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RESEARCH OF DEFORMATION OF MULTILAYERED PLATES ON UNDEFORMABLE FOUNDATION BY UNFLEXURAL SPECIFIED MODEL

O.G. Gurtovy*, S.O. Tynchuk

**National University of Water Management and Natural Resources Used, Rivne, Ukraine*
E-mail: O.G.Gurtovy@usuwm.rv.ua

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Abstract. *Stress-strain state (SSS) of multilayered plates on undeformable foundation is investigated. The settlement circuit of transverse loaded plate is formed by symmetrical attaching of a plate concerning a surface of contact to the foundation. The plate of the double thickness becomes bilateral symmetrically loaded concerning its median surface. It allows to model only unflexural deformation that reduces amount of unknown and the general order of differentiation of resolving system of the equations. The developed refined continual model takes into account deformations of transverse shear and transverse compression in high iterative approximation. Rigid contact between the foundation and a plate, and also shear without friction on a surface of contact of a plate with the foundation is considered. Calculations confirm efficiency of such approach, allowing to receive decisions which is qualitative and quantitatively close to three-dimensional solutions.*

1 INTRODUCTION

The multilayered locally loaded coverings in designs can be considered as thick plates on rigid linings. Receiving of exact three-dimensional solutions is possible only for the limited circle of problems at certain kinds of boundary conditions. Therefore use for receiving of solutions of the specified two-dimensional models that allows to expand a class of considered problems is actual. In works [1, 2] the continual specified model of the SSS of a plate which takes into account deformations of transverse shear and transverse compression in high degrees of iterative approximation is offered. The comparison of solutions of test problems of flexural and unflexural SSS of bilateral symmetric and antisymmetric loaded layered plates with the three-dimensional solutions received by a technique [3] is carry out in the same place. High accuracy of model in a wide range of geometrical and physical parameters of a plate is established. Further for the decision of the research problem of the SSS of a layered plate on undeformable foundation unflexural a component of the SSS is used.

2 THE MODEL OF PLATE ON UNDEFORMABLE FOUNDATION

In linear-elastic setting is constructed the refined model of the rectangular, transverse loaded, multilayered thick plate. The plate leans on not deformable basis. Layers of a plate are isotropic and transversely isotropic with any but constant thickness. Interlaminar contact is rigid.

Instead of a real design of a plate (fig. 1a) is offered to consider the settlement circuit of a plate which is twice thicker and which is formed by symmetric attaching of a plate concerning a surface of contact to the basis (fig. 1b). The plate becomes bilateral symmetrically loaded concerning its middle surface. Such circuit models shear without friction on a surface of contact of a plate with the basis. Rigid contact of a plate with the basis is modeled by introduction additional thin, but actually undeformable layer (fig. 1c). Axial modules of elasticity and shear module of this layer on a few orders exceed corresponding characteristics of a material of layers of a plate that provides its big relative rigidity.

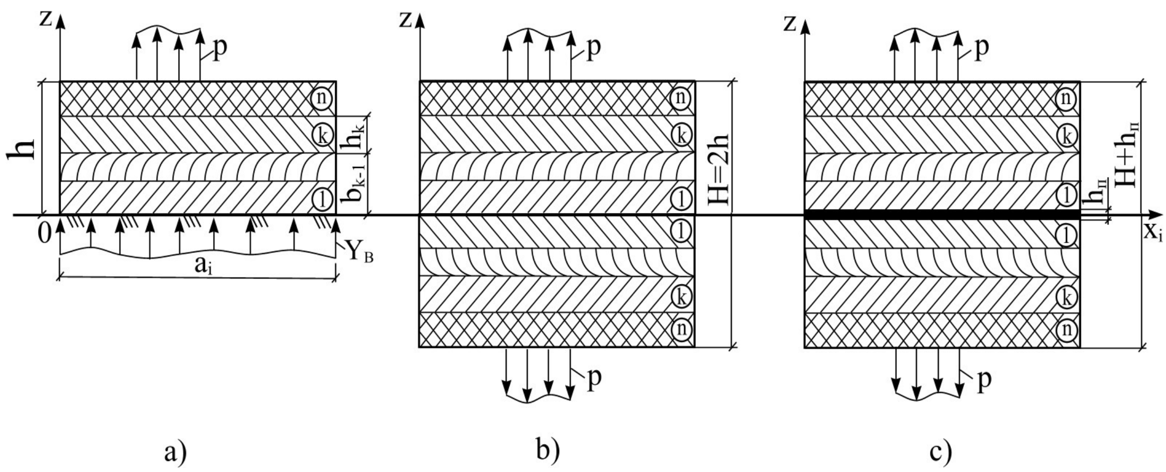


Fig. 1. Settlement schemas of a plate.

Stress-strain state (SSS) of the given plate consists with flexural and unflexural SSS [1]. For "attached" plate it is enough to consider only unflexural SSS. In this case, for example, approximation of distribution of stresses and deformations of transverse shear across the thickness of a plate it is enough to apply a number of sedate polynoms of transverse coordinate to only with unpaired degrees. By consideration flexural SSS it would be polynoms of a pair degree. Therefore the amount of unknown functions in a problem decreases a little.

3 THE MATHEMATICAL MODEL OF UNFLEXURAL SSS OF PLATE

Let us present the components of a vector of normal $u_3^{(k)}$ and tangential displacements as a sums of hypothetically well-founded (known) functions $\psi_{3t}^{(k)}$, $\psi_{ir}^{(k)}$ of transverse coordinate z and sought-for functions γ_t β_{ir}, v_i a coordinate surface $x_1 0 x_2$ (fig. 1), conterminous with a surface of contact of a plate and the foundation:

$$u_3^{(k)} = \psi_{3t,3}^{(k)}(z)\gamma_t + \psi_{33,3}^{(k)}(z)p\,; \qquad t=\overline{1,\,2}\,;\ \ i=\overline{1,\,2}\,;$$

$$u_i^{(k)} = v_i - \psi_{3t}^{(k)}(z)\gamma_{t,i} - \psi_{33}^{(k)}(z)p_{,i} - \psi_{ir}^{(k)}(z)\beta_{ir}\,; \qquad r=\overline{1,\,4}\,, \qquad (1)$$

where functions γ_t model influence transverse compression in the second iterative approximation, and β_{ir} are influence of transverse shear in the fourth approximation p is function of given load. Such degree of iterative approximation is proved in [1]. Hereinafter the superscript in brackets indicates the number of a layer, and subscript indicates directions of a coordinate axis. Differentiation on x_α are designated by the subscript after the comma.

Boundary conditions on a surface of contact of a plate with the foundation $z=0$ for all components of a vector of displacement u_α and for stresses ($\alpha,\beta=\overline{1,\,3}$) in the settlement circuit on fig. 1b by which sliding without friction and without avulsion is modeled has the form:

$$u_3^{(1)}=0\,; \qquad u_i^{(1)}\neq 0\,; \qquad \sigma_{33}^{(1)}=Y_B\,; \qquad \sigma_{i3}^{(1)}=0\,;\ \ i=\overline{1,\,2}\,, \qquad (2)$$

and are satisfied automatically with hypotheses (1). Here Y_B is reaction of repulse of the foundation on fig. 1a.

In case of rigid contact of a plate to the foundation (fig. 1c) on a surface $z=0$, and also on $z=\pm 0{,}5h_0$ - surfaces of the entered rigid layer will be approximately being satisfied the following boundary conditions:

$$u_3^{(1)}=0\,; \qquad u_i^{(1)}=v_i=0\,; \qquad \sigma_{33}^{(1)}=Y_B\,; \qquad \sigma_{i3}^{(1)}\neq 0\,. \qquad (3)$$

Using the Lagrange variational principle, Hooke's law and the Cauchy relations by a technique [2], is received the system of the resolving differential equations concerning the sought-for functions of displacements which have the form:

$$L_i^{(c)}(u_i) + L_{ir}^{(c)}(\beta_{ir}) + L_{3t}^{(c)}(\gamma_t) + L_{33}^{(c)}(p) + p\cdot(\psi_{33,3}^{(n)} - \psi_{33,3}^{(0)}) = 0\,; \qquad c=\overline{1,\,7}\,, \qquad (4)$$

where $L_{\alpha s}^{(c)}$ are differential operators of second and fourth orders on two variables with integrated characteristics of rigidity of a plate; t are amount of unknown functions of transverse compression χ_{3t}; $2\cdot r$ are amount of functions of transverse shear β_{1r}, β_{2r}; functions u_i are tangential displacement of coordinated surface x_i; c are the general number of unknown ($c=2+t+2\cdot r$). The general order of differentiation of system (4) does not depend on the numbers of layers. However, this order is high enough.

The developed specified model takes into account deformations of transverse shear and transverse compression in high iterative approximation. It allows receiving solutions which is qualitatively and quantitatively close to three-dimensional, for example, under boundary conditions such as Navier-type.

4 THE ANALYSIS OF ACCURACY OF MODEL

Calculation results of SSS homogeneous plate with $a/h=3$ at a cylindrical bend under action loads $p = p_0 \sin\frac{\pi x_1}{a}$ (fig. 2) is shown accuracy of model. Relative transverse rigidity is equal $E_3/E_1=100$, $G_{i3}/G_{12}=100$; Poisson's coefficients are $\nu_{12}=0{,}3$, $\nu_{i3}=0{,}001$.

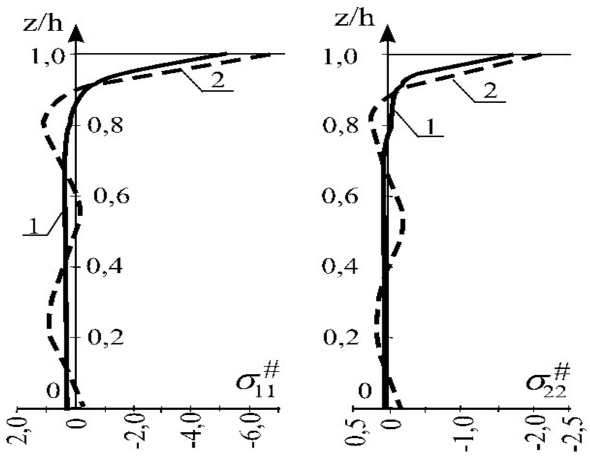


Fig. 2. The diagram of stresses $\sigma_{ii}^{\#} = 10\sigma_{ii}^{\max} / p_0$ along a plate thickness in its center.
1 - according to the exact three-dimensional solution; 2- the proposed model with $t=2$, $r=3$.

The SSS is localized in upper area of a plate, quickly fading along thickness. In the area of attenuation of SSS, approached solution is slightly deviates in relation to the exact decision received by us by a technique [2]. Similar character of SSS is received and in isotropic plate.

The account of deformations of transverse compression in model is obligatory. Otherwise we shall receive the trivial - zero solution. Function of repulse of the basis is received as an element of SSS of a plate, instead of additional required function of model.

The necessary numbers of required functions in model at significant decrease at relative transverse rigidity of layers are established by numerical analysis of SSS of plates and massifs.

It is visible (tab. 1), that in essentially thick square plates, for receiving error less than 5 % in comparison with the three-dimensional decision (T), enough one function of shear at $G/G'=1$, and at $G/G'=100$ it is necessary four functions of shear in each of orthogonal directions x_i. At a relation $E/E'=1$ it is enough to use one function of compression, and at $E/E'=100$ - it is necessary two. The analysis of SSS in a plate with relation has shown necessity of the same quantity of unknown functions whereas for a plate with $a/H=2{,}5$ there is an opportunity of reduction of their quantity. The three-dimensional solution is received by us by a technique [3].

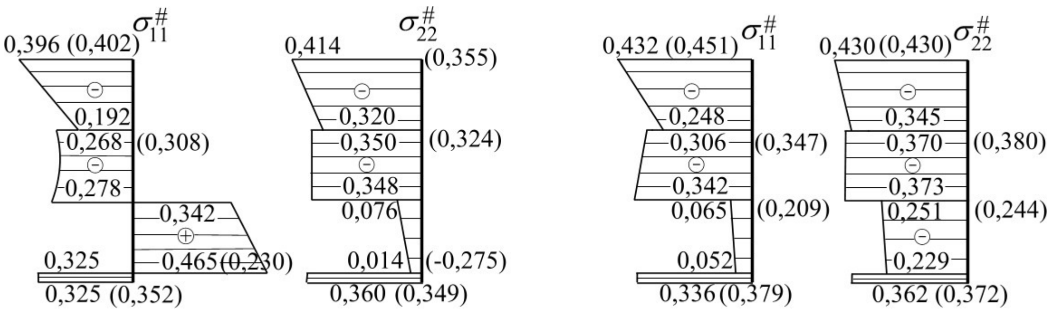
Table1.

The maximal deflections $u_3^{\#} = u_3^{\max} E / p_0 h$ and stresses $\sigma_{11}^{\#} = \sigma_{22}^{\#} = \sigma_{11}^{\max} / p_0$ in a homogeneous square plate ($a/H = 1{,}25$; $a/h = 2{,}5$)

Model	$A=0; B=0$		$A=0; B=2$		$A=2; B=0$		$A=2; B=2$	
	$10u_3^{\#}$	$10\sigma_{11}^{\#}$	$10u_3^{\#}$	$10\sigma_{11}^{\#}$	$10u_3^{\#}$	$10\sigma_{11}^{\#}$	$10u_3^{\#}$	$10\sigma_{11}^{\#}$
C_1S_{1i}	8,65	7,43	9,28	4,85	332,6	100,61	513,2	137,13
C_1S_{4i}	8,66	7,29	9,35	2,62	332,9	93,27	767,6	63,10
C_2S_{1i}	8,42	7,53	9,23	4,89	258,5	70,98	493,6	107,90
C_2S_{4i}	8,41	7,35	9,22	2,62	264,9	69,46	750,6	61,67
T	*8,04*	*5,80*	*9,07*	*2,16*	*262,8*	*68,58*	*747,6*	*54,20*

In tab. 1 it is designated $A = \log E/E'$ $B = \log G/G'$. At $A = 0$ is $\nu = 0{,}3$, at $A = 2$ are $\nu = 0{,}3$ $\nu'' = 0{,}1$ $\nu' = 0{,}001$. Here it is designated a degree μ of iterative approximation on compression as C_μ, and on shear as $S_{\mu i}$ in (each orthogonal direction x_i; $i = \overline{1,2}$).

Character of SSS in a real multilayered road covering of a rigid bridge design at rigid and at slippery contact between a plate and a design are shown. The stamp vertical evenly distributed load in the size 0,8×0,2 (м^2) and infinite plate are replaced with sinusoidal load on a plate with size 1,0×0,4 (м^2). Characteristics of layers: 1 is asphalt concrete: $E^{(1)} = 3 \cdot 10^3$ MPa, $\nu^{(1)} = 0{,}3$, $h^{(1)} = 4$ cm; 2 is asphalt concrete: $E^{(2)} = 2 \cdot 10^3$ MPa, $\nu^{(2)} = 0{,}3$, $h^{(2)} = 4$ cm; 3 is ferro-concrete $E^{(3)} = 12 \cdot 10^3$ MPa, $\nu^{(3)} = 0{,}3$, $h^{(3)} = 4$ cm; 4 is waterproof finish $E^{(4)} = 0{,}9 \cdot 10^3$ MPa, $\nu^{(4)} = 0{,}3$, $h^{(4)} = 0{,}5$ cm.



a) shear without friction b) rigid contact

Fig. 3. Maximal stresses in road clothes

At work of a plate of road clothes with shear without friction on a surface of contact of a plate with foundation (fig. 3a) in bottom carried layer of a covering significant stretching stresses that is inadmissible at operation of such designs are formed. Whereas at rigid contact of a plate to the foundation such stretching stresses are not formed (fig. 3b). In brackets the three-dimensional decision of a problem on is given.

Thus, the constructed specified mathematical model of SSS allows to receive exact enough solutions for homogeneous and layered plates (coverings) under various conditions of contact with undeformable foundation.

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