The mathematical simulation of the temperature fields of building envelopes under permanent frozen soil conditions

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Abstract. The physical-mathematical model of the thermal state of the aired technical underground taking into account the air exchange and design features of construction under permanent frozen soil conditions has been suggested. The computational scheme of the temperature fields prediction of building envelopes of projected buildings and soil under and nearby buildings has been developed. The numerical simulation of the temperature fields of building envelopes changes was conducted during a year. The results of the numerical simulation showed that the heat coming from the technical undergrounds and through the walls does not influence the temperature field of the soil neither under a building nor at a distance from it.

1. Introduction

The prediction of temperature fields in soils and adjoining building constructions is a complex heat engineering problem. It is conditioned by soils heterogeneity, their heat properties and ‘change of phase’ at their frost penetration. The permanent frozen soils demand a special attention because of the complex thermal processes of ‘freezing – defrosting’, the formation of the surface soil layer that has a positive temperature during a warm period, thawed soil that has relatively permanent temperatures during a year and a soil layer in a permanent frozen condition.

Very often, when designing and constructing buildings it is necessary to determine the temperature field of the soil. This task is very topical when designing warmth keeping of an underground lead-in water pipeline and a tapping of sewage disposal under conditions of relief with inequalities in the elevation in the site area when soil surface elevations and drops fluctuate. The knowledge of the temperature field in the zone of thermal influence of buildings could allow predicting the water cooling in pipes (including extremely low temperatures in winter) and to recommend the necessary measures in the pipe-line warming.

Underground piping under conditions of extremely low temperatures must be laid taking into account that not only permanent frozen soils influence the medium temperature and may cause pipe frost penetration freezing, but also pipes may cause the temperature rise of these soils. The consequences of natural temperature changes of permanent frozen soils can be unpredictable. The transition from the tough frozen state into a thawed one may entail the loss of lifting properties of piles of building foundations and it will be impossible to operate the construction.

Reviewing the theory of computation of thermal transfer in soils, the contribution of such authors as Bogoslovskiy, A.V. Gindoyan, A.G. Sander, A.A. Ioritis, G.V. Porhayev should be mentioned. Especially the works of Porhayev [1] should be marked as in his research he investigated the thermal interaction of constructions with permanent frozen soils. As G.V. Porhayev's research shows that the soil temperature on the building area differs from the soil temperature on the area free from engineering building.

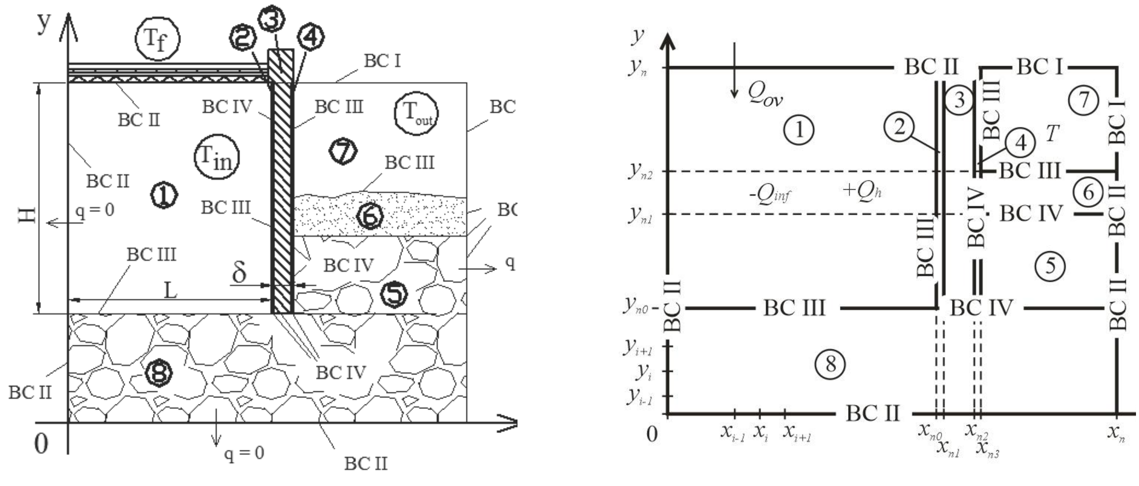
The contemporary authors such as D.A. Krylov [2-4], P.I. Dyachek [5], U.S. Melnikova [6] AA Sinitsyn [7] and N.I. Sydnyaev [8] should be marked. The foreign experts' works also deserve a special attention. These experts are Lunardini J. Virgil [9], Ebenezer E.K. [10], Ming Zhu and others [11-12].

2. The mathematical simulation of the thermal state of building envelopes deepened into soil

In a general case the problem of the calculation of heat loss through multilayer envelope buildings and the soil of the technical underground is three-dimensional. But, the length of buildings (especially of public buildings) is much greater than their width, that's why when developing the mathematical model of heat exchange in the technical underground a two- dimensional symmetrical problem of thermal conduction in a semi-limited mass with a set of boundary conditions is considered. Along with it, the inner zone of cellar L is separated from the external one by the wall section with thickness δ, m (Figure 1).

For the construction of the mathematical model of heat exchange of the technical underground in non-steady-state conditions the physical model including different media (air, snow, soil) and materials is suggested. Every medium or material having different thermal properties is isolated into a separate material layer: 1 – inner air, 2 – warmth-keeping layer, 3 – external wall, 4 – external layer of the wall envelope, 5 – soil out of a cellar, 6 – snow layer, 7 – air out of a technical underground, 8 – soil inside the technical underground, 9 – permanent snow layer above the blind area.

The schemes of physical and mathematical models with mentioned boundary conditions (BC) are in Figure 1.



a) b)

Figure 1. The mathematical model design scheme:
a) the physical model of the underground b) the mathematical model of the underground
1 – air inside a cellar; 2 – warmth-keeping; 3 – external wall (reinforced concrete); 4 - external hydroisolation layer; 5, 8 – soil; 6 – snow layer; 7 – air out of the technical underground; BC – boundary conditions.

It is worth mentioning that such problem statement allows providing enough 'flexibility' in calculations because there is a possibility of calculation of both cellars and technical undergrounds (Figure 1).

At this point it is sufficient to admit that layer 5 has the same thermal properties as layer 6. The heat transfer in multilayer building envelopes of the technical underground and in the solid media such as soil and snow is described with a three-dimensional nonlinear equation of the thermal conduction in the Cartesian coordinate system

$$\rho\, C_p \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda\, \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\, \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda\, \frac{\partial T}{\partial z}\right) + Q(x, y, z), \qquad (1)$$

$T = T(\bar{x}, \tau)$ – temperature; C, ρ – thermal capacity and density; λ – heat conduction coefficient; $Q(x, y, z)$ – power of inner heat sources; τ – time.

When constructing the mathematical model the following assumptions have been made:

– the thermal properties of materials (ρ_i, $C_{P,i}$, λ_i), building envelopes, snow and air are considered to be as constant;

– the thermal properties of the soil in layer 8 are assumed to be constant;

– the thermal properties of the soil in layer 5 are taken as constant, based on the results of the conducted research;

– the heat (Q_h) from the utility systems, the heat used for filtered cold air ($-Q_{inf}$) and the heat input through the ground floor (Q_{ov}) is considered equally allocated by the technical underground volume;

– the air temperature inside the ground floor room is equal to critical [13];

– in reference time $\tau = 0$ the temperature in all layers of the mathematical model is equal to the temperature in layer 7 (inner air);

– the coefficient of the heat transfer on the border 'snow layer - inner layer' is calculated in accordance with the recommendations for the definite region's climate conditions.

The equation (1) with proper initial and boundary conditions represents the problem of temperature allocation at each point of the considered area for each moment of time. It is suggested that the initial temperature allocation in the material, the temperature condition on its borders and the power of inner heat sources are known. The problem is symmetrical with respect to the *x*-axis, and in the direction normal to plane *x-y* it is semi-infinite because on these borders the heat streams are insignificant and not taken into account [15]. It allows solving the problem in a two - dimensional statement (figure 1).

Taking into account these assumptions the equation (1) will be the following:

$$\rho C_p \frac{\partial T}{\partial \tau} = \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + Q, \qquad (2)$$

C_p – heat capacity at constant pressure; ρ – material density; λ – heat conduction of the material; T – material temperature; τ – time; Q – inner sources of heat release (this component is taken into account only for layer 1, Figure 1); *x, y* – coordinates in horizontal and vertical directions.

The heat streams allocation on different material layers planes (Figure 1) is revealed from solving the equation (2) with the set of boundary conditions.

The boundary conditions for layers 1 and 2 (the air inside the technical underground, warmth-keeping layer) are:

$$\begin{cases} x = 0; y_{n0} \le y \le y_n; q = 0; \\ x = x_{n0}; y_{n0} \le y \le y_n; \alpha_{B1} \cdot (T_B - T_{x_{n0}}) = -\lambda \frac{\partial T_2}{\partial x}; \\ x_0 \le x \le x_{n0}; y = y_n; q = f(\tau, T_B); \\ x_0 \le x \le x_{n0}; y = y_{n0}; \alpha_{B2} \cdot (T_B - T_{y_0}) = -\lambda_8 \frac{\partial T_8}{\partial y},\ \lambda_8 = (\end{cases}$$

$$\begin{cases} x = x_{n0}; y_{n0} \le y \le y_n; \alpha_{B1} \cdot (T_B - T_{x_{n0}}) = -\lambda_2 \frac{\partial}{\partial} \\ x = x_{n1}; y_{n0} \le y \le y_n; T_2 = T_3; \lambda_2 \frac{\partial T_2}{\partial x} = \lambda_3 \frac{\partial T}{\partial x} \\ x_{n0} \le x \le x_{n1}; y = y_n; q = f(\tau, T_B); \\ x_{n0} \le x \le x_{n1}; y = y_{n0};\ T_2 = T_8; \lambda_2 \frac{\partial T_2}{\partial y} = \lambda_8 \frac{\partial}{\partial} \end{cases}$$

α_{B1}, α_{B2} – coefficients of the heat transfer beside the basement wall near the soil area, $W/(m^2 \cdot °C)$; λ_8 – coefficient of the soil heat transfer inside the technical underground, $W/(m^2 \cdot °C)$.

The boundary conditions for layers 3 and 4 (the building envelope of the technical underground and the finishing layer) are:

$$\begin{cases} x = x_{n1}; y_{n0} \le y \le y_n; T_2 = T_3; \lambda_2 \dfrac{\partial T_2}{\partial x} = \lambda_3 \dfrac{\partial T_3}{\partial x}; \\ x_{n1} \le x \le x_{n2}; y = y_{n0}; T_3 = T_8; \lambda_3 \dfrac{\partial T_3}{\partial y} = -\lambda_8 \dfrac{\partial T_8}{\partial y} \\ x_{n1} \le x \le x_{n2}; y = y_n; q = f(\tau, T_B); \\ x = x_{n2}; y_{n0} \le y \le y_n;\ T_3 = T_4; \lambda_3 \dfrac{\partial T_3}{\partial x} = \lambda_4 \dfrac{\partial T_4}{\partial x}, \end{cases}$$

$$\begin{cases} x = x_{n3}; y_{n0} \le y \le y_{n2}; \alpha_{H2} \cdot (T_H - T_{x_{n3}}) = -\lambda_4 \\ x = x_{n3}; y_{n1} \le y \le y_{n2}; T_4 = T_6; \lambda_4 \dfrac{\partial T_4}{\partial x} = \lambda_6 \dfrac{\partial}{\partial} \\ x = x_{n3}; y_{n0} \le y \le y_{n1}; T_4 = T_5; \lambda_4 \dfrac{\partial T_4}{\partial x} = \lambda_5 \dfrac{\partial}{\partial} \\ x = x_{n2}; y_{n0} \le y \le y_n; T_3 = T_4;\ \lambda_4 \dfrac{\partial T_4}{\partial x} = \lambda_3 \dfrac{\partial}{\partial} \\ x_{n2} \le x \le x_{n3}; y = y_n; q = f(\tau, T_B); \\ x_{n2} \le x \le x_{n3}; y = y_{n0}; T_4 = T_8; \lambda_4 \dfrac{\partial T_4}{\partial y} = \lambda_8 \dfrac{\partial}{\partial} \end{cases}$$

λ_2, λ_3, λ_4 – coefficients of the heat conduction of warmth-keeping concrete wall and the plaster; λ_5, λ_6, λ_8 – coefficients of the soil heat conduction out of the technical underground, snow and the soil inside of the technical underground, W/(m^2·°C); α_{H2} – coefficient of the heat conduction on the border 'the inner surface of the basement wall – inner air', W/(m^2·°C).

The boundary conditions for layers 5 and 6 (the soil out of the technical underground and the snow layer):

$$\begin{cases} x = x_{n3}; y_{n0} \le y \le y_{n1}; T_4 = T_5;\ \lambda_5 \dfrac{\partial T_5}{\partial x} = \lambda_4 \dfrac{\partial T}{\partial x} \\ x_{n3} \le x \le x_n; y = y_{n1}; T_5 = T_6; \lambda_5 \dfrac{\partial T_5}{\partial y} = \lambda_6 \dfrac{\partial T_6}{\partial y} \\ x = x_n; y_{n0} \le y \le y_{n1}; q = 0; \\ x_{n3} \le x \le x_n; y = y_{n0};\ T_5 = T_8; \lambda_5 \dfrac{\partial T_5}{\partial y} = \lambda_8 \dfrac{\partial T_8}{\partial y} \end{cases}$$

$$\begin{cases} x_{n3} \le x \le x_n; y = y_{n2}; \\ \alpha_{H1} \cdot (T_{y_{n2}} - T_H) = -\lambda_6 \dfrac{\partial T_6}{\partial y},\ \alpha_{H1} = f(v, \tau); T_H = f \\ x = x_n; y_{n1} \le y \le y_{n2}; q = -\lambda \dfrac{\partial T_6}{\partial x} = 0; \\ x_{n3} \le x \le x_n; y = y_{n1}; T_5 = T_6; \lambda_6 \dfrac{\partial T_6}{\partial y} = \lambda_5 \dfrac{\partial T_5}{\partial y}; \\ x = x_{n3}; y_{n1} \le y \le y_{n2}; T_4 = T_6; \lambda_6 \dfrac{\partial T_6}{\partial x} = \lambda_4 \dfrac{\partial T_4}{\partial x}, \end{cases}$$

λ_5 – coefficient of the soil heat conduction out of the technical underground, W/(m^2 ·°C); α_{H1} – coefficient of the heat conduction on the border "snow layer – inner air", determined in (10), W/(m^2·°C).

The boundary conditions for layers 7 and 8 (the air out of the technical underground and the soil inside of the technical underground):

$$\begin{cases} x = x_n; y_{n2} \le y \le y_n; q = 0; \\ x_{n3} \le x \le x_n; y = y_n; q = 0; \\ x = x_{n3}; y_{n2} \le y \le y_n; \alpha_{H2} \cdot (T_H - T_{x_{n3}}) = -\lambda_4 \dfrac{\partial T_4}{\partial x} \\ x_{n3} \le x \le x_n; y = y_{n2}; \alpha_{H1} \cdot (T_H - T_{y_{n2}}) = -\lambda_6 \dfrac{\partial T_6}{\partial y} \\ \alpha_{H1} = f(v, \tau); T_H = f(\tau). \end{cases}$$

$$\begin{cases} x_0 \le x \le x_{n0}; y = y_{n0}; \alpha_{B2} \cdot (T_B - T_{y_{n0}}) = -\lambda_8 \dfrac{\partial T_8}{\partial y}; \\ x_{n0} \le x \le x_{n1}; y = y_{n0}; T_2 = T_8;\ \lambda_8 \dfrac{\partial T_8}{\partial y} = \lambda_2 \dfrac{\partial T_2}{\partial y}; \\ x_{n1} \le x \le x_{n2}; y = y_{n0};\ T_3 = T_8; \lambda_8 \dfrac{\partial T_8}{\partial y} = \lambda \dfrac{\partial T_3}{\partial y}; \\ x_{n2} \le x \le x_{n3}; y = y_{n0};\ T_4 = T_8; \lambda_8 \dfrac{\partial T_8}{\partial y} = \lambda_4 \dfrac{\partial T_4}{\partial y}; \\ x_{n3} \le x \le x_n; y = y_{n0}; T_5 = T_8; \lambda_8 \dfrac{\partial T_8}{\partial y} = \lambda_5 \dfrac{\partial T_5}{\partial y}; \\ x = x_n; y_0 \le y \le y_{n0};\ \dfrac{\partial T_8}{\partial x} = 0; \\ x_0 \le x \le x_n; y = y_0;\ \dfrac{\partial T_8}{\partial y} = 0; \\ x = x_0; y_0 \le y \le y_{n0};\ \dfrac{\partial T_8}{\partial x} = 0. \end{cases}$$

In some authors' [1-3] opinion, at a distance from the construction the heat influence of a building becomes weaker and imperceptible. On these borders the heat exchange equals 0. On some borders of layers 5, 6 and 8 the boundary conditions of the second type (BCII) are taken into account. As additional heat sources *Q* the following components of heat balance of the technical underground are considered: Q_h – heat of the pipeline, Q_{ov} – heat coming from the underground floor, Q_{inf} – heat loss because of air exchange between the technical underground and the environment, W.

$$Q = Q_h + Q_{ov} - Q_{inf} \,. \tag{7}$$

Defining the coefficient of the heat transfer on different surfaces of building envelopes the empirical dependence (given in [14]) that describes the heat exchange under the natural convection conditions has been used. The average values of the heat transfer coefficients were assessed by the criteria equation for typical cases.

On account of low temperatures on the surfaces of the building envelopes and, therefore, heat proportion values, we neglect this component of the heat exchange.

Analysing the existing numerical methods of thermal problems solution, the problem of heat exchange of the attic space with the environment is solved using a broadly used sweep method with the implicit difference scheme [16].

The numerical solution of the above-mentioned problem has been implemented by means of the software package developed on the object-based framework Delphi 7.0. When creating the package a modular concept has been used.

A basic software unit realizes the solution of a parabolic type and a general form equation with the boundary conditions of 1 – 4 types in one layer and multilayer areas. It consists of smaller units of calculation of the implicit difference scheme coefficients and solves the difference equation system by the sweep method, tested on the famous analytical solutions.

For testing the module, the analytical solutions with their further comparison with the numerical solution results were obtained.

The results of comparison show that the maximum deviation of the numeric solution from the analytical one does not exceed 0.6 %. It proves the correctness of the numeric scheme choice and its implementation.

3. Conclusion

As a result of the research, the two-dimensional nonstationary symmetrical problem of the heat conduct in a semi-limited mass with a set of boundary conditions has been viewed. The physical-mathematical model of the heat exchange of the technical underground (with the influence of air exchange) and the snow layer out of a construction has been considered.

The results of the numeric simulation showed that the heat coming through the floor and the walls of the technical underground does not influence the temperature field of soils both under the construction and at a distance from it. This deduction fully meets the demands of buildings erection under the climatic conditions with permanent frozen soils [17], which "ban" the heat influence of designed constructions on the natural temperature background of soils.

In accordance with the research the temperatures in soils massive outside of the construction area are equal to their natural values.

The comparison of the results of the numeric solution of the software package (18) with the solution obtained from other thermal computer programs (e.g. Temper-3D) has shown their satisfactory coherence. Nevertheless, the program Temper-3D does not take into account the air exchange in the technical underground, which is very important for the formation of the temperature fields in the building envelopes and soil.

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