

# Convergence rates for inverse-free rational approximation of matrix functions

Carl Jagels, Thomas Mach, Lothar Reichel, Raf Vandebril

Department of Mathematics, Department of Computer Science

## Abstract

This article deduces geometric convergence rates for approximating matrix functions via inverse-free rational Krylov methods. In applications one frequently encounters matrix functions such as the matrix exponential or matrix logarithm; often the matrix under consideration is too large to compute the matrix function directly, and Krylov subspace methods are used to determine a reduced problem. If many evaluations of a matrix function of the form  $f(A)v$  with a large matrix  $A$  are required, then it may be advantageous to determine a reduced problem using rational Krylov subspaces. These methods may give more accurate approximations of  $f(A)v$  with subspaces of smaller dimension than standard Krylov subspace methods. Unfortunately, the system solves required to construct an orthogonal basis for a rational Krylov subspace may create numerical difficulties and/or require excessive computing time. This paper investigates a novel approach to determine an orthogonal basis of an approximation of a rational Krylov subspace of (small) dimension from a standard orthogonal Krylov subspace basis of larger dimension. The approximation error will depend on properties of the matrix  $A$  and on the dimension of the original standard Krylov subspace. We show that our inverse-free method for approximating the rational Krylov subspace converges geometrically (for increasing dimension of the standard Krylov subspace) to a rational Krylov subspace. The convergence rate may be used to predict the dimension of the standard Krylov subspace necessary to obtain a certain accuracy in the approximation. Computed examples illustrate the theory developed.

<b>Original language</b>	English
<b>Pages (from-to)</b>	291-310
<b>Number of pages</b>	20
<b>Journal</b>	<a href="#">Linear Algebra and Its Applications</a>
<b>Volume</b>	510
<b>State</b>	Published - Dec 1 2016

Jagels, C., Mach, T., Reichel, L., & Vandebril, R. (2016). *Convergence rates for inverse-free rational approximation of matrix functions*. *Linear Algebra and Its Applications*, 510, 291-310. DOI: [10.1016/j.laa.2016.08.029](https://doi.org/10.1016/j.laa.2016.08.029)