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INVESTIGATION OF MODELING ERRORS OF DIFFERENT RANDOM FIELD BASED WIND LOAD FORMULATIONS

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Abstract. *In this paper the influence of changes in the mean wind velocity, the wind profile power-law coefficient, the drag coefficient of the terrain and the structural stiffness are investigated on different complex structural models. This paper gives a short introduction to wind profile models and to the approach by Davenport A. G. to compute the structural reaction of wind induced vibrations. Firstly with help of a simple example (a skyscraper) this approach is shown. Using this simple example gives the reader the possibility to study the variance differences when changing one of the above mentioned parameters on this very easy example and see the influence of different complex structural models on the result. Furthermore an approach for estimation of the needed discretization level is given. With the help of this knowledge the structural model design methodology can be base on deeper understanding of the different behavior of the single models.*

1 INTRODUCTION

Studying the behavior of structures under wind loads a vary set of parameters are relevant in applying the load to the structure. This parameters are mostly experimentally determinate and are so not strictly exact. Using this parameters in computing the wind force acting on a building may cause structural damages or even total failure of the structure if not cared much about the the size and sensitivity of the different coefficients. But when setting up a model to design a building it's dimension and it's complexity is of major importance regarding the computational effort and of course if the model takes into account all phenomena and when changing parameters the model reaction behavior is the same for all different models. In the first part the model behavior for parameter changes is studied, in the second part the model quality itself is analyzed and an approach for estimation of the needed discretization level is given.

2 WIND LOAD

2.1 WIND VELOCITY PROFILES

For the wind load formulation a spectral approach suggested by Davenport [1] is used. The approach it self is nor changed or different possible spectra are use because the main goal of this study is to see if changes in the discretization of the random field have influence on to the result.

Davenport gives in the paper *Gust Loading Factors* [3] an approach how wind impacts can be applied to structures using velocity spectrums. Starting from the assumption the mean wind pressure is of the form, see in [3]

$$\bar{p}(Z) = \frac{1}{2}\,\rho\,\bar{V}_I^2\,C_P(Z) \tag{1}$$

in which $\bar{p}(Z)$ = mean pressure at point Z above the ground, $\bar{V}_I$ = mean wind velocity at the level of the top of the structure, ρ = air density ($1.26\ kg/m^3$, varies with the altitude above sea level, in this paper this influence is neglected, which it is generally) and $C_P(Z)$ = pressure coefficient of point Z. Suitable profiles for the mean wind velocity can be described as logarithmic or power-law profiles. The power-law used by Davenport in [3] is

$$\bar{V}_z = \bar{V}_{10}\left(\frac{z}{10}\right)^{\alpha} \tag{2}$$

with $\alpha = 0.16$ for country side and $\alpha = 0.40$ in the city center. This power-law will be used in this paper to describe the wind altitude velocity profile.

2.2 WIND SPECTRUM

The computation of the dynamic reaction of buildings under wind load in the frequency domain is suggested by Davenport [2]. Normally the goal of this method is to evaluate the size of a specific or a vary set of structural reaction values and the exceeding probability of the value for dynamic sensitive structures. The probability distribution used in the spectral method is already fixed with the standard Gaussian probability distribution. Initial step of the method is the analytical velocity power spectra suggested by Davenport in [1]

$$S_V(\omega) = \frac{4\kappa\bar{V}_{10}}{\omega}\frac{x^2}{(1+x^2)^{4/3}} \tag{3}$$

in which

$$x = L\frac{\omega}{\bar{V}_{10}} \quad (4)$$

and κ is a drag coefficient of the terrain an has an approximate value of 0.005 in open country and 0.05 in a city complex, $\bar{V}_{10}$ is the mean wind velocity at reference hight $10\ m$ above the ground and L a length scale of approximately $1200\ m$. This spectra was found by Davenport by studying a high number of measurements of high wind velocities. The invariant of equation (3) to the evaluation above the ground has to be given a special remark.

3 EVALUATION

In [4] and a more detailed explanation in [5] is given to the approach to compute wind induced structural reaction with the help of spectral load formulation. The used schema for the spectral wind induced reaction can be found in [4]. With the help of the power spectra velocity function $S_V(\omega)$ an the aero admittance function $H_a(\omega)$ the aero force spectrum can be expressed

$$S_P(\omega) = H_{a_i}(\omega) \cdot S_V(\omega) \quad (5)$$

In [4] for tall cylindric structures the aero admittance function is given

$$H_{a_i}(\omega) = \rho^2 C_{Di}^2 D_i^2 l_i^2 \bar{V}_i^2 \quad (6)$$

in which ρ is the air density ($1.26\ kg/m^3$, varies with the altitude above sea level, see note above) and C_{Di} is the pressure coefficient, D_i the dimension of the section, l_i length of the considered beam element and $\bar{V}_i$ is the mean wind velocity. All values are function of the ground distance z_i.

Additionally the function $H_{a_i}(\omega)$ can be multiplied with the square of the function $\chi_{F_i}(\omega)$.

$$\chi_{F_i}(\omega) = \frac{1}{1 + \frac{20\omega D_i}{4\pi \bar{V}_i}} \frac{1}{1 + \frac{8\omega D_i}{4\pi \bar{V}_i}} \quad (7)$$

This function takes into account the fluctuation of the wind force caused by instationary fluid flux. This function was not taken into account in the following examples.

To consider the wind load properly also the effect that the wind load in two different points i and j of the structure are not the same, neither in frequency nor in velocity, this means that a shift in the phase of the stochastic process related to the two points exist. The spectral relation for the same frequency ω and same velocity $\bar{V}_i$ between the two points can be represented with a cross spectral function $S_{V_{i,j}}(\omega)$. This cross spectral function exist of a real part (co spectra) and an imaginary part which can be neglected respect the real part, see [4]. The cross spectra

$$S_{V_{i,j}}(\omega) = R_{V_{i,j}}(\omega)\sqrt{S_{V_i}(\omega) S_{V_j}(\omega)} \quad (8)$$

with $R_{V_{i,j}}(\omega)$ as cross correlation coefficient, in [1] $R_{V_{i,j}}(\omega)$ is given

$$R_{V_{i,j}}(\omega) = e^{-k\frac{|z_i - z_j|\omega}{\bar{V}_{10}}} \quad (9)$$

in which k is an experimentally determinate value and is suggested to be 7.7, see [1]. Furthermore the cross correlation coefficient can be assumed to be the same vertically and horizontally.

It follows logically that the cross spectral function of the wind load results to

$$S_{P_{i,j}}(\omega) = H_{a_{i,j}}(\omega) \cdot S_{V_{i,j}}(\omega) \tag{10}$$

and the aero admittance function changes to

$$H_{a_{i,j}}(\omega) = \rho^2 C_{Di} D_i l_i \bar{V}_i \, C_{Dj} D_j l_j \bar{V}_j \tag{11}$$

Because not all structures can be modeled as a single degree of freedom system this approach has to be expand for multi degree of freedom systems [4]. For this purpose the equation of motion has to be expressed in matrix form, size of matrix is the number of degree of freedom of the structure. To solve the differential equation the modal analysis is advantageous which is based on the separation of the single equations. With the help of the impulse force in the frequency domain the square of the absolute structural reaction function for every $n-th$ natural vibration mode can be expressed, see [5]

$$|H_n(\omega)|^2 = \frac{1}{K_n^2 - [1 + (4\xi_n^2 - 2)(\omega/\omega_n)^2 + (\omega/\omega_n)^4]}. \tag{12}$$

In equation (12) ξ_n is the generalized damping coefficient for each mode.

The structure reaction spectral function $S_{y_r}(\omega)$for a certain point r is obtained out of the relation

$$S_{Y_r}(\omega) = \sum_n \phi_{rn}^2 |H_n(\omega)|^2 S_{P_n}(\omega) \tag{13}$$

ϕ_n is the $n - th$ normalized natural vibration mode vector and $S_{P_n}(\omega)$ is the generalized cross correlation spectral function for the wind load, in case of continues calculus the sums goes over to integrals.

$$S_{P_n}(\omega) = \sum_i \sum_j \phi_{in} \phi_{jn} S_{P_{i,j}}(\omega) \tag{14}$$

Integrating $S_{Y_r}(\omega)$ gives the variation of the wind induced deflection of the structure

$$\sigma_{S_{Y_r}}^2 = \int_0^\infty S_{Y_r}(\omega) d\omega. \tag{15}$$

With this approach the base for solving the problem is given.

4 EXAMPLE

To show the principle of this approach a skyscraper is considered to be exposed due to wind force acting on the broader side of the buildings facade. The structural model is in the first tentative a simple cantilever beam with constant mass and stiffness distribution, see Fig. (1). In the second tentative the structural model is a voluminous finite element model, as before with constant mass an stiffness distribution, see Table 2 for the used dimension of the building and the factors for the wind load approach.

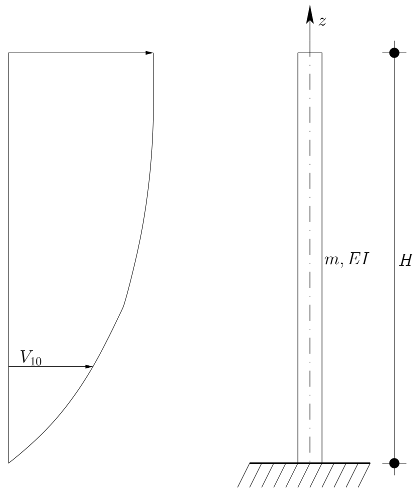

Table 1: Uniform cantilever beam

Table 2: Parameters

wind properties			
V_{10}	10.0	m/s	wind velocity
L	1200	m	gust length
ρ	1.26	kg/m^3	air density
κ	0.05	$-$	drag coefficient
α	0.30	$-$	power coefficient
k	7.7	$-$	correlation value

structure properties			
H	150.0	m	high
B/D	30.0/20.0	m	dimension
E	5e+7	N/m^2	young's modulus
m	351000	kg/m	mass per length
C_D	1.5	$-$	pressure coefcient
ξ	0.01	$-$	damping coefcient

4.1 DYNAMIC REACTION

For this analyzation two different models are used. One is a simple cantilever beam model and a volumes finite element model both with constant mass and stiffness disruption. The beam model was solved analytically, see below, the finite element model was solved with the finite element software package *SLang* developed at the Bauhaus Universität Weimar, Germany. For the spectral wind analysis three different discretization meshes are used, one with $2 * 8$, $4 * 16$ and $8 * 32$ number of elements $broad * high$.

Locking at the different models it get obvious that both has a different dynamic behavior out of it's model capacity. The beam model is not feasible to show torsion modes, which is easily represent with the finite element model. This phenomena is to be noticed very carefully, in this example we will see that it has no big impact, actually no impact at all because the first bending and the second torsion eigen mode can be considered separate and the torsion mode is not activated by the wind load spectrum. Another problem of the beam model is that only points on the building axis can be considered, with the finite element model any chosen point can be considered.

NB: Both models represent theatrically correct the behavior under wind load, but the similarity to the reality is very different in this case for both models.

4.1.1 Beam Model

The analytic results for the natural vibration mode and natural frequencies can be found in [5]. We find the frequency equation by solving the the equation system

$$1 + \cos \beta H \cosh \beta H = 0 \tag{16}$$

and following the natural frequencies

$$\omega_n = \frac{\beta_n^2}{H^2} \sqrt{\frac{EI}{m}} \tag{17}$$

an the natural vibration modes are

$$\phi_n(x) = C_1[\cosh \beta_n x - \cos \beta_n x - \frac{\cosh \beta_n H + \cos \beta_n H}{\sinh \beta_n H + \sin \beta_n H}(\sinh \beta_n x + \sin \beta_n x)]. \tag{18}$$

Only the first four natural vibration modes are used. This is already a assumption which is base on Fig. 1 which shows that the influence of the higher oder vibrations modes are negligible small relative to the first vibration mode. Sought at Eq. 3 we know that the influence of higher vibration modes decrease.

4.1.2 Mode Influence

The plots in Fig. 1 shows the values computed following Eq. (19)

$$\epsilon = \frac{\sigma^2_{S_{Y_r},n} - \sigma^2_{S_{Y_r},n-1}}{\sigma^2_{S_{Y_r},n-1}} \tag{19}$$

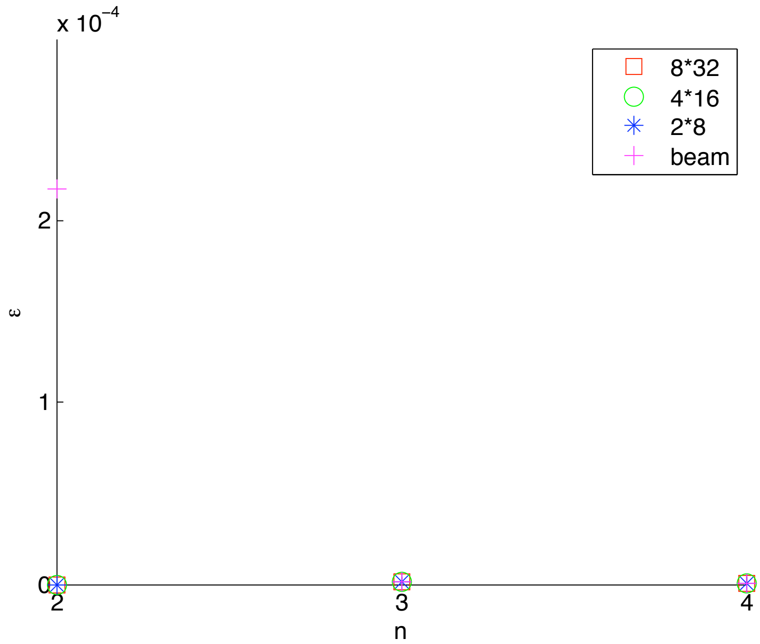

Figure 1: Influence of considered vibration mode amount

which describes the relative change of $\sigma^2_{S_{Y_r},n-1}$ to $\sigma^2_{S_{Y_r},n}$ as an absolute value. We see that most of all the first vibration mode is of importance, the higher vibration modes has only an effect if they are activated due to resonance phenomena.

Furthermore Fig. 1 shows that the influence for the beam and the here considered finite element models, is not quite good for the second mode and actually the second beam model mode is a bending mode and so has to be compared with the third mode of the finite element model which is a similar bending mode too. The fact that for the different discretization models the mode influence is the same shows the model to be consistent.

4.2 STRUCTURAL RESPONSE FUNCTION

Following the evaluation schema we obtain as result the structure reaction spectra $S_{Y_r}(\omega)$, see Fig.2, for the different structural and discretization models. As reference point we used the right top point of the building, see note above regarding the beam structural model.

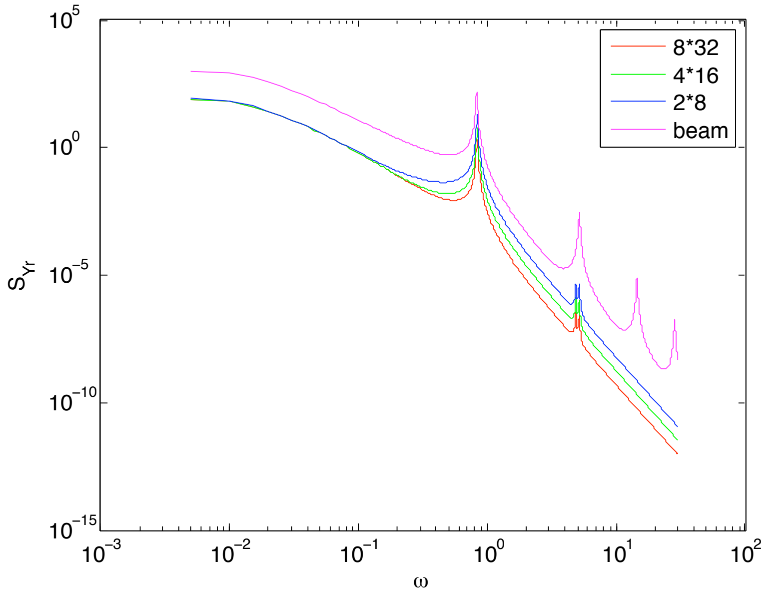

Figure 2: structure reaction spectra $S_{Y_r}(\omega)$

The statical deflection and mean μ, the standard deviation $\sigma_{S_{Y_r}}$ and the exceeding probability of the top displacement of the structure is given in Tab. 3. The target value for the computation of the exceeding probability p is $\mu_{max} = 0.75m$.

Table 3: Results

Model		Beam	$2*8$	$4*16$	$8*32$
deflection	μ $[m]$	0.5049	0.57363	0.57363	0.57363
standard deviation	$\sigma_{S_{Y_r}}$ $[m]$	4.8899	1.4160	1.2594	1.2117
exceeding probability	p $[-]$	0.4796	0.4504	0.4443	0.4421

In addition we see that increasing the model complexity, a finer discretization mesh, the structure reaction spectra $S_{Y_r}(\omega)$ converges to a certain common value. The function itself gets more closer to the pure wind spectra and the singularities caused by the structural reaction function get more sharper represented. Another advantage gets obvious from Fig. 2 is that the result merges from above due to the exact result. This means that the value obtained by a less complex model is larger then the value of a more complex one.

4.3 VARYING PARAMETERS

To study the influence of the parameters, the wind behavior coefficients V_{10}, α, κ and the structure parameter E are varied in an certain range. All the other parameter and coefficients are fixed at the value given in Tab. 2. This helps to see if the model complexity influence the model reaction and if models with different complexity behave different.

4.3.1 Wind Velocity

Beginning with the mean wind velocity V_{10} in the high $10\ m$ above the ground, we vary V_{10} from $0\ m/s$ up to $20\ m/s$. In Fig. 3 we see the relation between $\sigma_{S_{Y_r},V_{10}}/\sigma_{S_{Y_r},0} - 1$ as function of V_{10}.

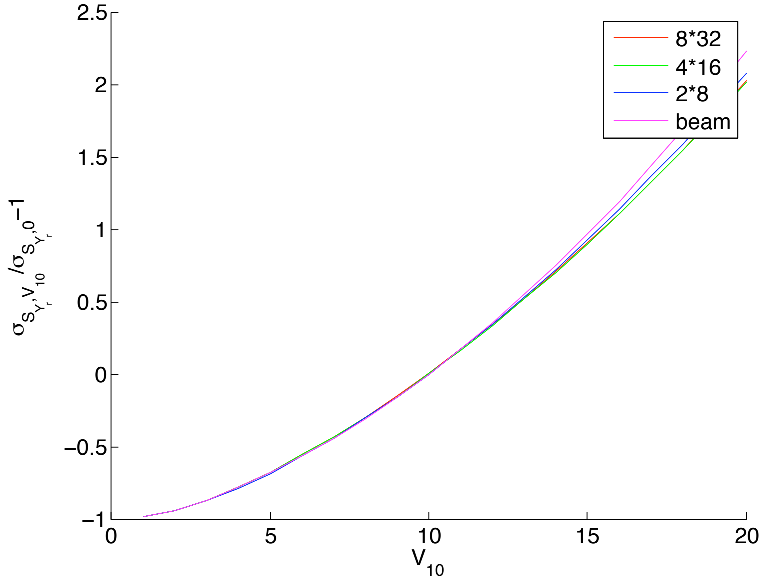

Figure 3: Relation between $\sigma_{S_{Y_r},V_{10}}/\sigma_{S_{Y_r},0} - 1$ as function of V_{10}

Looking at the model complexity we see the for the range $0\ m/s$ up to $15\ m/s$ the changes of $\sigma_{S_{Y_r},V_{10}}$ are the same for all different discretization models. Increasing the velocity even more the different model start to diverge. This shows that for high velocities more accurate models has to be used.

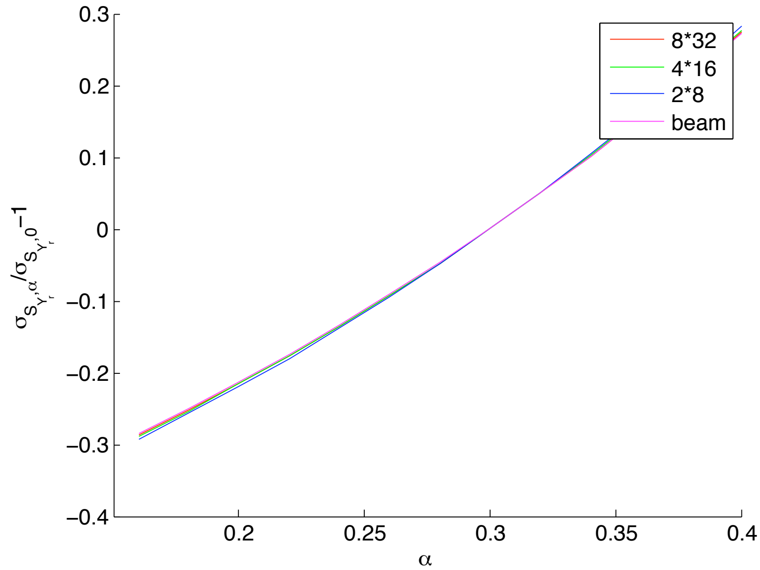

Figure 4: Relation between $\sigma_{S_{Y_r},\alpha}/\sigma_{S_{Y_r},0} - 1$ as function of α

4.3.2 Wind Law Power Factor

As second parameter we vary α from 0.16 to 0.40. Taking a careful look at Fig. 4 we see that the influence of model complexity is more or less negligible.

4.3.3 Wind Drag Coefficient

κ is varied from 0.005 to 0.05 and we see that their are no changes due to changes in the model complexity into the suggested range for κ.

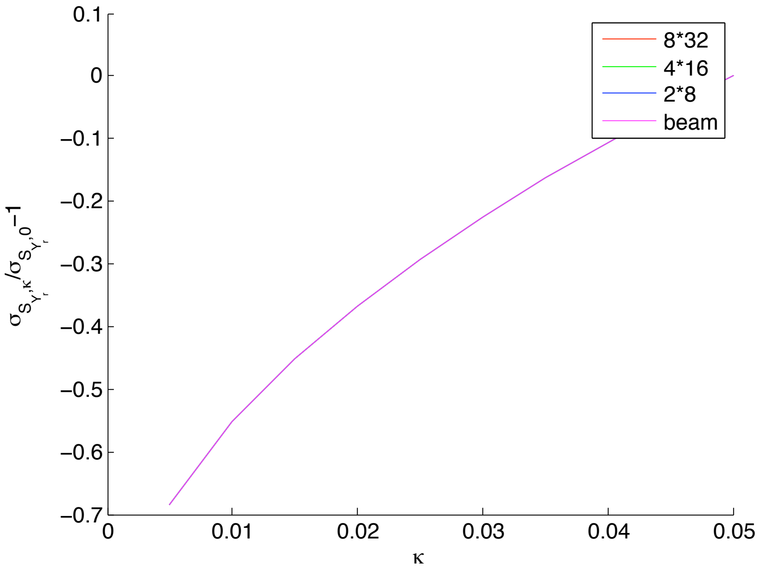

Figure 5: Relation between $\sigma_{S_{Y_r},\kappa}/\sigma_{S_{Y_r},0} - 1$ as function of κ

4.3.4 Structural Stiffness

To see how the structure's stiffness influences the structural reaction we vary E from $1e + 6\ Nm^2$ to $1e + 8\ Nm^2$ for the beam model. Logically by changing the structure's stiffness we change the natural frequency of the structure and so the structure reaction spectra $S_{Y_r}(\omega)$ varies significantly. In Fig. 6 we see the relation between $\sigma_{S_{Y_r},EI}/\sigma_{S_{Y_r},0}$ as function of young's modulus E.

What we see is that as long as the natural frequencies of the structure are out of range of the powerful gusts the differences in the stiffness has not so a big influence, but when moving towards the range of natural frequencies in the powerful regions of the wind velocity spectrum changes in the stiffness are getting dominant. Actually the variation $\sigma_{S_{Y_r},EI}$ degreases as stiffer the structure gets caused by the less strong activation power, see equation (14) and Fig. 2. But still the variation is very sensitive to changes in the structural stiffness EI.

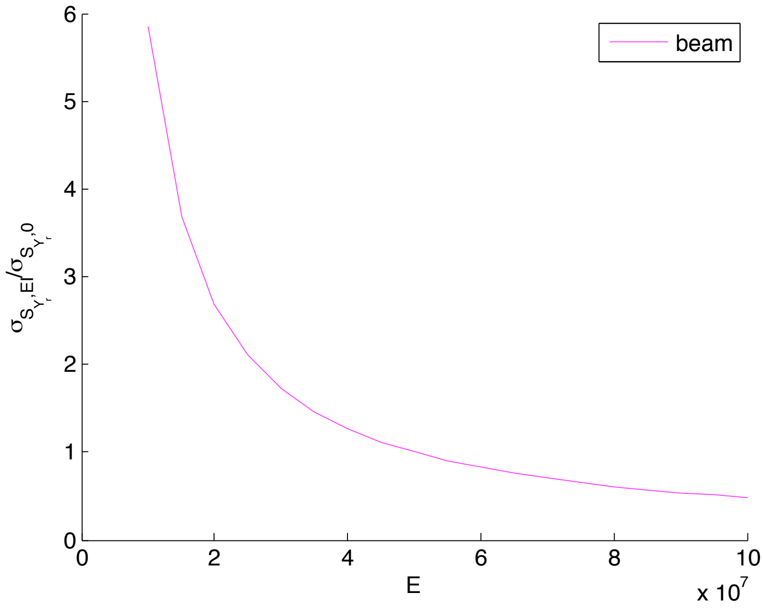

Figure 6: Relation between $\sigma_{S_{Y_r},EI}/\sigma_{S_{Y_r},0}$ as function of E

5 MODEL QUALITY, CONVERGENCE

Of corse one of the major interest is when using a discrete model how many discretization point has to be used. Here an approach for the beam model is show. The blue line in Fig. 7 shows the wind induced deflection $\sigma_{S_{Y_r}}$ here σ_N for any given discretization level N. This information is more or less easily computed for small numbers of N. The green line in Fig. 7 shows the relative error $\sigma_{N-1} - \sigma_N$, which degreases to zero very rapidly and converges to wards the x-axis.

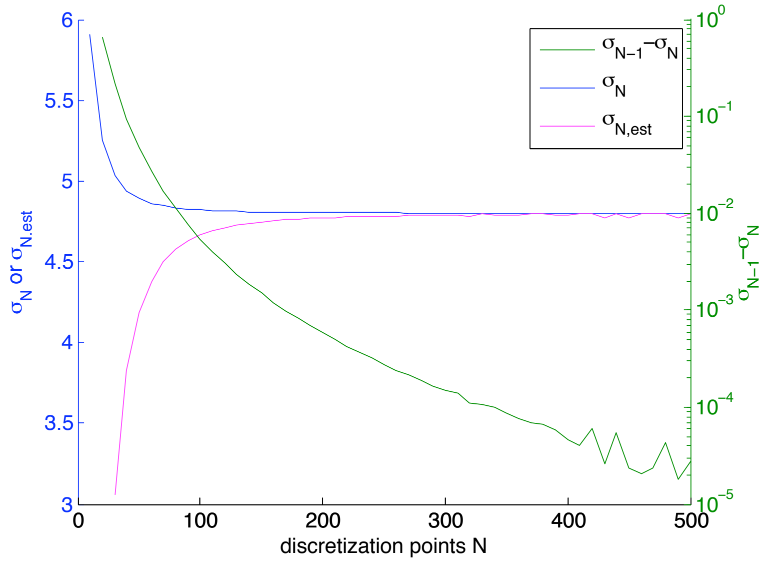

Figure 7: Convergence behavior of the beam model

Now what would be great is if for a certain N we could give a preferable small interval for the value of σ_N. This is achieved by trying to approximate the error value, the real value would be $\sigma_N - \sigma_\infty$. As long we do not now σ_∞ which of course is the exact value, we need an estimated lower bound value. What here is done is to evaluate approximately the value of the error $\sigma_N - \sigma_\infty$ by using a line going through the points σ_{N-1} and σ_N. Then every positive value for $N = 1 \ldots m$ of the line (m is the point where the line crosses the x-axis) is computed and summed up, this estimated error value shows to be always bigger than the real error value. So for every point N and knowing the value at point $N-1$ we can give an upper σ_N and lower $\sigma_{N,est}$ value, which are plotted as a magenta colored line in Fig. 7.

Having this information we know can easily decide if the chosen N is big enough or not looking at the resulted interval. The value of $\sigma_{N,est} \leq \sigma_\infty$ and so should not be used for further calculations.

The jumps in the relative error are due to numerical integration error, they are for $N > 400$ negligible small. But still the estimated value $\sigma_{N,est}$ fulfills all conditions and gives a very narrow interval which means that increasing N would have no sense any more.

6 CONCLUSIONS

What we see is that all studied different discretized models has more or less the same properties, but the simplest model, the beam model, is totally different and in this special case is not appropriate to represent the real behavior of the skyscraper.

What is very useful is that we see that for different mean wind velocities V_{10} different discretization levels are important to use if an approbate mapping of the reality is required. Of course we see that some parameters combined with the different models has no effect on the result. This knowledge is very important when starting to think about the model methodology.

The convergent studies in the last part of the paper gives a simple but very useful approach of estimating a lower bound value for the obtained result, so to be abel to verify the quality of the result and in case to increas the discretization level.

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