Methodology of strength calculation under alternating stresses using the diagram of limiting amplitudes

V V Konovodov[1], A V Valentov[2], I S Kukhar [1], O Yu Retyunskiy[2], A S Baraksanov[2]

[1] Novosibirsk State Agrarian University
147 Nikitin St, 630039 Novosibirsk, Russia; phone: (383) 267-35-07
[2] Yurga Institute of Technology, affiliate of Tomsk Polytechnic University
26 Leningradskaya St, 652050 Yurga, Russia; phone: (38451)-6-05-37

E-mail: valentov@mail.ru

Abstract. The work proposes the algorithm to calculate strength under alternating stresses using the developed methodology of building the diagram of limiting stresses. The overall safety factor is defined by the suggested formula.
Strength calculations of components working under alternating stresses in the great majority of cases are conducted as the checking ones.
It is primarily explained by the fact that the overall fatigue strength reduction factor ($K_{\sigma g}$ or $K_{\tau g}$) can only be chosen approximately during the component design as the engineer at this stage of work has just the approximate idea on the component size and shape.

1. Object of research
Let us consider defining safety factor under uniaxial stress state and under pure shear.

Pure shear occurs in the points of bar of circular cross section working in torsion. In most cases safety factor is defined by the suggestion that working stress cycle occurring in the component under study while its operation is similar to the limit cycle. It means that skewness coefficients R and parameters ρ of working and limit cycles are the same [1].

Safety factor can be most simply defined in case of symmetric cycle of stress variation, as fatigue endurance limits of material by these cycles are usually known. In cases with lack of experimental data on fatigue endurance limits of materials one can apply empirical relations between fatigue endurance limits by symmetric cycles and ultimate strength limit (σ_β) under tension.

Safety factor is a ratio of fatigue endurance limit defined for a component to the reference value of the maximum stress [1,2], occurring in the component critical point. Reference value is the value of stress defined by the basic formulas of materials strength, i.e. not considering the factors influencing the value of fatigue endurance limit (stress concentration, etc.).

Methods:
Thus, to define safety factor under symmetric cycles we obtain the following dependencies:
under bending:

$$n = \frac{\sigma_{-1g}}{\sigma_{\max}} = \frac{\sigma_{-1}}{K\sigma_g \cdot \sigma_{\max}}; \qquad (1)$$

under tension - compression:

$$n = \frac{\sigma_{-1pg}}{\sigma_{\max}} = \frac{\sigma_{-1p}}{K\sigma_g \cdot \sigma_{\max}}; \qquad (2)$$

under torsion:

$$n = \frac{\tau_{-1g}}{\tau_{\max}} = \frac{\tau_{-1}}{K\tau_g \cdot \tau_{\max}} \qquad (3)$$

There are several ways to build schematized diagram of limiting amplitudes [4,11]. However we need additional experimental data for that purpose. The only way where one can deal without the mentioned data is given in Fig. 1.

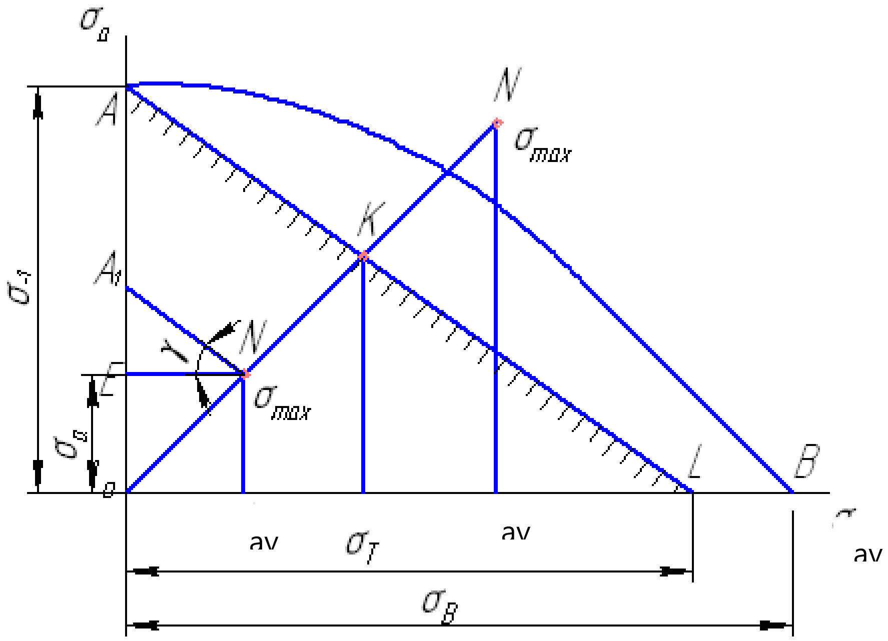

Fig.1. Schematized diagram of limiting amplitudes

Such diagram is used abroad, and economical feasibility of its application has been recently increasing with approximation of σ_T (due to application of different strengthening processing methods) to σ_B.

Using the conclusion suggested by M.N. Liuboshits [1,3], safety factor (or loss coefficient) by fatigue fracture for a cycle shown by point N is defined as the ratio:

$$n = \frac{OK}{ON} = \frac{OA}{OA_1} \qquad (4)$$

Following from Fig.1:

$$OA_1 = OE + EA_1 = \sigma_a + \sigma_{av} \cdot tg\gamma = \sigma_a + \psi_\sigma \cdot \sigma_{av},$$

where $tg\gamma = \psi_\sigma$.

Then we put the obtained values of OA and OA_1 into the ratio (4):

$$n = \frac{\sigma_{-1}}{\sigma_a + \psi_\sigma \cdot \sigma_{av}} \tag{5}$$

The similar case is with variables of shear stresses:

$$n = \frac{\tau_{-1}}{\tau_a + \psi_\sigma \cdot \tau_{av}} \tag{6}$$

The values ψ_σ and ψ_τ depend on the accepted calculation of schematized diagram of limiting amplitudes and on the component material.

To define safety factor for a definite component we need to take into account the influence of fatigue strength reduction factor $K\sigma_g$ ($K\tau_g$).

According to scientists [2,3,5,8] stress concentration, size effect and surface condition influence only the values of limiting amplitudes and show almost no influence on the values of limiting mean stresses. Thus, as accepted in practice of calculations fatigue strength reduction factor is referred only to amplitude cycle stress. Then the final formulas to define the safety factor by fatigue fracture can be as follows:

under bending:

$$n = \frac{\sigma_{-1}}{K\sigma_g \cdot \sigma_a + \psi_\sigma \cdot \sigma_{av}} \tag{7}$$

under torsion:

$$n = \frac{\tau_{-1}}{K\tau_g \cdot \tau_a + \psi_\tau \cdot \tau_{av}}. \tag{8}$$

Under tension-compression we should use the formula (7), but instead of σ_{-1} one should put fatigue endurance limit σ_{-1p} under symmetric cycle tension-compression.

Formulas (7) and (8) are true in all ways of schematization of diagram of limiting amplitudes, only the values of coefficients ψ_σ (ψ_τ) are changed.

Formula (7) is obtained for cycles with positive mean stresses ($\sigma_{cp} \geq 0$): for cycles with negative (compressive) mean stresses ($\sigma_{cp} < 0$) it should be assumed that $\psi_\sigma = 0$, i.e. proceed on the assumption that in compression zone the line of limiting stresses is parallel to X-axis.

Along with safety factor by fatigue fracture one should define safety factor by fluctuating stresses.

Under bending (or under tension-compression):

$$n_{\sigma_T} \frac{\sigma_T}{\sigma_{\max}} = \frac{\sigma_T}{\sigma_a + \sigma_{av}} \tag{9}$$

under torsion:

$$n_{\tau_T} = \frac{\tau_T}{\tau_{\max}} = \frac{\tau_T}{\tau_a + \tau_{av}} \tag{10}$$

The lowest of the safety factors defined by the formulas (7), (8) or (9), (10) should be accepted as the reference one.

In some simple cases it is possible to calculate fatigue resistance by admissible stress $[\sigma_R]$, corresponding to the specified characteristic of a cycle (ρ_σ or ρ_τ) [6,7]. Let us derive a formula for the admissible normal stress by the cycle with the characteristic ρ_σ. Suggesting that in formula (7) $n=[n]$, $\sigma_a=[\sigma_a]$; $\sigma_{av}=[\sigma_{av}]$, we obtain:

$$[n] = \frac{\sigma_{-1}}{K\sigma_g[\sigma_a] + \psi_\sigma[\sigma_{cp}]} = \frac{\sigma_{-1}}{[\sigma_a](K\sigma_g + \psi_\sigma \frac{[\sigma_{av}]}{[\sigma_a]})} = \frac{\sigma_{-1}}{[\sigma_{av}](K\sigma_g \frac{[\sigma_a]}{[\sigma_{av}]} + \psi_\sigma)}$$

Thus we get:

$$[\sigma_a] = \frac{\sigma_{-1}\rho_\sigma}{[n](K\sigma_g \cdot \rho_\sigma + \psi_\sigma)},$$

$$[\sigma_{cp}] = \frac{\sigma_{-1}}{[n](K\sigma_g \cdot \rho_\sigma + \psi_\sigma)},$$

but

$[\sigma_R] = [\sigma_a] + [\sigma_{av}]$,

and, therefore,

$$[\sigma_R] = \frac{\sigma_{-1}(\rho_\sigma + 1)}{[n](K\sigma_g \cdot \rho_\sigma + \psi_\sigma)} \tag{11}$$

The same way for calculation of admissible shear stress:

$$\tau_R = \frac{\tau_{-1}(\rho_\tau + 1)}{[n](K\tau_g \cdot \rho_\tau + \psi_\tau)} \tag{12}$$

We shall record without justification a dependence to define safety factor while bar operation under joint action of bending with torsion or torsion with tension (compression) or bending with torsion and tension (compression) [9,10,11,12], i.e. for those cases when plane stress state occurs in the component critical point. The overall safety factor in the mentioned cases is defined from the equation:

$$\frac{1}{n^2} = \frac{1}{n^2{}_\sigma} + \frac{1}{n_\tau^2} \tag{13}$$

Where n – the overall safety factor, n_σ – safety factor by normal stresses; n_τ – safety factor by shear stresses.

The same way we define the overall safety factor by yield stress, one just needs to substitute n_σ and n_τ with $n_{\sigma T}$ and $n_{\tau T}$ accordingly.

Formula (13) is applicable in case where normal and shear stresses in the verified point of a component are changed simultaneously, i.e. they reach their maximum and minimum values at the same time.

Results

Using system approach within the conducted studies enabled to formulate the fundamental principles of how to provide reliability of components and increase their fatigue strength.

Strength and fatigue properties of material show to some extent the fatigue properties of the ready components however they cannot serve the reliable basis to define fatigue limit of a definite kind of product operating under some dynamic conditions.

Thus to define safety factor one needs to have experimental data on components fatigue strength, including the ones by symmetric cycle, as the last parameter is included in the majority of dependencies connecting the limiting stresses under different coefficients of asymmetry.

In order to obtain fatigue characteristics by asymmetric cycles of loading, fatigue testings are conducted under different asymmetries of a cycle. Diagram of limiting amplitudes is built on the basis of testing results. The diagram characterizes the dependency between maximum and minimum limiting stresses $\tau_{maa,}\tau_{\min}$ (along the Y-axis) and the mean cycle stress (X-axis).

The limits of component fatigue strength by different coefficients of asymmetry r can be defined from diagram of limiting amplitudes, as well as fatigue safety factor, as the ratio of ultimately admissible stress τ_r, when the spring bears the basic number of loading cycles Nb to the maximum stress, acting in the spring under dynamic loading [8,9]:

$$n = \frac{\tau_r}{\tau_{\max}} \tag{14}$$

Conclusion

All the known ways of calculating fatigue strength of components come to defining fatigue safety factor. In this case the value of safety factor when calculating its fatigue strength depends on the accuracy of defining forces and stresses, the level of technology to recover components and other factors that can find application in calculation. Safety factors can vary from 1.3 to 3 due to the mentioned reasons.

Special research devoted to studies of strength in conditions of cyclic loading, and practice of machinery operation proves that the state of the surface layer of components defines their durability to a large extent as due to environmental influence and imperfect processing the surface has damages and micro roughnesses playing the role of stress concentrate. The largest influence of the surface layer condition on the fatigue strength is explained by the fact that the surface layers undergo the largest stresses while working under torsion and bending.

Thus understanding the existing knowledge of the influence of design and engineering and dynamic factors on the fatigue strength of machinery components one can conclude that applying processing procedures increasing the material quality and the state of surface layer can be an effective way to increase their fatigue resistance and service life. In order to predict the service life of the recovered machinery components it is necessary to develop methodology to evaluate their operational life depending on the recovery technology considering the fatigue characteristics of metal in the definite conditions of operation and define the influence of dynamic factors on the fatigue strength of a product.

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