POLISH JOURNAL OF PHILOSOPHY
Vol VIII, No. 1 (Spring 2014), 37-52.

Formal Representation of Proper Names in Accordance with a Descriptive Theory of Reference

Olga Poller
Jagiellonian University

Abstract. In this paper I present a way of formally representing proper names in accordance with a description theory of reference–fixing and show that such a representation makes it possible to retain the claim about the rigidity of proper names and is not vulnerable to Kripke's modal objection.

1. Introduction: What is Descriptivism?

Let me start with a brief explanation of what constitutes a descriptive theory of names. Descriptivism is a semantic theory which has its roots in the traditional Fregean approach to meaning. To explain his view on meaning Frege uses two metaphors–a meaning is a way of presenting an object (*Gegebenheitsweise*, 1918/1984, p. 359), and it is a way of determining an object (*Bestimmungsweise*, 1914/1980, p. 80). The idea behind this is that a speaker, by grasping one of the ways in which an object could be presented, has a way of determining that object. Any theory of names could be presented in a nutshell as answering two questions – one which concerns the semantic criterion for determining the reference of a proper name ('How is the reference of a proper name determined?'), and the other which concerns the semantic competence of the speakers ('What should a speaker know in order to know a name?'). And the traditional approach to meaning gives exactly the same answer to both these questions– it is information that constitutes the semantic criterion for determining the reference, and this information should be known by the speakers. Kripke's two ideas – that the descriptive information known by speakers does not determine the reference of a name and that the semantic criterion for determining the reference could not consist in descriptive information about the reference at all – were a turning point in thinking about names and about the semantic theory of names. After *Naming and Necessity* all modern descriptive theories of names could be divided into classical or non–classical, depending on their acceptance of either the Fregean or the Kripkean approach to meaning. Classical descriptive theories of names were defined by Kripke (1980, p. 71) as six theses I–VI. Theses I and II from Kripke's definition concern the speaker's belief, and it is required in thesis III that this belief should be true. Descriptivism is a theory of names which

DOI: 5840/pjphil20082219

postulates that the reference of a name is semantically determined via the satisfaction of descriptive properties. That is why on descriptive grounds a speaker's true belief is justified. A justified true belief is, in turn, classically understood as knowledge. Kripke stresses (1980, p. 73) that theses V and VI and the converses of these theses follow from theses I–IV. In this way Kripke's six theses could be reduced to the following two:

(1) The reference of a proper name is semantically determined via the satisfaction of a descriptive property;
(2) A speaker knows a proper name iff he knows a definite description determining the reference of the name.

I used 'knows' in thesis (2) for a speaker's justified true belief (the requirement of Kripke's I, II and III theses); in order that theses V and VI and their converses could follow from (1) and (2), I used 'iff' in thesis (2).

Kripke (1980, pp. 57–8) noticed that if we add possible worlds to semantics, thesis (1) could have two interpretations, so descriptivism could be understood either as a theory of meaning (theses (1M) and (2M) below), or as a theory of reference fixing (theses (1R) and (2R)):

Descriptive theory of meaning

(1M) The reference of a proper name in a possible world w is semantically determined via the satisfaction of descriptive property φ of definite description $\iota x.\varphi$ which fixes the reference of the name in world w;

(2M) A speaker knows a proper name iff he knows a definite description $\iota x.\varphi$ which fixes the reference of the name in world w.

Descriptive theory of reference–fixing

(1R) The reference of a proper name in a possible world w is semantically determined via the satisfaction of descriptive property φ of definite description $\iota x.\varphi$ which fixes the reference of the name in a distinguished world w^*;

(2R) A speaker knows a proper name iff he knows a definite description $\iota x.\varphi$ which fixes the reference of the name in a distinguished world w^*.

If a proper name is synonymous with a description then it has the very same modal profile as the description–with respect to possible worlds it changes its reference in exactly the same way as the description. But if a definite description is used only to fix the reference of a proper name in a distinguished (actual) world (1980, p. 55), then the proper name is a rigid designator–with respect to all worlds it designates the object which fulfils the descriptive content in the distinguished world. In this way the proponents of a descriptive theory of reference–fixing have an easier defense of their view against Kripke's objections, because they need not answer the modal argument raised by Kripke against (1M) –they need answer only the epistemic and semantic arguments against both (2M) and (2R) theses of descriptivism.

As I said, after *Naming and Necessity* modern descriptivists have a choice – they could accept the classical version of the descriptive theory of meaning or reference–fixing (as Glüer, Pagin (2006) or Chalmers (2006) did), or they could accept one or both Kripkean ideas and be proponents of non–classical descriptivism. For example, Kroon (1987) holds a version of the (1R) thesis, but rejects (2R): in his opinion, the reference of a proper name 'NN' is semantically determined via the satisfaction of the description 'the individual referred to by uses of the name NN from which I acquired the use of NN' (1987, p. 1 footnote 1), but speakers need not know this description (1987, p. 15). Such descriptivism is not vulnerable at all to Kripke's objections, but could hardly be used to solve puzzles with proper names because of the triviality of the reference–fixing description. Maybe that is why the majority of proponents of non–classical descriptivism (Jakson, 2010, Katz, 1994, Bach, 2002, Justice, 2001, Geurts, 1997) preferred to hold versions of the (2M) or (2R) theses, but reject theses (1M) or (1R). They accept both Kripkean ideas: that the semantic criterion of determining the reference does not consist in descriptive information about the reference (but in a kind of baptism), and that descriptive information possessed by speakers does not determine the reference, but they nevertheless require that speakers should know a description in order to know a name. That is why they are obliged to answer the semantic and epistemic arguments raised against descriptivism.

I present a formal representation of proper names in accordance with the first thesis of a descriptive theory of reference – only the first theses of both descriptive theories are semantic and concern the criterion for determining the reference (the second ones are pragmatic and concern the way of obtaining a proper name by the speakers). That is why the representation presented here can be used by both classical descriptivists and those from the non–classical descriptive camp who hold a version of the (1R) thesis. According to any version of the descriptive theory of reference, proper names designate rigidly and I need to show that the terms formally representing names are rigid designators. Besides giving proof that Kripke's modal argument does not affect the account presented here, I need to answer circularity objections raised against any version of descriptivism. I want to stress that I'm concerned here with the (1R) thesis only, and do not propose a developed theory of proper names (I leave open the question of how exactly the (2R) thesis could be formulated if we take into account the notion of time). This means that all objections raised against the (2R) thesis (Kripke's semantic and epistemic arguments) stay unanswered here (I answer these objections in (Poller *in preparation c*)).

The plan of this paper is as follows. In the next section I briefly explain how descriptions designate with respect to possible worlds and times. In section 3 I explain the key steps of the formal representation of proper names. In section 4 I give semantic and syntactic definitions of a language with new terms representing names, and finally in section 5 I prove that the new terms designate rigidly and have all the properties of rigid designators.

2. Descriptions and Time

In my formal representation of proper names I use a language with alethic and temporal modalities. As we noted earlier, taking into account possible worlds as parameters of evaluation results in a distinction between a descriptive theory of meaning and a descriptive theory of reference–fixing. Besides the notion of rigidity of proper names, two different ways of understanding (1) result in different understandings of the (2) requirement, that is, in the question of whether the speaker should know a reference–fixing description in *all* the parameters, or in only *one distinguished* parameter. If we add time as another parameter of evaluation, the divergences between the two theories become deeper. It seems that there is nothing controversial in adding the time parameter to (1M) (and sometimes (1M) is formulated with it, cf. Soames 2005, pp. 14–15), but we should be more careful with (1R). With a time parameter it could become (1Rt):

(1Rt) The reference of a proper name in a possible world w and a time t is semantically determined via the satisfaction of descriptive property φ of definite description $\iota x.\varphi$ which fixes the reference of the name in a distinguished world w^* and a time t.

I will briefly explain why in (1Rt) we talk about a distinguished world w^* and a time t instead of talking about a distinguished world w^* and a distinguished time t^*. Conventionally, definite descriptions are formally represented as iota–terms formed via applying the ι–operator to a formula φ, and they designate with respect to a parameter of evaluation if there is only one object which fulfils φ in a set assigned to the parameter (otherwise iota–terms fail to designate, Fitting, Mendelsohn, 1998, pp. 254, 104). Iota–terms designate contingently with respect to possible worlds and if you add time as another parameter of evaluation they would designate contingently also with respect to times. Take, for example, 'the Pope'. This expression designates different people with respect to different times in our world (or fails to designate). In order to designate somebody in particular by 'the Pope' expression, we need to fix a time–parameter, for example, by adding 'present' to it (getting 'the present Pope'), or expressing a time explicitly ('the Pope in 1967'). Now the expressions 'the present Pope' or 'the Pope in 1967' designate exactly one person with respect to our world and any time. Note that we treat 'present' as an indexical 'now' which fixes a time–parameter (cf. Kaplan, 1989, p. 545), but 'present' could also be treated as an operator of present time which operates on times of evaluation. If 'present' is understood in the latter way, then the definite descriptions with it ('the present Pope') also designate contingently with respect to times (cf. Fitting, Mendelsohn, 1998, pp. 238–9) and do not designate somebody in particular. I prefer the former way of understanding definite descriptions, and I represent them as a special kind of iota–terms of the form '$\iota x.[_i]\varphi$', where '$[_i]$' is a notational variant of $\mathbf{then}_i$ operator ('true at t_i') taken after (Rini, Cresswell, 2012). Time operator $[_i]$ fixes a time of evaluation, so a definite description $\iota x.[_i]\varphi$ designates with respect to a world w and any time t the very same object designated by iota–term $\iota x.\varphi$ and for a world w and time t_i $\left(I^g_{\langle w,t\rangle}(\iota x.[_i]\varphi) = I^g_{\langle w,t_i\rangle}(\iota x.\varphi)\right)$. That is why in (1Rt) we talk about any time t, not distinguished time t^*.

3 The Formal Representation: Key Steps

Let us have a look once again at thesis (1Rt). According to it, the reference of a proper name is semantically determined via the satisfaction of a definite description connected with the name. This means that the interpretation of a term which formally represents a proper name (I will call it 'a name–term') depends on the interpretation of a description which, in turn, depends on interpretations of the predicates it contains. In other words, interpretation of name–terms and descriptions is calculated and depends on the interpretation of predicate constants.

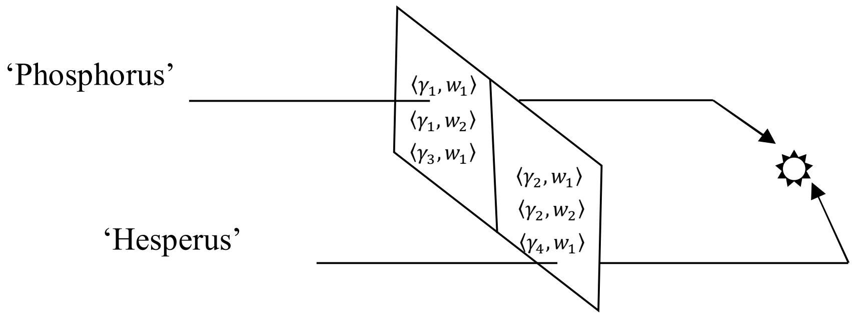
According to a descriptive theory of reference, one (or some) definite description expressing a contingent property was used by speakers to fix the reference of a name (cf. Kripke 1980: 75–6, 78). But the majority of model domains contain no speakers, and that is why there is no difference in them between descriptions which fix a reference of a proper name and descriptions which identify the reference (that is why this difference would not be formally represented).

A set of properties which the referent of a proper name has could differ with respect to a time and a possible world. Nevertheless, every description connected with a proper name NN should contain one common property–a property of being called [*n*], where [*n*] is a sound or an inscription. Let me give an example. Take any proper name, say 'John'. You know that every person with this name has a property, that is, of being called [dʒɒn]. This knowledge is trivial and could be obtained without any reference to extra–linguistic facts (cf. Recanati, 1993, pp. 161–2). Kripke (1980, p. 70) objected that a descriptive theory of reference is circular because "[…] whatever this relation of *calling* is really what determines the reference and not any description like 'the man called 'Socrates''." We avoid the circularity argument by treating [dʒɒn] as a physical object (a sound or an inscription) which belongs to a model domain, not to a language. It is used as a mark to distinguish somebody (cf. Mill, 1889/2011, p. 41), but the property of 'being called [dʒɒn]' is not sufficient to determine the reference because a lot of people are called so.

To represent proper names formally we need a language with a set of distinguished predicates (N_1, N_2, N_3, …) which we will read as 'called α', 'called β', etc., where 'α' and 'β' stand for strings of sounds or inscriptions (arguments supporting such a view on verbs of naming could be found in (Matushansky, 2008) and in (Geurts, 1997)). I will use the symbol '$!\,x.\varphi$' for iota–terms $\iota x.\varphi$ with only one variable x which occurs free in φ. All descriptions $!\,x.[_i]\varphi$ which we connect with a name–term have a form of $!\,x.[_i]\left(N_j(x) \wedge Q(x)\right)$, where '$N_j$' is a distinguished predicate and 'Q' is a 1–place undistinguished, e.g. 'the (present) president called [oʊˈbɑːmə]'. A set of such descriptions will be called '$\Gamma_{\mathcal{L}}$' (see section 4, *Def.*VI.$S(a)$).

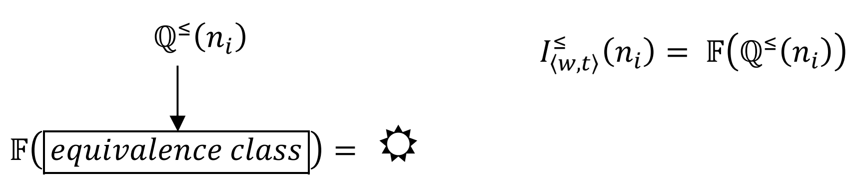
The other objection of circularity concerns the structure of descriptions connected with a name. Kripke (1980, p. 83) noticed that people connect definite descriptions with names which in turn contain other proper names, so both the descriptive theory of meaning and the descriptive theory of reference fixing lead to a vicious circle (cf. Devitt, Sterelny, 1999, p. 55). In order to avoid this circularity we need to be sure that the definite descriptions connected with a proper name contain no proper names. That is why we consider two languages, $\mathcal{L}$ and $\mathcal{L}^+$ ($\mathcal{L} \subset \mathcal{L}^+$). Let me start with language $\mathcal{L}$. All the terms it contains are variables and definite descriptions (we will use definite descriptions from $\mathcal{L}$ to give interpretation of name–terms from $\mathcal{L}^+$). The idea behind the formal representation of proper names in accordance with descriptive theory of reference is simple: we let name–terms (formally representing names) designate through equivalence classes of descriptions which designate one and the same object and contain one and the same distinguished predicate (relation $\mathbb{R}$, see section 4, *Def.*VI.$S(c)$). We need this last requirement in order to be able to distinguish two co–referring but distinct proper names formally (to represent them as two different name–terms). Descriptions designate different objects with respect to different worlds, so we need to define an equivalence relation not on a set of descriptions $\Gamma_{\mathcal{L}}$ but on a set of pairs containing a description and a world in which this description designates (set Δ, see section 4, *Def.*VI.$S(b)$). For example, take two descriptions, 'the planet called [fɒs fər əs]' and 'the planet called [hɛs pər əs]' (I will use the symbol 'γ_1' for the former and the symbol 'γ_2' for the latter description). Both descriptions, γ_1 and γ_2, designate in our world w, but pairs $\langle \gamma_1, w \rangle$, $\langle \gamma_2, w \rangle$ will belong to different equivalence classes because γ_1 contains the predicate 'called [fɒs fər əs],' while γ_2 contains a different predicate, i.e., 'called [hɛs pər əs]'. This idea is represented schematically in *Graph 1* below:

Graph 1



In order to define an interpretation of a name–term n_i I need two functions – one which connects n_i with an equivalence class (function $\mathbb{Q}^{\leq}$, *Def.*VI.$S(e)$), and the other which takes an equivalence class and gives the object designated by every description in the class (function $\mathbb{F}$, *Def.*VI.$S(d)$). I presented this idea schematically in *Graph 2* below:

Graph 2



In effect, name–terms designate rigidly (I will return to this question in section 5) and are not synonymous with descriptions (this is exactly what a descriptive theory of reference postulates). The interpretation of a name–term and the notion of satisfaction of a formula with a name–term are defined in the following way (*Def.*VI. $S15$, *Def.*VI. $S16$):

Interpretation of a name–term

if n_i is a name–term and $\Gamma_{\mathcal{L}} \neq \emptyset$, then $I^{\leq}_{\langle w,t \rangle}(n_i) = \mathbb{F}(\mathbb{Q}^{\leq}(n_i))$; if $\Gamma_{\mathcal{L}} = \emptyset$, then n_i fails to designate in $\mathfrak{M}^{\leq}$ (at any $\langle w', t' \rangle$);

Satisfaction of a formula with a name–term

if a term s designates at $\langle w, t\rangle$ in $\mathfrak{M}^{\leq}$ with respect to g, then $\mathfrak{M}^{\leq g\ w\ t} \vDash (\lambda x.\varphi)(s)$ iff $\mathfrak{M}^{\leq g\ \binom{d}{x} w\ t} \vDash \varphi$, where $d = I^{\leq g}_{\langle w,t\rangle}(s)$; if a term s fails to designate at $\langle w, t\rangle$ in $\mathfrak{M}^{\leq}$ with respect to g, then $\mathfrak{M}^{\leq g\ w\ t} \nvDash (\lambda x.\varphi)(s)$.

A name–term–if it designates–designates rigidly (see section 5 for a proof), and in the case it does designate, the definition of satisfaction of a formula containing this name–term is substantively simplified:

$\mathfrak{M}^{\leq g\ w\ t} \vDash (\lambda x.\varphi)(n_i)$ iff $\mathfrak{M}^{\leq g\ \binom{d}{x} w\ t} \vDash \varphi$, where $d = I^{\leq}_{\langle w,t\rangle}(n_i)$.

It follows from this definition that only the referent is relevant for sentence truth–conditions (irrespectively of the descriptions identifying it). That is why a formula with a name–term could be satisfied even in cases where the referent of the term belongs to the extension of a predicate, but does not satisfy any descriptions in the time and the world of evaluation. Note that the interpretation of a name–term is 'calculated' and there are such models in which name–terms do not designate. In such cases objects from the model domain cannot be distinguished with respect to their properties (this means that definite descriptions do not designate in these models, so $\Gamma_{\mathcal{L}} = \emptyset$). Now let me give definitions of languages $\mathcal{L}$ and $\mathcal{L}^{+}$.

4. The Formal Representation: Definitions

The languages $\mathcal{L}$ and $\mathcal{L}^{+}$ are based on a first–order predicate logic with identity and descriptions (I followed Fitting, Mendelsohn 1998). I will skip all the standard definitions and present the definitions that are specific for a formal representation of names.

Let me start with language $\mathcal{L}$, which contains only two sorts of terms, variables and iota–terms.

Definition I*: Alphabet of* $\mathcal{L}$

A first–order language $\mathcal{L}$ contains the following symbols: sentential connectives $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\sim$; quantifiers $\exists$, $\forall$; an infinite set of individual variables x_1, x_2, x_3, ...; an infinite set of predicate constants P_1, P_2, P_3, ..., with a positive integer (an arity), assigned to each of them; the identity sign $=$; the definite descriptions operator ι; the abstraction operator λ; temporal operators of past **P** and future **F**; an infinite set of temporal operators $[_i]$ ('true at t_i'), where $i \in \mathbb{N}$; modal operators $\Box$, $\Diamond$; an infinite set of distinguished predicate constants N_1, N_2, N_3, ...; a set of numerical symbols for natural numbers; the left parenthesis (, the right parenthesis).

Definition II*: Syntax of* $\mathcal{L}$

Predicate constants and the predicate abstracts defined below are predicates of $\mathcal{L}$. The notions of a variable ($R1$), a predicate constant ($R2$), an atomic formula ($R3$), $\sim\varphi$ ($R4$), $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$, $(\varphi \leftrightarrow \psi)$ ($R5$), $\mathbf{P}\varphi$, $\mathbf{F}\varphi$, $[_i]\varphi$ ($R6$), $\Box\varphi$, $\Diamond\varphi$ ($R7$), $\forall_x\varphi$, $\exists_x\varphi$ ($R8$), $\iota x.\varphi$ ($R9$), $(\lambda x.\varphi)$ ($R10$), $(\lambda x.\varphi)(s)$ ($R11$) are defined in a standard way (compare (Fitting, Mendelsohn, 1998): $R1$–definition 12.1.1 point 1 (p. 248), $R3-R5$ and $R7$, $R8$–definitions 4.1.1, 4.1.2 points 1–5 (pp. 81–2), $R9$–definition 12.1.1 point 4 (p. 249), $R10$–definition 9.4.2 point 6A (p. 196), $R11$–definition 9.4.2 point 6B (p. 197) and definition 12.1.1 point 10 (p. 249), $R6$–definition (27) from (Rini, Cresswell, 2012, p. 90)).

Definition III*: Semantics of* $\mathcal{L}$

A varying domain first–order model $\mathfrak{M}$ for $\mathcal{L}$ is a structure $\mathfrak{M} = \langle \mathcal{D}, T, <, W, I \rangle$, such that:

– $\mathcal{D}$ is a domain function mapping pairs of possible worlds and times $\langle w, t \rangle$ to non–empty sets. The domain of the model is the set $\cup \{\mathcal{D}_{\langle w,t \rangle} : w \in W, t \in T\}$. We write $\mathcal{D}_{\mathfrak{M}}$ for the domain of the model $\mathfrak{M}$ and $\mathcal{D}_{\langle w,t \rangle}$ for a value of the function $\mathcal{D}$ for an argument $\langle w, t \rangle$;

– T is a set of natural numbers and $<$ ('earlier then') is a linear order defined on elements of T (a set $(T, <)$ is thought as the flow of time);
– W is a non–empty set of possible worlds;

– I is a function which assigns an extension to each pair of an atomic predicate of $\mathcal{L}$ and a pair $\langle w, t \rangle$, where $w \in W, t \in T$, in the following way:

– if Q is a *n*–place predicate constant, then $I_{\langle w,t \rangle}(P) \subseteq \mathcal{D}^n_{\mathfrak{M}}$;

– $I_{\langle w,t \rangle}(=) = \{\langle d, d \rangle \in \mathcal{D}_{\mathfrak{M}}\}$;

let g be a variable assignment (a mapping that assigns to each free variable x some member $g(x)$ of the model domain $\mathcal{D}_{\mathfrak{M}}$) and let $I^g_{\langle w,t \rangle}$ be a function which assigns an extension to each pair of an atomic predicate or a term of $\mathcal{L}$ and a pair $\langle w, t \rangle$, where $w \in W, t \in T$:

– if x is a variable, then $I^g_{\langle w,t \rangle}(x) = g(x)$ for any $\langle w, t \rangle$;

– $I \subseteq I^g$ for any g;

the notion of interpretation of terms other than variables, the interpretation of predicates, and the satisfaction of formulas in $\mathfrak{M}$ are defined as follows:

$S1$. if Q is a *n*–place predicate constant and $y_1, \ldots, y_n$ are variables, then $\mathfrak{M}^{g\,w\,t} \models Q(y_1, \ldots, y_n)$ iff $\langle g(y_1), \ldots, g(y_n) \rangle \in I_{\langle w,t \rangle}(Q)$;

the notions of satisfaction of $\sim\varphi$ $(S2)$, $(\varphi \wedge \psi)$ $(S3)$, $(\varphi \vee \psi)$ $(S4)$, $(\varphi \rightarrow \psi)$ $(S5)$, $(\varphi \leftrightarrow \psi)$ $(S6)$, $\mathbf{P}\varphi$ $(S7)$, $\mathbf{F}\varphi$ $(S8)$ are defined in a standard way;

$S9$. if φ is a formula, then $\mathfrak{M}^{g\ w\ t_j} \vDash [_i]\varphi$ iff $\mathfrak{M}^{g\ w\ t_i} \vDash \varphi$;
the notions of satisfaction $\Box\varphi$ $(S10)$, $\Diamond\varphi$ $(S11)$, $\forall_x\varphi$ $(S12)$, $\exists_x\varphi$ $(S13)$ are defined in a standard way;

$S14$. if $\mathfrak{M}^{g\binom{d}{x}w\ t} \vDash \varphi$ for exactly one $d \in \mathcal{D}_{\mathfrak{M}}$, then $I^{g}_{\langle w,t\rangle}(\iota x.\varphi) = d$; if it is not the case that $\mathfrak{M}^{g\binom{d}{x}w\ t} \vDash \varphi$ for exactly one $d \in \mathcal{D}_{\mathfrak{M}}$, then $\iota x.\varphi$ fails to designate at $\langle w,t\rangle$ in $\mathfrak{M}$ with respect to g;

$S15$. if a term s designates at $\langle w,t\rangle$ in $\mathfrak{M}$ with respect to g, then $\mathfrak{M}^{g\ w\ t} \vDash (\lambda x.\varphi)(s)$ iff $\mathfrak{M}^{\leq g\binom{d}{x}w\ t} \vDash \varphi$, where $d = I^{\leq g}_{\langle w,t\rangle}(s)$; if a term s fails to designate at $\langle w,t\rangle$ in $\mathfrak{M}$ with respect to g, then $\mathfrak{M}^{g\ w\ t} \nvDash (\lambda x.\varphi)(s)$.

I will use the symbol '$!x.\varphi$' for the special case of $\iota x.\varphi$ terms with only one variable x which occurs free in φ. There are no free variable occurrences in $!x.\varphi$ and due to this if $I^{g}_{\langle w,t\rangle}(!x.\varphi)$ is defined, then $I^{g}_{\langle w,t\rangle}(!x.\varphi) = I^{g'}_{\langle w,t\rangle}(!x.\varphi)$ for any assignments g and g'. That is why instead of '$I^{g}_{\langle w,t\rangle}(!x.\varphi)$' we will write '$I_{\langle w,t\rangle}(!x.\varphi)$', which should be understood as '$I^{g}_{\langle w,t\rangle}(!x.\varphi)$' where g is any assignment.

Now I will expand language $\mathcal{L}$ to $\mathcal{L}^+$ by adding name–terms. I will skip all syntactical and semantic definitions of $\mathcal{L}^+$ duplicating the definitions of $\mathcal{L}$, and will write below only the new ones.

Definition IV*: Alphabet of* $\mathcal{L}^+$

A first–order language $\mathcal{L}^+$ contains all symbols of $\mathcal{L}$ with the addition of an infinite set of name–terms $\mathcal{N} = \{n_1, n_2, n_3, \ldots\}$.

Definition V*: Syntax of* $\mathcal{L}^+$

The syntax of $\mathcal{L}^+$ is the same as the syntax of $\mathcal{L}$ with the addition of the following clause: name–term n_i is a term with no free variable occurrences.

Definition VI*: Semantics of* $\mathcal{L}^+$

Let $\mathfrak{M} = \langle \mathcal{D}, T, <, W, I\rangle$ be a model of $\mathcal{L}$. A varying domain first–order model $\mathfrak{M}^{\leq}$ for $\mathcal{L}^+$ is a structure $\mathfrak{M}^{\leq} = \langle \mathcal{D}, T, <, W, I^{\leq}\rangle$, where $I^{\leq} \upharpoonright \mathcal{L} = I$.

Using the already defined properties of $\mathfrak{M}$ (*Definition* III), we define the following sets, relations, and functions.

$S(a)$: *set* $\Gamma_{\mathcal{L}}$

Set $\Gamma_{\mathcal{L}}$ is a set of iota–terms $!x.[_i]\varphi$ of $\mathcal{L}$. $\iota x.[_i]\varphi \in \Gamma_{\mathcal{L}}$ iff 1) there is a world $w \in W$ such that for every time $t \in T$ $!x.[_i]\varphi$ designates at $\langle w,t\rangle$ in

$\mathfrak{M}$; 2) $\varphi = \big(N_i(x) \wedge Q(x)\big)$, where N_i is a distinguished predicate and Q is an undistinguished predicate. (I will use symbols 'γ_i', 'γ_j' for members of $\Gamma_{\mathcal{L}}$)

$S(b)$: *set* Δ

$\Delta \subseteq \Gamma_{\mathcal{L}} \times W$. $\langle \gamma_i, w \rangle \in \Delta$ iff for any time $t \in T$ $I_{\langle w,t \rangle}(\gamma_i)$ is defined.

$S(c)$: *relation* $\mathbb{R}$

$\mathbb{R} \subseteq \Delta^2$. $\langle \gamma_i, w \rangle \mathbb{R} \langle \gamma_j, w' \rangle$ iff for any time $t \in T$ $I_{\langle w,t \rangle}(\gamma_i) = I_{\langle w',t \rangle}\big(\gamma_j\big)$ and there is the same predicate N_k in γ_i, γ_j.

Let Δ / $\mathbb{R}$ be a partition of set Δ by equivalence relation $\mathbb{R}$ and $[\langle \gamma_i, w \rangle]_{\mathbb{R}}$ be an equivalence class from Δ / $\mathbb{R}$.

$S(d)$: *function* $\mathbb{F}$
$\mathbb{F}$: Δ / $\mathbb{R} \rightarrow \mathcal{D}_{\mathfrak{M}}$. For any $[\langle \gamma_i, w \rangle]_{\mathbb{R}} \in \Delta$ / $\mathbb{R}$, $\mathbb{F}([\langle \gamma_i, w \rangle]_{\mathbb{R}}) = d$, where for any time $t \in T$ $d = I_{\langle w,t \rangle}\big(\gamma_j\big)$ for any $\langle \gamma_j, w \rangle \in [\langle \gamma_i, w \rangle]_{\mathbb{R}}$.

Let $\leq$ be any well–ordered relation on a set Δ / $\mathbb{R}$ and let $\langle \Delta/\mathbb{R}, \leq \rangle$ be a well–ordered set.

$S(e)$: *function* $\mathbb{Q}^{\leq}$

$\mathbb{Q}^{\leq}$: $\mathcal{N} \rightarrow \Delta$ / $\mathbb{R}$. Function $\mathbb{Q}^{\leq}$ for an argument n_i gives an equivalence class $[\langle \gamma_i, w \rangle]_{\mathbb{R}}$ in the following way:

– for n_1 $\mathbb{Q}^{\leq}$ gives the least element of $\langle \Delta/\mathbb{R}, \leq \rangle$;

– for every next element of $\mathcal{N}$ (with respect to an index), $\mathbb{Q}^{\leq}$ gives the next element of $\langle \Delta/\mathbb{R}, \leq \rangle$;

– in the case there are no next elements in $\langle \Delta/\mathbb{R}, \leq \rangle$, then for a next element of $\mathcal{N}$, $\mathbb{Q}^{\leq}$ gives the least element of $\langle \Delta/\mathbb{R}, \leq \rangle$.

Semantic rules $S1.- S14.$ of language $\mathcal{L}^+$ are the same as rules $S1.- S14.$ of language $\mathcal{L}$ (except for talking about $I^{\leq}$ instead of I);

$S15.$ if n_i is a name–term and $\Gamma_{\mathcal{L}} \neq \emptyset$, then $I^{\leq}_{\langle w,t \rangle}(n_i) = \mathbb{F}\big(\mathbb{Q}^{\leq}(n_i)\big)$; if $\Gamma_{\mathcal{L}} = \emptyset$, then n_i fails to designate in $\mathfrak{M}^{\leq}$ (at any $\langle w', t' \rangle$);

$S16.$ if a term s designates at $\langle w, t \rangle$ in $\mathfrak{M}^{\leq}$ with respect to g, then $\mathfrak{M}^{\leq g\, w\, t} \vDash (\lambda x. \varphi)(s)$ iff $\mathfrak{M}^{\leq g \binom{d}{x} w\, t} \vDash \varphi$, where $d = I^{\leq g}_{\langle w,t \rangle}(s)$; if a term s fails to designate at $\langle w, t \rangle$ in $\mathfrak{M}^{\leq}$ with respect to g, then $\mathfrak{M}^{\leq g\, w\, t} \nvDash (\lambda x. \varphi)(s)$.

5. Answering the Modal Argument

We noted in section 1 that, according to a descriptive theory of reference, a proper name is a rigid designator (see (1R)). Let me present a proof that name–terms of $\mathcal{L}^+$ are obstinately rigid (they designate the same object with respect to all pairs world–time, irrespectively of its existence in possible worlds and times (cf. Kripke, 1980, pp. 48–9, 78; Salmon, 2005, p. 34)). For names being rigid designators means that any two co–designative name–terms should be necessarily co–designative (Kripke, 1980, pp. 3, 4, 109, 143) and interchangeable in modal contexts (1980, p. 21 footnote 21). Besides these two properties of rigid designators, formulas with a name–term and a modal operator in different scopes should have equivalent truth–conditions (1980, p. 12 footnote 15). After presenting a proof that name–terms are rigid, I will prove that they have all these properties of rigid designators.

Let me start with the proof showing that name–terms are obstinately rigid. I noted in section 4 that $\mathcal{L}^+$ is based on a first–order predicate logic with identity and descriptions taken after (Fitting, Mendelsohn, 1998). Fitting and Mendelsohn (1998, pp. 211, 213, 217) defined a term's rigidity as the equivalence of truth–conditions of formulas with the term and □–operator in different scopes, $\mathfrak{M}^{\leq g\ w\ t} \vDash (\lambda x.\Box\varphi)(s) \leftrightarrow \Box(\lambda x.\varphi)(s)$. This formula is valid iff s is a obstinately rigid designator for which scope does not matter. Let me prove that this formula holds for a name–term.

Theorem I

For any name–term n*, a possible world* w*, a time* t*, an assignment* g*, a formula* φ*, and a model* $\mathfrak{M}^{\leq}$*, the following formula is valid:* $\mathfrak{M}^{\leq g\ w\ t} \vDash (\lambda x.\Box\varphi)(n) \leftrightarrow \Box(\lambda x.\varphi)(n)$.

Proof

I. Let us assume that n designates in $\mathfrak{M}^{\leq}$ with respect to $\langle w, t\rangle$ (which means that $\Gamma_{\mathcal{L}} \neq \emptyset$).

$\mathfrak{M}^{\leq g\ w\ t} \vDash (\lambda x.\Box\varphi)(n) \leftrightarrow \Box(\lambda x.\varphi)(n)$ **iff** (*Def.*VI. $S6$)

$\mathfrak{M}^{\leq g\ w\ t} \vDash (\lambda x.\Box\varphi)(n)$ and $\mathfrak{M}^{\leq g\ w\ t} \vDash \Box(\lambda x.\varphi)(n)$ or

$\mathfrak{M}^{\leq g\ w\ t} \nvDash (\lambda x.\Box\varphi)(n)$ and $\mathfrak{M}^{\leq g\ w\ t} \nvDash \Box(\lambda x.\varphi)(n)$ **iff** (*Def.*VI. $S16$, $S10$)

$\mathfrak{M}^{\leq g\binom{d}{x} w\ t} \vDash \Box\varphi$, where $d = I^{\leq}_{\langle w,t\rangle}(n)$ and for every world $w' \in W$ $\mathfrak{M}^{\leq g\ w' t} \vDash (\lambda x.\varphi)(n)$ or

$\mathfrak{M}^{\leq g\binom{e}{x} w\ t} \nvDash \Box\varphi$, where $e = I^{\leq}_{\langle w,t\rangle}(n)$ and there is a world $w'' \in W$ such that $\mathfrak{M}^{\leq g\ w'' t} \vDash (\lambda x.\varphi)(n)$ **iff** (*Def.*VI. $S16$, $S10$)

for every world $w''' \in W$ $\mathfrak{M}^{\leq g \binom{d}{x} w''' t} \vDash \varphi$, where $d = I^{\leq}_{\langle w,t\rangle}(n)$ and

for every world $w' \in W$ $\mathfrak{M}^{\leq g \binom{d'}{x} w' t} \vDash \varphi$, where $d' = I^{\leq}_{\langle w',t\rangle}(n)$ or

there is a world $w'''' \in W$ such that $\mathfrak{M}^{\leq g \binom{e}{x} w'''' t} \nvDash \varphi$, where $e = I^{\leq}_{\langle w,t\rangle}(n)$ and

there is a world $w'' \in W$ such that $\mathfrak{M}^{\leq g \binom{e'}{x} w'' t} \nvDash \varphi$, where $e' = I^{\leq}_{\langle w'',t\rangle}(n)$.

Due to $I^{\leq}_{\langle w,t\rangle}(n) = I^{\leq}_{\langle w',t\rangle}(n)$ for any w and w', $d = e = d' = e'$. Let us assume that the first part of the disjunction is false. Let us show that the second part of the disjunction is true under this assumption. $d = d'$, which means that the conjunction in the first part of the disjunction is equal to the requirement for formula φ to be satisfied with respect to any $\langle w, t\rangle$. The falsity of this requirement means that there is a world w' such that φ is not satisfied with respect to $\langle w', t\rangle$. $e = e'$, which means that the conjunction in the second part of the disjunction is equal to the requirement such that there is a world w'' such that formula φ is not satisfied with respect to $\langle w'', t\rangle$. We assumed that the first part of the disjunction is false and it follows from this assumption that the second part of the disjunction is true. If we assume that the second part of the disjunction is false we get the same result.
II. Let us assume that n does not designate in $\mathfrak{M}^{\leq}$ with respect to $\langle w, t\rangle$ (which means that $\Gamma_{\mathcal{L}} = \emptyset$).

$\mathfrak{M}^{\leq g\, w\, t} \vDash (\lambda x. \Box\varphi)(n) \leftrightarrow \Box(\lambda x. \varphi)(n)$ **iff** (*Def.*VI. $S6$)

$\mathfrak{M}^{\leq g\, w\, t} \vDash (\lambda x. \Box\varphi)(n)$ and $\mathfrak{M}^{\leq g\, w\, t} \vDash \Box(\lambda x. \varphi)(n)$ or

$\mathfrak{M}^{\leq g\, w\, t} \nvDash (\lambda x. \Box\varphi)(n)$ and $\mathfrak{M}^{\leq g\, w\, t} \nvDash \Box(\lambda x. \varphi)(n)$.

n does not designate in $\mathfrak{M}^{\leq}$ with respect to $\langle w, t\rangle$, so $\mathfrak{M}^{\leq g\, w\, t} \nvDash (\lambda x. \Box\varphi)(n)$ (*Def.*VI. $S16$). This means that the first part of the disjunction is false. In order to check if the initial formula is satisfied we need to check if the second part of the disjunction is satisfied or not. The first part of the conjunction in it is true, so we need to check if also the second part of the conjunction is true.

$\mathfrak{M}^{\leq g\, w\, t} \nvDash \Box(\lambda x. \varphi)(n)$ **iff** (*Def.*VI. $S10$) there is a world $w' \in W$ such that $\mathfrak{M}^{\leq g\, w'\, t} \nvDash (\lambda x. \varphi)(n)$.

n does not designate in $\mathfrak{M}^{\leq}$ with respect to any $\langle w'', t\rangle$, so n does not designate in $\mathfrak{M}^{\leq}$ with respect to $\langle w', t\rangle$. This means (*Def.*VI. $S16$) that $\mathfrak{M}^{\leq g\, w'\, t} \nvDash (\lambda x. \varphi)(n)$, so the initial formula is satisfied. ■

Let n_i, n_j stand for any name–terms. Assuming that they co–designate,

$I^{\leq}_{\langle w,t\rangle}(n_i) = I^{\leq}_{\langle w,t\rangle}(n_j)$, it follows from *Theorem* I and *Def.*VI. $S16$, $S10$ that the following formulas (a)– (d) have equal truth–conditions:

(a) $\quad \Box(\lambda x.\lambda y.(x = y)(n_i))(n_j)$

(b) $\quad \lambda x.(\Box\lambda y.(x = y)(n_i))(n_j)$

(c) $\quad \lambda y.\left(\Box\lambda x.(x = y)(n_j)\right)(n_i)$

(d) $\quad \lambda x.(\lambda y.\Box(x = y)(n_i))(n_j)$

Equality of the truth–conditions formally represents the claim about the interchangeability of co–designative names in modal contexts and the insensitivity of modal operators to scopes. We get a similar result if we change the modal operator in formulas (a)– (d) to temporal operators of past **P** and future **F** (*Theorem* I and *Def.*VI. $S16$, $S7$, $S8$).

6. Concluding Remarks

I presented a way of formally representing proper names in accordance with a description theory of reference fixing. Such a representation makes it possible to keep the thesis about the rigidity of proper names and to answer objections against the (1R) thesis (the circularity objections). It can be used either by classical descriptivists or those of non–classical descriptivists who hold a version of the (1R) thesis. The main advantage of this representation is that it connects different descriptions with co–designative but different proper names, and this feature could be used in solving puzzles with proper names (see Poller *in preparation a and b*).

Acknowledgements

I am especially indebted to Katarzyna Kijania–Placek who provided advice and encouragement at every step of the preparation of this paper. I owe many thanks to Professor Adam Olszewski for his remarks about the formal representation presented here. I have also benefited greatly from many discussions with members of the Department of Epistemology, Institute of Philosophy, Jagiellonian University and especially with Professor Tomasz Placek, Leszek Wroński, Jacek Wawer, Juliusz Doboszewski and Maja (Sherlock) Białek. A version of this paper was presented at two meetings of the Department of Epistemology IF UJ (in 2011 and 2012) and I should like to thank the audience and especially Professor Jan Woleński for valuable comments on these occasions.

This project was supported by Ministry of Science and Higher Education of Poland, DEC–2012/05/N/HS1/01429.

References

Bach, K. (2002). Giorgione was So–called Because of His Name. *Philosophical Perspectives 16*, 73–102.

Frege, G. (1914/1980). VIII/12 Frege to Jourdain undated. In McGuiness, B. (ed.) *Philosophical and Mathematical Correspondence* (pp. 78–80). Oxford: Basil Blackwell.

Frege, G. (1918/1984). Logical Investigations. I: Thoughts. In McGuiness, B. (ed.) *Collected Papers on Mathematics, Logic, and Philosophy* (pp. 351–372). Oxford: Basil Blackwell.

Chalmers, D. (2006). "The Foundations of Two–Dimensional Semantics." In García–Carpintero M., & Maciá J. (eds.) *Two–Dimensional Semantics* (pp. 55–140). Oxford: Oxford University Press.

Devitt, M., Sterelny, K. (1999). *Language and Reality. An Introduction to the Philosophy of Language*. Oxford: Wiley Blackwell.

Fitting, M., Mendelsohn, R. L. (1998). *First–Order Modal Logic.* Dordrecht: Kluwer Academic Publishers.

Geurts, B. (1997). Good News about the Description Theory of Names. *Journal of Semantics 14*, 319–348.

Glüer, K. & Pagin, P. (2006). Proper Names and Relational Modality. *Linguistics and Philosophy 29(5)*, 507–535.

Jackson, F. (2010). *Language*, *Names and Information.* Singapore: Wiley Blackwell.

Justice, J. (2001). On Sense and Reflexivity. *The Journal of Philosophy 98(7)*, 351–364.

Katz, J. J. (1994). Names Without Bearers. *The Philosophical Review 103(1)*, 1–39.

Kripke, S. (1980). *Naming and Necessity.* Cambridge, Massachusetts: Harvard University Press.

Kroon, F. W. (1987). Causal Descriptivism. *Australasian Journal of Philosophy 65(1)*, 1–17.

Matushansky, O. (2008). On the Linguistic Complexity of Proper Names. *Linguistics and Philosophy 31(5)*, 573–627.

Mill, J. S. (1889/2011). *A System of Logic*. South Australia: eBooks@Adelaide, University of Adelaide.

Poller, O. (*in preparation a*). *Descriptive Content of Names as Predicate Modifiers.*

Poller, O. (*in preparation b*). *All the Superhero's Names.*

Poller, O. (*in preparation c*). *Obrona Deskrypcyjnej Teorii Odniesienia Nazw Własnych*. [Defence of a Desriptive Theory of Reference of Proper Names] In Polish

Recanati, F. (1993). *Direct Reference. From Language to Thought.* Oxford: Basil Blackwell, 1994.

Rini, A. A., Cresswell, M. J. (2012). *The World–Time Parallel: Tense and Modality in Logic and Metaphysics.* New York: Cambridge University Press.

Soames, S. (2005). *Reference and Description. The Case Against Two–Dimensionalism.* Princeton, Oxford: Princeton University Press.