Load analysis of the planetary gear train with intermediate rollers. Part 2

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Abstract. The paper presents the load analysis of the planetary gear train with intermediate rollers based on solving a statically indeterminate problem including contact area problems.

1. Introduction

The planetary gear train with intermediate rollers possess a high loading capacity due to the multiple contact between their members. In order to estimate the loading capacity, the force distribution in the system members and, thereby, the statical indeterminateness of the system should be measured.

The dependencies between the geometry and linkage parameters of the planetary gearing system are described in work [1].

2. Effective forces in members of the planetary gearing with intermediate rollers of a unit length

Figure 1 presents the interaction between the planetary gearing members under the carrier loading conditions induced by load moment M_2. Let us calculate the forces acting on the intermediate roller in the form of a circular cylinder. First, we shall consider the planetary gearing design shown in Figure 1,*a*.

For simplification, let M_2 be 1. Then force N_2 (a response of the carrier to the intermediate roller) can be obtained from

$$N_2 = \frac{M_2}{h} = \frac{M_2}{s\sin\psi} = \frac{1}{s\sin\psi}.$$

From the condition of projection equality

$$N_1\cos\gamma = N_2\cos\psi\ ,$$

response N_1 of the annular gear is given by

$$N_1 = \frac{1}{s\cos\gamma\,\mathrm{tg}\psi}.$$

The response of a separator plate equals the sum of force projections:

$$N_3 = N_1\sin\gamma - N_2\sin\psi = \frac{\mathrm{tg}\gamma}{s\,\mathrm{tg}\psi} - \frac{1}{s}.$$

The iteration of all the operations given above results in forces applied to the intermediate roller for the planetary gearing design shown in Figure 1b:

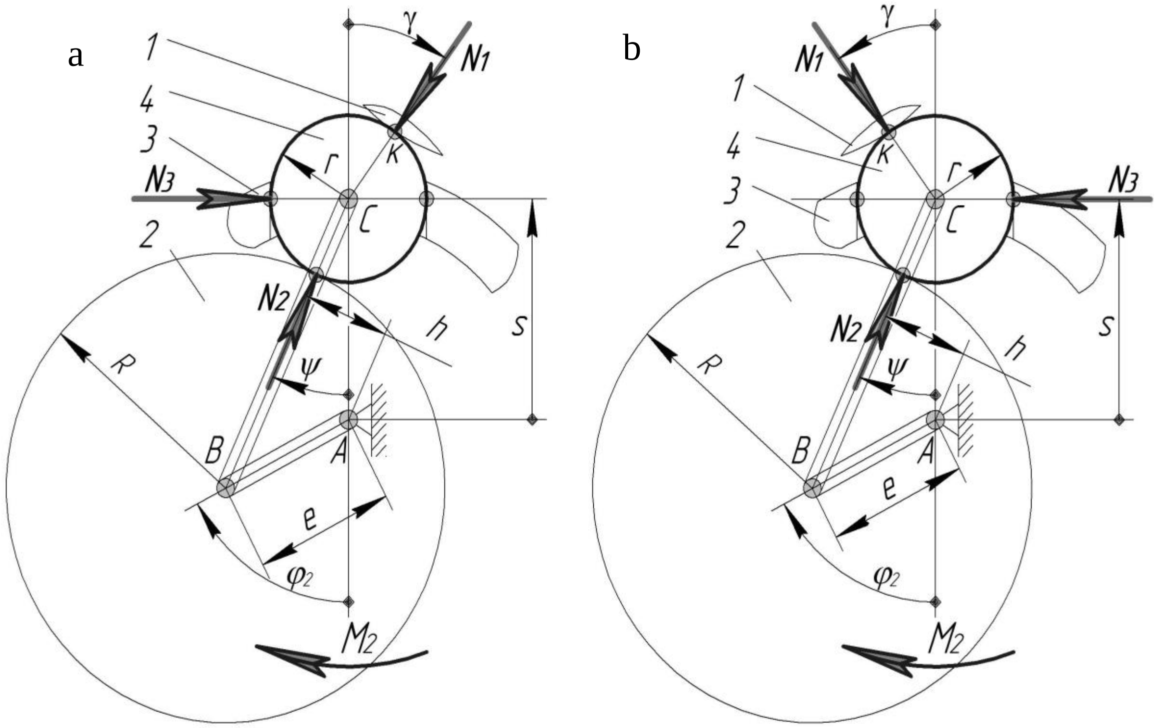

Figure 1. Schematics of loading conditions for the intermediate roller: *a* – the number of intermediate rollers is less than that of teeth by one; *b* − the number of intermediate rollers is greater than that of teeth by one; *1* – the annular gear; *2* – the carrier; *3* – the separator plate; *4* – the intermediate roller.

$$N_2 = \frac{M_2}{h} = \frac{M_2}{s\sin\psi} = \frac{1}{s\sin\psi};$$

$$N_1 = \frac{1}{s\cos\gamma\,\mathrm{tg}\psi};$$

$$N_3 = N_1\sin\gamma + N_2\sin\psi = \frac{\mathrm{tg}\gamma}{s\,\mathrm{tg}\psi} + \frac{1}{s}.$$

3. Statically indeterminate problem

In case the annular gear and the separator plate are stationary, load moment M_2 of the carrier, as shown in Figure 2, results in plastic deformation at the place of engagements that, in turn, leads to increment $\delta\varphi_2$ of the rotation angle. The angle increments are common for all members (an annular gear, a carrier, a separator plate, an intermediate roller) participating in the multiple contact in the planetary gearing system. Taking this principle into account, the force distribution can be detected between the members of the system.

Let us calculate increment $\delta\varphi_2$ of the rotation angle of the carrier. Figure 2 shows the positions of the carrier and the intermediate roller before and after loading for two abovementioned design variants of the planetary gearing system.

Let us consider the planetary gear train shown in Figure 2,*a*.

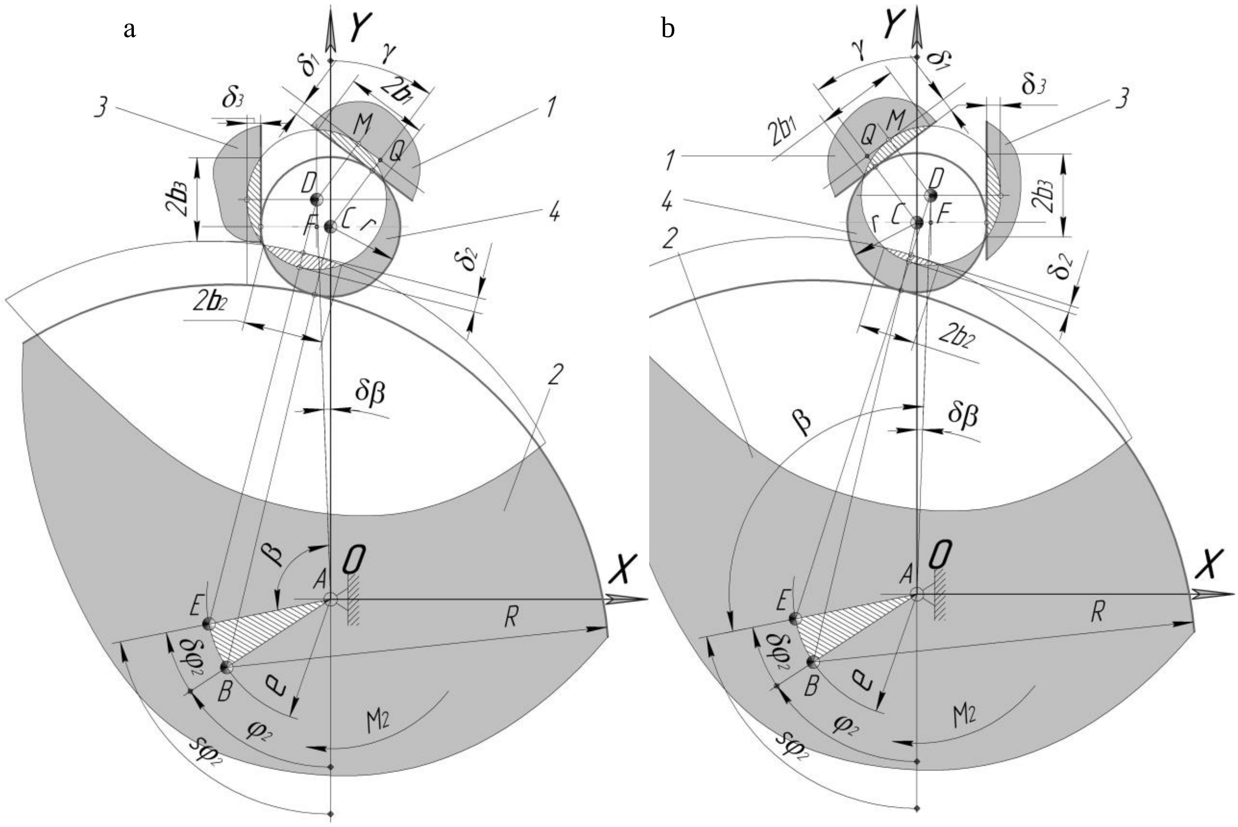

Figure 2. Schematics of loading conditions for system members in two design variants: *B*, *E* – positions of the carrier geometrical centre before and after loading, respectively; *C*, *D* – positions of the intermediate roller before and after loading, respectively; δ_1, b_1 – engagement and half-width of the contact area, respectively, between the intermediate roller and the annular gear tooth; δ_2, b_2 – engagement and half-width of the contact area, respectively, between the intermediate roller and the carrier; δ_3, b_3 – engagement and half-width of the contact area, respectively, between the intermediate roller and the separator plate.

3.1. A new position of centre D of the intermediate roller

The vector equality has form

$$\overrightarrow{CF}+\overrightarrow{FD}=\overrightarrow{CQ}+\overrightarrow{QM}+\overrightarrow{MD}, \tag{1}$$

the components of which are expressed by basis vectors $\boldsymbol{i}$ and $\boldsymbol{j}$ in the $S(Oxy)$ coordinate system and equal:

$$\overrightarrow{CF}=-\delta_3\boldsymbol{i}; \tag{2}$$

$$\overrightarrow{FD}=FD\,\boldsymbol{j}; \tag{3}$$

$$\overrightarrow{CQ}=\left(R+\delta_1\right)\left(\sin\gamma\,\boldsymbol{i}+\cos\gamma\,\boldsymbol{j}\right); \tag{4}$$

$$\overrightarrow{QM}=QM\left(-\cos\gamma\,\boldsymbol{i}+\sin\gamma\,\boldsymbol{j}\right); \tag{5}$$

$$\overrightarrow{MD}=-R\left(\sin\gamma\,\boldsymbol{i}+\cos\gamma\,\boldsymbol{j}\right). \tag{6}$$

Transformations of the set of equations (1)-(6) yields

$$FD=\delta_1\cos\gamma+\left(\delta_1\sin\gamma+\delta_3\right)\mathrm{tg}\gamma. \tag{7}$$

As shown in Figure 2a the radius-vector of new centre *D* in the accepted coordinate system is

$$\overrightarrow{AD}=-\delta_3\boldsymbol{i}+\left(FD+S\right)\boldsymbol{j}\,\boldsymbol{.} \tag{8}$$

3.2. *Increment of the rotation angle of the carrier*

According to Figure 2a,

$$\overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD}. \tag{9}$$

Angle β can be obtained from (9):

$$\beta = \arccos\left(\frac{AD^2 + AE^2 - ED^2}{2AD \cdot AE}\right), \tag{10}$$

and

$$ED = R + r - \delta_2; \tag{11}$$

$$AE = e; \tag{12}$$

$$AD = \sqrt{\delta_3^2 + \left[S + \delta_1 \cos\gamma + \left(\delta_1 \sin\gamma + \delta_3\right)\mathrm{tg}\gamma\right]^2}. \tag{13}$$

The latter equation is based on (7) and (8).

Based on dependence

$$\overrightarrow{AD} \cdot \boldsymbol{j} = AD \cos\delta\beta,$$

Let us obtain $\delta\beta$:

$$\delta\beta = \mathrm{arctg}\frac{\delta_3}{S + \delta_1 \cos\gamma + \left(\delta_1 \sin\gamma + \delta_3\right)\mathrm{tg}\gamma}. \tag{14}$$

At the same time, the new position of the carrier is defined by angle

$$s\varphi_2 = \pi - \beta - \delta\beta, \tag{15}$$

while the angle increment of the carrier rotation is

$$\delta\varphi_2 = s\varphi_2 - \varphi_2. \tag{16}$$

Thus, to detect angle inclement $\delta\varphi_2$, it is expedient to regard (16) in combination with (10)-(15). The parameters of S and γ were obtained in work [1].

Note that calculations for the design variant presented in Figure 2b differ from those presented in Figure 2a only by the angle $s\varphi_2$ value:

$$s\varphi_2 = \pi - \beta + \delta\beta.$$

Calculations of engagement parameters $\delta_1, \delta_2, \delta_3$ are somewhat difficult. Design formulas given in [2-4], are not universal and intended rather for particular cases. Thus, for example, in case the radius of one of the profile surface approaches infinity (a cylinder/plane transfer), these formulas produce erroneous outcomes.

Moreover, the profile surface curvature of the annular gear (Figures 1 and 2) ranges from one sign to another in different sections. Therefore, to simplify the problem solving, formulas given in [4] are used for the schematics of cylinder/plane engagement with substituting radius R by that of relative curvature R_n with a positive sign:

$$\delta = 0{,}5795\frac{q}{E}\left(0{,}41 + \ln\frac{2R_n}{b}\right),$$

where q is the specific force due to pressure; E is the Young's modulus similar to both of engaged members; b is the half-width of the contact area.

The above-stated equation also implies the equality of Poisson numbers, namely: $\mu = 0.3$.

The half-width of the contact area has form

$$b = 1{,}523\sqrt{\frac{qR_n}{E}},$$

while the highest contact stress is

$$\sigma_{\max} = 0{,}418\sqrt{\frac{qE}{R_n}}.$$

The main difficulties of the problem solving are connected with cumbersome calculations and impossibility of the analytical study of the profile surface curvature. In this connection, the computer program has been developed on the Rad Studio XE8 environment basis for the numerical implementation of this problem. The curvature radius of the teeth profile surface can be obtained using two chords between three points on it and the sine theorem [5, 6]. The developed computer program allows solving a wide range of problems. The simplest problem algorithm is suggested so as to determine the total rotation moment of the carrier accounting for the condition of restricting the highest contact stress of the given geometry and properties of materials of the planetary gear train.

The problem solving is based on the method of successive approximations, which includes:

– assignment of a random variable of angle increment $\delta\varphi_2$;

– definition of load moment M_2 in all engaged members; maximum contact stresses, i.e. $\sigma_{max}^{(1-4)}$ between the surfaces of teeth and the intermediate roller; $\sigma_{max}^{(2-4)}$ between the surfaces of the carrier and the intermediate roller; $\sigma_{max}^{(3-4)}$ between the surfaces of the separator plate and the intermediate roller;

– control of the condition of restricting highest contact stress $\sigma_{max} \le [\sigma]$. In satisfying this condition, the total rotation moment of the carrier is obtained, which equals to the sum of all moments obtained in the previous paragraph, and also the sum of forces having effect on each member. Otherwise, the calculations must be done anew taking into account the improved value of angle increment $\delta\varphi_2$.

Note that the obtained results depend on the member arrangement in the planetary gearing system, i.e. angle φ_2 (engagement phases). To avoid this dependence, the averaged values of parameters should be determined:

$$SM_2 = \frac{\sum_{i=1}^{n} M_{2i} \cdot Z}{2n},$$

where SM_2 is the averaged total moment of the carrier; n is the number of calculations at $\varphi_2 \in (0, \pi)$; M_{2i} is the moment at the i-th step of calculations; Z is the number of intermediate rollers;

$$S\sigma_{max} = \frac{\sum_{i=1}^{n} \sigma_{max\,i}}{n},$$

where $S\sigma_{max}$ is the average value of the maximum stress between the contact areas (indices are omitted); σ_{maxi} is the contact stress at the i-th step of calculations;

$$SN_x = \frac{\sum_{i=1}^{n} N_{xi} \cdot Z}{2n},$$

$$SN_y = \frac{\sum_{i=1}^{n} N_{yi} \cdot Z}{2n},$$

where SN_x and SN_y are the components of the total force on axes Ox and Oy, respectively; N_{xi} and N_{xy} are the force components on axes Ox and Oy at the i-th step of calculations.

4. Example of load analysis

The initial data: R = 104 mm; r = 5 mm; e = 2.5 mm; Z = 31; Z-1 is the number of teeth in the annular gear; E = 200000 MPa; μ = 0.3; 1 mm link height; M_2 = 5600 N·mm.

Results of calculations obtained using the developed computer program are as follows. The moment of rotation of the separator plate is 173458.1 N·mm.

The average contact stress between the intermediate roller and
– the annular gear is 1500 MPa;
– the carrier is 1195 MPa;
– the separator plate is 871.5 MPa.

The force acting on:
– the carrier is 2239.9 N;
– the annular gear is 2590.5 N;

– the separator plate is 1329.6 N.

5. Conclusions

1. The results of multiple calculations have showed that the most load-subjected member is an intermediate roller/tooth of the annular gear.
2. The suggested calculation methodology and the computer program have been successfully used in designing drivers of different functions at Tomsk industrial enterprises.

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