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Radiation of the magneto-crystalline undulator

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Abstract. The radiation at grazing incidence of relativistic positively charged particles on the crystal surface in the presence of magnetic field is studied theoretically. The magnetic field is supposed to be parallel to the surface. Dependent on the initial conditions the particle can be captured in the channeling mode and perform periodic oscillations along the surface of the crystal. The spectrum, angular distribution and polarization of radiation are calculated. The emission spectrum of a single particle is discrete and it extends up to very large numbers of harmonics. If the magnetic field is much weaker than the electric field of atoms, the frequency range of radiation of the particle beam does not depend on magnetic field and is defined solely by the energy of the particles and by the surface averaged potential, though the frequency of the first harmonic is defined only by the magnetic field. In case of channeled positrons the characteristic energy of the emitted photons is of order $10\gamma^{3/2}$ (eV), where γ is the particle relativistic factor. The main part of radiation is bound to a narrow cone and is polarized largely orthogonal to the surface of the crystal.

1. Introduction

New sources of X-ray and gamma radiation, consisting of the particle accelerators and precisely oriented mono-crystals are created on the basis of the channeling phenomenon. There is interest in developing a radiation source using the periodically deformed crystals. See, for example, the recent papers [1, 2]. The channeled positrons are bound to the bent atomic crystal planes, and emit monochromatic radiation of a frequency which depends on the particles energy and the period of the crystal plate bending. Such a "crystalline undulator" was first proposed in [3, 4], where the deformation of the crystal was proposed to be performed by an ultrasonic wave. There are also a number of projects of multi-crystal undulators where the particle is deflected successively in the opposite directions at passing through a series of mutually oriented ultrathin crystals (see, for example, [5]). The radiation generated in the crystal undulators can be harder than one can get in a magnetic undulator at the same energy of the positrons, since the period of the deformed crystal may be much smaller than the period of undulator magnetic field.

A "magneto-crystalline undulator" which produces electromagnetic radiation by means of charged particles moving near the flat surface of a crystal in a magnetic field parallel to the surface was proposed in [6]. The positively charged particles at grazing incidence to the surface of the crystal are reflected from the surface and returning back by the magnetic field, if the field is orthogonal to the average velocity of the particles. The quantum energy levels of the particle transverse motion at the surface channeling in a magnetic field, and the possible frequency of radiation were also found in [7].

Interaction of the particles with the crystal surface in the presence of a magnetic field can be observed when the crystal is mounted on the beam path in the chamber of a circular accelerator. The dynamics of the beam in an accelerator at its interaction with a radiator which is thin in profile and extended along the beam, in case of grazing incidence geometry is not investigated in accelerator physics. An effect of asymmetric generation of the X-ray radiation produced at grazing interaction of 33 MeV electrons with 50 mm Si plate in a magnetic field have recently been observed in experiments reported in [8].

Here we report our theoretical investigation on radiation at the surface channeling of fast positrons. It is assumed that the crystal is placed in a uniform magnetic field parallel to the crystal surface. The initial velocity of the particle is directed so that the Lorentz force exerted by the magnetic field is directed to the surface of the crystal. The result of the action of two competing forces – the force caused by the magnetic field and the repulsive force of the crystallographic plane, is that the particle oscillates in the vicinity of the surface.

2. The particle dynamics

Let the axis Z be directed along the vector of the magnetic field, and the axis Y be orthogonal to the surface of the crystal directed outward (figure 1). The axis X lies in the crystal surface so that the vector $\boldsymbol{v}$ of the particle initial velocity is lying in the plane XY. The crystallographic plane forming the surface of the crystal has the coordinates $y = 0$. In order to improve the

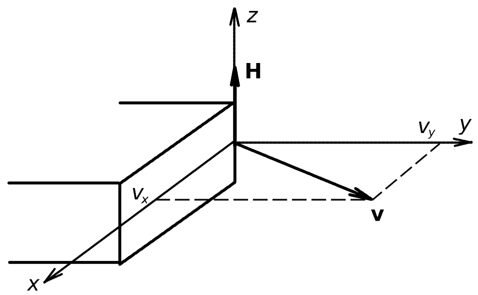

Figure 1. The coordinate system

possibility of analytical solution of the problem we simplify the potential of the electric field of the crystallographic plane as follows:

$$U(y) = U_0 e^{-y/a}.$$

where U_0 is the averaged potential of atomic plane, and a is the screening radius.

The electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ fields in the halfspace $y > 0$ can be written as

$$\boldsymbol{H} = (0, 0, H), \quad \boldsymbol{E} = (0, E(y), 0), \quad E(y) = -\frac{dU(y)}{dy} = \frac{U_0}{a} e^{-y/a},$$

where H is a constant. Relativistic equations of motion of a particle in this field have the form

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = \frac{e}{c}\dot{y}H, \quad \frac{\mathrm{d}p_y}{\mathrm{d}t} = eE(y) - \frac{e}{c}\dot{x}H. \tag{1}$$

Here p_x, p_y are projections of the particle momentum on the coordinate axes:

$$p_i = \gamma m_0 \dot{x}_i, \quad x_i = x, y, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta^2 = \frac{\dot{x}^2 + \dot{y}^2}{c^2},$$

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m_0 and e are the mass and the charge of the particle, c is the speed of light. We assume that $\dot{y}$ is much less then $c\gamma^{-1}$. Integration of the equations (1) gives

$$\dot{x} = \omega y + V_1, \tag{2}$$

$$\frac{m\dot{y}^2}{2} = \mathcal{E}_0 - \frac{eU_0}{a} e^{-y/a} - m\omega\left(\frac{\omega y^2}{2} + V_1 y\right). \tag{3}$$

Here $m = m_0\gamma$, $\omega = eH/mc$, $\mathcal{E}_0$ and $V_1 = v_{x0} - \omega y_0$ are constants of integration. The last equation describes the one-dimensional motion of a particle in direction of the Y axis in the field with effective potential energy

$$U_{ef} = \frac{eU_0}{a} e^{-y/a} + m\omega\left(\frac{\omega y^2}{2} + V_1 y\right).$$

The integral of motion $\mathcal{E}_0$ plays the role of "transverse" energy of the particle. It is convenient to introduce the dimensionless quantities

$$V = \frac{U_{ef}}{eU_0}, \quad \chi = \frac{y}{a}, \quad \eta = \frac{aH}{U_0} = \frac{amc^2}{ReU_0},$$

where $R = c/\omega$ is a parameter that determines the strength of the magnetic field. It is approximately equal to the particle orbit radius in this field. The parameter η is the ratio of the force acting on the particle by the magnetic field to the electrostatic repulsion force of the crystallographic plane. The dimensionless potential V with this notations takes the form

$$V = e^{-\chi} + \chi\eta\beta_0 + \frac{a}{R}\beta_0\eta\chi(\chi - 2\chi_0), \tag{4}$$

where $\chi_0 = y(0)/a$. Since we are only interested in periodic motion in the Y-direction, we can take as a starting point, without loss of generality, the point where y has a maximum. Then $v_{0y} = 0, v_{0x} = v_0 = c\beta_0$, where v_0 is the initial velocity of the particle. We put $\beta_0 = 1$ where it is not critical. The value of a/R is much smaller than unity. The value of η in normal conditions is also small. For example, if $a = 1$ Å, $H = 10^4$ Oe, $eU_0 = 30$ eV, we have $\eta = 10^{-2}$. Consequently, the terms containing a/R in the formula for the potential energy can be neglected. As a result, the interaction potential takes the form

$$V = e^{-\chi} + \chi\eta. \tag{5}$$

A plot of this function is shown in figure 2. The minimum of potential lies in the plane with coordinate $\chi = \chi_m$ and has a value of V_m:

$$\chi_m = -\ln\eta, \quad V_m = \eta(1 - \ln\eta). \tag{6}$$

In the considered case $\eta \ll 1$ we have: $\chi_m \gg 1$ and $V_m \ll 1$. This means that the coordinate y_m of the potential minimum is located at a considerable distance from the atomic plane ($y_m \gg a$). This makes it possible to split the space above the surface of the crystal into two areas: $y < y_m$ and $y \geq y_m$. We assume that potential in region $y < y_m$ is represented by the potential of atomic plane, and in the area $y \geq y_m$ by potential of the magnetic field. Such combined potential has the form

$$V(\chi) = \begin{cases} e^{-\chi}, & \chi \leq \chi_m, \\ \eta\chi, & \chi \geq \chi_m. \end{cases} \tag{7}$$

Hence, we have to solve the equations of motion in two separate regions: $\chi \leq \chi_m$ and $\chi \geq \chi_m$ and take the final coordinates in the first region as the initial coordinates for the second region. The solution of equations (2) and (3) in the region $\chi \geq \chi_m$ has the form

$$x = v_0 t, \quad y = y_0 - \frac{1}{2}R(\omega t)^2. \tag{8}$$

And in the region $\chi \leq \chi_m$

$$x = v_0 t, \quad y = 2a \ln \frac{\cosh \alpha(t - t_0)}{\sqrt{\varepsilon}}, \tag{9}$$

with

$$\alpha = \omega\sqrt{\frac{y_0 R}{2a^2}}, \quad \varepsilon = \frac{\mathcal{E}_0}{eU_0} \tag{10}$$

and integration constant t_0, defined by condition of the trajectory continuity in the plane $y = y_m$:

$$t_0 = t_1 + \frac{1}{\alpha}\,\mathrm{Arth}\,\frac{\sqrt{\chi_0^2 - \chi_1^2}}{\chi_0}, \quad t_1 = \sqrt{\frac{2a(\chi_0 - \chi_1)}{c\omega}}, \tag{11}$$

where t_1 is the time moment when the particle crosses the plane $y = y_m$. Actually, if we neglect the short-range potential in region $y > y_m$, the particle moves in magnetic field along an arc of a circle. But in accepted approximation this arc is equivalent to osculating parabola (8). One period of the trajectory of a particle moving under the law (8) and (9) is shown in figure 3.

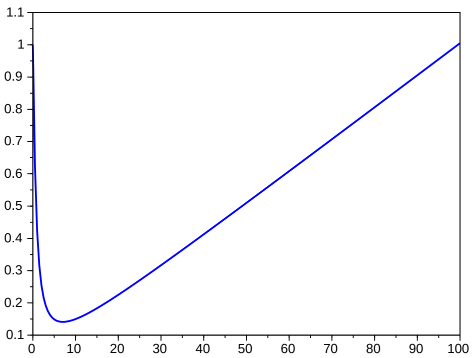

Figure 2. Potential $V(\chi)$, plotted according to equation (5) for $\eta = 0.01$.

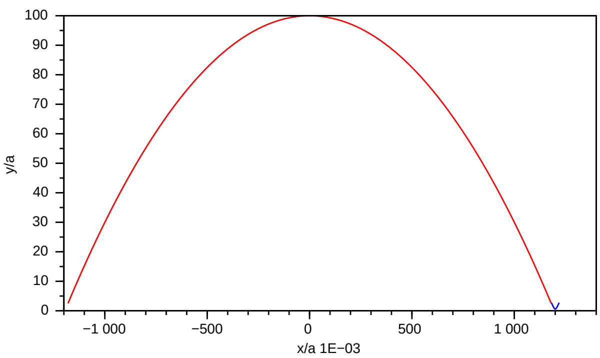

Figure 3. One period of trajectory of a charged particle. $H = 50$ kOe, $mc^2 = 40$ MeV, $eU_0 = 30$ eV, $a = 1$ Å, $\varepsilon = 0.5$.

The trajectory period $T = 2t_0$ in approximation $\eta \ll 1$ consists mainly of the time spent by the particle in the magnetic field: $T \approx 2t_1$. This follows from equations (10) and (11) — as $\alpha \gg \omega$, the particle spends much less time $2(t_0 - t_1)$ in the electric field of the surface atomic layer than in the magnetic field. It is also seen in figure 3.

3. Radiation

Let us denote by $\omega_0 = 2\pi/T$ the frequency of the particle oscillation in y direction. Since the particle motion is periodic, we use the formulae obtained in the theory of undulator radiation (see., e.g. [9, 10]). In particular, the spectral and angular distribution of the energy $\mathrm{d}\mathcal{E}$, emitted in a solid angle $\mathrm{d}\Omega$, in the frequency range $\mathrm{d}\tilde{\omega}$ is defined by expression

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}\Omega \mathrm{d}\tilde{\omega}} = \frac{e^2\gamma^6(1 - \boldsymbol{\beta n})}{c\omega_0^2}\frac{\sin^2 \pi\nu N}{\sin^2 \pi\nu}(\rho_\sigma + \rho_\pi)|\dot{\boldsymbol{\beta}}(\nu)|^2, \tag{12}$$

where $\boldsymbol{n}$ is the unit vector in the direction of radiation, N is the number of complete periods of the particle oscillations, $\dot{\boldsymbol{\beta}}(\nu)$ is the Fourier transform of acceleration, ρ_σ and ρ_π are the functions defining pattern of mutually orthogonal polarization components of radiation. Namely, ρ_σ defines the angular distribution of radiation polarized parallel to the plane of the particle trajectory. The frequency of the first harmonic is determined by period of the trajectory shifted by the Doppler effect, and ν indicates the number of harmonic. We define the direction of radiation $\boldsymbol{n}$ and the unit vectors of linear polarization $\boldsymbol{n}_\sigma$ and $\boldsymbol{n}_\pi$, in a spherical coordinate system as shown in figure 4. Functions ρ_σ and ρ_π in this case are of the form

$$\rho_\sigma = \frac{8(1-\psi^2\cos 2\phi)^2}{(1+\psi^2)^5}\,, \quad \rho_\pi = \frac{8\psi^4 \sin^2 2\phi}{(1+\psi^2)^5}, \tag{13}$$

where $\psi = \vartheta\gamma$. A detailed analysis of the polarization components ρ_σ and ρ_π and its graphical representation can be found in [9, 10]. Calculating the Fourier components of the particle acceleration and substituting the result into expression (12), we obtain

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}\Omega\mathrm{d}\tilde{\omega}} = \frac{2e^2\gamma^4\omega^2(1+\psi^2)}{c\omega_0^2}\left|\frac{\xi\nu}{\sinh\xi\nu} - \frac{\sin\omega_0 t_1\nu}{\omega_0 t_1 \nu}e^{i\pi\nu}\right|^2 \frac{\sin^2\pi\nu N}{\sin^2\pi\nu}(\rho_\sigma+\rho_\pi), \tag{14}$$

where $\xi = \pi^2 a/y_0$. The first term containing the hyperbolic sine is responsible for radiation in the electric field of the crystallographic plane. Since $\xi \ll 1$, this term is significantly different from zero at high harmonic numbers $\nu \sim \xi^{-1} \gg 1$. The emission spectrum (14) consists of narrow lines at the integer values of ν. The envelope of the spectrum is shown in figure 5.

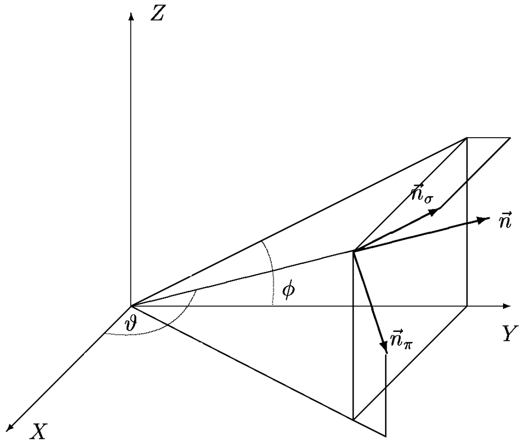

Figure 4. Unit vectors in the spherical coordinate system.

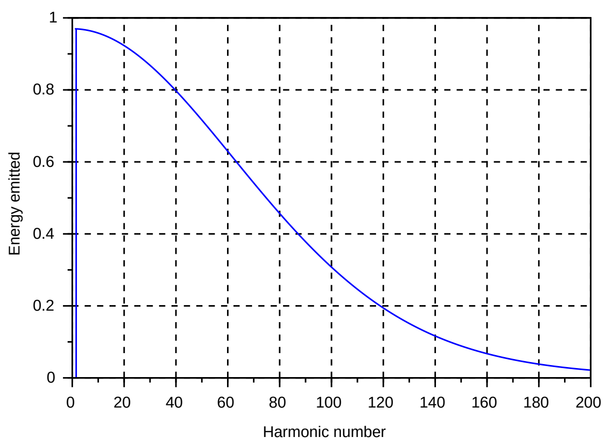

Figure 5. The radiation spectrum of a single particle. H = 10 kOe, mc^2 = 40 MeV, eU_0 = 30 eV, $a = 1$ Å, $y_0 = 500$ Å.

The angular spectral density of radiation of a beam of charged particles can be obtained by averaging of expression (14) over the transverse energy of the particles. Suppose that the beam of particles is of uniform distribution and insides on the crystal parallel to its surface. We assume that the particles perform very large number of oscillations, and proceed to the limit $N \to \infty$. The spectrum in this case becomes discrete and ν takes only integer values. The second term in (14) vanishes, as $\Omega_0 t_1 n \approx \omega_0 t_0 n = \pi n$, and the angular spectral density becomes

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}\Omega\mathrm{d}\tilde{\omega}} = \frac{2e^2\gamma^4 L\sqrt{y_0}(1+\psi^2)}{\pi^2 c R^{3/2}}\frac{\xi^2\nu^2}{\sinh^2\xi\nu}(\rho_\sigma+\rho_\pi),$$

where $L = 2N\sqrt{2y_0 R}$ is the channeling length (the projection of the trajectory on the x axis). Next we average this equation over the initial coordinate y_0 of the particle. Using notation $x = \xi\nu = \nu\pi^2 a/y_0$ we obtain the spectral and angular distribution of the emitted energy of the beam per particle

$$\frac{\overline{\mathrm{d}\mathcal{E}}}{\mathrm{d}\Omega\mathrm{d}\tilde{\omega}} = \frac{2e^2\gamma^4 L\sqrt{2b}(1+\psi^2)}{\pi^2 c R^{3/2}}(\rho_\sigma + \rho_\pi)S(z), \tag{15}$$

where b is the maximum possible value of the initial position, in which the particle is captured into channeling. The value of b can be found from condition that the transverse energy of the particle is less than the potential barrier $V = 1$. It follows then from equation (7) that $b = a/\eta$. The emission spectrum is determined by the spectral function $S(z)$

$$S(z) = z^3 \int\limits_z^{kz} \frac{\mathrm{d}x}{x^2 \sinh^2 x}, \quad z = \frac{\pi\tilde{\omega}\sqrt{aR\eta}(1+\psi^2)}{2^{3/2}c\gamma^2}, \quad k = \sqrt{\frac{b}{y_m}}. \tag{16}$$

The graph of the spectral function $S(z)$ for $k = 10$ is shown in figure 6. The emission spectrum

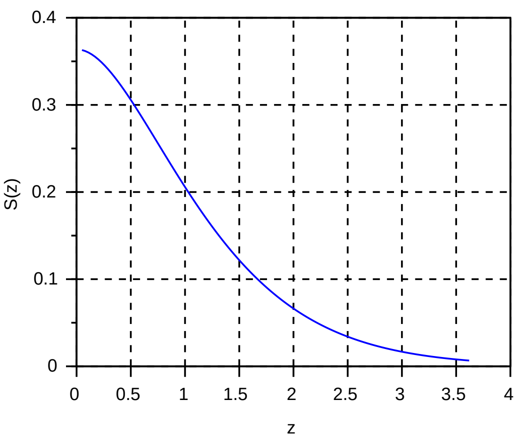

Figure 6. The spectral function $S(z)$ for $k = 10$.

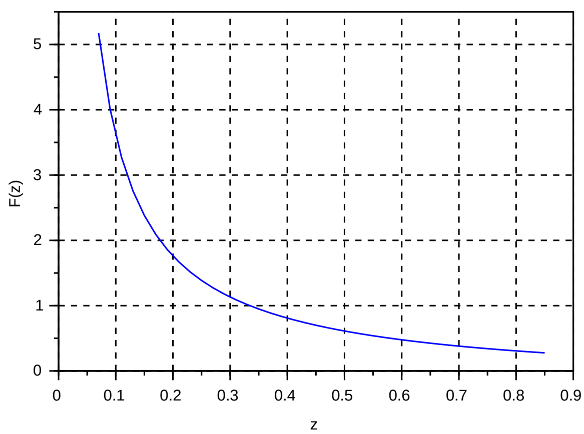

Figure 7. The normalized photonic spectrum $F(z)$ for $k = 10$.

extends from zero to a frequency $\tilde{\omega}_{cr}$ corresponding to the values of $z \sim 1$ or

$$\tilde{\omega}_{cr} \sim \frac{c\gamma^2}{\sqrt{aR\eta}} = \frac{c}{a}\sqrt{\frac{eU_0}{m_0 c^2}}\,\gamma^{3/2}. \tag{17}$$

As one can see, this frequency does not depend on the magnetic field. This is due to the fact that most of the energy of radiation is generated during reflection of the particle from the surface. In this process the particle reverses its transverse velocity in a very short period of time in comparison with the period of motion T. The duration of the braking process in the vicinity of crystal plane is of order of α^{-1}. Taking into account the Doppler effect, the characteristic frequency of the radiation is $\gamma^2\alpha$. For example, for parameters $eU_0 = 30$ eV, $a = 1$ Å, and $\gamma = 100$ the photon energy is equal to $\hbar\tilde{\omega}_{cr} \approx 15$ keV. The photon energy increases with the particle energy as $\gamma^{3/2}$.

If we devide expression (15) by $\hbar\tilde{\omega}$ where $\hbar$ is the Planck's constant, we get the number of photons N_f per unit frequence per unit solid angle

$$\frac{\mathrm{d}N_f}{\mathrm{d}\Omega\mathrm{d}(\hbar\tilde{\omega})} = \frac{e^2\gamma^2 L\sqrt{ay_m\eta}(1+\psi^2)^2}{\pi\hbar^2c^2R}(\rho_\sigma + \rho_\pi)F(z), \tag{18}$$

where $F(z) = S(z)/z$ can refered to as a normalized photonic spectrum. The function $F(z)$ is plotted in figure 7.

4. Conclusion

Under assumption that the magnetic field is much weaker than the average electric field of atoms in the surface layer, the main part of the particle trajectory lies exterior to the screening radius of the surface layer, and represents an arc of a circle, on which the particle moves in the magnetic field. The frequency of the first harmonic of radiation is determined by the length of the arc of the circle. The main part of radiation is generated during the sharp braking of the particles in the electric field of atoms forming the surface. The radiation frequency is inversely proportional to time spent by the particle within the range of screening radius and is much greater than the frequency of the first harmonic. The spectrum extends to the harmonic numbers of order y_0/a, where a is the screening radius, and $y_0 \gg a$ is the maximal distance between the particle and the crystal surface.

The frequency of radiation at the first harmonic and the number of harmonics forming the spectrum are dependent on the magnetic field. However, the spectral range in which radiation is generated by a parallel beam of particles does not depend on the magnetic field, and is determined only by the energy of the particles and the average potential of the surface layer of atoms. Angular distribution and polarization of radiation are typical for radiation of an ultra-relativistic particle moving along a flat trajectory and oscillating with a small amplitude: the radiation is concentrated in a narrow cone in the direction of the average velocity and polarized mainly in the plane of the trajectory.

Radiation effects caused by the surface channeling were also observed at grazing incidence of the electrons on the surface of silicon crystal placed into a betatron chamber [8]. However, the theory developed in this paper is not applicable in this case because interaction of negatively charged particles with the surface is more complicated.

Acknowledgments

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