Numerical Simulation of Slip flow Heat Transfer in a Microtube

Thesis submitted in partial fulfillment of the requirements for the degree of

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in

Mechanical Engineering

(Specialization: Thermal Engineering)

by

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Certificate

This is to certify that the work in the thesis entitled ***Numerical Simulation of Slip flow Heat Transfer in a Microtube*** by ***S R Akhil Krishnan*** is a record of an original research work carried out by him under my supervision and guidance in partial fulfillment of the requirements for the award of the degree of Master of Technology with the specialization of **Thermal Engineering** in the department of **Mechanical Engineering**, National Institute of Technology Rourkela. Neither this thesis nor any part of it has been submitted for any degree or academic award elsewhere.

Place: NIT Rourkela
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Abstract

The isothermal wall convective heat transfer characteristics of the model fluid flow in 2-D micro tube under hydro dynamically fully developed conditions was investigated. In which it covers the slip flow regime $0.01 < Kn < 0.1$, where Kn , the Knudsen number is defined as the ratio of molecular mean free path and the hydraulic diameter of the micro tube . So in case of velocity-slip flow regime solving the fluid flow and heat transfer governing equation can no longer be sufficient for simulating the flow. We will require incorporating slip velocity and temperature jump boundary conditions. Slip velocity and temperature boundary conditions were derived by Maxwell and Smoluchowski respectively.

Investigation has been done in a commercial CFD package FLUENT slip and temperature jump boundary condition is taken into account through program called udf (User Defined Function). The validity of UDF is done by forward backward approach in which outcome of the UDF is considered as the input for the FLUENT inbuilt shear stress slip model. The error calculated in slip velocity by forward-backward approach is 1.485

The main interest is to determine effect of Knudsen number on various flow variable. Also the heat transfer rate , skin friction coefficient ,axial wall shear stress , slip velocity , Nusselt number , heat transfer coefficient and wall fluxes with varying Knudsen Number and Reynold Number is also studied.

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List of Abbreviations

α	:	Thermal Diffusivity
λ	:	Free mean path
θ	:	Dimensionless Temperature
μ	:	Dynamic Viscosity , kg/m-s
ν	:	Kinematic viscosity . m^2/s
σ	:	Molecular diameter ,m
ρ	:	Density ,kg/m^3
a	:	Velocity of sound , m/s
Br	:	Brinkman number
C_p	:	Specific heat , J/kg-K
d	:	Diameter ,m
D_h	:	Hydraulic diameter, m
h	:	Heat transfer coefficient, W/m^2K
k	:	Thermal conductivity, W/m-K
Kn	:	Knudsen Number
L	:	Length of pipe, m
Ma	:	Mach number
Nu	:	Average Nusselt number
P	:	Pressure, Pa
R	:	Tube radius , m
r	:	Radial coordinate , m
Re	:	Reynolds Number
T	:	Fluid Temperature, K
Ti	:	Fluid inlet temperature , K
u	:	Axial velocity , m/s
u_i	:	Slip velocity , m/s
x	:	Axial coordinate ,m
F_m	:	Momentum accommodation coefficient
F_T	:	Thermal accommodation coefficient

Chapter 1

Introduction

Chapter 1

Introduction

1.1 Background

In last two decades study of compact devices has been emphasized . We all are aware of the fact that heat generated in compact devices possessing high flux may cause damage of the various components of devices. To avoid the damage of component there must be incorporation of heat removal mechanism.

Modern technologies capable of increasing the energy efficiency in form of transportation, housing, and appliance systems, by reducing energy consumption and positively impact the global environment

Micro devices having dimensions of the order of micron are being designed for use in cooling of integrated circuits, biochemical applications , micro electro mechanical system and cryogenics. The trend of miniaturization has significantly enhanced the problem associated with overheating of ICs. It require effective cooling which is used to remove the heat generated from a relative low surface area. This require effective thermal design increasing sales of circuit integration of electronic component accompanied which in turn reduces the size of IC chips tremendously which leads to larger increment of heat generation dissipation . And also the implementation of efficient cooling techniques for integrated circuit IC chips is one of the important applications of micro scale heat transfer.

Micro tube heat sinks are the ultimate solution for removing these high amounts of heat. The removal of heat is done by forced convection generally using liquid flow . Numerical methods using fluent code are reported for predicting the flow and temperature of working fluid ,which is useful for designing of thermal chips.

1.2 Motivation and Application

Micro scale heat transfer has drawn much attention due to their high potential for cooling high power microelectronic and application in biomechanical and aerospace industries. Heat released from micro devices is now becoming a cutting edge field since the performance of the device used in primarily determined by flow and temperature fields. The benefit of using micro tube heat sinks is that its having high heat transfer area per unit volume. Heat transfer in a micro scale mainly takes place via conduction and convection modes.

The construction of the tube have a greater influence on the convective heat transfer characteristics. Hence , to design effective micro tube heat sinks , design parameters like pressure required circulating the cooling fluid , flow rate ,hydraulic diameter of the tube , temperature of the fluid and the tube wall and the number of tube has to be considered. To make the system effective and cheaper ,these parameters have to be optimized. The use of convective heat transfer in micro tubes to cool electronic chips . Both analytical and experimental works were being done to determine the heat transfer at micro scale for liquids and gases . However ,none of them has been able to come to a general conclusion. Experimental studies in the literature have showed that many micro channel flow and heat transfer phenomena cannot be governed by classical theories of transport phenomena.e.g. , in micro tube transition from laminar flow to turbulent flow occurs much earlier (Re =200) ; the relationship between the friction factor and the Reynolds number for microtube flow are very different from that in classical theory of fluid mechanics.

Study of gaseous flow done and it showed that although continuum assumption is invalid for the phenomena of rarefaction for slip flow conditions . It is also found that gas velocity not exactly zero and there is a finite amount of temperature jump between wall and adjacent gas , which is contemporary to fluid flow in macro tube.Current problem incorporate Slip velocity and temperature jump boundary conditions derived by Maxwell and Smoluchowski. Based upon this boundary condition code is written and hooked into FLUENT . Now we can simulate microtube and also design them for these micro applications .

1.3 Slip boundary conditions

The slip phenomena plays an important role in modelling liquid flow through micro channel. Helmoltz and Von Piotrowski had found the slip between a solid and liquid surface and later Brodman verified their results. Analytic slip-flow studies have traditionally been used on the continuum form of Navier -Stokes equations and energy equation with the slip-flow effects concentrated in the additional terms in the tangential velocity and temperature conditions.

These conditions were representing the velocity slip and temperature jump conditions at the gas surface interface .Slip flow condition have been well elaborated by by introducing the concept of slip length,which is the distance behind the solid-liquid interface at which the velocity extrapolates to zero. Liquid flow in a micro channel becomes fully developed after a short entrance length so that it can be modified as a two dimensional flow .

The slip velocity at the wall as given by Navier is

$$V_x = L_s * (\frac{\partial V_x}{\partial \eta})_{wall} \tag{1.1}$$

Where Ls is the slip length ; n is the normal coordinate pointing inward from the channel wall.

Slip velocity boundary condition as derived by Maxwell is :

$$U_{fluid} - U_{wall} = \frac{2 - F_v}{F_v} * \lambda * (\frac{\partial U}{\partial Y})_{wall} \tag{1.2}$$

the above condition is valid for both gas as well as liquid flow. Where the Fv is the momentum accommodation coefficient . is the mean free path and (U/Y) is velocity gradient. We can model the slip from various correlation given in different literature but present study is completely based upon Maxwell derived slip formulation.

1.4 Literature Review

The literature survey is giving a brief description of the analysis performed in single phase forced convection in microtubes.

1. Orhan Aydin ,Mete Avci : They analyzed analytically laminar forced convection heat transfer of a Newtonian fluid in a micro-channel between two parallel plates. Current investigation includes viscous dissipation effect , velocity slip effect and temperature jump at the wall. Analysis is done under hydro-dynamically and thermally fully developed condition. Either hot wall or cold wall condition is discussed for constant heat flux and constant wall temperature .The relative variation of Nusselt number with respect to Knudsen number and Brinkman number are analytically revealed .

2. Nicolas G. Hadjiconstantinou ,Olga Simek : Their findings covers the slip flow regime for isothermal wall convective heat transfer characteristic of a gaseous flow in a 2 D micro and nano channel , where flow is hydrodynamically and thermally fully developed. They use axial heat conduction to calculate the nusselt number .Incorporation of the phenomena of axial heat conduction in the continuum model is necessary since small scale internal flows are typically The characterisation of finite Peclet number is incorporated by phenomena of axially heat conduction in the continuum model . Investigation explores that the prediction is matching with the

DSMC results for slip flow. It is revealed in the study that with increasing Knudsen number Nu number decreases in hydro dynamically and thermally fully developed condition in both slip flow and transition flow regime . At Kn=50 Nusselt number is found to increase by axial heat conduction. They have also investigate a micro tube and found qualitatively similar prediction for slip flow heat transfer in circular tube. This investigation gave new amplitude to slip flow convective heat transfer in a micro and nano channel.

3. Gokturk tunc , Yoldiz Bayazitoglu : They have also investigated slip flow convective heat transfer in a rectangular micro-channel under thermally and hydro-dynamically developed flow condition. The H-2 boundary condition condition is applied at the walls of the channel. The velocity profile for rectangular is unknown for the slip flow condition the momentum equation is used for solving the velocity and then the obtained velocity profile is incorporated in the energy equation . Integral transform technique is used two times once for determining velocity and second for temperature.

4. Chien-hsin , chen : They included the viscous dissipation which is not considered by many researchers. They analyzed forced convection flow in a microchannel with isothermal condition at the wall, as their investigation lies in slip flow regime ($0.001 < Kn < 0.1$) incorporation of both velocity slip and temperature condition at the wall are necessary , hence they included the velocity slip and temperature jump at the wall. Energy equation was solved for developing temperature field with the help of finite integral transform .And also increasing Knudsen number is indicating increasing slip velocity at the wall surface in turn to reduce the friction factor. Effects of parameter like Kn , b and Brinkman number on the heat transfer characteristic are mentioned .By fixing Br the Nusselt number may be either lower or higher than those of the continuum regime depending upon the effects of Knudsen number and b. At a known Kn the variation of surface Nusselt number increases when b becomes large which results into a shortening of the thermal entrance length. Thermally fully developed nusselt number

decreases with increasing b irrespective of Knudsen number .

5. Ho-Eyoul Jeong , Jae-Tack Jeong : In their work hydro dynamically developed convective flow through a micro-channel with isothermal wall or constant heat flux boundary condition is considered. They analyzed extended Graetz problem in micro channel with Eigen function expansion for solving the energy equation .The effects of slip velocity and temperature boundary condition on the micro channel wall are explored . Stream wise conduction and viscous dissipation are also taken into consideration. The dependence of Nusselt number distribution on dimensionless number like Peclet number , Brinkman number and Knudsen number also analyzed in their work . The fully developed nusselt number for both boundary conditions is computed in terms of dimensionless numbers like Reynold number , Peclet number , Knudsen number ,Brinkman number etc

6. Tuckerman and Pease : They reported that $790W/cm^2$ heat flux dissipation is possible from the water cooled micro-channel heat sink without a phase change . They predicted that h for laminar flow via micro-channels might be higher than for turbulent flow through conventionally sized micro-channels.

7. Wu and Little : They have done experiments to determine flow and heat transfer characteristics of gaseous flows in micro-channels. They found a difference from the data obtained by experiments and also transfer of flow from laminar to turbulent is occuring at lower Reynold number and it results in improving heat transfer.

8. Pfahler : Experimentally investigated the fluid flow in rectangular micro-channels with cross section of 80 to $7200 micro - m^2$ .Their aim was to determine the length scales at which the continuum assumptions breakdown. They relate that large flow channels are relating with the predictions of the Navier-Stokes equations ,.And for the smallest of their channels, a significant deviation from the Navier-Stokes predictions was observed . They founded out the h for Nitrogen gas in micro tubes with dia of 3 to 81 m for the case

of laminar and turbulent flows. In turbulent flow the measured h were larger than the macro tubes.The experimentally determined values for turbulent flow heat transfer coefficients is seven times larger than the ones which are obtained by the relations for turbulent flows in macro-channels.

9. Randall f.Barron , X.M.Wang and Roberto Warington : The conventional problem thermally developing heat transfer in laminar flow through a circular tube formulated by Graetz to include effect of slip flow occurring in low pressure gases in a micro tube at ordinary pressure is considered. A different method was used to determine the Eigen value . Eigen values were determined for Knudsen number lying in the range 0 to 1. Correlations were made to show the effect of slip flow on h. Integral transforms technique was incorporated to solve heat transfer for hydro-dynamically developed laminar , steady state flow in micro-tube with constant heat flux and isothermal wall jump are considered. For both fluid heating and cooling case effect of viscous heating is investigated. Prandtl number investigation has explored that as we enhance the Pr the temperature jump effect disappears when Nu number increases . In other way for higher Prandtl number fluid temperature jump can be neglected.

10. Orhan Aydin , Mete Avci: In current study cases of Hydro dynamically and thermally fully developed flow is determined for two different thermal boundary condition. Constant heat flux and Constant wall temperature. Either wall cooling or wall heating case is considered. Analytically it is determined that Nu=f(Br,Kn) .Here Br dependence on Nu number was discussed in terms of energy balance.

11. Wang and Peng : They investigated the influence of liquid flow, thermal parameters , generalized size and structure on the single phase forced convection characteristics for methanol and de-ionized water through micro-channels with rectangular cross section . The convective heat transfer coefficient was found to be highly associated with liquid flow. They got that in

the transition region for single phase heat transfer coefficient exists beyond it is nearly independent of the wall temperature.

12. Bestok et al.: The high order velocity slip and temperature jump boundary condition were determined :

$$u_g - u_w = \frac{2-\sigma_V}{\sigma_V} * Kn * (\frac{\partial u}{\partial \eta})_s + \frac{(Kn)^2}{2} * \left(\frac{\partial^2 u}{\partial^2 \eta}\right)_s + \frac{(Kn)^3}{6} * \left(\frac{\partial^3 u}{\partial^3 \eta}\right)_s \tag{1.3}$$

$$T_g - T_w = \frac{2-\sigma_T}{\sigma_T} * \frac{2\gamma}{\gamma+1} * \frac{1}{Pr} * Kn * (\frac{\partial T}{\partial \eta})_s + \frac{(Kn)^2}{2} * \left(\frac{\partial^2 T}{\partial^2 \eta}\right)_s + \frac{(Kn)^3}{6} * \left(\frac{\partial^3 T}{\partial^3 \eta}\right)_s \tag{1.4}$$

13. Peng et al. : Rectangular micro-channels with hydraulic diameter ranging from 133 to 367 micro-meters were used in the experiments . Steady state and fully developed flow conditions were investigated. They found that turbulent flow is initiated at smaller Reynold number and the transition range is not as wide in conventional channels. Transition starts at Re =200-700 and the transition Re decreases with decreasing hydraulic diameter. Flow friction also decreases with decreasingD_h until the aspect ratio equals to 0.5. Else decreasing the channel size results in higher friction values . Experimental Nusselt number values for laminar flow were smaller than the predicted and they were dependent on the Reynolds number , Re proportional to $Nu^{0.62}$, unlike conventional channels. h for both laminar and turbulent flow changes significantly for different values for hydraulic diameter and aspect ratio. They reported an optimum value of aspect ratio as 0.75 for laminar and 0.5-0.75 for turbulent flows.

14. Peng et al .(1994 a,b) : They done experimentally the forced convection characteristics of H_2O flowing micro-channels of rectangular shape with diameters of 0.133-0.3667 mm and aspect ratios in the range of 0.333-1. Two thermocouples were used for testing T_f and 3 thermocouples were used for measuring Tw . For all their test conditions , their dimensionless distance was such that the flow was fully developed . A h is calculated for the micro-channel , defined as

$$h = q/(Tw - Tf_{in}) \tag{1.5}$$

where T_w is the value measured at the downstream end. They relate their data in the with the Sieder Tate equations for laminar flow. But it did not match well with quantitatively with their experimental data , they suggest a new correction and compared with their data. However neither their new correlation nor this equation qualitatively agree with the unusual behaviour of $Nu/Pr^{0.33}$ receding with increasing Re , exhibited by their experimental data. They also measured the flow friction to analyze the heat transfer regimes , and to explore the physical aspects of convection. Their data analysis revelaed that the geometric parameters , plays a very important role on the heat transfer characteristics. For the laminar heat transfer regime , Nu is proportional to $Re^{0.62}$,while the turbulent heat transfer case exhibited the typical relationship between Nu and Re , but with a different empirical constant . The empirical constants for both these regimes were tabulated as a function of the geometric parameters.

15. Peng et al.(1995a): They also tested microgrooves having rectangular shape built into stainless steel plates with methanol as a working fluid to determine the effects of aspect ratio, fluid velocity and temperature and channel configuration.It was observed that , due to the large heat fluxes , liquid temperature along the channel changes significantly , which causes a significant thermo physical property variation. In this case , the Reynolds number was different at the exit than that at the inlet. This directly influenced the transfer of flow from from laminar to turbulent and thus the amount of heat transfer. Liquid velocity is another important factor that improves the cooling capacity.

16. Peng and Peterson : They informed experiments to understand the physics of the single-phase convective heat transfer in micro-channels of varying size. The influence of the flow characteristics , thermo fluid conditions and channel geometry was analyzed. Their experiments showed a Nusselt number reduction with Re increases in the laminar flow regime, for which they didnot give an explanation. In the turbulent flow regime , Nu increased with

increasing Re .Liquid inlet temperature and velocity also has a significant influence on the h .In general , a small inlet temperature and a large velocity result in higher heat transfer coefficients .For the same temperature and velocity values , different sized channels cause different flow modes , heat transfer regimes and therefore different heat transfer capacities.

17. Orhan Aydin and Mete Avci : In another study of them a theoretical analysis is presented for hydro dynamically developed , thermally developing and steady laminar forced convection heat transfer of a Newtonian fluid in the entrance region of a micro tube. Present work also includes viscous dissipation effect. Velocity slip and temperature jump at the wall , two different boundary conditions are taken into account a) The constant heat flux (H-1 type) and b) the constant wall temperature(T type).

18. V.P . Tyagi and K.M.Nigam : Present work for steady state heat transfer in slug flow in a circular tube uses both Laplace transform in axial coordinate and Gaierkins technique with constant physical properties and negligible axial heat conduction. Calculation reveals that the 2nd approximation is relating with the known exact analytical solution

19. Antonio Barletta and Enzo zanchini : Current work deals with the slug flow in a circular duct. Effect of viscous dissipation is considered in thermal entrance region.. Current work deals with the boundary condition a) Uniform wall heat flux b) a linearly varying wall heat flux c) an exponentially varying wall heat flux. In wall heat flux situation it is explored that viscous dissipation inversely relates with Nu number , which indicates Nu number decreases with increasing viscous dissipation . In linearly or exponentially increasing wall heat flux viscous dissipation affect the local Nu number only on the thermal entrance region and become negligible in the fully developed region.

20. A. Haji-Sheikh , Donald E.Amos and J.V. Beck : This paper deals with conjugate heat transfer where conduction is also included with convective

heat transfer . Computed analytical result reveals that close to thermal entrance region heat conduction dominates and the local heat flux becomes independent of velocity.

21. Kalkac and Yener : With increasing Kn, the Nusselt number goes on decreasing. And in the slip flow regime , Nusselt was reduced to forty percent . They also determined that the rarefaction is increasing with the increase in entrance length , that denotes that the thermally fully developed flow cant be achieved so fastly in case of conventional channels. The following formula shows the relationship between the and the and Kn :

22. Xu et al. (1999) : Experimentally and analytically the investigation of liquid flow in micro-channels were done. Micro-channels with diameters of 50 and 300 micrometre laminar case and Reynold numbers of 50 and 1500 was considered.It was found that the results were deviating for diameters less than 100 micrometre from Navier Stokes predictions. And also found that this deviation was independent on the Reynolds number and also they suggested the use of a slip boundary conditions ,which can estimate the velocity of fluid at the wall by the velocity gradient at the wall.

23. Yu and Ameel (2001) : Studied laminar slip flow forced convection in micro-channels which are rectangular in shape analytically by deploying an integral transform technique which can solve the energy equation , assuming hydro flow dynamically fully developed flow. Results are given in terms of the fluid mixed mean temperature , both local and fully developed mean Nusselt numbers. Due to effects of rarefaction and the fluid/wall interaction heat transfer is found to increase , decrease or remain unchanged compared to non slip flow conditions.

24. Toh et al. (2002) :In this work 3 D fluid flow and heat transfer phenomena inside heated micro-channels is investigated by using a finite volume method . And it is checked by comparing the predicted local thermal resistances with available experimental data.. It was obtained that at the lower-Reynolds

number , the heat input lowers the frictional losses and also water temperature increases , which leads to smaller frictional; losses and also decrease in the viscosity.

Chapter 2

Mathematical Model

Chapter 2

Mathematical Model

2.1 Theory:

There are basically two ways of fluid modelling .i.e. Molecular Models and Continuum models . In Molecular Models , the fluid really is a collection of molecules and in case of Continuum models there is continuous matter and indefinitely divisible. The Molecular Modelling is subdivided into deterministic methods and probabilistic ones , while in the Continuum models the velocity , density , pressure etc are defined at every point in space and time , and conservation of mass , energy and momentum is leading to a set of nonlinear partial differential equations(Navier Stokes).

Navier stokes based fluid dynamics solvers are often inaccurate when applied to MEMS . This inaccuracy stems from their calculations of molecular transport effects ,such as viscous dissipation and thermal conduction and also mean velocity and temperature which are the bulk flow quantities. This approximation of micro-scale phenomena with macro-scale information fails as the characteristic length of the gaseous flow (L) approaches the average distance travelled by molecules between collisions (mean path ,) .The ratio of these quantities is known as Knudsen number.

Kn $= \lambda/D_h$

where $\lambda = \dfrac{KT}{\sqrt{2}P\sigma^2 3.147}$

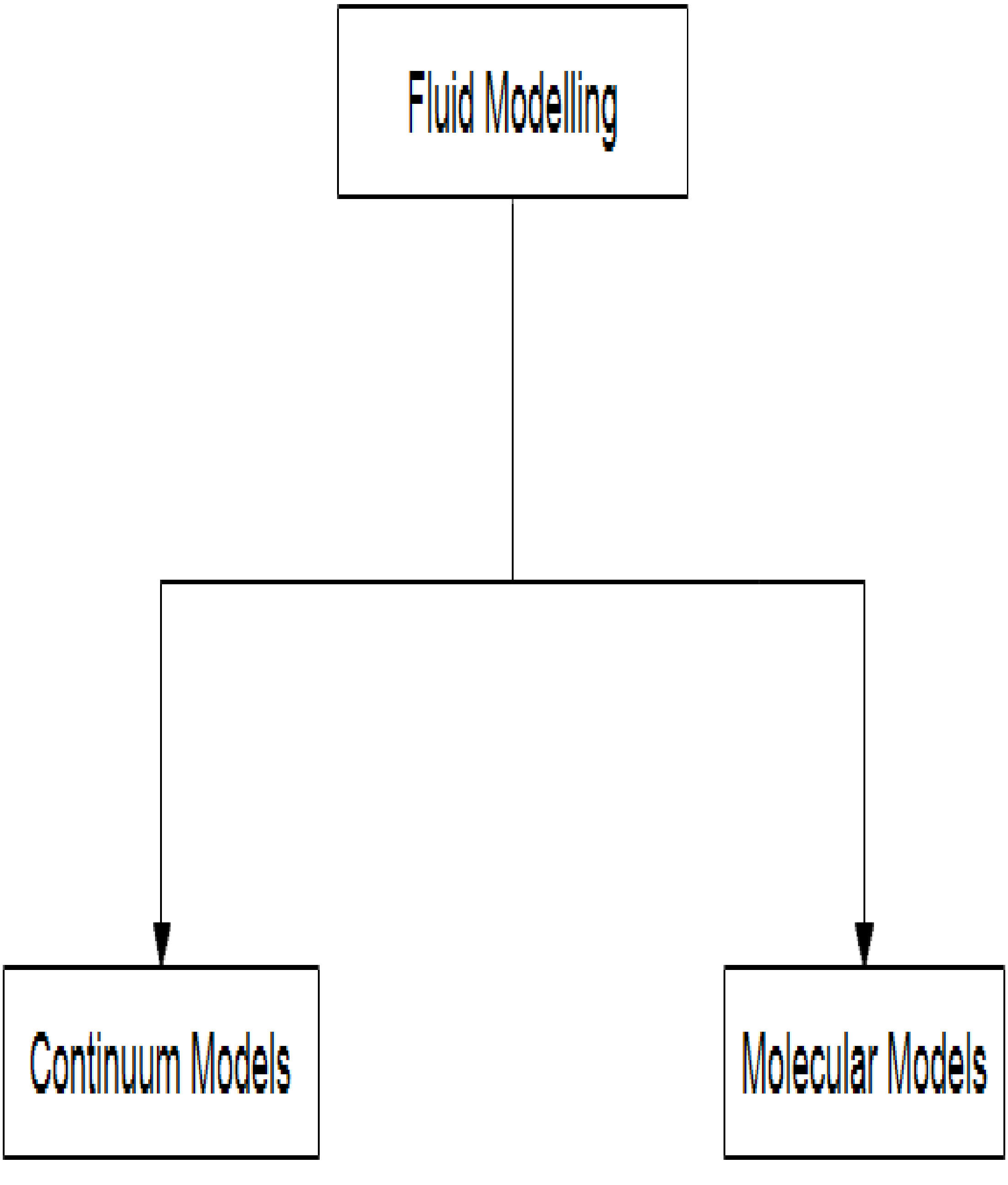

Figure 2.1: Fluid Modelling Classification

This is valid for an ideal gas model as rigid sphere , is the molecular diameter and k is Boltzmann constant . Generally the traditional continuum approach is valid with boundary conditions as long as Kn is less than 0.1.

Table 2.1: Knudsen number Regimes

Knudsen Number	**Regimes**
Kn <0.001	Continuum
0.001<0.1	Slip flow
0.1<10	Transition
Kn>10	Free Molecule

The Navier-Stokes equations can be taken into account when $\lambda < L$. And if it is violated , then the flow is no longer near equilibrium and the no-slip velocity condition are no longer valid. Similarly , the linear relation between heat flux and temperature gradient and the no-jump temperature condition at a solid-fluid interface are no longer accurate when mean free path is not much smaller than L.

The fluid is considered to be continuum when the $Kn < 0.001$,and , the fluid is considered to be a free molecular flow when the Knudsen number is greater than equal to 10 . For $(0.001 < Kn < 0.1)$ is the near continuum region. Knudsen number can also be expressed in terms of two important dimensionless numbers of fluid mechanics .

$$\text{Kn} = \sqrt{\frac{3.14 y Ma}{2Re}}$$

$$\text{Re} = \frac{\rho V L}{\mu}$$

And the Mach Number is the ratio of flow velocity to the speed of sound.

$$\text{Ma} = \frac{V}{a}$$

Where V is the characteristic velocity of the fluid.

The Mach Number is defined as the ratio of inertial forces to elastic ones and is a dynamic measure of fluid compressibility . From the kinetic theory of gases , the mean free path is related to the viscosity as follows

$\nu = \frac{\mu}{\rho} = \frac{\lambda V}{2}$

Where is the dynamic viscosity ,and V is the mean molecular speed which is somewhat higher than the sound speed ,a_o

V=$\sqrt{\frac{8}{3.14y}}a_o$

2.2 Governing Equations:

For steady 2D , incompressible flows having properties which are constant in Cartesian Coordinates ,Navier-Stokes equations , the continuity ,momentum and energy equations can be written as

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(vr) = 0 \tag{2.1}$$

Momentum Equations:

X component

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\Big(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\Big) \tag{2.2}$$

Y component

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\Big(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\Big) \tag{2.3}$$

Energy Equations

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\varphi}{\rho c} + \alpha\Big(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\Big) \tag{2.4}$$

where

$$\varphi = 2\mu\Bigg[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\Big(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\Big)^2 + \frac{1}{3}\Big(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\Big)^2\Bigg]$$

Momentum equations in x and r directions under the conditions specified above are :

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\Big\{\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) + \frac{\partial^2 u}{\partial x^2}\Big\} \tag{2.5}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\Big\{\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial (rv)}{\partial r}) + \frac{\partial^2 v}{\partial x^2}\Big\} \tag{2.6}$$

Energy Equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right) + \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \varphi \tag{2.7}$$

where

$$\varphi = 2\mu\Big[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial r}\right)^2 + \Big(\frac{v}{r}\Big)^2\Big]$$

the Navier- Stokes equation reduces to :

$$-\frac{1}{\rho}\frac{dP}{dx} + \nu\ \frac{1}{r}\ \frac{d}{dr}\,(r\ \frac{du}{dr}) = 0 \tag{2.8}$$

The pressure must be constant across any cross section perpendicular to the flow:

$$-\frac{1}{\rho}\frac{\partial P}{\partial r} = 0 \tag{2.9}$$

Thus general governing equation

$$\frac{1}{\mu}\frac{dP}{dx} = \frac{1}{r}\frac{d}{dr}\,(r\ \frac{du}{dr}) \tag{2.10}$$

$atr = R \rightarrow u = u_{fluid}$

$atr = 0 \rightarrow u = finite$

where u_s represents slip velocity Energy Equation reduces to

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\nu}{c_p}\left(\frac{du}{dr}\right)^2 \tag{2.11}$$

at r = R $\rightarrow T = T_{fluid}$

at r=0 $\rightarrow \frac{\partial T}{\partial r} = 0$

at x=0$\rightarrow T = T_(i)$

2.3 Boundary Conditions:

In practice , the no-slip/no-jump condition leads to fairly accurate predictions as long as Kn is less than 0.001 (for gases). Beyond that , the collision frequency is simply not high enough to ensure equilibrium and a certain degree of tangential velocity slip and temperature jump must be allowed

2.4 Slip Velocity:

When rarefaction of gases comes into role no slip boundary condition at the wall are no longer sufficient . Rarefaction of gases directly depends upon Knudsen number with increase in the Knudsen Number .Rarefaction effect increases and the no-slip assumption are invalid in those circumstances. Hence application of slip at the wall becomes necessary at higher rarefaction in turn at higher Knudsen number .Maxwell derived slip velocity correlation as :

$$U_{fluid} - U_{wall} = \frac{2 - F_v}{F_v} * \lambda * (\frac{\partial U}{\partial Y})_{wall} \tag{2.12}$$

Here F_v is Tangential Moment Accomodation Coefficient and for air is equal to 1.

2.5 Temperature Jump:

During a rarefield gas flow through a duct the temperature of the duct wall and the fluid layer just adjacent to the wall is not exactly equal. The difference in the temperature of the wall and adjacent fluid layer is known as temperature jump. The basic cause of the jump is the rarefaction effect which increases with increase in Knudsen number. At higher Knudsen number rarefaction effect will be high thus the collision between molecules will be less which in turn produces a high temperature gradient between wall and adjacent fluid layer. In macrochannel where characteristic dimension is larger than molecular mean free path which results in lower Knudsen Number .We do not require temperature jump boundary condition to correctly simulate the flow but in case of micro channel where characteristic dimension is larger than molecular mean path which results in lower Knudsen number. We do not require temperature jump boundary condition to correctly simulate the flow but in case of micro channel where characteristic dimension is comparable to molecular mean free path resulting high Knudsen number .Application of temperature jump boundary condition is must to correctly the fluid flow numerically and analytically. Temperature jump condition derived by Smoluchowski is :

$$T_{fluid} - T_{wall} = \frac{2 - F_T}{F_T} * \frac{2\gamma}{\gamma + 1} * \frac{1}{Pr} * Kn * \lambda * (\frac{\partial T}{\partial y})_{wall} \tag{2.13}$$

Here F_T is thermal accommodation coefficient . Its numerical value lies in the range of 0 to 1 and for fluid having Prandtl No 0.7 (Ex. Air) value of thermal accommodation coefficient is 1. Temperature jump boundary condition can be neglected for high Prandtl No fluid like water because temperature jump incorporate a little little change in heat transfer rate for high Prandtl no. fluid

Thus solving the governing momentum equation and energy equation in cylindrical coordinate with general boundary condition .Slip flow boundary condition and Temperature jump boundary condition yields the following result :

For 2 dimensional steady state constant wall temperature and fully developed laminar flow through a micro tube with constant hydro and thermo-physical properties , the fully developed velocity profile for slip profile can be written as :

Velocity profile

$$u = \frac{2u_m[1 - \left(\frac{r}{R}\right)^2 + 4Kn]}{1 + 8Kn} \tag{2.14}$$

$$Nu = \frac{h_x D_h}{k} \tag{2.15}$$

2.6 Analytical Result:

The empirical relation gives the brief idea about heat transfer phenomena and flow variables in the 2D steady state , laminar , isothermal wall and fully developed flow in a micro-tube.

Variation of fully developed Nusselt number with increasing the Knudsen number can be observed in the table below . It clearly indicates that with increase in the Knudsen number fully developed Nusselt number is decreasing .

Table 2.2: Fully Developed Conditions , Laminar flow Nu values[35]

Knudsen Number	**Numerical Nu**	**Analytical Nu**
0	3.6566	3.6751
0.02	3.3527	3.3675
0.04	3.0627	3.0745
0.06	2.8006	2.8101
0.08	2.5689	2.5767
0.1	2.3659	2.3723

2.7 Analytical Conclusions:

The above mathematical model results very complex and astonishing facts about a micro tube. Analytical and experimental results reveal the following observation :

1. Solving the momentum equation and energy equation with proper slip boundary condition is required to correctly model the flow in microtube where Knudsen number is high in comparison to macro tube.

2. Pr number plays a significant role in micro -tube analysis as it directly affects the temperature jump magnitude. With increase in Pr number temperature gradient temperature gradient between the wall and fluid decreases. Hence high Pr number fluid diminishes the applicability of temperature jump boundary condition with small inaccuracy in the prediction of flow variables and dimensionless number.

3. The fundamental correlation for macro-tube is not applicable in micro-tube. Transition of laminar zone to turbulent zone cannot be predicted by fundamental correlation of macro-tube. It has been observed that the transition of laminar to turbulent flow takes place at lower Re value (Re=200) in comparison to the conventional size tube where transition takes place at higher Re number (Re=2300-4000).

4. Pr number being fixed Knudsen number affects the prediction of dimensionless number and variable up to a larger extent. It has been found tha the nusselt number inversely relates with the Nusselt number which reveals that with the increase in the Knudsen number Nu number is decreasing

Chapter 3

Problem Formulation

Chapter 3

Problem Formulation

3.1 Problem Description

In present simulation a 2D ,laminar and steady state flow through a micro-tube has been analysed . Dimensions of the microtube having order of micrometre. Diameter of the micro-tube is 2e-5 and length of the tube is 10.9e-3 m . The walls of the tube are considered to be at constant temperature .The geometry of the fig is as shown

Fluid is entering in the tube from the inlet of the tube at ambient temperature 300K and leaving the tube from outlet where gauge pressure is zero. Flow is considered to be pressure driven . The walls of the tube are assumed at a constant temperature of 335 K.

Above stated geometry is created in ANSYS. The complete simulation is done in Fluent and results have been analysed for flow variable like pressure , temperature and velocity at different cross section of the pipe .Some of the important dimensionless number like Nusselt number , Knudsen number which plays a significant role in convective heat transfer have been computed and plotted in the graphical form. Complete investigation covers the study of Nusselt number , axial wall shear stress , convective heat transfer coefficient , skin friction coefficient and slip velocity.

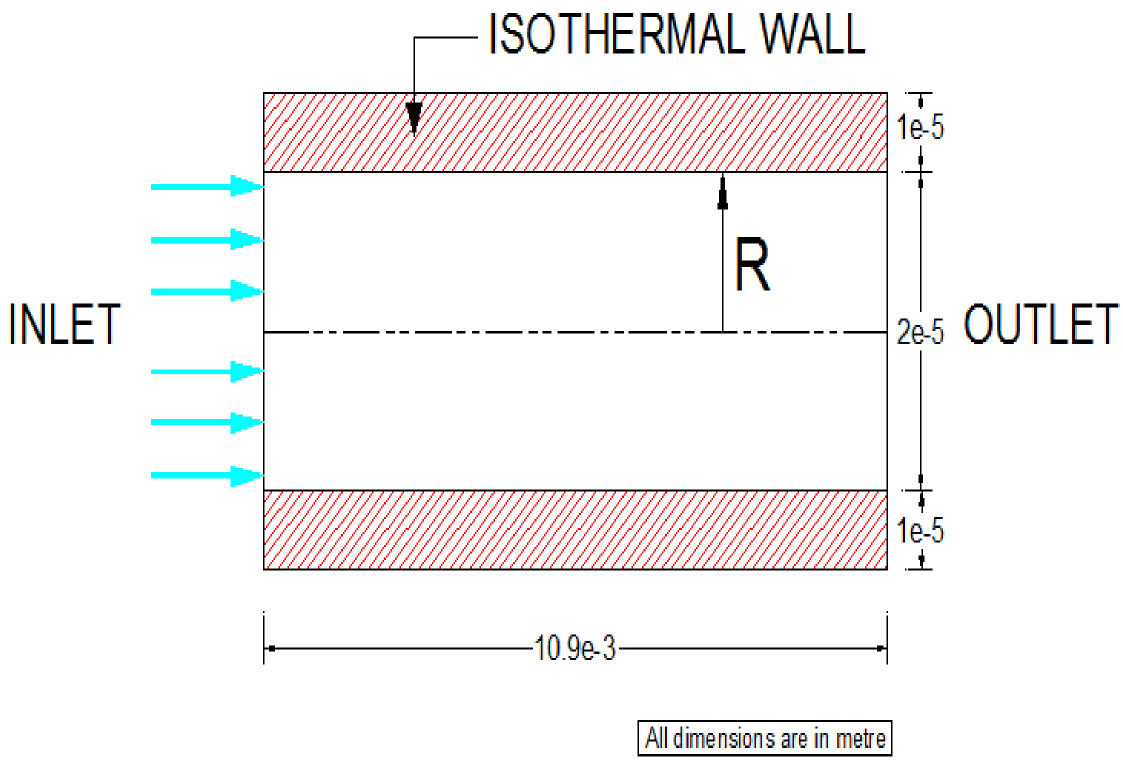

Figure 3.1: Physical Geometry of Problem

3.1.1 Simulation Approach :

The above 2D problem geometry is created by using ANSYS 13.0 . Then the geometry is meshed (4000*10) into smaller cells. Higher the no. of cells the greater is the accuracy with more rigorous calculations and lower convergence limit.Complete meshed geometry is shown in fig 3.2. Now specifying the boundary conditions and choosing a suitable solver we initialize the problem and then run the iteration for the solution.The convergence of residuals is shown in fig 3.3:

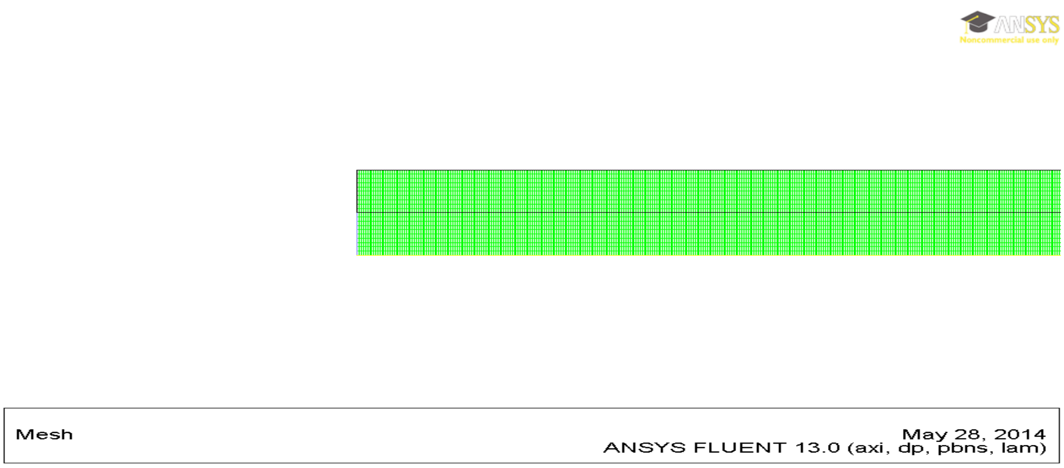

Figure 3.2: Complete Meshed Geometry

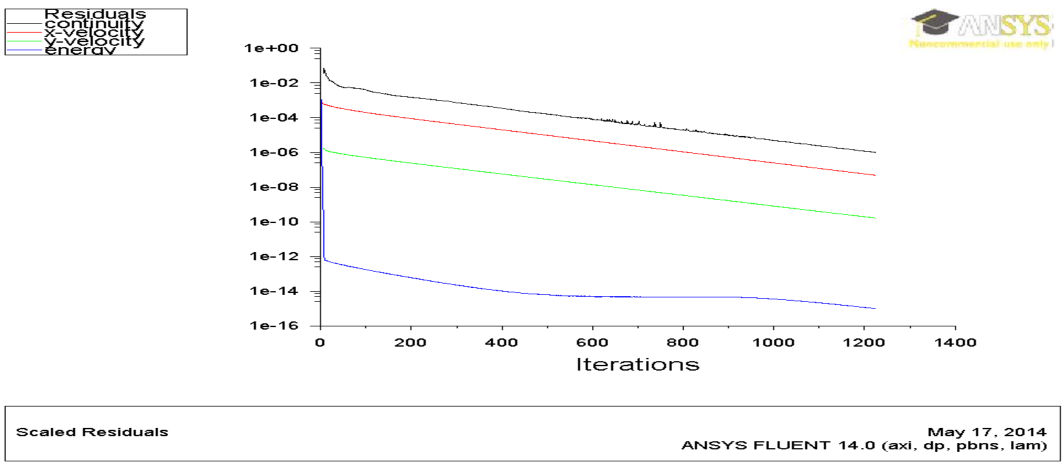

Figure 3.3: Convergence Of Residuals

3.1.2 Velocity Simulation

As discussed above the mesh fileis imported into ANSYS 14 and the boundary values are specified.The working fluid considered is Air with outlet pressure of 1 atm .

(a) The inlet velocity 5.5m/s is considered for simulation .This corresponds to a Reynolds number equal to 10.

(b) At the wall the slip velocity is accounted by implementing the slip velocity user defined function .

The solver specified uses a pressure correction based iterative SIMPLE algorithm with the discretization done with 1st order upwind scheme. Problem is initialized with velocity inlet boundary condition and iteration is performed. The resultant velocity distribution obtained in the microtube is shown in fig 3.4 :

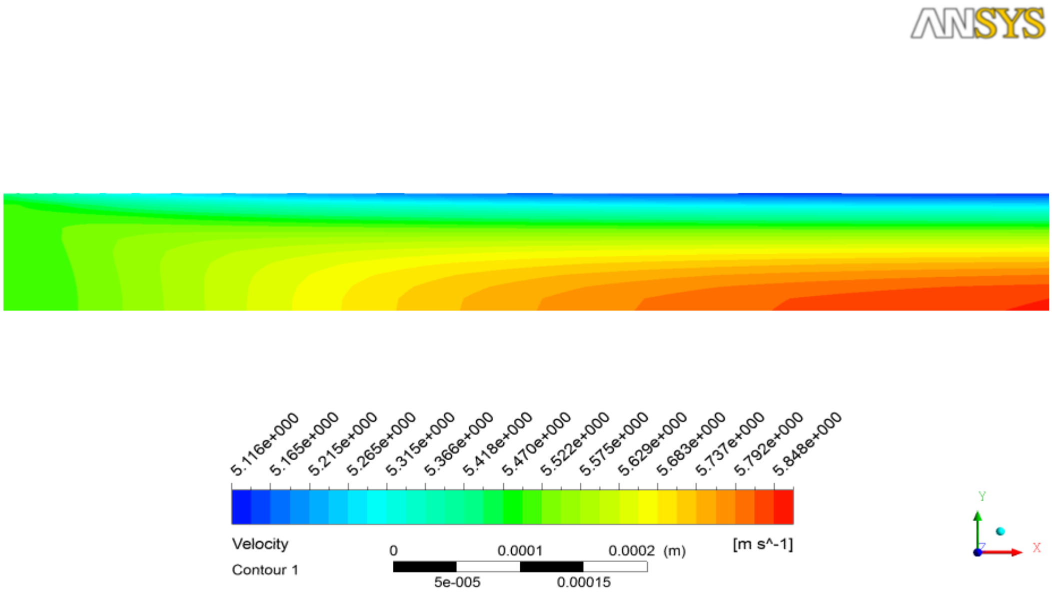

Figure 3.4: Velocity Contour

3.2 Temperature Simulation

For the same dimensions of the micro-tube we consider the boundary walls to be at a temperature of Twall (=335k). The inlet temperature of the free stream be Ti (=300K).The above parameters are fed to the software and thus in addition to flow equation , energy equation is also solved.

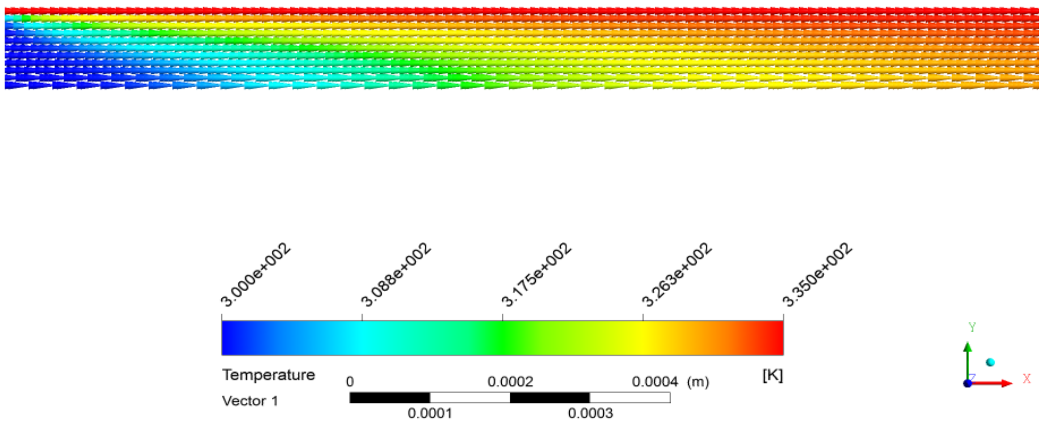

Figure 3.5: Velocity Vectors Colored by Static Temperature

3.3 Program Validation

Validity of program is done using a forward backward approach which considers the output of the udf as an input to Fluent inbuilt shear stress velocity model. It is found that slip velocity computed from udf and fluent based model are approximately equal and the error calculated in slip velocity from both the cases is found to be 1.485

(a) Rounding off error causes the slip velocity deviation in the two cases .The data computed from Fluent for shear stress is not exactly utilized as an input to shear stress slip velocity model. Hence there is a little difference in slip velocity computed from both cases .

(b) In the entrance region axial shear stress is not constant , actually it is decreasing rapidly but we have neglected the effect of axial shear stress in entrance region for simplification .Thus this assumption also plays role to some extent in the slip velocity error.

Velocity profile of the two cases merges with each other and graph for other flow variables and dimension less number merges for both the situation. Thus we conclude that our code is written correctly .We can see from both the fig that slip velocities form the two cases almost collapse with each other in turn two plots merges into one , representing our code to be correct.

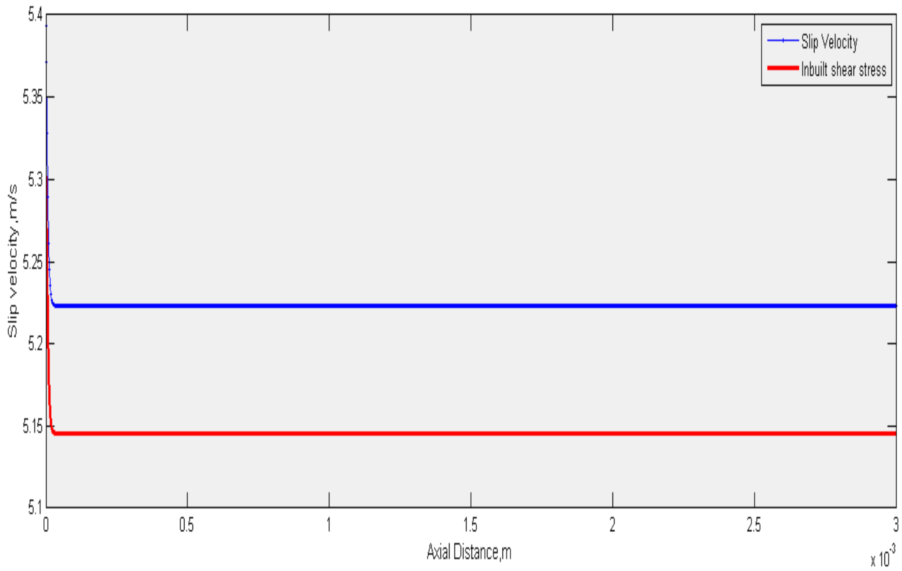

Figure 3.6: Slip Velocity comparison for Code validation

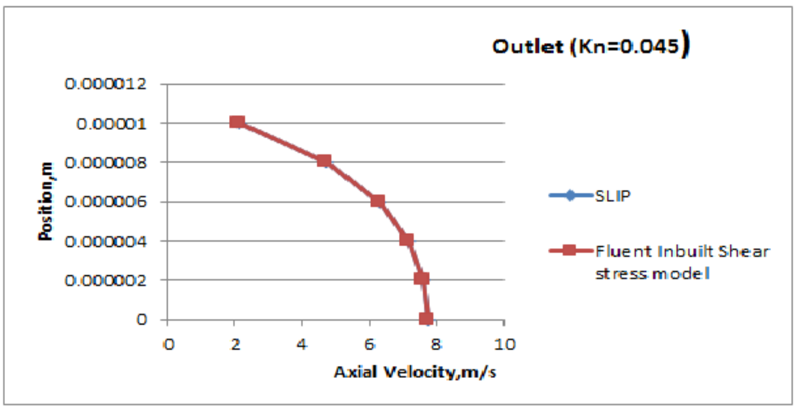

Figure 3.7: Velocity profile comparison for Code validation

Chapter 4

Results and Discussion

Chapter 4

Results and Discussion

4.1 Grid Test

Considering the various geometry meshing size we have seen that slip velocity marginally deviates ,thus we conclude that our geometry is grid independent.

Table 4.1: Grid Independence Test Results

Grid Size	**Slip Velocity**
200*5	2.86
400*5	2.648
200*10	1.56
1000*10	1.46
4000*10	1.36

4.2 Effect of Slip

It is clear from the literature review that slip velocity and temperature jump boundary condition is necessary to simulate the flow. Below listed plots were representing the effect of effect of implementing the Maxwell based slip velocity and Smoluchowski based temperature udf

4.2.1 Slip velocity

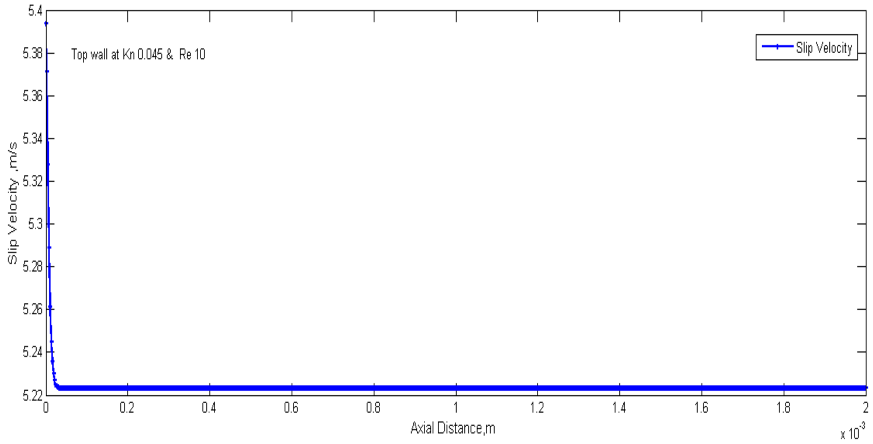

Figure 4.1: Slip Velocity at Kn 0.045

4.2.2 Axial Wall Shear Stress

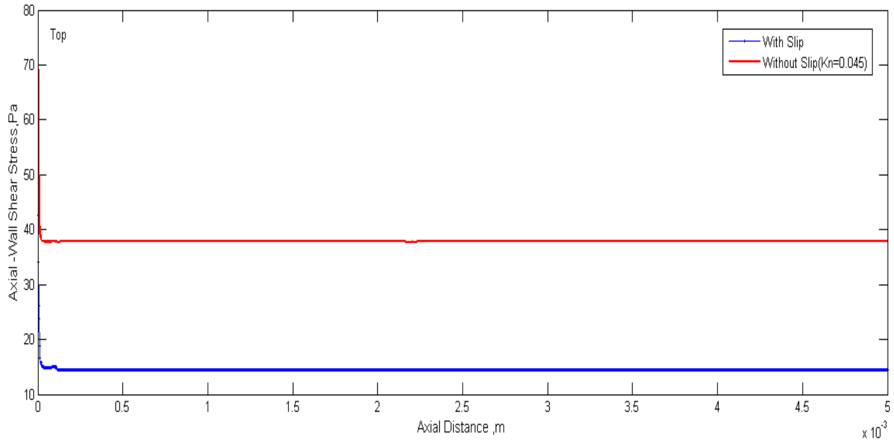

Figure 4.2: Axial wall shear stress comparison

4.2.3 Skin Friction Coefficient

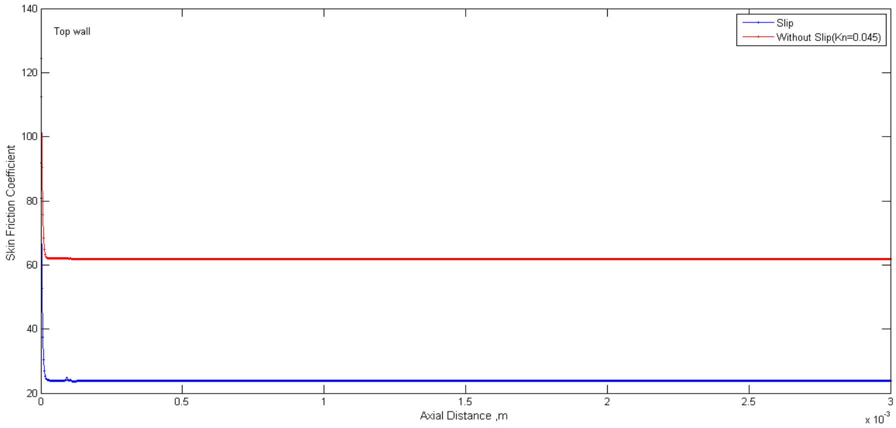

Figure 4.3: Skin Friction Coefficient

4.2.4 Surface Nusselt Number

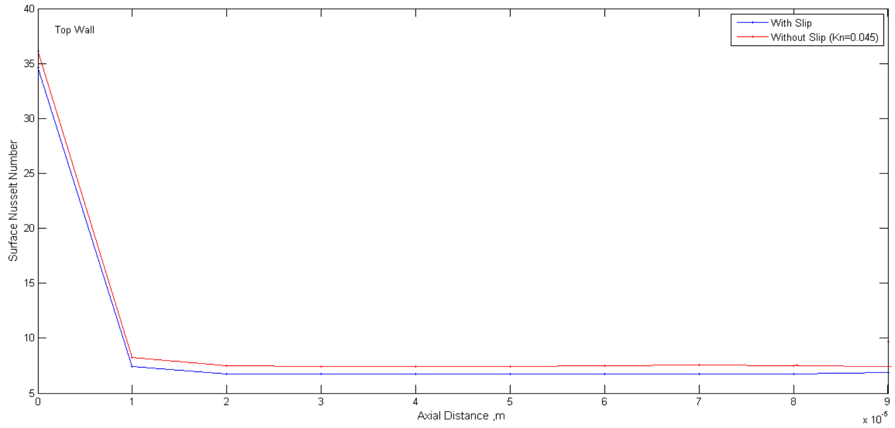

Figure 4.4: Surface Nusselt Number

4.2.5 Surface Heat Transfer Coefficient

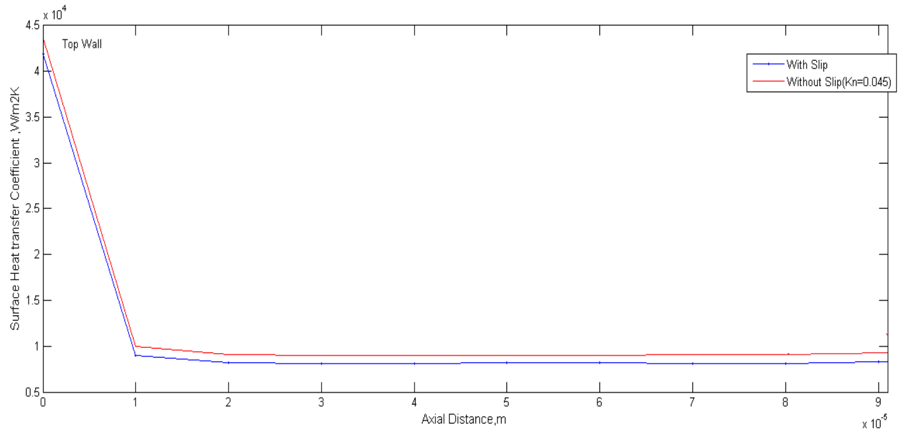

Figure 4.5: Surface Heat Transfer Coefficient

4.2.6 Centreline Temperature

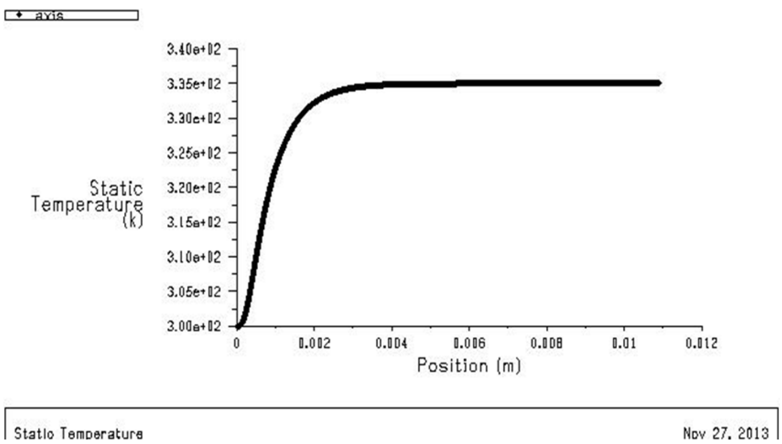

Figure 4.6: Centreline Temperature

4.3 Effect of Knudsen Number

4.3.1 Slip velocity

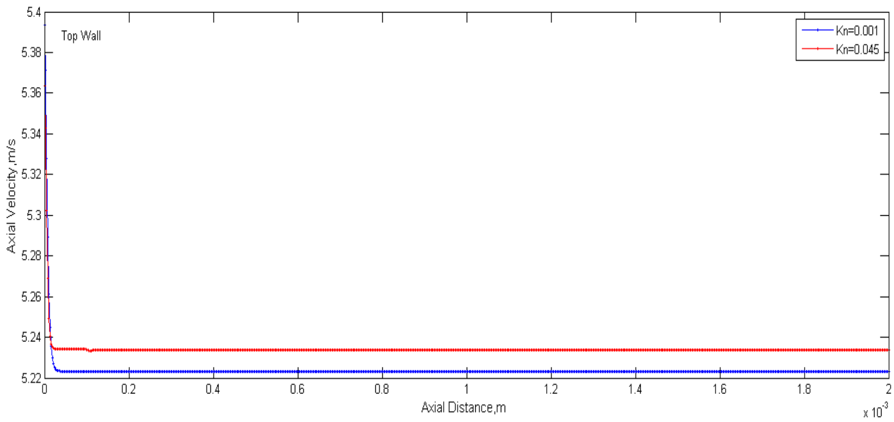

Figure 4.7: Slip Velocity Comparison at different Knudsen Number

4.3.2 Axial Wall Shear stress

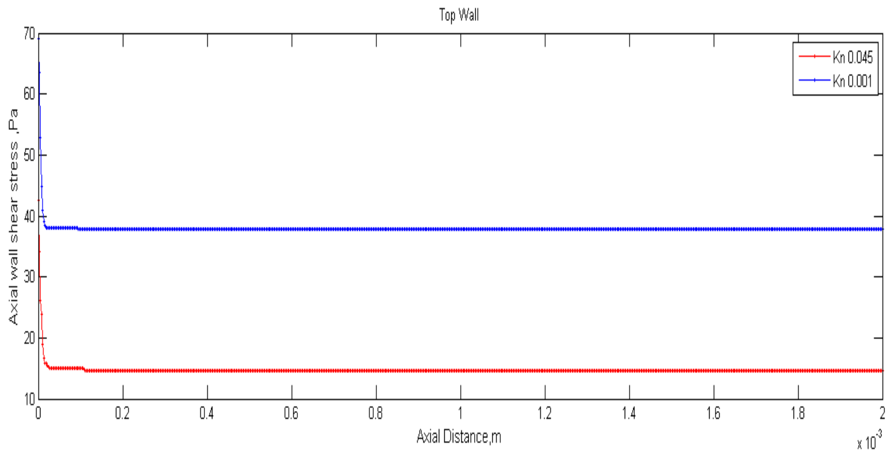

Figure 4.8: Axial Wall Shear stress Comparison at different Kn no.

4.3.3 Heat Transfer Coefficient Behaviour

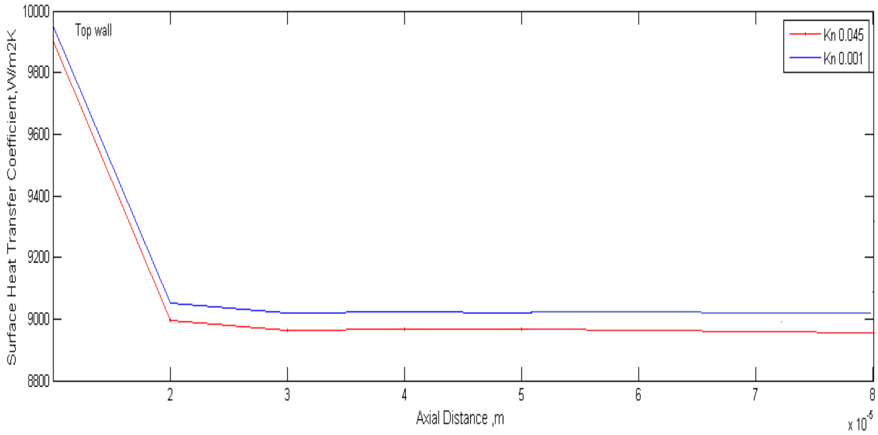

Figure 4.9: Surface Heat Transfer Coefficient at different Kn no.

4.3.4 Surface Nusselt Number

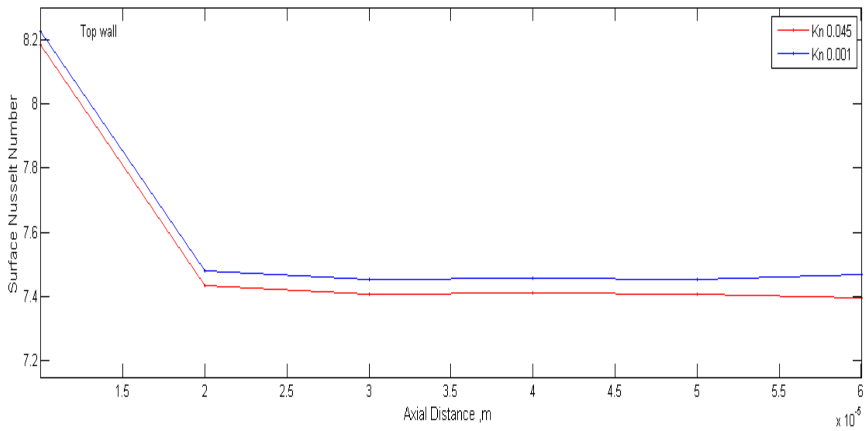

Figure 4.10: Surface Nusselt Number at Different Knudsen Number

4.4 Effect of Reynolds Number

4.4.1 Slip Velocity

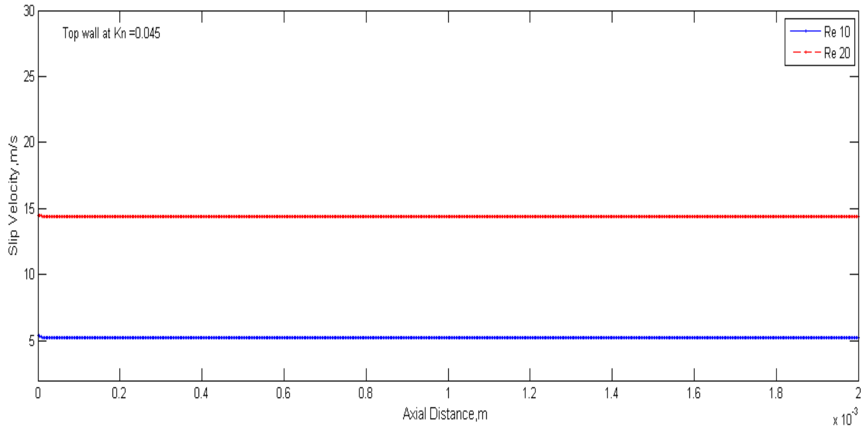

Figure 4.11: Slip Velocity at different Re no.

4.4.2 Axial Wall Shear stress

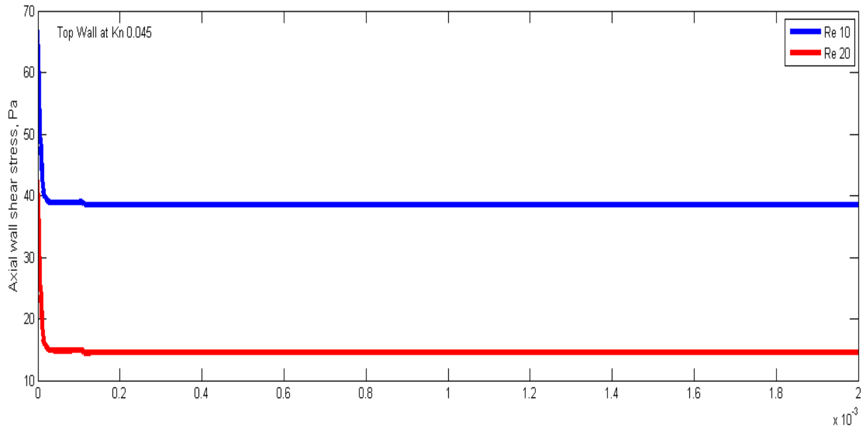

Figure 4.12: Axial Wall Shear stress at different Re no.

4.4.3 Heat transfer Coefficient Trend

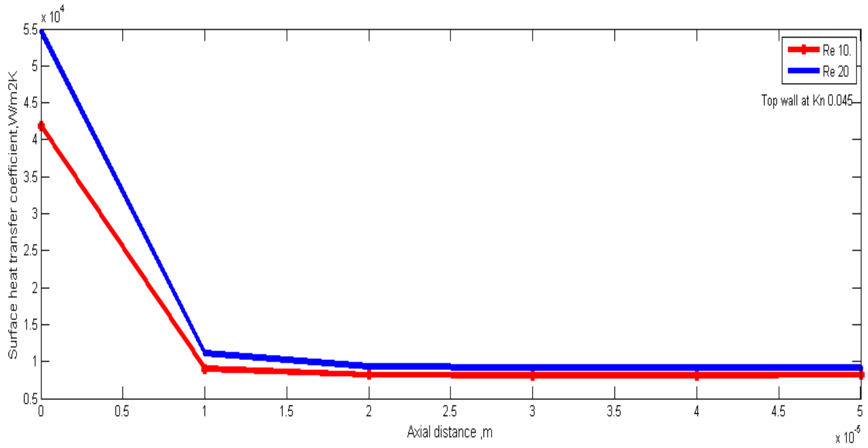

Figure 4.13: Heat transfer Coefficient at different Re no.

4.4.4 Surface Nusselt Number Trend

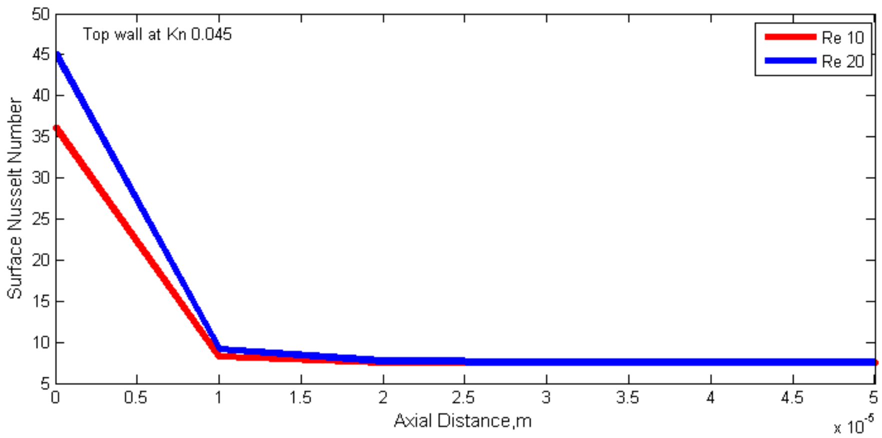

Figure 4.14: Surface Nusselt Number at different Re no.

4.4.5 fRe Vs Axial distance

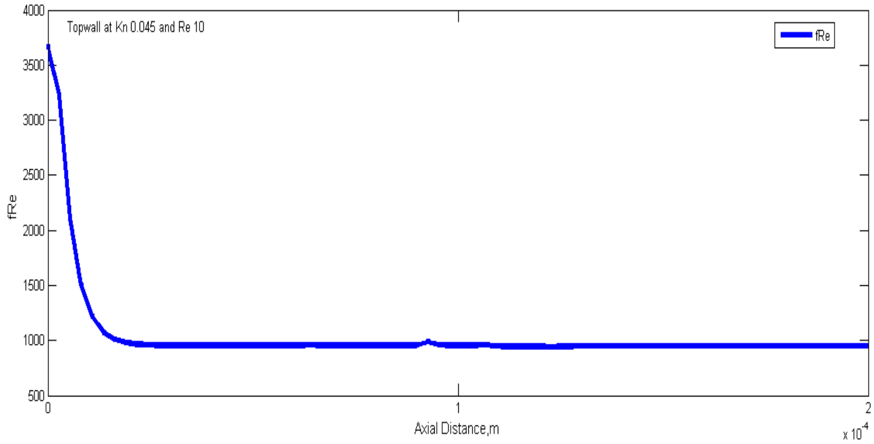

Figure 4.15: fRe Vs Axial distance

(a) Velocity contour shown in fig 3.4 clearly indicates that velocity is highest at the centre and it is decreasing continuously as we move from axis to the wall ,ȦT the entrance region velocity is changing at axis but once the flow is developed it remains constant throughout the tube.

(b) Fig 3.5 is showing the distribution of temperature inside the micro-tube. It is clear that the temperature vector plot that heat is transferring from wall to fluid hence temperature of wall is increasing and at the end of the micro tube whole fluid attains the temperature of the wall.

(c) Fig 3.6 represents the slip velocity computed from two different method i.e through slip velocity udf and through fluent inbuilt shear stress model which do not have capability of varying Knudsen number .It is clear from the graph that slip velocity that slip velocity plot merge together indicating our code to be right.

(d) In Fig 3.7 velocity profile computed from two models is compared and it is found that two plot overlap one over other which validates our code to be right.

(e) Here in fig 4.1 ,effect of implementing the user defined function is shown . Without slip the velocity at the wall comes out to be zero but as we implemented the slip user defined function initially velocity at the wall is high and then decreases rapidly and attains a constant value very quickly.

(f) Fig 4.2 represents the Axial wall shear stress in y coordinate and axial length of micro tube in x coordinate is considered .Here we can judge that the trend of the plot is similar to the slip velocity plot . It is indicated by the present fig that as we implement our udf of slip axial wall shear stress is reducing as shown with the red curve.

(g) Fig 4.3 represents the skin friction coefficient and it is found that the trend of the plot is similar to the slip velocity and applying the udf reduces the skin friction coefficient as represented by red curve.

(h) Fig 4.4 depicts the behaviour of Surface Nusselt Number with respect to the axial length of the micro-tube.It is found that initially the value of surface Nusselt number is very large and it rapidly decreases in the entrance region but after some length it decreases slowly and at the end of the pipe attains a value around 3.86, this curve is not enough accurate in terms of data but depicts the effect of implementing the slip reduces the Nusselt number . Fig 4.5 finds the same explanation as in place of Surface Nusselt number heat transfer characteristic is taken.

(i) Fig 4.6 the temperature of the centreline is drawn with respect to the axial length. In the beginning length temperature of the centreline increases linearly as heat transfer from the wall to the fluid , once the centreline attains the temperature of wall there is no further increase in temperature and curve becomes parallel to axis representing no further increase in temperature.

(j) Fig 4.7 represents the effect of changing the Knudsen number on slip velocity while Reynolds number is being fixed and it is found that with increasing the Reynolds number slip velocity is also increasing.

(k) Fig 4.8 represents the effect of varying the Knudsen number on axial wall shear stress and trend of the plot is showing that shear stress decreases with increase in the Knudsen number.

(l) Fig 4.9 4.10 depicts the variation in Nusselt number and heat transfer coefficient respectively with changing the Knudsen Number and indicates reduction of both Nusselt number and heat transfer coefficient with increase in the Knudsen number.

(m) Fig 4.11 represents the effect of changing the Reynolds number on slip velocity while Knudsen number is being fixed and it is found that with increasing Re number slip velocity is also increasing.

(n) Fig 4.12 represents the effect of varying the Reynolds number on axial wall shear stress and the trend of the plot is showing that it is decreasing with increasing the Reynolds number.

(o) Fig 4.13 and 4.14 depicts the variation in heat transfer coefficient and Nusselt number respect with changing the Reynolds number and indicates enhancement of both Nusselt number and heat transfer coefficient with increase in Reynolds number.

(p) Fig 4.15 is representing the effect of varying the fRe along the axial distance and the trend of the plot is showing that it is decreasing with the increase in the axial distance.

Chapter 5

Conclusion and Future Work

4.5 Conclusion

Present work provides the trends of flow variable and dimensionless number by varying the Reynolds number and Knudsen number .Observing the plots for various parameters and comparing them with the analytical solution it is concluded that flow variable are correctly predicted in terms of data and trend but dimensionless number and parameters derive from flow variable are correct in terms of nature and trend but do not accurately predict the exact data value. The complete essence of current work can be summarized as follows:

i. Effect of UDF section represents implementing the UDF reduces the heat transfer phenomena and Surface Nusselt Number .Implementation of Slip decreases the slip flow regime where value of Knudsen number is high.

ii. Effect of Knudsen Number reveals the fact that increasing the Knudsen number increases the slip velocity at the wall but decreases the axial wall shear stress, heat transfer coefficient and Surface Nusselt number . Centerline temperature is find to increase linearly and then attaining a constant value.

iii. Effect of Reynolds Number : It reveals the fact that increasing the Reynolds number enhances the heat transfer and parameter related to heat transfer like surface Nusselt number and Heat transfer coefficient.

4.6 Future work

The geometric parameters of the tube have a significant influence on the convective heat transfer characteristics. Hence , to design effective micro tube heat sinks , design parameters like pressure required circulating the cooling fluid , flow rate ,hydraulic diameter of the tube , temperature of the fluid and the tube wall and the number of tube has to be considered. To make the system effective and cheaper ,these parameters have to be optimized .

i. It is observed that temperature jump effect is almost negligible in high Prandtl number fluid like water (Pr =14).Deviations in the flow and convective heat transfer characteristics have been observed for ionic fluids in micro-channels , the reasons is due to the presence electrostatic charge difference between the surface and the fluid. So the presence of the electrostatic force field and the double layer is found to influence the velocity field and this influence is to be incorporated in the analysis of the flow of ionic fluids in micro-tube and this can be done by introducing an additional term which represents the electric body force due to the electric double layer , in the momentum equations .

ii. Study of thermal creep is an important phenomenon which increases or decreases slip velocity depending upon heating or cooling of fluid .

iii. Change in thermo physical property with temperature and pressure can be included by writing the UDF.

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