# Mathematical Simulation of Contaminant Flow in Closed Reservoir

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Abstract. A mathematical model of the propagation in flooded mine lightweight contaminant due to allocation of groundwater is considered. Mathematical model was based on an analysis of experimental data and using concept and methods from reactive media mechanics. The boundary-value problem is solved numerically using the finite volume method. The distribution of fields of velocities and concentration of impurity particles in a flooded mine have been obtained at different times. These results can be used to analyze mining water treatment process due to environment and evaluate its further possible improvements.

#### **1. Introduction**

Water body pollution by mining and quarry waters is a typical problem for Kuzbass (Kemerovo region, Russia) and many other mining regions [1–3]. Mining waters usually contain particles of coal dust, clay, calcium compounds, magnesium, oil products, etc. Light substances (which density is less than water density) such as oil products accumulate on water surface while other particles remain suspended or sediment gradually. The problem of mining water treatment by pumping into abandoned mines and further use of it after precipitation of impurities (for heavy particles) or impurity floating up (for light particles) is of great interest. This methodology was presented in [4]. The model in this paper is based on laminar flow. In fact, estimates indicate that for a considered area is turbulent. The model which is being reported here is complex, and includes particles. The paper considers fluid flow containing impurity particles in a flooded mine.

### 2. Physical and mathematical setting

To analyze float impurities distribution a square form mine is under consideration. It has a ledge at the top (shown in figure 1).

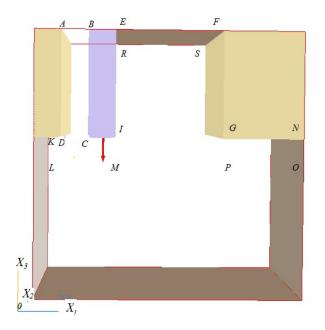


Figure 1. Computational domain scheme.

Mathematically the following differential equation system for turbulent flow should be solved in the following way:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}(-\overline{\rho u_i' u_j'}) - \rho SC_d u_i \left| \vec{u} \right| - \rho g_i,$$
(2)

$$\rho(\frac{\partial Y_k}{\partial t} + u_1 \frac{\partial Y_k}{\partial x_1} + (u_3 - u_{3k}) \frac{\partial Y_k}{\partial x_3}) = \frac{\partial}{\partial x_i} (-\overline{\rho Y'_k u'_j}),$$
(3)

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(u_i \rho k) = \frac{\partial}{\partial x_i} \left[ \left( \frac{\mu_i}{\sigma_k} + \mu \right) \frac{\partial k}{\partial x_i} \right] - \mu_i \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial u_j} \right) \frac{\partial u_i}{\partial x_j} - \beta \rho g_i \frac{\mu_i}{\Pr} \frac{\partial T}{\partial x_i} - \rho \varepsilon, \tag{4}$$

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_i}(u_i\rho\varepsilon) = \frac{\partial}{\partial x_i} \left[ \left( \frac{\mu_i}{\sigma_{\varepsilon}} + \mu \right) \frac{\partial\varepsilon}{\partial x_i} \right] + C_1 \frac{\varepsilon}{k} (G_k + G_k) - C_2 \rho \frac{\varepsilon^2}{k},$$
(5)

$$p = \rho R_0 T \sum_k \frac{Y_k}{M_k}, \ \vec{g} = (0, g), \ u_{3k} = \frac{g d_k^2}{18\nu} (\frac{\rho_k}{\rho} - 1).$$
(6)

Here, t,  $x_i$  – time and spatial coordinates (i=1, 3);  $u_i$  – velocity vector projection on the corresponding axis of Cartesian reference system, p – pressure; g – gravitational acceleration,  $R_0$  – absolute gas constant,  $M_k$  – molecular weight of k – component,  $\rho$  – density of fluid and particles mixture, v – kinematic viscosity coefficient,  $D_t$  – turbulent diffusion coefficient,  $d_k$ ,  $\rho_k$ ,  $u_{3k}$  - diameter, density and velocity of particle settling [5],  $Y_k$  – mass concentrations of k – component (k=1 – water, 2 – solid particles);  $\mu_t = \rho C_{\mu} k^2 / \varepsilon$  - coefficient of turbulent viscosity,  $k = \overline{u'_i u'_i}/2$  - turbulent kinetic energy;  $\varepsilon$  – its dissipation,  $C_{\mu}$ ,  $\mathcal{O}_k$ ,  $\mathcal{O}_{\varepsilon}$ ,  $C_1$ ,  $C_2$  – empirical constants, and  $G_k$ ,  $G_B$  -turbulence caused by forced convection and natural convection.

#### 3. Numerical solution and results

Based on mathematical formulation of the problems (1) - (6) numerical calculations were made to determine the pattern of float impurity distribution process in a flooded mine with the help of **PHOENICS** [6–7].

Vector fields of velocity and impurity distribution at different time moments were obtained as the result of numerical integration of equation system (1) - (5). Side walls are considered not to influence the impurity distribution process and fluid flow. Thus the problem is solved in the two-dimensional domain  $X_1OX_3$ . A mine (length -10 meters horizontally, height-3 meters) is under consideration (figure 1.). Underground water without any impurity enters the domain. Impurity concentration equals 1 inside the domain. Particles size is  $d_k = 5 \cdot 10^{-5}$  m. Impurity particles density is 500 kgs/m<sup>3</sup> that is two times less than water density. The velocity of groundwater inflow from the upper layers is 0.1 m/s. Figure 2 shows the distribution of impurity concentration at different time moments.

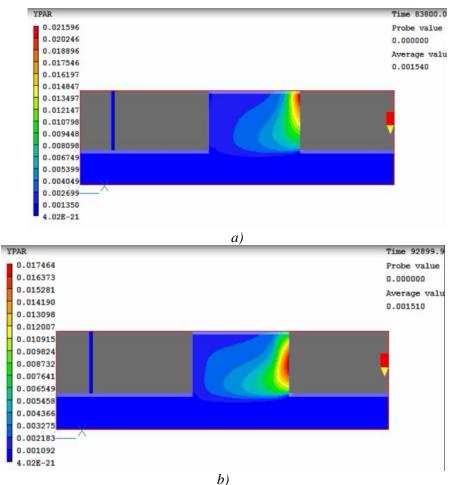


Figure 2. Impurity distribution: a) t=83800 sec, b) t=92899 sec.

Figure 2 shows, that as time goes some part of the impurities transferred by a flow leaves a mine, while the remaining part is moved to a dirt collector by a vortex. In case impurity particles density is halved (as little as 250 kgs/m<sup>3</sup>) the distributions of impurity are numerically calculated at different time moments (figures 3–5). These figures show that the flow becomes stable and impurities accumulate in the upper part of the domain as time goes. It happens faster compared with the previous case because the particles density is two times less.

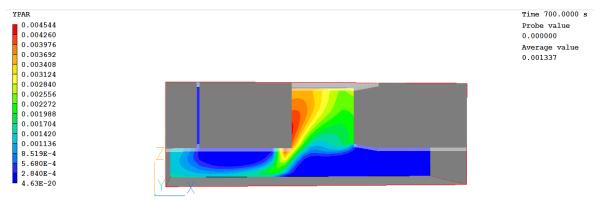
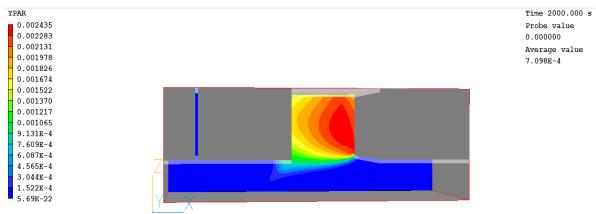
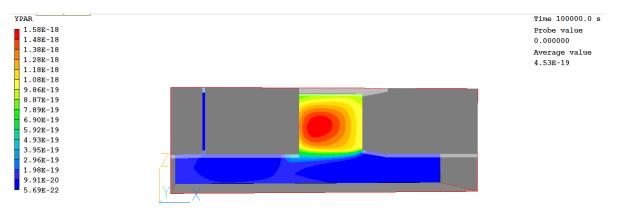


Figure 3. Distribution of impurity which density is  $250 \text{ kgs/m}^3$  (t=700 sec).

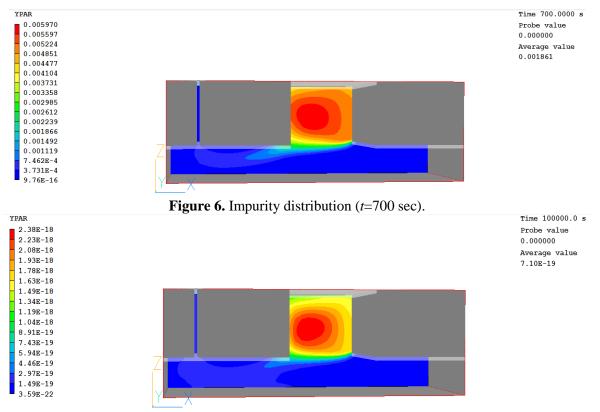


**Figure 4.** Distribution of impurity which density is  $250 \text{ kgs/m}^3$  (*t*=2000 sec).



**Figure 5.** Distribution of impurity which density is  $250 \text{ kgs/m}^3$  (*t*=10000 sec).

Later the inflow velocity was halved as compared with the previous case (0.05 m/s). When compare the results of impurity distribution (figures 6, 7) with the previous case at the same moment of time (figures 4, 5 correspondingly) it is obvious that the flow becomes stable and impurity concentration increases faster in the upper part of the domain.



**Figure 7.** Impurity distribution in a flooded mine (*t*=10000 sec).

Based on the results of the numerical calculations, the way velocity of underground water inflow and parameters of impurity particles influence mining water treatment can be identified. The mathematical model presented in this paper can be used to analyze mining water treatment process due to environment and evaluate its further possible improvements.

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