OPTIMIZATION OF PRODUCTION PLANNING IN UNDERGROUND MINING

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF TECHNOLOGY
IN
MINING ENGINEERING

BY
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DEPARTMENT OF MINING ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY
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CERTIFICATE

This is to certify that the thesis entitled **"OPTIMIZATION OF PROCUCTION PLANNING IN UNDERGROUND MINING"** submitted by **Sri JITESH GANGAWAT** in partial fulfilment of the requirements for the award of Bachelor of Technology degree in Mining Engineering at the National Institute of Technology, Rourkela is an authentic work carried out by him under our supervision and guidance.

To the best of our knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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ACKNOWLEDGEMENT

ABSTRACT

Use of Integer programming (IP) or mixed integer programming (MIP) for formulation of mine optimization problem is best suited modelling approach for underground mining. Optimization algorithm for underground stope design problems cannot be generalised as geotechnical constraints for each method is different. This project concentrates on optimization model for open stoping mining method. The stope design model maximizes Net cash flow of the stope while adhering to the stope constraints. The methodology considers open stoping sequence, in which every block is moved towards the cross-cuts at the lower level. In this thesis, stopes are designed to maximize the undiscounted cash flow from the stope after satisfying stope height and extraction angle constraints. An integer programming formulation is developed and solved using CPLEX solver for single stope. The proposed algorithm is solved for first stope and then blocks for the crown pillar for first stope is identified. After eliminating the first stope and respective crown pillar data from the data set, algorithm is solved again for the second stope from the remaining data set. After stope design, production scheduling is done by applying heuristic approaches. Blocks from the stopes are extracted heuristically satisfying extracting angle, mining and processing constraints. Initially blocks from the first stope are selected and then to fulfil the constraints, some of the blocks from the second stope are selected. A study is carried out on the part of the Zinc mine data of India which contains 4992 number of blocks. Total 3 numbers of stopes are designed. The NPV of the considered data is found to be 7313.346 million rupees in 3 periods with total tonnage of 1.103 million tonnes. Metal content in 3 periods is found to be 86.485 thousand Tonnes. The overall dilution is found to be 3.82% with average dilution of 2.692%.

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Chapter 1 : Introduction

GENERAL

OBJECTIVE

1. INTRODUCTION

1.1 GENERAL

Mining is the process of excavating a material which is naturally occurring from the earth to get profit. Underground method is used when ore body is thin or narrow and extends much below surface (Newman et al., 2010). In underground mining, development required to reach the orebody is much and increases the production time from the start of the development (Haycocks, 1992). Effective mine planning is required to generate profit (Carlyle et al., 2001). Mine production scheduling is an important problem in mine planning (Gershon, 1983). Main objective of the production scheduling is to maximize cash flow and it is also the main objective of all the mining industries. Before the use of computers in planning, it was done manually and based on the sections and graph created by the persons, this work takes a lot of time and is bit tedious and efficiency also depends on the individuals. But now days, operation research techniques are used to define or optimize any process or activities (Newman et al., 2010). These techniques are faster and more work is being carried out to decrease the computational time. Solutions obtained from these techniques are close to the real values.

Production scheduling can be defined as the allocation of resources/reserves over a time period with particular sets of constraints (Martinez et al., 2011). Mine production scheduling is an optimization process which assigns the extraction sequence of mining blocks based on the constraints which incorporate method of mining, slope, stope size, etc such that it maximizes the net present value (NPV) of a mine (Kuchta et al., 2003). There are two types of mine production scheduling: long-term production scheduling and short-term production scheduling (Nehring et al., 2012). Long-term production scheduling is done over life of the mine. In Short-term production scheduling, result generated from the long-term production scheduling is broken down according to task or time period that need to be studied and different sets of constraints active in short-term is applied minimize the deviations from the pre-defined capacities.

Generally, production schedule is based on the block model of the ore body generated by the interpolation techniques, such as Kriging (Hugh and Davey, 1979), from the drillhole sample data. Block model can be transformed into the economic block model knowing mining cost, metal price, metal recovery and ore density (Lane, 1988). This model is considered to be fair

representation of the ore body. Then this block model is used for optimization of production scheduling. Scheduling problem mainly consists of 3 steps: (a) deciding the extraction sequence of blocks satisfying slope and mining method constraints and produces the optimum Net Present Value (NPV), (b) designing different mining phases based on the optimum sequence, and (c) optimizing the production schedule and cut off grades (Menabde et al., 2004). The NPV of this optimum schedule found is taken as a main criterion of the viability of the project.

Use of Integer programming (IP) or mixed integer programming (MIP) for formulation of mine scheduling optimization problem is best suited modelling approach. In this approach, model includes objective function which tries to maximize NPV over the pre decided time period and constraints which take in geotechnical considerations, mining method considerations, mining and processing capacities, cut off grades, etc (Lamghari et al., 2010). Geotechnical constraints on the stope shape pertain to angle of repose of the material, minimum and maximum stope dimensions (Smith, 2003). As it is known that, geotechnical constraints vary according to the mining methods used. All methods have different-different approaches. Therefore, a general optimization algorithm cannot be defined for all underground mining methods. In open pit optimization, lower level blocks are linked with the blocks of upper level based on the wall angles of open pit. Lerchs-Grossman algorithm (Lerchs and Grossman, 1965) and network flow (Picard, 1976) concepts are the best approaches to solve these problems. These techniques cannot be applied directly to the underground mining problems, but can be used if all the constraints leaving the slope constraint are embedded in the objective problem itself (Dagdelen and Johnson, 1986). As in open pit problems, every block is moved towards the free surface and in this case the free surface is ground. Likewise in underground problems, an initial free surface is created & differs in every method, can be in form of raise or ore pass or cross-cuts, and every block is pushed towards the free surface. This translates the problem into a network flow problem which can be solved by following open pit techniques (Bai, 2013).

Underground stope design contains stope extraction angle constraint, stope height constraint. This problem cannot be solved by either using Lerchs-Grossman algorithm or using network flow concepts due to non-unimodularity of constraint matrix. That is why in this project, problem is divided in two stages. First stope design problem is formulated as Integer Programming (IP) which is solved using branch and cut algorithm. Then production scheduling problem on the generated solution is solved by applying heuristic approaches.

Heuristic approaches give solution which is close to the optimal solution but is not an optimal solution; however computational time can significantly be reduced.

1.2 OBJECTIVE

1. To optimally design stopes for open stoping method based on constraints of stope extraction angle and stope height.
2. To do production scheduling of the generated stopes by applying heuristic approaches.

Chapter 2 : Literature review

PREVIOUS RESEARCH

OPEN STOPING-UNDERGROUND MINING METHOD

2. LITERATURE REVIEW

2.1 Previous research

Use of operations research (OR) techniques in mine planning is widely used. Most of the models developed so far are for open-pit mines which only solve the part of the long-term planning problem (Newman et al., 2010). Optimization of either open-pit or underground problems has been divided in two parts: the ultimate pit problems which determines the final pit in open-pit case or stope design problem which optimizes the stopes; and the production scheduling problems which determines the extraction sequence of the blocks with respect to time period.

Use of OR techniques started with solving the ultimate pit problem. Kim (1978) presents the classic moving cone algorithm which selects a block as a reference and expands upwards based on the sloping criteria. This algorithm gives sub-optimal solution only. But the algorithm given by the Lerchs and Grossmann (1965) that gives the optimal solution. For solving the production scheduling problem, Gershon (1983) gives mixed integer programming formulation, Towlanski et al. (1996) presents use of dynamic programming approach, and Caccetta and Hill (2003) presents an algorithm which uses branch and cut. Kuchta et al. (2003) presents the MIP model for underground mine which aims at minimizing the deviations from the pre-defined targets. Likewise Martinez et al. (2011) present a solution approach which optimizes the long- and short-term production scheduling. They developed an optimization based decomposition heuristics which gives better and faster solutions. Bakhtaver et al. (2012) present a (0-1) integer programming model which optimizes the transition from open-pit to underground mining.

Research on sequencing models for underground mining is relatively new. Earlier models use linear programming and simulation to determine production schedules and decisions related to ore extraction. Jawed (1993) uses linear programming to minimize the deviations from the mining capacities subjected to operational constraints. Author considered room and pillar mining method as a reference for effective design. Carlyle et al. (2001) presents a mixed integer programming model which considers several planning constraints. This model was applied to only one sector of the underground platinum mine. Smith et al. (2003) uses mixed integer programming (MIP) for life of mine planning which is aimed at maximizing the cash flow based on detailed production scheduling study and the operational constraints. But the

solution time for solving all the instances took sound amount of time. Newman et al. (2007) uses a small model in which time period is aggregated and then solved using heuristic approaches. This aggregated model serves as a base and the information gained from this model is used to solve the original model which tries to minimize the deviation from the planned production quantities. Grieco et al. (2007) presents a probabilistic mixed integer programming model to optimize the underground open stoping situations. This methodology includes location, size, and number of stopes with uncertainty in grade and acceptable risk levels. Epstein et al. (2012) presents a methodology based on the multicommodity network flow which considers that open pit and underground deposits share multiple downstream processing plants over the time horizon. This model tries to integrate several mines and then optimize them. Author uses block caving method as a reference for optimization of the underground mine. Nehring et al. (2012) presents a methodology for sublevel stoping method which tries to integrate the short term and medium term production plans by combining the short term objective (to minimize the deviation from the target) and medium term objective (to maximize the net present value). Bai et al. (2013) presents an algorithm for sublevel stoping method which tries to optimize stope design. Methodology is based on the location of vertical raise and then conversing the blocks towards it. Optimization program is transformed to a maximum flow over the graph problem by adding source and sink node.

2.2 Open stoping method

Underground mining methods are generally categorised in 3 categories: unsupported methods, supported methods, and caving methods. Generally mining method is determined by the geotechnics and not by the OR techniques. Mining method determination depends on size of the orebody, shape of the orebody, and characteristics of the ore and the surrounding rock (Newman et al., 2010).

Open stoping underground methods are used where orebody strength vary from moderate to strong with low discontinuities. Hang wall and foot wall strength need to be good and does not require more than incidental support (Haycocks, 1992).

In open stoping method, first of all levels are created which act as a haulage road and divides the orebody in stoping blocks as shown in Figure 2.1 Stoping blocks are extracted according to the sequence method used and mainly depends on the orientation and thickness of the orebody. For extraction of the stoping blocks, they are divided into panels and the ore pass and raises are created for access between them.
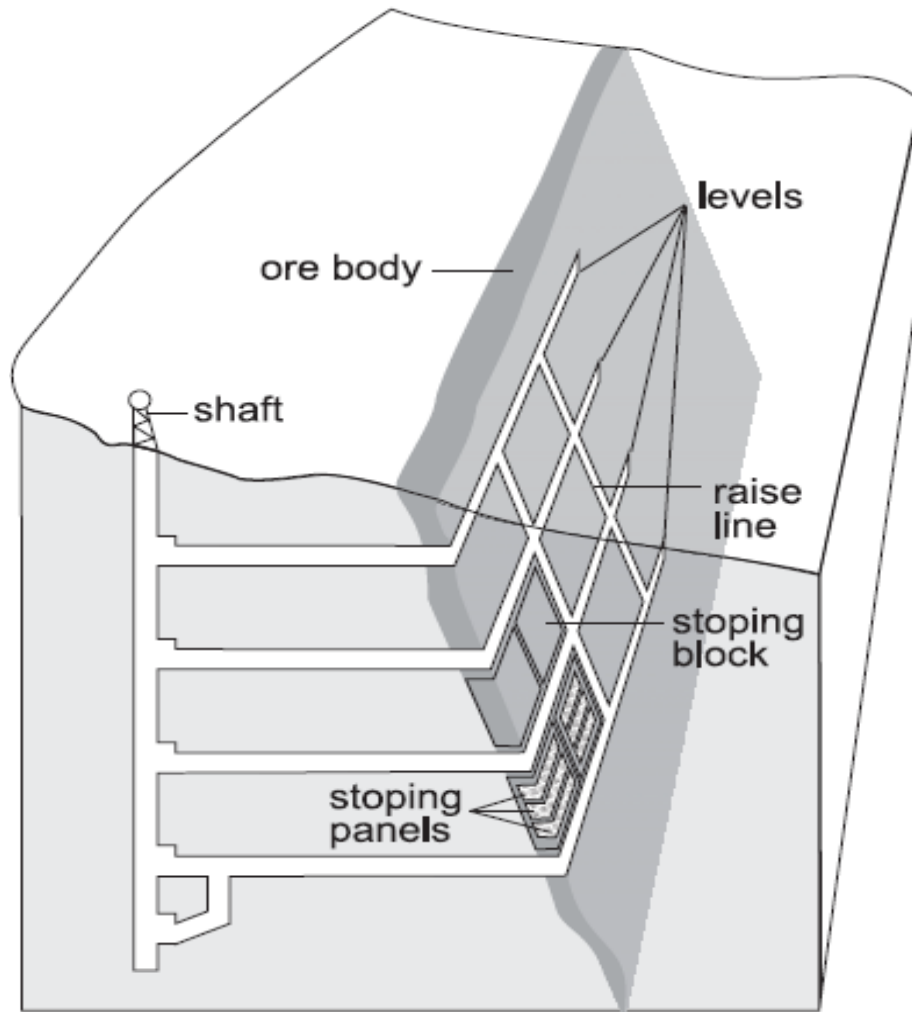
Figure 2.1: Underground mine layout (Bley et al., 2012)

Figure 2.2 shows the flow in an underground mine. First of all broken ore is loaded from the draw points along parallel crosscuts. Load haul dumpers (LHD) are used in this operation and the crosscuts have enough space for smooth working of LHDs. LHDs dumped the material in the ore passes which opens in the internal crusher. Internal crusher is used for size reduction and the material is then transported to the processing plant outside mine through the shaft and then by the train (Epstein et al., 2012).

Figure 2.2: Description of the underground mining operations (Epstein et al., 2012)

Chapter 3 : Methodology

INTRODUCTION

ASSUMPTIONS

NOTATIONS

OBJECTIVE FUNCTION

CONSTRAINTS

3. METHODOLOGY

3.1 Introduction

In optimization problem formulation, first of all objective function is formed, transferring the required problem in the form of a mathematical equation. Then to meet th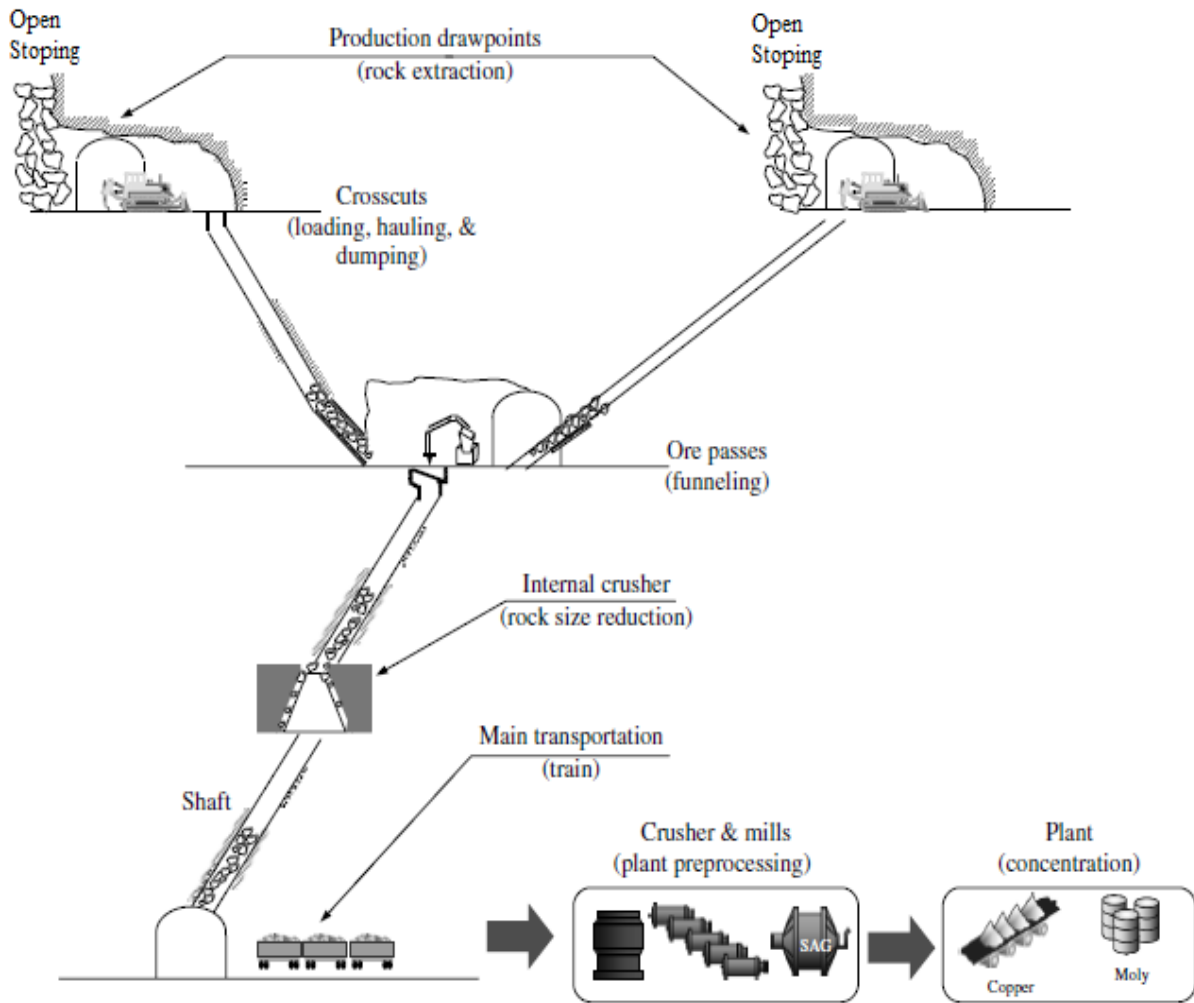e different criteria, constraints are formed and expressed in the manner of mathematical equations. Constraints help in determining the feasible solution. In this project, the underground stope design optimization problem is formulated as Integer Programming (IP).

3.2 Assumptions

Before formulating the underground mine production scheduling problem, number of things are assumed. These are:

1. Access to the ore body is fully developed and is in working order.
2. Development required for production activities like drilling, blasting, loading, unloading is done before hand and is in working order.
3. Open stoping mining method is used for extraction.
4. Size of the stoping blocks is known based on the thickness of the orebody, depth of the orebody.
5. Size of the crown pillar is fixed and is decided by the stress analysis of the different openings and extractions in different manner as decided by the geotechnical parameters.

3.3 Notations

3.3.1 Constants

$G[i,j,k]$	Grade value of the $(i, j, k)^{th}$ block (%)
OP	Selling price of the metal (rupees per tonne)
PC	Cost of processing of the ore (rupees per tonne)
MC	Cost of mining of the ore (rupees per tonne)
BM	Tonnage of single Block (tonne)
M	Number of blocks in X direction
N	Number of blocks in Y direction
O	Number of blocks in Z direction
MN	Minimum number of blocks in Z direction in one stoping block

MX	Maximum number of blocks in Z direction in one stoping block
COG	Cut off grade for a particular type (%)
MIN_M	Minimum tonnage that need to be mined in one period (tonne)
MAX_M	Maximum tonnage that need to be mined in one period (tonne)
MIN_P	Minimum tonnage that need to be processed in one period (tonne)
MAX_P	Maximum tonnage that need to be processed in one period (tonne)

3.3.2 Decision variables

X [i, j, k]   1   if $(i, j, k)^{th}$ block is in stope

                   0   otherwise

3.4 Objective function

Main objective of the mining companies is to maximize the profit from different-different activities. Keeping that in mind, objective function in this project tries to maximize the Net cash flow from single stope. The profit from single block can be calculated by the economic function as follows (Lane, 1988):

$$P_i = d_i * v_i * (g_i * r * s - c)$$

Where, 'i' represents the block, d_i is the density of the block, v_i is the block volume, g_i is the average grade of block, r is the recovery, s is the unit selling price, and c is the unit mining and processing costs. Present value of the money received at the end of each period can be calculated as follows:

$$PV = \frac{M}{(1 + i)^t}$$

Where, M is the amount money received at the end of tth period, 'i' is the interest rate, and t is time period index.

Based on the above concepts, the objective function which is aimed at maximizing cash flow from single stope is as follows:

$$\max f(x) = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{O} \left\{ \left(\left(R * \frac{G[i, j, k]}{100} * OP - PC \right) - MC \right) * X[i, j, k] * BM \right\}$$

Note that the development costs i.e. costs to create the way in to the stopes, are not considered in cash flow calculations as these costs are assumed to be same for all possible stope.

3.5 Constraints

3.5.1 Stope extraction angle constraint

To reach a given block, all the blocks lying below it as shown in the Figure 3.1 referred to as its predecessors, need to be extracted first.
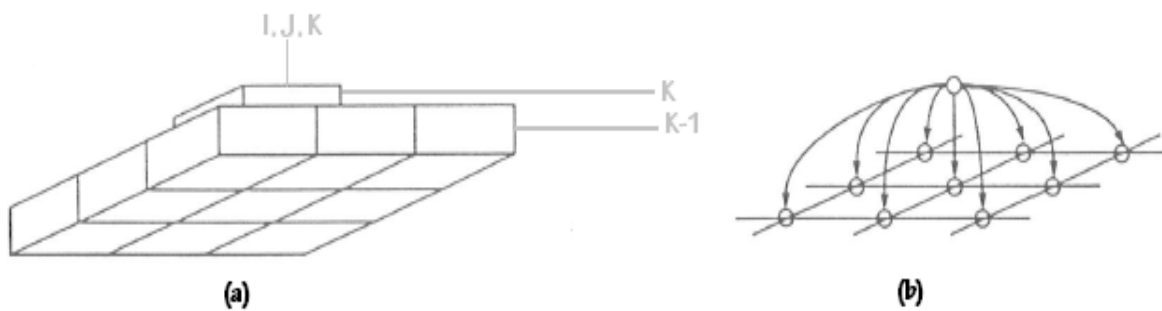
Figure 3.1: Position of predecessor blocks (a) 3-D view (b) node view

This constraint ensures that a block is mined only after its predecessor blocks are mined. Notations of the predecessor block for a particular block (i, j, k) is given in the Figure 3.2.

i+1,j-1,k-1	i+1,j,k-1	i+1,j+1,k-1
i,j-1,k-1	i,j,k-1	i,j+1,k-1
i-1,j-1,k-1	i-1,j,k-1	i-1,j+1,k-1

Figure 3.2: Notation of 9 predecessor blocks with their position

Mathematical form of this constraint is as follows:

$$9 * X[i,j,k] - \sum_{\alpha \in p} \sum_{\beta \in p} X[i+\alpha, j+\beta, k-1] \qquad \forall i,j,k$$

Where, p is the set which contains {-1, 0, 1}.

For blocks which reside in sides, only six blocks will act as a predecessor block. If they reside in left hand side then (j-1) blocks will not be there and if they reside in right hand side then (j+1) blocks will not be there. For corner blocks, only four blocks need to be mined first.

3.5.2 Minimum Stope height constraint

Extraction of each and every block in underground mining based on the technical and economic point of view is conditional upon extracting all the blocks which are adjacent to it. It can be say that a block is mined only if the other blocks adjacent to it have the potential to be extracted.

This constraint ensures that a given stope contains at least minimum number of blocks. The mathematical equation governing this constraint is as follows:

$$\sum_{u=0}^{MN-2} X[i,j,k+u] - X[i,j,k+MN-1] - X \le MN-2$$

$$\forall i,j,k = \{1,2,\dots,O-MN+1\}$$

3.5.3 Maximum Stope height constraint

Dimension of a stope must be restricted through the adjacent blocks in vertical direction from the technical point of view. If not restricted then roof failure and sagging problem might arise and can cause accidents.

This constraint restricts the number of blocks within a given stope, so it does not get past the maximum allowable limit. The mathematical logic for this constraint is as follows:

$$\sum_{v=0}^{MX} X[i,j,k+v] \le MX \qquad \forall i,j,k = \{1,2,\dots,O-MX\}$$

Chapter 4 : Case study

4. CASE STUDY

The case study data are collected from an Indian zinc mine which is located at the location $(24^0 57`, 74^0 08`)$. Strike length of the mine is 4.5 km with width ranging from 2 m to 40 m. Total of 128 boreholes for exploration purpose and 800 m of exploratory mining is done. The orebody is in 2 parts and designated as north lode and south lode. The cut-off grade is 3% (Zn+Pb). Average zinc content is found to be 5.5 % and lead content to be 1.2 % in the north lode and 1.37 % Pb and 5.9 % Zn in the southern part of the deposit. In the ore, zinc is the main base metal which is followed by the lead and copper and small quantities of silver, arsenic, and mercury. The stope design study is carried out on the eastern part of the mine and data lies in the square of area 0.194 km^2.

Data contains total of 4992 blocks. There are total of 12 blocks in X direction, 16 blocks in Y direction, and 26 blocks in Z direction. The dimension of a single block is 2.5 m * 5 m * 8.33 m. Specific gravity of the ore is considered to be 3. Height of the crown pillar is considered to be around 25 m. So, 3 blocks are left as a crown pillar.

Figure 4.1 shows X-Z plane of the data and colour variation is according to the grade value of that specific block whereas Figure 4.2 shows 3-D view of the data.

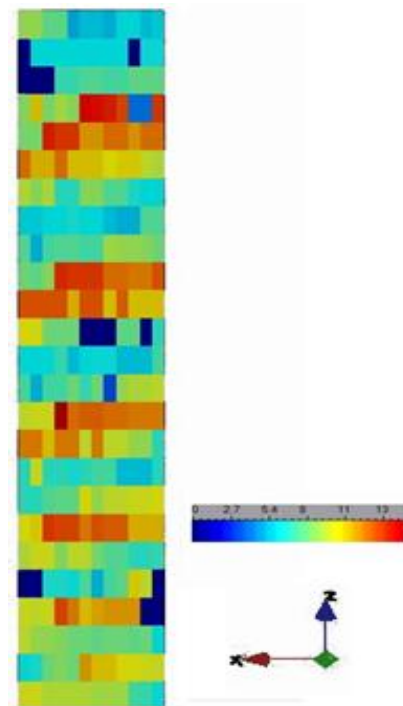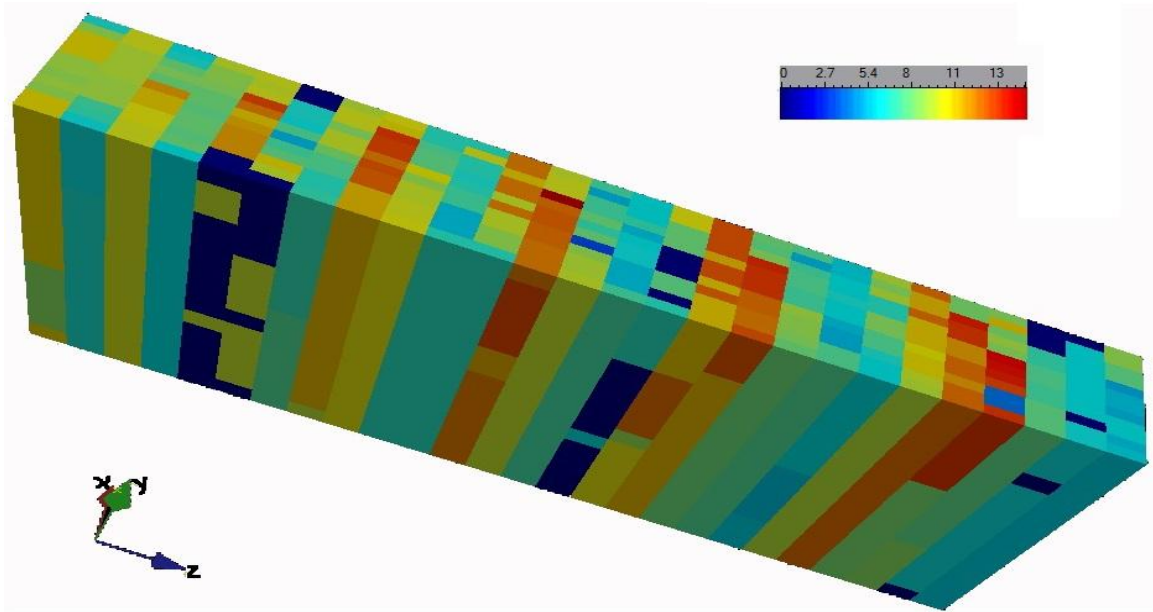**Figure 4.1: X-Z view of the data**

Figure 4.2: 3-D view of the data

Figure 4.3 shows the histogram of the grade value of the data. It contains number of blocks for different-different range of grade value. Given data is normally distributed based on the visual inspection. It shows that data contains more number of blocks for the grade ranging from 5-9 %.
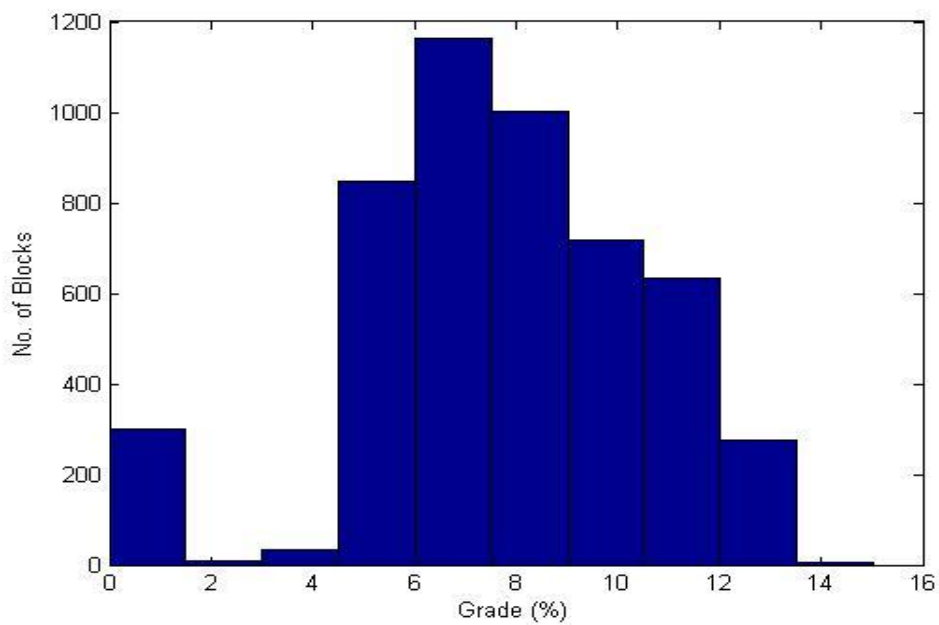

Figure 4.3: Histogram of the grade

Table 4.1: Tonnage for dfferent-different cut-off grrades

Cut -off grade (%)	Total tonnage (MTs)
0	1.56
1.5	1.469
3	1.466
4.5	1.457
6	1.199
7.5	0.8
9	0.728
10.5	0.433
12	0.183
13.5	0.097

Table 4.1 shows tonnage value with respect to the cut-off grade value. This data with grade-tonnage curve is helpful in determining the feasible cut-off grade and respective tonnage that is available for extraction and processing. Figure 4.4 shows the grade-tonnage curve of the data of part of the zinc mine. This graph shows the quantity of material that is available if that grade is selected as a cut off grade. This helps in economical analysis of the data. For a cut-off grade 3 % that is used in planning at the mine, tonnage available for processing is 1.466 Million tonnes out of 1.56 million tonnes.
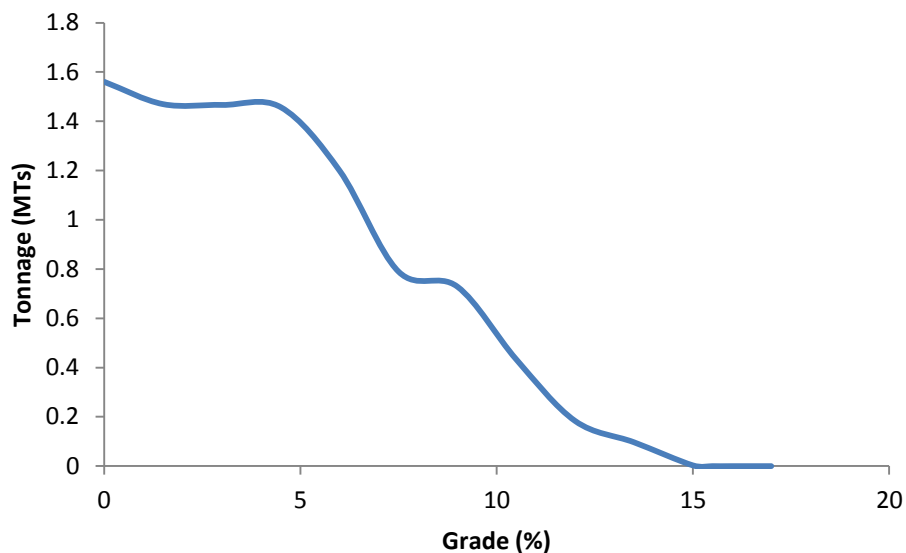

Figure 4.4: Grade-Tonnage curve

Chapter 5 : Results and Discussions

STOPE DESIGN FORMULATION

PRODUCTION SCHEDULING FORMULATION

5. RESULTS AND DISCUSSIONS

There are total of 4992 blocks in the part of the zinc mine of india of size 2.5 m * 5 m * 8.33 m. Blocks are arranged in the manner like 12 blocks in X direction, 16 blocks in Y direction, and 26 blocks in Z direction. Tonnage of the single block is calculated to be 312.5 T as specific gravity is considered to be 3. Selling price of the Zinc metal is considered as given by the HZL circular on 10-2-2014 regarding price of the commodities (www.hzlconnect.com). The mining cost and the processing cost for the underground mining is calculated based on the information given by the O'Hara et al. (1992). As per the information obtained from the zinc mine of India, the crown pillar thickness is assumed to be 25 m i.e. equal to the combined height of the 3 blocks.

Table 5.1: Constant parameters and their values

Parameters	Values
Mining cost	2100 Rs/ton of ore
Processing cost	1000 Rs/ton of ore
Recovery	0.9
Selling price	155000 Rs/ton of metal
Specific gravity	3
Block Mass	312.5 tonne

Value of some of the constant parameters is given in the Table 5.1 and some of the values taken is as follows: minimum and maximum height of the single stope to be 33.3 m and 58.3 m respectively, minimum and maximum mining values to be 0.24 MT and 0.42 MTs respectively, minimum and maximum metal production values to be 35 thousand tonne and 20 thousand tonne respectively.

5.1 Stope design formulation

Problem is written in the ZIMPL (Koch, 2004) which is then solved with the CPLEX (IBM, 2012) commercial solver. This algorithm is valid for optimization of single stope only. So, algorithm is solved first for first stope starting from the base of the data set and proceeding upwards. Formulation is started from below because the location of the crusher and skip is near this starting point. After obtaining the result, the first stope data is then eliminated from the problem set. After leaving the crown pillar as required, the algorithm is solved again for

second stope from the remaining data set. This process is repeated till the block count reaches maximum. The result obtained is that data contains 3 stopes.

Cash flow, tonnage, dilution, and metal content of each stope are calculated using the solution obtained. Dilution refers to the waste material that is mined with the ore and not separated from the ore material during the operation. It is mixed with the ore and sent to the material. Dilution increases the tonnage to be mined while decreasing the grade of the material that is to be processed. It can be calculated as follows:

$$Dilution = \frac{waste\ material}{ore\ material + waste\ material} * 100$$

Table 5.2: Values of the different parameters from the optimal solution

Stope Number	Tonnage of stopes (MTs)	Cash flow from stopes (Crore Rs.)	Dilution (%)	Metal content of stope (thousand Ts)
1	0.308	256.7	1.724	25.22
2	0.392	307.1	1.836	30.66
3	0.403	305.1	7.364	30.61

Table 5.2 shows the value of the different parameters that is determined from the optimum solution data. This shows that third stope contains more tonnage and first stope contains less tonnage, it is because of the fact that number of blocks whose economic value is less is aggregated at the top of the first stope. From the data, it is seen that metal content of the third stope is less as compared to second stope despite of more tonnage. It is because of the dilution in the third stope is more as compared.

Table 5.3: Stope parameters

Stopes	No. of blocks in a stope	Minimum height (m)	Maximum height (m)	No. of waste blocks
1	986	33.333	50	17
2	1253	50	58.333	23
3	1290	41.667	58.333	95

Table 5.3 shows different parameters of stopes like minimum and maximum height, number of ore blocks and waste blocks. This data is useful in production planning and profit

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calculation. From the data it is seen that the solution is following maximum height value of 33.333 m and minimum height value of 58.333 m. So, it can be said that the solution is following minimum and maximum height constraints.
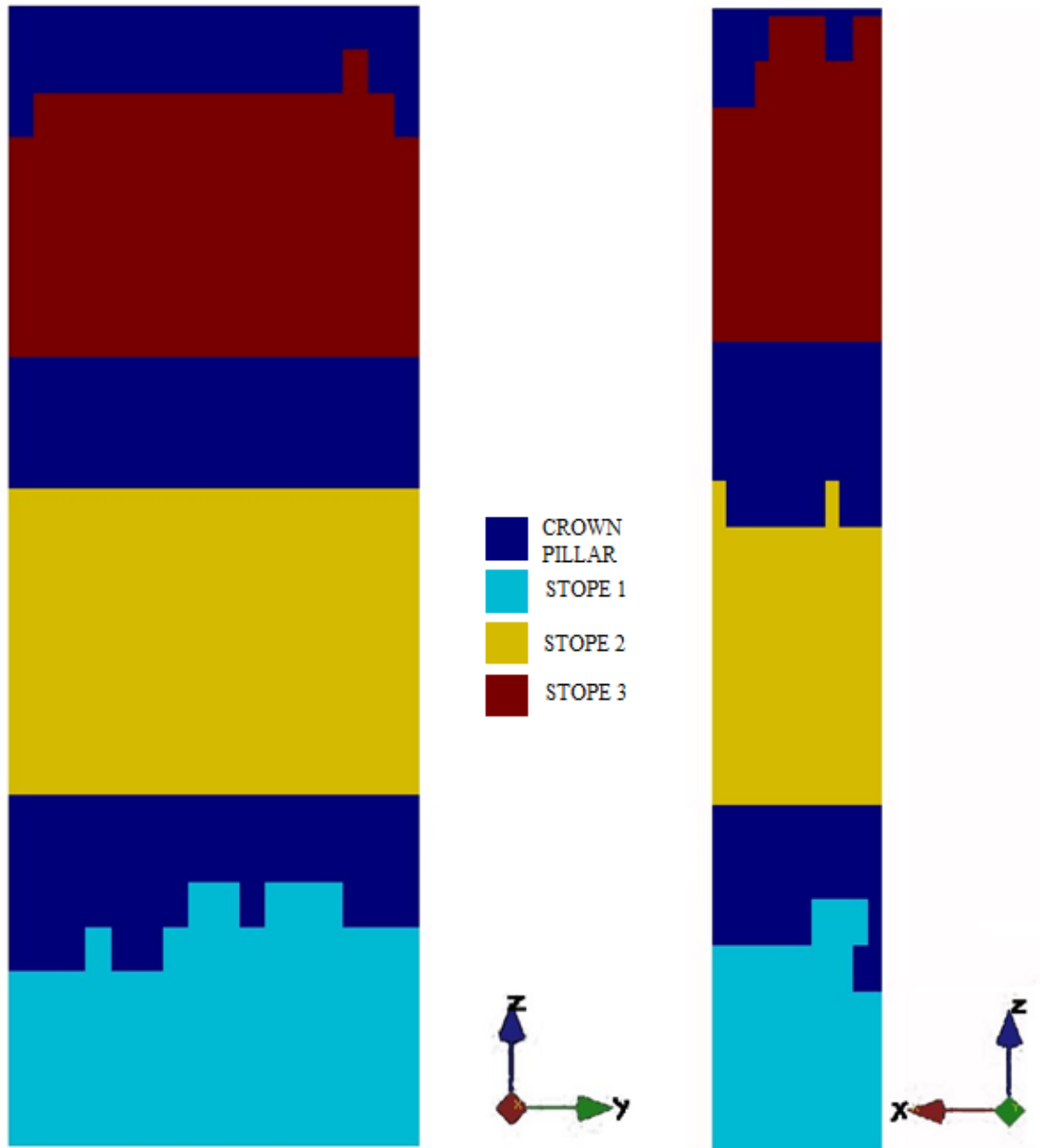

Figure 5.1: Y-Z and X-Z view of the final solution

Final solution gives the block identification against stope number which is then plotted in SGeMS (Remy et al. 2008). Final solution is the one which gives maximum profit though satisfies all the constraints. The generated optimum solution satisfies all the constraints. It also gives the information regarding which block to be extracted first.
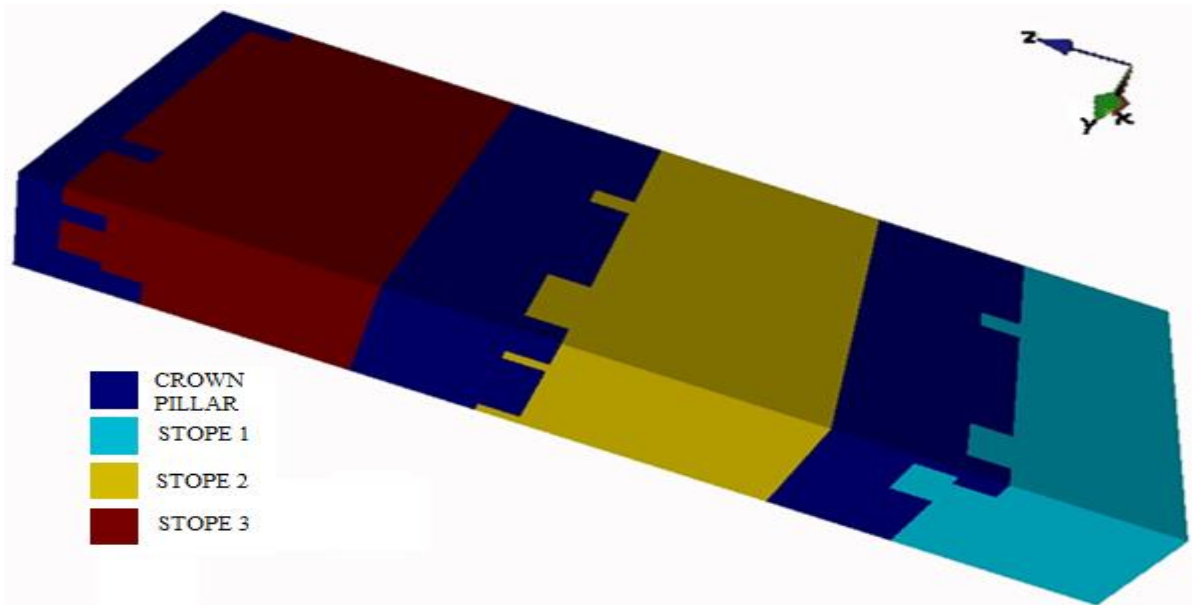
Figure 5.2: 3-D view of the final solution

Figure 5.1 and Figure 5.2 shows the 3-D view and section of the data that are in 3 stopes. The lighter blue colour represents blocks that are in first stope. The yellow colour blocks are in second stope, and the dark red colour blocks are in third stope. The deep blue colour represents the blocks which are left because either the block economic value is less than zero or they fall in the crown pillar rows which are left for support. From the Figure 5.2, it is seen that the solution is following all the constraints like minimum height, maximum height, and stope extraction angle constraint. Hence, the generated solution is a feasible solution.

5.2 Production scheduling formulation

Solution from the stope design formulation is then used in the production scheduling problem. Production scheduling problem is applied using heuristic approaches in which first stope is extracted first and some of the blocks from second stope is extracted to satisfy the mining constraints and processing constraints.

5.2.1 Mining Constraint

The total tonnage that is going to be mined during each period should reach at least a minimum value to avoid an unbalanced flow during the periods. If mined below minimum then machines are going to be idle which is not favourable from the economic point of view.

This constraint ensures that minimum amount is extracted in each period and the mathematical logic for the same is as follows:

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{O} BM * X[i,j,k,t] \geq MIN_M \qquad \forall t$$

On the other hand, the tonnage extracted should not be greater than the mining equipment capacity available in that period. This constraint restricts the tonnage that is to be extracted in the given period. The mathematical equation governing this constraint is as follows:

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{O} BM * X[i,j,k,t] \leq MAX_M \qquad \forall t$$

5.2.2 Processing constraint

The blocks which contain grade value greater than cut off grade value i.e. ore blocks are only sent to the processing plants and other blocks are stacked there as an overburden or waste material or sub-grade material.

So, the material reaching the processing plant from the mined out material should be at least equal to the minimum amount required for continuous run of the processing plant during each period. The mathematical logic which governs this constraint is as follows:

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{O} BM * Y[i,j,k,t] \geq MIN_P \qquad \forall t$$

All the machines have the upper limit beyond which they cannot run. So, the material reaching the processing plant should not exceed the capacity of the processing plant. The mathematical equation which restricts the amount of material required to feed the processing plant is as follows:

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{O} BM * Y[i,j,k,t] \leq MAX_P \qquad \forall t$$

Table 5.4: Values of the different production scheduling parameters

Periods	Tonnage (MTs)	Discounted Cash Flow (Crore Rs.)	Dilution (%)	Metal produced (thousand Ts)
1	0.419	312.6	1.714	33.93
2	0.419	268.4	3.505	32.48
3	0.264	150.3	7.447	20.07

Parameters value that is generated from the production scheduling problem is given in the Table 5.4. It is good thing that the dilution is less in initial periods as compared to other periods as it will not decrease the quantity of metal produced from the mined quantities and the profit is also maximum. As per the minimum, maximum mining constraints, and sequencing constraints, first stope and some part of the second stope is going to be extracted in first period. In second period, the left part of second stope and some part of third stope is scheduled to be extracted. In third period, remaining blocks of the third stope is to be mined as shown in Figure 5.3.

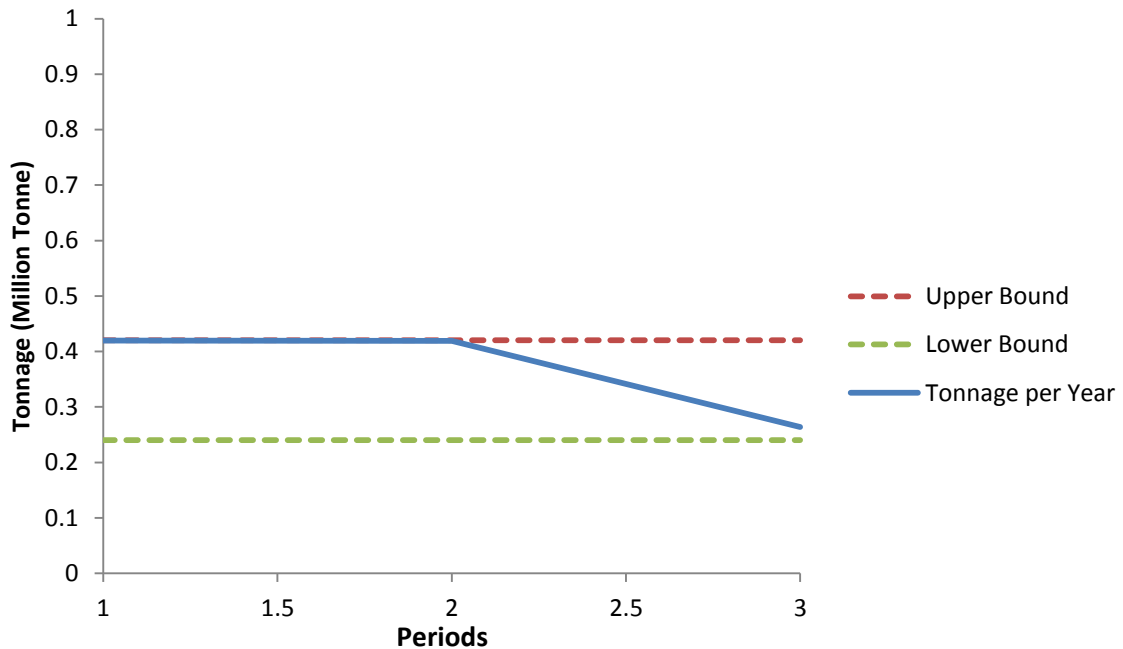**Figure 5.3: X-Z section of the data showing production scheduling**

Figure 5.4: Tonnage extracted for different periods

Figure 5.4 shows tonnage with respect to the periods. It shows that the tonnage extracted is between minimum mining value of 0.24 MT and maximum mining value of 0.42 MT. In third period, the tonnage extracted is very low and it is because of the fact that only that much quantity is left.
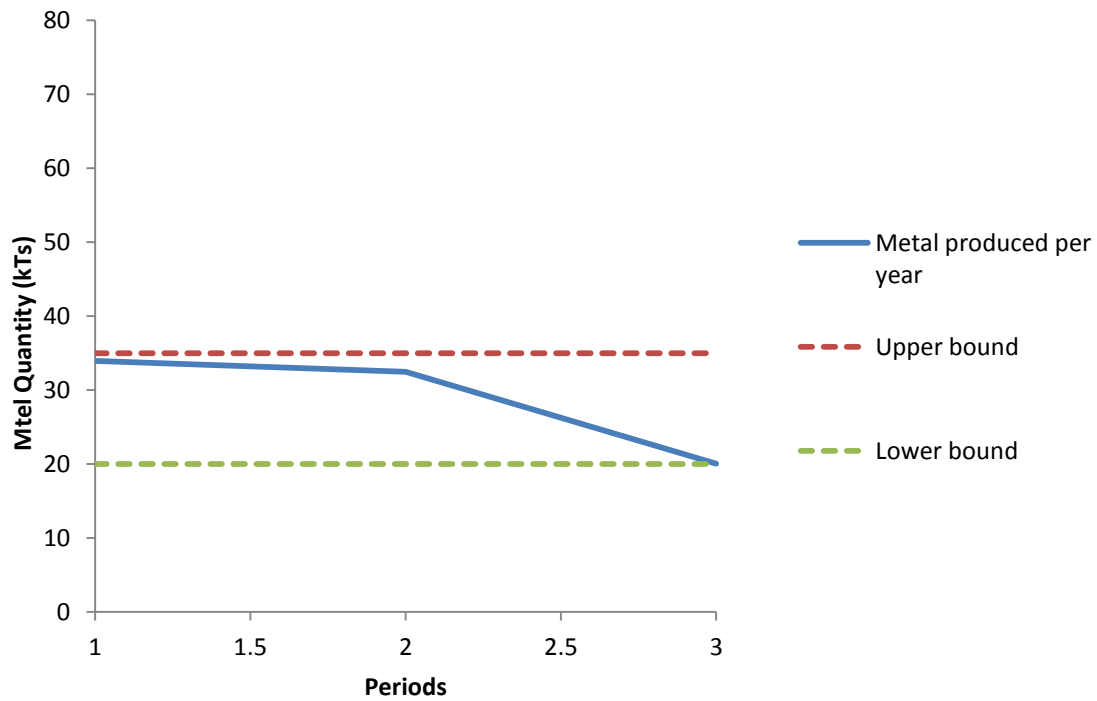

Figure 5.5: Graph showing metal production in each period

Figure 5.5 shows the metal production with respect to the periods. It shows that maximum quantity of metal is produced in initial period and decreases in subsequent periods. Because of which maximum cash flow occurs in initial period and fulfils the requirement of the initial investment.
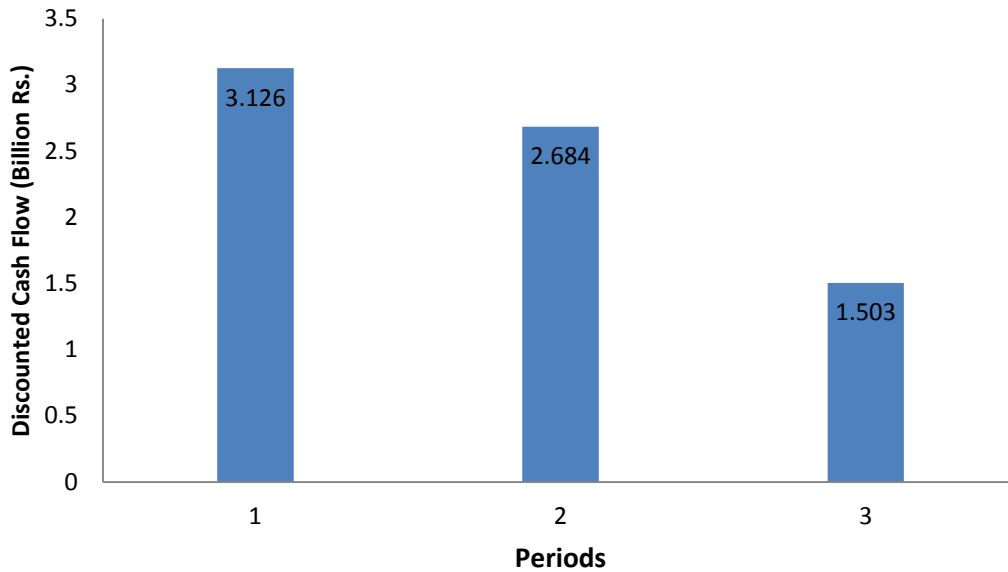
Figure 5.6: Bar graph showing DCF in each period

Discounted cash flow (DCF) is the present value of the cash flow in a particular time period. Cash flow refers to the net outflow and inflow of money which occurs during a specific time period (Hustrulid, 1998). Figure 5.6 shows the bar graph of the discounted cash flow of each particular period. It shows maximum DCF in first period because money is required at the start of the mine to keep it running for subsequent periods.
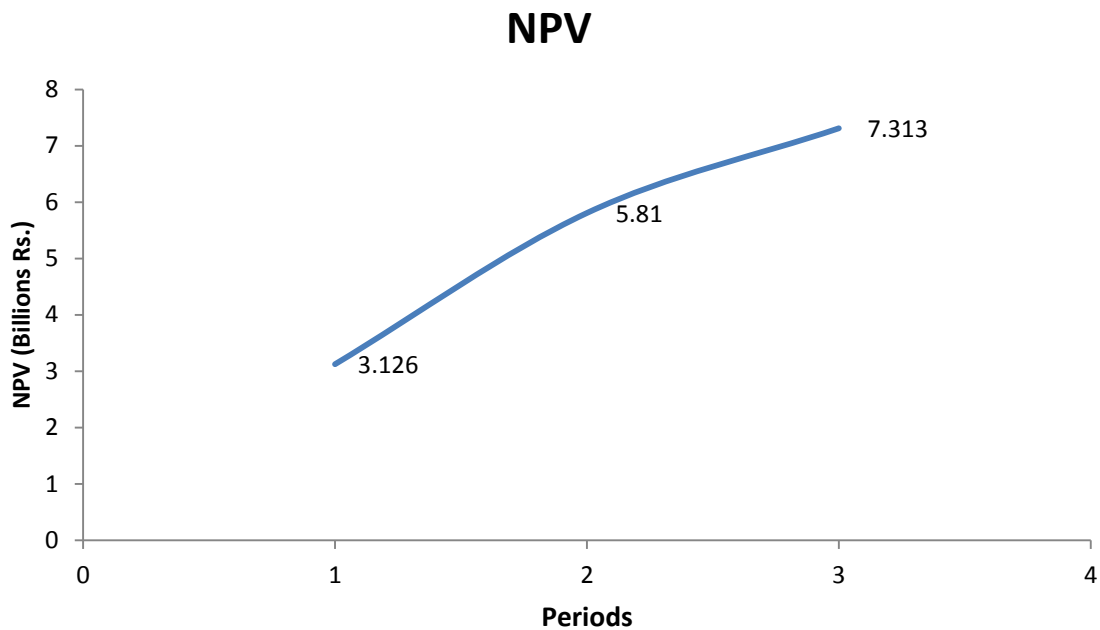

Figure 5.7: Graph showing cumulative NPV over the periods

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Net present value (NPV) is sum of the discounted cash flow in each period which is net of incoming and outgoing money. Figure 5.7 shows the NPV which is always rising upwards, if it starts declining then the mining is discontinued from that point onwards. In general, NPV starts from negative value and then touches maximum value but in this project, it starts positive. It is because development rate and development cost is not taken into account in calculation of the discounted cash flow. The NPV for 3 periods is calculated to be 731.3 Crore Rupees.

Figure 5.8 shows the cummulative value of the dilution over the periods. To follow all the constraints like slope constraint, height constraint, it becomes necessary to mine some of the low grade ore or waste material with ore material and that is why dilution occurs. It can be decreased by adopting selective mining. It decreases the grade of the ore that is to be processed while increasing the quantity to be mined.
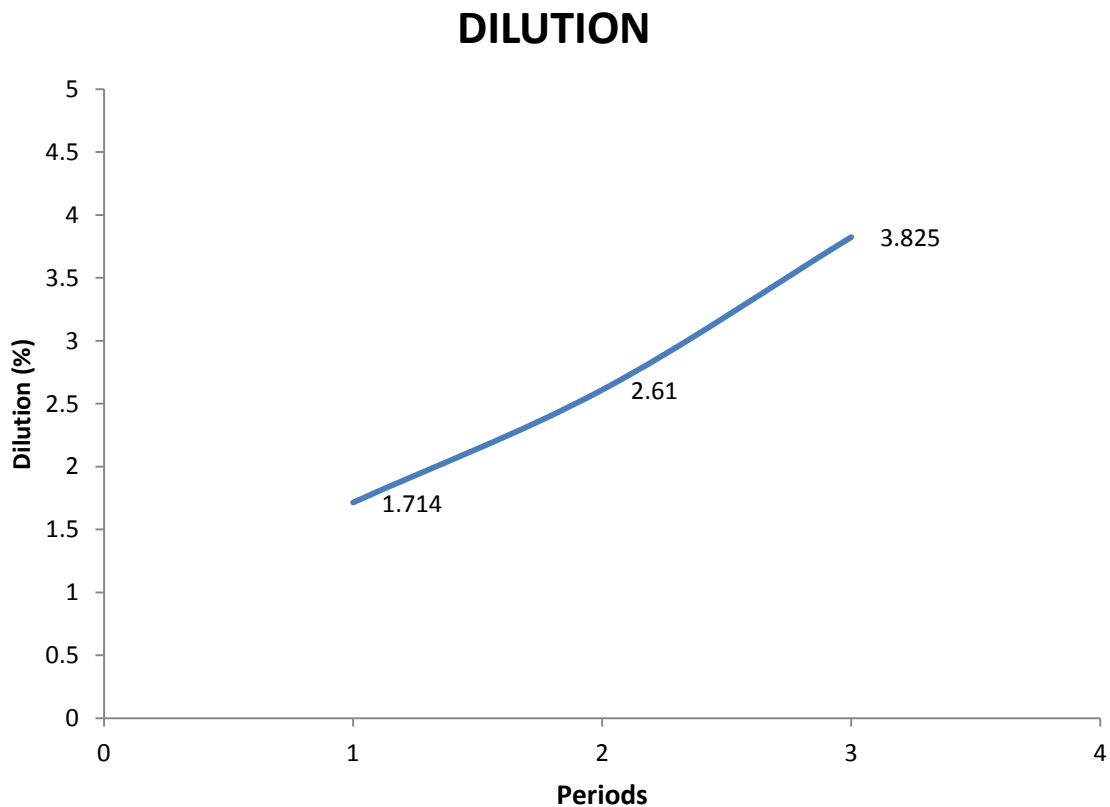
DILUTION

Figure 5.8: Graph showing the cumulative dilution over the periods

Chapter 6 : Conclusions

6. CONCLUSIONS

This project work has presented an algorithm which is in the form of integer programming for optimization of stope design in underground mine for open stoping method. For optimization problem, objective function of maximizing cash flow from the single stope of the mine is accompanied by the constraints like stope extraction angle and stope height. Solution of the stope design is then used in the production scheduling problem which is solved heuristically. Production starts from the lower block of the first stope and proceeds upwards and if mining constraint is not satisfied then some of the blocks of second stope from lower side is also mined in first period. This process continues till all the stopes are mined.

Algorithm is then solved using part of the zinc mine of India. Data has 4992 blocks. Problem is formulated using ZIMPL which is then solved using CPLEX commercial solver. Algorithm is applicable for design of 1 stope only, so eliminating data from the problem set each time after the design to run the algorithm again for next stope design. Then obtained solution is used for production scheduling which is solved heuristically.

The results obtained after solving the algorithm, are used to calculate the net cash flow and the dilution from each stope. Total of 3 stopes are designed. Total cash flow from all designed stopes is found to be 868.8 Crore rupees. Production scheduling result gives NPV value to be 731.3 Crore rupees and dilution to be 3.83 % respectively for 3 periods. It is satisfactory in nature. From the above result it is seen that, this algorithm is applicable in stope design.

In this project, the cost of development is not included in the objective function. If that can be included then the NPV generated will be close to the real time solution. Further, optimizing short-term planning of the present data will reduce the deviations from the pre-defined capacities.

Chapter 7 : References

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