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Modeling of a single-cycle current generator while forming a quasi-sinusoidal current

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Abstract. The paper presents the results of investigation of the influence of the output voltage magnitude on the operating frequency of the switch in the single-cycle quasi-sinusoidal current generator circuit. Analytical expressions for calculating the time parameters for transients in the circuit under given assumptions have been obtained. The results presented in the paper can be used in the design of converters of this type.

1. Introduction

Sometimes, in order to improve the efficiency of electrochemical processes it is reasonable to use power supplies based on a quasi-sinusoidal asymmetrical current generator [1, 2]. To design a current generator for a rational choice of the element base one must follow the requirements for frequency properties and determine the switching losses of the circuit to achieve the corresponding calculated ratio. The expressions to calculate these parameters have been obtained in [3], but they are transcendental, inconvenient for practical use, since a special mathematical approach is required to obtain a certain result. To simplify the calculation procedure, it is appropriate to obtain more simple approximate analytical expressions that enable the calculations of the switching losses with acceptable errors and set the requirements for their frequency properties.

The quasi-sinusoidal asymmetrical current generator circuit is quite complex [1]; however, to solve the problem it is enough to consider the process of one half-wave current generation, which is realized in a simplified version of the circuit presented in Figure 1a.

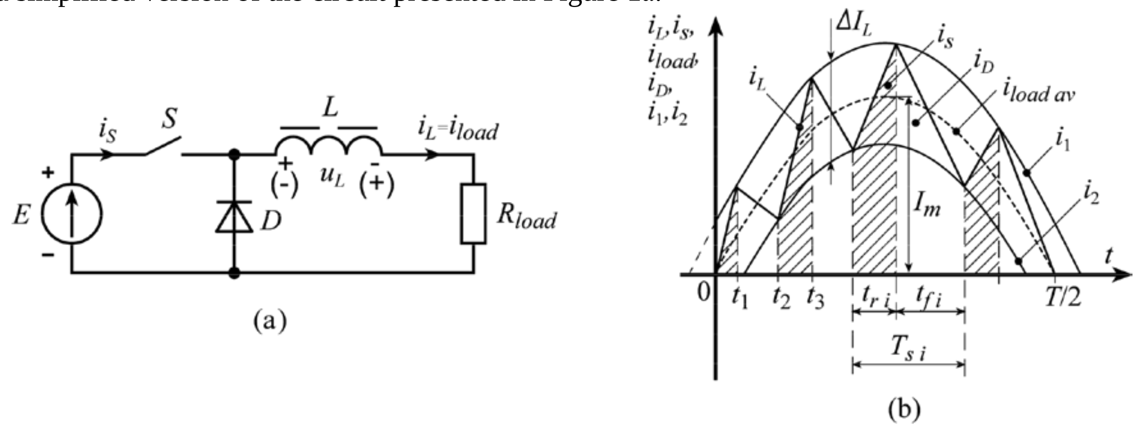

Figure 1. Single-cycle current generator circuit (a) and current waveforms (b).

2. Modeling

The process of generating a quasi-sinusoidal load current is realized by means of regulating turn-on and turn-off states of switch S according to the given law of durations and, respectively, of the inductor current L. This process is illustrated with the waveforms of currents shown in Figure 1b.

For clarity, the switching frequency is chosen to be relatively low. Further we assume the following: voltage source E, diode D, inductor L and switch S are ideal; inductor L is a linear element; R_{load} is a constant resistive load; duration of the current i-cycle of switch $T_{s\,i}$ is much smaller than the period of the formed sine-wave, i.e. $T_{s\,i} << T$; during the current cycle of the switch the output voltage does not change, i.e. $U_{load\,i} \approx$ const; the load current varies sinusoidally, i.e. its ripples caused by the switch commutations are infinitesimal; in the load there is a half wave of sinusoid with amplitude I_m, angular frequency ω and period T, which is the average value of load and inductor currents, are formed as follows:

$$i_{load\ av}(t) = i_{L\ \mathrm{av}}(t) = I_m \sin\omega t;$$

The load current changes between the upper $i_1(t)$ and lower $i_2(t)$ threshold levels, limiting the inductor current ripples with respect to $i_{load\ av}(t)$:

$$\left.\begin{aligned} i_1(t) &= 0.5\Delta I_L + i_{load\ av}(t) = 0.5\Delta I_L + I_m \sin\omega t, \\ i_2(t) &= -0.5\Delta I_L + i_{load\ av}(t) = -0.5\Delta I_L + I_m \sin\omega t \end{aligned}\right\}, \tag{1}$$

with $\Delta I_L = i_1(t) - i_2(t)$ – is the required value of inductor current ripples.

Let us consider the principle of the circuit operation. Let us assume that at time moment $t = 0$ switch S is turned on. Voltage E is applied to series-connected L and R_{load} and diode D, providing an off-state of D. At this moment, inductor current $i_L(t)$ and the load current are zero, therefore voltage E is applied to the inductor with polarity shown in Figure 1a (without brackets). Under given assumptions current $i_L(t)$ begins to increase linearly. The inductance of the inductor is chosen so that the slew rate of inductor current $i_L(t)$ would be a bit higher than the slew rate of load current $i_{load\ av}(t)$. At moment t_1 the inductor current reaches upper threshold level $i_1(t_1)$, and switch S is turned off. The polarity of the voltage across inductor L is changed to the opposite polarity (shown in brackets in Figure 1a). Flowing through the load and the freewheeling diode, the inductor current decreases linearly. Reaching lower threshold level $i_2(t)$ at moment t_2, switch S is turned on again and the inductor current increases once again. Then all the processes, described above, repeat cyclically. Thus, as a result of a large number of operation cycles of the switch, the current is generating in the load. The average (approximated) value of this current corresponds to a half-wave of a sinusoidal signal (in Figure 1b it is shown by the dashed line).

Let us suppose that during time interval $0 – T/2$ N cycles are needed for the formation of a half-wave current, and each cycle consists of two transients: rise and fall times of the inductor current, respectively. Let us denote the number of the current cycle as i, where $i = 1…N$ – an integer. Let us assign alphabetic indexes 'r' or 'f' indicating the rise or fall time of $i_L(t)$ for current, voltage and time parameters respectively; the numerical index corresponds to the number of the current cycle.

To receive the basic ratios we need to perform the analysis of transients in the circuit. Let us consider an i-cycle of the switch operation (Figure 1a), occurring at moment t_i – a fixed moment of time period $0 – T/2$, during which the i-cycle of the generator operation takes place.

Taking into account the assumptions that the load current at this point has a definite value we obtain

$$i_{load\ i}(t_i) = I_m \sin\omega t_i\,.$$

At the stage when the inductor current increases, switch S is turned on in the mentioned cycle, the constant voltage with positive polarity is applied to the inductor (polarity is shown in Figure 1a without brackets)

$$U_{Lr\ i} = E - i_{load\ i}(t_i) R_{load} = \mathrm{const}\,. \tag{2}$$

When the switch is turned off during the fall time in the same cycle the current is turned on through freewheeling diode *D*, and the inductor voltage polarity is reversed (voltage polarity is shown in Figure 1a in brackets) and becomes equal to

$$U_{Lf\,i} = -i_{load\,i}(t_i)R_{load} = -U_{m\,load}\sin\omega t_i = \mathrm{const}\,. \tag{3}$$

It is known that the relation between voltage and current of the inductor is as follows

$$u_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t}\,, \tag{4}$$

from which it follows that if the voltage applied to the inductor is constant, the current of the inductor changes according to the linear law:

$$i_L(t) = I_L(0) + \frac{U_L}{L}t\,,$$

where $I_L(0)$ – the initial condition for the transient process.

Taking into account this equation and expressions (2) and (3) the inductor current change during the *i*-cycle can be written in the following form (the start of timing is moved to the switching time)

$$i_{Lr\,i}(t) = I_{Lr\,i}(0) + \frac{U_{Lr\,i}}{L}t = I_{Lr\,i}(0) + \frac{E - U_{m\,r}\sin\omega t_i}{L}t\,,$$

$$i_{Lf\,i}(t) = I_{Lf\,i}(0) + \frac{U_{Lf\,i}}{L}t = I_{Lf\,i}(0) - \frac{U_{m\,load}\sin\omega t_i}{L}t\,,$$

where $U_{m\,load} = I_m R_{load}$ – the average amplitude of the load voltage.
Initial conditions for rise and fall times, taking into account equation (1), are determined by expressions:

$$I_{Lr\,i}(0) = i_2(t_i) = -\frac{\Delta I_L}{2} + I_{m\,load}\sin\omega t_i\,,$$

$$I_{Lf\,i}(0) = i_1(t_i) = \frac{\Delta I_L}{2} + I_{m\,load}\sin\omega t_i\,.$$

In formula (4) the differentials for time and current as increments with constant inductor voltage

$$U_L(t) = L\frac{\Delta i_L}{\Delta t} = \mathrm{const}\,; \tag{5}$$

Let us denote $\Delta t = t_{r\,i}$ for the rise time, $\Delta t = t_{f\,i}$ for the fall time, and substituting these parameters into (5) and using expressions (2) and (3), after transformations we obtain:

$$t_{r\,i} = \frac{\Delta I_L L}{E - U_{m\,load}\sin\omega t_i} = \frac{K_{rip}\tau U^*}{1 - U^*\sin\omega t_i}\,,$$

$$t_{f\,i} = \frac{\Delta I_L L}{\left|-U_{m\,н}\right|\sin\omega t_i} = \frac{K_{rip}\tau}{\sin\omega t_i}\,,$$

$$T_{s\,i} = t_{r\,i} + t_{f\,i} = \frac{K_{rip}\tau}{\sin\omega t_i - U^*\sin^2\omega t_i}\,,$$

$$f_{s\,i} = \frac{1}{T_{s\,i}} = \frac{\sin\omega t_i - U^*\sin^2\omega t_i}{K_{rip}\tau}\,,$$

where $U^* = U_{m\,load}/E$ – the normalized amplitude of the output voltage; $\tau = L/R_{load}$ – the time constant of the circuit; $K_{\mathrm{rip}} = \Delta I_L/I_m$ – the ripple factor of the load and inductor current.

Let us consider the obtained parameters in a dimensionless form:

$$t_{r\,i}{}^* = \frac{t_{r\,i}}{T} = \frac{K_{rip}\cdot U^*}{\delta\cdot\left(1 - U^*\sin\omega t_i\right)}\,, \tag{6}$$

$$t_{f\,i}* = \frac{t_{f\,i}}{T} = \frac{K_{rip}}{\delta \cdot \sin \omega t_i}, \tag{7}$$

$$T_{s\,i}* = \frac{T_{s\,i}}{T} = \frac{K_{rip}}{\delta \cdot \left(\sin \omega t_i - U * \sin^2 \omega t_i\right)}, \tag{8}$$

$$f_{s\,i}* = \frac{f_{s\,i}}{f} = \frac{\delta(\sin \omega t_i - U * \sin^2 \omega t_i)}{K_{rip}}, \tag{9}$$

where $\tau* = \tau/T$ is the relative value of the time constant; $\delta = 1/\tau* = T/\tau$ is the inverse value of the relative value of the time constant or the damping ratio of the transient showing how the period of a sine wave exceeds the time constant; $T_{s\,i}* = 1/f_{s\,i}* = T_{s\,i}/T$ is the relative value of the cycle duration; $t_{f\,i}* = t_{f\,i}/T$ is the relative value of the fall time of the inductor current; $f_{s\,i}* = f_{s\,i}/f = 1/T_{s\,i}*$ is the relative value of the local switching frequency.

3. Discussion

The most important thing for practice is the information about local switching frequency f_s*of the duration of half-period of the generated sine waves, allowing determination of the requirements for the frequency properties of the switches, and estimation of the value of the switching losses in them.

Approximate expression (9) gives a possibility to simply see the tendency of the operating frequency change of the switch during the half-period of the sine wave while changing the normalized amplitude of output voltage U*. In Figure 2 one can see the ratios obtained with expression (9). It is seen that three local extremums (2 high and 1 low) are observed at values of $U* > 0.5$; and when $U* \leq 0.5$ there is only one local extremum and it is the maximum.

Taking into account that the maximum operating frequency of the switch is an important value from the point of view of selecting elements, it is advisable to determine the values of these extremums. Having differentiated equation (9) by the parameter $\omega t = \nu$ and having brought the result to zero, we will find the roots of the resulting equation corresponding to the required extremums:

$$\nu_1 = \pi/2; \tag{10}$$

$$\nu_2 = \arcsin\left(1/2U*\right); \tag{11}$$

$$\nu_3 = \pi - \arcsin\left(1/2U*\right). \tag{12}$$

The analysis of expressions (10) – (12) has shown that roots ν_2 and ν_3 exist only for values of $U* \geq 0.5$, and root ν_1 exists for all values in the range of $0 \leq U* \leq 0.5$.

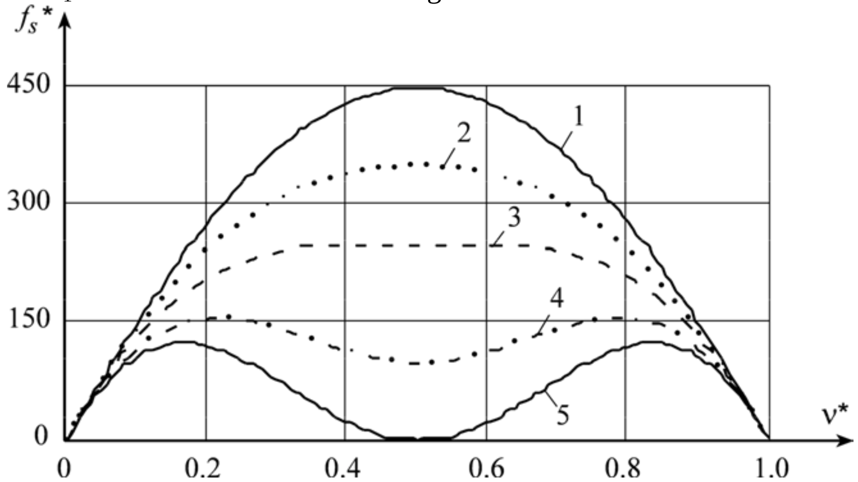

Figure 2. Dependences of the relative value of the local switching frequency on the relative value of the current phase $\nu*=\omega t/\pi$ when $K_{rip} = 0.2$, $\delta = 100$ and various U*: 1) $U* = 0.1$; 2) $U* = 0.3$; 3) $U* = 0.5$; 4) $U* = 0.8$; 5) $U* = 1$.

We will obtain the expressions for calculating the relative value of the switching frequency at the extreme points. To find out the extremums of the frequency when $U^* \geq 0.5$ we include ν_1, ν_2, ν_3 in equation (9). After transformations for the first and the second maximums of the switching frequency we obtain

$$f_{s\,max1}{*} = \frac{f_{s\,max1}}{f} = \frac{f_{s\,max2}}{f} = \frac{\delta}{4U^* K_{rip}}, \tag{13}$$

and the expression for the local minimum is

$$f_{s\,min}{*} = \frac{f_{s\,min}}{f} = \frac{\delta(1-U^*)}{K_{rip}}. \tag{14}$$

Inserting ν_1 to equation (9), we obtain the expression for determining the only maximum of the operating frequency of the switch when $U^* \leq 0.5$:

$$f_{s\,max}{*} = \frac{f_{s\,max}}{f} = \frac{\delta(1-U^*)}{K_{rip}}. \tag{15}$$

Expressions (14) and (15) are identical. This gives us possibility to claim that when U^* changes from 0 to 1, the maximum goes into the minimum, and the boundary case occurs under $U^* = 0.5$, with $f_{s\,min}{*} = f_{s\,max}{*}$.

Equations (6) – (15) are convenient for analysis, though approximated, and may be used for ‘quality’ assessment of the impact of any parameter on the transient process duration.

A comparison of the results of maximum frequency calculations carried out using approximate formulas with the results obtained by using the exact formulas [1] showed that the error does not exceed 10% if the number of commutate cycles of the switch is $N \geq 12$ when any U^* and $K_{\mathrm{rip}} \leq 0.3$.

4. Conclusion

The analysis of the single-cycle current generator allowed deriving approximate calculated ratios specifying the time parameters of transients in the circuit.

These relatively simple expressions made it possible to identify the tendencies and nature of changes of the time parameters of the transients in the circuit.

The investigation results show that under $U^* > 0.5$ the function of the relative value of the local switching frequency versus the relative value of the current phase has three local extremums – two maximus and one minimum; when $U^* < 0.5$ the function has the only extremum (maximum), exceeding the maximum frequency value, which is peculiar when $U^* > 0.5$.

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