

# Parameters assessment of the inductively-coupled circuit for wireless power transfer

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**Abstract.** In this paper, a wireless power transfer model through the example of inductively-coupled coils of irregular shape in software package COMSOL Multiphysics is studied. Circuit parameters, such as inductance, coil resistance and self-capacitance were defined through electromagnetic energy by the finite-element method. The study was carried out according to Helmholtz equation. Spatial distribution of current per unit depending on frequency and the coupling coefficient for analysis of resonant frequency and spatial distribution of the vector magnetic potential at different distances between coils were presented. The resulting algorithm allows simulating the wireless power transfer between the inductively coupled coils of irregular shape with the assessment of the optimal parameters.

## 1. Introduction

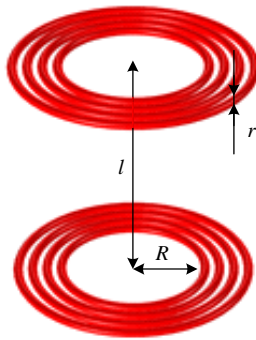
Wireless power transfer technologies are of great practical interest especially in the development of intelligent systems [1]. The purpose of this study is the optimum parameters assessment of the inductively-coupled circuit, such as inductance, coil resistance and self-capacitance for a wireless power transfer.

Statement of the research problem resides in estimation of the optimum parameters of the inductive coil corresponding to wireless power transfer with maximum efficiency at a distance not exceeding the size of the room [2]. The frequencies range is given with step  $f=10^5$  Hz from initial value  $f_0=100$  Hz to finite value  $f_k=600$  MHz, because the radio frequency (until 300 GHz) electromagnetic field effect is not dangerous for human health [3]. The generator wavelength at a finite value of frequency  $f_k$  is equal to  $\lambda = c/f_k = 0.5$  m, where  $c$  – light velocity.

## 2. Technique

Parameters will be estimated in the system of two inductively-coupled helical coils having  $N=4$ ,  $N$  - number of turns, which are the source and device coils, respectively (Figure 1). In the simulation, we present a helical coil in the form of four rings for the calculation convenience. Sine voltage with amplitude  $V=1$  V is applied to the inductive coil of the source. Both coils are made of cooper wire with radius  $r=0.3$  m, the internal radius of the helical coil smaller turn is equal to  $R=0.25$  m, the distance between inductive coils varies in the range of 0.5-2 meters maintaining the power transfer to the distance exceeding the radius of coils by the factor of 8.





**Figure 1.** The modeled system of two inductive coupled coils:  $l$  – distance between coils;  $r$  – radius of cooper wire;  $R$  – radius of coil.

### 3. Mathematical model description

When voltage is applied to the high-frequency range of the transmitter, coil high-frequency currents will flow, so we use Maxwell equations system as a basis for calculation of coil parameters [4].

The equation for direct current, connecting flux density  $\mathbf{B}$  and magnetic vector potential  $\mathbf{A}$ , is the following:

$$\mathbf{B} = \nabla \mathbf{A} = \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{pmatrix} = \mathbf{i} \left( \frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \right) + \mathbf{j} \left( \frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z \right) + \mathbf{k} \left( \frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \right). \quad (1)$$

The equation for direct current, connecting magnetic force  $\mathbf{H}$  and current density vector  $\mathbf{J}$ , is the following:

$$\nabla \mathbf{H} = \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{pmatrix} = \mathbf{i} \left( \frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y \right) + \mathbf{j} \left( \frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z \right) + \mathbf{k} \left( \frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x \right) = \mathbf{J}. \quad (2)$$

Using (1) and (2), we can receive Poisson equation for magnetic vector potential  $\mathbf{A}$  through current density vector  $\mathbf{J}$  as follows:

$$\Delta \mathbf{A} = \frac{\partial^2}{\partial x^2} \mathbf{A} + \frac{\partial^2}{\partial y^2} \mathbf{A} + \frac{\partial^2}{\partial z^2} \mathbf{A} = \mathbf{J}. \quad (3)$$

For alternative current, using Helmholtz equation and (3) we can receive the following expression:

$$(j\omega\sigma - \omega^2\epsilon_0\epsilon)\mathbf{A} + \nabla \mathbf{H} = \mathbf{J}, \quad (4)$$

where:  $\omega = 2\pi f$  – angular frequency;  $\sigma$  – wire conductivity;  $\epsilon$  – dielectric permeability;  $\epsilon_0$  – permittivity of vacuum;  $\mathbf{A}$  – magnetic vector potential;  $\mathbf{H}$  – magnetic force;  $\mathbf{J} = \nabla \times (\mu^{-1} \nabla \times \mathbf{A})$  – current density vector.

Helmholtz equation (4) can be rewritten through voltage  $V$ , applied to the coil:

$$(j\omega\sigma - \omega^2\epsilon)\mathbf{A} + \nabla \times (\mu^{-1} \nabla \times \mathbf{A}) = \frac{\sigma V}{2\pi r}. \quad (5)$$

The current in the coil of wire, using (5), can be expressed as:

$$I = \pi \int_0^R \mathbf{J} r^2 dr.$$

Magnetic field energy is given by [2]:

$$W = 2\pi \int \mathbf{A} \cdot d\mathbf{r}.$$

Circuit parameters, such as inductance, coil resistance and capacitance were defined as follows:

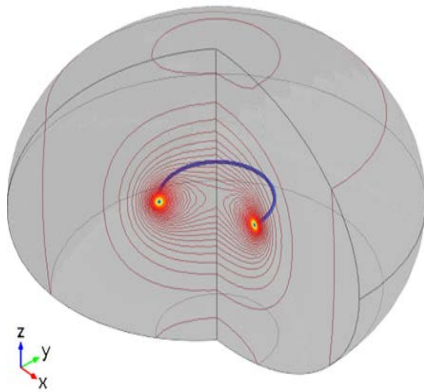
$$L = \frac{2W}{|I|^2}, \quad R = \operatorname{Re}\left(\frac{V}{I}\right), \quad C = \frac{2W_e}{|V|^2}, \tag{6}$$

where:  $W_e$  – electrical field energy.

**4. Experimental part**

The direct current is assumed to be  $I=1$  A for inductance calculation. Distribution of vector magnetic potential  $\mathbf{A}$  of the inductive coil with  $N=1$  was calculated in COMSOL Multiphysics and is shown in Figure 2.

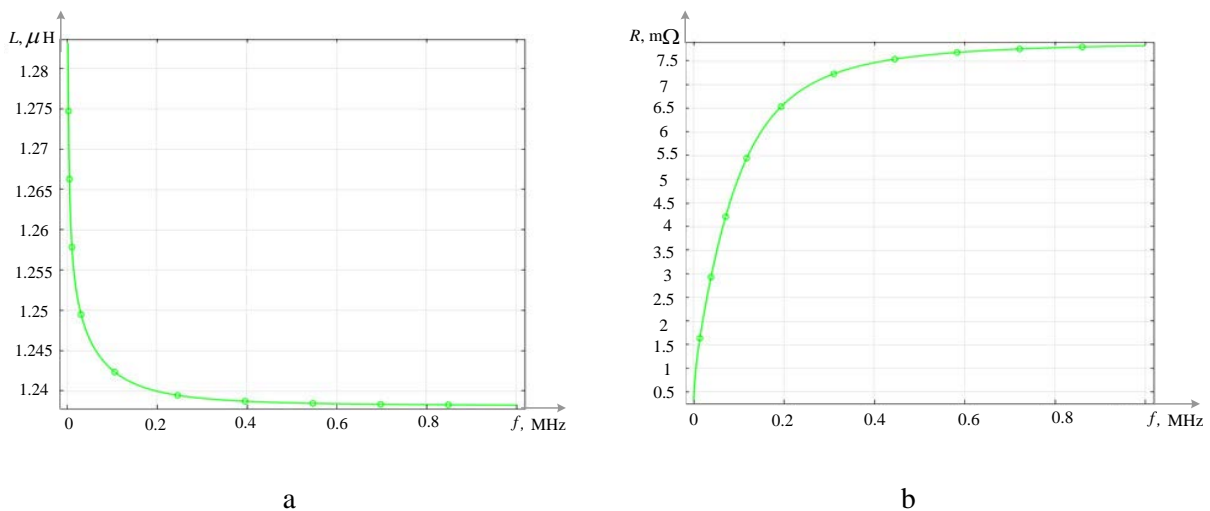
Inductance calculation with  $N=1$  has been checked on the basis of [5], using  $L = \mu_0 R \left( \ln \frac{8R}{r} - 2 \right)$ .



**Figure 2.** Distribution of vector magnetic potential  $\mathbf{A}$  for the inductive coil with  $N=1$

The inductance and the resistance of the irregular shape coil with  $N=4$  as a function of the frequency were calculated according to the same algorithm (Figure 2), using (5) and (6) by the finite-element method in COMSOL Multiphysics.

Calculation results are presented in Figure 3.



**Figure 3.** The frequency characteristic for the inductive coil: a) inductance  $L(\omega)$ ; b) coil resistance  $R(\omega)$

Figure 3, *a* shows that coil inductance decreases sharply and has a constant value in a high-frequency band, which is  $L=1.238 \mu\text{H}$  according to (6) due to saturation of the material. Figure 3, *b* shows that coil resistance increases and has a constant value in a high-frequency band, which is  $R=8 \text{ m}\Omega$ .

Poisson equation for the electric field in Cartesian coordinates for coil self-capacitance calculation is given by:

$$\Delta\varphi = \frac{\partial^2}{\partial x^2}\varphi + \frac{\partial^2}{\partial y^2}\varphi + \frac{\partial^2}{\partial z^2}\varphi = -\frac{\rho}{\varepsilon\varepsilon_0}. \quad (7)$$

The direct voltage is assumed to be  $V=1 \text{ V}$  for self-capacitance calculation, which is independent of frequency. Self-capacitance calculation with  $N=1$  has been checked on the basis of [6], using

$$C_0 = 4\pi^2\varepsilon R \cdot \left(\ln\left(\frac{8R}{r}\right)\right)^{-1}. \quad (8)$$

The simulation result in COMSOL Multiphysics coincided with the reference data. The self-capacitance of the irregular shape coil with  $N=4$  was calculated through electric field energy according to (6) and (7), which is  $C_0=0.21 \text{ pF}$ .

Resonant frequency was defined as  $f_0=1/2\pi(LC)^{-1}$  and is equal to  $f_0=312 \text{ MHz}$ . Thus, the resonant wavelength is  $\lambda_0=0.96 \text{ m}$ .

Spatial distribution of current per unit  $I(\omega, k)$  for the inductive coil as a function of angular frequency  $\omega$  and coupling coefficient  $k$  for analysis of resonant frequencies range  $f_0$  may be presented as follows:

$$I(\omega, k) = \varpi(\omega) \cdot \frac{1 + \frac{\omega_0^2 \cdot k^2 \cdot L(\omega)^2}{R(\omega)^2}}{\left(1 + j \cdot \frac{X(\omega)}{R(\omega)}\right)^2 + \frac{\omega^2 \cdot k^2 \cdot L(\omega)^2}{R(\omega)^2}}, \quad (8)$$

where:  $\varpi(\omega) = \frac{\omega}{\omega_0}$  – relative angular frequency;  $\omega_0$  – resonant angular frequency;

$k = M(\omega) / \sqrt{L_1(\omega) \cdot L_2(\omega)} = M(\omega) / L(\omega)$  – coupling coefficient;  $M(\omega)$  – mutual inductance;

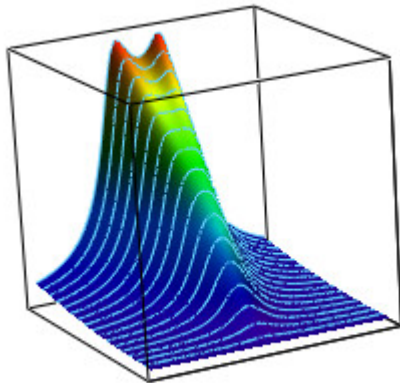
$X(\omega) = \omega \cdot L(\omega) - \frac{1}{\omega \cdot C}$  – equivalent reactive resistance [7].

Expression (8) can be rewritten through general detuning  $\xi$ , which is absolutely dependent on quality factor  $Q$ :

$$I(\omega, k) = \varpi(\omega) \cdot \frac{1 + (k \cdot Q)^2}{(1 + j \cdot \xi)^2 + (k \cdot Q)^2}, \quad (9)$$

where:  $\xi = \frac{X(\omega)}{R(\omega)} = Q \cdot \left(\varpi(\omega) - \frac{1}{\varpi(\omega)}\right)$  – general detuning;  $Q = \frac{\omega_0 \cdot L(\omega)}{R(\omega)}$  – quality factor.

Expression (9) shows that the effect of circuit parameters on the form of resonance curve  $I(\omega, k)$  is taken into consideration fully by quality factor  $Q$  [8]. Spatial distribution of current per unit  $I(\omega, k)$  for the inductive coil is presented in Figure 4.

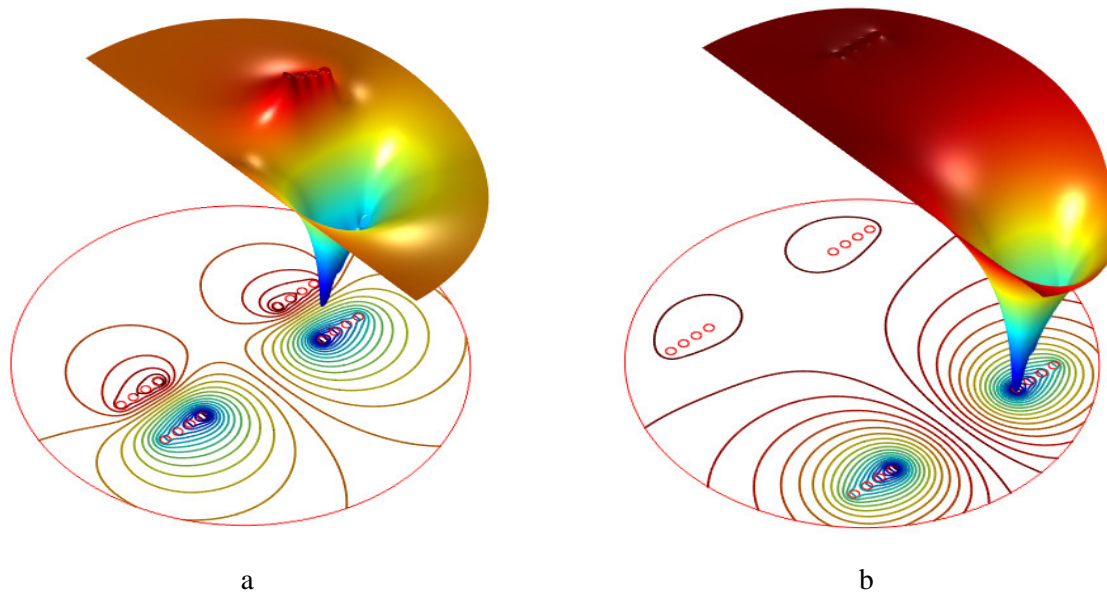


**Figure 4.** Spatial distribution of current per unit  $I(\omega, k)$  for the inductive coil

Figure 4 shows a possibility to estimate the role of losses in the circuit and to select only natural resonance frequency from the entire frequencies range in  $k \cdot Q \ll 1$  (weak-coupling regime) or two resonance frequencies in  $k \cdot Q \gg 1$  (strong-coupling regime). Optimal parameter estimation of the inductively-coupled circuit has allowed one to establish which part of the power is transferring to the receiving coil when the circuit is tuned to resonance [9, 10].

**5. Simulation of the wireless power transfer in COMSOL Multiphysics**

The system is modeled and calculated in software package COMSOL Multiphysics on the basis of Helmholtz equation (4). Sine voltage with amplitude  $V=1$  V and resonant frequency  $f_0$  was supplied on the source inductive coil. Results of simulation for different distance of the coils between each other as a function of the resonant wavelength  $\lambda_0$  are shown in Figure 5.



**Figure 5.** Spatial distribution of vector magnetic potential  $\mathbf{A}$ : a)  $l = \frac{\lambda_0}{2}$ ; b)  $l = \frac{4 \cdot \lambda_0}{2}$

Figure 5 shows that the field pattern is symmetric, so we can plot half of the field pattern of vector magnetic potential spatial distribution  $\mathbf{A}$  relative to the central axis of the coil. It may be noted that if the distance between coils increases, the power transfer decreases (Figure 5, b) [11].

Thus, a research transition phase about a wireless power transfer was presented.

The power is the energy change in time, so power equation  $P(\omega)$ , transferring to the receiving coil (Figure 5), is given by:

$$P(\omega) = \frac{dW(\omega)}{dt} = \frac{dW(\omega)}{d\omega} \frac{d\omega}{dt} = -\omega^2 \frac{dW(\omega)}{d\omega}. \quad (9)$$

We believe that the optimum parameters of the inductively-coupled circuit and coupling coefficient  $k$ , which depend on the distance between coils  $l$ , relationship of diameters of the primary and secondary coils, and also quality factor  $Q$ , can appreciably improve the power (9) transfer distances [12]. The suggested algorithm could be useful in practical applications for further study.

## 6. Conclusion

The model for wireless power transfer research through the example of inductively-coupled coils of irregular shape in software package COMSOL Multiphysics was suggested.

Circuit parameters, such as inductance, coil resistance and self-capacitance were defined through electromagnetic energy by the finite-element method. The coil inductance was determined on the basis of calculation of the magnetic vector potential succeeded by calculation of magnetic field energy. The self-capacitance was calculated through electric field energy.

Spatial distribution of current  $I(\omega, k)$  per unit in dependence to frequency  $\omega$  and coupling coefficient  $k$  for analysis of resonant frequency  $f_0$  and spatial distribution of vector magnetic potential  $\mathbf{A}$  at different distances  $l$  between coils were presented. The study was carried out according to Helmholtz equation.

The resulting algorithm allows us to simulate the wireless power transfer between the inductively coupled coils of irregular shape with the assessment of the optimal parameters for improving the efficiency of resonant energy transfer.

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