19th International Conference on the Application of Computer
Science and Mathematics in Architecture and Civil Engineering
K. Gürlebeck, T. Lahmer and F. Werner (eds.)
Weimar, Germany, 04–06 July 2012

DEPENDENCY OF THE INFLUENCE OF INPUT PARAMETERS OF BVI MODELS ON THE INITIAL EXCITATIONS AND SPEED RANGES OF THE VEHICLE

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Keywords: sensitivity analysis, functional input, joint meta-modeling, bridge-vehicle interaction.

Abstract. *Bridge vibration due to traffic loading has been subject of extensive research in the last decades. Such studies are concerned with deriving solutions for the bridge-vehicle interaction (BVI) and analyzing the dynamic responses considering randomness of the coupled model's (BVI) input parameters and randomness of road unevenness. This study goes further to examine the effects of such randomness of input parameters and processes on the variance of dynamic responses in quantitative measures. The input parameters examined in the sensitivity analysis are, stiffness and damping of vehicle's suspension system, axle spacing, and stiffness and damping of bridge. This study also examines the effects of the initial excitation of a vehicle on the influences of the considered input parameters. Variance based sensitivity analysis is often applied to deterministic models. However, the models for the dynamic problem is a stochastic one due to the simulations of the random processes. Thus, a setting using a joint meta-model; one for the mean response and other for the dispersion of the response is developed. The joint model is developed within the framework of Generalized Linear Models (GLM). An enhancement of the GLM procedure is suggested and tested; this enhancement incorporates Moving Least Squares (MLS) approximation algorithms in the fitting of the mean component of the joint model. The sensitivity analysis is then performed on the joint-model developed for the dynamic responses caused by BVI.*

1 INTRODUCTION

Structural systems can be represented by various mathematical models implemented and solved using complex computer codes, which may be referred to as numerical models. Developed numerical models are often employed to identify influential input parameters that affect responses of interest. This identification is carried out by methods of sensitivity and uncertainty analyses. Such studies can be used for model validation, model calibration, and decision making processes. Variance sensitivity analysis is one of the efficient methods to study global sensitivities where main and higher order effects of input parameter can be quantitatively measured. Furthermore, sampling-based techniques for estimating the variance-based sensitivity indices are frequently used for complex engineering problems for their ease in implementation. However, their main drawback is the demand of high computational time. Therefore, meta-models are used as an alternative for the complex computer codes, and the sensitivity analysis is performed efficiently on fitted meta-models [1]. Such a procedure applies to deterministic models that produces always the same output for the same set of input. Unfortunately, this is not the case in all engineering applications where random processes or unknown input parameters may be over-looked.

Stochastic computer codes are the ones were simulations of random processes are included in every run of the analysis, thus, the output values depend on the realizations of these random processes. Applying variance-based sensitivity analysis for such models is a challenge and has attracted the attention of researchers in the recent years. *Tarantola et al.* [2] suggested a solution to consider the effects of random processes by introducing a scalar input parameter ($\xi \sim U(0,1)$) controlling the simulation of random processes and including them in the computer model, this require performing the sensitivity analysis directly on the numerical model, which means high computational time. More recently, [3] introduced building a joint model for the output of stochastic computer codes. The joint model is then used for the sensitivity analysis. The procedure for joint modeling followed by [3] is within the framework of generalized linear models. Later the same authors suggested non-parametric models, e.g. generalized additive models and joint Gaussian process modeling, as they proved to be more efficient in estimating the sensitivity indices [4]. A similar idea is to be used and tested for the engineering problem at hand.

The engineering problem of interest is bridge-vehicle interaction. There has been an increasing attention to develop procedures for solving the bridge-vehicle interaction, which is encouraged by the advent computational power of digital computers and the increasing number and weights of vehicles traveling on bridges. Therefore, researchers and modelers had been concerned with deriving solutions of the dynamic problem of bridge-vehicle interaction. *F. Yang et al.* [5] and [6] reviewed the different methods with their corresponding mathematical and computational descriptions. Moreover, probabilistic studies had also been employed to assess the effects of random input parameters and road unevenness on the dynamic response. *Hwang and Nowak* [7] presented a procedure to calculate statistical parameters for the dynamic loading of bridges. These parameters were based on surveys and tests and included vehicle mass, suspension system, tires and road roughness, which were simulated by stochastic processes. *Kirkegaard and Nielsen* [8, 9] studied the randomness of vehicle input parameters and the randomness of road unevenness in two separate studies. One conducted for vehicle input parameters and the other for the effects of random road profiles on the dynamic response of highway bridges. Moreover, solutions for the statistical characteristics of a bridge's response to the passage of a vehicle over a random rough surface have been of interest in a number of research works, such as [11, 12, 13]. More recently, [14] considered both the randomness of the vehicle input parameters and road unevenness, and calculated the statistical characteristics of the bridge response by using the random variable functional moment method.

This study aims to extend probabilistic studies and use them for purposes of sensitivity and uncertainty analyses. One of the main challenges of such an analyses is considering the effect of road unevenness on the variance of the bridge displacements. Further, the effect of the initial excitations on the influence of vehicle and bridge dynamics on variances of bridge displacements is also examined.

The first section of the paper deals with the general description of the enhanced generalized linear models and their use to determine the sensitivity indices followed by presenting the main solution algorithm of the bridge-vehicle interaction. An academic example is illustrated to validate the presented approach followed by the application on the influences of vehicle dynamics, bridge dynamics and initial excitations on the variance of a bridge's response.

2 ENHANCED GENERALIZED LINEAR MODELS

The numerical models of interest are the ones that are stochastic in nature, e.g. having functional inputs which cannot be captured by scalar ones. For such a problem and where no replications is preferred for computational time reduction, it is useful to model both the output's mean and variance jointly, which leads to the use of generalized linear models. Generalized linear models generalizes linear regression by allowing the linear model to relate to the response variable using a link function and by allowing the variance at each observation point to be a function of its prediction at the same position. Each generalization model has three components; response variable distribution, linear predictor, link function. A full description of the such models and their extension can be found in [15, 16]. In short the followings describe the mean and dispersion components of the joint model.

$$
\begin{aligned}
E(Y_i) &= \mu_i, \quad \eta_i = g(\mu_i) = \Sigma_j x_{ij}\beta_j; &(1)\\
var(Y_i) &= \phi_i v(\mu_i), &(2)
\end{aligned}
$$

where $(Y_i)_i = 1, \ldots, n$ are random variables with mean μ_i; x_{ij} are samples of covariate vectors X_j; β are the regression coefficients; η_i is the linear predictor of the mean; $g(.)$ is the link function; ϕ_i is the dispersion parameter, and $v(.)$ is the variance function. The dispersion is assumed to vary and dependent on the predicted mean values, hence a model is built for ϕ_i:

$$
\begin{aligned}
E(d_i) &= \phi_i, \quad \xi_i = h(\phi_i) = \Sigma_j u_{ij}\gamma_j; &(3)\\
var(d_i) &= \tau v_d(\phi_i), &(4)
\end{aligned}
$$

where $(d_i)_i = 1, \ldots, n$ are estimates of dispersion, error of prediction is used; u_{ij} are samples of covariate vectors U_j; γ are the regression coefficients; ξ_i is the linear predictor of the dispersion; $h(.)$ is the link function; τ is a constant, and $v_d(.)$ is the variance function for dispersion. The choice of the linear predictor has a strong effects on the joint model and its quality. In the case where the distribution for the responses of the mean model is chosen to be normal, an identity link function follows the choice of this distribution and (1) becomes $E(y) = \mu$, which is the general formulation of simple linear regression.

This paper is concerned with enhancing the procedure of generalized linear models by using local approximation algorithms for the predictor of the mean. Not only a weighting function dependent on the variance distribution is introduced to estimate the regression coefficients β but also a weighting function dependent on the position of the approximation point relative to the observation (support) points is used. Moving least squares (MLS) is proposed to be the predictor in GLM procedure.

Furthermore, the base of the variance model is the squared residuals $\epsilon_i^2 = (y_i - \mathbf{X_i}\beta)^2$ for the $i = 1, 2, \ldots, n$, where n is number of observations. For the mean component of joint model, the meta-models coefficients β are evaluated as

$$\beta = (X'V^{-1}X)^{-1}X'V^{-1}y, \tag{5}$$

where $Var(\epsilon) = V_{n\times n}$ with $\sigma_i^2 = e^{\grave{u}_i\gamma}$. It can be noticed that the maximum likelihood (MLE) estimator of β involves γ through V matrix and the MLE of γ clearly involves β since the data in the variance involves β. As a result an iterative procedure is carried out.

The enhanced GLM procedure using MLS is as follows:

1. Ordinary linear regression models is used to obtain β_0 for the mean model $y_i = X_i'\beta_0 + \epsilon_i$
2. β_0 is used to compute n residuals, $\epsilon_i = y_i - X_i'\beta_0$
3. The residuals ϵ_i^2 are used as data to fit the variance model with regressors u and a log link function, the regression coefficients γ are determined
4. The variance weighting matrix V is formulated to be used in updating β_0 to β_1 for the iteration step
5. The moving least squares (MLS) is concurrently applied on the approximation point
6. Step 2 is repeated with the updated data, and analysis is continued till convergence

The Gaussian weighting function is used for MLS algorithm, which is an exponential function described as

$$w_G(s) = e^{-s^2/\alpha^2}, \tag{6}$$

with α as a shape factor and $s = \|x - x_i\|/D$, where s is the normalized distance between the approximation point and the supporting point considered and D is the influence radius. Furthermore, for the above procedure cross validation is used to find the residuals, which eliminates the over-fitting of noise in the fitting

3 SENSITIVITY INDICES

Sensitivity analysis is the study of how uncertainties or variances in the output of a model is apportioned to uncertainties or variances of the inputs. Variance based methods have been chosen due to their independence from the investigated model, and the influence of groups or sets of input parameters may be examined. Moreover, such an analysis provides the importance ranking of the input parameters as well as quantifying their contribution to the output variance [17]. The main idea of variance-based methods is to estimate the amount of variance that would disappear if the true value of the input parameter X_i is known. This can be described by the conditional variance of Y fixing X_i at its true value $V(Y|X_i)$, and is obtained by varying over all parameters, except X_i. Since the true value of X_i in complex engineering problems is unknown, the average of the conditional variance for all possible values of X_i is used, i.e. $E(V(Y|X_i))$. Having the unconditional variance of the output $V(Y)$ and the expectation of the conditional variance $E(V(Y|X_i))$, the following relation holds, which is known as the law of total variance:

$$V(Y) = V(E(Y|X_i)) + E(V(Y|X_i)), \tag{7}$$

From equation (7) the variance of the conditional expectation $V(E(Y|X_i))$ is determined. This term is often referred to as the main effect, as it estimates the main effect contribution of the X_i to the variance of the output. Normalizing the main effect by the unconditional variance $V(Y)$ results in:

$$S_i = \frac{V(E(Y|X_i))}{V(Y)} \tag{8}$$

The ratio S_i is known as a first order sensitivity index [18], which is also known as the importance measure [19]. The value of S_i is less than 1, further the sum of all first order indices corresponding to multiple input parameters is an indicator of the additivity of the model. The model is considered additive when the sum equals to one (no interactions between the input parameters), and non-additive when the sum is less than one. Hence, the difference $1 - \sum S_i$ is an indicator for the presence of interactions between the input parameters. For example, the interaction between two parameters X_i and X_j on the output Y in terms of conditional variance is expressed as:

$$V_{ij} = V(E(Y|X_i, X_j)) - V(E(Y|X_i)) - V(E(Y|X_j)), \tag{9}$$

where $V(E(Y|X_i, X_j))$ describes the joint effect of the pair (X_i, X_j) on Y. This is known as a second order effect. Higher order effects can be computed in a like manner. The total effect index S_{Ti} is used to represent the total contribution of the input parameter X_i to the output, i.e. the first order effects, in addition to all higher order effects.

The above formulations applies to deterministic computer codes, where the same set of data produces the same output repeatedly. However, such a statement cannot be said when considering functional inputs, e.g. random processes, for the numerical models at hand, which are called stochastic models as mentioned before.

The work of [4] suggested the family of generalized linear models (GLM) and generalized additive models (GAM) to model the mean and dispersion of model's output and use the joint model to estimate the sensitivity indices. This general approach is adopted in this study using GLM with moving least squares MLS as the fitting algorithm of the mean component of join model, having the identity as the link function.

The procedure starts with assuming the existence of an uncontrollable input parameter X_ϵ in addition to the scaler inputs $X = (X_1, X_2, \ldots, X_k)$. Thus, the output of the numerical models can be written as $Y = f(\mathbf{X}, X_\epsilon)$. The joint meta-models using enhanced GLM are used to formulate the relation for the mean (f_m) and dispersion (f_{ds}) with respect to the scaler inputs $(\mathbf{X})$, which can be written as [4]:

$$f_m(\mathbf{X}) = E(Y|\mathbf{X}) \tag{10}$$

$$f_{ds}(\mathbf{X}) = V(Y|\mathbf{X}) \tag{11}$$

The total variance of Y is defined by (7), hence, the sensitivity indices for the scalar inputs can be estimated on the mean component of the joint model using classical sampling methods having $S_i = V_i(f_m)/V(Y)$. At the same time the dispersion component of the joint model f_{ds} is developed. $E(V(Y|\mathbf{X}))$ presents the expected value of the variance caused by X_ϵ and its interaction with $\mathbf{X}$, thus, the total effect sensitivity index of X_ϵ is estimated as $S_{T\epsilon} = E(V(Y|\mathbf{X}))/V(Y)$.

4 MODELING OF BVI

The engineering problem of interest is the vibration of bridges caused by a moving heavy vehicle. A general description of the vehicle and the bridge models as well as the used solution algorithms are explained.

4.1 Vehicle model

The equations of motion for the vehicle can be written in the following general form:

$$\mathbf{M}_v \ddot{\mathbf{U}}_v + \mathbf{C}_v \dot{\mathbf{U}}_v + \mathbf{K}_v \mathbf{U}_v = \mathbf{P}_v, \tag{12}$$

where $\mathbf{M}_v$ is the mass matrix of the vehicle, $\mathbf{C}_v$ is the damping matrix of the vehicle, $\mathbf{K}_v$ is the stiffness matrix of the vehicle, $\mathbf{P}_v$ is the dynamic force vector of the vehicle, and $\mathbf{U}_v$ is the generalized coordinate vector describing the dynamics of the vehicle model (degrees of freedom).

The chosen vehicle model is an eight-degree-of-freedom model representing a typical configuration of a common heavy truck traveling on road networks [20]. The vehicle consists of a two-axle tractor and a three-axle semi-trailer linked by a hinge. It is assumed that the three axles of the semi-trailer share the rear static load equally since load-sharing mechanisms are common in multi-axle heavy vehicle suspensions [21]. The generalized coordinates used to describe the vehicle dynamics are tractor vertical displacement y_T, tractor pitch angle θ_T, semi-trailer vertical displacement y_S, semi-trailer pitch angle θ_S, tractor front unsprung mass vertical displacement y_1, tractor rear unsprung mass vertical displacement y_2, and semi-trailer unsprung masses vertical displacements $y_3,\ y_4$, and y_5, as shown below:

$$\mathbf{U}_v = \left\{ \begin{array}{cccccccc} y_T & \theta_S & \theta_S & y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \right\}^T \tag{13}$$

The mass, damping and stiffness matrices can be found in [20].

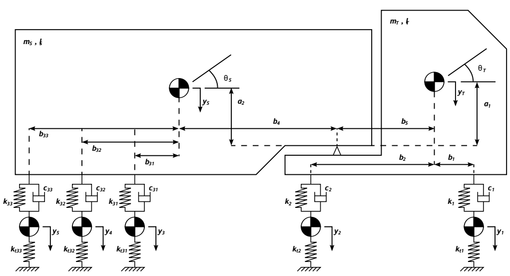


Figure 1: Schematic for the five-axle vehicle model

The interaction force F_i^{int} can be expressed as:

$$F_i^{int} = k_{ti} \left[y_i(t) - y_b(x_i, t) - r_i(t) \right], \;\; i = 1,\ 2,\ 3,\ 4,\ 5 \tag{14}$$

where $y_b(x_i, t)$ and $r_i(t)$ are the displacements of the bridge and road unevenness respectively, at the contact point corresponding to the i^{th} axle at instant t.

The vibration of such a heavy vehicle has two distinctive frequency ranges; the first range is 1.5 Hz to 4 Hz, representing the sprung mass bounce involving some pitching, and the second range is 8 Hz to 15 Hz, representing the unsprung mass bounce involving suspension pitch modes [21].

4.2 Bridge model

The equations of motion of the bridge considering time varying forces can be expressed in the following matrix notation:

$$\mathbf{M}_b\ddot{\mathbf{U}}_b + \mathbf{C}_b\dot{\mathbf{U}}_b + \mathbf{K}_b\mathbf{U}_b \;=\; \mathbf{P}_b \;, \tag{15}$$

with $\mathbf{M}_b$, $\mathbf{C}_b$, $\mathbf{K}_b$ are the mass, damping and stiffness matrices of the bridge, $\ddot{\mathbf{U}}_b$, $\dot{\mathbf{U}}_b$, $\mathbf{U}_b$ are the accelerations, velocities and displacements of the bridge, and $\mathbf{P}_b$ is the vector of forces acting on each bridge node at time t, which has two components, as shown below:

$$\mathbf{P}_b \;=\; \mathbf{F}^g + \mathbf{F}^{int} \;, \tag{16}$$

where $\mathbf{F}^g$ is the force acting on the bridge due to the weight of the vehicle, which is independent of the interaction, and $\mathbf{F}^{int}$ is the time-variant force acting on the bridge, which depends on the interaction between the bridge and the vehicle. The damping of the bridge is assumed to be viscous, which means that it is proportional to the nodal velocities.

4.3 Bridge-vehicle interaction

The equations of motion for the vehicle and the bridge are written as (12) and (15), respectively. Assuming perfect contact, the solution of these equations is governed by satisfying the compatibility equation and imposing the equality of displacement at the contact point, as expressed below:

$$y_w(x_i,t) \;=\; y_b(x_i,t) + r_i(t) \;, \tag{17}$$

where $y_w(x_i,t)$ is the displacement of the tire of the vehicle at i^{th} contact point at instant t. In addition, the force equilibrium conditions at the contact point i must be satisfied, which can be shown as:

$$P_b^i \;=\; F_i^g + F_i^{int} \;, \tag{18}$$

where F_i^g is the static weight of the i^{th} axle and F_i^{int} is the interaction force at the i^{th} axle. The i^{th} contact point usually does not coincide with the a DOF of the bridge model. Therefore, the forces F_i^g and F_i^{int} are converted to equivalent nodal forces associated with the bridge's DOF.

The solution algorithm described in [10] is used in the analysis. It is a non-iterative solution conditioning over a sufficiently small time step. With such a time step, the force acting on the vehicle at the current time step is estimated from the previous step. The choice of the time step should be *small enough* to capture the highest desired frequency of the bridge, the vehicle passage, and the excitation from road unevenness. Moreover a factor of $\frac{1}{10}$ is introduced into the Δt selected to secure reasonable integration accuracy.

In general, many DOFs are involved in the FE model of the bridge system, but only the first modes of vibration make the significant contribution to the dynamic response. Therefore, the modal superposition method has been used to solve the equations of motion of the bridge, which reduces the computational effort considerably, which is regarded as advantageous [22].

4.4 Road unevenness

Road unevenness is often treated as a realization of a stationary Gaussian homogeneous random process described by its power spectral density function in space domain $S_{f_0 f_0}(\kappa)$ with κ as the wavenumber [23].

However, the dynamic analysis is performed in time domain, and a description of the road unevenness in time domain is needed. Therefore, the temporal power spectral density function $S_{f_0 f_0}(\omega)$ is to be computed. Assuming a constant speed for the vehicle v, $S_{f_0 f_0}(\omega)$ and $S_{f_0 f_0}(\kappa)$ can be related using the following:

$$S_{f_0 f_0}\left(\omega = v\kappa\right) \;=\; \frac{1}{v} S_{f_0 f_0}(\kappa) \tag{19}$$

When performing the analysis in time domain, one can deduce that the excitation of the vehicle due to road unevenness can be described as non-stationary when the vehicle speed is time dependent [25]. Even when the speed is constant and the vehicle excitation is stationary, the dynamic responses of the bridge are non-stationary due to the movement of the vehicle [11]. This observation is of importance in deriving the stochastic characteristics when the dynamic problem is solved in frequency domain.

The model for generating realizations of road unevenness is a series of cosine terms with random phase angles, and described in (20).

$$\begin{aligned} f(t) &= \sum_{k=0}^{N_d-1} \left[C_k cos(\omega_k t + \Phi_k) \right] \ , \\ \omega_k &= \omega_l + k\Delta\omega \ , \\ k &= 0, 1, 2, \ldots, N_d - 1 \ , \end{aligned} \tag{20}$$

where Φ_ks are independent random phase angles uniformly distributed in the range $[0, 2\pi]$ and C_ks are random variables following Rayleigh distribution with a mean value of $\beta_k \sqrt{\frac{\pi}{2}}$ and a variance of $\beta_k^2 (2 - \frac{\pi}{2})$ taking β_k as $\sqrt{S_{FF}(\omega_k)\Delta\omega}$. S_{FF} is the one sided power spectral density function (PSD) used to describe the road unevenness. Further, the realized road surfaces reflect the prescribed probabilistic characteristics of the random process accurately as the number N_d gets larger.

It is noticed from (20) that the PSD is discretized into temporal frequency bands of a width of $\Delta\omega$, and the corresponding discretized frequencies are used in the realization of the stochastic process. However, the entire frequency domain of the PSD cannot be used in the realization for mathematical and physical reasons [26]. For the realizations of road surfaces, cut-off frequencies are needed. The discretizing frequency band is defined as

$$\Delta\omega \;=\; (\omega_u - \omega_l)/N_d \ , \tag{21}$$

with ω_u and ω_l (rad/s) as the upper and the lower cut-off frequencies. The long wavelength irregularities correspond to low frequency components in the time domain and short wavelength irregularities correspond to high frequency components [27].

5 NUMERICAL EXAMPLES

5.1 Ishigami test function

The described joint modeling combined with sensitivity analysis have been applied to Ishigami function [24]:

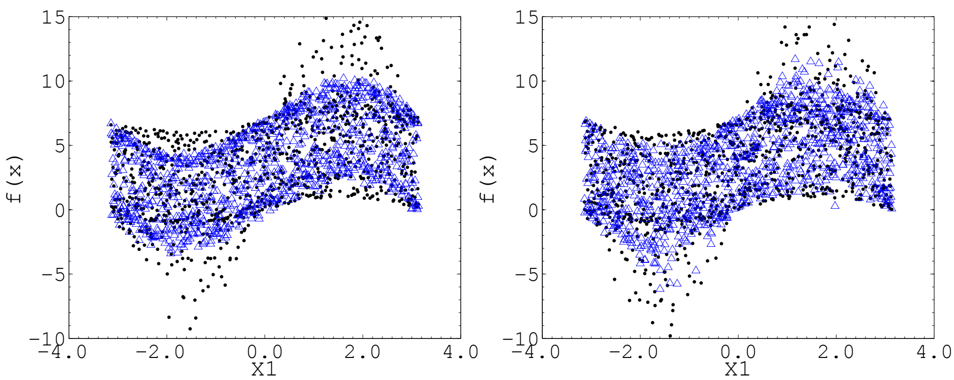
$$Y(X1, X2, X3) = \sin(X_1) + a\sin(X_2)^2 + bX_3^4\sin(X_1), \tag{22}$$

where $a = 7$, $b = 0.1$, and $X_i \sim U[-\pi;\pi]$ for $i = 1,2,3$. The sensitivity indices for this function are well documented in [1]. A similar setting that of [4] is used in this example. The input parameters X_1 and X_2 are considered as the known input parameters, whereas X_3 is an uncontrollable or unknown input parameter which is not considered in the joint modeling of the function's output.

For Joint modeling support samples are obtained by running a Monte Carlo simulation. One thousand samples of (X_1, X_2, X_3) are simulated to obtain the support observations. Latin hypercube is used for efficiency in sampling. A joint model using the enhanced GLM procedure is applied, the properties of the fit for the mean and dispersion of the joint models are given in Table 1. As mentioned before, cross validation has been used in the determinations of the predictions errors which are used in GLM procedure. Fig. 2 depicts the better fit when GLM procedure is used in building the meta model.

Table 1: Properties of the fitted joint model for Ishigami function

	Formula
Joint GLM enhanced with MLS	$f_m = f(X_1, X_2^2, X_1^3, X_2^4)$ radius$_{MLS} = 0.77$, $\alpha_{MLS} = 0.4$ $f_d = f(X_1^2, X_2^3)$



(a) using MLS (b) using GLM procedure with MLS

Figure 2: Comparison between fitting the data with and without GLM procedure: • observations (supports) △ approximated

The sensitivity indices are calculated using the developed joint model, thousands of samples are run on the joint model to ensure convergence in the estimation of the sensitivity indices. A comparison between the exact indices [1], estimated by [4], and the procedure suggested here are shown in Table 2. The results by [4] were estimated using generalized additive models that employs spline smoothing algorithms based on 100 repetitions of the joint models fitting process. Whereas, the suggested enhanced GLM with MLS has been performed using 10 repetitions of the joint models fitting process.

Table 2: Exact and estimated sensitivity indices

SI	Exact [1]	Joint GAM [4]	Joint GLM$_{MLS}$
S_1	0.314	0.310	0.32
S_2	0.442	0.452	0.41
S_{T3}	0.244	0.236	0.22

It can be noticed from Table 2 that an agreement in the estimated indices exists, which proves the efficiency of the application of the proposed joint models and their use in estimating the sensitivity indices.

5.2 Effects of bridge-vehicle interaction

The engineering problem at hand is the effects of bridge and vehicle dynamics on the bridge displacements considering the excitations of the vehicle due to road unevenness, which can be described as an uncontrollable parameter rendering the dynamic model to stochastic.

The vehicle model presented by [20] is used. The characteristics of the vehicle are found in [29]. The bridge model is a single span simply supported beam model for the Pirton Lane Highway bridge in Gloucester (United Kingdom) [21]. The bridge has a length $L = 40$m, an estimated mass per unit length of $m = 12000$ kg/m and a bending stiffness of EI = $1.26 \times 10^5\ MNm^2$. The bridge's first natural frequency is $f_1 = 3.20$ Hz with a modal damping ratio $\zeta_1 = 0.02$.

Road unevenness is considered in the dynamic analysis, its realization follows (20) where $\omega_l = 1.74$ rad/s and $\omega_u = 75.54$ rad/s with $\Delta\omega = 0.104$ rad/s. The dynamic model's output is the displacements at mid-span that are normalized by the corresponding static displacements, which is known as the Dynamic Incremental Factor (DIF).

A sensitivity analysis is carried out to identify the influence of the input parameters of the vehicle dynamics; stiffness (k_t) and damping (c_t) of suspension system and spacing of fifth axle (S), and the bridge dynamics; flexural stiffness (EI) and damping ratio (ζ). Further, the excitation of the vehicle by the road profile of the approach leading to the bridge is examined and its effects on the variance of the dynamic response is studied.

In order to build the joint model, 1000 random vectors of input parameters are generated and the dynamic model is run for each sample to obtain the support observations. The uncontrollable input parameter X_ϵ represents the random processes of road unevenness. For the scaler input parameters (X), the first order indices are determined from the mean component of joint model, whereas, for X_ϵ the total effect index is estimated. The results are presented for two speeds, these are critical speeds derived for the examined bridge and vehicle models, which had been documented in a previous work by the author [30]. The critical speeds for the vehicle are 57km/h and 84km/h; these speeds cause the highest dynamic effects on the bridge.

Two scenarios are examined; one considers the vehicle traveling over the bridge with initial excitation (WI), and another ignores the initial excitation (WoI). The corresponding sensitivity indices are presented in Table 3. It can be noticed that the initial excitation has a limit influence on the identified input parameters from the vehicle and bridge dynamics affecting the variance of the bridge's displacement. Whereas, studying the effect of the speed on the sensitivity indices in Table 4 one can see that higher speeds shadow the influence of vehicle dynamics and power the influence of road unevenness on the variance of the bridge's displacement. In other words, the higher the speed, the higher is the amplification in the dynamic response, however, the scatter of the output is also higher. Such an observation is of significance in the modeling of the dynamic problem as more attention must be given when higher speeds are considered in the analysis as higher variations in the response are expected.

Table 3: Estimated sensitivity indices estimated for the displacements due to a vehicle traveling at 57km/h

	1st order WoI	1st order WI
k_s	0.14	0.16
c_s	0.03	0.02
S	0.00	0.02
EI	0.09	0.09
ζ	0.01	0.03
$S_{T\epsilon}$	0.72	0.68

Table 4: Estimated sensitivity indices estimated for the displacements considering the initial excitations by the approach

	1st order v_{cr}=57km/h	1st order v_{cr}=84km/h
k_s	0.16	0.05
c_s	0.02	0.03
S	0.02	0.01
EI	0.09	0.05
ζ	0.03	0.10
$S_{T\epsilon}$	0.68	0.80

6 CONCLUSIONS

The study is concerned with performing sensitivity analysis for responses retrieved from stochastic models of bridge-vehicle interaction. The main presented and tested methods are based on building a joint model using GLM procedure and enhancing the fitting by suggesting MLS approximation algorithms within the framework of GLM. Hence, a meta-model is built for the mean and the dispersion jointly. The described method is applied on an academic example and proved efficient. Later it has been used for the engineering problem of interest. It can be said that considering the initial excitation of the vehicle by road unevenness of the bridge's approach has a limited effect on the identified parameters from the vehicle and bridge dynamics affecting the variances of the bridge's displacement. However, the speed has a prominent effect as higher speeds leads to higher amplifications in the bridge's displacements accompanied with higher variances caused mainly by the uncontrollable parameter of road unevenness, which has been qualitatively measured.

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