

A Single-Loop DC Motor Control System Design with a Desired Aperiodic Degree of Stability

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Abstract. The application of the original analytical approach for PI-controller synthesis of a stable second-order plant is considered. This approach allows finding controller parameters without any intensive computing by using the direct expressions. The plant model is obtained on the basis of identification, which is based on the automated real-interpolation method. The results of natural experiments are given.

1. Introduction

The design of automatic control system regulators is one of the most important problems in the control theory. Initial data for the design solution are the mathematical description of the plant and some demands of the designed system functioning. There are a lot of approaches to the automatic control systems design [1, 2]. There is a huge class of automatic control systems, which require aperiodic transient responses, that is, without oscillations. It is also known that a control system of any order and complexity may be described with a tolerable accuracy rate by a system of a reduced order (first or second) [3]. Therefore PI-controller parameters determination for a DC motor control providing the desired behavior of a designed system is considered in this work.

2. Problem Formulation

DC motor 'Dynamo' Sliven PIVT6-25/3 A with a stationary velocity sensor is used as a plant. The motor has the following characteristics: source voltage, 30 V; rotation speed, 3000 rpm; rotating torque, 0.11 Nm; starting current, 4.5 A; electromechanical time constant, 25 msec; electrical time constant, 2 msec; mass moment of inertia, $5.1 \cdot 10^{-5}$ kg m²; velocity sensor constant, 3 V / 1000 min⁻¹. A microcircuit chip Pololu High-Power Motor Driver 18v 15 is used as a motor driver.

The view of the control system block diagram is represented in Figure 1.

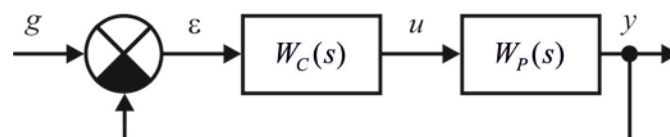


Figure 1. Control system block diagram

PI-controller transfer function is

$$W_C(s) = \frac{k_1 s + k_0}{s} \quad (1)$$



where k_0 is an integral coefficient and k_1 is a proportional coefficients.

The plant transfer function is represented by the following expression:

$$W_p(s) = \frac{B(s)}{A(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}, \quad n \geq m. \quad (2)$$

It is known that the condition of the aperiodic transient response is such location of closed loop system poles, at which a real dominant pole is the nearest to the imaginary axis, and all other so-called 'free' poles are moved at the sufficient distance $\gamma = [\eta, \alpha]$ to the left and are located in the truncated segment Γ (Figure 2). Therefore, the problem is to place real dominant pole α of the closed loop system in the coordinate $(-1; 0)$ and other free poles on the real axis past the boundary $\eta = 7$.

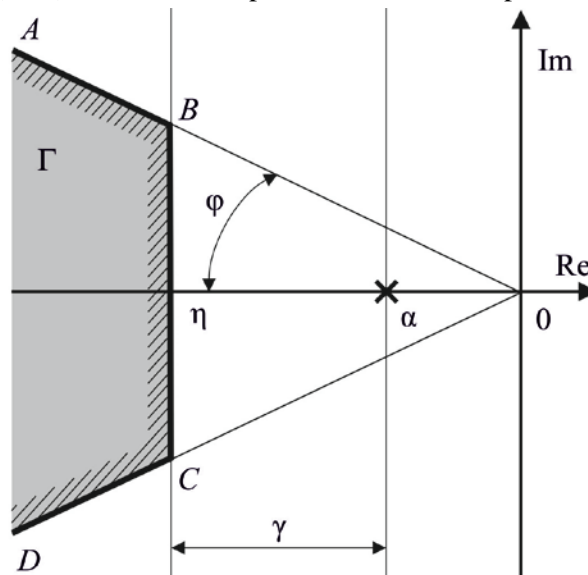


Figure 2. The location of a real dominant pole and free poles of a closed loop system

3. Plant Identification

On the first stage it is necessary to obtain the mathematical description of the plant. It is suggested to make parametric identification, which is mainly used for stable objects with a non-zero static gain.

The plant model is formed on the basis of experimental data applying the approach represented in [4]. The approach is based on the real-interpolation method, which discretizes continuous functions and reconstructs them in a continuous form with the help of interpolation.

As a result of an input test step signal the vector of the plant output data under a load is obtained. In consequence the following model of the plant is obtained:

$$W_{p1}(s) = \frac{4.321 \cdot 10^{-3} s + 5.126 \cdot 10^{-2}}{6.206 \cdot 10^{-2} s^2 + 1.130 s + 1} \quad (3)$$

It is known that the results of the controller design on the basis of root approaches may have some errors [5]. It is strongly connected with a presence of zeros in the transfer function (2) numerator. This fact may influence in an unforeseen manner the control system behavior and performance measures in the time domain. That is why it is offered to take the transfer function (3) without zeros in the numerator into consideration.

$$W_{p2}(s) = \frac{5.126 \cdot 10^{-2}}{6.206 \cdot 10^{-2} s^2 + 1.130 s + 1} \quad (4)$$

Figure 3 shows step responses of transfer functions (3) and (4), and it is vivid that both plants have similar dynamics.

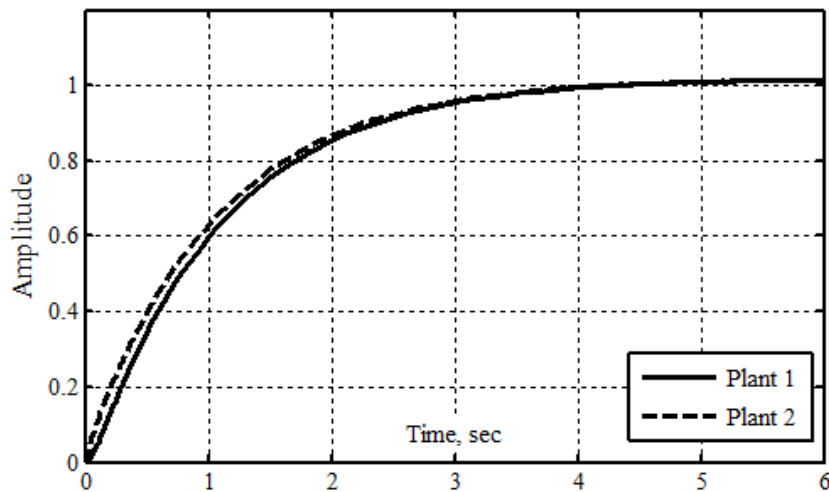


Figure 3. Transfer functions of $W_{p1}(s)$ and $W_{p2}(s)$

4. Controller Design

A controller design method is based on the decomposition of the closed loop system characteristic equation $A(s) = \sum_{i=0}^n a_i s^i, a_i > 0$ into the dominate polynomial $Q(s) = s + \alpha$, which carries the

information about location of the dominate pole and the free polynomial $P(s) = \sum_{i=0}^{n-1} p_i s^i, p_i > 0$, which roots are free poles. The result of the decomposition is a residue of division $R = a_0 - \alpha p_0$. On the basis of R , controller parameter k_0 can be expressed. In general terms the equation may be written as follows: $A(s) = Q(s) \cdot P(s) + R$.

If the characteristic polynomial order of the closed loop system is $n = 3$, then the free polynomial order is $n - 1 = 2$. On the basis of the second order equation solution the analytical expressions connecting PI-controller parameters k_0 and k_1 with free polynomial parameters p_i and parameter η the following form can be obtained:

$$\begin{cases} p_1^2 - 4p_0p_2 > 0; \\ p_0 \geq \eta p_1 - \eta^2 p_2. \end{cases} \quad (5)$$

The verification of design results is conducted through the synthesized closed loop system characteristic equation roots definition.

5. Results of the Design

Let us define the PI-controller parameters for the plant (4). Substituting numeric values in expressions (5), the acceptable region of $k_1 \in (88.353; 91.515]$ is obtained. Let us suppose that $k_1 = 91.515$, then, on the basis of division residue R , $k_0 = 90.181$, the poles are $s_1 = -1$, $s_2 = -8.59$, $s_3 = -8.63$ and zero is $N_1 = -0.985$.

In case of the transfer function (TF) (3) the impact of the zero numerator does not allow placing the closed loop system poles in accordance with requirements: $s_1 = -1$, $s_2 = -3.73$, $s_3 = -19.8$, $N_1 = -0.985$, $N_2 = -11.9$.

Step responses of the synthesized systems are represented in Figure 4.

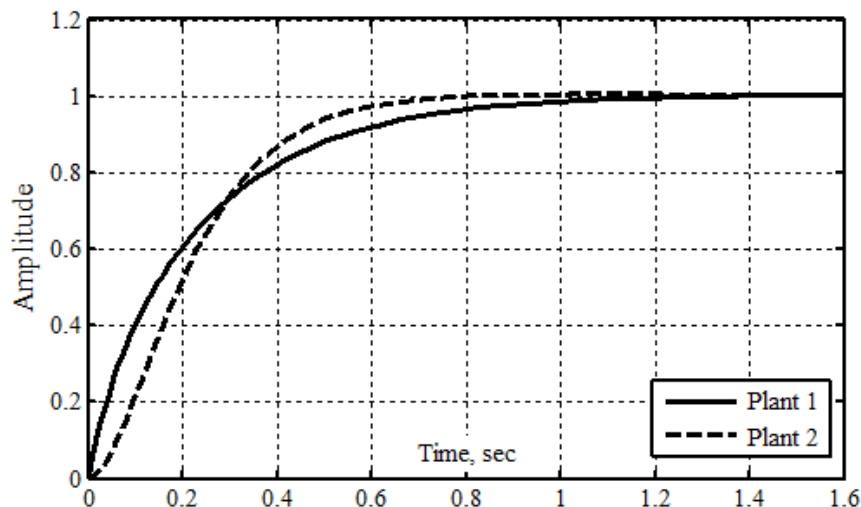


Figure 4. Step responses of the synthesized systems

On the basis of step responses of the synthesized closed loop system (Figure 4) the system performance measures can be estimated. Therefore, for plant 1 there is an overshoot $\sigma = 0.13\%$ and the settling time is $t_s = 0.9$, sec, for plant 2 there is $\sigma = 0.54\%$ and $t_s = 0.6$, sec.

Table 1. Performance measures of identified and synthesized systems.

	DC motor model		Designed Control System	
	$W_{p1}(s)$	$W_{p2}(s)$	$W_{p1}(s)$	$W_{p2}(s)$
	TF with zero	Reduced TF	TF with zero	Reduced TF
Overshoot, %	0	0	0.135	0.548
Settling time, sec	4.25	4.17	0.911	0.624

As we can see from the table the overshoot of synthesized system $W_{p2}(s)$ is negligibly higher than $W_{p1}(s)$ overshoot, but at the same time $W_{p1}(s)$ settling time is inconsiderably longer than $W_{p2}(s)$ settling time.

6. Conclusion

As a result of experiments with the DC motor control system it is possible to conclude that both (full and reduced) synthesized systems introduce acceptable dynamics and performance measures. But the introduced method of PI-controller synthesis has some restrictions on its application in the plants with zeros in the transfer function numerator. This fact is inessential in terms of low-order control systems because it does not influence the dynamics of the synthesized system critically.

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