

EVALUATION OF SOIL-STRUCTURE INTERACTION MODELS USING DIFFERENT MODEL-ROBUSTNESS APPROACHES

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Abstract. *The aim of this study is to show an application of model robustness measures for soil-structure interaction (henceforth written as SSI) models. Model robustness defines a measure for the ability of a model to provide useful model answers for input parameters which typically have a wide range in geotechnical engineering. The calculation of SSI is a major problem in geotechnical engineering. Several different models exist for the estimation of SSI. These can be separated into analytical, semi-analytical and numerical methods. This paper focuses on the numerical models of SSI specific macro-element type models and more advanced finite element method models using contact description as continuum or interface elements. A brief description of the models used is given in the paper. Following this description, the applied SSI problem is introduced. The observed event is a static loaded shallow foundation with an inclined load. The different partial models to consider the SSI effects are assessed using different robustness measures during numerical application. The paper shows the investigation of the capability to use these measures for the assessment of the model quality of SSI partial models. A variance based robustness and a mathematical robustness approaches are applied. These different robustness measures are used in a framework which allows also the investigation of computational time consuming models. Finally the result shows that the concept of using robustness approaches combined with other model–quality indicators (e.g. model sensitivity or model reliability) can lead to unique model–quality assessment for SSI models.*

1 INTRODUCTION

Soil-Structure Interaction can be defined as a certain kind of structure which is embedded within the soil. Hereby, the interaction between both materials reaction to each other is important. Typical topics for SSI analysis are deep foundations, shallow foundations and excavations, geosynthetic reinforcements to name a few [1]. SSI effects are important topics which have to be considered in a wide range of geotechnical and structural engineering applications. SSI can be modeled using one or more of a high amount of approaches which have been published in the past. The different available models can be split into analytical, semi-analytical and numerical models considering SSI effects in diverse ways.

The major problem in dealing with SSI effects is the great amount of model and the rising question of what the best suitable model is, in regards to model-robustness, model-uncertainty and/or model-complexity. These different model attributes are important to quantify to select the most suitable model [2]. [1] states that it is important to consider the best suitable model to allow predictions and back calculations. [3] points out that less attention is paid to validate the models and investigate their capability for reliable simulation results. [4] shows a benchmark test where it is obvious that the model choice and modeling techniques have a major influence on the results which consider SSI effects.

Recent approaches are presented to assess the modelquality in geotechnical engineering [5] to validate the use of the constitutive soil models. In general, it can be pointed out that there is great need to continue this work, in particular for different SSI models. These different SSI models are so-called ill-posed problems, because for the identification of the most suitable model it is important to consider changes in the structure as well as in the soil. This paper focuses on the use of numerical finite element models and so called macro-element approaches. These models are introduced briefly.

The purpose of this paper is to clarify the use of such different SSI models, taking into account the model-robustness. Therefore a variance-based model robustness and a mathematical robustness approaches are used. These two slightly different ways for the model robustness are applied to a shallow foundation with a transient inclined static loading. Thus a scheme is proposed to evaluate the model robustness especially for boundary value problems which have a large computational time. For this statistical analysis, different methods presented which can be used to evaluate the most influencing parameter to the model response are presented. Following this preliminary study, a meta-model is generated for the further evaluation of the model-robustness. During the consideration of this meta-model the mathematical model robustness is evaluated.

2 SOIL-STRUCTURE INTERACTION MODELING

In this section the different SSI modelling techniques are briefly introduced. The main focus is on the Finite-Element Method, but the SSI modeling using a macro-element approach is also captured.

2.1 Modelling SSI with macro-element approach

A macro element approach is a simplified approach which combines the soil half-space, interface between the soil and the structure, and the foundation into on single model which can be solved using a parabolic function. A general description of different kinds of macro-elements to calculate SSI effects are given in [6].

A number of parallel studies have been conducted on the subject of macro-element modeling for a static loaded strip footing by e.g. [7, 8, 9]. Using an incremental plasticity model consisting of constitutive description to account for the interaction between the forces of structure and the plastic displacement, it is possible to use an elasto-plastic strain hardening macroelement which can predict the behaviour of SSI [7]. Some examples are shown by [10, 11]. These macro-element use a yield function Eq. (1) given by [7]

$$f(\mathbf{Q}, \rho_c) = h^2 + m^2 - \xi^2 [1 - \text{frac}\xi\rho_c]^{2\beta} = 0 \quad (1)$$

Where $h = H/(\mu V_m)$, $m = M/(V_m)$, $\xi = V/V_m$, ρ_c is a loading history parameter, H the horizontal load, M the generated moment, V the vertical force, V_m the maximum vertical load capacity of the macroelement, μ the slope of failure envelope in the H-V plane, ψ Slope of the failure envelope in the M-V plane, B width of the foundation and β is a constitutive parameter which controls the shape of the failure envelope. The plastic potential can be written as developed by [7]:

$$g(\mathbf{Q}) = \lambda^2 h^2 + \chi^2 m^2 - \xi^2 [1 - \text{frac}\xi\rho_g]^{2\beta} = 0 \quad (2)$$

Where $\lambda = \mu/\mu_g$ and $\chi = \psi/\psi_g$. Both of these parameters must be determined experimentally. If $f(\mathbf{Q}, \rho_c) = g(\mathbf{Q})$ the flow rule is associated. Both functions can be used with an incremental plasticity scheme to calculate the displacement in respect to the load. For the interested reader, refer to [10, 7] for a detailed description.

2.2 Modelling with Finite Elemente Methode SSI

Generally, if a structure is loaded, relative movements with regards to the soil can occur. Therefore, the use of conventional finite elements can create compatibility problems prohibiting relative movements into soil structure interaction modeling. Due to discretization as shown in Figure 1a the nodal compatibility in the finite elements method is constrained, such that the soil and the structure move together. To prevent this occurrence, so-called interface or joints elements could be used. Particular advancement also is that it is possible to use a different material formulation for this interaction zone (e.g. maximum wall friction angle). Another important point is that with such elements it is possible to allow separation or sliding.

Undergoing research to use finite element analysis to investigate SSI has been considered since the early 70s. There are different proposed methods and models used. The different groups are tackled here in the following:

1. node to node contact
2. using conventional continuum finite elements (e.g. [12, 13])
3. zero thickness / thin layered interface elements (e.g. [14, 18, 16])

Figure 1 shows the different types of SSI modelling. These three methods are used in a finite element analysis. In the following a brief description follows the methodology and the constitutive models.

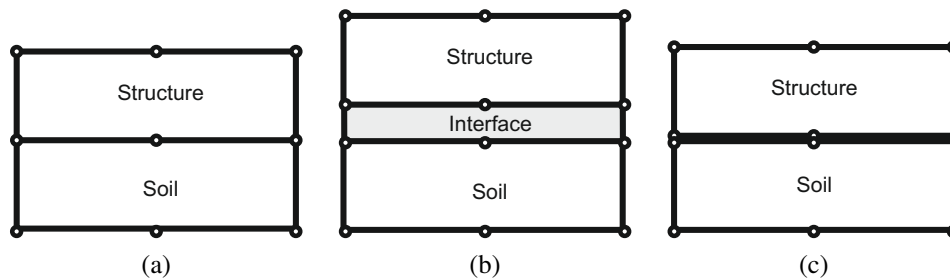


Figure 1: (a) node to node contact (b) use of continuum element as interface (c) use of interface element according to [17]

All of these different techniques can be used under conditions where their utilization is justified.

Node to node contact

Modeling the SSI using only node to node contact is a rough method with which to model the phenomena which appear these transition zones. Clearly there is a big disadvantage in using such kinds of SSI models due to the fact that the nodes for the soil and structure must fulfill the nodal compatibility. This is due to the fact that if the structure is loaded, relative movements can occur [17]. Due to the discretization shown in Fig. 1(a) the nodal compatibility in the finite elements method is constrained such that the soil and structure tend to move together.

Also, it is important to point out that the use of such a modeling technique for SSI can lead to unrealistic high failure loads. This is due to the fact that at corner points singularity points, will occur which reduces the accuracy of the global finite element mesh [18].

The use of these modeling methods can not be recommended but it is quite often that in practical engineering applications using interface elements are forgotten or the effect is underestimated by the engineer.

Using conventional finite elements

[12] proposes the use of conventional finite elements (Fig. 1(b)) in cases where the slip of a foundation structure must not be considered. [12, 13] shows that the conventional finite element formulation is able to predict SSI effects in efficient quality. If the finite element model should be used to model slipping of the structure in a great range over the soil it is not suitable for this purpose [12].

Another advantage is also that it is possible to use a different material formulation for this interaction zone (e.g. maximum wall friction angle). As with interface elements it is possible to use the same constitutive material formulation than in the surrounding soil.

Zero-thickness interface elements

[15] shows the first use of special interface elements in finite element analysis which can model discontinuity like joints in rock mechanics. This pioneer work from [15] are followed by a huge amount of different proposed interface element formulation from 6-10 node isoparametric interface element to different material descriptions from linear elastic material behavior to bi-linear models of the *Mohr-Coulomb* friction material model to more advanced material models like Damage models [20], Critical State Soil Mechanics [21] framework and advanced elasto-plastic formulations [19].

The use of thin continuum interface elements (e.g. [16, 22]) for soil-structure interaction can lead to problems due to the fact that the thickness of the interface is unknown and the determination of the input parameters used is difficult without conducting special laboratory tests [19]. Special interface formulation are developed which ensure of singular points at the corner of SSI modelling [18].

The interface models used are in respect to the finite element formulation in the commercial software used. 6 or 15 noded triangular elements are used in this publication. The associated interface elements are 6 or 10 noded joint elements. Both types of elements for the continuum and interface are shown in Fig. 2. The rate of interface traction t and the displacement discontinuity Δu is a combination of linear elasticity and perfect plasticity therefore the elasto-plastic relationship can express the following equation:

$$t = \mathbf{D}_c^e \Delta u^e = \mathbf{D}_c^e (\Delta u - \Delta u^p) \quad (3)$$

Eq. (3) relates the objective rate of traction t to the relative displacement over the full length of the interface surface Δu^e . The following D-Matrix can be generated for isotropic linear elastic behavior (4):

$$\mathbf{D}_c^e = \begin{bmatrix} k_s & 0 \\ 0 & k_n \end{bmatrix} \quad (4)$$

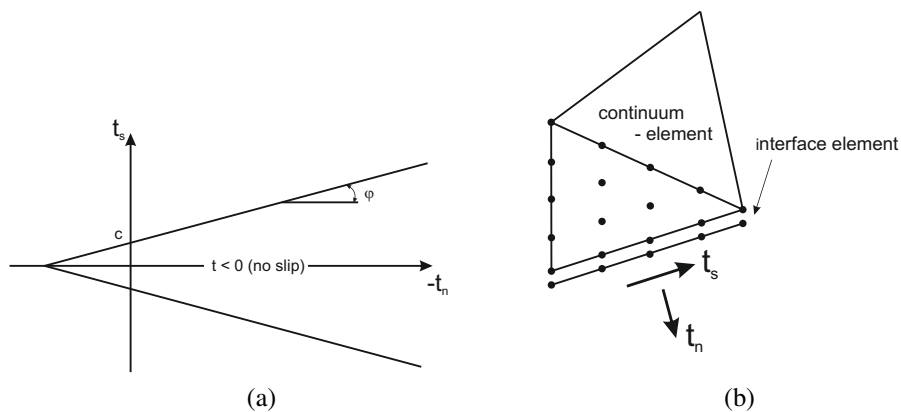


Figure 2: (a) Yield function and (b) interface element assembly according to [18]

where the interface stiffness is related to the mean element length l , G the shear modulus of the soil and ν Poisson's ratio.

$$k_s = \mu (G/l) \quad k_n = \mu G / (1 - 2\nu) \quad (5)$$

In the present study only a simple *Mohr-Coulomb* yield function is considered. This yield function is combined with a non-associated plastic potential. Fig. 2a shows a representation of the yield function in a t_s - t_n space. The yield function and plastic potential can be described with:

$$f(t) = t_s + t_n \tan \varphi_c - c_c \quad g(t) = t_s + t_n \tan \psi_c \quad (6)$$

Where φ_c is the friction angle and c_c the adhesion in the interface zone. For the plastic potential ψ_c is the dilation angle which controls the plastic dilation. As described in [23] the use of a non-associated flow rule $\varphi_c > \psi_c$ prevents a unrelastic high plastic dilation. For some special cases this can result in an overestimation of the contact pressure and consequently of the shear strength. Using the plastic potential the plastic slip can be derived using the following equation:

$$\Delta u^p = \alpha \lambda (\partial g / \partial t) \quad (7)$$

where α is a coefficient defined as:

$$\alpha = 0 \quad \text{if} \quad f < 0 \text{ or } [\partial f / \partial t]^T \mathbf{D} \Delta(u) < 0 \quad (8)$$

$$\alpha = 1 \quad \text{if} \quad f = 0 \text{ with } [\partial f / \partial t]^T \mathbf{D} \Delta(u) \geq 0 \quad (9)$$

using this switch on and off coefficient α the multiplier λ can be solved with the help of the consistency equation $f = 0$. To solve these constitutive equations a *Newton-Cotes* algorithm is applied. Considering this integration scheme, a lumped interface stiffness matrix can be achieved.

The advantages of using this interface elements is that it is not necessary to care about the interface thickness and the special needed material parameters which are uncertain and difficult to obtain by using conventional laboratory tests.

3 MODEL – ROBUSTNESS

Model–robustness is defined as the capability of a model to given consistent output over the full range for which it is generated of possible input parameters. Two different types of robustness measures are used. Both of these robustness approaches are based on global model–robustness.

3.1 Variance-based robustness measure

The variance based robustness measure is also called *Taguchis* robustness and is defined as an adapted ”signal-to-noise” ratio discussed in [24] and can be expressed in Eq. (10):

$$T = -10 \log \left(\frac{1}{\sigma_Y^2} \right) \quad (10)$$

Where the standard derivation σ_Y^2 is used to estimate the model-robustness. The robustness measure by *Taguchi* has some drawbacks described in [24].

3.2 Mathematical robustness approach

The principal idea of the mathematical robustness approach is directly derived from the definition of model-robustness, that the model will generate an input which is completely conneted to the output. As an example the change in input will be related to the change of output. Therefore the input-output relationship of the SSI models will be investigated. There can be three different model robustness classes defined. Robust, partial robust and non-robust models as shown in Fig. 3.

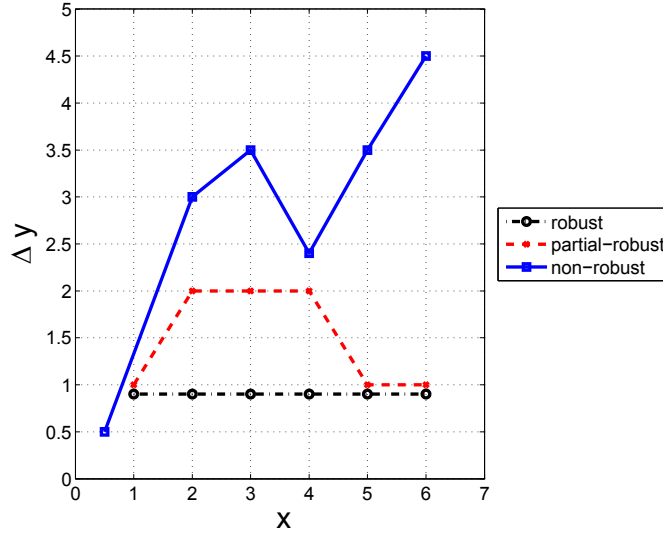


Figure 3: Input-Output relation for **robust**, **partial-robust** and **non-robust** model

Plotting $\Delta y - x$ diagram shows different possible cases. If the plot shows a straight horizontal line the input-output relation is linear and it is a quite robust model. Shows the input-output relation some fluctuations, it is partial robust. The third case is that if the model shows a complete irregular behavior for the output it can be called non-robust. If $\Delta y = 0$ the parameter does not influence on the output. This means the parameter has no influence on the model response.

To calculate this, the model input will be split in n numbers of intervals. Using n intervals for all important input variables used. These are used to compute all possible combinations as model response. The number of combinations will be quite high, as the following Eq. 11 shows:

$$nC_i = n^{np} \quad (11)$$

where nC_i is the amount of different possible combinations and np are the number of parameters. For all these combinations the global response of the model is computed. Therefore

one parameter is frozen and all other combinations are computed.

$$E(Y|X_i) \quad (12)$$

From these output the mean (μ_Y) is calculated for all combinations. Using these mean values the Δy for every single parameter interval is calculated by:

$$\Delta y = E\left(\hat{Y}_n|\mathbf{X}_i\right) - E\left(\hat{Y}_{n+1}|\mathbf{X}_i\right)$$

These Δy are plotted against the parameter input x to estimate the robustness of a model graphically.

4 METHODOLOGY FOR THE ASSESSMENT OF MODEL – ROBUSTNESS

The first step in the determination of the robustness measures are the concept as show in Fig. 4. The different step as shown in Fig. 4 are explained in the next paragraphs.

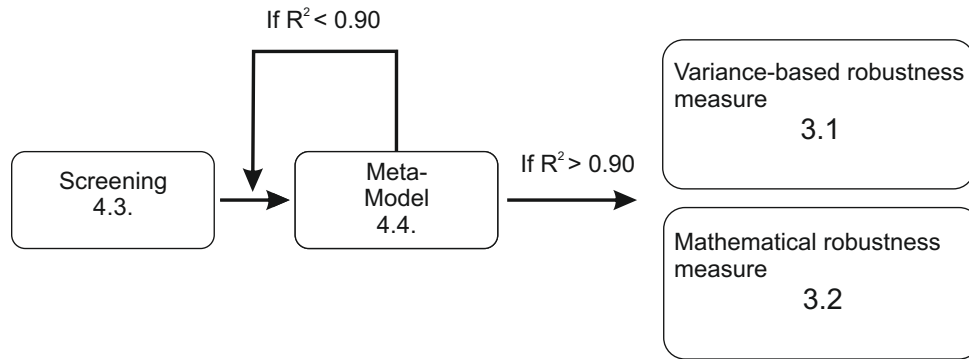


Figure 4: Methodology for the assement of the model–robustness

4.1 Application to an inclined loaded shallow foundation

The application for the comparison of these different robustness approaches described above is a shallow foundation which is loaded with an inclined load. Figure 5 shows the boundary conditions.

This boundary value problem is assumed for all the 4 different partial models. For the three different finite element models, the contact description is different according to section 2.2. Model 1 (M1) uses a simple node–node contact, Model 2 (M2) uses an interface element and Model 3 (M3) uses the continuum approach. The macro–element model is the fourth type of model which is used. To exclude mesh dependencies and errors due to other modeling issues, some comparison with different meshes are conducted in order to ensure that such effects could not happen. Please note that these investigations are not shown here.

The gray scaled areas in Fig. 5 are the meshed parts of the model. For the discretization, 15-noded triangular elements with a fourth order interpolation for displacements are used. These elements have 12 Gauss points (stress points) for each element (also shown in Fig. ??).

The model has a plane strain boundary condition with the width of 9.00 m and height of 4.00 m. Fig. 5 uses the same element formulation for the beam embedded in the soil. For the material

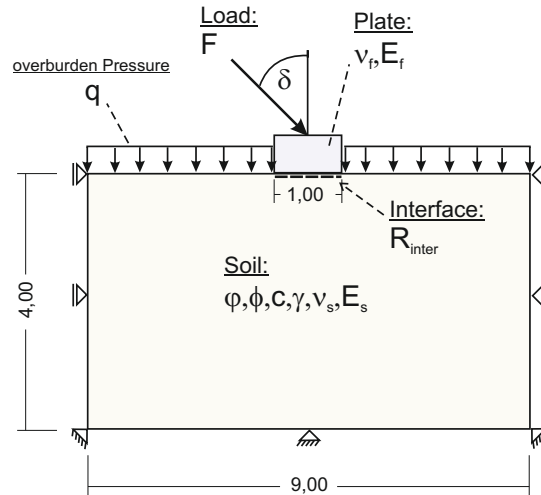


Figure 5: Applied boundary value problem

model of the foundation a simple linear-elastic "Hookes law" model is chosen. This requires 2 material parameter. The Young's Modulus E_f and the Poisson's ratio ν_f . The constitutive material model for the soil is the *Mohr-Coulomb* model which is a classical model used in geotechnical engineering. It is a linear-elastic perfectly plastic model with a fixed yield surface in principal stress space. For a more detailed description see for example [6]. The model uses five different parameters to describe the constitutive behavior of soils and rocks in a wide range. The different parameters are φ the friction angle, ψ dilatancy angle, c cohesion, E_s Youngs modulus and ν_s Poissons ratio. For a more detailed description, [6] is referenced.

Considering this boundary value problem, the SSI models are applied. Consequently, this means that three finite element analysis and one macro-element calculation is conducted. The macroelement approach is also applied for the shown boundary in Fig. 5. Due to its modelreduction it is not explicitly drawn here, for a pictorial representation is referred to [8].

For comparison of the different models a fixed load level of 75 kN is applied and all results are shown in respect to the displacement by this load level.

4.2 Parameters used in the different SSI models

The parameters which are used are listed in the following Tab. 1. The sampling of the parameter is performed as uniform distributed sampling as described by [25]. The meta model sampling is also uniformly distributed. These uniform distribution is chosen to hold each variable for the model input independent of each other.

4.3 Preliminary Study for Elementary Effects

The different models were first screened with the help of a screening technique proposed by [25]. The idea of this is to search the elementary effects of the entire design space. Therefore, a randomized sampling plan is generated where so-called Elementary Effects (EE) can be calculated using Equation (13). With the help of these EE the input parameters can be ranked in order of their importance. These EEs can not give a statement of the importance quantifiably. The basic assumption from [25] is that the objective function from the underlying computational model is deterministic.

Table 1: Parameter input for the analysis

Parameter [unit]	Baseline	Minimum	Maximum
Friction angle φ [°]	35	28	45
Dilation angle ψ [°]	5	0	10
Cohesion c [kN/m ³]	1	0.0001	5
Poissons ratio (soil) ν_s [-]	0.25	0.2	0.38
Youngs modulus (soil) E_s [kN/m ²]	30000	20000	45000
Unit weight (soil) [kN/m ³]	18	15	19.5
Youngs modulus (foundation) E_f [MN/m ²]	30000	20000	100000
Poissons ratio (foundation) ν_f [-]	0.45	0.3	0.495
Friction coefficient R_{inter} [-]	0.8	0.3	1
Load inclination angle δ [°]	22.5	0.1	45

Soil	Foundation	Interaction	Load
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To estimate EEs the mean and standard deviation is calculated. These indicators shows the importance to the global response and the non-linearity / interaction. In computational regard this scheme is quite efficient because with the help of one simulation run two sensitivity values can be examined.

$$d_i(\mathbf{x}) = \frac{y(x_1, x_2, \dots, x_{i-1}, x_i + \Delta, \dots, x_k) - (y(\mathbf{x}))}{\Delta} \quad (13)$$

Where $\Delta = \xi / (p - 1)$, $\xi \in \mathbb{N}$ and $\mathbf{x} \in D$ such that the components $x_i \leq 1 - \Delta$ and $\mathbf{x} \in D = [0, 1]^2$ for scaling issues, k amount of input variables, p number of discrete values along each dimension. The basic idea of a parameter in regarding to [25] is that a parameter with a large measure of central tendency indicates a major influence to the objective function. Further, [25] considers that a significant measure of spread indicates that the variable is involved in non-linear effects and/or interacts with other parameters.

With estimation of the sample mean and the sample standard deviation, for a set of $d_i(x)$ values overspanned to the design space. Of major importance for this purpose is to generate a sampling plan that each evaluation of the objective function f participates in the estimation of two elementary effects. Therefore the sampling has to give us a defined number r elementary effects for each variable. A more detailed discussion is given by [25].

The sampling can generate with the help of Eq. (14).

$$\mathbf{B}^* = (1_{k+1,1} \mathbf{x}^* + (\Delta/2) [(2\mathbf{B} - 1_{k+1,k}) \mathbf{D}^* + 1_{k+1,k}]) \mathbf{P}^* \quad (14)$$

Here, B is the basic sampling matrix, P^* is the random permutation matrix and D^* randomly generated matrix with +1 or -1 on the diagonal. For the calculation of r EEs for each variable, the screening plan is built from r random orientations using Eq. (15):

$$\mathbf{X} = \begin{bmatrix} \mathbf{B}_1^* \\ \mathbf{B}_2^* \\ \dots \\ \mathbf{B}_r^* \end{bmatrix} \quad (15)$$

The advantage of the method presented is to generate a more efficient and accurate response surface with a less of computational time. The result of these screening is shown as a bar plot in Fig. 6. Therefore, the parameters scaled to show the influence of the different parameters correlated under each other. These scaling is done with the following Eq. 16:

$$EE_i^{scale} = \frac{\mu_Y}{\max(\mu_Y^2)} + \frac{\sigma^2}{\max(\sigma_Y^2)} \quad (16)$$

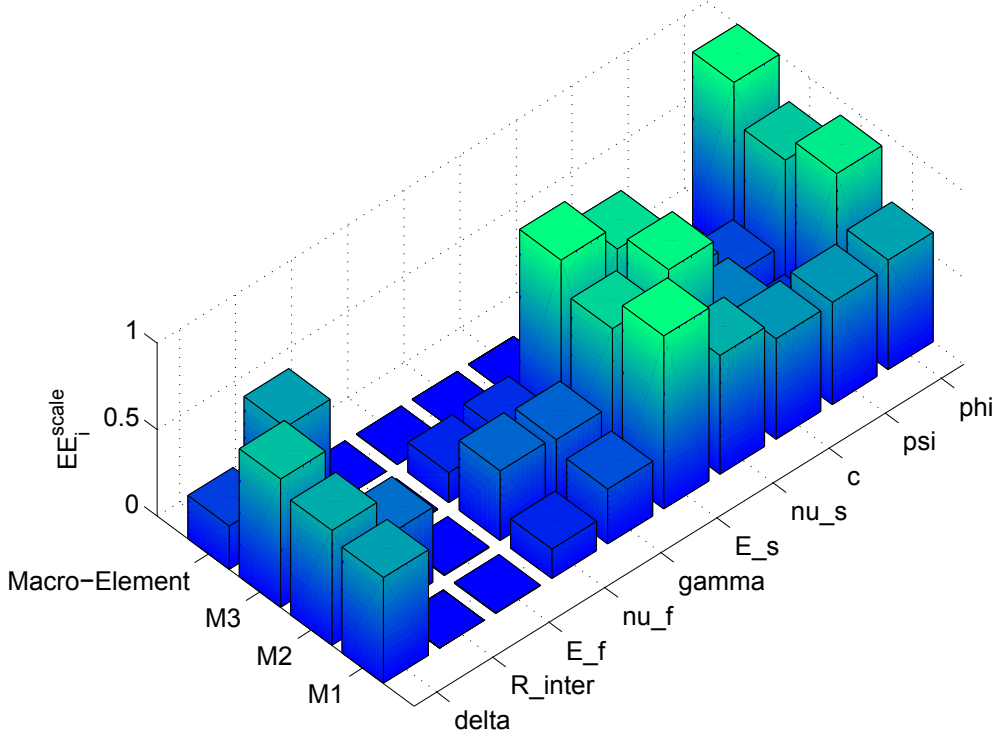


Figure 6: Scaled Elementary Effects

4.4 Meta-Modeling

Due to the computational time consuming finite element models, this models are replaced by different meta-models. For these, only those from the screening important model input parameters are considered. For the investigation of the robustness criterion it was important that the model input are considered as uncorrelated input, to generate also uncorrelated meta-models. To control and build the response surfaces a multi-linear regression was used. The object function of the observed model will be idealized by the following equation. For the regression linear, quadratic and mixed terms Eq. 17 are used:

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_{P_k} X_{P_k} + \beta_{11} X_{11}^2 + \beta_{22} X_{22}^2 + \cdots + \beta_{P_k P_k} X_{P_k P_k}^2 + \beta_{12} X_1 X_2 + \cdots + \beta_{P_k-1 P_k} X_{P_k-1} X_{P_k} + \mathbf{e} \quad (17)$$

Here, \hat{Y} is the regression equation for the approximation of the model response, β the regression coefficient, X_i the i -th parameter set. The regression coefficients β are calculated using Eq. (18):

$$\beta = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y} \quad (18)$$

For the control of correctness of the used response surface the coefficient of determination R^2 is calculated using Eq. 19.

$$R^2 = 1 - \left(\frac{SS_E}{SS_T} \right) \quad (19)$$

If the coefficient of determination is smaller than 0.90 the number of samplings must be higher to reach a higher accuracy of the meta-models.

The different coefficient of determination are shown in Fig. 7a. For the generation of the meta-model, 500 samples are used. The macro-element is not replaced by a meta-model. The plane in the bar plot shows the value of the coefficient of determination. All meta-models can be used with a number of 500 samples.

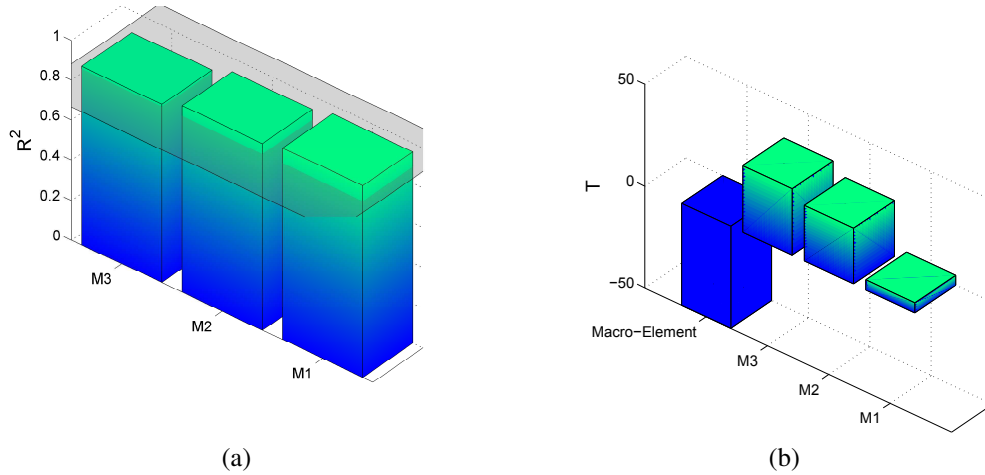


Figure 7: (a) R^2 for meta-models (b) Results for the *Taguchi* robustness

4.5 Robustness approaches

At the end of the concept show in Fig. 4 both robustness approaches are investigated, to investigate the model-robustness.

Results: Variance-based robustness measure

Fig. 7b visualizes the results for the variance based robustness measure. Based on the *Taguchi* robustness measure it can be stated that the most robust model is the model M3 followed by M2.

Results: Mathematical based robustness approach

Currently for the mathematical robustness, there are no clear mathematical formulation to express the robustness of a model in scalar existence. However, generally the concept for the

evaluation of the model–robustness presented above can identify the model–robustness for each model parameter independently (see Fig. 8).

In Fig. 8a the results for all four different models are shown for the load inclination angle. In general can be stated that all models have parts where they are robust. In Fig. 8b shows the results for the friction angle φ .

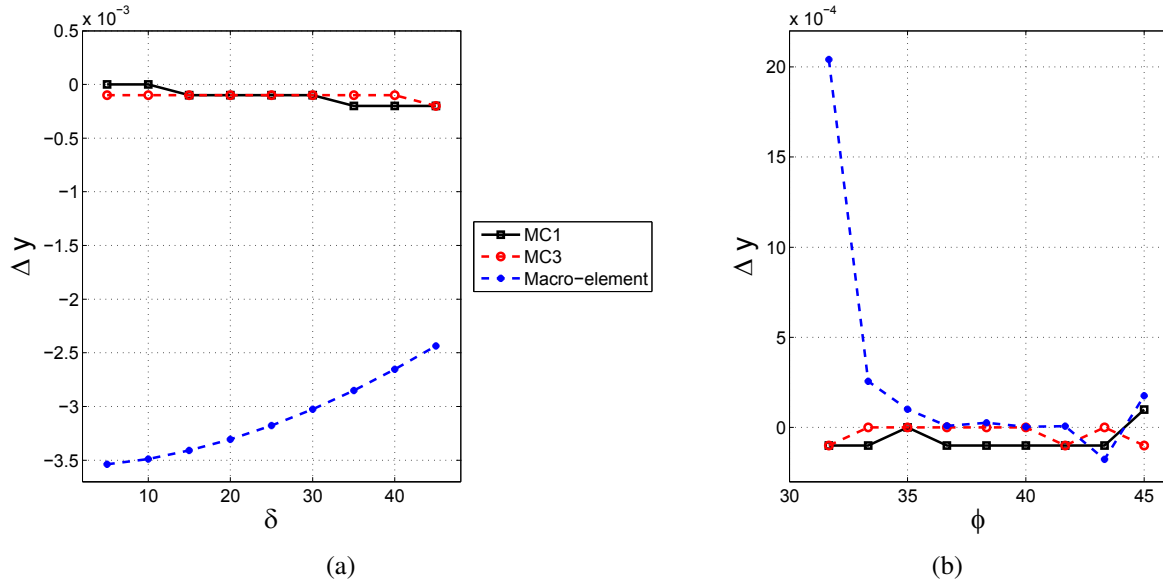


Figure 8: (a) mathematical robustness inclination angle δ (b) mathematical robustness for the friction angle φ

5 CONCLUSION

Both of these robustness approaches can be applied to soil–structure coupling models. In the case of the *Taguchi* robustness measure, the different models shows really different results. The mathematical robustness approach have to be completed. This approach has to be refined as a mathematical formulation. This will helps to compare models in the sense of model–robustness and model–quality. Furthermore, some effort has to be done on the question of which model delivers a reference model robustness and what happens if the models tends to be show a non-linear behavior.

This paper shows the applicability of using two different model robustness assement approachs for the model robustness. This paper concludes with an outlook on the development of a straight-forward mathematical formulation of model–robustness.

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