

# The frequency-domain method of calculation for the pulsed electromagnetic field in a conductive ferromagnetic plate

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**Abstract.** The technique for parameters determination of magnetic skin effect on ferromagnetic plate at a specified pulse of magnetic field intensity on the plate surface is proposed. It is based on a frequency-domain method and could be applied for a pulsing transformer, a dynamoelectric pulse generator and a commutating inductor that contains an imbricated core. Due to this technique, such plate parameters as specific heat loss energy, the average power of this energy and the plate temperature raise, the magnetic flux attenuation factor and the plate q-factor could be calculated. These parameters depend on the steel type, the amplitude, the rms value, the duration and the form of the magnetic field intensity impulse on the plate surface. The plate thickness is defined by the value of the flux attenuation factor and the plate q-factor that should be maximal. The reliability of the proposed technique is built on a common frequency-domain usage applicable for pulse transient study under zero boundary conditions of the electric circuit and the conformity of obtained results with the sinusoidal steady-state mode.

## 1. Introduction

Many of today's electrophysical devices contain an imbricated core run under the pulsed electromagnetic field (EMF). Pulsing transformers, dynamoelectric pulse generators and commutating inductors provide examples [1-5, 10]. It is necessary to take uneven distribution of the pulsed magnetic flow in ferromagnetic plates of the imbricated core (magnetic skin effect) into account. To calculate its parameters, it is possible to solve EMF equations by a numerical technique. However, this technique is quite cumbersome so it makes determination of the pulsed EMF influence on imbricated cores too complicated [6-8]. On the other hand, the using of the frequency-domain method simplify computations and allows us to consider the shape, amplitude and duration of pulsed EMF at its diffusion in the ferromagnetic plate [9]. Consequently, such task as calculation of the magnetic skin effect in the ferromagnetic plate due to the frequency-domain method and choosing the plate thickness with respect to the pulsed EMF shape, duration and amplitude seems to be a vital one and reflects the purpose of the paper.

## 2. Assumptions

The following assumptions were used to obtain calculating formula.

1. Isolated from each other, ferromagnetic plates of the imbricated core at the same initial



temperature of  $\theta_0$  ( $^{\circ}\text{C}$ ) have the same values of absolute permeability  $\mu_a$  (H/m), specific conductivity  $\gamma$  ( $1/\Omega\cdot\text{m}$ ), volume density  $\rho$  ( $\text{kg}/\text{m}^3$ ) and specific heat capacity  $C_T$  ( $\text{J}/\text{kg}^{\circ}\text{C}$ ).

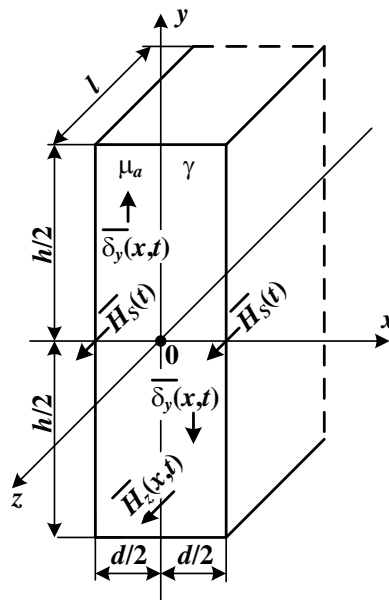
2. Quite short EMF pulse of  $\tau$  (s) heats ferromagnetic plates only by eddy currents (hysteresis neglected). It is adiabatic heating and heat transfer with the environment that has been neglected. Moreover, identical temperature rise  $\vartheta$  ( $^{\circ}\text{C}$ ) of plates due to the pulsed EMF influence is minor in comparison with initial temperature  $\theta_0$  and does not lead to the change of  $\mu_a, \gamma, \rho, C_T$ .
3. Thickness  $d$  (m) of ferromagnetic plates is much smaller than its height  $h$  (m) and length  $l$  (m), namely  $d \ll h$  and  $d \ll l$ , so we will proceed from the conception of the plane one-dimensional electromagnetic wave penetrating from the opposite sides to conductive ferromagnetic plates along the X axe (Figure 1) perpendicular to plate lateral surfaces.
4. The EMF pulse is prescribed as an identical magnetic field intensity on plates lateral surfaces  $H_S(t)$  – i.e. pulse time function  $t$  with duration  $\tau$  and directed along the Z axe (Figure 1).
5. According to the frequency-domain method, the definable magnetic field in conductive ferromagnetic plates has zero initial conditions [9] that is the magnetic field in plates is completely absent (magnetic field intensity and induction are equal to zero).

### 3. The calculation method

Within these assumptions, let us consider the EMF influence on one ferromagnetic plate. Let us suppose that the plate lateral surface is an  $x0y$  plane of Cartesian coordinates as it is shown in Figure 1. Then, E-vector  $\vec{E}$ , H-vector  $\vec{H}$  and current density vector  $\vec{\delta}$  of the plane one-dimensional electromagnetic wave have unit components each of which is dependent on the  $x$  coordinate and the  $t$  time [9]:

$$\vec{E} = E_y(x,t) \cdot \vec{1}_y; \quad \vec{H} = H_z(x,t) \cdot \vec{1}_z; \quad \vec{\delta} = \delta_y(x,t) \cdot \vec{1}_y,$$

where  $\vec{1}_y, \vec{1}_z$  – lie in  $y$  and  $z$  axis unit vectors, respectively (Figure 1).



**Figure 1.** The conductive ferromagnetic plate:  $H_S(t) = H_z(\pm d/2, t)$  – the magnetic field intensity on the plate surface.

In this case, EMF diffusion in the plate is described by the following equations [10]:

$$\frac{\partial^2 H_z(x,t)}{\partial x^2} = \mu_a \gamma \frac{\partial H_z(x,t)}{\partial t}; \delta_y(x,t) = \gamma E_y(x,t) = -\frac{\partial H_z(x,t)}{\partial x}, \quad (1)$$

where boundary and zero conditions are

$$H_y(\pm d/2, t) = H_s(t); H_y(x, 0) = 0. \quad (2)$$

Let us find a spectral function of the magnetic field intensity on the conductor surface by the spectral (frequency-domain) method of unilateral forward Fourier transformation:

$$H_{sm}(j\omega) = \int_0^\infty H_s(t) \cdot \exp(-j\omega t) dt = H_{sm}(\omega) \cdot \exp[j\varphi_s(\omega)]; j = \sqrt{-1}, \quad (3)$$

where  $H_{sm}(\omega)$  and  $\varphi_s(\omega)$  – the amplitude and the phase response of the magnetic field intensity pulse on the conductor surface as a function of cyclic frequency  $\omega$ .

Then, equation (1) becomes:

$$\frac{d^2 H_{zm}(x, j\omega)}{dx^2} = j\omega \mu_a \gamma H_{zm}(x, j\omega); \delta_{ym}(x, j\omega) = -\frac{dH_{zm}(x, j\omega)}{dx}, \quad (4)$$

where  $H_{zm}(x, j\omega)$  and  $\delta_{ym}(x, j\omega)$  – the complex amplitude of the magnetic field intensity and the eddy current density as a function of the  $x$  coordinate and cyclic frequency  $\omega$ .

Considering conditions (2, 3), the solutions of equations (4) will be:

$$H_{zm}(x, j\omega) = \frac{H_{sm}(j\omega)}{\cosh[0.5 \cdot p(j\omega) \cdot d]} \cdot \cosh[p(j\omega) \cdot x]; \quad (5)$$

$$\delta_{ym}(x, j\omega) = -\frac{p(j\omega) \cdot H_{sm}(j\omega)}{\cosh[0.5 \cdot p(j\omega) \cdot d]} \cdot \sinh[p(j\omega) \cdot x]; p(j\omega) = \sqrt{j\omega \mu_a \gamma}. \quad (6)$$

Next, it is possible to determine the magnetic field density and the eddy current density in the plate:

$$H_z(x, t) = \frac{2}{\pi} \int_0^\infty \left\{ \operatorname{Re} [H_{zm}(x, j\omega)] \cdot \cos(\omega t) \right\} d\omega; \quad (7)$$

$$\delta_y(x, t) = \frac{2}{\pi} \int_0^\infty \left\{ \operatorname{Re} [\delta_{ym}(x, j\omega)] \cdot \cos(\omega t) \right\} d\omega. \quad (8)$$

As a result, specific heat loss energy  $W_T$  (J/kg), the average power of its energy  $P_T$  (W/kg) and the plate temperature rise caused by eddy currents (8) will be:

$$W_T = \frac{1}{\gamma \rho d} \int_0^\infty \left[ \int_{-0.5d}^{0.5d} \delta_y(x, t)^2 dx \right] dt; P_T = W_T / \tau; \vartheta = W_T / C_T. \quad (9)$$

In virtue of (7), let us find the pulsed magnetic flux in plate  $\Phi(t)$  and its rms value  $\Phi$  (Wb/m):

$$\Phi(t) = \mu_a \int_{-0.5d}^{0.5d} H_z(x, t) dx; \Phi = \sqrt{\frac{1}{\tau} \int_0^\tau \Phi(t)^2 dt}, \quad (10)$$

and calculate the values in the absence of eddy currents and the rms value of density on plate surface  $H_s$  that are then used for determination of magnetic flux attenuation factor  $K_\Phi$  with regard to eddy currents:

$$\Phi_s(t) = \mu_a H_s(t) d; H_s = \sqrt{\frac{1}{\tau} \int_0^\tau H_s(t)^2 dt}; \Phi_s = \mu_a H_s d; K_\Phi = \frac{\Phi}{\Phi_s}. \quad (11)$$

Taking into account (7, 9), we will define the average specific magnetic field energy in plate  $W_M$  (J/kg) and the ratio of  $W_M$  and  $W_T$  that we will call plate q-factor  $K_w$ :

$$W_M = \frac{\mu_a}{2\rho\tau d} \int_0^\tau \left[ \int_{-0.5d}^{0.5d} H_z(x,t)^2 dx \right] dt; \quad K_w = \frac{W_M}{W_T}. \quad (12)$$

Let us examine the sinusoidal steady-state mode for comparison under:

$$H_s(t) = H_m \sin(\omega_0 t + \varphi), \quad (13)$$

when the complex amplitude of magnetic field intensity  $\dot{H}_{zm0}(x)$  and eddy current density  $\dot{\delta}_{ym0}(x)$  with respect to (5, 6) when  $\omega = \omega_0$  are rated as follows:

$$\dot{H}_{zm0}(x) = \frac{H_m e^{j\varphi} \cdot \cosh\left[x\sqrt{j\omega_0\mu_a\gamma}\right]}{\cosh\left[0.5d\sqrt{j\omega_0\mu_a\gamma}\right]} = \left|\dot{H}_{zm0}(x)\right| \cdot \exp\left[j\psi_H(x)\right]; \quad (14)$$

$$\dot{\delta}_{ym0}(x) = -\frac{\sqrt{j\omega_0\mu_a\gamma} \cdot H_m e^{j\varphi} \cdot \sinh\left[x\sqrt{j\omega_0\mu_a\gamma}\right]}{\cosh\left[0.5d\sqrt{j\omega_0\mu_a\gamma}\right]} = \left|\dot{\delta}_{ym0}(x)\right| \cdot \exp\left[j\psi_\delta(x)\right], \quad (15)$$

In other words, magnetic field intensity  $H_z(x,t)$  and eddy current density  $\delta_y(x,t)$  will be:

$$H_z(x,t) = \left|\dot{H}_{zm0}(x)\right| \cdot \sin\left[\omega_0 t + \psi_H(x)\right]; \quad \delta_y(x,t) = \left|\dot{\delta}_{ym0}(x)\right| \cdot \sin\left[\omega_0 t + \psi_\delta(x)\right]. \quad (16)$$

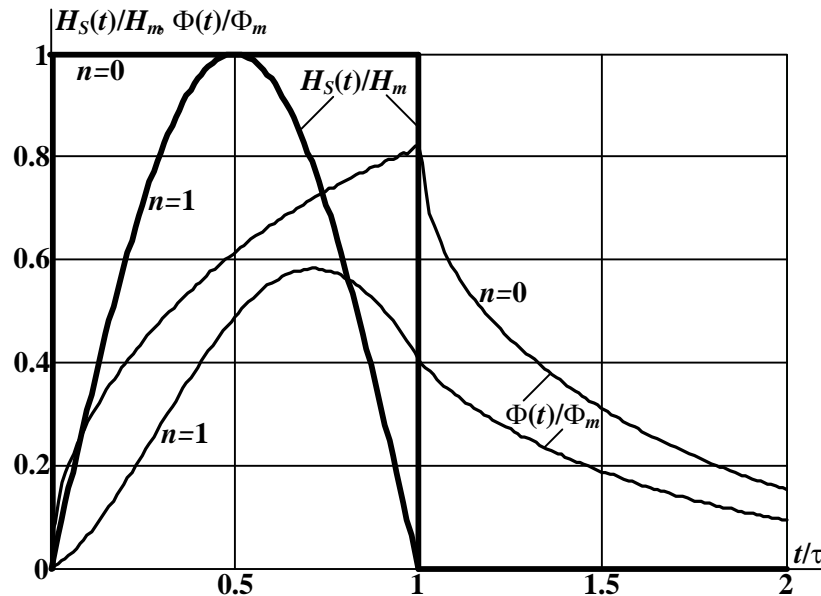
Due to substitution (14-16) into (9-12), let us determine EMF integral characteristics in the sinusoidal steady-state mode.

#### 4. Calculation results

Calculation of the magnetic skin-effect for the electrical steel plate of type 2411 and thickness  $d=0.5$  (mm) with the intensity pulse of magnetic field equal to:

$$H_s(t) = \begin{cases} H_m \sin^n\left(\frac{\pi}{\tau}t\right) & \text{at } 0 < t < \tau; \\ 0 & \text{at } t > \tau \end{cases} \quad (17)$$

using formulas (3, 5-17) and replacement of integrals by sums have been carried out. Steel parameters at  $\mu_0 = 4\pi \cdot 10^{-7}$  (H/m) are shown in table 1. Calculation results under various magnetic field intensity amplitudes  $H_m$  (17) are listed in tables 2, 3, 4. In addition, the sinusoidal steady-state mode at  $n=1$ ;  $\omega_0 = 2\pi f = \pi/\tau$  (tables 2, 3) and the pulse mode at  $\tau=0.1; 1$  (ms);  $n=0;1$  (table 4) have been computed. Specific time dependence of pulsed intensity  $H_s(t)$  (17) and magnetic field flux  $\Phi(t)$  (10) for the steel plate of type 2411 at  $H_m=1000$  (A/m);  $\tau=0.1$  (ms);  $d=0.5$  (mm);  $\Phi_m = \mu_a H_m d = 0.685$  (mWb/m) are shown in Figure 2.



**Figure 2.** Specific time dependence of pulsed intensity  $H_s(t)$  and magnetic field flux  $\Phi(t)$  for the steel plate of type 2411:  $H_m = 1000$  (A/m);  $\tau = 0,1$  (ms);  $d = 0,5$  (mm);  $\Phi_m = \mu_a H_m d = 0,685$  (mWb/m)

**Table 1.** Cold-rolled isotopic electric steel of type 2411 (silicon 2.8-3.8%)

$H_m$ , A/m	$\mu_r = \mu_a / \mu_0$	$B_m = \mu_a H_m$ , T	$\theta_0$ , °C	$\gamma$ , $1/\Omega \cdot m$	$\rho$ , $kg/m^3$	$C_T$ , $J/kg^\circ C$
1000	1090	1.37				
2500	477.6	1.50	20	$2 \cdot 10^6$	7650	472.3
10000	135.3	1.70				

We can see that such characteristics as specific heat loss energy  $W_T$ , average power of this energy  $P_T$  and plate temperature  $\vartheta$  grow with the amplitude  $H_m$  growth and the decline in duration  $\tau$  of magnetic field intensity (17) (tables 2, 4). It is obviously that the use of the steel plate of type 2411 of thickness  $d = 0,5$  (mm) is most likely for the pulse mode at such  $H_m$ ,  $n$ ,  $H_s$  and  $\tau$  values when  $K_\Phi$  and  $K_W$  are maximal (table 4):  $H_m = 10000$  (A/m);  $n = 1$ ;  $\tau = 1$  (ms).

**Table 2.** The sinusoidal steady-state mode of the steel plate of type 2411 and thickness  $d = 0,5$  (mm)

$H_m$ , A/m	$f/\tau$ , Hz/ms	$W_T$ , J/kg	$P_T$ , W/kg	$\vartheta$ , °C	$K_\Phi$	$K_W$
1000	500/1	0.051	50.56	$1.07 \cdot 10^{-4}$	0.988	0.862
	5000/0.1	0.097	970.74	$2.06 \cdot 10^{-4}$	0.474	0.156
2500	500/1	0.063	62.99	$1.33 \cdot 10^{-4}$	1	1.951
	5000/0.1	0.334	3340	$7.07 \cdot 10^{-4}$	0.751	0.230
10000	500/1	0.082	81.56	$1.73 \cdot 10^{-4}$	1	6.876
	5000/0.1	0.760	7604	$1.61 \cdot 10^{-3}$	0.977	0.698

**Table 3.** The sinusoidal steady-state mode of the steel plate of type 2411 at frequency  $f=5000$  (Hz)

$H_m$ , A/m	$K_\phi/K_w$				
	$d=0.05$ (mm)	$d=0.15$ (mm)	$d=0.25$ (mm)	$d=0.35$ (mm)	$d=0.5$ (mm)
1000	1 / 8.534	0.992 / 0.956	0.895 / 0.362	0.714 / 0.213	0.474 / 0.156
2500	1 / 19.476	1 / 2.167	0.984 / 0.789	0.921 / 0.416	0.751 / 0.230
10000	1 / 68.748	1 / 7.640	1 / 2.753	1 / 1.408	0.977 / 0.698

**Table 4.** The pulse mode of the steel plate of type 2411 and thickness  $d=0.5$  (mm)

$H_m$ , A/m	$\tau$ , ms	$H_s/H_m$ , A/m	$n$	$W_T$ , J/kg	$P_T$ , W/kg	$\vartheta$ , °C	$K_\phi$	$K_w$
1000	1	0.5	4.837	0.053	53.49	$1.13 \cdot 10^{-4}$	0.963	0.390
		0.707	1	0.043	43.15	$9.14 \cdot 10^{-5}$	0.994	1.021
		1	0	0.177	176.51	$3.74 \cdot 10^{-4}$	0.971	0.476
	0.1	0.5	4.837	0.047	468.74	$9.92 \cdot 10^{-5}$	0.518	0.167
		0.707	1	0.068	682.48	$1.44 \cdot 10^{-4}$	0.603	0.274
2500	1	0.5	4.837	0.079	78.84	$1.67 \cdot 10^{-4}$	1	0.772
		0.707	1	0.058	58.10	$1.23 \cdot 10^{-4}$	1	2.133
		1	0	0.468	468.02	$9.91 \cdot 10^{-4}$	0.997	0.517
	0.1	0.5	4.837	0.178	1778	$3.76 \cdot 10^{-4}$	0.724	0.201
		0.707	1	0.224	2240	$4.74 \cdot 10^{-4}$	0.816	0.379
10000	1	0.5	4.837	0.107	107.33	$2.27 \cdot 10^{-4}$	1	2.615
		0.707	1	0.079	78.63	$1.66 \cdot 10^{-4}$	1	7.190
		1	0	1.812	1812	$3.84 \cdot 10^{-3}$	1	0.621
	0.1	0.5	4.837	0.742	7421	$1.57 \cdot 10^{-3}$	0.944	0.338
		0.707	1	0.629	6286	$1.33 \cdot 10^{-3}$	0.985	0.855
		1	0	2.207	22070	$4.67 \cdot 10^{-3}$	0.959	0.462

## 5. Conclusion

1. The technique for parameters determination of the magnetic skin effect on the ferromagnetic plate at a specified pulse of magnetic field intensity on the plate surface is proposed. The technique is based on a spectral method and could be applied for a pulsing transformer, a dynamoelectric pulse generator and a commutating inductor that contains an imbricated core.
2. Due to this technique such plate parameters as specific heat loss energy, the average power of this energy and the plate temperature raise, the magnetic flux attenuation factor and the plate q-factor could be calculated. These characteristics depend on the steel type, the amplitude, the rms value, the duration and the form of magnetic field intensity impulse on the plate surface.
3. In virtue of the minor temperature rise of the given type of the plate under one impulse, its considering is necessary only under numerous EMF impulses.
4. The plate thickness is governed by the value of the flux attenuation factor and the plate q-factor that should be maximal.
5. The reliability of the offered methodology is built on a common frequency-domain usage applicable for pulse transient study under zero boundary conditions of the electric circuit and conformity of the obtained results with the sinusoidal steady-state mode.

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