DYNAMIC ANALYSIS OF CANTILEVER BEAM AND ITS EXPERIMENTAL VALIDATION

A Thesis submitted in partial fulfillment of the requirements for the Degree of

Bachelor of Technology
In
Mechanical Engineering

By

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CERTIFICATE

This is to certify that the thesis entitled, **"Dynamic analysis of cantilever beam and its experimental validation"** submitted by **SUBHRANSU MOHAN SATPATHY** and **PRAVEEN DASH** in partial fulfillment of the requirement for the award of Bachelor of Technology degree in Mechanical Engineering at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

Date: 12 May, 2014

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INDEX

FIGURES

1. INTRODUCTION

Beam is a inclined or horizontal structural member casing a distance among one or additional supports, and carrying vertical loads across (transverse to) its longitudinal axis, as a purlin,girder or rafter. Three basic types of beams are:

(1) Simple span, supported at both ends

(2) Continuous, supported at more than two points

(3) Cantilever, supported at one end with the other end overhanging and free.

There exist two kinds of beams namely Euler-Bernoulli's beam and Timoshenko beam. By the theory of Euler-Bernoulli's beam it is assumed that

- Cross-sectional plane perpendicular to the axis of the beam remain plane after deformation.
- The deformed cross-sectional plane is still perpendicular to the axis after deformation.
- The theory of beam neglects the transverse shearing deformation and the transverse shear is determined by the equation of equilibrium.

In Euler – Bernoulli beam theory, shear deformations and rotation effects are neglected, and plane sections remain plane and normal to the longitudinal axis. In the Timoshenko beam theory, plane sections still remain plane but are no longer normal to the longitudinal axis.

1.2 Objective and Scope of work

In this paper, we will be formulating the equations of motion of a free cantilever beam. The natural frequency of continuous beam system will be found out at different variables of beam using ANSYS 14.0. The results will be compared further using experimentation by free vibration of a cantilever beam. Using those results, we will be able to compare the parameters in Euler-Bernoulli and Timoshenko beam.

2. LITERATURE SURVEY

An exact invention of the beam problem was first studied in terms of general elasticity equations by Pochhammer (1876) and Chree (1889). They deduced the equations that describe a vibration of a solid cylinder. However, it is impractical to solve the full problem because it results in more information than actually needed in applications. Therefore, approximate solutions for transverse displacement are adequate. The beam theories under consideration all generate the transverse displacement equations as a solution.

It was documented by the early investigators that the bending effect is the single most important factor in a transversely vibrating beam. The Euler Bernoulli model takes into account the strain energy due to the bending effect and the kinetic energy due to the lateral displacement. The Euler Bernoulli model goes back to the 18th century. Jacob Bernoulli (1654-1705) first revealed that the curvature of an elastic beam at any point is relational to the bending moment at that point. Daniel Bernoulli (1700-1782), nephew of Jacob, was the first one who framed the differential equations of motion of a vibrating beam. Later, Jacob Bernoulli's theory was acknowledged by Leonhard Euler (1707-1783) in his investigation of the shape of elastic beams subjected to various loading conditions. Many advancements on the elastic curves were deduced by Euler. The Euler-Bernoulli beam theory, sometimes called the classical beam theory, Euler beam theory, Bernoulli beam theory, or Bernoulli and Euler beam theory, is the most commonly used because it is simple and provides realistic engineering approximations for many problems. However, the Euler Bernoulli model slightly overestimates the natural frequencies. This problem is aggravated for the natural frequencies of the higher modes. Also, the prediction is more focused for slender beams than non-slender beams.

Timoshenko (1921, 1922) suggested a beam theory which adds the effect of Shear as well as the effect of rotation to the Euler-Bernoulli beam. The Timoshenko model is a major enhancement for non-slender beams and for high-frequency responses where shearing and rotary effects are considered. Following Timoshenko, several authors have deduced the frequency equations and the mode shapes for various boundary conditions.

The finite element method devised from the need of solving complex elasticity and structural analysis equations in civil and aeronautical engineering. Its improvement could be traced back to the work by Alexander Hrennikoff (1941) and Richard Courant (1942). While the approach used by these pioneers is different, they all stick to one essential characteristic: mesh discretization of a continuous domain into a set of discrete subdomains, usually called elements. Starting in 1947, Olgierd Zienkiewicz from Imperial College collected those methods together into what is called the Finite Element Method, building the revolutionary mathematical formalism of the method.

Hrennikofs work discretizes the domain by using a lattice analogy, while Courant's approach divides the domain into finite triangular sub regions to solve second order elliptic partial differential equations (PDEs) that arise from the problem of torsion of a cylinder. Courant's effort was revolutionary, drawing on a large body of earlier results for PDEs developed by Rayleigh, Ritz, and Galerkin.

Development of the finite element method began in the middle to late 1950s for air frame and structural analysis and gained momentum at the University of Stuttgartthrough the work of John Argyris and at Berkeley through the work of Ray W. Clough in the 1960's useful in civil engineering. By late 1950s, the key concepts of stiffness matrix and element assembly existed essentially in the form used today. NASA sent a request for proposals for the development of the finite element software NASTRAN in 1965. The method was again provided with a arduous mathematical foundation in 1973 with the publication of Strang and Fix *An* Analysis *of The Finite Element Method*, and has since been generalized into a branch of applied mathematics for numerical modeling of physical systems in a wide variety of engineering disciplines, e.g., electromagnetism and fluid dynamics.

ANSYS, Inc. is an engineering simulation software (computer-aided engineering, or CAE) developer that is headquartered south of Pittsburgh in the Southpointe business park in Cecil Township, Pennsylvania, United States.

ANSYS offers engineering simulation solution sets in engineering simulation that a design process requires. Companies in a wide variety of industries use ANSYS software. The tools put a virtual product through a rigorous testing procedure such as crashing a car into a brick wall, or running for several years on a tarmac road before it becomes a physical object.

3. Numerical formulation

1. **Formulation:**

EULER BERNOULLI BEAM:

For stiffness matrix:

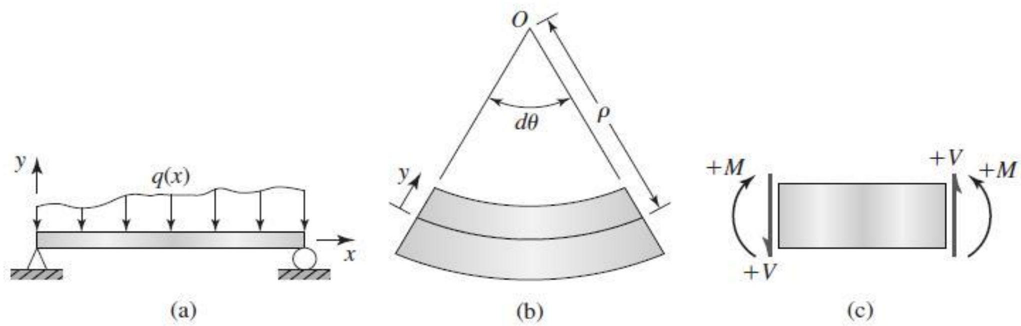


Fig 1: (a) Simply supported beam subjected to arbitrary (negative) distributed load.(b) Deflected beam element. (c) Sign convention for shear force and bending moment.

The bending strain is:

$$\in = (ds - dx)/dx = -\frac{y}{\rho}$$

The radius of curvature of a curve is given by:

$$\rho = \frac{\left[1+\left(\frac{dv}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2v}{dx^2}}$$

The higher order term can be neglected.

Timoshenko beam:

The shearing effect in Timoshenko beam element:

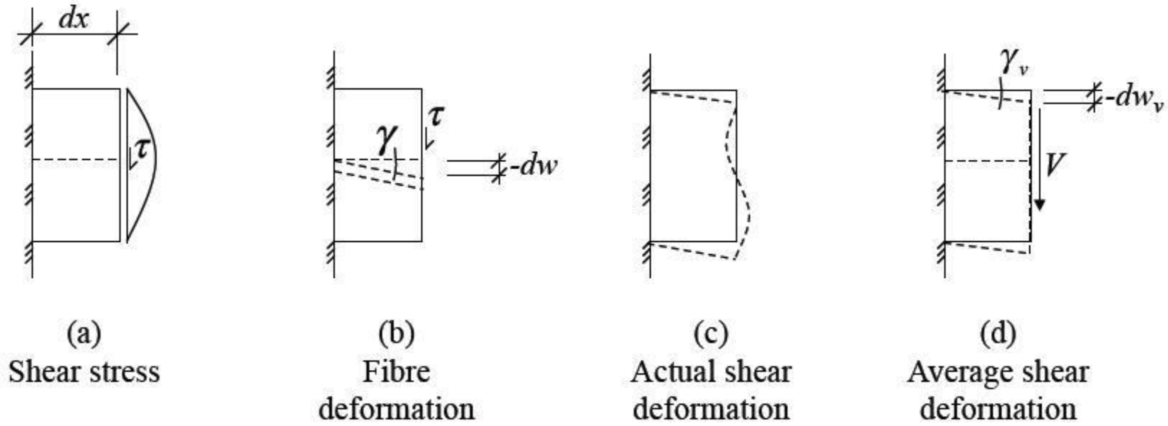
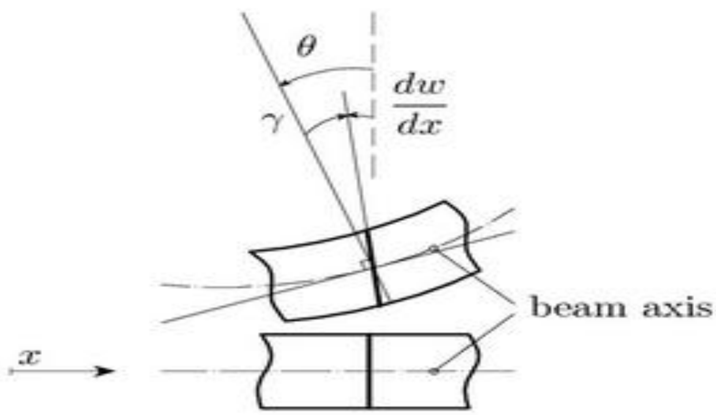


Fig 2: Shearing effect in Timoshenko beam

Considering an infinitesimal element of beam of length δx and flexural rigidity *EI,* we have



4. Modal analysis using ANSYS 14.0

Problem Specification

Considering an aluminum cantilever beam with given dimensions we have,

Length	4 m
Width	0.346 m
Height	0.346 m

The aluminum used is given by the following properties.

Density	2,700 kg/m^3
Young's Modulus	70x10^9 Pa
Poisson Ratio	0.35

Pre-Analysis

The following given equations have the frequencies of the modes and their shapes and have been deduced from Euler-Bernoulli Beam Theory.

$$\omega_n = \alpha_n^2 \sqrt{\frac{EI}{ml^3}}$$

$$n = 1, 2, 3......$$

$$\alpha_n = 1.875, 4.694, 7.855.....$$

$$m = \rho V = \rho.l.h.w$$

$$I = \frac{wh^3}{12}$$

$$\omega_1 = 17.8$$

$$\omega_2 = 111.5$$

$$\omega_3 = 312.1$$

$$\frac{d^2}{dx^2}\{EI(x)\frac{d^2Y(x)}{dx^2}\} = \omega^2 m(x)Y(x)$$

Conclusion from modal analysis

The results found are presented in the subsequent jpg files.

Verification and Validation

For our verification, we will focus on the first 3 modes. ANSYS uses a different type of beam element to compute the modes and frequencies, which provides more accurate results for short and stubby beams. However, for these beams, the Euler-Bernoulli beam is invalid for higher order modes.

Comparison with Euler-Bernoulli Theory

From the Pre-Analysis, we obtained frequencies of 17.8, 111.5 and 312.1 Hz for the first three bending modes. The ANSYS frequencies obtained for the first three bending modes are 17.7, 107.0 and 285.2 Hz. In the ANSYS results, the third mode cannot be considered as bending mode. This fourth mode given by ANSYS is the third bending mode. The results show percentage variances of 0.6%, 4.2% and 8.7% between ANSYS results and the theory. Thus the ANSYS outcomes equal quite fit with Euler-Bernoulli beam theory. The ANSYS beam element formulation utilized here is built on Timoshenko beam theory that comprises shear-deformation effects (which was ignored in the Euler-Bernoulli beam theory).

Comparison

Next, we check our results with a refined mesh. We have run the simulation for 25 elements a replacement for 10. Succeeding the steps drawn in the section of refining mesh of the verification of cantilever beam, we refined the mesh.

We have meshed the beam with 25 elements yielding the subsequent modal frequencies:

Tabular Data

	Mode	Frequency [Hz]
1	1.	17.68
2	2.	107.03
3	3.	179.16
4	4.	285.07
5	5.	318.23
6	6.	524.21

The modal frequencies are close to ones computed with the mesh of 10 elements, giving that our explanation is mesh converged

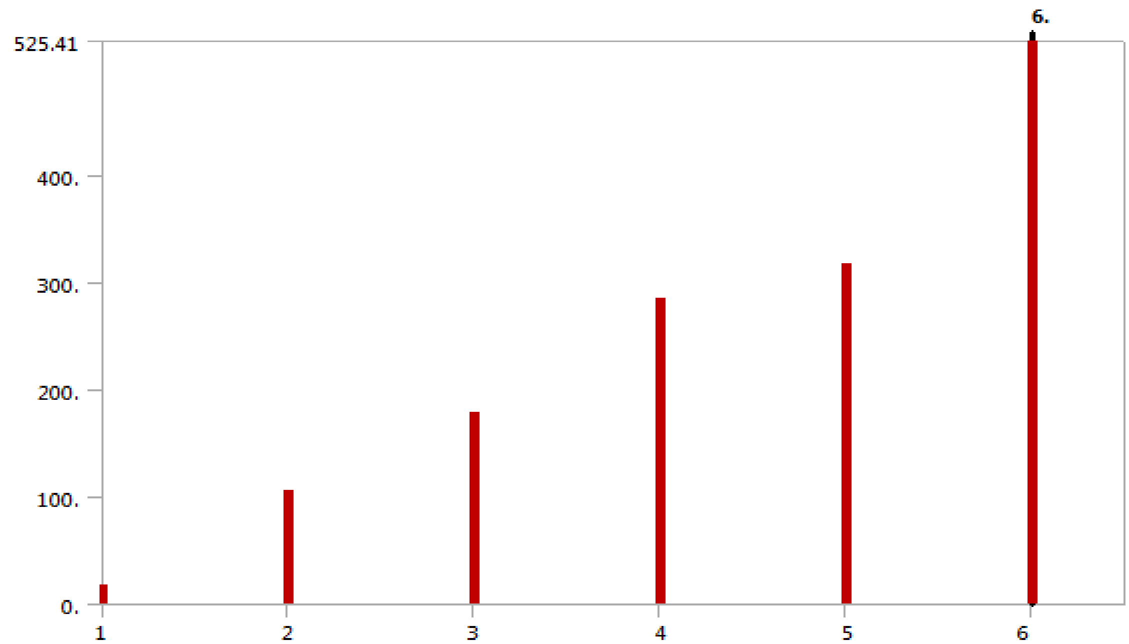


Fig 3: Graphical representation of the modal frequencies

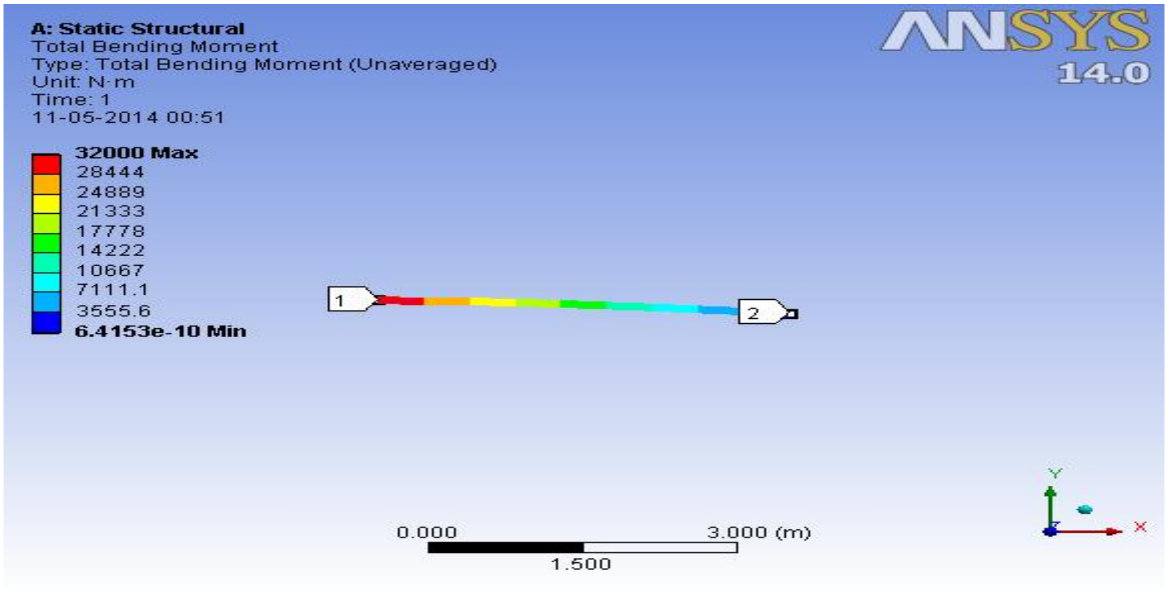


Fig 4: Total bending moment

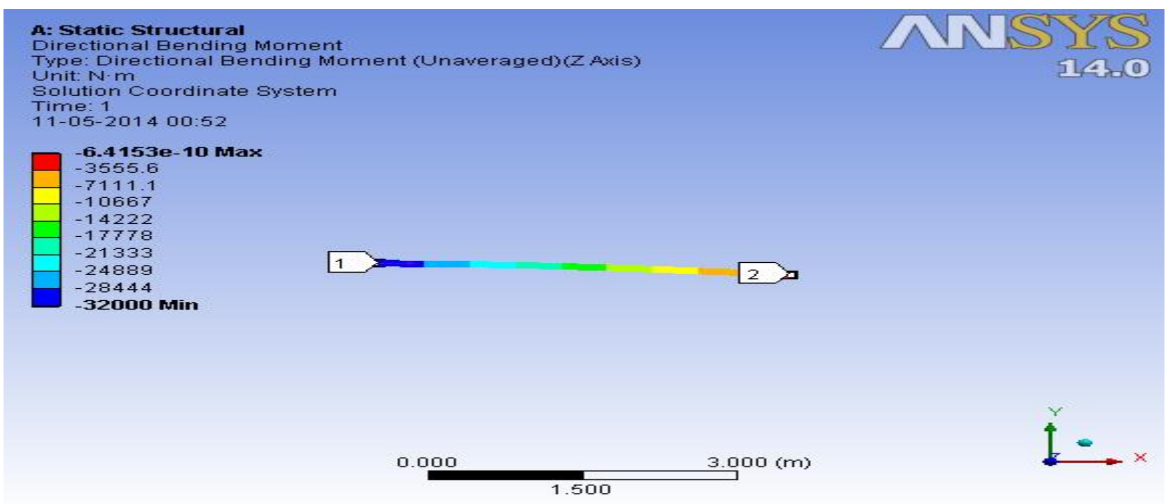


Fig .5 Directional bending moment

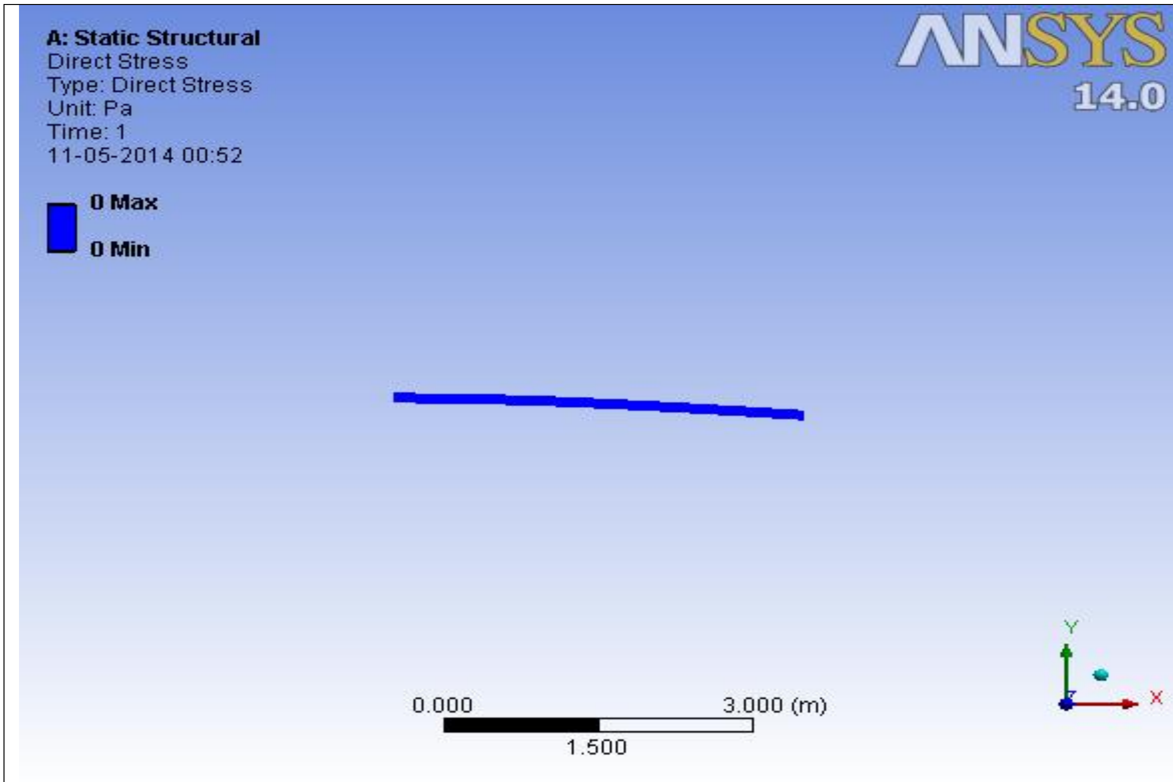


Fig 6: direct stress

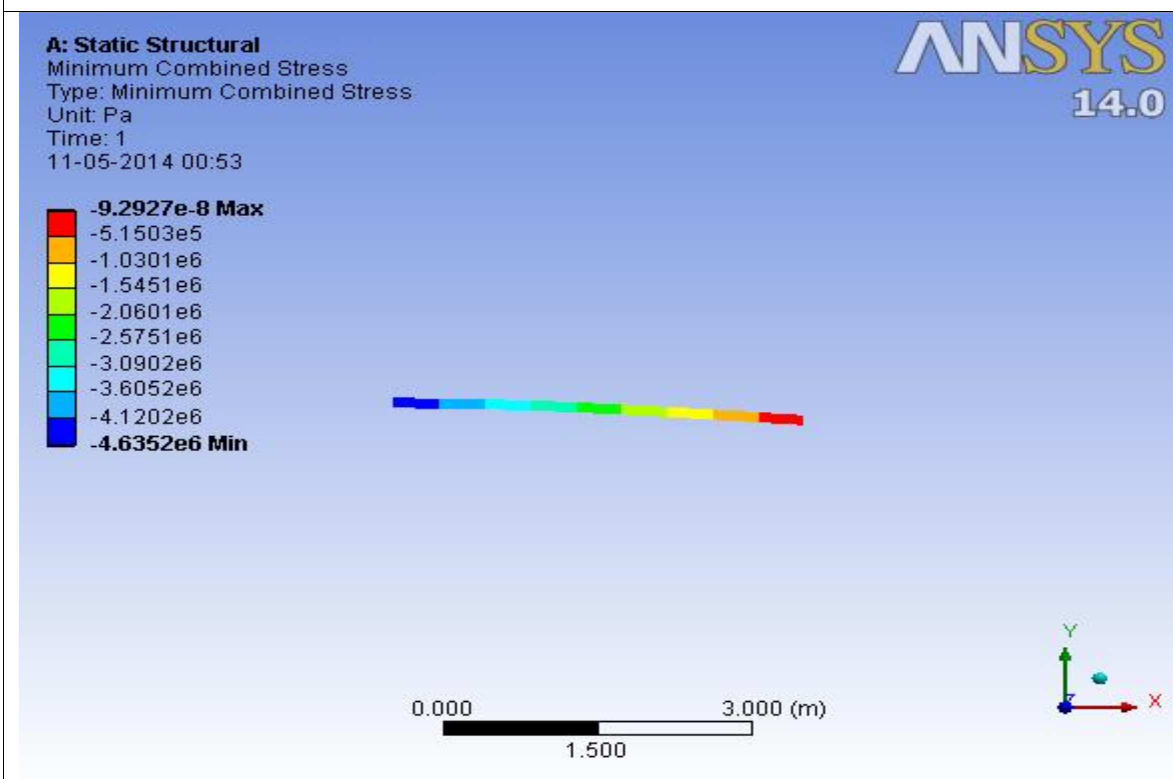


Fig 7: minimum combined stress

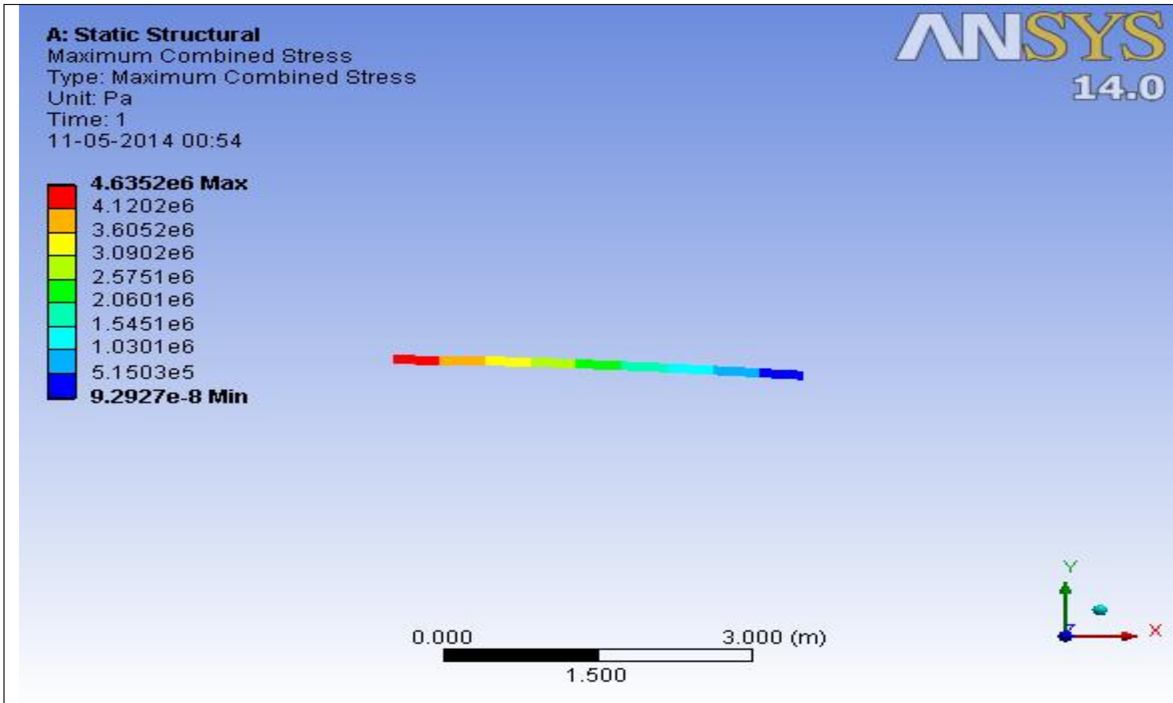


Fig 8: maximum combined stress

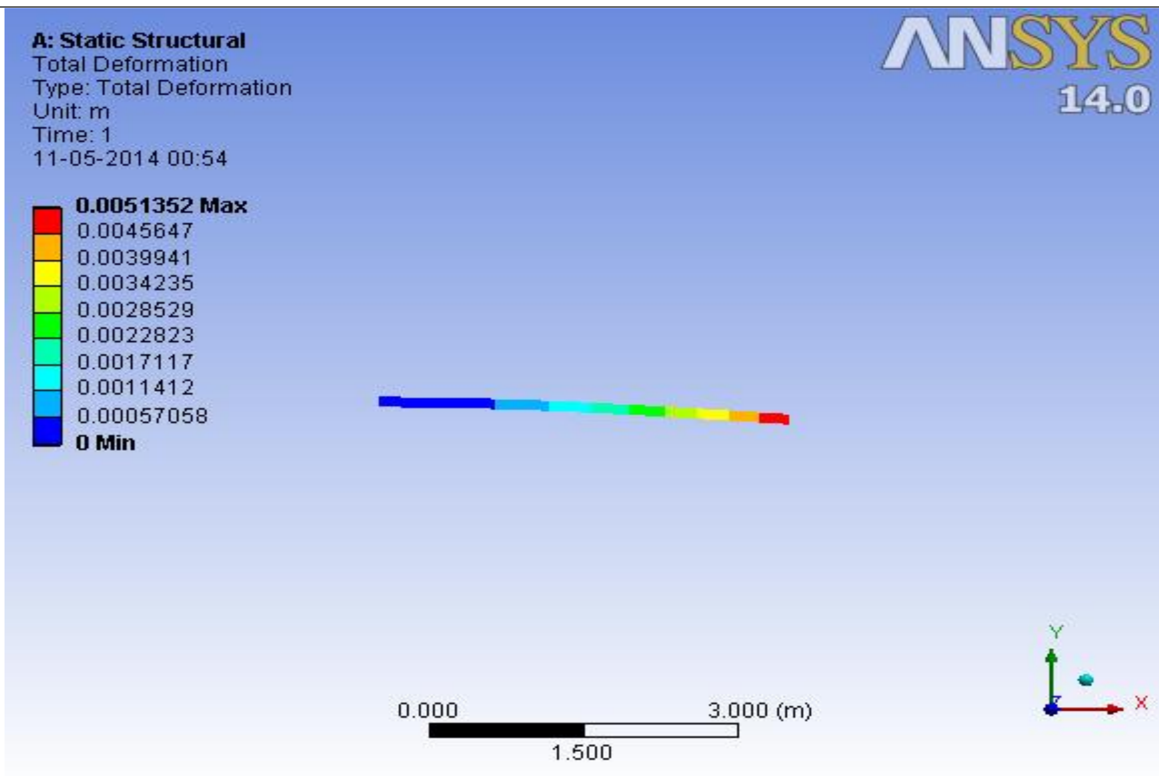


Fig 9: total deformation

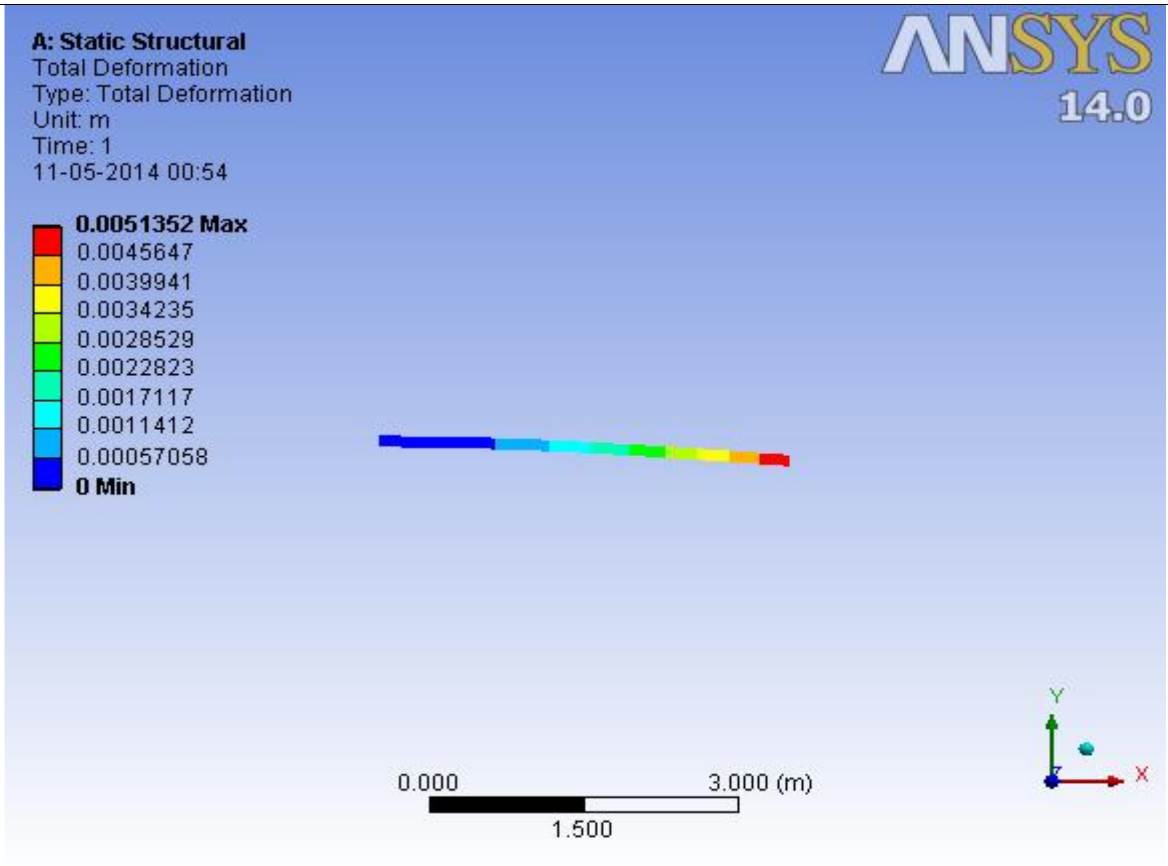


Fig no.10: Maximum Bending Stress

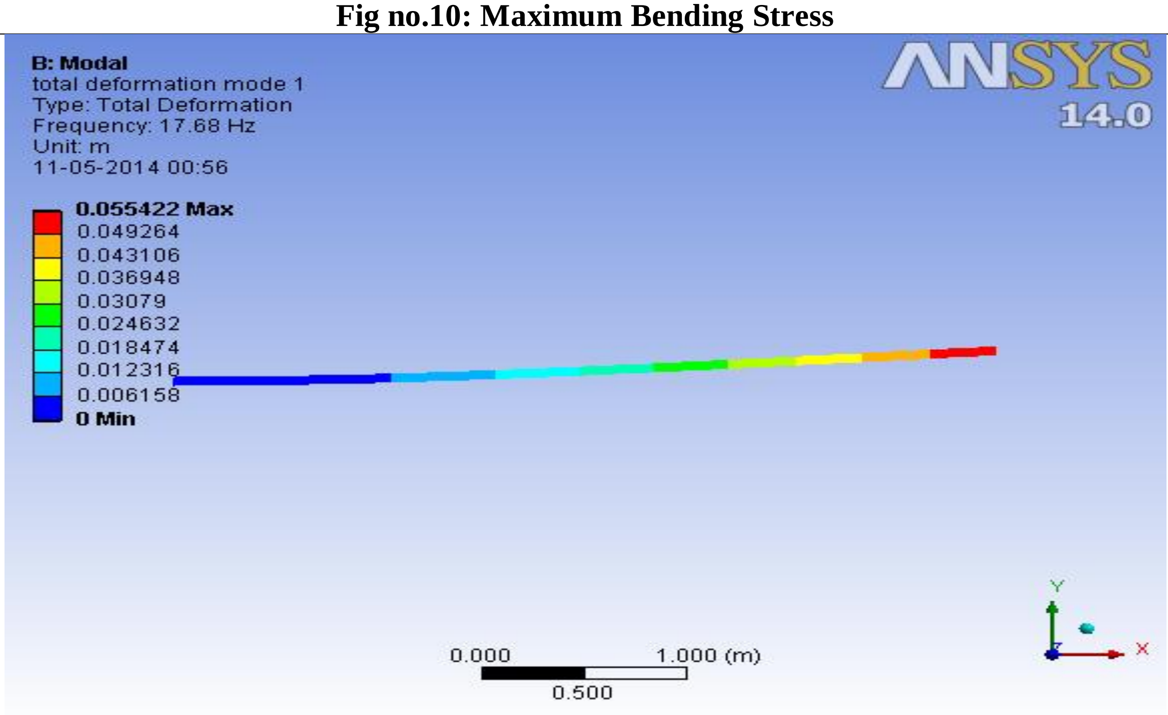


Fig 11: total deformation mode 1

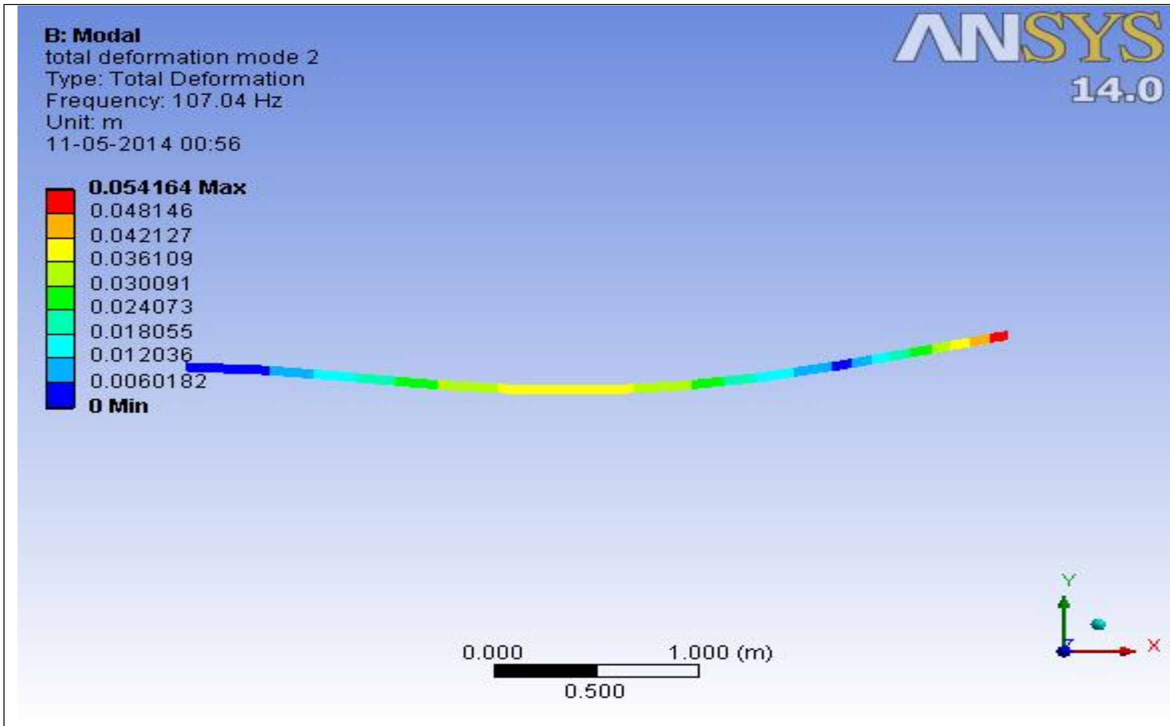


Fig 12: Total Deformation Mode 2

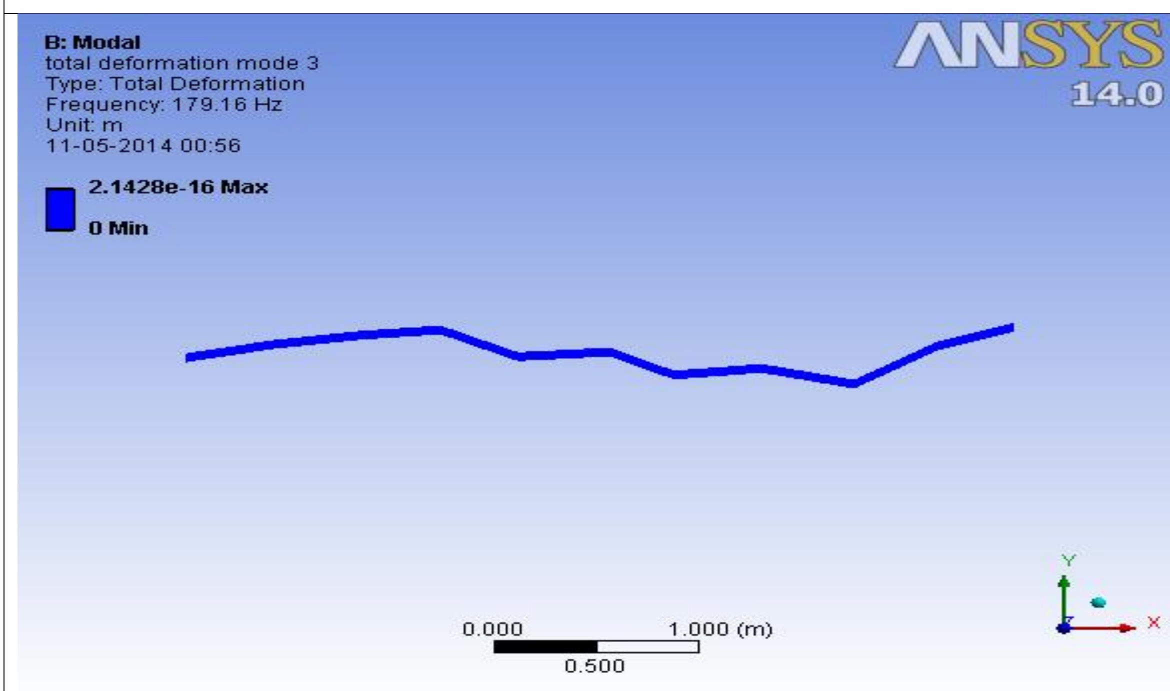


Fig 13: Total Deformation Mode 3

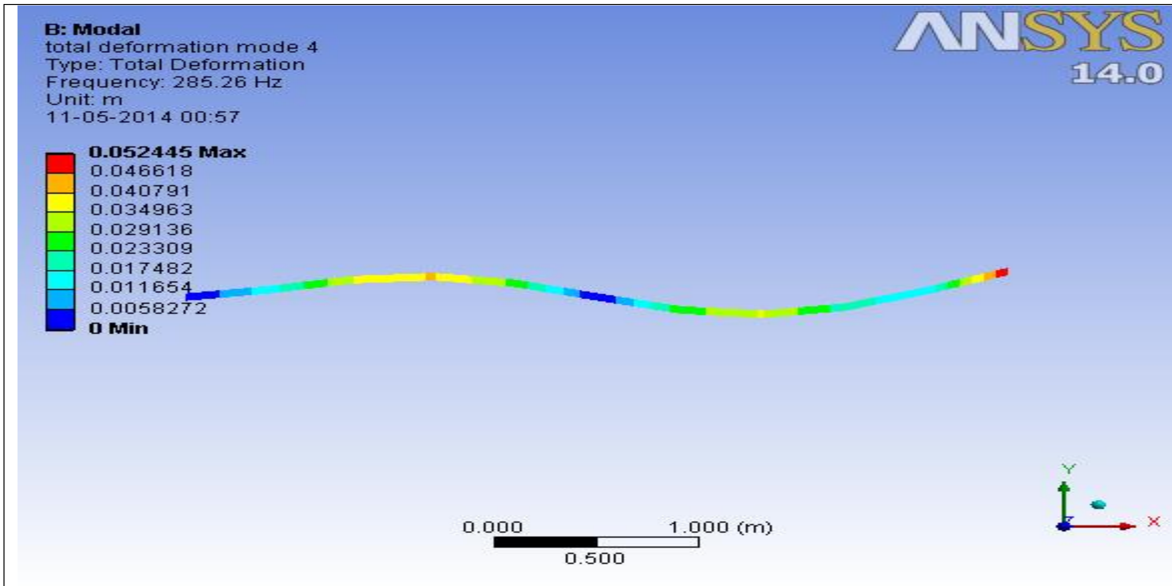


Fig 14: Total deformation mode 4

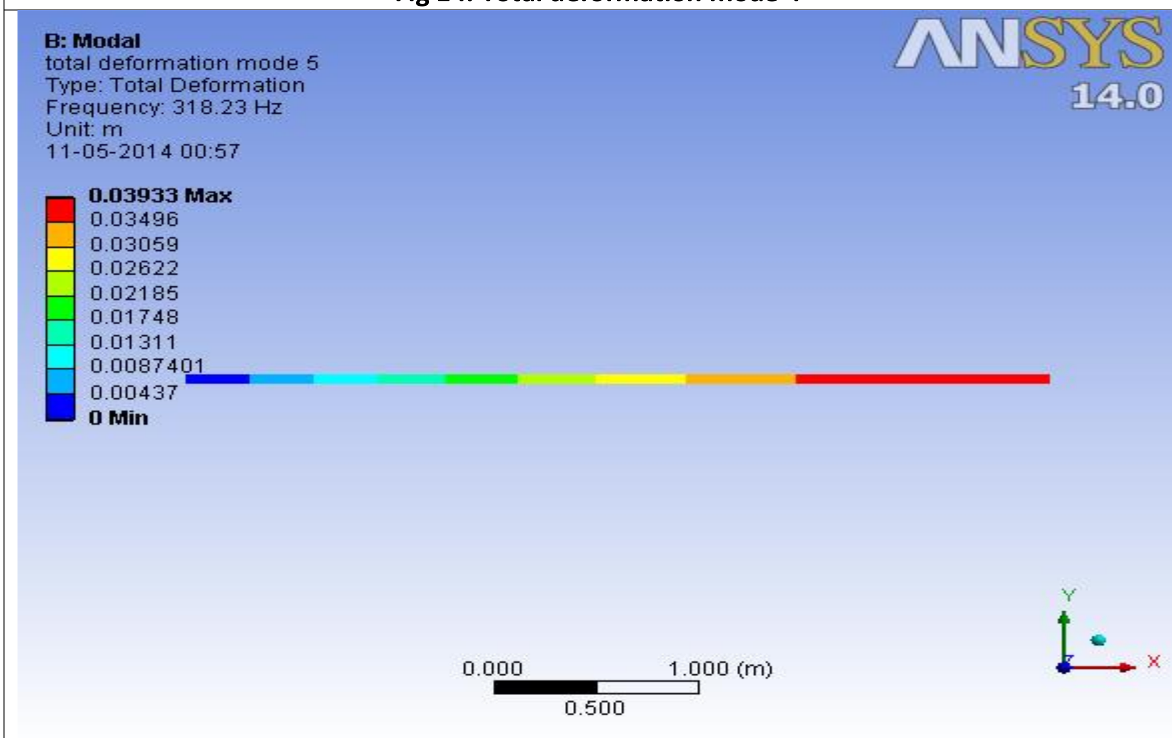


Fig 15: Total deformation mode 5

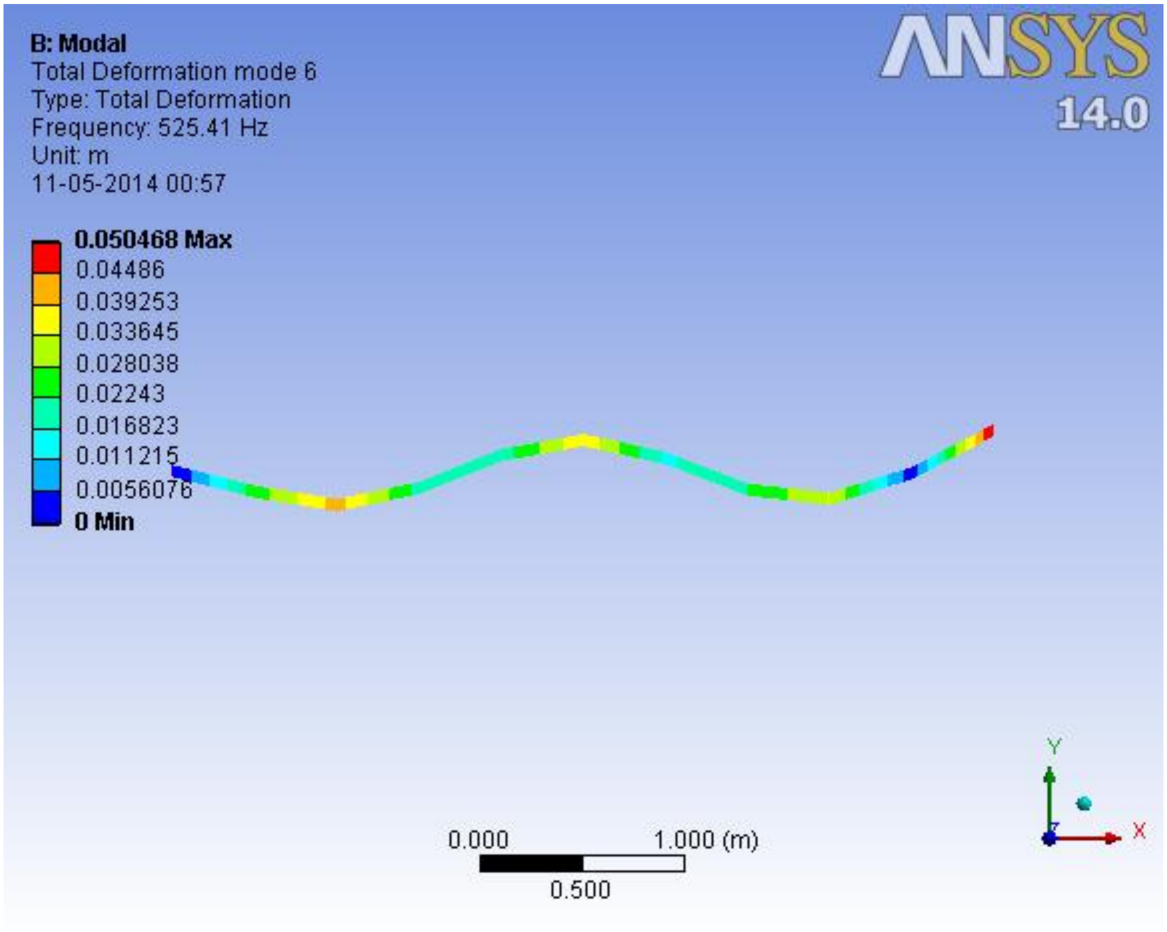


Fig 16: total deformation mode 6

5. EXPERIMENTAL VALIDATION

Free vibration of a continuous beam system

Objectives of the experiment

To deduce natural frequencies up to the second mode of a cantilever beam experimentally; and to observe the system response subjected to a small initial disturbance.

Introduction

Free vibration takes place when a system oscillates under the action of forces integral in the system itself due to initial deflection, and under the absence of externally applied forces. The system will vibrate at one or more of its natural frequencies, which are properties of the system dynamics, established by its stiffness and mass distribution.

In case of continuous system the system properties are functions of spatial coordinates. The system possesses infinite number of degrees of freedom and infinite number of natural frequencies.

In actual practice there exists some damping (e.g., the internal molecular friction, viscous damping, aero dynamical damping, etc.) inherent in the system which causes the gradual dissipation of vibration energy, and it results in decay of amplitude of the free vibration. Damping has very little influence on natural frequency of the system, and hence, the observations for natural frequencies are generally made on the basis of no damping. Damping is of great significance in restraining the amplitude of oscillation at resonance.

The comparative displacement alignment of the vibrating system for a particular natural frequency is known as the Eigen function in continuous system. The mode shape of the lowest natural frequency (i.e. the fundamental natural frequency) is termed as the fundamental (or the first) mode frequency. The displacements at some points may be zero which are called the nodal points. Generally nth mode has (n-1) nodes excluding the end points. The mode shape varies for different boundary conditions of a beam.

Mathematical analysis

For a cantilever beam exposed to free vibration, and the system is considered as continuous system considering the beam mass as distributed along with the stiffness of the shaft, the equation of motion can be written as given by the following equations (Meirovitch, 1967),

$$\frac{d^2}{dx^2}\{EI(x)\frac{d^2Y(x)}{dx^2}\} = \omega^2 m(x)Y(x)$$

Where, *E* is the modulus of rigidity of beam material, *I* is the moment of inertia of the cross section of the beam, *Y*(*x*) is displacement in *y* direction at distance *x* from fixed end, *ω* is the circular natural frequency, *m* is the mass per unit length, *m = ρA(x)* , *ρ* is the density of the material, *x* is the distance measured from the fixed end.

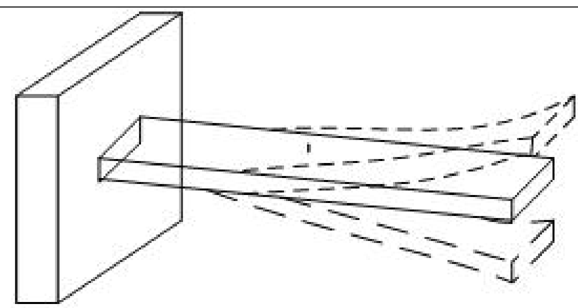


Fig 17: The beam under free vibration for cantilever case

Fig.17 shows a cantilever beam having rectangular cross section, which is subjected to bending vibration by giving a small initial displacement at the free end; and Fig. 18 depicts a cantilever beam under the free vibration.

The boundary conditions for a cantilever beam (Fig. 17) are given by;

$$at \quad x = 0, Y(x) = 0, \frac{dY(x)}{dx} = 0$$

$$at \quad x = l, \frac{d^2Y(x)}{dx^2} = 0, \frac{d^3Y(x)}{dx^3} = 0$$

For a uniform beam under free vibration from equation, we get

$$\frac{d^4Y(x)}{dx^4} - \beta^4 Y(x) = 0$$

With $\beta^4 = \frac{\omega^2 m}{EI}$

The mode shapes for a continuous cantilever beam is given as

$$f_n(x) = A_n\left\{(\sin\beta_n L - \sinh\beta_n L)(\sin\beta_n x - \sinh\beta_n x) + (\cos\beta_n L - \cosh\beta_n L)(\cos\beta_n x - \cosh\beta_n x)\right\}$$

Where

$$n = 1, 2, 3.....\infty \quad and \quad \beta^4 L = n\pi$$

The circular natural frequency ω_{nf} given in closed form, from above equation of motion and boundary conditions can be written as,

$$\beta^4 = \frac{\omega^2 m}{EI}$$

where,

$$\alpha = 1.875, 4.694, 7.885$$

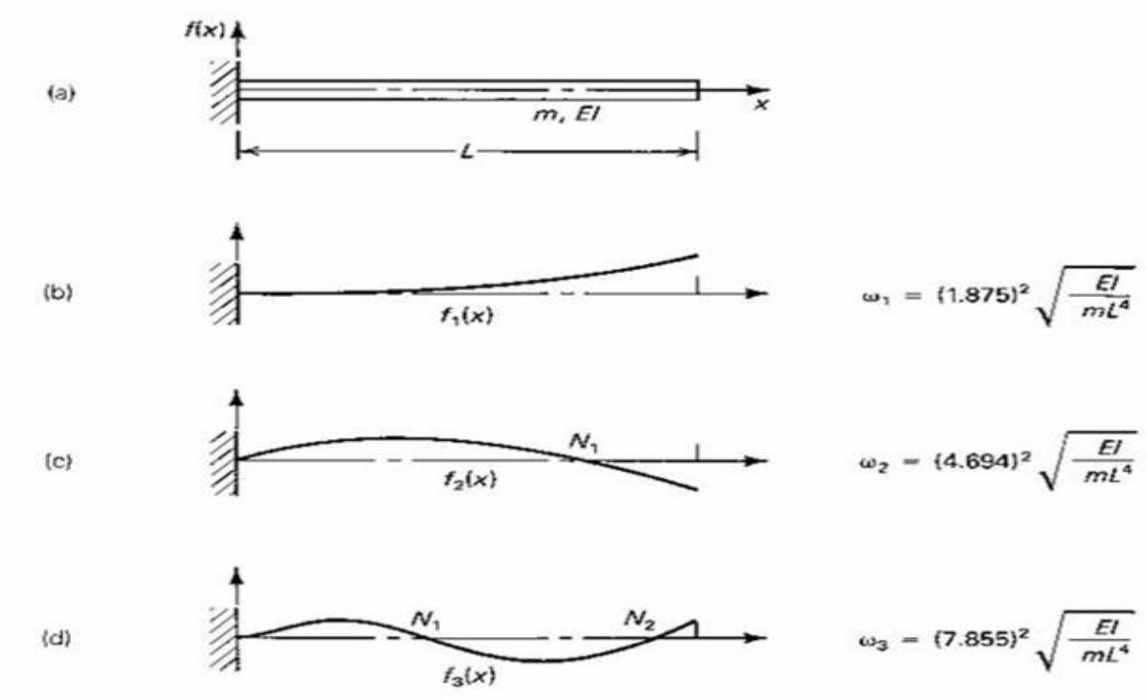


Fig 18: mode shapes

Calculation of experimental natural frequency

To observe the natural frequencies of the cantilever beam subjected to small initial disturbance experimentally up to third mode, the experiment was conducted with the specified cantilever beam specimen.The data of time history (Displacement-Time), and FFT plot was recorded. The natural frequencies of the system can be obtained directly by observing the FFT plot. The location of peak values relates to the natural frequencies of the system. Fig. below shows a typical FFT plot.

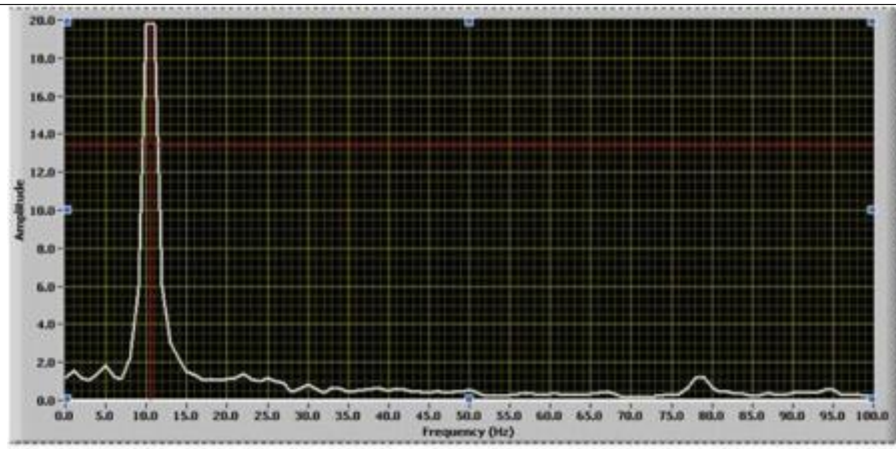


Fig 19: Typical example of a FFT

Experimental Setup

In our experiment we will use **digital phosphor oscilloscope (model DPO 4034)** for data acquisition.

Accelerometer is a kind of transducer to measure the vibration response (i.e., acceleration, velocity and displacement). Data acquisition system acquires vibration signal from the accelerometer, and encrypts it in digital form. Oscilloscope acts as a data storage device and system analyzer. It takes encrypted data from the data acquisition system and after processing (e.g., FFT), it displays on the oscilloscope screen by using analysis software.

Fig 20: Experimental setup for a cantilever beam

Fig. shows an experimental setup of the cantilever beam. It includes a beam specimen of particular dimensions with a fixed end and at the free end an accelerometer is clamped to measure the free vibration response. The fixed end of the beam is gripped with the help of clamp. For getting defined free vibration cantilever beam data, it is very important to confirm that clamp is tightened properly; otherwise it may not give fixed end conditions in the free vibration data.

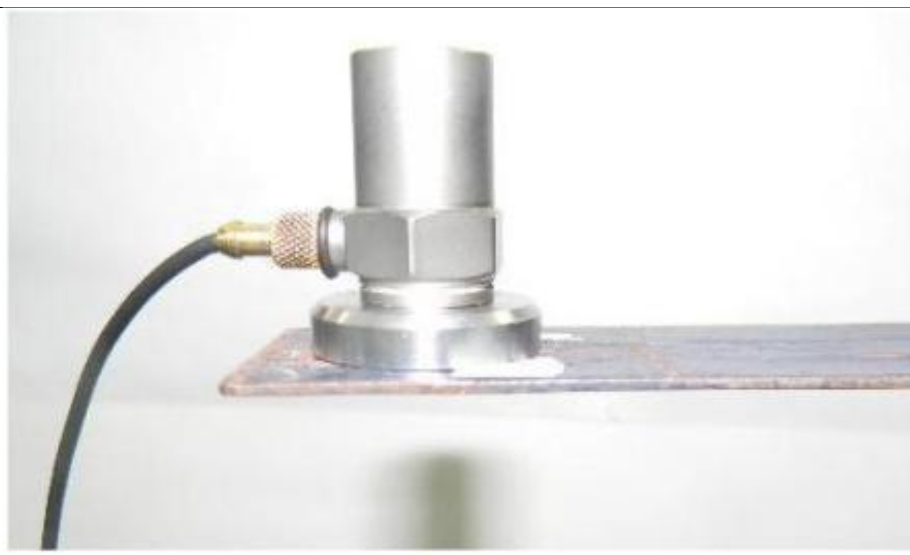


Fig 21: A Closed View Of Accelerometer

Experimental Procedure

1. A beam of a particular material (steel, aluminum or copper), dimensions (L, w, d) and transducer (i.e., measuring device, e.g. strain gauge, accelerometer, laser vibrato meter) was chosen.

2. One end of the beam was clamped as the cantilever beam support.

3. An accelerometer (with magnetic base) was placed at the free end of the cantilever beam , to observe the free vibration response (acceleration).

4. An initial deflection was given to the cantilever beam and allowed to oscillate on its own. To get the higher frequency it is recommended to give initial displacement at an arbitrary position apart from the free end of the beam (e.g. at the mid span).

5. This could be done by bending the beam from its fixed equilibrium position by application of a small static force at the free end of the beam and suddenly releasing it, so that the beam oscillates on its own without any external force applied during the oscillation.

6. The free oscillation could also be started by giving a small initial tap at the free end of the beam.

7. The data obtained from the chosen transducer was recorded in the form of graph (variation of the vibration response with time).

8. The procedure was repeated for 5 to 10 times to check the repeatability of the experimentation.

9. The whole experiment was repeated for different material, dimensions, and measuring devices.

10. The whole set of data was recorded in a data base.

Results

Good agreement between the theoretically calculated natural frequency and the experimental one is found. The correction for the mass of the sensor will improve the correlation better. The present theoretical calculation is based on the assumption that one end of the cantilever beam is properly fixed. However, in actual practice it may not be always the case because of flexibility in support.

The experimental values obtained are 5.21 Hz and 32.4 Hz for first and second modes respectively.

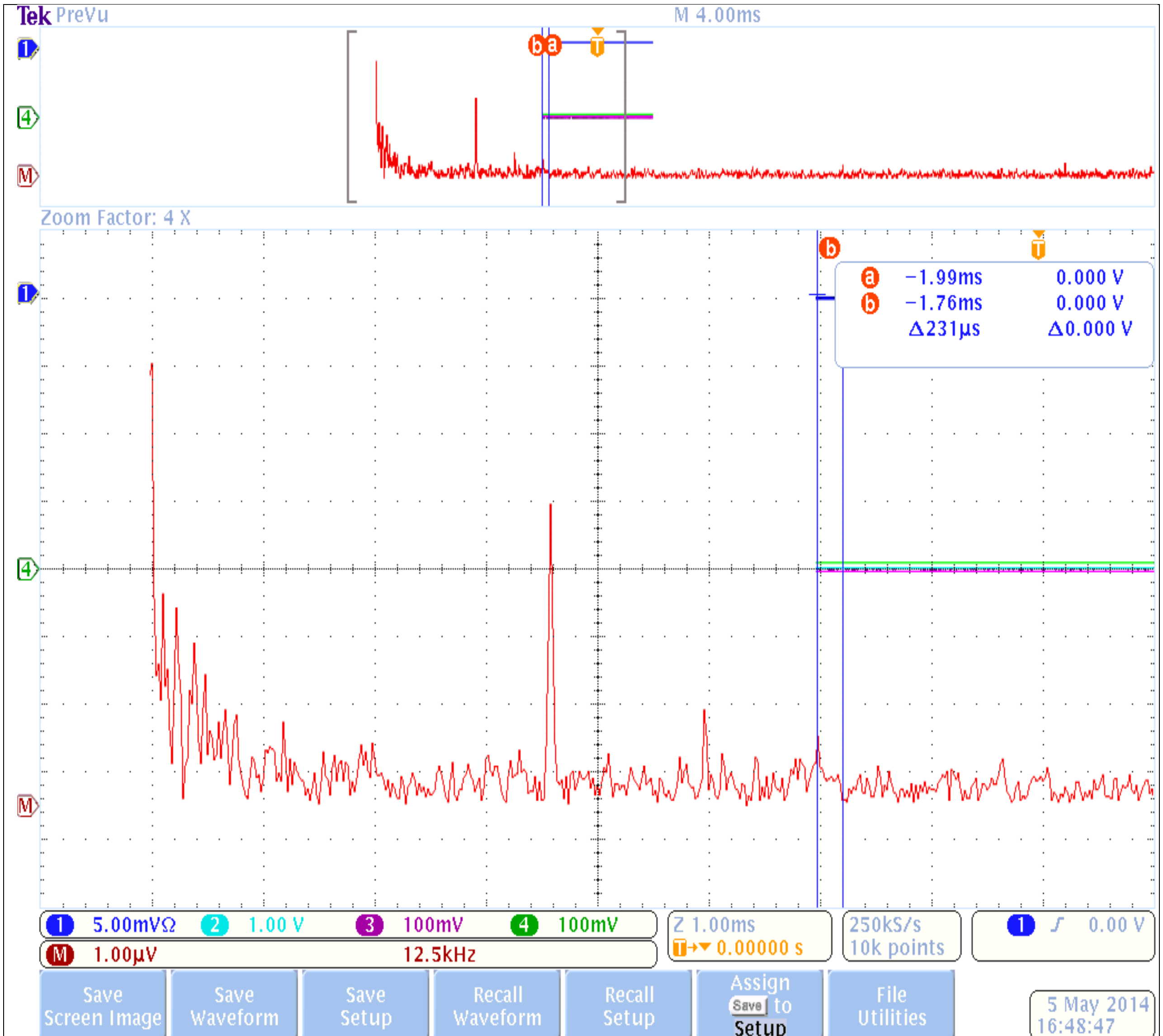


Fig 22: FFT plot obtained

Verification and validation

A mild steel beam that is clamped at one end, with the following dimensions.

Length	0.48 m
Width	0.032 m
Height	0.002 m

The mild steel used for the beam has the following material properties.

Density	7856 kg/m^3
Young's Modulus	210x10^9 Pa
Poisson Ratio	0.3

The theoretical values of the natural frequencies were found to be 4.56 Hz and 28.55 Hz for first and second mode with an error of 14.3% and 13.48% respectively.

6. CONCLUSION

In this report, we compared the Euler-Bernoulli and Timoshenko models by using ANSYS and experimentally .The equation of motion and the boundary conditions were obtained and the natural frequencies were also obtained for different modes.

It can be found out that Euler-Bernoulli equation is valid for long and slender beams where we neglect shear deformation effects and rotational effects. Timoshenko beam theory is valid for short and clubby beams. In this model shear deformation is taken into account.

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