Integrated Method – the Optimum Way to Improve the Quality of Frequency Response Characteristics of the Space Vehicle Attitude Control System

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Abstract. The integrated method applied to the design of technical systems is a process in which various project, calculation and verification procedures are interconnected and interrelated. The results of procedures are used in a certain sequence, thus ensuring maximum reachable optimality of the system being designed.

Introduction

Executive element of the attitude control system is one of the space vehicle's technical systems. At the moment one of the most popular type of executive elements is the one based on the torque-controlled reaction wheel motor (RWM). Generally RWM (Fig. 1) comprises a solid rotor with expressed rotor rim *1*, fitted onto the supports *2*, an electric motor *3* with rotor attached to the reaction wheel and stator attached to the to the base *4* [1].

Methods. Experimental Procedure

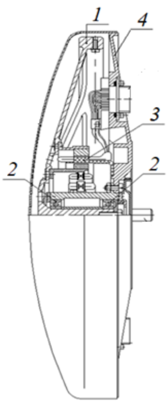

Figure 1. RWM design

Design of RWM, as with any other electromechanical system, starts with calculation of major operation parameters and, subsequently, of required mass-dimensional characteristics. RWMs are designed based on the classical mathematical model shown in the form of Equations (1)–(11). They demonstrate interrelations of its operating parameters.

Due to toughening of the requirements imposed on executive elements of attitude control systems, the mathematical model was subjected to expansions with regard to critical angular velocity, induced perturbated oscillations, resonant angular velocity of the wheel and self frequencies of the system parts.

$$H = \frac{1}{32}\pi\gamma h \Omega D_{w^4}\left(1+k^4\right); \quad \text{kinetic momentum} \quad (1)$$

$$m = \frac{\gamma h\left(R^2 - r^2\right)}{2}; \quad \text{wheel mass} \quad (2)$$

$$N = \frac{1}{\Omega}\left(\frac{C}{Q_l}\right)^{3,3}; \quad \text{operation life} \quad (3)$$

$$P = 1.028 M_c \Omega 10^5 \eta; \quad \text{power consumption} \quad (4)$$

$$M_r = \left(M_c + M_{bb} + M_a\right); \quad \text{resistive torque} \quad (5)$$

$M_c = \frac{dH}{dt} = J\Omega$	wheel control torque	(6)
$T = 4T_{const}$	run-up time	(7)
$M_{bb} = 1.11K\left(1 + \frac{D_{inner}}{d_r} Q_l\right);$	frictional torque of ball bearings	(8)
$M_a = 2\pi\rho C_{L(T)}\Omega^2;$	aerodynamic resistance torque	(9)
$C_L = 0.53R_e^{-0.5};$	aerodynamic coefficients for laminar and turbulent flows	(10)
$C_T = 0.0287R_e^{-0.2};$		(11)

$$\omega_{cr} = \frac{1}{\sqrt{0.5(m\alpha - J_Z\beta) + \sqrt{0.25(m\alpha - J_Z\beta)^2 + mJ_Z(\alpha\beta - \xi^2)}}}$$

with

$$\alpha = \frac{ab^2}{3EI_1} + \frac{b^3}{3EI_2},\ \beta = \frac{a}{3EI_1} + \frac{b}{EI_2},\ \xi = \frac{ab}{3EI_1} + \frac{b^2}{2EI_2}$$

critical angular velocity of WM (12)

$$\vartheta = \frac{\left[Gl - (J_Z + J_X)\Omega^2\right]\left(J_Z - J_E\right)\varepsilon\Omega^2}{(Gl - J_Y\Omega^2)(Gl - J_X\Omega^2) - (J_Z\Omega^2)^2}\sin\Omega t \quad (13)$$

$$\psi = -\frac{\left[Gl - (J_Z + J_Y)\Omega^2\right]\left(J_Z - J_E\right)\varepsilon\Omega^2}{(Gl - J_Y\Omega^2)(Gl - J_X\Omega^2) - (J_Z\Omega^2)^2}\cos\Omega t \quad (14)$$

induced oscillations occurring along two orthogonal axes of WM

$$\omega_r = \pm\sqrt{\frac{Gl\left(J_X + J_Y\right) \pm \left[Gl\left(J_X + J_Y\right) - \frac{GlJ_XJ_Y}{J_X + J_Y}\left(1 - \mu^2\right)\right]}{2J_XJ_Y\left(1 - \mu^2\right)}}$$

resonant angular velocity of the wheel (15)

$f_m = 0.5\left[1 - \left(\frac{2d_{ball}}{D + d}\right)cos q\right]\Omega$	vibratory frequency due to misbalanced ball bearing cage	(16)
$f_{inner} = 0.5\left[1 + \left(\frac{2d_{ball}}{D + d}\right)cos q\right]z\Omega$	vibratory frequency due to defective inner ring of the ball bearing	(17)
$f_{outer} = 0.5\left[1 - \left(\frac{2d_{ball}}{D + d}\right)cos q\right]z\Omega$	vibratory frequency due to defective outer ring of the ball bearing	(18)
$f_b = \left(\frac{d + D}{4d_{ball}}\right)\left[1 - \frac{4d^2_{ball}}{(D + d)^2}cos^2 q\right]\Omega$	vibratory frequency due to defective rolling element of the ball bearing	(19)
$f_0 = \frac{d + D}{2d_{ball}}z\Omega$	vibratory frequency due to rough or oval-shaped rolling elements of the ball bearing	(20)
$f_Z = 0.5z\Omega$	vibratory frequency due to play in the ball bearing	(21)

with R for external radius of the wheel; r for internal radius of the wheel; $k=R/r$; m for RWM weight; γ for specific density of the wheel material, h for rim height; D_w for wheel diameter; Ω for angular velocity of rotation; C for ball bearings performance factor; Q_l for load; η for efficiency; T_m for time constant; K for coefficient of rolling friction; D_{inner} for inner diameter of the ball bearing outer ring; ρ for density of the media around the wheel; R_e for Reynold's number; D_0 for diameter through the centers of the ball bearing rolling elements; z for quantity of rolling elements (balls); d_b for rolling element (ball) diameter; α, β, ξ for influence coefficients, defined by the wheel and shaft design; a for distance from the left support to the wheel; b for distance between the shaft supports; G for wheel weight; EI_1 and EI_2 for flexural stiffness of the shaft section; J_X, J_Y, J_Z for inertia of WM along the corresponding axes; J_E for RWM equatorial moment of inertia; l for RWM's center-of-gravity shift in relation to its suspension point; ε for angle of deviation between the axis of principal moment of inertia and the spinning axis; μ for ratio of axial and equatorial moments of inertia; D for diameter of the ball bearing outer ring ; d for diameter of the ball bearing inner ring ; z for quantity of rolling elements (balls); d_b for rolling element (ball) diameter; q for contact angle of the rolling elements.

The value of the critical angular velocity of the reaction wheel is defined by the dimensions of the design elements that belong to the Equation 12. When calculating this parameter, it is important to fulfill the following condition: the value shall be outside the range of operating angular velocities of the reaction wheel [2, 3].

An operating RWM is generally a hydraulic motor installed into the cardan suspension. That is why it is necessary to consider induced oscillations [4] that occur along the two nominal orthogonal axes of RWM. Nutation oscillations are defined relatively to the axes, refer to expressions (Eq. 13–14).

Amplitudes of induces oscillations of RWM can be great in case of resonance with denominator of trigonometric values of expressions (Eq. 13–14) turning to zero:

$$(Gl - J_B \Omega^2)(Gl - J_C \Omega^2) - (J\Omega^2)^2 = 0 \,. \tag{22}$$

Then the angular velocity Ω of the wheel spin, when resonance occurs, is defined by expression (Eq. 15).

In this expression μ is the coefficient defined as relation of the RW moments of inertia along the three orthogonal axes:

$$\mu^2 = \frac{J_Z^2}{J_X J_Y} \;. \tag{23}$$

Analysis of expression (Eq. 14) shows that with $\mu<1$ the radicand will always have two positive values. Consequently, in this case, the increase of the rotor self-rotation angular velocity Ω results in double vanishing of denominator in expressions (Eq. 13–14), which is indicative of two resonant values for Ω.

If $\mu>1$, the denominator of expressions (Eq. 13–14) will go to zero only with one of the angular velocity values Ω. Thus, at the rotor run-up RWM will go though resonance only once.

In the limiting case, with $\mu=1$, the denominator of expressions (Eq. 13–14) will also vanish when $\Omega=\infty$.

Expressions (Eq. 13–14) demonstrate that the amplitudes of induced oscillations go to zero with $\Omega=0$.

The RWM wheel is usually supported by ball bearings. The parameters for RWM with ball bearing supports are mainly governed by the structural dimensions of the ball bearing elements (Eq. 16–21), as well as relation of axial and equatorial moments of inertia attributable to the RWM rotating parts (reaction wheel, electric motor rotor, reaction wheel shaft (Eq. 14–15)) [4, 5].

It is obvious that the mathematical model of RWM is quite sophisticated and cannot be based on calculation expressions. Calculation stage of RWM designing process should also include modelling and analysis with the assistance of up-to-date CAD systems.

At the present moment there is set trend, established in the world of space vehicles designing: WM weigh decrease, on the one hand, and provision of proper strength characteristics, on the other hand. That is why integrated method is the optimum way to improve the quality of frequency response characteristics of RWM.

When designing RWM the following stages of the integrated method must be observed [6]:

I. calculation using mathematical model;

II. designing and prototyping;

III. experimental run up of the prototype;

IV. analysis of the results and correcting the engineering documentation.

One of the structural schemes to implement this method applied to the design of the space vehicle attitude control system is shown in Figure 2.

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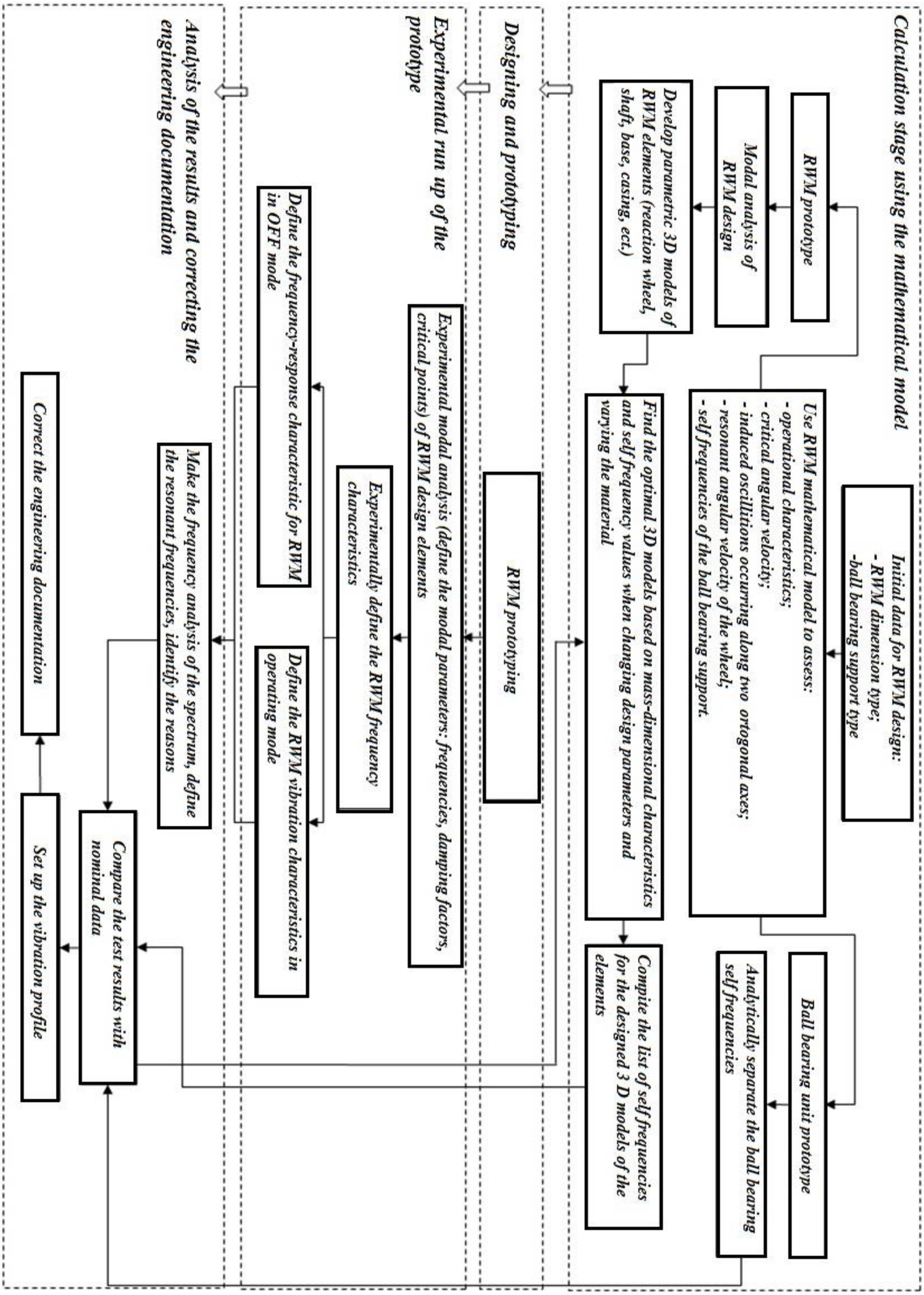


Figure 2. Structural scheme of the integrated method implementation

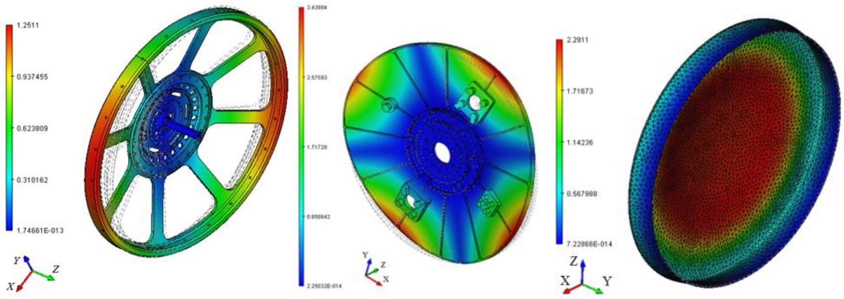
The proposed integrated method has a number of specific features: taking into account the experience of previous engineering works, interconnecting of the RWM calculation and engineering with RWM prototyping and test run-ups.

When seeking the RWM weigh minimization, we shall correlate its value with RWM mathematical model parameters, in order that the stiffness of the RWM in general and, consequently, the self frequencies of the system parts, the shift of the critical velocity to the reaction wheel angular velocity range, are not affected by, for instance, decreased sections of load-bearing elements, addition of supplementary weight reduction openings, etc [7].

And the most effective way to optimize the whole set of characteristics is to vary the system parameters while designing it.

Results

The results of integrated method application when designing RWM are shown in Figure 3. Here we can see parametrical 3D models of RWM main elements - reaction wheel (*a*), base (*b*) and casing (*c*) with the forms of oscillations associated with the first self-frequencies [8].



a *b* *c*

Figure 3. Forms of oscillations of RWM parts at the first self-frequency
a – reaction wheel; *b* – base; *c* – casing

The points cited above indicate that the RWM designing is quite a complicated technical task involving the need to combine dozens of operating parameters.

The frequency analysis of designed 3D models of the system elements enables to make the necessary corrections to the engineering documents at the stage of designing, thus complying with the task of satisfying the specifications.

Conclusion

The requirements imposed nowadays on the set of executive element characteristics dictate the necessity to naturally fit their mathematical models (via dedicated CAD systems) into the stage of RWM design. Literally, the whole technical level is formed at the design stage.

The closed-loop cycle of this set of theoretical and experimental researches allows formulating an integrated method aimed at improvement of the frequency response characteristics of executive devices and, as a consequence, of the quality of tasks fulfilled by the space vehicle.

The universal character of the developed method expands the number of application opportunities when designing any complex technical systems.

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