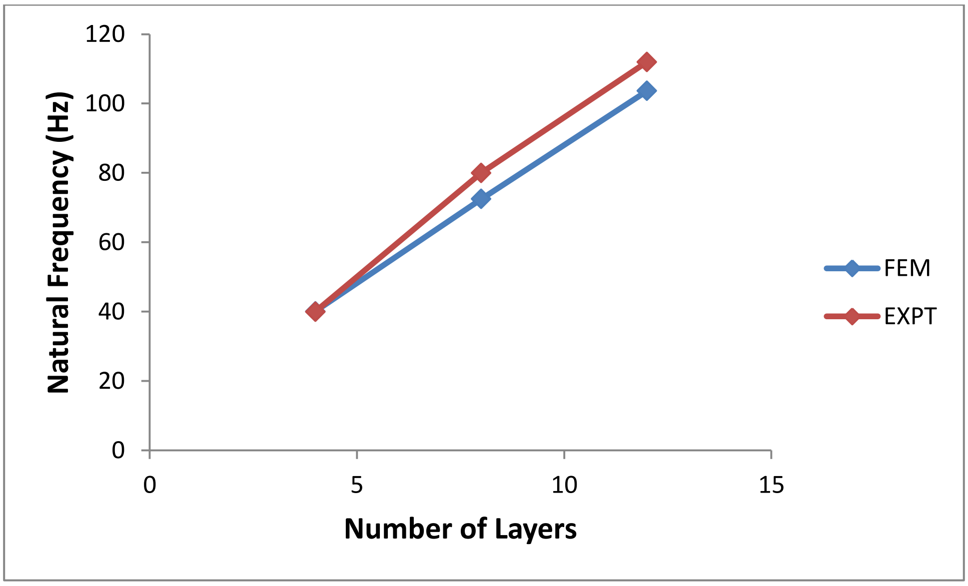


Fig 30: Variation of fundamental natural frequency with number of layers for cylindrical shell with 25% delamination

5.2.2.4 Effects of aspect ratio:

Three different aspect ratios such as a/b=1, a/b=1.5, and a/b=2 are taken in this present work to examine the influence of aspect ratio on vibration characteristics of 25% delaminated composite cylindrical shells. For different aspect ratios, the shell dimension varied. For aspect ratio i.e. a/b=1, both length and width are considered as 235mm. For a/b=1.5, length and width are taken as 237mm and 158mm respectively. Similarly for a/b=2, length and width are considered as 236mm and 118mm respectively. The variations of fundamental frequency with aspect ratios for both numerically and experimentally are depicted in Fig 31. It is found that the experimental fundamental natural frequency of 25% delaminated composite shell with F-F-F-F boundary condition for 1.5 & 2.0 aspect ratio increases by 40% and 85% respectively as compared to 1.0 aspect ratio. Also it indicates that with increase in aspect ratio, the fundamental natural frequency of a delaminated composite shell increases.

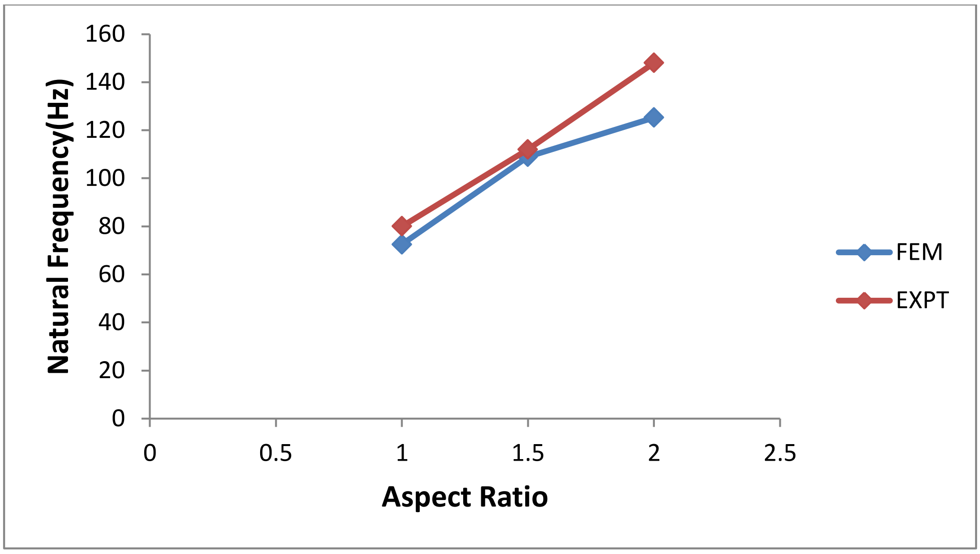


Fig 31: Variation of fundamental natural frequency with aspect ratio for cylindrical shell with 25% delamination

5.2.2.5 Effects of boundary condition:

In order to carry out the influence of boundary conditions on natural frequencies of delaminated shells, three types of boundary conditions are considered in this investigation, i.e. F-F-F-F (four edges free end), F-S-F-S (two opposite edges free and other two edges simply supported), and C-C-C-C (four edges clamped). Here specimen of eight layered cross ply composite shell having 25% delamination was taken for the study of influence of boundary conditions. Both numerical and experimental natural frequencies of composite cylindrical shell with 25% area delamination at different boundary conditions are presented in Table 7.

Table 7: Natural frequencies (Hz) of composite cylindrical shell (R=1.2625m) with 25% delamination at different boundary condition

Boundary conditions	Experimental Results			Numerical Results		
	1st	2nd	3rd	1st	2nd	3rd
F-F-F-F	80	168	232	80	105	198
S-F-S-F	52	92	232	54	98	231
C-C-C-C	356	536	612	357	453	520

From this Table, It is found that the first, second and third mode frequencies were least for S-F-S-F (two opposite edges simply supported and other two edges free) boundary condition and the maximum for C-C-C-C boundary condition. The fundamental frequency for 25% delaminated shell at F-F-F-F and S-F-S-F boundary conditions are decreased by 77.52% and 86.39% respectively with respect to C-C-C-C boundary condition. This experimental result represents that the natural frequency of delaminated shells are significantly dependent on the boundary conditions.

5.2.3 Pulse report:

The Natural frequencies of the free vibration analysis are found out experimentally by using pulse software. Typical pulse reports for the laminated composite shell (for a/b ratio 1.0, and 25% of delamination) are shown in Fig: 32 and Fig: 33. The peaks of the FRF shown in Fig: 29 give the different natural frequencies of vibration. The coherence shown in Fig: 33 gives indication of the accuracy of measurement.

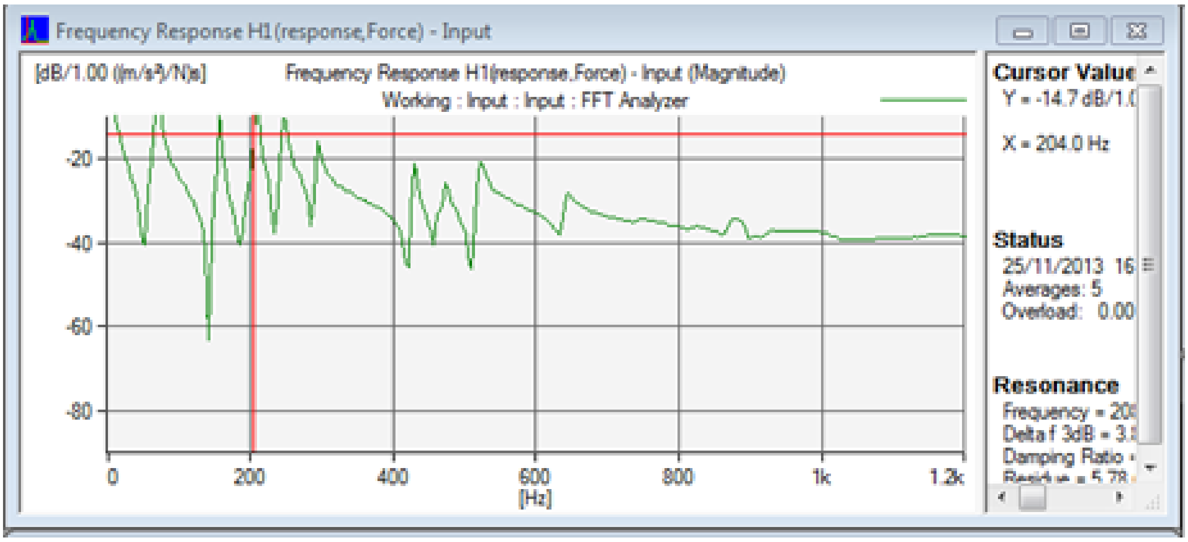


Fig 32: Frequency response function spectrum (In X-axis: Frequency in Hz, In Y-axis: Acceleration per force (m/s2)/N)

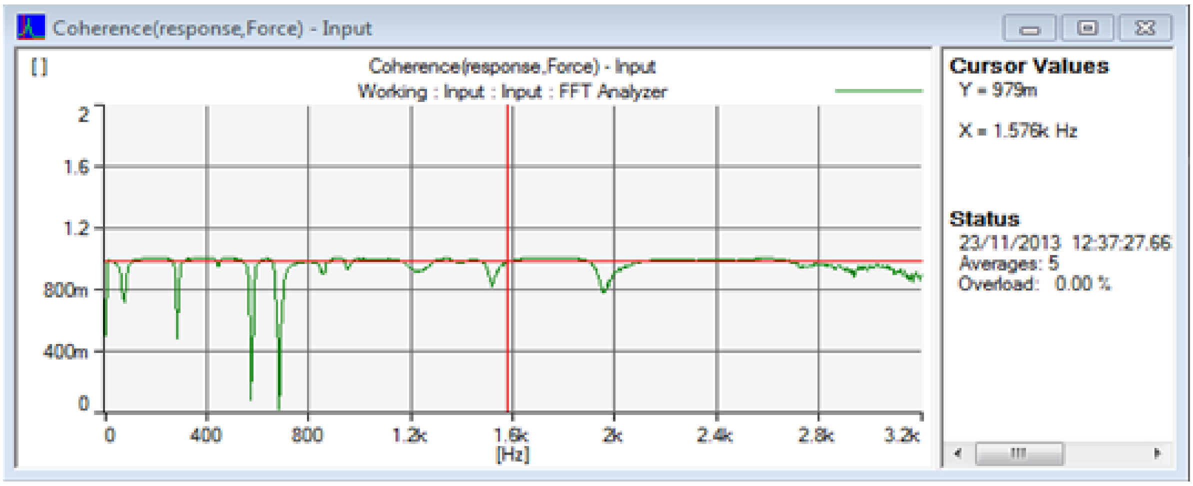


Fig 33: Coherence (Response, Force)

CHAPTER-6

CONCLUSION

The present investigation deals with the study of the free vibration characteristics of delaminated composite shells. The formulation has been developed by using finite element approach for studying the instability region of mid plane delaminated composite shell. Both numerical and experimental results are obtained for the free vibration characteristics of delaminated composite shells with the effects of different parameters like delamination size, curvature to side ratio, aspect ratio, ply orientation, number of layers of lamina, fiber orientations, boundary conditions etc. From this investigation, we found the following concluding remarks during the analysis of natural frequency of delaminated composite cylindrical shells.

- A detailed formulation is presented and a program is developed in MATLAB environment for the free vibration of delaminated composite shell
- Both numerical predictions and experimental results on vibration of shell panels agrees well.
- The natural frequency of shells increases as compared to plates due to curvature of panel.
- Due to decrease in radius of curvature to side ratio, the frequency of vibration of composite shell increases further.
- There is a decrease in natural frequency of composite shell with increase in percentage of delamination irrespective of all the boundary conditions in woven fiber composite specimens due to reduction in stiffness.
- The fundamental frequency of cylindrical shell with fixed delamination increases due to increase in number of layers due to bending stretching coupling.
- It is observed that there is an increase in vibration characteristic of composite shell with increase in aspect ratio.

- The natural frequencies of delaminated shells also vary with different boundary condition. It is observed that the fundamental frequency for C-C-C-C boundary condition is maximum due to restraint at the edges. However, the S-F-S-F supported delaminated shells shows least frequency among the three conditions tested.

From the above discussion, the vibration behaviour of delaminated shells is influenced by the geometry, material, boundary conditions, the size and position of the delamination. The presence of delamination weakens the structure from the point of view of vibration behavior and causes resonance due to reduction in natural frequency. So delaminations play a critical role on the vibration behavior of the structures. So the designer has to be careful while dealing with structures subjected to delamination. This can be used to the advantage of tailoring during design of delaminated composite structure. The vibration results of the delaminated composite panels can be used as a tool for structural health monitoring, identification of delamination location and extend of damage in composites and also helps in assessment of structural integrity of composite structures.

FUTURE SCOPE OF RESEARCH

In the present investigation, the natural frequency of delaminated composite shell was determined numerically and experimentally. The effects of various parameters like delamination area, ply lay-up, aspect ratio, different curvature to side ratio, boundary condition was studied. The further scope of the present study can be extending as follows.

- The present study deals with the square size delamination. This may be extended for different shape of delamination.
- For single delaminated shell, the delamination is considered here is mid plane. So arbitrary location may be taken into account for further extension.
- Free vibration of multiple delaminated shells may be extended.
- The present study is based on linear range of analysis. It can also be extended for nonlinear analysis.
- The effects of damping on instability regions of delaminated composite shells can be extended.
- Free vibration of composite shells with delamination due to hygrothermal effects can be studied.
- Buckling analysis and dynamic stability of delaminated composite shells can be studied.

REFERENCES

1. Acharyya A.K., Chakravorty D., Karmakar A. (2009) "Bending characteristics of delaminated composite cylindrical shells." *Journal of Reinforced Plastics and Composites*, 8, 965-978.
2. Alijani F., Amabili M., Bakhtiari-Nejad F. (2011) "Thermal effects on non-linear vibrations of functionally graded doubly curved shells using higher order shear deformation theory." *Composite Structures*, 93, 2541-2553.
3. ASTM D 2344/ D 2344M (2006) Standard test method for short-beam strength of polymer matrix composite materials and their laminates, Annual Book for ASTM Standards, American Society for Testing and Materials.
4. ASTM D 3039/D 3039 M (2008) Standard Test Method for Tensile Properties of Polymer Matrix Composite Materials.
5. Bert C.W., Kumar M.(1982) "Vibration of cylindrical shells of bimodulus composite materials." *Journal of Sound and Vibration*, 81, 107-21.
6. Bhattacharya P., Suhail H., Sinha P. K. (2002) "Finite element analysis and distributed control of laminated composite shells using LQR/IMSC approach," *Aerospace Science and Technology*, 6, 273–281.
7. Civalek O. (2013) "Vibration analysis of laminated composite conical shells by the method of discrete singular convolution based on the hear deformation theory." *Composites*, 1001-009.

8. Dasgupta, A., Huang, K.H .(1997) "A layer-wise analysis for free vibration of thick composite spherical panels*", Journal of Composite materials*, 31,658.

9. Ganapathi M., Patel B.P., Pawargi D.S. (2002) **"**Dynamic analysis of laminated cross-ply composite non-circular thick cylindrical shells using higher-order theory". *International Journal of Solids and Structures*, 39, 5945-5962.

10. Jinhua Y., Yiming F. (2007) "Analysis of dynamic stability for composite laminated cylindrical shells with delamination." *Composite structures,* 78, 309-315.

11. Jones R.M. (1975) "Mechanics of Composite Materials, McGraw Hill, New York

12. Ju F., Lee H.P., Lee K.H.**(**1995) "Finite element analysis of free vibration of delaminated composite plate." *Composite Engineering*, 5,195-209.

13. Kim J. S., Cho M.(2003) "Efficient higher order shell theory for laminated composites with multiple delamination*." AIAA Journal*, 41, 941-50.

14. Kouchakzadeh M.A. and Shakouri M. (2014) "Free vibration analysis of joined cross ply laminated conical shells." *International Journal of Mechanical Sciences,* 78, 118-125.

15. Kurpa L., Shmatko T., Timchenko G. (2010) "Free vibration analysis of laminated shallow shells with complex shape using the R-function method." *Composite Structures*, 93, 225-233.

16. Latifa S .K., Sinha, P. K.(2005) "Improved Finite Element Analysis of multi-layered, Doubly Curved Composite Shells," *Journal of Reinforced Plastics and Composites*, 24, 385.

17. Lam K. Y., Loy C. T.(1995) “Analysis of rotating laminated cylindrical shells by different thin shell theories.” *Journal of sound and vibration*, 186, 23-35.

18. Lam K.Y., Quin W.(2000). “Free vibration of symmetric angle plies thick laminated composite cylindrical shells.” *Composites,* 31, 345–354.

19. Lee SY., Chung DS.(2010) ”Finite element delamination model for vibrating composite spherical shell panels with central cut-outs”. *Finite Elements in Analysis and Design,* 46, 247–256.

20. Librescu, Khdeir and Frederick.,(1989). “Shear deformation theory of laminated composite of shallow shell type panels and their response analysis.” I: Free vibration and buckling. *Acta Mechnica,* 76, 1-33

21. Liu M. L., and Yu J. (2003). “Finite element modelling of delamination by layer wise shell element allowing for interlaminar displacements.” *Composite Science and Technology,* 63, 517-529.

22. Mochida Y., Ilanko S., Duke M., Narita Y. (2012) “Free vibration analysis of doubly curved shallow shells using the Superposition-Galerkin method.” *Journal of Sound and Vibration,* 331, 1413-1425.

23. Nanda N., Sahu S.K.(2012). “Free vibration analysis of delaminated composite shells using different shell theories”. *International Journal of Pressure Vessels and Piping,* 98,111-118.

24. Narita Y., Ohta Y.(1993). "Finite element study for natural frequencies of cross ply laminated cylindrical shells*." Composite Structures*, 26, 55-62.

25. Nguyen-van H, Mai-duy N., Tran-cong T. (2008)" Free vibration analysis of laminated plate/shell structures based on FSDT with a stabilized nodal-integrated quadrilateral element*", Journal of Sound and Vibration* ,313, 205–223.

26. Oktem A.S., Chaudhuri R.A. (2009) "Higher-order theory based boundary-discontinuous Fourier analysis of simply supported thick cross-ply doubly curved panels," *Composite Structures*, 89, 448–458.

27. Parhi P.K., Bhattacharyya S.S., Sinha P.K.(2001) "Hygrothermal effects on the dynamic behavior of multiple delaminated plate and shell*" Journal of Reinforced Plastics and Composites,* 20, 1276.

28. Park T., Lee S.Y**.**(2009) "Parametric instability of delaminated composite spherical shells subjected to in-plane pulsating forces." *Composite Structures*, 91, 196-204.

29. Qatu M. S., Sullivan R. W. and Wang W.(2010) "Recent research advances on the dynamic analysis of composite shells." *Composite Structures*, 93, 14–31.

30. Sahu S.K., Datta P.K.(2001) "Parametric Resonance Characteristics of Laminated Composite Doubly Curved Shells Subjected to Non-Uniform Loading". *Journal Of Reinforced Plastics And Composites*, 20, 18.

31. Sahu S.K., Datta P.K.(2002) "Dynamic stability of curved panels with cutouts." *Journal of Sound and Vibration*, 251, 683-96.

32. Sai Ram K.S., Sreedhar Babu T.(2002) " Free vibration of composite spherical shell cap with and without a cutout." *Computers and Structures,* 80, 1749–1756.

33. Singh A. V.(1999) "Free vibration analysis of deep doubly curved sandwich panels" *Computers & Structures* ,73, 385-394.

34. Soldatos K.P.(1984) "A comparison of some shell theories used for the dynamic analysis of cross ply laminated circular cylindrical panels." *Journal of Sound and Vibration,* 97, 305-19.

35. Swamy Naidu N.V., Sinha P.K.(2007) "Nonlinear free vibration analysis of laminated composite shells in hygrothermal environments." *Composite Structures,* 77, 475–483.

36. Yang J., Fu Y.(2006) "Delamination growth of laminated composite cylindrical shells" *Theoretical and Applied Fracture Mechanics,* 45,192–203.

37. Zhao X., Ng T.Y., Liew K.M.(2004) "Free vibration of two sides simply supported laminated cylindrical panels." *International Journal of Mechanical Sciences,* 46, 123–142

APPENDIX

Delamination Modelling

For single mid plane delamination with different sizes like 0, 6.25%, 25%, 56.25% of the total shell area is considered. The delamination sizes are assumed to be increase from the centre of the laminate and can be located anywhere along the thickness of the laminate. The composite shells with different percentage of delamination, without delamination and mid plane single delamination are shown in the Fig: 34 to Fig: 39.

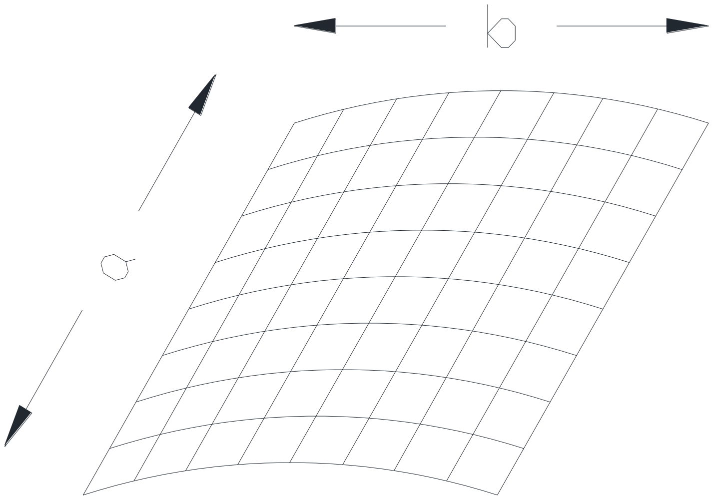


Fig 34: Shell with zero delamination

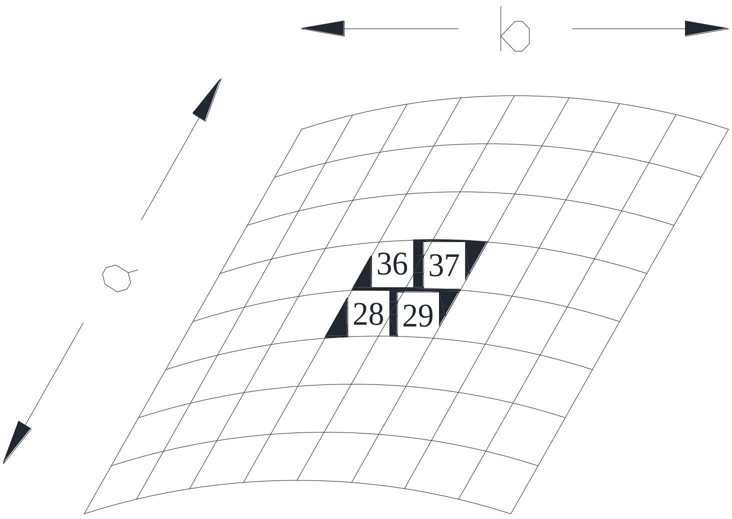


Fig 35: Shell with 6.25% central area delamination

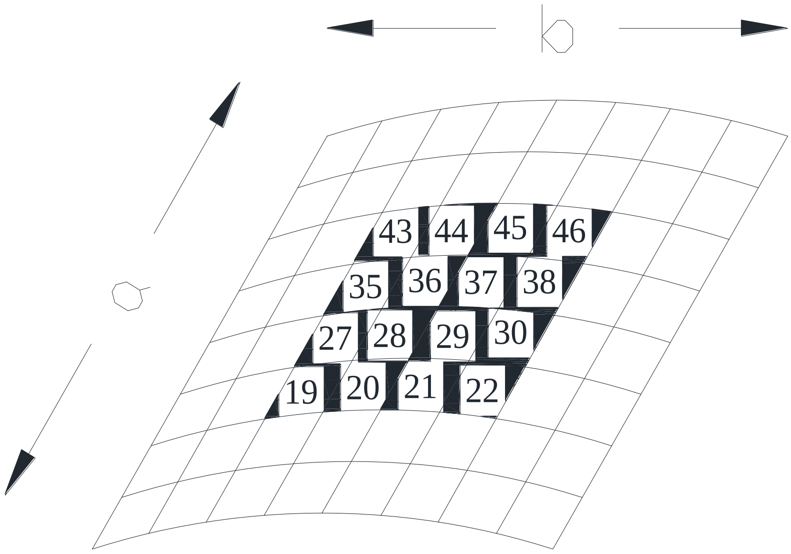


Fig 36: Shell with 25% central area delamination

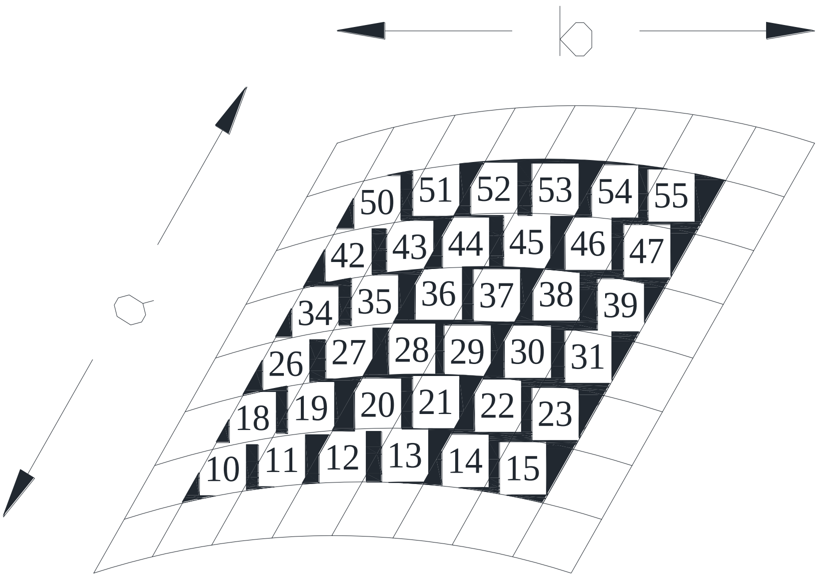


Fig 37: Shell with 56.25% area central delamination

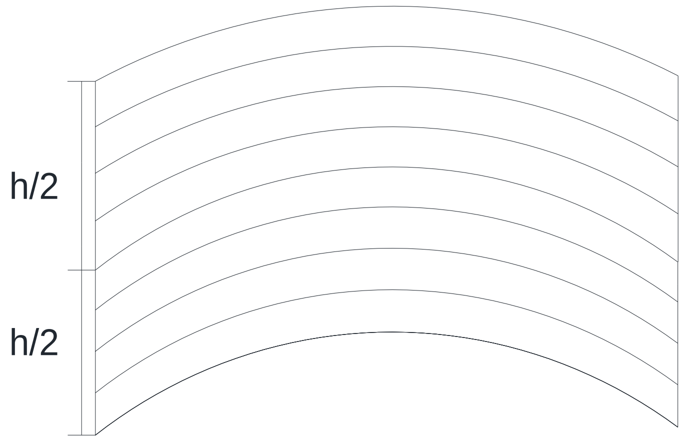


Fig 38: Eight layered laminate without delamination

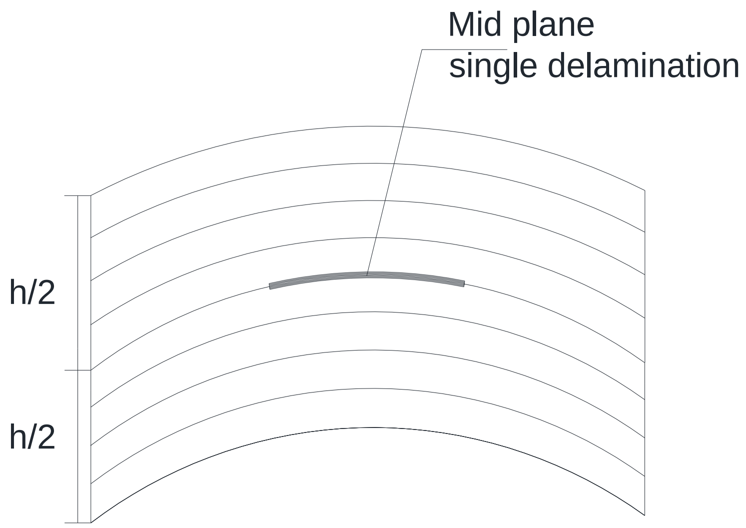


Fig 39: Eight layered laminate with mid plane single delamination