Features of X-ray Absorption Densitometry of Large-size Objects with Variable Thickness

S Osipov[1], S Chakhlov[2], O Osipov[3], S Badin[4] and A Abashkin[5]

[1]Leading reseacher, NDT Institute, Tomsk Polytechnic University, Russia
[2]Head laboratory of technical tomography and introscopy, NDT institute, Tomsk Polytechnic University, Russia
[3]Engineer-reseacher, NDT Institute, Tomsk Polytechnic University, Russia
[4]Postgraduate, NDT Institute, Tomsk Polytechnic University, Russia
[5]Undergraduate, NDT institute, Tomsk Polytechnic University, Russia
E-mail: osip1809@rambler.ru

Abstract. Features of formation and processing of the primary radiometric signals in the digital high-energy X-ray absorption densitometers for the homogeneous objects with variable thickness are examined. The densitometer's equation based on the polynomial approximation of the object's ray thickness dependence from its mass thickness is proposed. Guidance to select the capacity of the analog-digital converter is given. There is one example of the densitometer's equation coefficients calculation to examine the carbon, aluminum and steel wares with the mass density from 15 to 80 g/cm^2. It was shown that disagreement of the experimental and estimated values of the ray thickness for the similar mass thicknesses of the testing object is conditioned by the scattered radiation. On the high-energy digital radiography set with the X-ray source – the betatron MIB-4.5/9 the accuracy of the experimental estimation of the density was within 0.0086 g/cm^3 for the steel ware thickness from 25 to 100 mm.

1. Introduction

The problem of calculation of density and values related with density is typical for many branches of science, industry, construction engineering and it is solved efficiently by methods based on the gamma radiation attenuation measurement [1, 2]. Currently the digital radiography methods with X-ray sources are widely using to measure the density [3−5]. There are some factors [6−7] whose essentially affect value of the systematic inaccuracy of the testing objects (TO) parameters calculation for X-ray or gamma radiation absorption. During examination of TO with variable thickness the most important is the effect of X-ray radiation beam hardening which influence is reduced by few modes. The X-ray computational tomography uses the compensating filters [8], but this method is hard-to-use. The dual energy method (DEM) allows to calculate the density and the material's effective atomic number of TO at the same time [9]. The spectrometric method [10] is based on the energy spectrum recording. This method allows to minimize the analyzed effect influence on the parameter's estimation accuracy by the X-ray absorption method. The spectrometric version of the counting X-ray registration mode is differed by low performance. The analyzed effect compensation method based on the preliminary calibration is most productive and simple in implementation. Until now, there is a lack of discussion about features of the generation and processing of primary radiometric signals for the digital X-ray absorption densitometers of the large-size homogeneous objects with variable thickness.

2. Generation and processing of the radiometric information in X-ray absorption densitometers

The simplified geometry of the X-ray absorption densitometer is shown on figure 1. As example of the TO fragment there is selected a rectangular parallelepiped with dimensions $H \times A \times B$.

Initial data. The point source of high-energy X-ray radiation with energy spectrum $f(E,E_0)$, where E_0 is the maximal energy is located on distance F from scintillation detector. The axis of symmetry of the bar-shaped scintillation detector is directed to the radiation source, lateral dimensions are $a \times b$, the thickness is h. The geometric parameters of the examination setup are satisfied the following conditions $F >> h >$ (a and b). The slit collimator has length L, thickness D and gap width d. The TO thickness H is varied in the range from H_{min} to H_{max}.

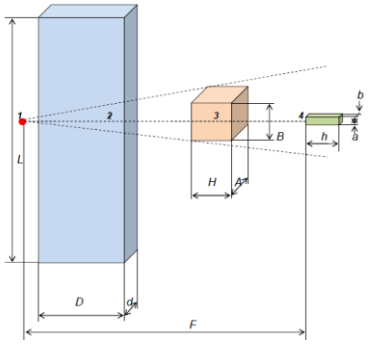

Figure 1. Geometry of X-ray absorption densitometer: 1 – radiation source; 2 − slit collimator; 3 – testing object; 4 – scintillator.

2.1. Generation, digitization and calibration of the measuring signals

The generation of the analog radiometric signal. The energy lost in scintillation detector by the X-ray photons is converted to the optical radiation. The optical radiation is transformed to the current by the photo-detector. The analog signal value on the radiometric detector output is depended from TO thickness H and its density ρ, the maximal energy of X-ray radiation E_0, the scintillator thickness and its material. The value ρH is the product of density ρ on thickness H is called the mass density and is measured in g/cm^2. The analog signal on the radiometric detector output $J(E_0,\rho H)$ equals to sum the signal $I(E_0,\rho H)$ defined by absorbed energy of the registered photons and by the own detector noises p

$$J(E_0,\rho H) = I(E_0,\rho H) + p\,. \tag{1}$$

The level of the own detector noise p can be estimated without X-ray radiation or with absolute detector protection from photon radiation. Just second approach confirms the correctness of $p=J(E_0,\infty)$.

Besides the own detector's noises level p it is interesting the output signal of the radiometric detector produced without the TO as provided by (1) and this signal logically can be denoted $J(E_0,0)$.

Digital signal generation. The output detector signal is digitized by the analog-digital converter (ADC). The main ADC feature is its capacity K. The sampling rate Δ is defined by the ADC capacity K and by the variation interval of the digitized signal ($\min J$, $\max J$). Evidently that the analog signal value is maximal for H=0. The zero level is selected as the minimal value of the analog signal. Hence it follows that the sampling rate is defined by the expression

$$\Delta = \frac{C_k J(E_0,0)}{2^K - 1}, \tag{2}$$

here C_k, C_k >1 is a coefficient which allow possible variations of the analog signal maximum.

The transformation of the analog signal J to the digital signal J_d is described by formula

$$J_d = \left[\frac{J}{\Delta}\right], \tag{3}$$

where $[x]$ is integer part of number x.

The primary calibration of the measuring signal. On first stage of the primary calibration two parameters are defined: the own noise level of the radiometric detector $J_d(E_0,\infty)$ and the signal level without the TO $J_d(E_0,0)$. The mentioned levels were estimated many times therefore we can consider that errors of the sample mean values $\overline{J_d(E_0,\infty)}$ and $\overline{J_d(E_0,0)}$ are close to zero. The second stage is the

"black" calibration which reduced to subtraction value $\overline{J_d(E_0,\infty)}$ from the digital measuring signal. On third stage the "black" calibrated digital measuring signal is normalized on the "black" calibrated digital signal without the TO. On fourth stage we find the logarithm of the "black" and "white" calibrated measuring signal. The total signal transformation is described by the expression

$$Y_d(E_0,\rho H)=\ln\frac{J_d(E_0,0)-J_d(E_0,\infty)}{J_d(E_0,\rho H)-J_d(E_0,\infty)}. \quad (4)$$

The value $Y_d(E_0,\rho H)$ called the ray thickness of the TO for the X-ray radiation with maximal energy E_0. The dependence of the ray thickness Y_d from the mass thickness ρH is the backgroud for all following analysis of the X-ray absorption densitometer.

2.2. The equation of X-ray absorption densitometer

The most common case of the equation connecting the material density, the TO thickness and the TO ray thickness expresses like that

$$P(\rho H)=Y, \quad (5)$$

here P is function of the object mass thickness ρH for the fixed value E_0. For the monotone continuous function P there exist the inverse function P^{-1}. The equation to calculate density ρ has form

$$\rho=\frac{P^{-1}(Y)}{H}. \quad (6)$$

The function P^{-1} can be created by the testing results of the specially designed graded calibration sample. This calibration sample must be made from the same material as TO. As the testing results of the calibration sample there are generated a set of pairs $(\rho H_i,Y_i)$, $i=1\ldots n$, here n is number of steps in the graded calibration sample. We shall examine the polynomial approximations of the function $P^{-1}(Y)$.

The systematic inaccuracy of the density estimation conditioned by the approximation error of P^{-1} by some type function is depended from the type of function G, the object mass density range, the approximation function parameters number, the number and the calibration sample step levels. The influence of the abovementioned factors on the density precision can be examined by experiment.

The criterion of the maximal pointwise deviation of the calculated density values from the nominal ones is the most reasonable

$$\Delta_\rho=\max_{\rho H_{\min}\le\rho H\le\rho H_{\max}}\left|\frac{G(Y)}{H}-\rho\right|. \quad (7)$$

The selection of approximation type $G(Y)\approx P^{-1}(Y)$ is impossible without the analysis of functions $P(\rho H)$ the experimental dependencies of TO ray thickness Y from the mass thickness ρH.

3. The dependence $P(\rho H)$ calculation

3.1. The formula to calculate the ray thickness of the testing object

For the fixed maximal X-ray radiation energy E_0 the formula to calculate the TO ray thickness Y with the mass thickness ρH are

$$Y=F(\rho H)=\ln\int_0^{E_0}\frac{\overline{E}_{ab}(E)}{E}f(E,E_0)\left(1-e^{-\mu_{sc}(E)h}\right)dE-\ln\int_0^{E_0}\frac{\overline{E}_{ab}(E)}{E}f(E,E_0)e^{-m(E)\rho H}\left(1-e^{-\mu_{sc}(E)h}\right)dE, \quad (8)$$

here $\overline{E}_{ab}(E)$ is the mean value of the energy loosed by the registered photon in scintillator; $m(E)$, $\mu_{sc}(E)$ are the mass and linear attenuation coefficients of the photon radiation with energy E by the TO material and by the scintillator material, correspondingly. The expression (8) is valid for the sampling rate $\Delta=0$. The calculation for the values of the sampling rate Δ must allow the analog to digital signal conversions that based on expressions (3), (4).

The usage of the tables of the interaction cross-section of gamma radiation with material [11] and the spline-interpolation is reasonable to calculate the ray thickness by the formula (8). Note that value $\overline{E}_{ab}(E)$ is depended from photon energy, material and sizes of scintillator [12], and for the detectors with small lateral dimensions or small thickness it is necessary take into account not only the

secondary photon leakage, but also the secondary electron leakage [13]. The dependencies $\overline{E}_{ab}(E)$ are defined in the first approximation by the tables [11]. The relation $\overline{E}_{ab}(E)/E$ is no other than the part of energy lost in the detector by the registered photon.

The selection of ADC capacity is need to convert the analog signal to digital ones.

3.2. The selection of ADC capacity

One of the component of the density estimation error is the analog to digital signal conversion error Δd. The maximal density measurement error $\Delta\rho$ is defined on design phase of the densitometer. It is logical to claim the restriction $\Delta_d << \Delta_\rho$, for example, $\Delta_d<0{,}3\Delta_\rho$ for all range of the thickness H. The ADC capacity for the examined task is adequate if satisfy the condition

$$\left[\frac{J(E_0,(\rho+\Delta_d)H_{max})(2^K-1)}{C_k J(E_0,0)}\right]-\left[\frac{J(E_0,\rho H_{max})(2^K-1)}{C_k J(E_0,0)}\right]\geq 1. \qquad (9)$$

Remind the $[x]$ is integer part of the number x.

Let test the feasibility of (9) for the following conditions: K=16; the steel ware with mass thickness ρH_{max}=80 g/cm^2; the steel density 7.86 g/cm^3; E_0=4.5 and 9 MeV; Δ_d=0.002 g/cm^3; C_k=1.5. For those conditions the left part of inequality (9) is equal 1 for E_0=4.5 MeV and 2 for E_0=9 MeV, that is ADC capacity is enough for the example requirements.

3.3. The example of Y(ρH) calculation

To analyze the function type $P^{-1}(Y)$ the calculations of dependencies $Y(\rho H)$ in respect to the objects from carbon, aluminum and iron for $0<\rho H\leq$ 80 g/cm^2 were done. The maximal energy of X-ray radiation E_0 is changed in range from 2 up 9 MeV, that is in the high X-ray energy range. The radiometric detector was done from $CdWO_4$ with thickness 45 mm. The ADC capacity was K=16. The energy spectrum $f(E,E_0)$ of the X-ray source is described by the function Schiff's [14], E_0>1 MeV.

The typical dependencies of $Y(\rho H)$ are shown on figure 2.

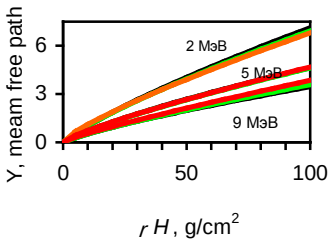

Figure 2. The typical dependencies of $Y(\rho H)$: ▬ − carbon; ▬ − aluminum; ▬ − iron.

From the analysis of the data presented on figure 2 can draw two main conclusions:

1. The functions $Y(\rho H)$ are smooth and monotone increasing in all examined range of the maximal X-ray energies.

2. There is a value E_0 for that the deviation of dependencies $Y(\rho H)$ is minimal for different materials.

The first conclusion allows use the polynomials to approximate the dependence $P^{-1}(Y)$. The second conclusion can be used to design the universal densitometers without the high accuracy requirement.

4. The study of the polynomial approximations of $P^{-1}(Y)$ function

Let analyze the approximation of table values $(\rho H_i, Y_i)$, i=1…n of the function $P^{-1}(Y)$ by the polynomial power of k. In this case the function $P^{-1}(Y)$ is described by the expression

$$P^{-1}(Y)\approx\sum_{j=0}^{k}a_j Y^j, \qquad (9)$$

here a_j, j=0…k are the polynomial coefficients.

The coefficients a_j, j=0…k of the polynomial regression to approximate the dependence $P^{-1}(Y)$ are founded by the least square method (LSM)

$$\min_{a_0,a_1,\ldots,a_k} \sum_{i}^{n} \left(\sum_{j=0}^{k} a_j Y_i^j - \rho H_i \right)^2 \tag{10}$$

After use the polynomial approximation of the function $P^{-1}(Y)$ the equation (6) reduce to

$$\rho = \frac{\sum_{j=0}^{k} a_j Y^j}{H}. \tag{11}$$

Calculation results were analyzed to estimate the approximation accuracy $P^{-1}(Y)$ by polynomial power of k=3. To do that we compute the polynomial coefficients a_i, i=0…k (11) by the LSM (10) and the approximation accuracy was estimated for the TO mass thickness $15<\rho H\leq 80$ g/cm^2. The analysis of calculated data allows to draw a conclusion about the applicability of the polynomial approximation to build the densitometer equation by the X-ray absorption method for the maximal X-ray energies under study and the mass densities of the TO from 15 to 80 g/cm^2. The polynomial power of 3 or more can be recommended. The systematic inaccuracy of the density due to the lack of approximation is not exceed for carbon 0.0015 g/cm^3, for aluminum 0.0028 g/cm^3 and for iron 0.0127 g/cm^3. The indicated inaccuracies are satisfactory for most practical applications.

The abovementioned conclusions must be verified experimentally.

5. The experiment

5.1. The experimental estimation of dependence Y(ρH) for the steel

The experimental verification of the dependence $Y(\rho H)$ was carried out on the digital radiography system with the high energy X-ray source – the portable impulse betatron MIB−4/9. The detector characteristics are specified in the description of the calculation example. The geometry of measurement is closed to the layout described on figure 1. The narrow X-ray beam is shaped by the slit collimator. The width of the collimator gap equals d=5 mm. The collimator thickness is D=300 mm. The distance from the radiation source to the TO is about 3000 mm. The dependencies $Y(\rho H)$ were estimated for the maximal X-ray energies E_0=4.5 and 9 MeV. The steel objects with densities ρ=7.85 g/cm^3 were examined. Figure 3 shows the experimental dependencies $Y(\rho H)$ for the steel fragments of the TO and the maximal X-ray energies E_0=4.5 and 9 MeV. The selection of the TO material was due to the high quality of the rolled iron. To the experimental results on figure 3 were added the dependence $Y(\rho H)$ calculated by the formula (8). The analysis of the results presented on figure 3 can draw a conclusion about underestimation of the ray thickness values of the steel fragments in the experiment. We suggest a hypothesis that the most likely reason of this underestimation is an insufficient collimation of the X-ray source.

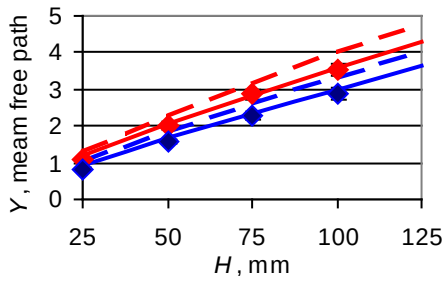

Figure 3. Typical dependencies of $Y(\rho H)$ for steel: E_0=4.5 MeV — — is the calculation result; ♦ is the experimental result; —— is the calculation result with correction for the scattered radiation; E_0=9 MeV — — is the calculation result; ♦ is the experimental result; —— is the calculation result with correction for the scattered radiation.

To validate the abovementioned hypothesis we made a series of calculations of the X-ray energy accumulation coefficient B_E by Monte Carlo method for the measurement geometry of the high energy digital radiography system of the Tomsk Polytechnic University (see figure 1). The initial data for calculation B_E are: F=4200 mm; D=400 mm; d=5 mm; A=200 mm; B=200 mm; H is a parameter

variable from 25 to 125 mm; a=5 mm; b=6 mm; h=45 mm. Accounting the scattering leads to transformation of the calculated dependency $Y(\rho H)$ to $Y^*(\rho H)$

$$Y^*(\rho H) = Y(\rho H) - \ln B_E(\rho H). \quad (12)$$

Figure 3 shows the dependencies $Y^*(\rho H)$ produced from the primary calculation dependencies by the formula (12). The comparison of the calculated curve $Y^*(\rho H)$ and the experimental values of the ray thickness of the TO fragments validates the hypothesis.

5.2. Experimental equations for X-ray transmission densitometer

The table 1 includes the coefficients of the polynomial power of three, approximated the experimental dependencies (11) for the steel TO fragments for the maximal X-ray high energy radiation E_0=4.5 and 9 MeV. Also the table has values of the limiting approximation error Δ_ρ.

Table 1. The experimental values of equation (11) coefficients a_0, a_1, a_2, a_3 and Δ_ρ for the steel

E_0, MeV	Parameters				
	a_0	a_1	a_2	a_3	Δ_ρ
4,5	–5.707	25.57	–3.216	0.764	0.0008
9	–3.184	28.35	–1.694	0.5581	0.0086

The data analysis for table 3 draws a conclusion about applicability of the equation (11) with four parameters for the practical estimation of the TO material density by the X-ray absorption method.

6. Conclusion

The results of the theoretical and experimental researches allow draw a conclusion about possibility to measure the material density by the X-ray high energy method with the systematic error 0.01 g/cm^3. This method can be used to examine the large-size testing objects with variable thickness. It was shown that the visible shift of theoretical and experimental dependencies of the TO ray thickness from the mass thickness is due to the insufficient collimation of the high energy X-ray source.

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