

# **TIME DOMAIN EVALUATION OF MULTILEVEL CONVERTERS VOLTAGE AND CURRENT QUALITY**

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**Submitted in fulfilment of the requirements  
for the degree of Masters of Science**



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# Declaration

I hereby, declare that this manuscript, entitled “Time Domain Evaluation of Multilevel Converters Voltage and Current Quality”, is the result of my own work except for quotations and citations, which have been duly acknowledged.

I also declare that, to the best of my knowledge and belief, it has not been previously or concurrently submitted, in whole or in part, for any other degree or diploma at Nazarbayev University or any other national or international institution.

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Signature: 

Date: 03/02/2017

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# List of Abbreviations

THD	Total Harmonic Distortion
SHE	Selective Harmonic Elimination
MLC	Multilevel Converter
PWM	Pulse Width Modulation
SVPWM	Space Vector PWM
NMS	Normalized Means Square
MPC	Model Predictive Control
NPC	Neutral Point Clamped
CHB	Cascaded H-Bridge



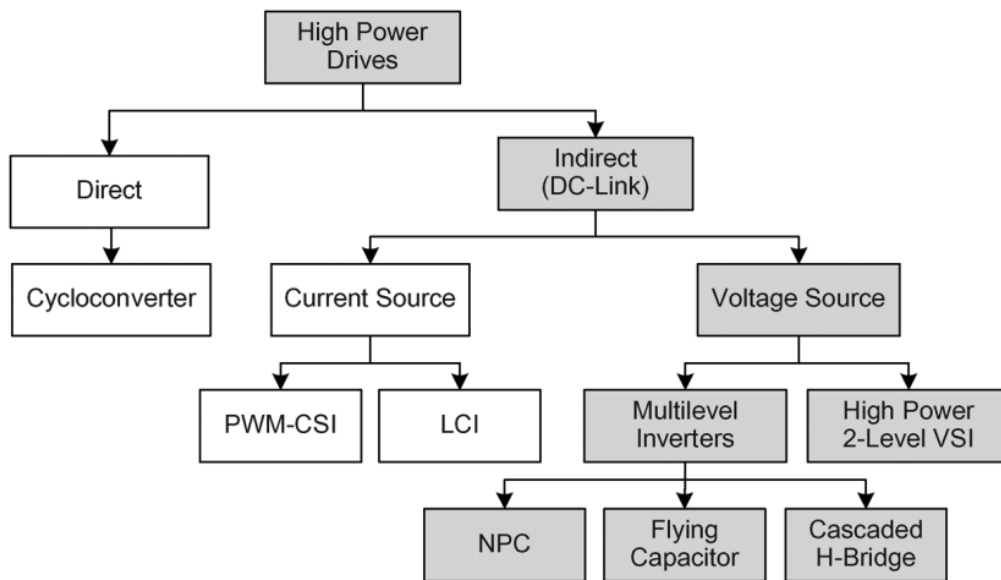
# Abstract

Multilevel converters are power conversion devices consisting of voltage sources and semiconductor switches. This work was motivated by the fact that majority of the conducted research estimates converter quality using frequency-domain approach, which requires tones of calculation. As an alternative for frequency-domain approach, time-domain evaluation method, developed in a recent time, was selected. This method brings novelty in deep-seated method of convertor evaluation, based on frequency-domain. General goal of this thesis is to demonstrate competency and applicability of time-domain optimization method on online adaptive inverter systems. Work will be focused on development of algorithm for calculation of optimal parameters for single phase CHB converter. Modulation optimization and local minimum finding algorithms are developed and discussed in this work. Moreover, combination of SHE and THD minimization techniques was designed and simulated. Obtained results coincide with those presented so far. However, majority of the results are novel, and has not presented to community yet. Future work will focus on development of the optimization algorithm for three-phase inverters.

# 1. Introduction

Converters play crucial role in power electronics devices used for power processing and control. Their importance is a result of power devices and systems diversity, as various systems have different voltage/current frequency, amplitude, etc [1]. As a result, power conversion is a necessary part of power processing. Due to upcoming development of smart grid systems, converters become an essential unit of load adaptive systems. For this purpose, fast and precise online processing is required, which is hardly achieved for present modulation methods.

Classification of converters is represented in Figure 1-1 [2].



*Figure 1-1. Classification of converters*

Modulation is control process of power electronic devices, which are part of power converters. Optimal algorithm MPC were developed in 1980s, and aimed to

minimize switching losses, optimize voltage/current waveform, decrease waveform distortion, etc [3]. Models vary in calculation processes, implementation on different hardware, etc. Majority of the techniques uses different assumptions and merits, e.g. current and torque distortion [4], and, as a result, comparison of these methods is hardly imaginable. Despite the progress in research during last years, agile optimal modulation technique is yet to be found in future.

## 2. Literature Review

Series of semiconductor switches connected to voltage DC sources is a key part of MLCs. Altogether, circuit makes power conversion by generating staircase voltage wave form. Maximum high voltage output is the sum of DC sources, whereas intermediate values depends only on sources connected to it [5].

Multilevel circuit topologies differ based on application. However, three are widely spread: Cascaded H-bridge (CHB), Neutral Point Clamped (NPC), and Flying Capacitor (FC) [6][7][8]. Although these topologies are popular, they have drawbacks. CHB requires large isolation transformer, that makes it useless in offshore and small scale systems. However, it is still useful for high power systems in industry [9]. NPC is inapplicable for higher-order multilevel topologies as it has large losses on diodes. Whereas FC requires high switching frequencies, which cannot be used at high power systems.

Optimization of modulation methods bases on different performance factors. Some of them are switching losses, stator current and electromagnetic torque distortion, switching frequency, THD [10]. Switching losses can be minimized by two ways, either minimize switching frequency, or minimize losses that occur as a result of switching. In [3] current and torque minimization under several modulation techniques was tested, the results are presented in Figure 2.

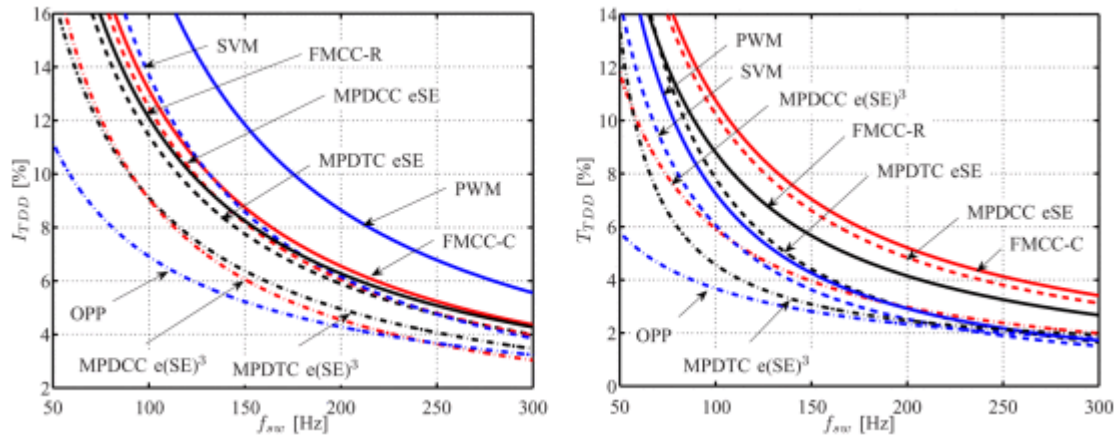


Figure 2-1. Stator current (left) and electromagnetic torque (right) distortion vs switching frequency[2]

Distortion in both cases converges at long switching horizon. Different methods can be used for different applications, and one of the factors can be sacrificed to enhance performance of another.

During last two decades engineers proposed several PWM methods for MLCs [11]. Methods of PMW control can be divided into three groups: SPWM, SHEPWM and triangular carrier PWM [12][13][14]. SPWM is easy to implement and control. However, it gives high output voltage THD. SHEPWM allows to obtain lower switching frequency, it helps to reduce voltage distortion, but compromises current one. SVPWM offers better waveform output and lower harmonic content.

All aforementioned modulation techniques use frequency spectra calculation for estimation of performance indicators. Majority of the conducted research uses frequency domain in evaluation of performance merits [15]. [16] demonstrates the

work, where crucial role is given to the idea that time averaging of a square waveform signal, influences on output signal quality. Similar results were obtained in [17], where calculation was based on infinite sum Fourier series.

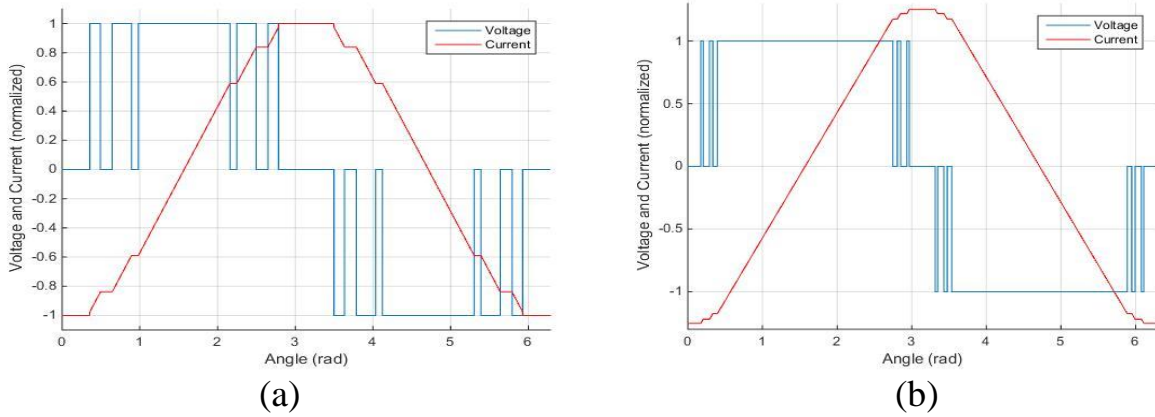
Time domain evaluation can become a fast computational technique that facilitate computation/calculation process that eliminates necessity of harmonic calculation. Altogether, swiftness and easiness of this method can be used for online adaptive control of power systems, which is very important due to development of smart grid systems and applications [18].

This master thesis work will focus on time-domain optimization techniques used for simple topologies, and simple modulations, e.g. staircase modulation. Results will be compared with existing works.

### 3. Methodology

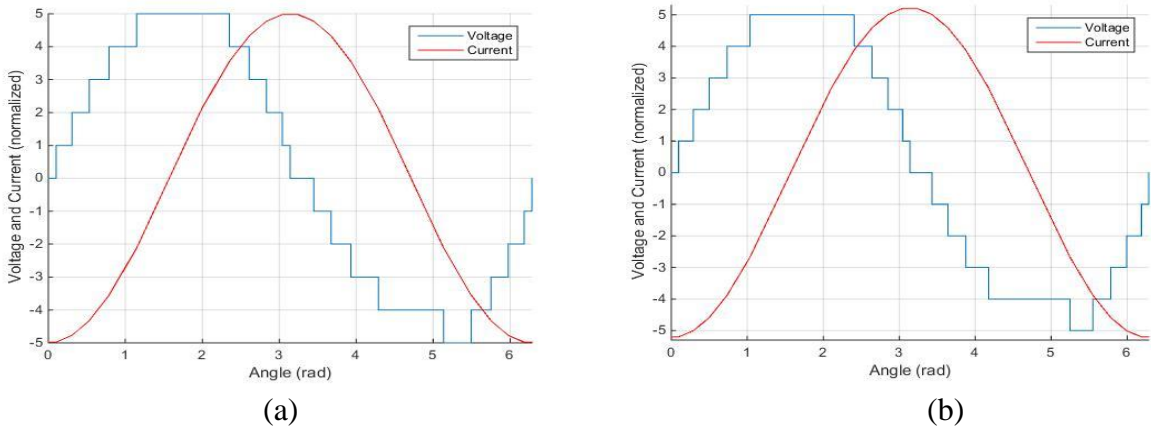
The method of optimization of voltage/current THD will be discussed further. Mainly, three major points are in the area of interest: 1. Optimization algorithm of arbitrary number of angles; 2. Determination of local and global minima of THD; 3. Optimization of THD and addition of SHE technique. For this purpose, *fmincon* MATLAB tool was used. The following single-phase inverters with inductance dominant RL-load will be considered: 2-level with multiple switching angles, and multilevel with staircase modulation. Load can be approximated as purely inductive to make time constant greater than switching intervals. The main idea behind developed algorithm is calculation of optimal switching angles at which normalized mean square error of the voltage or current waveform decreases, and as a result voltage/current distortion will be minimum. The main advantages of the time domain approach over the frequency domain is lower complexity of calculations, e.g. no necessity in knowledge of Fast Fourier Transform, and higher calculation accuracy. In frequency domain distortion is mainly calculated by the ratio of the sum of higher harmonics over fundamental frequency, and limited number of harmonics may be a potential source of inaccuracy. For 2-level, voltage/current were assigned to 0 and 1. In staircase modulation voltage/current switches from 0 to next until the last level is achieved, last level number equals number of angle plus one. Only non-negative was

considered at level number calculation. Figure 3-1 demonstrates current optimized voltage & current waveforms for 2-level modulation for different modulation indices. Modulation index change leads to change in optimal switching angles, and as a result shape of the waveform.



**Figure 3-1. Current optimized voltage & current waveform for 2-level modulation with 5 switching angles (a) modulation index = 1, (b) modulation index = 1.2**

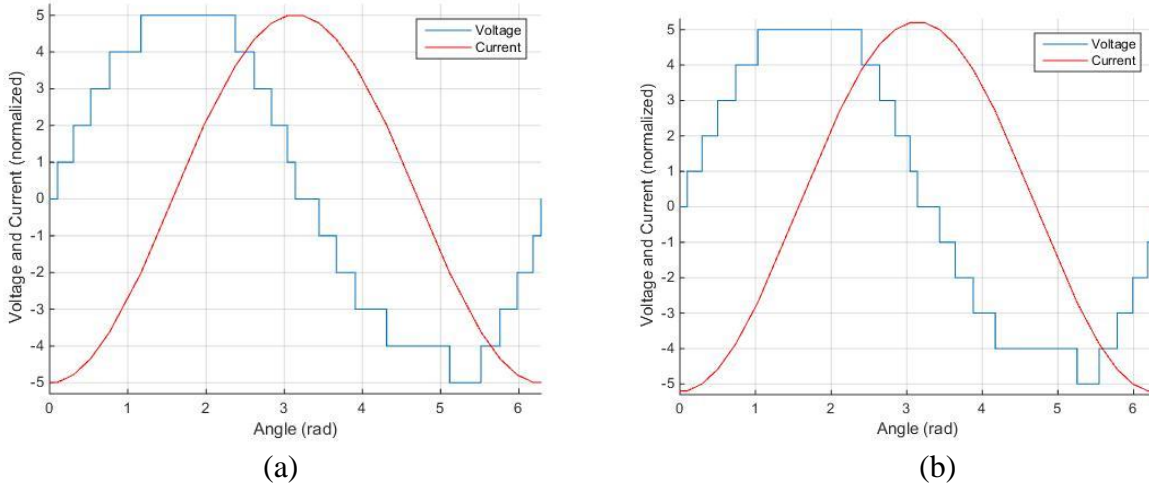
Figure 3-2 shows voltage optimized voltage and current waveforms for 6-level staircase modulation. It can be seen that the waveforms have more smooth shape compared to 2-level modulation.



**Figure 3-2. Voltage optimized voltage & current waveforms for 6-level staircase modulation (a) modulation index = 5, (b) modulation index = 5.2**



Figure 3-3 demonstrates voltage optimized voltage and current waveforms.



**Figure 3-3. Current optimized voltage & current waveforms for 6-level staircase modulation (a) modulation index = 5, (b) modulation index = 5.2**

Modulation index used was from  $0$  to  $4/\pi$  for 2-level, and from  $0$  to  $4n/\pi$  for staircase modulation, where  $n$  is the number of angles. Simulation step was chosen to be  $0.01$ . *fmincon* tool was used to solve systems of nonlinear equations under predefined constraints. In time domain, THD calculation is based on evaluation of normalized mean square error:

$$THD(m) = \frac{\sqrt{2 \cdot NMS}}{m} \cdot 100\% , \quad (1)$$

where *NMS* is normalized mean square error, and *m* is modulation index

*NMS* is calculated as Fourier series expansion over a period of  $2\pi$ . Using quarter-wave symmetry, calculation of *NMS* is done over  $\pi/2$  period:

$$NMS(m) = \frac{2}{\pi} \int_0^{\pi/2} v^2(t) dt - \frac{1}{2} m^2 \quad (2)$$

Voltage waveform is written as a piecewise function, squaring the waveform and integrating over  $\pi/2$  period allows to calculate normalized mean square error.

Voltage NMS calculation for staircase 6-level modulation is given below:

$$NMS(m) = 25 - \frac{2(\alpha_1 + 3\alpha_2 + 5\alpha_3 + 7\alpha_4 + 9\alpha_5)}{\pi} - \frac{m^2}{2} \quad (3)$$

Due to purely inductive load current waveform may be calculated as integral of the voltage waveform over the given period of time (4):

$$V = L \frac{dI}{dt} \quad (4)$$

Then NMS of the current waveform can be calculated by integration of weighted average value of the current waveform, which is demonstrated in (5), over the given period of time

$$\text{Waveform average} = \frac{I_1^2 + I_1 I_2 + I_2^2}{3} \quad (5)$$

where  $I_1$  – initial current value at given time interval,  $I_2$  – final current value

Optimization is done under a main constraint, which is a constraint function for the first harmonic, and is calculated through Fourier series. For 2-level modulation:

$$m = \frac{4}{\pi} \sum_{i=1}^n (-1)^{i+1} \cos(a_i) \quad (6)$$

For 2-level modulation with 5 switching angles equation (6) becomes:

$$m = \frac{4}{\pi} (\cos(\alpha_1) - \cos(\alpha_2) + \cos(\alpha_3) - \cos(\alpha_4) + \cos(\alpha_5)) \quad (7)$$

and for multilevel staircase modulation:

$$m = \frac{4}{\pi} \sum_{i=1}^n \cos(a_i) \quad (8)$$

For 6-level staircase modulation equation (8) becomes:

$$m = \frac{4}{\pi} (\cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3) + \cos(\alpha_4) + \cos(\alpha_5)) \quad (9)$$

Angles constraints used at both 2-level and multilevel staircase modulation is straightforward one, the following switching angle must be greater than previous:

$$\alpha_1 < \alpha_2 < \alpha_3 < \dots < \alpha_n \quad (10)$$

For staircase modulation there is a different constraint that does not allow signal to be overmodulated:

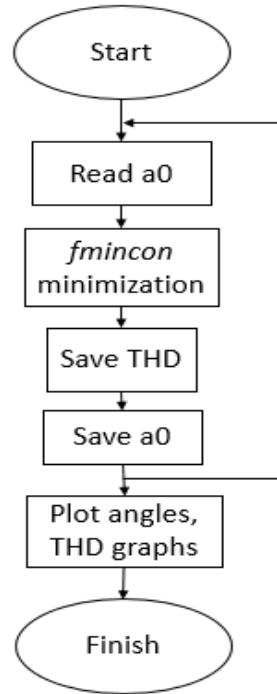
$$m \sin(\alpha_n) < n \quad (11)$$

From (9) following constraints are developed:

$$0 < \alpha_1 < a \sin\left(\frac{1}{m}\right) < \alpha_2 < a \sin\left(\frac{2}{m}\right) < \alpha_3 < \dots < a \sin\left(\frac{n-1}{m}\right) < \alpha_n < \frac{\pi}{2} \quad (12)$$

Optimization is based on initial preassigned values of switching angles. After optimization is done for one loop, obtained values are saved and used as initial

conditions for the next iteration. General algorithm of optimization process is represented on the flowchart below:



*Figure 3-4. Algorithm flowchart*

Note, that optimal angles are saved after every iteration and used as initial conditions for next iteration. Initial conditions for first loop are set manually.

The last step is introduction of SHE algorithm. It is done by adding additional constraints for odd harmonics. Equations make sum of all harmonics of the same order equal to zero.

$$0 = \sum_{i=1}^n \cos(k \cdot \alpha_i), \quad (12)$$

where  $k$  is a number of odd harmonics  $k=3,5,7,\dots$

The result range of modulation index decreases as additional SHE constraints are added. Thus solutions do not exist at the whole range at which optimization was defined.

## 4. Results

Further simulation results methods described in previous chapter will be demonstrated. Time-domain optimization method for single-phase inverter with 2-level and staircase multilevel modulation for several switching angles will be shown.

### 2-level modulation

For 2-level modulation with multiple switching angles optimization algorithm was used for current only, because at voltage optimization, addition of switching angles introduces new harmonics, and as voltage is very sensitive to it, optimization is hardly achievable.

#### Current optimization

Current optimization with 2-level modulation for 5-7 angles, and SHE implementation will be shown. Optimal current switching angles and THD graphs:

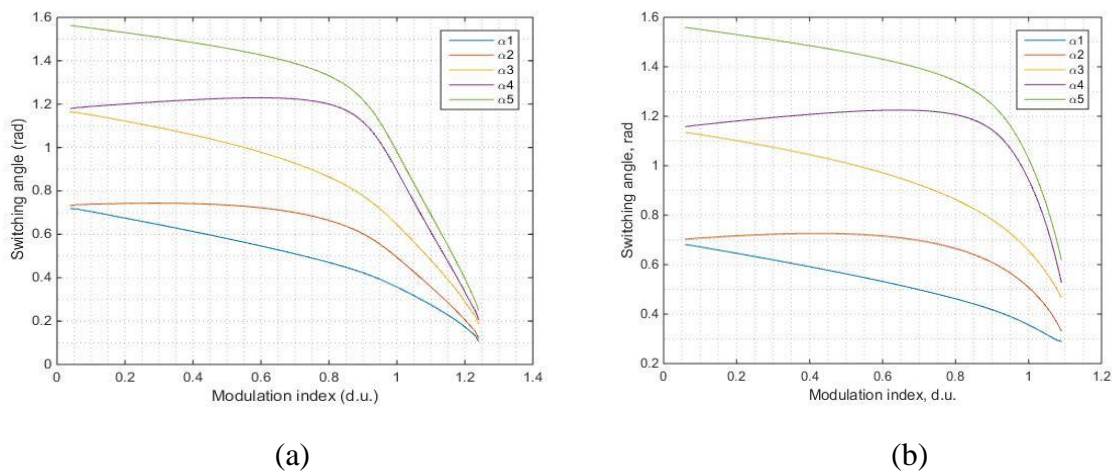
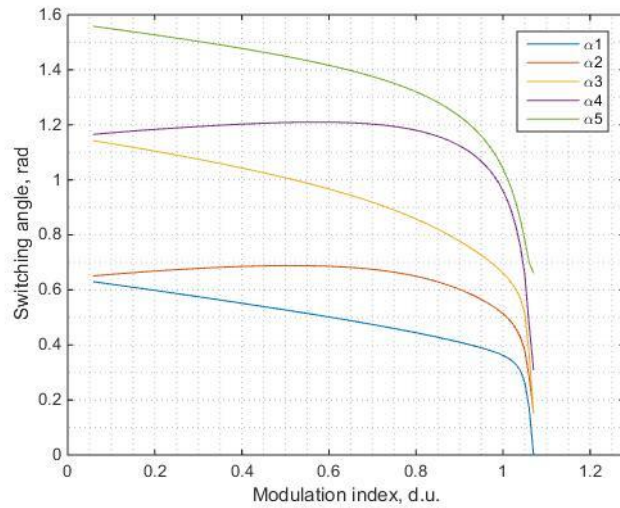
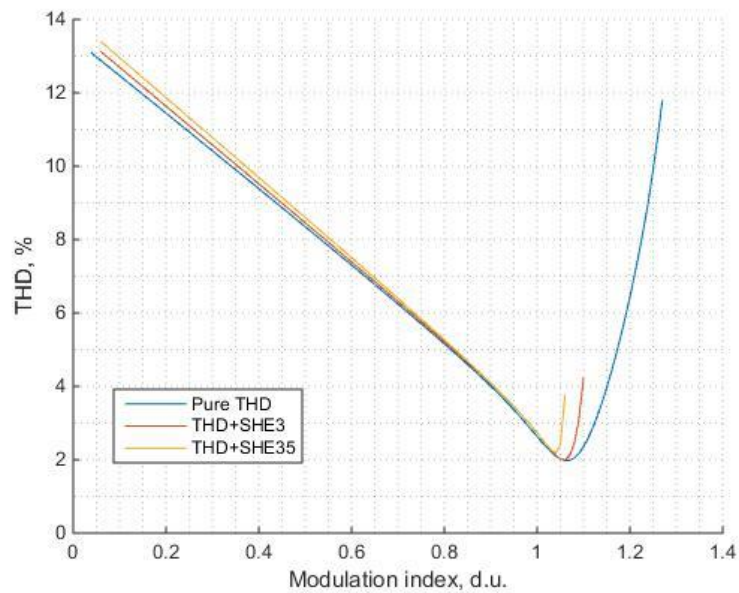


Figure 4-1. 2-level, 5-angles, optimal current switching angles (a) pure THD, (b) SHE3



*Figure 4-2. 2-level, 5-angles, optimal current switching angles for SHE 3,5*



*Figure 4-3. 2-level, 5-angle, current THD*

As can be noticed, addition of harmonics constraints (SHE), shrinks modulation range. Shrinking is small if small number of harmonics is eliminated, but further harmonic elimination leads to serious shrinking of modulation index

range. On Figure 4-3. it can be seen that solutions for SHE 3,5,7 and SHE 3,5,7,9 were not found, although analytically they should be achievable.

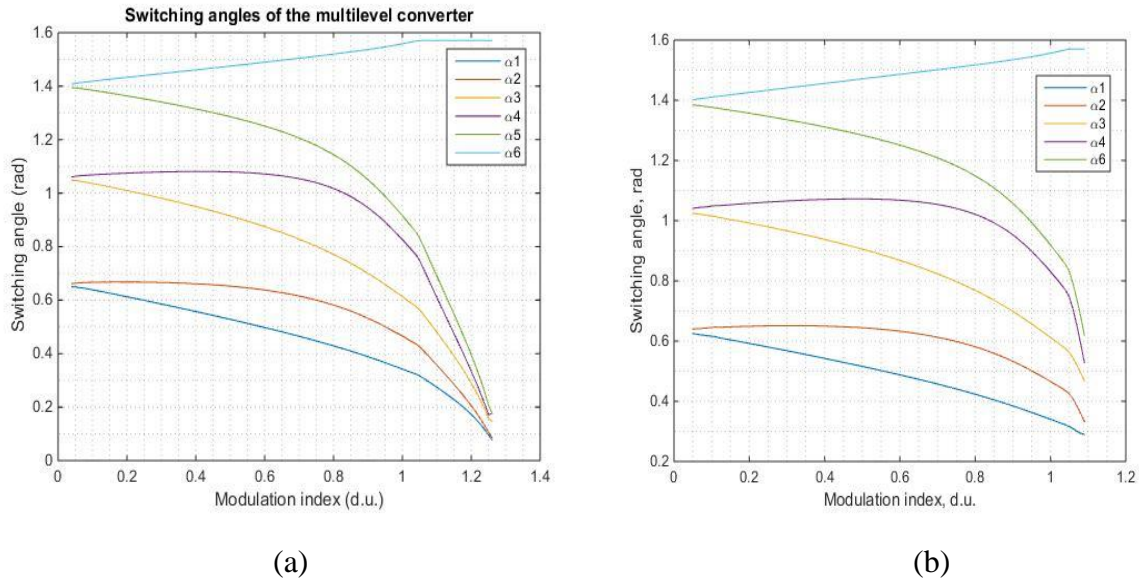


Figure 4-4. 2-level, 6-angle, optimal current switching angles (a) pure THD, (b) SHE 3

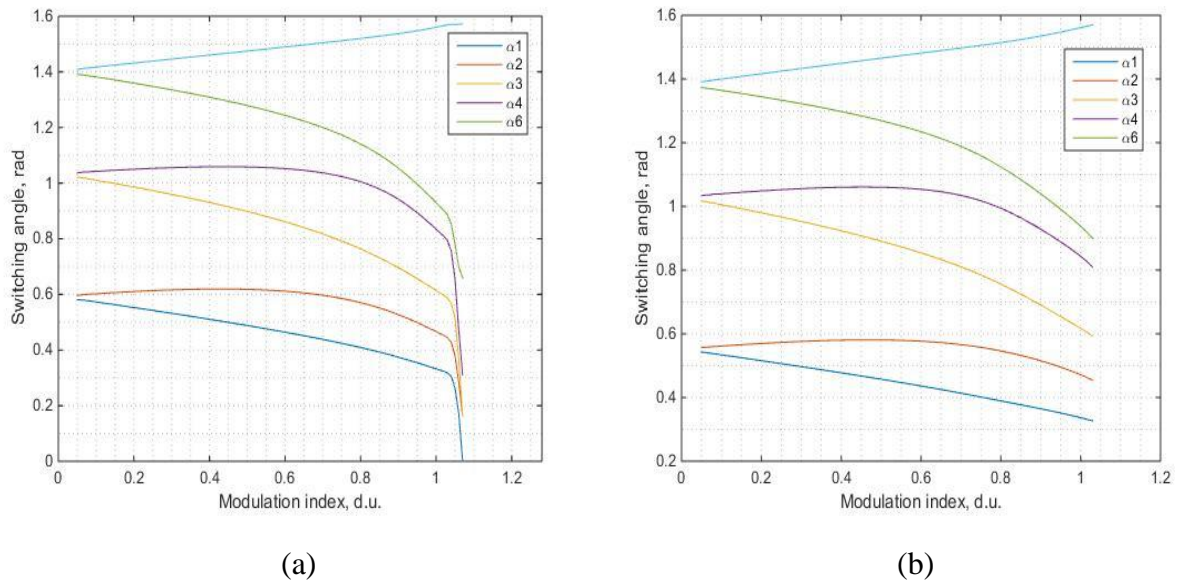
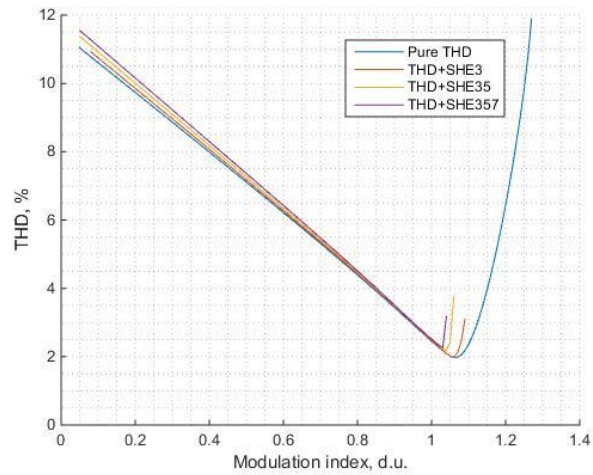
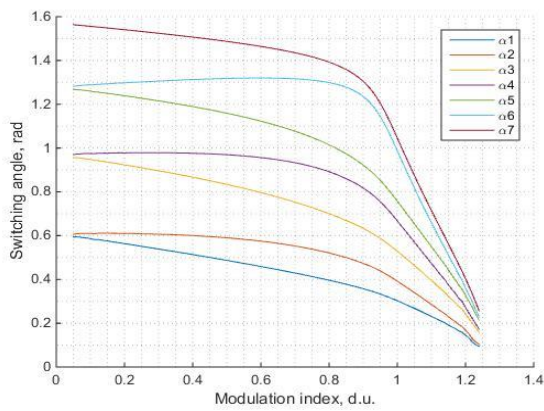


Figure 4-5. 2-level, 6-angle, optimal current switching angles (a) SHE 3,5, (b) SHE 3,5,7

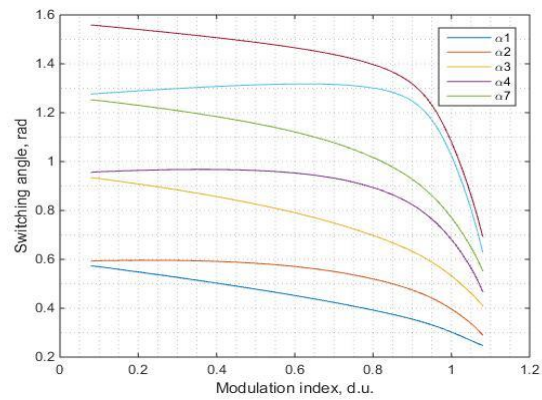




**Figure 4-6. 2-level modulation, 6-angle, current THD**

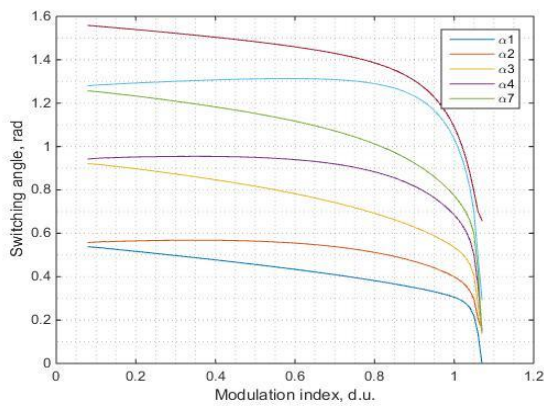


(a)

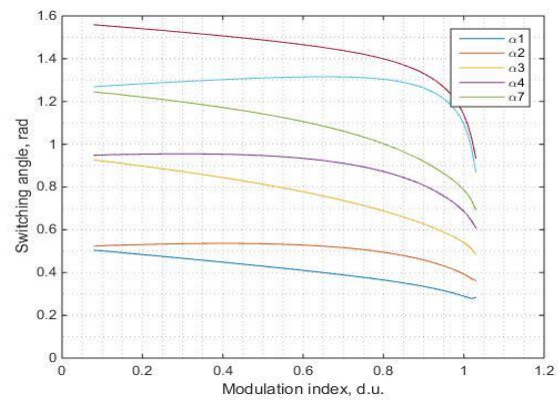


(b)

**Figure 4-7. 2-level, 7-angle, optimal current switching angles: (a) pure THD, (b) SHE 3**

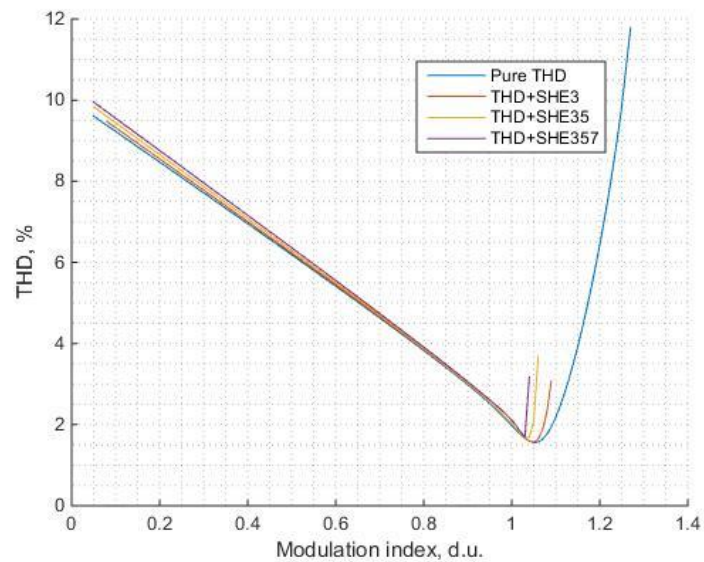


(a)



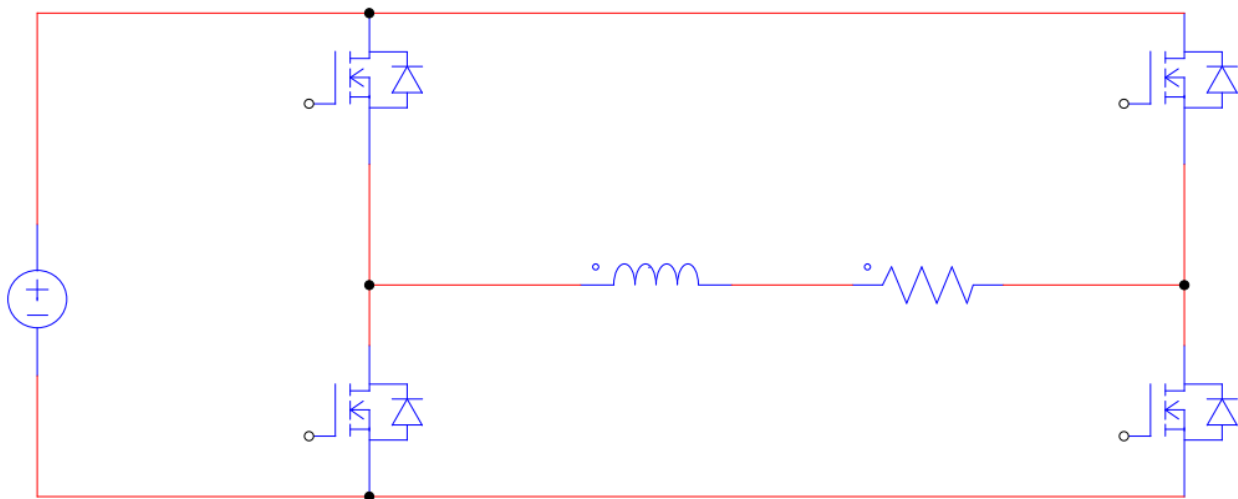
(b)

**Figure 4-8. 2-level, 7-angle, optimal current switching angles: (a) SHE 3,5, (b) SHE 3,5,7**



*Figure 4-9. 2-level, 7-angle current THD*

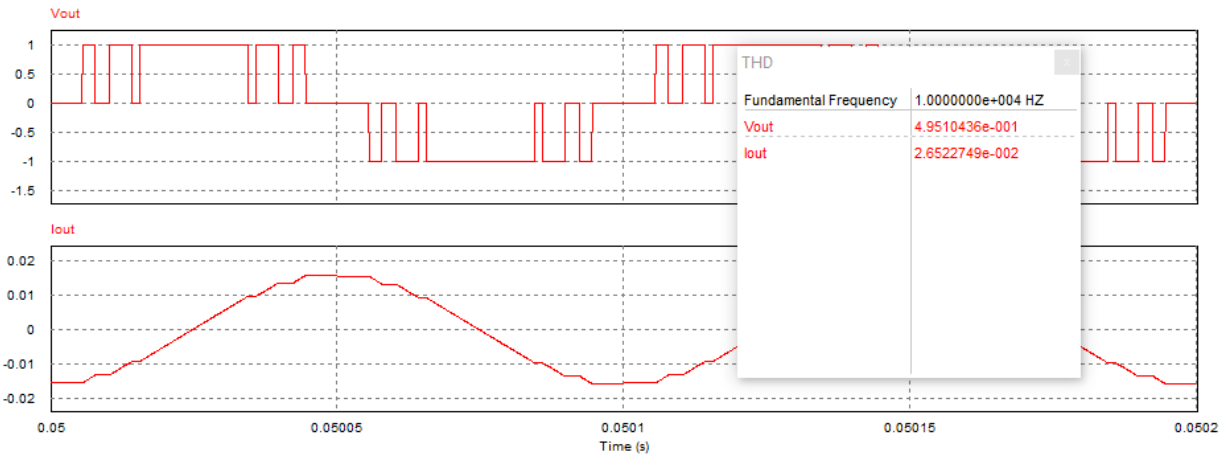
Figure 4-10 shows schematic diagram of single phase 2-level (non-negative levels considered only).



*Figure 4-10. 2-level, single phase converter schematic*

Figure 4-11 shows PSIM simulation results for 2-level 5 angle modulation with modulation index of 1; as can be seen, the waveform resembles that showed in Figure 3-1(a). Current THD calculated during simulation is approximately equal

to 2.65%, that matches with theoretically calculated value for pure THD, showed in Figure 4-3.



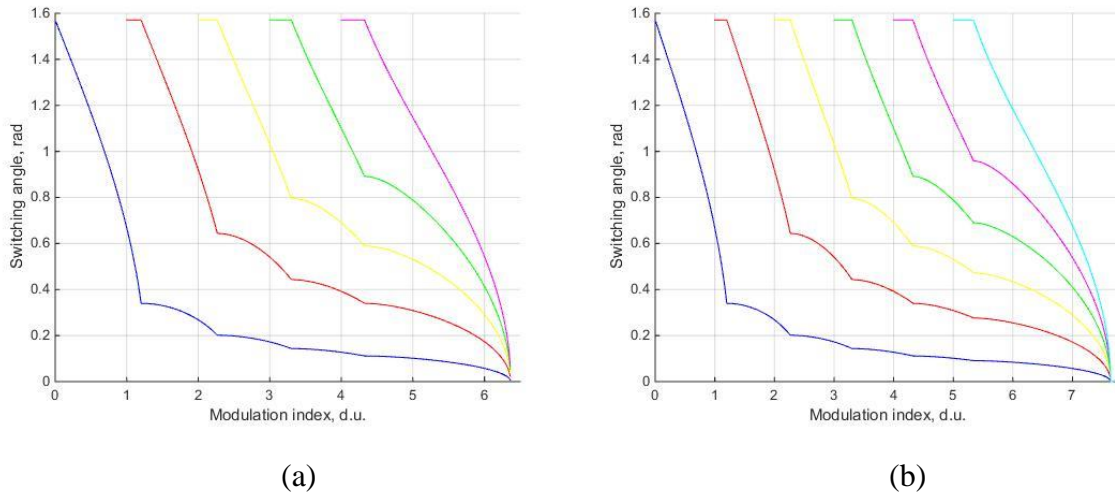
*Figure 4-11. 2-level, 5-angle current optimized voltage & current waveforms, modulation index = 1*

## 4.2. Staircase modulation

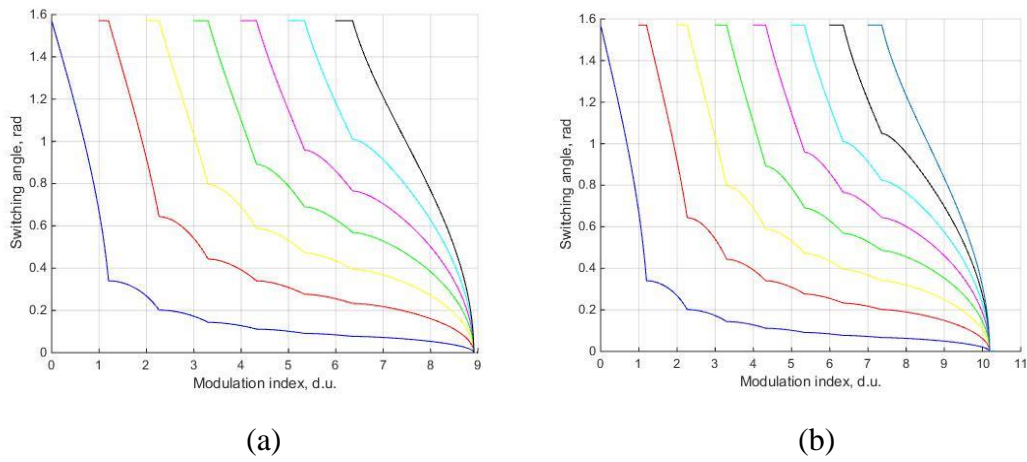
### 4.2.1. Voltage optimization

Results of voltage optimization algorithm for 5-9 angles (6-10 levels).

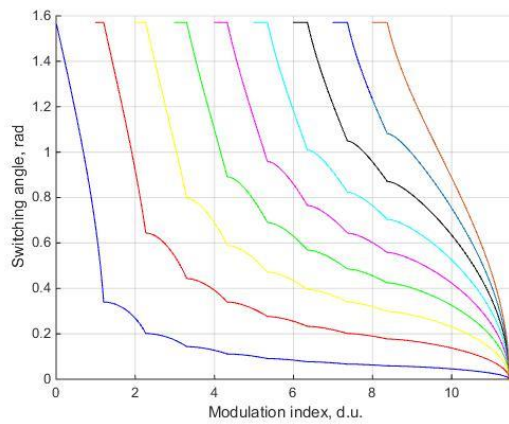
Optimal voltage switching angle for multilevel staircase modulation and THD graph are demonstrated next.



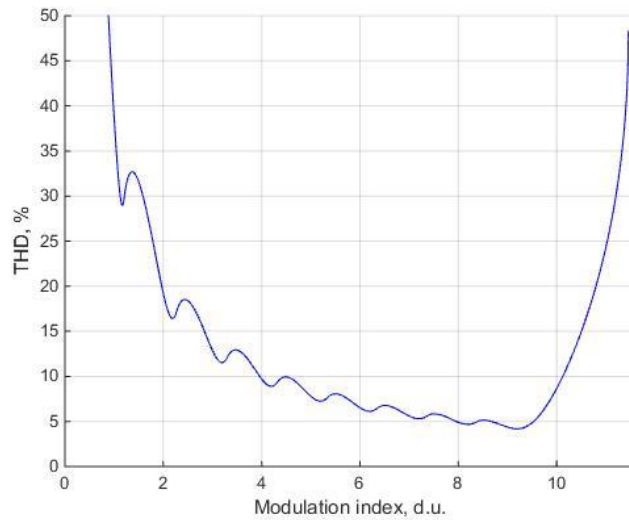
**Figure 4-12. Optimal voltage angles for staircase modulations: (a) 6-level, (b) 7-level**



**Figure 4-13. Optimal voltage angles for staircase modulations: (a) 8-level, (b) 9-level**



**Figure 4-14. Optimal voltage angles for 10 level staircase modulations**



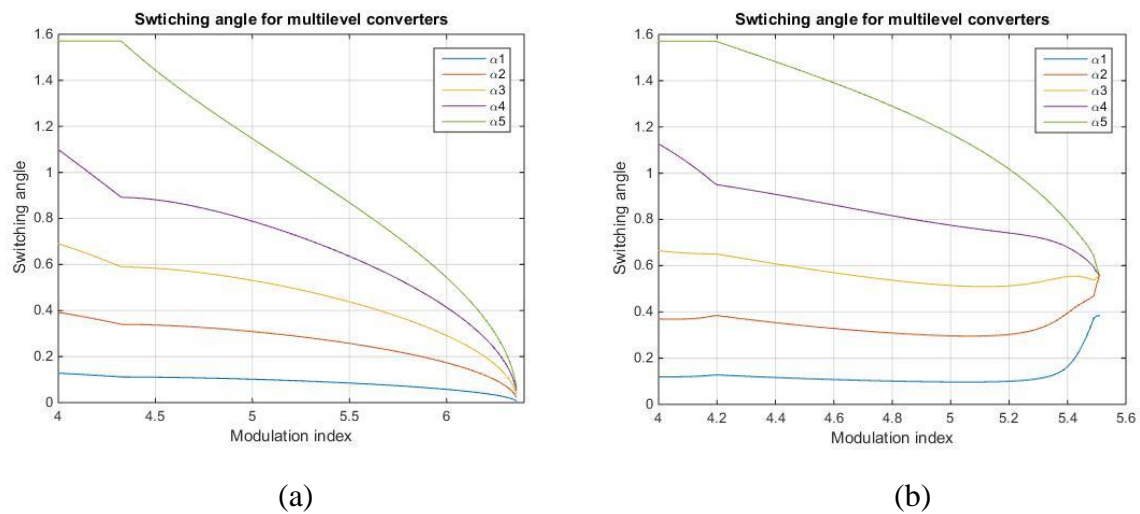
*Figure 4-15. 10-level, optimal voltage THD*

From Figures 4-10:12 it can be seen that switching angles are similar and distributed across whole defined modulation range. Figure 4.13 demonstrates optimal voltage THD for 10-level modulation. THD curve experiences exponential decay with fluctuations. Local and global minima with corresponding modulation indices, and switching angles are shown in Table 1.

*Table 1. Voltage THD minima (angles represented in radians)*

Modulation index	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	$\alpha_8$	$\alpha_9$	THD <sub>v</sub> (%)
1.17	0.4055	-	-	-	-	-	-	-	-	28.96
2.19	0.2242	0.7302	-	-	-	-	-	-	-	16.42
3.2	0.1546	0.4801	0.8786	-	-	-	-	-	-	11.53
4.2	0.1183	0.362	0.6313	0.9725	-	-	-	-	-	8.903
5.2	0.09576	0.2909	0.4985	0.7333	1.036	-	-	-	-	7.257
6.19	0.08061	0.244	0.4144	0.5988	0.8106	1.088	-	-	-	6.129
7.19	0.06943	0.2097	0.3543	0.5071	0.6744	0.8682	1.124	-	-	5.306
8.2	0.06086	0.1835	0.309	0.4398	0.5793	0.733	0.912	1.149	-	4.679
9.19	0.05435	0.1637	0.2751	0.3901	0.5108	0.6405	0.7841	0.9525	1.177	4.186

Results of THD minimization combined with SHE optimization algorithm are presented next. To the author's best knowledge, results of THD minimization combined with SHE was used for up to 5 angles [16]. Elimination of odd harmonics (up to 9<sup>th</sup> harmonic for 5 angles) was also demonstrated. Further, results for 5-9 angles (6-10 levels) and graphs of switching angles and THD are demonstrated. Graphs show shrinking of modulation range, while harmonics are eliminated. Note that for several levels, solutions for THD combined with SHE were not found for large number of eliminated harmonics.



**Figure 4-16. 6-level, optimal voltage angles: (a) pure THD, (b) SHE3**

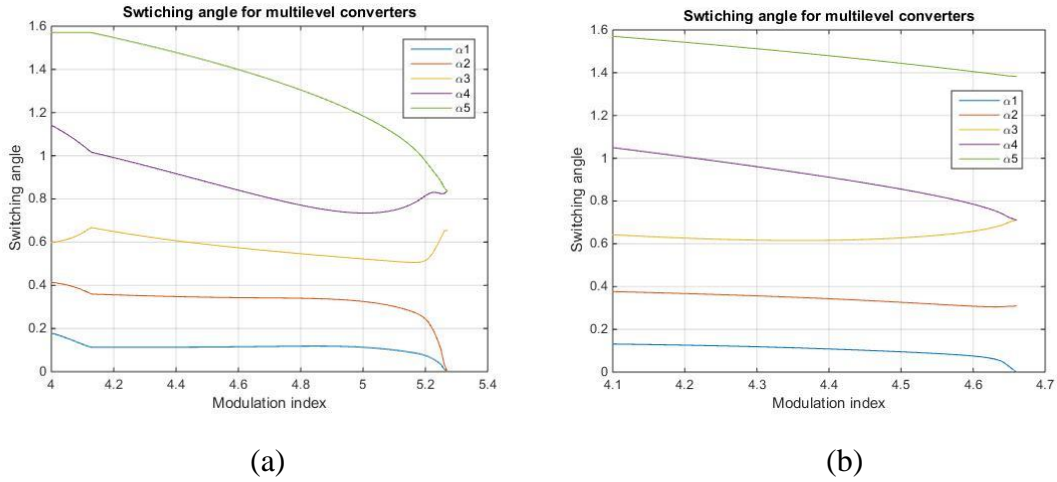


Figure 4-17. 6-level, optimal voltage angles: (a) pure SHE 3,5, (b) SHE 3,5,7

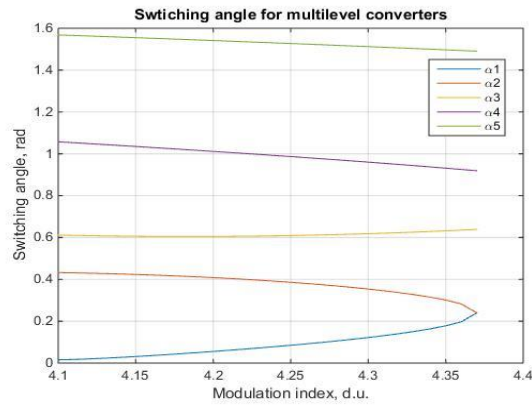


Figure 4-18. 6-level, optimal voltage angles for SHE 3,5,7,9

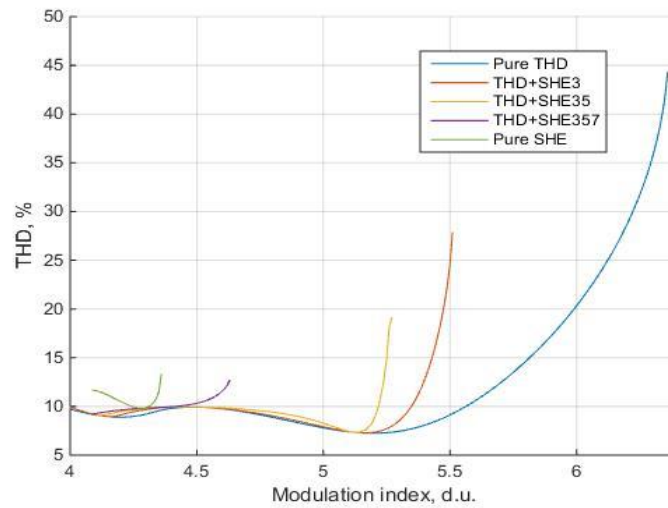
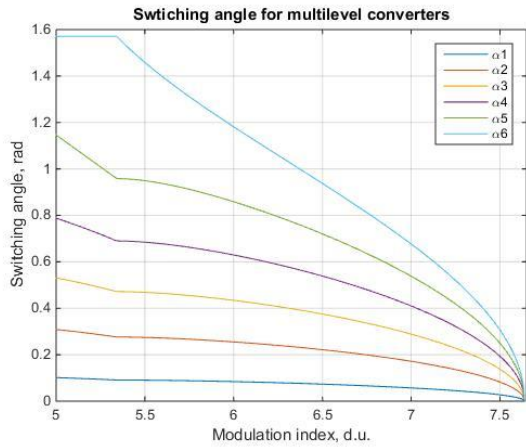
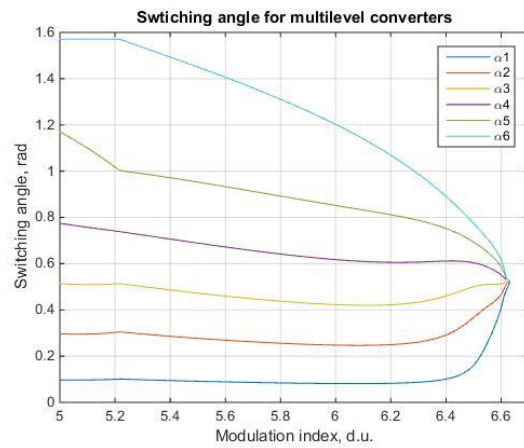


Figure 4-19. 6-level, optimal voltage THD

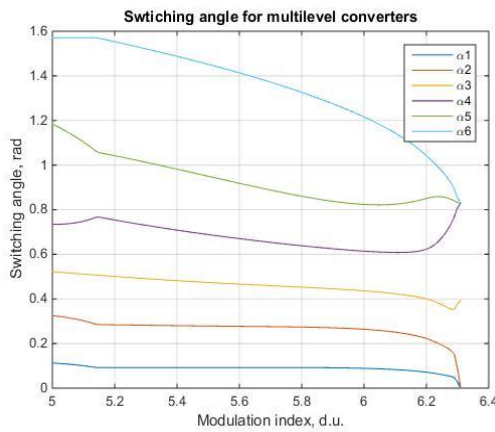


(a)

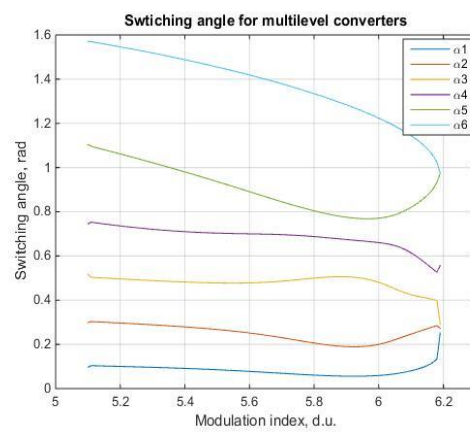


(b)

Figure 4-20. 7-level, optimal voltage angles: (a) pure THD, (b) SHE 3

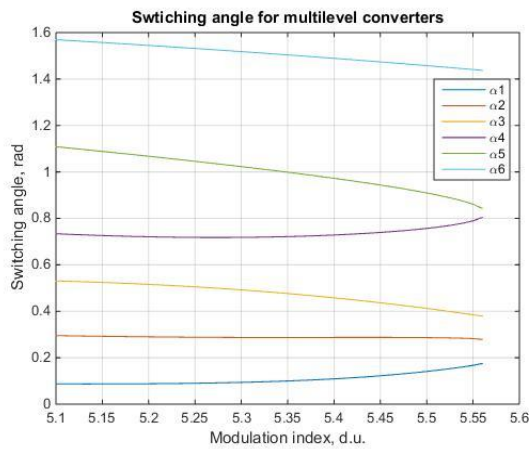


(a)

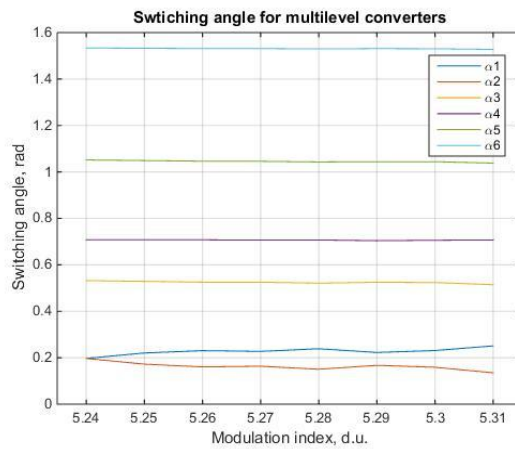


(b)

Figure 4-21. 7-level, optimal voltage angles: (a) SHE 3,5, (b) SHE 3,5,7



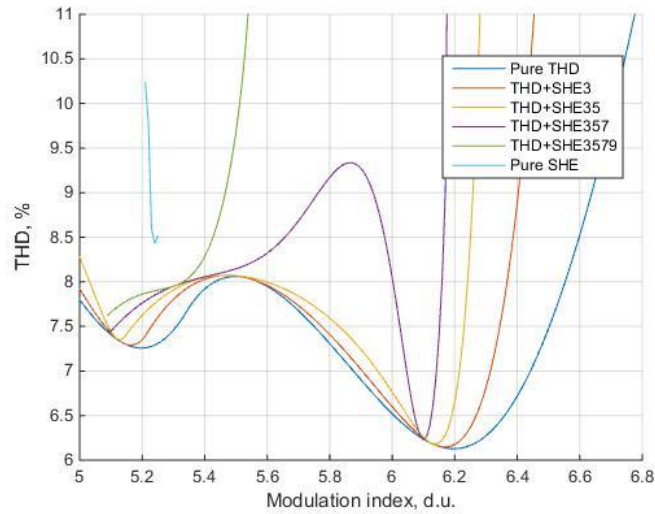
(a)



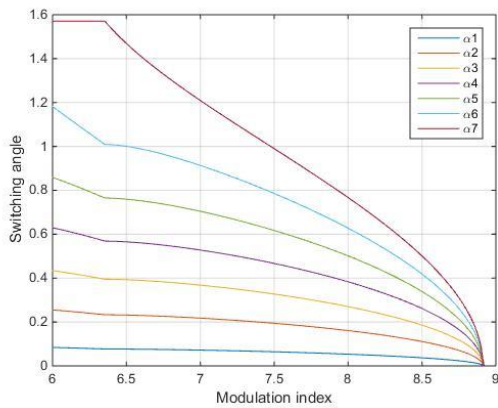
(b)

Figure 4-22. 7-level, optimal voltage angles: (a) SHE 3,5,7,9, (b) pure SHE

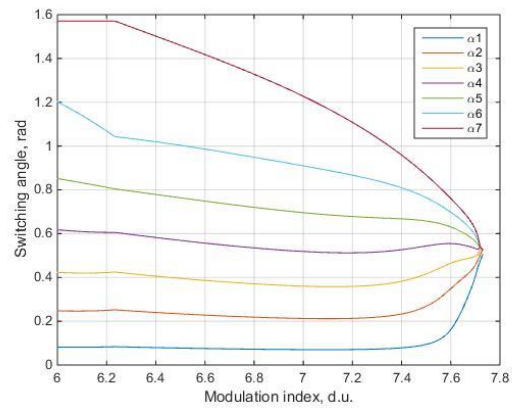




**Figure 4-23. 7-level, optimal voltage THD**

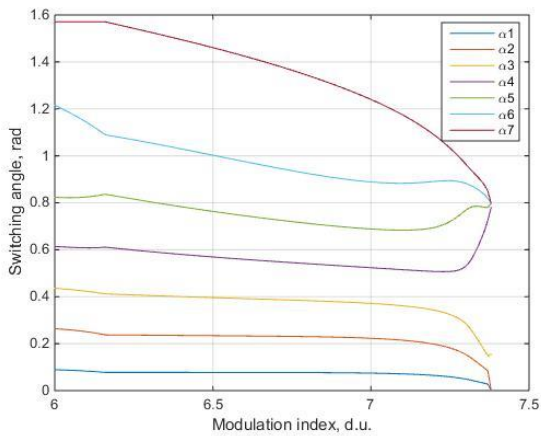


**(a)**

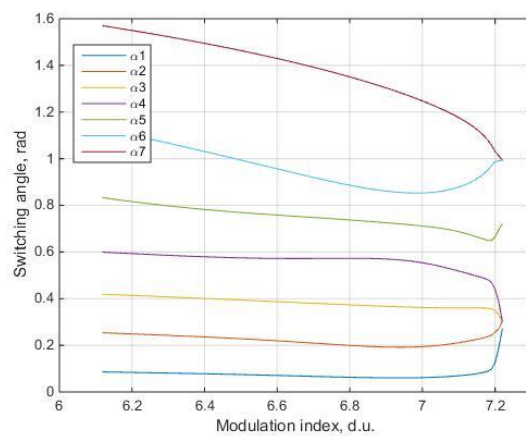


**(b)**

**Figure 4-24. 8-level, optimal voltage angles: (a) pure THD, (b) SHE 3**



**(a)**



**(b)**

**Figure 4-25. 8-level, optimal voltage angles: (a) SHE 3,5, (b) SHE 3,5,7**

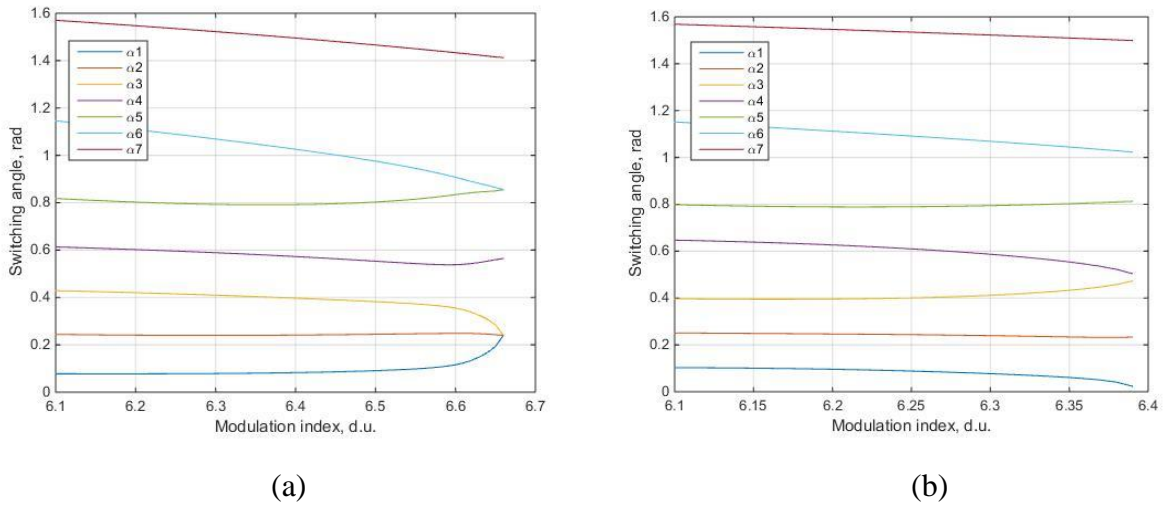


Figure 4-26. 8-level, optimal voltage angles: (a) SHE 3,5, (b) SHE 3,5,7

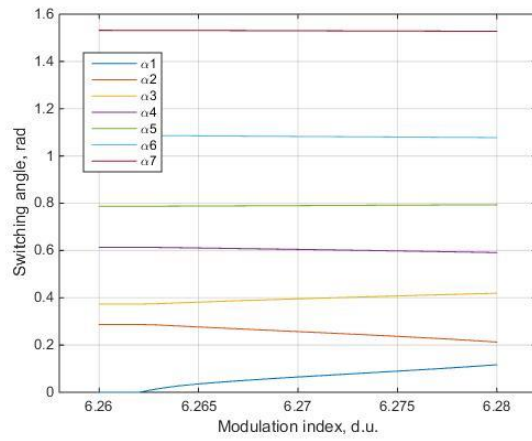


Figure 4-27. 8-level, optimal voltage angles for pure SHE

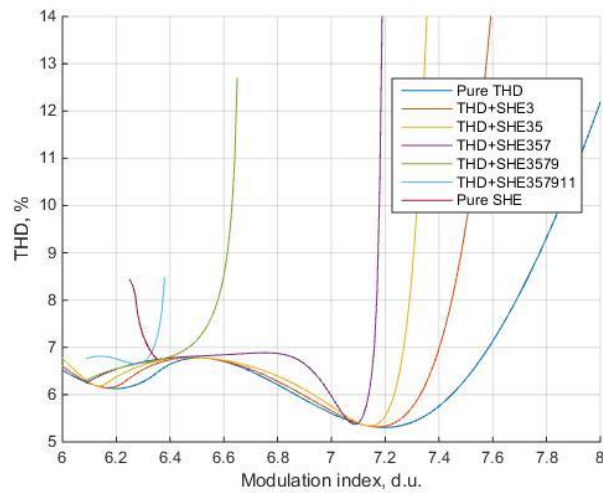
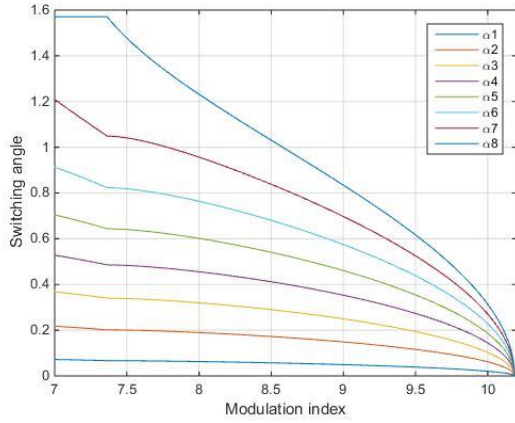
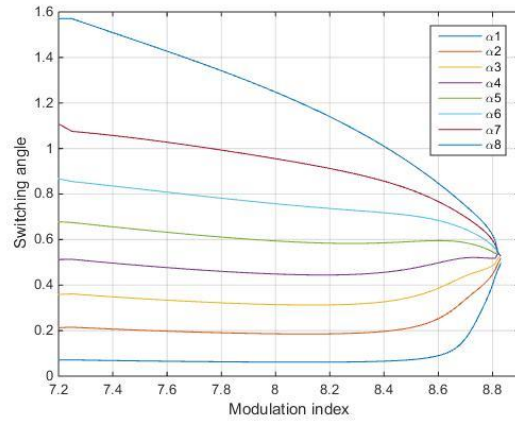


Figure 4-28. 8-level, optimal voltage THD

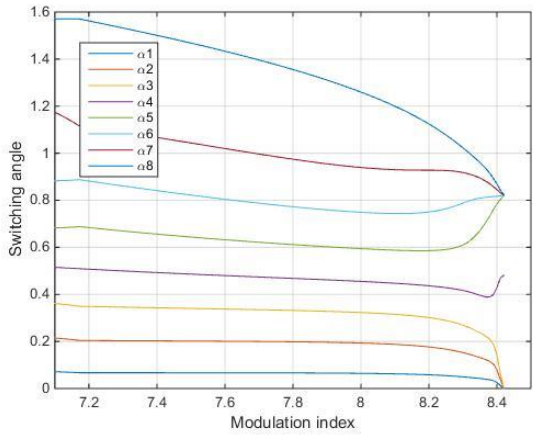


(a)

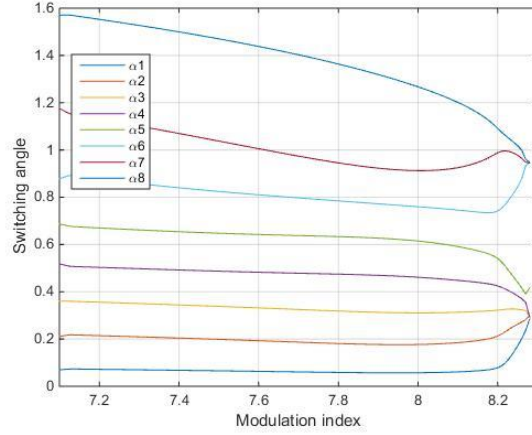


(b)

**Figure 4-29. 9-level, optimal voltage angles: (a) pure THD, (b) SHE 3**

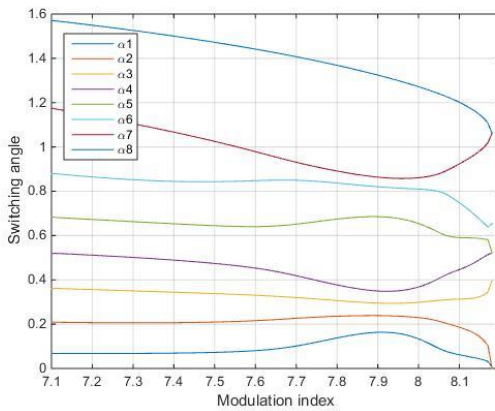


(a)

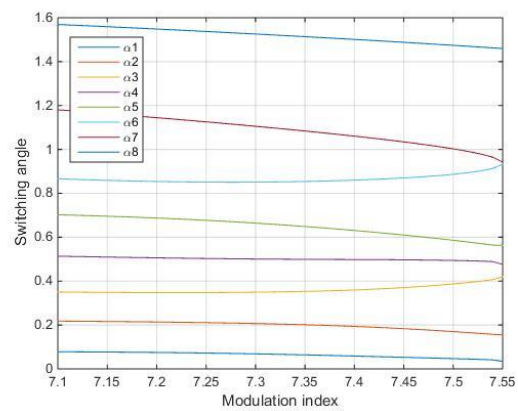


(b)

**Figure 4-30. 9-level, optimal voltage angles: (a) SHE 3,5, (b) SHE 3,5,7**



(a)



(b)

**Figure 4-31. 9-level, optimal voltage angles: (a) SHE 3,5,7,9, (b) SHE 3,5,7,9,11**

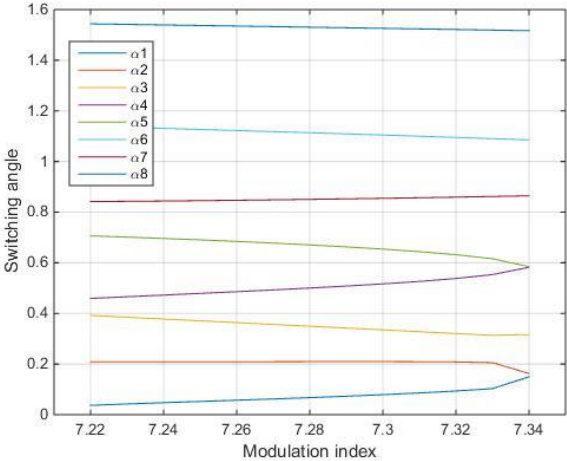


Figure 4-32. 9-level, optimal voltage angles for SHE 3,5,7,9,11,13

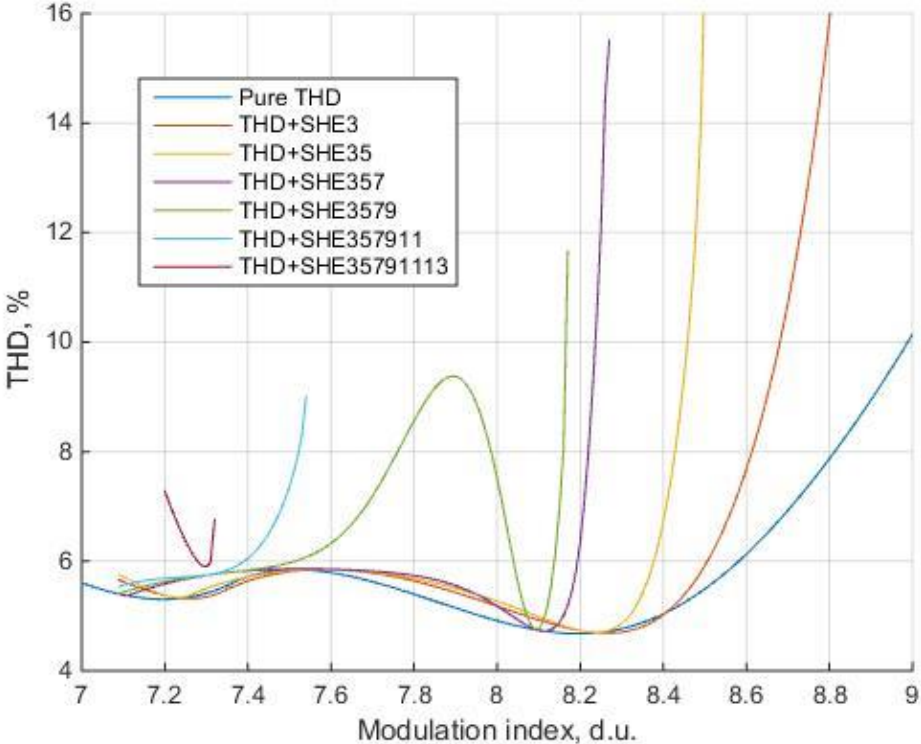
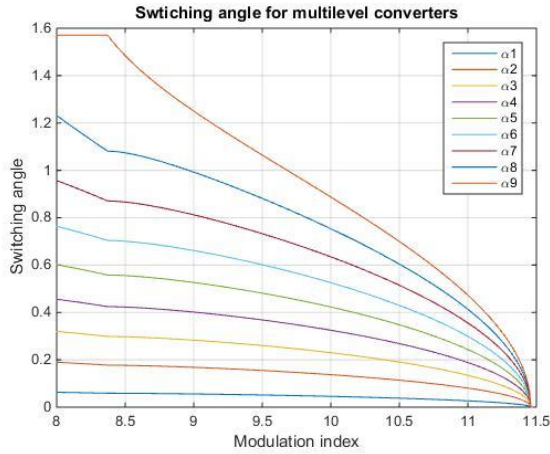
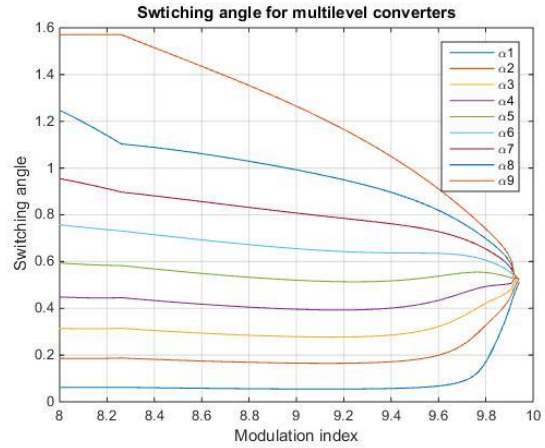


Figure 4-33. 9-level, optimal voltage THD

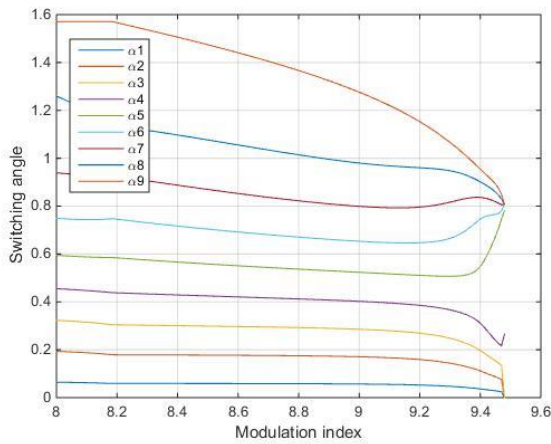


(a)

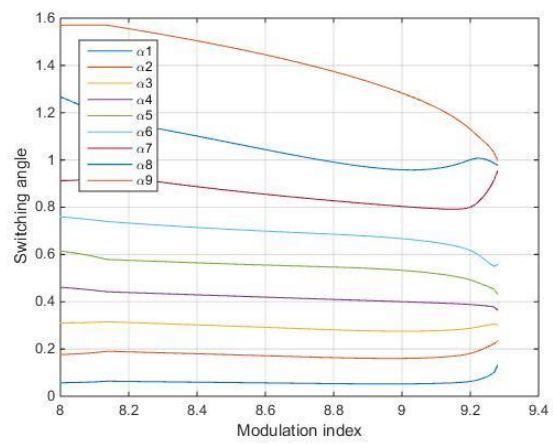


(b)

Figure 4-34. 10-level, optimal voltage angles: (a) pure THD, (b) SHE 3

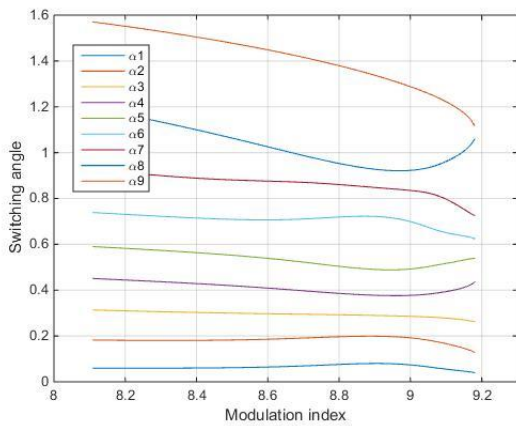


(a)

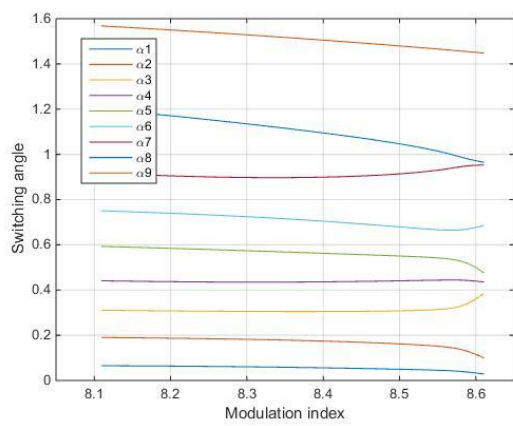


(b)

Figure 4-35. 10-level, optimal voltage angles: (a) SHE 3,5, (b) SHE 3,5,7

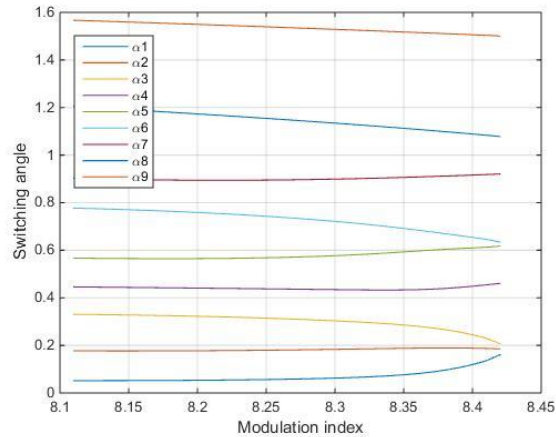


(a)

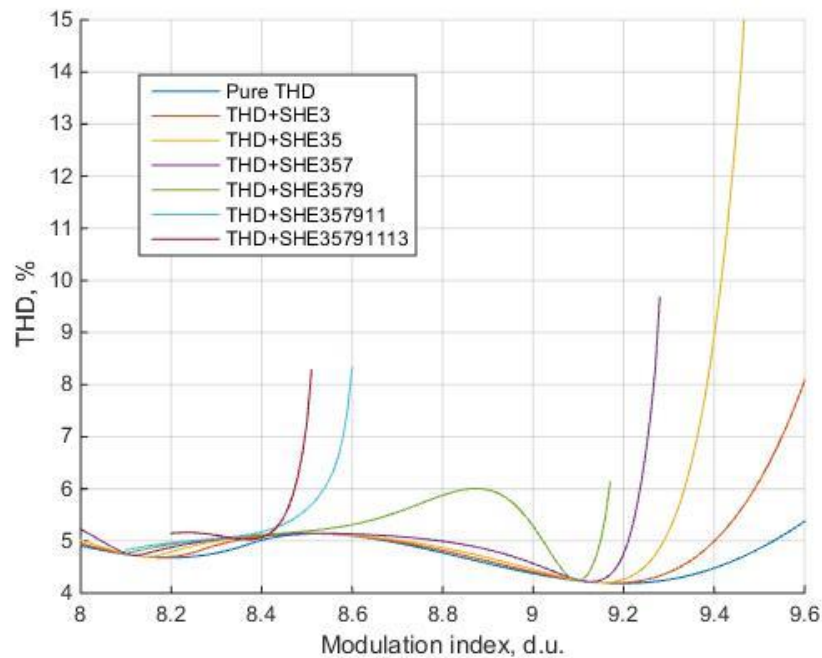


(b)

Figure 4-36. 10-level, optimal voltage angles: (a) SHE 3,5,7,9, (b) SHE 3,5,7,9,11



*Figure 4-37. 10-level, optimal voltage angles for SHE 3,5,7,9,11,13*



*Figure 4-38. 10-level, optimal voltage THD*

Figure 4-39 demonstrates 6-level (non-negative levels considered only) single phase inverter schematic. Figure 4-40 shows voltage-optimized voltage and current waveforms, and THD value with modulation index of  $m = 5$ . The value for voltage THD equals 8.47%, and matches to theoretically calculated voltage THD value showed in Figure 4-15.

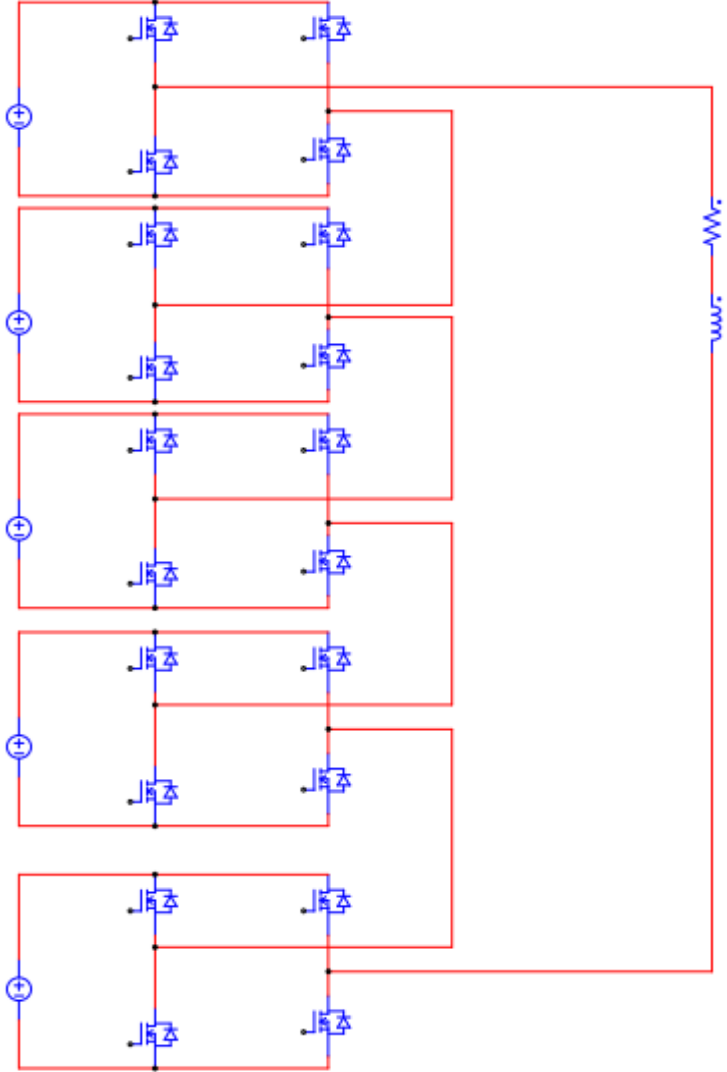


Figure 4-39. 6-level, single phase inverter schematic

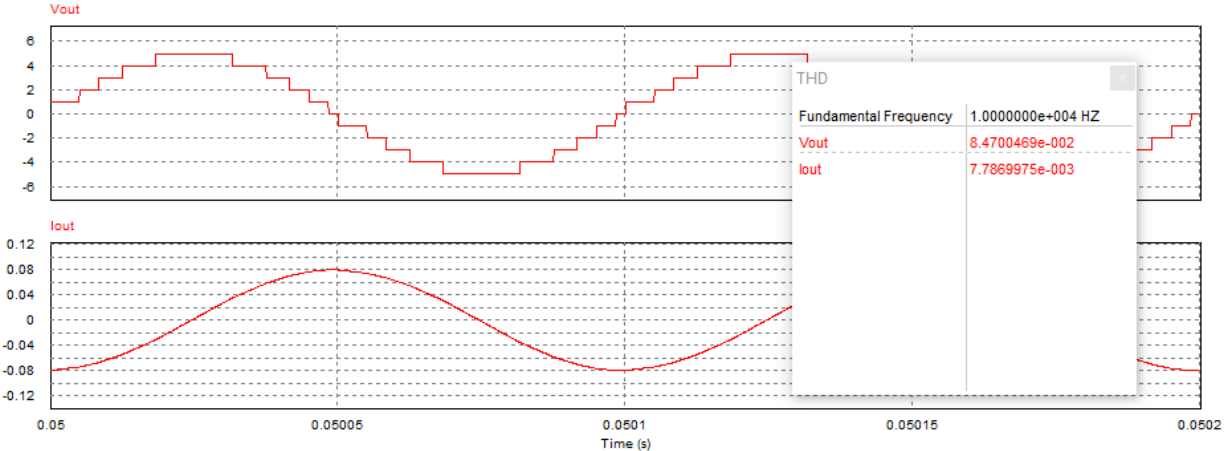
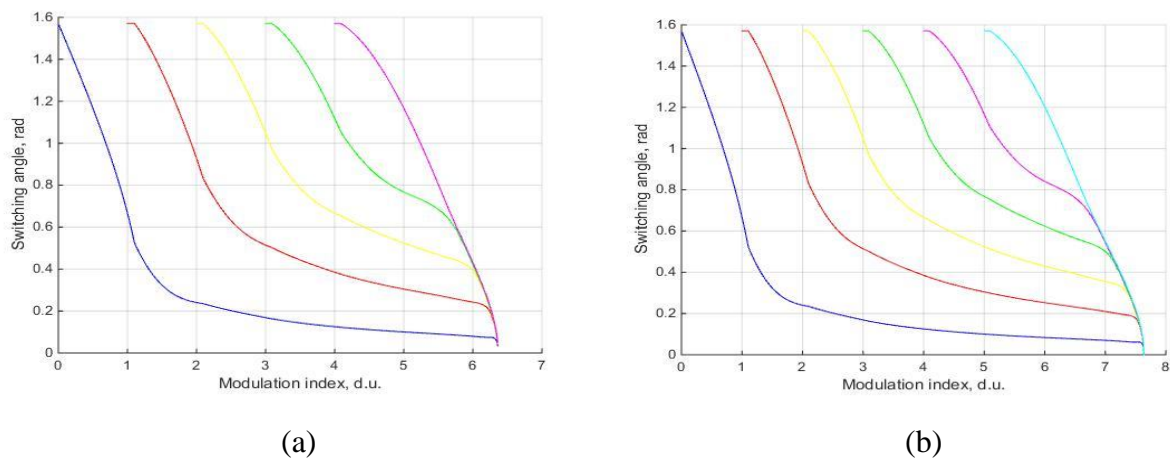


Figure 4-40. Voltage optimized 6-level, voltage & current waveforms, modulation index = 5

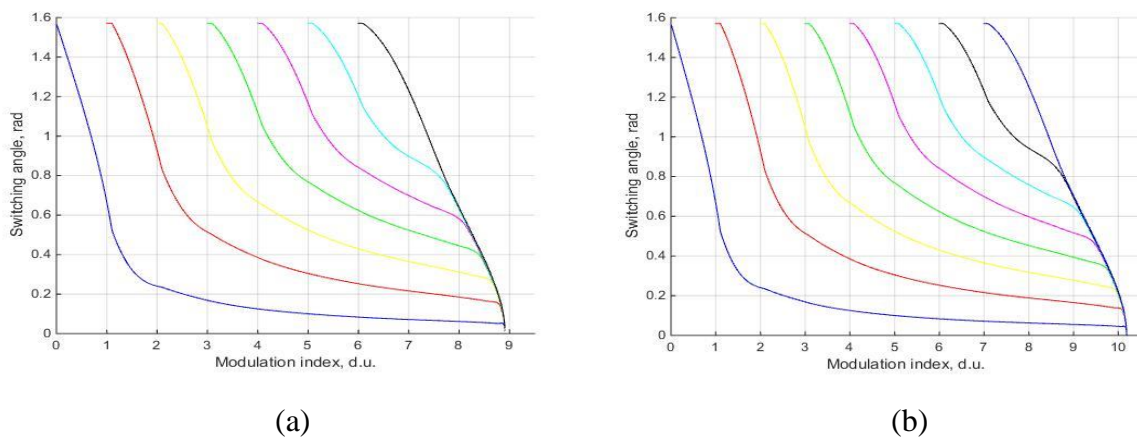
Optimal voltage and current switching angles are approximately equal for multilevel staircase modulation. Thus, current THD calculated value from Figure 4-40 (THD = 0.77%), can be used for comparison. It is clear that it matches with current THD predicted value showed in Figure 4-44, at  $m = 5$ .

Further, the same types of results described for voltage, are demonstrated for current optimization algorithm.

## Current



**Figure 4-41. Optimal current angles staircase modulations: (a) 6-level, (b) 7-level**



**Figure 4-42. Optimal current angles staircase modulations: (a) 8-level, (b) 9-level**



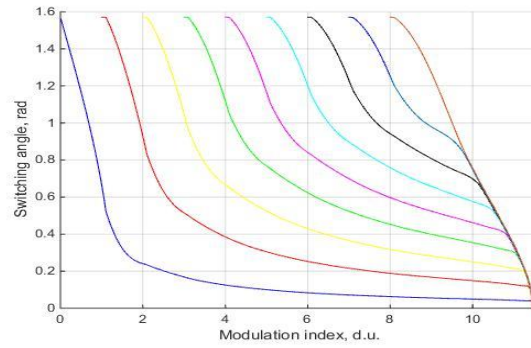


Figure 4-43. Optimal current angles for 10 level staircase modulation

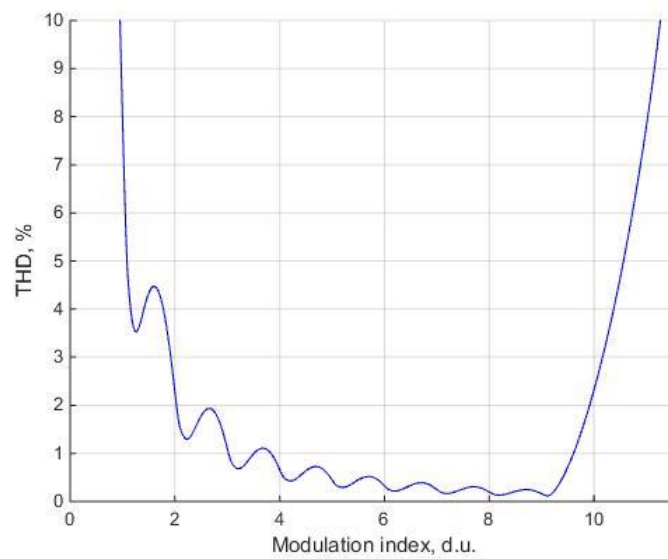


Figure 4-44. 10-level, optimal current THD

Table 2. Current THD minima (angles presented in radians)

Modulation index	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	$\alpha_8$	$\alpha_9$	THD <sub>i</sub> (%)
1.26	0.4259	-	-	-	-	-	-	-	-	3.528
2.23	0.2231	0.7535	-	-	-	-	-	-	-	1.293
3.22	0.1561	0.4819	0.9032	-	-	-	-	-	-	0.682
4.21	0.119	0.3644	0.6333	0.9949	-	-	-	-	-	0.4266
5.21	0.09634	0.2919	0.5006	0.7344	1.055	-	-	-	-	0.2948
6.2	0.08078	0.2443	0.415	0.6	0.81	1.103	-	-	-	0.2171
7.2	0.0696	0.2098	0.3547	0.5075	0.6754	0.8672	1.137	-	-	0.1672
8.19	0.06121	0.184	0.3105	0.4413	0.5819	0.7363	0.9148	1.168	-	0.1335
9.11	0.05511	0.1653	0.2782	0.3941	0.5168	0.6481	0.7946	0.9648	1.215	0.1178

Figure 4-40 of optimal current THD resembles THD graph for current.

However, it demonstrates lower THD. Generally, optimization of current results in lower current THD compared to voltage THD at voltage optimization.

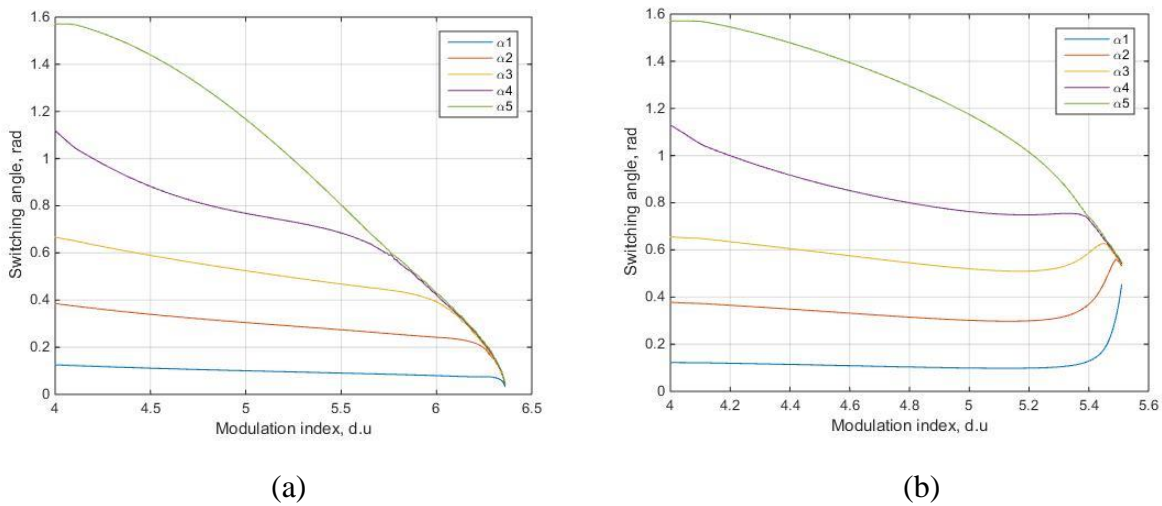


Figure 4-45. 6-level, optimal current angles: (a) pure THD, (b) SHE 3

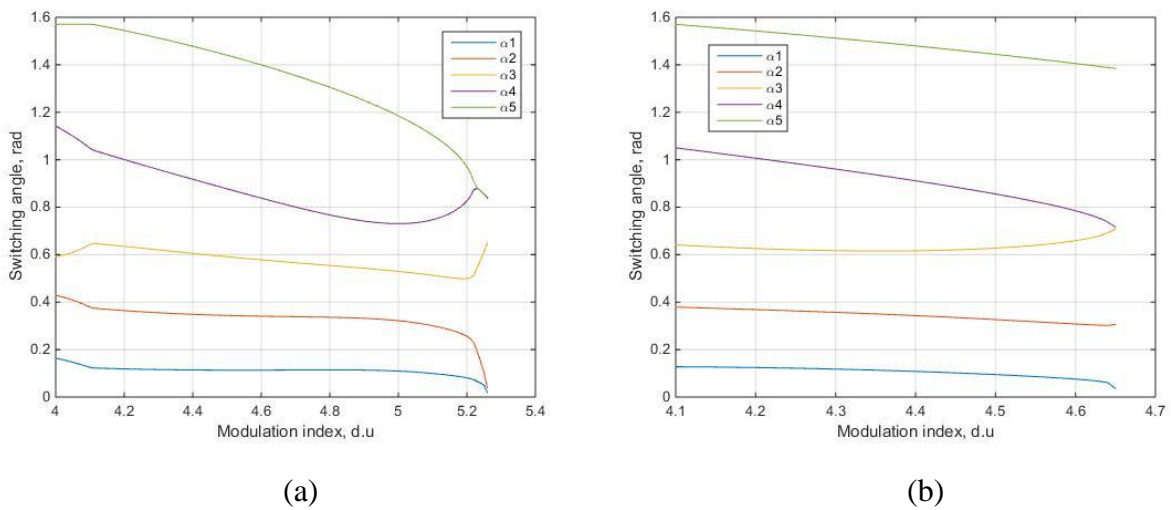


Figure 4-46. 6-level, optimal current angles: (a) SHE 3,5, (b) SHE 3,5,7

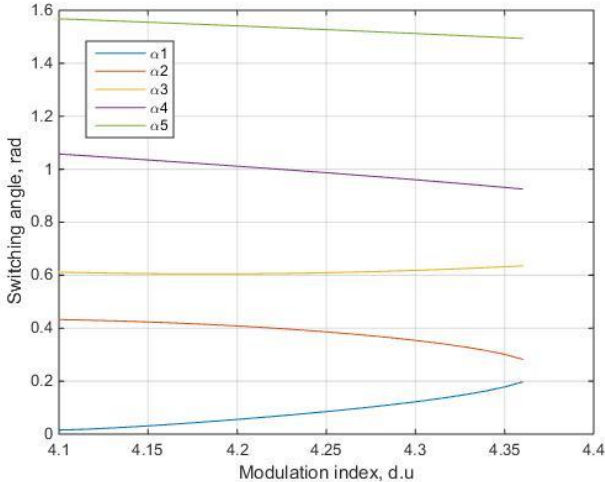


Figure 4-47. 6-level, optimal current angles for pure SHE

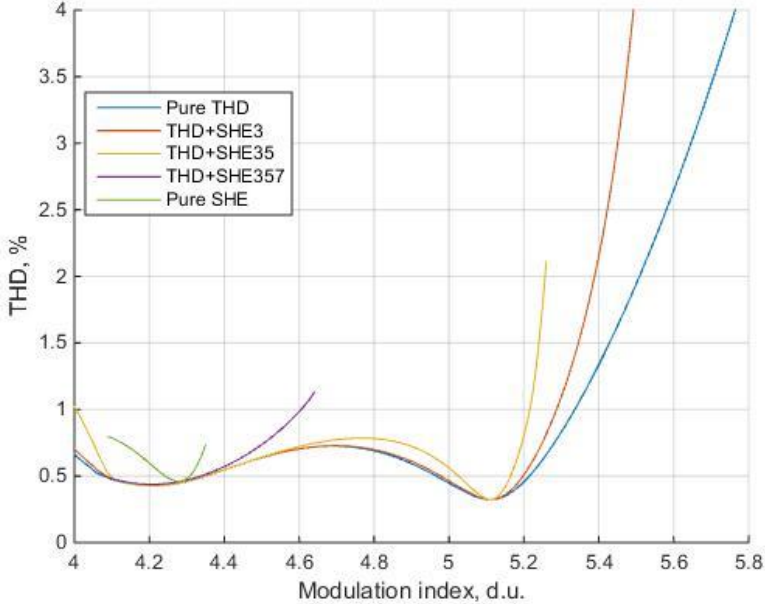


Figure 4-48. 6-level, optimal current THD

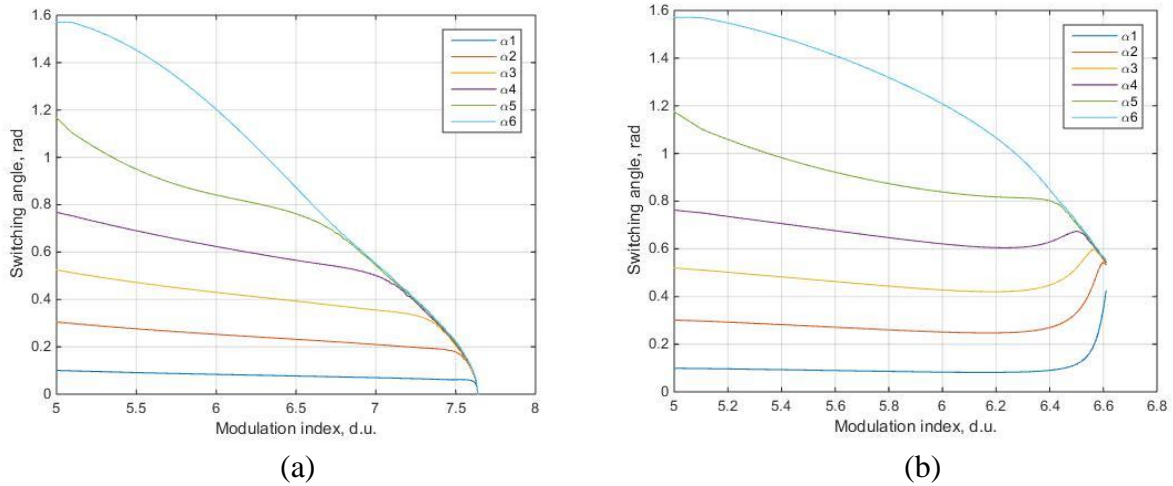


Figure 4-49. 7-level, optimal current angles: (a) pure THD, (b) SHE 3

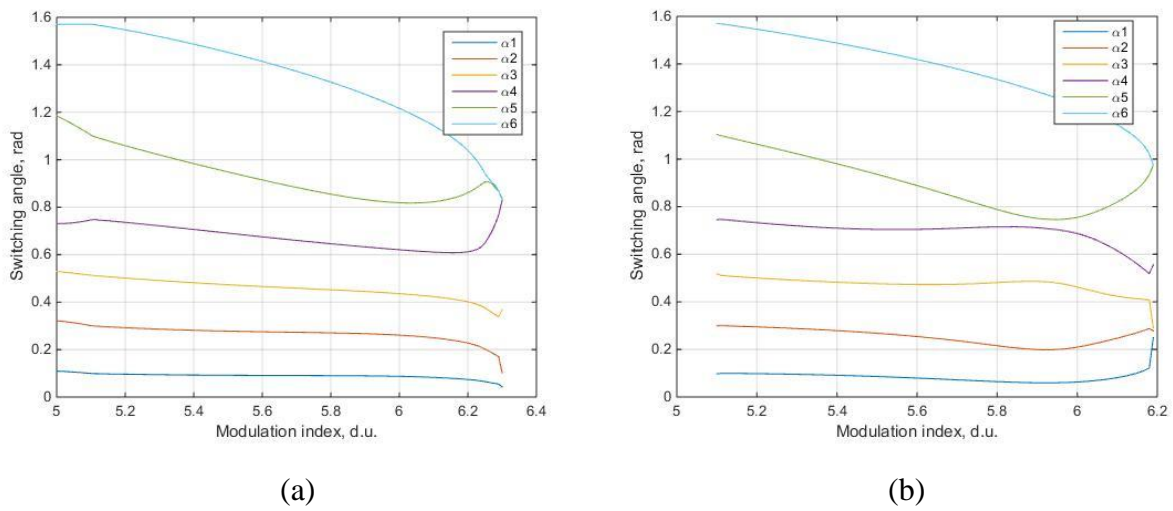


Figure 4-50. 7-level, optimal current angles: (a) SHE 3,5, (b) SHE 3,5,7

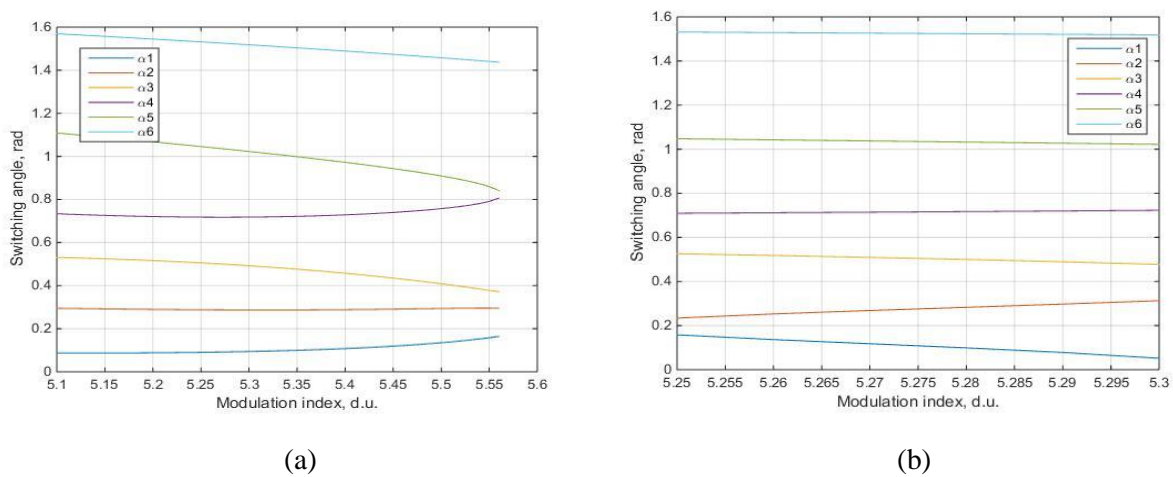


Figure 4-51. 7-level, optimal current angles: (a) SHE 3,5,7,9, (b) pure SHE

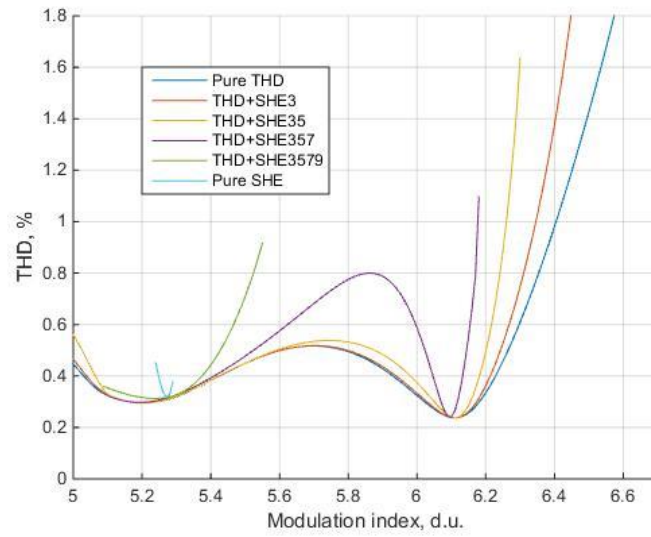
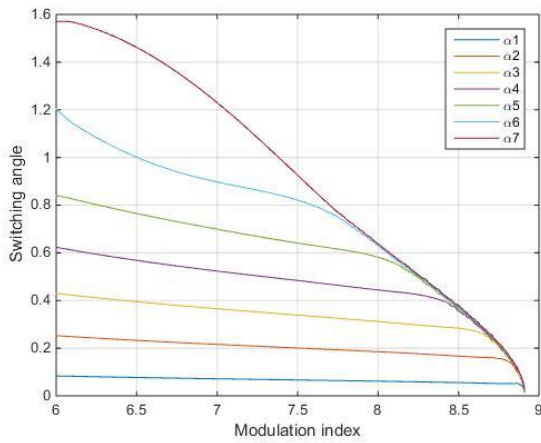
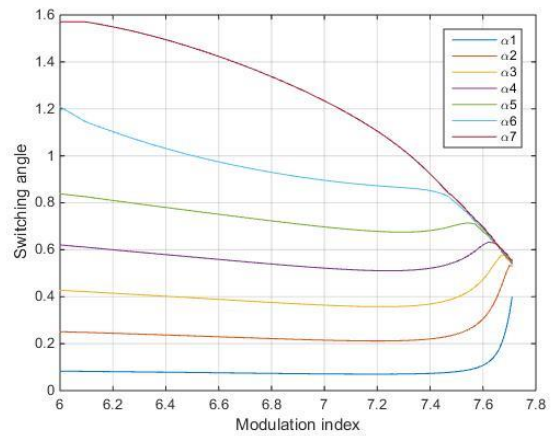


Figure 4-52. 7-level optimal current THD

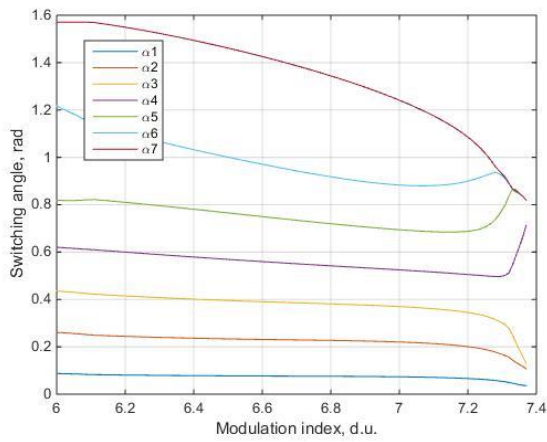


(a)

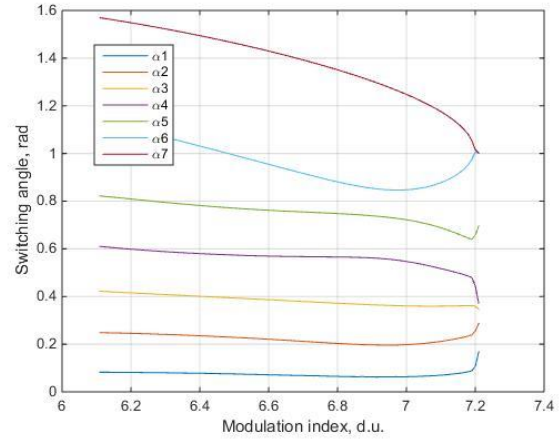


(b)

Figure 4-53. 8-level, optimal current angles: (a) pure THD, (b) SHE 3

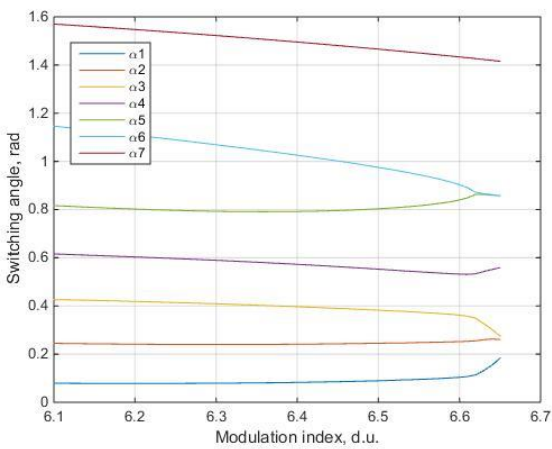


(a)

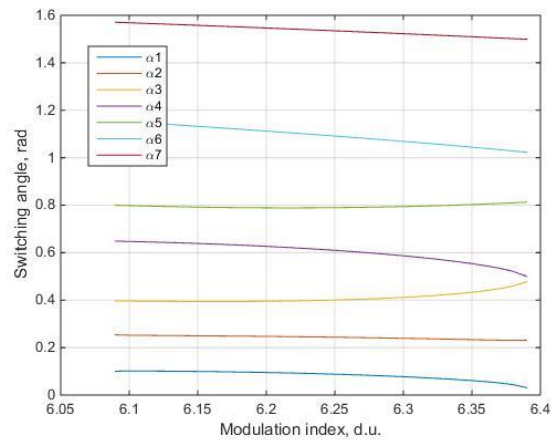


(b)

Figure 4-54. 8-level, optimal current angles: (a) SHE 3,5, (b) SHE 3,5,7



(a)



(b)

Figure 4-55. 8-level, optimal current angles: (a) SHE 3,5,7,9, (b) SHE 3,5,7,9,11

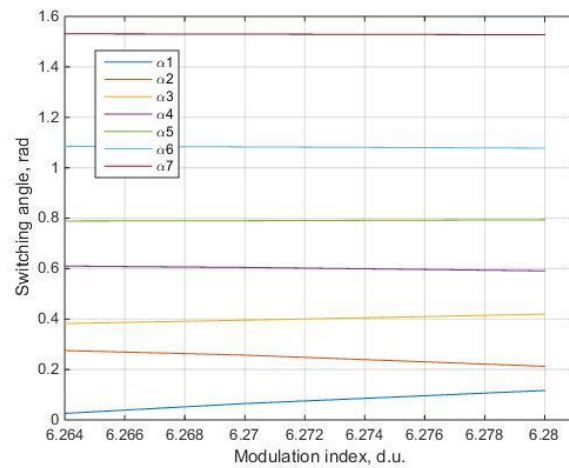


Figure 4-56. 8-level, optimal current angles for pure SHE

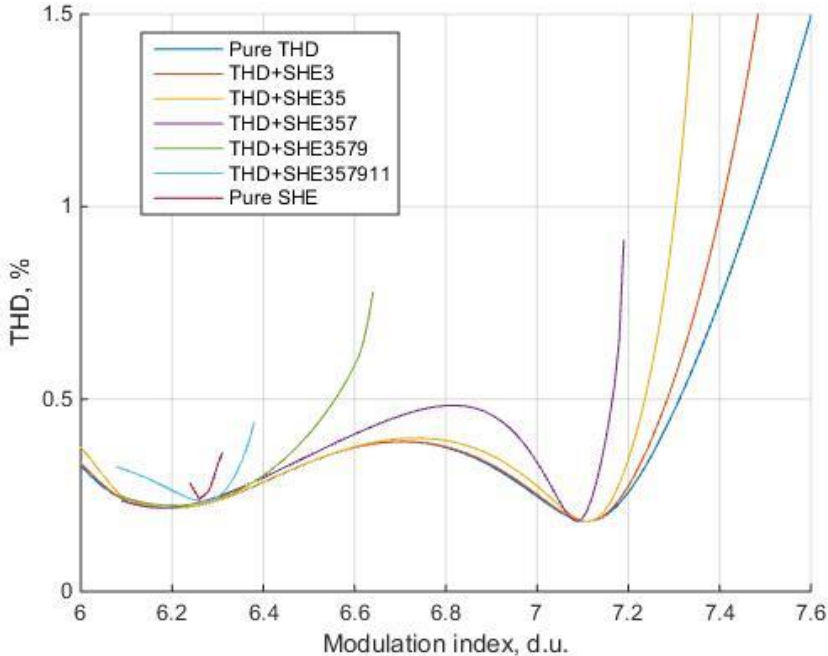
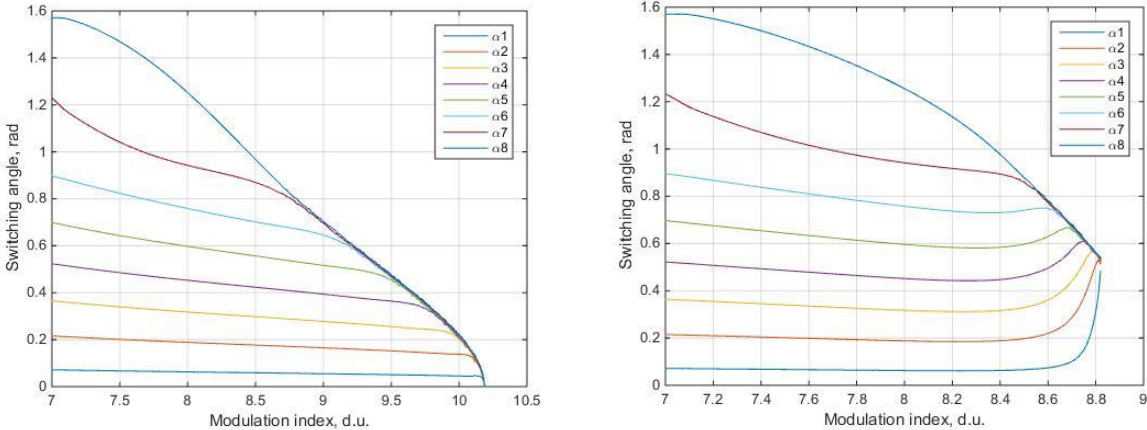


Figure 4-57. 8-level, optimal current THD



(a) (b)

Figure 4-58. 9-level, optimal current angles: (a) pure THD, (b) SHE 3

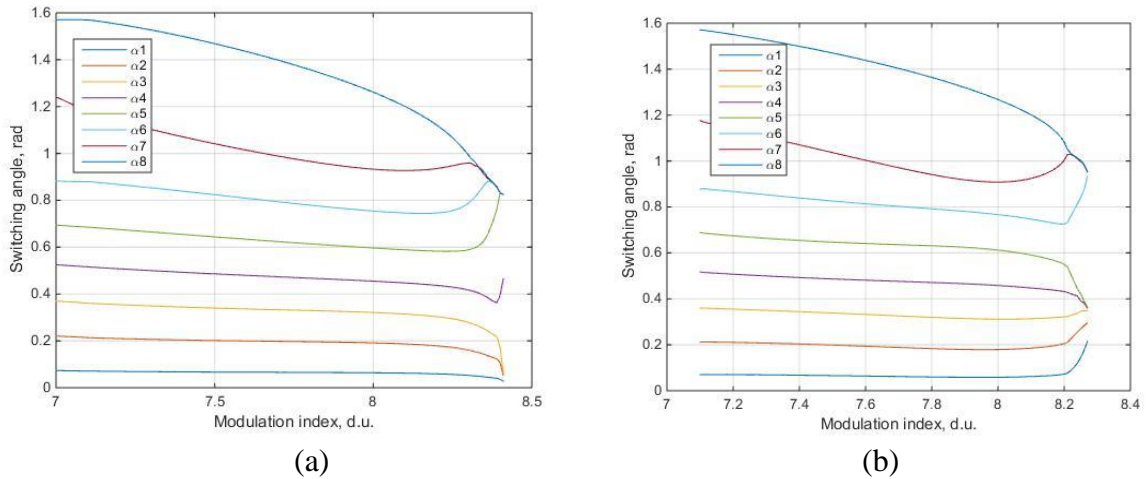


Figure 4-59. 9-level, optimal current angles: (a) SHE 3,5, (b) SHE 3,5,7

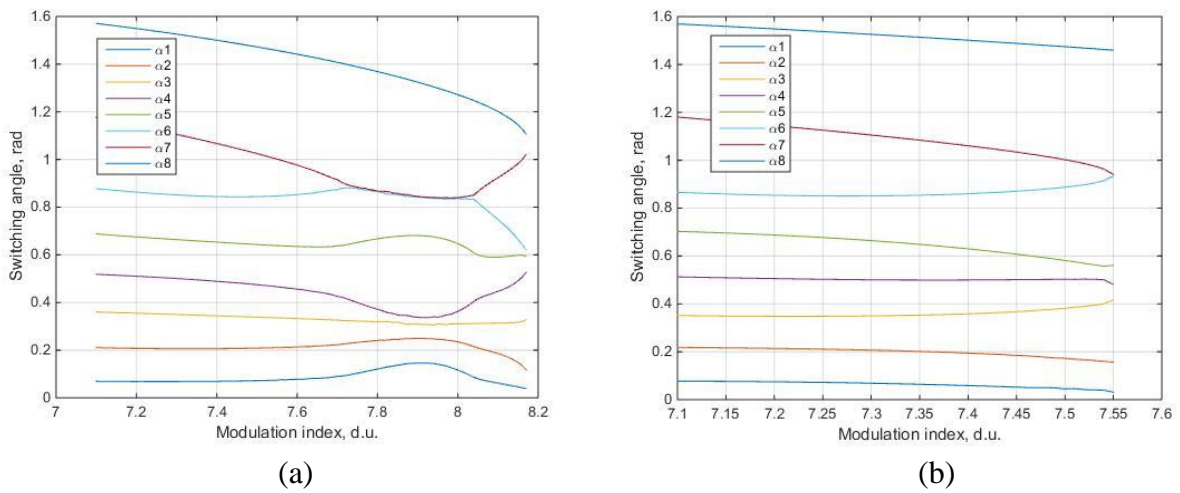


Figure 4-60. 9-level, optimal current angles: (a) SHE 3,5,7,9, (b) SHE 3,5,7,9,11

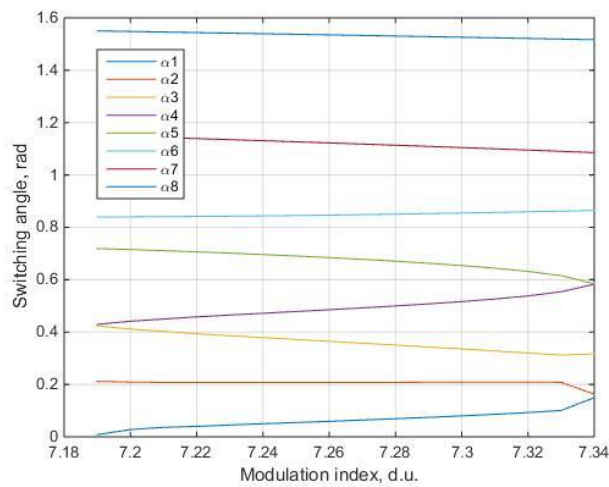


Figure 4-61. 9-level, optimal current angles for SHE 3,5,7,9,11,13



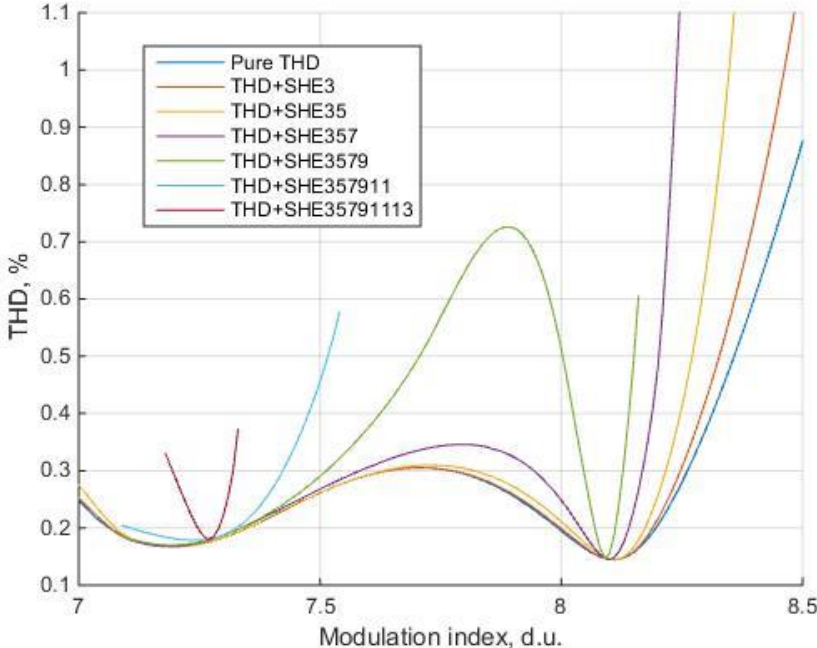


Figure 4-62. 9-level, optimal current THD

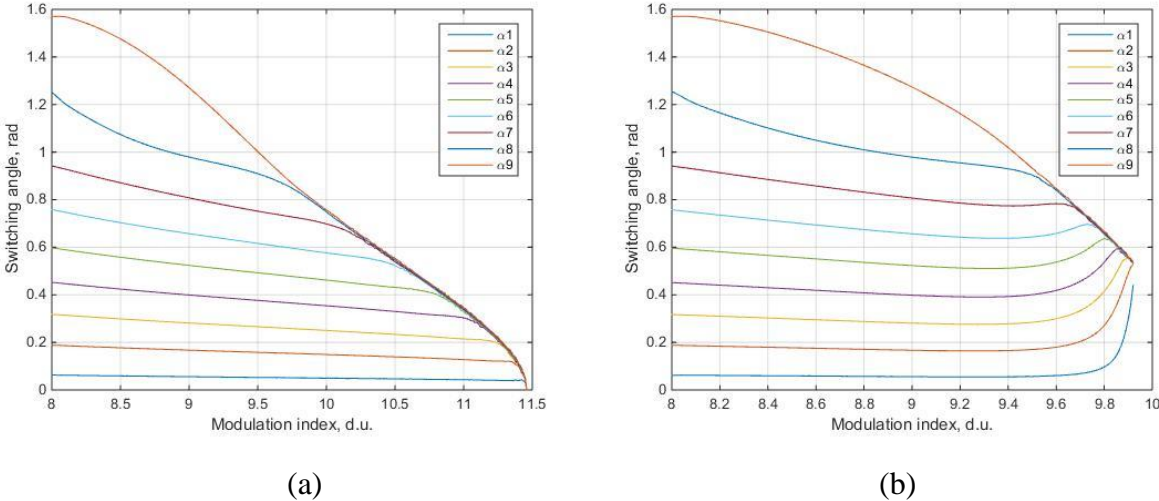
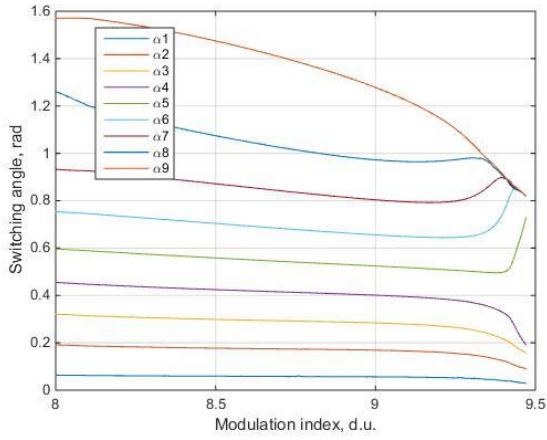
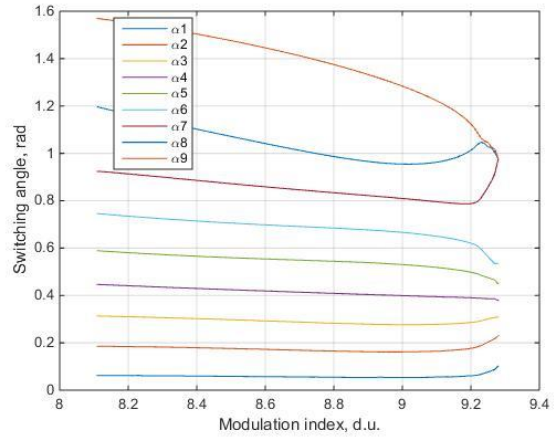


Figure 4-63. 10-level, optimal current angles: (a) pure THD, (b) SHE 3

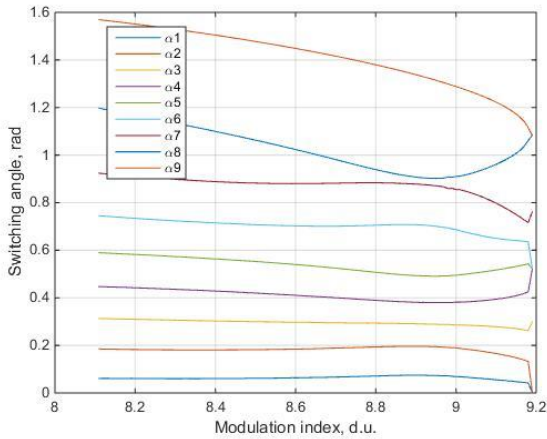


(a)

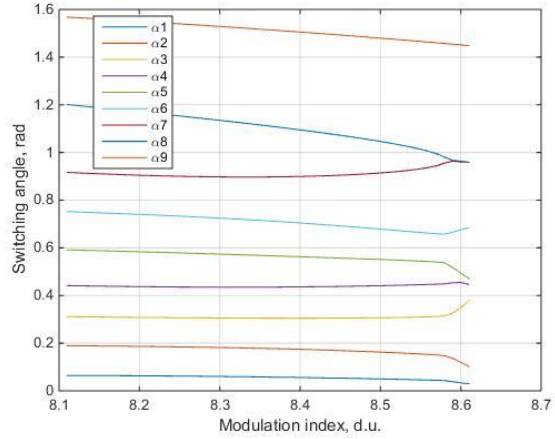


(b)

**Figure 4-64. 10-level, optimal current angles: (a) SHE 3,5, (b) SHE 3,5,7**

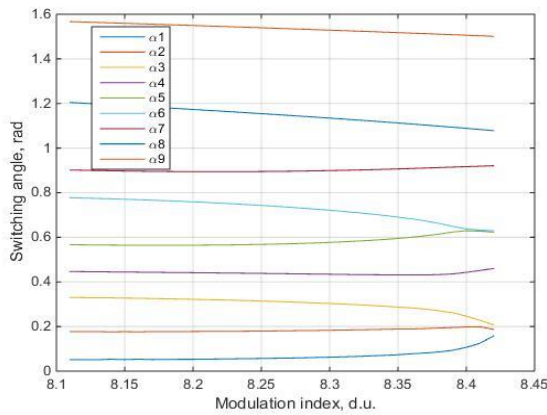


(a)

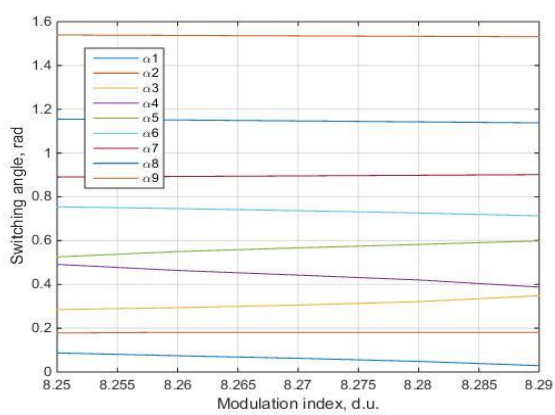


(b)

**Figure 4-65. 10-level, optimal current angles: (a) SHE 3,5,7,9, (b) SHE 3,5,7,9,11**



(a)



(b)

**Figure 4-66. 10-level, optimal current angles: (a) SHE 3,5,7,9,11,13, (b) SHE 3,5,7,9,11,13,15**

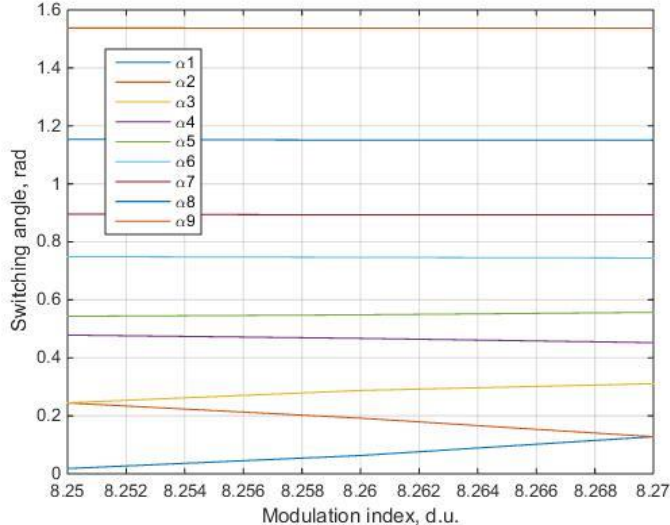


Figure 4-67. 10-level, optimal current angles for pure SHE

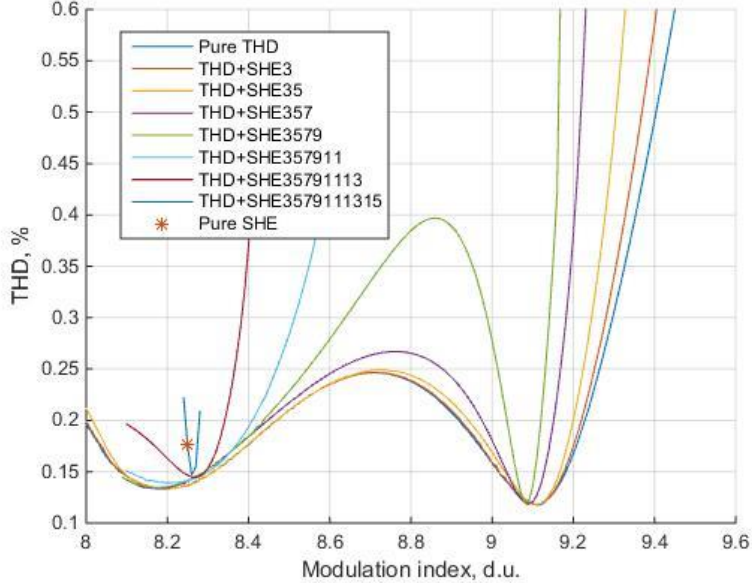


Figure 4-68. 10-level, optimal current THD

## 5. Discussion

Results of optimization algorithm proposed in this thesis were demonstrated in previous chapter. In order to confirm correctness of the results they were compared to those demonstrated in [16], and simulation results. Larger number of angles was introduced in this thesis, and their results of optimization resembles general pattern, and allows to conclude that obtained results are correct. It can be noticed that increase of number of switching angles reduces THD at 2-level modulation. As well addition of levels at staircase modulation also decreases voltage and current THD. It can be explained by the fact that larger number of switching angles results in smaller normalized mean square error, and as a result smaller THD. It should be mentioned that optimization of current results in lower current THD compared to voltage THD at voltage optimization, and as a result current optimization is more preferable. Introduction of SHE algorithm increases THD of both voltage and current waveforms. However, elimination of harmonics positively affects on switching losses. Combination of THD minimization with SHE algorithm leads to shrink in available modulation range, as range of real solutions shrinks with introduction of SHE constraints, and for proper work elimination of large number of harmonics is not recommended. Simulation time is in the range of 5-15 seconds, depending on the number of angles and simulation step. This proves possibility of usage of time-domain optimization algorithm in

online applications. Simplification of calculation method will facilitate computation load, and as a result leads to decrease in computation time.

## 6. Conclusion

The main idea behind this work was to demonstrate potential of time-domain evaluation techniques to be used in calculation of performance indicators.

Theoretical part was discussed in literature review, and modulation method was chosen from that. Methodology describes optimization algorithm used for 2-level and multilevel staircase modulation. Results part demonstrates simulation outcome and demonstrates obtained optimal parameters. Obtained results were analyzed in Discussion part, and in the end summarized. To conclude with, time-domain optimization proved itself as accurate and fast optimization method.

Matlab codes of optimization algorithm of 2-level and staircase current modulation for 5 angles is attached in Appendix. Future work will be focused on implementation of current optimization method for 3-phase inverters, and hardware testing of results obtained during this work.

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# Appendix

## Appendix 1.

### Matlab code for 2-level, 5 angles current modulation

```

mmi5 = 0.01:0.01:1.27; % modulation index range
b0 = [0.1; 0.4; 0.7; 1; 1.3]; % predefined initial conditions

for i=1:length(mmi5)

    m = mmi5(i);
    A = [-1 0 0 0 0;
         1 -1 0 0 0;
         0 1 -1 0 0;
         0 0 1 -1 0;
         0 0 0 1 -1;
         0 0 0 0 1];
    B = [0; 0; 0; 0; 0; pi/2];

    options = optimoptions('fmincon','Algorithm','interior-point');
    b_out = fmincon(@(b) myfun(b,m),b0,A,B,[],[],[],[], @ (b)
mycon(b,m),options); % function of constrained minimization

    NMSI5 = 2*(b_out(1)^3/3 + b_out(1)^2*(b_out(2)-b_out(1)) + (b_out(1)^2 +
(b_out(1)-b_out(2)+b_out(3))^2 +...
    b_out(1)*(b_out(1)-b_out(2)+b_out(3))*(b_out(3)-b_out(2))/3 +
(b_out(1)-b_out(2)+b_out(3))^2*(b_out(4) - b_out(3))+...
    ((b_out(1)-b_out(2)+b_out(3))^2 + (b_out(1)-b_out(2)+b_out(3)-
b_out(4)+b_out(5))^2+(b_out(1)-b_out(2)+...
    b_out(3))*(b_out(1)-b_out(2)+b_out(3)-b_out(4)+b_out(5)))*(b_out(5)-
b_out(4))/3 + (b_out(1)-b_out(2)+b_out(3)-b_out(4)...
    +b_out(5))^2*(pi/2-b_out(5)))/pi-m^2/2; % NMS calculation formula

    b0 = [b_out(1); b_out(2); b_out(3); b_out(4); b_out(5)]; % obtained
optimal values are saved and used for next iteration

    b51(i) = b_out(1);
    b52(i) = b_out(2);
    b53(i) = b_out(3);
    b54(i) = b_out(4);
    b55(i) = b_out(5);

    a51(i) = pi/2-b_out(5);
    a52(i) = pi/2-b_out(4);
    a53(i) = pi/2-b_out(3);
    a54(i) = pi/2-b_out(2);
    a55(i) = pi/2-b_out(1);

```

```

    NMSI5_out(i) = NMSI5;
    THDI5_Iout(i)=sqrt(2*NMSI5_out(i))/m*100; % THD calculation
end

```

## Mycon

```

function [c, ceq] = mycon(b,m)

c = [];
ceq(1) = 4*(sin(b(5))-sin(b(4))+sin(b(3))-sin(b(2))+sin(b(1)))/(pi)-m;
%equality constraint
ceq(2) = sin(3*b(5))-sin(3*b(4))+sin(3*b(3))-sin(3*b(2))+sin(3*b(1)); %SHE
constraint for 3rd harmonic
ceq(3) = sin(5*b(5))-sin(5*b(4))+sin(5*b(3))-sin(5*b(2))+sin(5*b(1));
ceq(4) = sin(7*b(5))-sin(7*b(4))+sin(7*b(3))-sin(7*b(2))+sin(7*b(1));
ceq(5) = sin(9*b(5))-sin(9*b(4))+sin(9*b(3))-sin(9*b(2))+sin(9*b(1));
end

```

## Myfun

```

function f = myfun(b,m)
f = 2*(b(1)^3/3 + b(1)^2*(b(2)-b(1)) + (b(1)^2 + (b(1)-b(2)+b(3))^2 + ...
    b(1)*(b(1)-b(2)+b(3))*(b(3)-b(2))/3 + (b(1)-b(2)+b(3))^2*(b(4) -
b(3))+...
    ((b(1)-b(2)+b(3))^2 + (b(1)-b(2)+b(3)-b(4)+b(5))^2+(b(1)-b(2)+...
    b(3))*(b(1)-b(2)+b(3)-b(4)+b(5))*(b(5)-b(4))/3 + (b(1)-b(2)+b(3)-
b(4))...
    +b(5))^2*(pi/2-b(5)))/pi-m^2/2;
end

```

## Appendix 2

### Matlab code of 5 angle (6-level) current optimization

```

mmi5 = 4.1:0.01:4.36;

b0 = [0.1; 0.4; 0.7; 1; 1.3];

for i=1:length(mmi5)

    m = mmi5(i);

    A = [-1 0 0 0 0;
        1 -1 0 0 0;
        0 1 -1 0 0;
        0 0 1 -1 0;
        0 0 0 1 -1;
        0 0 0 0 1];

```

```

B = [0;0;0;0;0; pi/2];

options = optimoptions('fmincon','Algorithm','interior-point');
b_out = fmincon(@(b) myfun(b,m),b0,A,B,[],[],[],[], @ (b)
mycon(b,m),options);

NMSI5 = 2*(25*b_out(1)^3/3 +
(25*b_out(1)^2+(b_out(1)+4*b_out(2))^2+5*b_out(1)*(b_out(1)+4*b_out(2)))*(b_o
ut(2)-b_out(1))/3 + ...

((b_out(1)+4*b_out(2))^2+(b_out(1)+b_out(2)+3*b_out(3))^2+(b_out(1)+4*b_out(2)
))* (b_out(1)+b_out(2)+3*b_out(3)))*(b_out(3)-b_out(2))/3 + ...

((b_out(1)+b_out(2)+3*b_out(3))^2+(b_out(1)+b_out(2)+b_out(3)+2*b_out(4))^2+(
b_out(1)+b_out(2)+3*b_out(3))*(b_out(1)+...
b_out(2)+b_out(3)+2*b_out(4)))*(b_out(4)-b_out(3))/3 +
((b_out(1)+b_out(2)+b_out(3)+2*b_out(4))^2+...

(b_out(1)+b_out(2)+b_out(3)+b_out(4)+b_out(5))^2+(b_out(1)+b_out(2)+b_out(3)+
2*b_out(4))*(b_out(1)+b_out(2)+b_out(3)+...
b_out(4)+b_out(5))*(b_out(5)-b_out(4))/3 +
(b_out(1)+b_out(2)+b_out(3)+b_out(4)+b_out(5))^2*(pi/2-b_out(5))/pi-m^2/2;

b0 = [b_out(1); b_out(2); b_out(3); b_out(4); b_out(5)];

b51(i) = b_out(1);
b52(i) = b_out(2);
b53(i) = b_out(3);
b54(i) = b_out(4);
b55(i) = b_out(5);

a51(i) = pi/2-b_out(5);
a52(i) = pi/2-b_out(4);
a53(i) = pi/2-b_out(3);
a54(i) = pi/2-b_out(2);
a55(i) = pi/2-b_out(1);

NMSI5_out(i) = NMSI5;
THDI5_Iout(i)=sqrt(2*NMSI5_out(i))/m*100;
end

```

## Mycon

```

function [c, ceq] = mycon(b,m)

c = [];
ceq(1) = 4*(sin(b(1))+sin(b(2))+sin(b(3))+sin(b(4))+sin(b(5)))/(pi)-m;
ceq(2) = sin(3*b(5))+sin(3*b(4))+sin(3*b(3))+sin(3*b(2))+sin(3*b(1));
ceq(3) = sin(5*b(5))+sin(5*b(4))+sin(5*b(3))+sin(5*b(2))+sin(5*b(1));
ceq(4) = sin(7*b(5))+sin(7*b(4))+sin(7*b(3))+sin(7*b(2))+sin(7*b(1));
ceq(5) = sin(9*b(5))+sin(9*b(4))+sin(9*b(3))+sin(9*b(2))+sin(9*b(1));

```

end

## Myfun

```
function f = myfun(b,m)
f = 2*(25*b(1)^3/3 + (25*b(1)^2+(b(1)+4*b(2))^2+5*b(1)*(b(1)+4*b(2)))*(b(2)-
b(1))/3 + ...

((b(1)+4*b(2))^2+(b(1)+b(2)+3*b(3))^2+(b(1)+4*b(2))*(b(1)+b(2)+3*b(3)))*(b(3)
-b(2))/3 + ...

((b(1)+b(2)+3*b(3))^2+(b(1)+b(2)+b(3)+2*b(4))^2+(b(1)+b(2)+3*b(3))*(b(1)+...
b(2)+b(3)+2*b(4)))*(b(4)-b(3))/3 + ((b(1)+b(2)+b(3)+2*b(4))^2+...

(b(1)+b(2)+b(3)+b(4)+b(5))^2+(b(1)+b(2)+b(3)+2*b(4))*(b(1)+b(2)+b(3)+...
b(4)+b(5)))*(b(5)-b(4))/3 + (b(1)+b(2)+b(3)+b(4)+b(5))^2*(pi/2-
b(5))/pi-m^2/2;
end
```