Features of control systems analysis with discrete control devices using mathematical packages

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Abstract. The article contains presentation of basic provisions of the theory of automatic pulse control systems as well as methods of analysis of such systems using the mathematical software widespread in the academic environment. The pulse systems under research are considered as analogues systems interacting among themselves, including sensors, amplifiers, controlled objects, and discrete parts. To describe such systems, one uses a mathematical apparatus of difference equations as well as discrete transfer functions. To obtain a transfer function of the open-loop system, being important from the point of view of the analysis of control systems, one uses mathematical packages Mathcad and Matlab. Despite identity of the obtained result, the way of its achievement from the point of view of user's action is various for the specified means. In particular, Matlab uses a structural model of the control system while Mathcad allows only execution of a chain of operator transforms. It is worth noting that distinctions taking place allow considering transformation of signals during interaction of the linear and continuous parts of the control system from different sides. The latter can be used in an educational process for the best assimilation of the course of the control system theory by students.

1. Introduction

Development of the computer technique and hardware of digital processing of signals which has been observed in recent years led to widespread use of digital microprocessor devices as the part of systems of automatic control of technological processors [1]. However, despite advantages of the use of digital devices in the control systems: low cost, flexibility of solvable tasks, possibility of implementation of complex corrective dynamic elements [2], there are complexities in analysis and simulation of a similar class of systems [3].

Specifics of digital control inevitably lead to the fact that a controlling signal on the controller output exists only in separate moments of time, that is, it has an impulse character [4]. The latter leads to appearance of additional restrictions which are superimposed on parameters of the control system and which non-compliance can lead to loss of stability [5]. At the same time, parameters of a control sequence of pulses have a significant effect on control quality.

In this connection, analysis of pulse control systems (PCS) is the most important task of the modern theory of control, and the ability to reproduce it in practice is a necessary competence of experts in the field of automation [6]. Despite the fact that this task is studied in detail and, in general, is solved [7], practical analysis of PCS still represents complexity for students as it requires understanding of transformations taking place in the control circuit as well as application of specialized software [6].

Such application is difficult in the context of work with the mathematical apparatus describing PCS. The classical method of description of the considered class of systems is compilation and

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solution of difference equations [8] that is in many respects similar to description of the continuous systems by means of differential equations which in many respects are similar to classical methods of solution of differential equations. In spite of the fact that the solution of state equations (both analytical, and numerical ones) does not represent complexity, it cannot find wide practical application [6]. What is explained by the difficulties connected with obtaining difference equations from differential equations.

This work is directed towards selection of basic theoretical data and working methods with mathematical software Matlab and Mathcad widespread in the academic environment, which are necessary and sufficient for formation of practically significant competences of the analysis of elementary PCS during the study. Results of this work can be used, first of all, in an educational process.

2. Pulse control systems

The main difference between PCS and continuous systems is the passage of the signal on a circuit only in certain discrete moments of time. Therefore, the continuous part reacts only to the discrete values of the continuous signal in the moments of quantization $n \cdot T$ $[4, 6, 9]$. As a result, to describe PCS one uses the mathematical apparatus of lattice functions $x(nT)$ (where n is integer) instead of continuous functions $x(t)$ [11]. At that, if quantization interval T is set, then the lattice function is uniquely determined with function $x(t)$.

The ratio between lattice function x (n) and its difference $\Delta^k x(n)$ defines the difference equation [6]. If this ratio is linear, then the difference equation is called the linear one. It can be presented in a general view as [6]:

$$a_k \cdot \Delta^k(n) + a_{k-1} \cdot \Delta^{k-1}(n) + \cdots + a_o \cdot q(n) = f(n), \quad (1)$$

or, expressing differences directly through the required functions,

$$b_k \cdot q(n+k) + b_{k-1} \cdot q(n+k-1) + \cdots + b_0 \cdot q(n) = f(n), \quad (2)$$

where $f(n)$ is the known lattice function, $q(n)$ is the required lattice function representing the solution of the difference equation. This difference equation, containing $q(n)$ and $q(n+k)$ in an explicit form, is called the difference equation of the k^{th} order.

Systems of the difference equations find wide application for description of PCS and serve as an analog of the systems of the differential equations describing the state of the continuous systems. [8].

The typical and the most distributed in practice task of analysis of PCS is the task of analysis of single-circuit linear PCS which flow diagram is presented in figure 1 [4, 6].

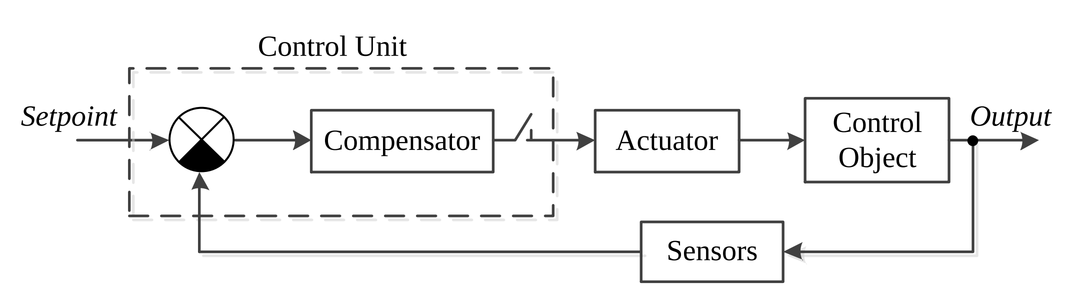

Figure 1. Single-circuit linear pulse system.

Single-circuit PCS can be presented as a combination of digital and continuous subsystems interacting with each other. The continuous part contains a control object as well as intensifying and executive devices [4,6]. The digital part, as a rule, is the control device and integrates the functional elements participating in impulse transformation of a signal.

In the course of transformation of the continuous signal into the discrete one, impulse element executes two operations – quantization on time and pulse modulation [4, 6]. The first one is that the output signal of the impulse element appears in certain discrete, usually equidistant moments $t_i = iT$. As a result [4,6] of impulse modulation, any parameter of the output pulse changes depending on the value of the incoming signal. Depending on the changeable parameter of the pulse [6], one distinguishes different types of modulation: pulse width modulation (PWM), amplitude-pulse modulation and others. The simplest one, from the point of view of implementation, and therefore widespread is PWM.

3. Analysis of impulse control systems

To research linear PCS [4], one uses usually transfer functions (TF) which are a ratio of operator images of output and input signals of the system. As signals exist in PCS in moments of time nT, so they are discrete, TF are called impulse ones and are defined as [4,10]:

$$W(z) = \frac{X(z)}{G(z)}. \tag{3}$$

For lattice functions, it is possible to apply discrete Laplace transform [4, 6]:

$$F(p) = \sum_{i=0}^{\infty} f(i) \cdot e^{-piT}. \tag{4}$$

In case if (4) is converged when $Re(p) < \infty$, original $f(t)$ corresponds to some image $F(p)$. We understand z-transform [1, 4, 6] as an image of the function defined as follows:

$$F(z) = \sum_{i=0}^{\infty} f(i) \cdot z^{-i}. \tag{5}$$

The image exists and corresponds to the original if row (5) converges. Discrete transforms are connected by relation $e^{pT} = z$. The formula of z-transform can be written also for the continuous function in the form of [1, 4, 6]:

$$F(z) = \sum_{i=0}^{\infty} f(t_i) \cdot z^{-i},\ t_i = iT. \tag{6}$$

According to (5) and (6), transitional function $w(t)$ [4] has to serve as original $W(z)$:

$$W(z) = \sum_{i=0}^{\infty} w(iT) \cdot z^{-i}. \tag{7}$$

3.1. Obtainment of impulse transfer function

Let us consider the single-circuit PCS presented in figure 2 [4, 6]. The impulse element in the system is located after comparing the added device. A continuous part contains strengthening devices and the control object. Correcting devices are connected in the feedback circuit.

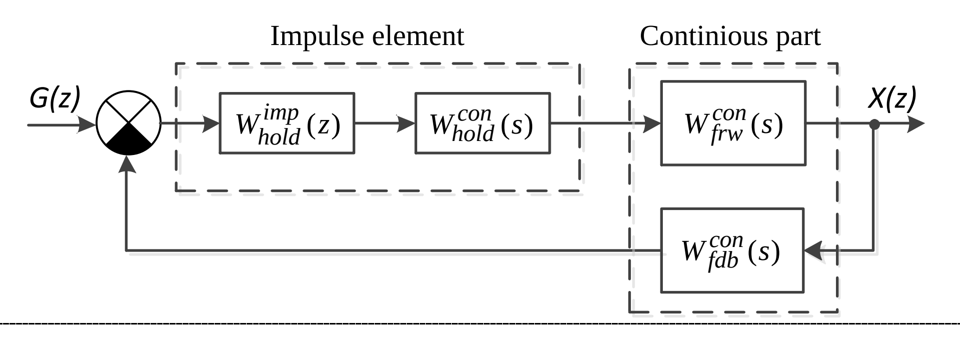

Figure 2. Single-circuit linear impulse system.

To estimate stability of PCS and to determine its quality measures, a TF for the open-loop system should be used. The output signal of continuous system $y(t)$ under zero initial conditions is defined as convolution of the input $x(t)$ signal of a preset value and weight function $w(t)$:

$$y(t) = \int_0^t w(t-\tau)x(\tau)d\tau. \tag{8}$$

The weight function is the original of TF of continuous system $W_{loop}(s)$ so it is connected with this system with inverse Laplace transform. Thus, to pass from continuous Laplace transform to discrete transform, it is enough to do the following actions [6]:

1. To define TF of the continuous part of open system $W_{loop}(s)$, taking into account zero-order hold:

$$W_{loop}(s) = W_{hold}^{con}(s) \cdot W_{frw}^{con}(s) \cdot W_{fdb}^{con}(s). \tag{9}$$

2. With the help of inverse Laplace transform (according to (9)), it is possible to find impulse TF of the open-loop system:

$$w(t) = L_{-1}\{W_{\text{loop}}(s)\}. \tag{10}$$

3. To determine the weight sequence of the system as:

$$w(nT) = w(t)|_{t=Tn}. \tag{11}$$

4. To apply direct discrete Laplace transform:

$$W^0{}_{\text{loop}}(z) = Z\{w(nT)\}. \tag{12}$$

5. To obtain TF of the PCS to multiply the increase of expression (12) on $W_{hold}^{con}(z)$:

$$W_{\text{loop}}(z) = W^0{}_{\text{loop}}(z) \cdot W_{hold}^{imp}(z). \tag{13}$$

3.2. Obtaining transfer function in Matlab

Matlab is the software, being widespread in the academic environment and finding its application when solving automatic control tasks [12,13]. It is explained, on the one hand, by a big set of the modules and instruments focused on the solution of various tasks of the analysis and synthesis of control systems, and on the other hand – by convenience of application of a graphical environment of imitating Simulink modeling for solution of these problems [7].

As example, let us consider the application of Matlab to obtain discrete TF of the linear control system presented in figure 3. Values of parameters of TF of the elements which are parts of the system are given in table 1.

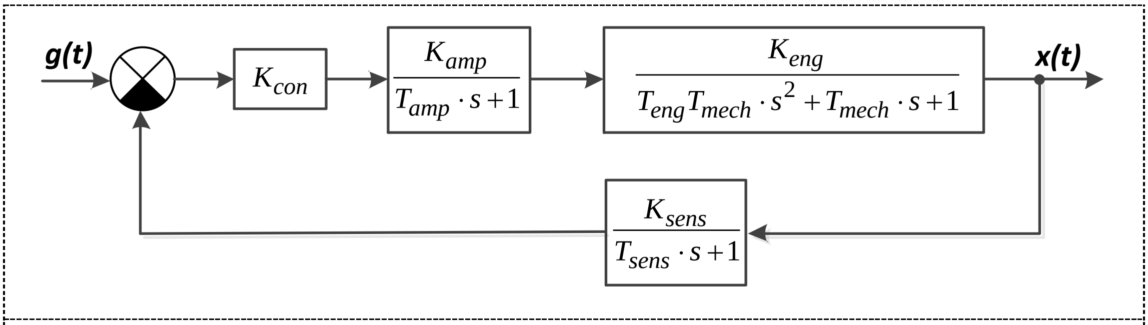

Figure 3. Block diagram of control system.

As it was noted earlier, an essential analysis stage is obtaining of impulse TF of open-loop system $W_{loop}(z)$. For this purpose, one can use the Control System Toolbox module containing a necessary set of functions.

Table 1. Values of parameters of control system.

Parameter	Value
K_{con}	20.00
K_{amp}	18.75
K_{eng}	1.60
K_{sens}	0.02
T_{amp}	0.06
T_{eng}	0.02
T_{mech}	0.30
T_{sens}	0.01
T_{samp}	0.157

The solution of the set problem is reduced to running the following program:

```
>>Kcon=20;Kamp=18.75;Keng=1.6;Ksens=0.02;Tamp=0.06;
  Teng=0.02;Tmech=0.3;Tdc=0.01;
>>  Wcon=tf([Kcon]); Wd=tf([Keng],[Teng*Tmech Tmech 1]);
    Wamp=tf([Kamp],[Tamp 1]); Wsens=tf([Ksens],[Tsens 1]);
>>Wloop=Wcon*Wamp*Weng*Wsens

Transfer function:
                              12
---------------------------------------------------
3.6e-006 s^4 + 0.0006 s^3 + 0.0276 s^2 + 0.37 s + 1

>> Wzloop=c2d(Wloop,0.157)

Transfer function:
    2.665 z^3 + 2.087 z^2 + 0.0386 z + 2.198e-006
---------------------------------------------------------
z^4 - 0.6428 z^3 + 0.042 z^2 - 2.848e-005 z + 4.325e-012

Sampling time: 0.157
```

It is worth noting that when allowing one to work with control systems models at the higher level, Matlab conceals the mathematical aspect of the made transformations, which understanding is essential for students, and for users.

3.3. Obtaining transfer function in Mathcad

To study the mathematical aspect of the problem, the use of Mathcad environment, which allows carrying out symbolical transformations and has a visual graphical interface, seems preferable [14]. The program fragment in Mathcad solving the set problem is presented further.

$$K_{con} := 20 \quad K_{amp} := 18.75 \quad K_{eng} := 1.6 \quad K_{sens} := 0.02$$

$$T_{amp} := 0.06 \quad T_{eng} := 0.02 \quad T_{mech} := 0.3 \quad T_{sens} := 0.01 \quad T_{samp} := 0.157$$

$$W_{amp}(s) := \frac{K_{amp}}{T_{amp} \cdot s + 1} \quad W_{eng}(s) := \frac{K_{eng}}{T_{eng} \cdot T_{mech} \cdot s^2 + T_{mech} \cdot s + 1}$$

$$W_{imp}(z) := \frac{z-1}{z} \quad W_{sens}(s) := \frac{K_{sens}}{T_{sens} \cdot s + 1}$$

$$W_{loop}(s) := W_{amp}(s) \cdot W_{eng}(s) \cdot W_{sens}(s) \quad w(t) := W_{loop}(s) \text{ invlaplace}$$

$$w(n) := w(t) \text{ substitute}, t = T \cdot n \quad W_{tmp}(z) := w(n) \text{ ztrans}$$

$$W_{zloop}(z) := W_{tmp}(z) \cdot W_{imp}(z) \text{ simplify}$$

$$W_{zloop}(z) = \frac{2.665 \cdot z^3 + 2.087 \cdot z^2 + 0.039 \cdot z + 2.202 \cdot 10^{-6}}{z^4 - 0.643 \cdot z^3 + 0.042 \cdot z^2 - 2.848 \cdot 10^{-5} + 4.337 \cdot 10^{-12}}$$

4. Conclusion

The present work provides a summary of some provisions of control theory and of basic control systems of research methods using mathematical packages Mathcad and Matlab, being necessary and sufficient to solve tasks of the analysis of the simplest single-circuit PCS.

Simultaneous application of various software for research allows one to consider interaction of discrete and continuous parts in PCS from different viewpoints and, thus, to form corresponding competences during the study. Besides, the use of special instruments to solve typical tasks of automation promotes acquisition of skills in software automating engineering calculations by students.

Importance of the PCS analysis is caused by wide distribution of digital controllers. The task of the analysis of single-circuit linear PCS considered in the work, in spite of triviality, is important from the practical point of view as typical tasks of digital control of technological objects can be reduced to it.

The materials provided in this work, as well as the method based on parallel presentation of theoretical data, analytical calculations, numerical and structural models were used during composition of the education guidance for the discipline "Automation of design of control systems and means" for the master's degree program of Tomsk Polytechnic University.

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