NSU Florida
NOVA SOUTHEASTERN UNIVERSITY

Transformations

Volume 2
Issue 2 *Winter 2016*

Article 2

12-27-2016

Associating Mathematics to its History: Connecting the Mathematics we Teach to its Past

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Introduction

> *"I am sure that no subject loses more than mathematics*
> *by any attempt to dissociate it from its history."*
>
> --Glashier

The above quote has certainly been a complaint by many in the western world studying mathematics. Many learners are taught mathematics through a drill and practice and sometimes a constructivist manner using manipulatives, yet seldom do our young people learn the history as to where algebra came from, or why geometry is important for logic, or what brought about peoples' interest in improving their chances of winning at a game of craps as it relates to data analysis and probability. The math we now use and teach is usually always taught in isolation from its history.

Around the world demographics are changing considerably with increased globalization. Many countries are affected by larger and larger populations of immigrants, whether in Europe from Morocco and northern African countries, or South Africa from people from Zimbabwe, or Malaysia from Myanmar and neighboring countries, or Canada and the USA from Central and South America countries. Math can be made more meaningful for our students when educators make meaningful historical and cultural connections to the math they are learning while valuing their heritage. The study of the cultural and historical contexts of ancient civilizations can be an intriguing way to introduce students to the evolution and logic of today's mathematics (Bidwell, 1993; McNeill, 2001; Wilson, 2001; Zaslavsky, 2002; Farmers & Powers, 2005). Students can benefit greatly by learning about their ancestors' math, other cultures symbols, and even how zero was represented in another civilization. The purpose of this paper is to: 1) to briefly explore the ways mathematics can relate its history; 2) to provide suggestions for infusing strategies into planning and teaching; 3) to make recommendations to infuse more historical and cultural connections into the math curriculum; and 4) to suggest the use of math manipulatives and webquests as part of math instruction.

Connecting Mathematics Content to its History

Today in education there is a big push for getting more of our young people to pursue careers in Science, Technology, Engineering, and Mathematics (STEM) fields. STEM has been the buzzword for several years now. Slykhuis, Martin-Hansen, Thomas, & Barbatom (2015) believe that history is a critical force that needs to be infused within the instruction of the STEM fields. As the quote above implies, no other subject loses more than mathematics by any attempt to dissociate it from its history. Many teachers when they teach math very rarely talk about the history that lead to the discovery of Algebra or the use of or development of Napier Rods. In research by McGee and Hostetler (2014) they contend that teachers need to draw on historical and contemporary narratives to position social justice and history issues in mathematics and social studies education. As researchers in these two fields, they envision greater possibilities for the advancement of knowledge, and we envision learning from inequalities and resisting oppression by nurturing deeper, more explicit connections between mathematics and social studies.

Panagiotou (2011) has found that many researchers have discussed the question "why" we should use the history of mathematics to mathematics education. This researcher claims that knowing "how" to introduce history into mathematics lessons is a more difficult step. Finding that however, that only a limited number of articles contain instructions on how to use the material, as opposed to numerous general articles suggesting the use of the history of mathematics as a didactical tool. The Panagiotou article focuses on converting the history of logarithms into material appropriate for teaching students of 11th grade, without any knowledge of calculus. History uncovers that logarithms were invented prior of the exponential function and shows that the logarithms are not an arbitrary product, as is the case when we leap straight in the definition given in all modern textbooks, but they are a response to a problem. This research describes step by step the historical evolution of the concept, in a way appropriate for use in class, until the definition of the logarithm as area under the hyperbola. Next, they present the formal development of the theory and define the exponential function. The teaching sequence has been successfully undertaken in two high school classrooms in their research. This paper expands on this work by offering many possible ways to incorporate both history and math concepts within the instruction of mathematics to off more opportunities for infusing history into mathematics instruction. The remainder of this paper provides examples of history within the common math strands to infuse more history within math instruction and Appendix A provides a detailed sample math lesson infusing history in a math lesson.

Historical Examples by Mathematical Strands

800's B.C.E.

Number Sense

The Hindu-Arabic numerals that are used internationally today originally derived from the Indian numerals. The Indian numerals were adopted by the Persians in India, and passed on to the Arabs further west. The numerals were modified in shape as they were passed along, and developed their European shapes by the time they reached North Africa. From there they were transmitted to Europe in the Middle Ages. The use of Arabic numerals spread around the world through European trade, books, and colonialism. Students may enjoy exploring different number systems from different cultures.

The Hindu-Arabic numbers were eventually adopted by the Arab world in the 9th Century (800-900 C.E./B.C.E.). At this time there was some movement within Europe. It was during this century that Norseman left Scandinavia, in longboats, and eventually landed in Ireland. In Asia, Buddhism, originated from India, is growing in China. However, there were attacks on Buddhism from Taoists, and although Buddhism would survive in China, it eventually would become secondary to Confucianism during the 9th Century. In the America this would be the last of the Mayan empire, as most of the Mayan cities were abandoned by 900 A.D. (B.C.E.). (Smitha, 2010). See Appendix A for a sample math lesson incorporating Mayan History.

1700's C.E.

Another activity for our intermediate and middle/high students may be to make and use Napier Bones/Rods in the class which are from the 1700's created by a John Napier from Scotland. The students can use the rods to better understand math, place value, mental math, and lattice multiplication. They can learn that these math tools were somewhat of walking calculators of the times used to do quick multiplications. Throughout history numbers have been created and used from most cultures as a way to bring order to our chaotic world. Social Studies and history teachers may reinforce contributions of cultures in developing their own number systems.

Measurement

The Metric System originally from France, around the 1700's, it may be good to have students explore what was going on in history during this time? Too, even in the 1970's big push in the USA to adopt. Before units of measurement were created, primitive societies often used parts of their bodies as measurement tools (i.e. foot, index finger, tip of chin to outstretched fingers). The Greeks developed the "foot" as the fundamental unit of length. Legends say that the unit was actually based on the actual measurement of Hercules' foot. Romans were responsible for creating the measurement of the mile. The French created a standard unit of measurement called the metric system in 1790. In 1824, the English Parliament legalized a new standard yard which had been made in 1760. It was a brass bar containing a gold button near each end. A dot was engraved in each of these two buttons. These two dots were spaced exactly 1 yard apart. Since colonialists brought with them the measuring methods of their homeland, confusing and contradictory measuring systems came to America. It wasn't until 1832 that Congress created the legal American Yard Standard. In 1866 Congress passed a bill which permitted the use of the metric system in the United States. Then in 1975 Congress passed the Metric Conversion Act and in 1976 the U.S. Metric Board was appointed. While students learn to measure, it may be nice for them to know some of the history as to where the Metric System came from or who invented the foot. Please see Appendix B for several resources for incorporating more history into the math curriculum.

Data Analysis and Probability

Data Analysis and Probability has its roots in Italy, the Italians were interested in ways to improve chances of winning card and dice games, many contributions to probability came from Cardano, an Italian mathematician. Probability theory before 1750 was inspired mainly by games of chance. Dicing, card games, lotteries, public and private, were important social and economic then as today. During the 1650-1700, the age of Scientific Revolution, some of the biggest contributors were Galileo and Newton. The period from 1708-1718 is often referred to as the "great leap forward" because of the numerous important contributions to the subject. In the years before the Great War of 1914-18 probability and statistics were expanding in all directions. During the war research in statistics and probability almost stopped as people went into the armed services or did other kind of war work. In the latter 1980's, the invention of the computer progressed many mathematicians research such as analysing larger data sets.

The 18th Century was one of the more progressive centuries in our past, not only in mathematics. The Industrial Revolution began in the coal mines and textile industries during the 1700's, however its importance grew during the 1800's. In Russia, we see Catherine II (The Great) become Czarina and she attempted to continue the trend of modernizing and strengthening Russia. In Colonial America, revolution was brewing, as King George III was systematically evoking more taxes on Colonial Americans to pay for the French and Indian War. This eventually led to the Declaration of Independence, American Revolution, Treaty of Paris, and the Constitution. France was involved in their own revolution in the latter part of the century. This contributed to Napoleon coming to power. There were other important events from the 1700's: James Watt invented the Steam Engine, the first Encyclopedia was published, Benjamin Franklin began publishing *Poor Richard's Almanac,* Beethoven's first printed works, Mozart's *Don Giovanni*, and at the end of the century the Rosetta Stone was discovered in Egypt, which allowed for the understanding of the Egyptian Hieroglyphics. (McCollom, No date)

Geometry

In Greece around 2000B.C. E, Geometry was considered to be the most important branch of mathematics. In fact, geometry was so important in Greece and the Mediterranean regions like Alexandria that even over the entrances of universities, they would place, "Let no man ignorant of geometry enter here." Geometry surrounds us, our world is geometrical. Not only is 2-D and 3-Dd geometry important, but what is consider to be very important is the deduction, logic, and proof that can applied. As early as 2000B.C.E. Ancient Egyptians, during the Middle Period, demonstrated their knowledge of geometry through construction projects (such as the pyramids). There are also clay tablets from the Babylonians which contain writings of problems that connected the Pythagorean relationship. In 400B.C.E. Euclid of Alexandria, a Greek mathematician, wrote an important book titled *The Elements*, which has formed the basis for most of geometry studied in schools today. A second major publication came from David Hilbert in 1899 titled Grundlagen der Geometrie, which "marks the start of the move towards axiomatising mathematics, or to be more precise, towards giving formal, "meaningless" axiom systems as the basis of each mathematical discipline and eventually of all mathematics" (Gray, 1998). In the 20th century, Donald Coxeter is regarded as a major synthetic geometer. He worked most of his life in Canada and made contributions to the theory of polytopes, non-Euclidean geometry, group theory and combinatorics.

1800-1600 B.C.E.

Algebraic Thinking

The word "algebra" comes from Arabic word "al-jabr" (restoration), its origins can be traced to the ancient Babylonians, who developed an advanced arithmetical system with which they were able to do calculations in an algorithmic fashion. The Babylonians developed formulas to calculate solutions for problems typically solved today by using linear equations, quadratic equations, and indeterminate linear equations. There are numerous people throughout different cultures that contributed to the history of algebra. Some of them include: Omar Khayyam (Persian) who created the foundations of algebraic geometry, Persian mathematician, Sharaf al-Dīn al-Tūsī, found algebraic and numerical solutions to various cases of cubic equations, Chinese mathematician Zhu Shijie, solved various cases of cubic, quartic, quintic and higher-order polynomial equations using numerical methods, and The idea of a determinant was developed by Japanese mathematician Kowa Seki in the 17th century. Students should be encouraged in classrooms while learning about algebra to explore the origins of Algebra and mathematicians like Fibonacci, to explore the exciting patterns and generalizations that exist, to use formulas, and hence solve algebra equations/functions seeing practical applications to algebra in our world today.

The mathematics of the Old Babylonian Period was more advanced than the mathematics from Egypt. It may have been possible because of the order instilled by Hammurabi. During the 18th Century B.C.E. Hammurabi conquered the Fertile Crescent. He ruled an area from the Mediterranean to the Persian Gulf. He also instituted his code of law, "Hammurabi's Code", during this period. During this period Babylonians recognized that the evening star (Venus) and the morning star were the same star. They believed the movement of the planets was to be interpreted by kings. It led to an understanding of weather patterns. It is believed that it was through their understanding of mathematics that allowed for their beginning comprehension of Astronomy. (Magruder,2011).

Covering Math and Social Studies Standards

The National Council for the Social Studies has provided 10 Themes as a way of categorizing information about the human experience. This paper addresses two themes: Time, Continuity, and Change; and Science, Technology and Society. (NCSS, 2010) The National Council of Teachers of Mathematics (NCTM, 2000) has identified five mathematics strands as follows: Number Sense and Operations, Measurement, Geometry and Spatial Sense, Algebraic Thinking, and Data Analysis and Probability, all have been addressed in this paper. Please see Appendix B for history resources broken down by the math strands providing a means to incorporating more history into the teaching of mathematics.

Math lessons with historical tidbits interweaved into the math lessons can bridge the appreciation for what they are learning while making better connections for our students. Various resources can be used to modify and enhance such historical math lessons to meet diverse populations of students. Other examples may include: in a formal algebra class it may be nice to explore the father of Algebra, Abu Ja'far Muhammad, and talk about the locations in Uzbekistan, Iraq, Iran where it originated and why, or to explore Euclid as the father of Geometry and share with students what was happening in Greece and Alexandria, Egypt and why geometry was considered the most important subject of those times, particularly due to its logic. Teachers may enjoy teaching using the Napier Bones to teach and reinforce multiplication and share how in Scotland the Napier Bones were the walking calculator of the times in the 17 and 18th century. (See Appendix A for sample lesson plan.) Mathematics was developed and expanded out of a need and as we all know today, we use it daily, it is hence critical that we strive to weave some of the history of this important subject into the teaching of the material so our young people see more meaning and richness in what they are learning.

Summary

Teachers from around the world in today's mathematics classroom are confronted with the challenge of meeting the needs of students with diverse needs and backgrounds. The NCTM *Standards* and the literature on diverse learners suggest that all students may benefit from strategies which promote cultural and historical connections and the use of technologies and manipulatives which focus upon the active engagement of students through exploration and communications.

In a passionate plea for bridging the culture gap in our classrooms, Moore (1994) proposes that "Mathematics is definitely not culture-free...no mathematics teacher could even contemplate seriously taking only the values of his culture and a textbook which is a product of his culture and imposing both himself and the textbook upon individuals possessed by a culture that diverges from his in any significant area." (p. 13).

Teachers who use a variety of resources and who incorporate innovative ideas into their teaching in order to make learning more meaningful will find students more interested in mathematics. Bridging the cultural gap and infusing history in mathematics instruction will profit all students as they becoming more understanding, appreciative, and tolerant of one another and each other's cultures while better understanding and having an appreciation for where ideas and math content came from to begin with.

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Appendix A
Sample Lesson Plan on Mayan Math

Developing Teaching Units Using Mayan Mathematics and Historical Chronology.

The following are suggestions for incorporating the historical context and symbolic notation system of the Mayans into a teaching unit. The lessons might be taught from an interdisciplinary, integrated curriculum perspective, and modified to meet age appropriate needs of the students. This lesson too can serve in reinforcing concepts related to place value and number systems with different bases in mathematics, with historical chronology and historical analysis and inquiry. The strategies specifically include effective learning techniques for ESOL students (English for Speakers of Other Languages).

Objective(s):

Students will:

- explore similarities and differences of number systems from other cultures, particularly the Mayan system.
- calculate place values with base-10 and base-20 systems.
- reinforce student understanding of place value.

- apply student technology skills.
- develop an appreciation for the culture and math of the Mayans.
- describe how people in the past lived.
- use a variety of sources to learn about the past.
- research and analyze past periods
- formulate questions about the past.

Menu of Suggested Activities:

Menu of Motivation (Initiating) Activities

1) Pairs of students (one of which is an ESOL student) will take a webquest and visit websites to read about Mayan numerical systems. Students will identify and list in chronological order important dates as they work through the webquest.
2) Students view the video, *Mystery of the Maya*.
3) Students will identify and list in chronological order important dates.
4) Students meet in discussion groups. Possible topics for discussion might be:
    a) defining terms such as decimal system, non-decimal system, place value.
    b) describe similarities and differences between (among) systems.
    c) organization of the chronological dates.
    d) formation of a historical question about the Mayans.

Menu of Core Activities

1) Create Mayan "manipulatives" to use in Mayan calculations.
2) Calculate and solve problems using their Mayan manipulatives.
3) Create and solve coded puzzles (see Figure 3).
4) Read literature which relates to cultural differences in mathematics (particularly Mayan Mathematics). A suggested books would be *Arithmetic in Maya*, and *Skywatchers of Ancient Mexico.*
5) Develop and refine discussion groups and paired activities. Possible questions for discussion group:
    - Can you translate Mayan numbers into our numeral system. How about translating our numbers? into Mayan numbers.
    - Why do you think the Mayans chose a base-20 numeral system?
    - How does the Mayan system compare to that of the Egyptians or Romans?
    - Why do you think the Mayans chose a seashell to symbolize zero? What symbol for zero would you choose? Why?
6) Practice writing Mayan numerals 0 to 100.

7) Write reflective essays on web site visits.
8) Create a fictional number system with a unique base and symbols.
9) Keep a journal of math activities and ideas.
10) Illustrate Mayan mathematics with a selected artistic medium such a magazine photo collage, penciled sketch, etc.
11) Locate additional books and websites about Mayan math and other math systems.
12) Invite parents and selected community guest speakers who are knowledgeable about the Mayan or other number systems (in Lake Worth, FL we have the Guatemalan Mayan Center).

13) In cooperative groups discuss what things were like then.
14) Identify important dates during the Mayan period.
15) Chronologically list important dates.
16) Using chart paper, create a timeline.
17) Identify a Conceptual title for the timeline.
18) Compare the timelines within groups. Have groups discuss their important dates and conceptual title.
(As a whole group)
19) Using the timelines, and secondary resources about the Maya, the students will create an "Historical Question".
20) Teacher and students will create research teams to answer the "Historical Question"
21) Students, in research groups, will present findings to their peers.

Bibliography of resources for Lessons
Books:
Aveni, A. (1980). *Skywatchers of ancient Mexico*. Austin, Texas: University of Texas Press.
Sanchez, G. (1971). *Arithmetic in Maya*. San Diego, CA: Author.

Video/Film:
Howell, B. (1995). *Mystery of the Maya.* In Canada: Developed jointly from The National Film Board of Canada and The Canadian Museum of Civilization and in México: The Instituto Mexicano de Cinematografía.
Internet Websites for Webquest:
Mayan Mathematics at:
http://www.hanksville.org/yucatan/mayamath.html
http://hanksville.phast.umass.edu/Yucatan/mayamath.html
http://www.michielb.nl/maya/math.html
http://mathforum.org/k12/mayan.math/
Mayan Numbers at
http://www.niti.org/mayan/lesson.htm
http://www.vpds.wsu.edu/fair_95/gym/UM001.html
http://www.cmcc.muse.digital.ca/membrs/civiliz/maya/mmc05eng.html
History of the Mayans at:
http://www.cancunsteve.com/mayan.htm
http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Mayan_mathematics.html

http://www.civilization.ca/civil/maya/mmc05eng.html

Diversity, Interdisciplinary, and Heritage Connections

The lesson suggestions include provisions which are appropriate for all students and specially ELL students. Realia and demonstrations develop vocabulary through webquests, literature, and study of artifacts of the culture. Prior knowledge and background are enriched through the study of the historical context of the evolution of number systems while developmentally appropriate activities using manipulatives provide concrete examples which reinforce concept development. Exploring learning through various media such as drawing, painting, sketching, and creating collages addresses learning styles and promote creativity. Discussion about readings, webquests/field trips (actual or Internet), activities, and guest speakers prompt analytical and critical thinking as well as metacognition by encouraging students to verbalize their perceptions of learning. Interdisciplinary and cultural connections are established through historical and literature readings, discussions of economic and marketplace functions, and explorations of artistic and scientific contributions such as the Mayan calendar.

Appendix B

Annotated Bibliography of Mathematics/History Resources

Number Sense

Wardley, P, and P. White. (2003). THE ARITHMETICKE PROJECT: A COLLABORATIVE RESEARCH STUDY OF THE DIFFUSION OF HINDU-ARABIC NUMERALS.. *Family and Community History*, *6* (1), pp. 5-13.
Focuses on the Arithmeticke Project, a collaborative research study by Family and Community Historical Research Society members in England and Ireland designed to test the findings of a research on the diffusion of *Hindu-Arabic* numerals. Insight on the *Hindu-Arabic* numeral system, sources and methodology of the collaborative research study, and analysis of findings from the probate inventories.

Allard, Andre. "Hindu-Arab Roots of Medieval Europe." UNESCO Courier 46 (1993): 34-37. 8 Oct. 2009
Contends a written numeration system was transmitted to the West from India through the Arab

world. How counting frames (abacus) came to be superseded in the West by figures; The fingers a long-time rival of counting system; House of Wisdom, Baghdad, promoted cultural exchange with India in the ninth century; Influence of mathematical treatises written in 12th century.

Websites:

Sarcone, Gianna A. "Numbers' & Numeral systems' history and curiosities." Archimedes Lab. 1997. 8 Oct. 2009. <http://www.archimedes-lab.org/numeral.html>.
This is a very informative website that traces the history of numbers and shows visual representations of numbers that were used in Chinese, Mayas, Egyptians, and Europeans. Today's numbers, also called Hindu-Arabic numbers, are a combination of just 10 symbols or digits: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. These digits were introduced in Europe within the XII century by Leonardo Pisano (aka Fibonacci), an Italian mathematician. Before adopting the Hindu-Arabic numeral system, people used the Roman figures instead, which actually are a legacy of the Etruscan period.

O'Conner, J J. "Arabic mathematics: forgotten brilliance?" Arabic Mathematics. Nov. 1999. 8 Oct. 2009. <http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Arabic_mathematics.html>.
This website covers from the end of the eighth century to about the middle of the fifteenth century. The regions from which the "Arab mathematicians" came was centered on Iran/Iraq but varied with military conquest during the period. At its greatest extent it stretched to the west through Turkey and North Africa to include most of Spain, and to the east as far as the borders of China.

Books:

Smith, David E, and Louis Karpinski. The Hindu-Arabic Numerals. Boston: Dover Publications, 2004
Two distinguished mathematicians unite many fragmentary narrations in this concise history of the origin and development of Hindu-Arabic numerals. They recount the labors of scholars who studied the subject in different parts of the world, assess the historical testimony, and draw conclusions from the evidence.

Menniger, Karl, and Paul Broneer. Number words and number symbols: a cultural history of numbers. Cambridge: Dover Publications, 1992.
A classic study documenting the origin and meaning of numbers in various cultures. "The historian of mathematics will find much to interest him here both in the contents and viewpoint, while the casual reader is likely to be intrigued by the author's superior narrative ability." — *Library Journal.*

Measurement

Websites:

National Institute Of Standards And Technology, (Sep.). In *A Brief History of Measurement Systems* . Retrieved Oct. 7, 2009, from standards.nasa.gov/history_metric.pdf
Man understandably turned first to parts of his body and his natural surroundings for measuring instruments. Early Babylonian and Egyptian records, and the Bible, indicate that length was first measured with the forearm, hand, or finger and that time was measured by the periods of the sun, moon, and other heavenly bodies. The "yard" as a measure of length can be traced back to early Saxon kings. They wore a sash or girdle around the waist that could be removed and used as a

convenient measuring device. The word "yard" comes from the Saxon word "gird" meaning the circumference of a person’s waist. In 1790, in the midst of the French Revolution, the National Assembly of France requested the French Academy of Sciences to "deduce an invariable standard for all the measures and all the weights." From this Commission, the metric system was created and the rapid spread of the system coincided with an age of rapid technological development.

(2007, Nov. 18). In *A World History of Measurement*. Retrieved Oct. 7, 2009, from http://www.cftech.com/BrainBank/OTHERREFERENCE/WEIGHTSandMEASURES/MetricHistory.html

Although this website does not give an author (just says “Cool Fire Technology”) it offers the most comprehensive historical approach to measurement. Includes the history of how Babylonians, Egyptians, Greeks, Romans, French, British, and American’s all used and developed a measuring system. At the end of the web-site it lists significant dates in U.S. history.

Articles:

Nelson, Robert A. "Guide for Metric Practice." Physics Today 52 (1999): 11-13. 7 Oct. 2009 <http://web.ebscohost.com/ehost/detail?vid=11&hid=12&sid=43fc4956-b7fe-4863-9bfe-1338b0fc9c02%40sessionmgr4&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#db=aph&AN=2131986>.

This article discusses the origination of the International System of Units, and the Meter Convention that took place in Paris, May 20, 1875. This article also discusses the steps necessary in order to change any metric units. Today 28 states are members of the General Conference of Weights and Measures.

Books:

Roche, John. The Mathematics of Measurement: A Critical History. London: The Athlone Press, 1998.

The Mathematics of Measurement is a historical survey of the introduction of mathematics to physics and of the branches of mathematics that were developed specifically for handling measurements, including dimensional analysis, error analysis, and the calculus of quantities. Using an interdisciplinary approach and the insights provided by historical studies, Roche clarifies well-known difficulties in the mathematics of measurement, some of which have plagued scientists for over a century. The book is primarily intended for physicists and scientists from related disciplines such as mathematicians or meteorologists, however, the level and breadth of the treatment should also make it interesting for advanced undergraduates in these fields, as well as for historians and philosophers of science.

Heath, Robin, and John Michelle. The Lost Science of Measuring the Earth: Discovering the Sacred Geometry of the Ancients. Kempton: Adventures Unlimited Press, 2006.

Two leading researchers into ancient wisdom demonstrate that the earth’s dimensions were accurately known prior to 3000 BC. These astonishing findings, available for the first time, include a system of surveying and measure based on simple numerical and geometrical rules documents that remnants of this science still existed in medieval times when it became lost.

Geometry

Books:

Mlodinow, L. (2001). *Euclide’s Window: The story of geometry from parallel lines to*

Hyperspace. New York: Touchstone.
Narration of five diverse principles that define geometry's history. Mlodinow begins in the ancient world with examples in Euclidean geometry. He then moves into analytical geometry, developed by René Descartes in the early 1600s. Discovered in the 1800s by Carl Friedrich Gauss, Johann Bolyai, and Nikolay Ivonovich Lobachevsky, non-Euclidean geometry is also discussed. The book includes Albert Einstein's fourth dimension to space-time, discovered in the early 1900s, which shows that the presence of matter affects geometry by warping space and time. Mlodinow brings us to the present day in his discussions of string theory and M-theory, which suggest that space and time don't actually exist but are approximations of complex occurrences.

Heilbron, J.L. (2000). *Geometry Civilized: History, Culture and Technique*. Oxford: Oxford University Press.
With concise discussions and carefully chosen illustrations the author brings the material to life by showing what problems motivated early geometers throughout the world. Geometry Civilized covers classical plane geometry, emphasizing the methods of Euclid but also drawing on advances made in China and India. It includes a wide range of problems, solutions, and illustrations, as well as a chapter on trigonometry, and prepares its readers for the study of solid geometry and conic sections.

Articles:
Henk, J.M. (1993). 'The Bond with Reality Is Cut': Freudenthal on the Foundations of Geometry around 1900. *Educational Studies in Mathematics*, 25(1/2), 51-58.
This article discusses Freudenthal's contribution to our understanding of the development of geometry at the turn of the century and his style in historical research. Freudenthal published several articles on the history of geometry around 1900, in particular on Hilbert's innovative approach to the foundations of geometry.

Gray, J. (1998). The Foundations of Geometry and the History of Geography. *Mathematical Intelligencer*, 20(2), doi: 03436993
In this article, Gray examines look, first, at the early years *of* the axiomatisations *of geometry*; second, at the effect this philosophy *of geometry* had on the writing *of* the *history of* mathematics at the time. Gray analyzes the work of Moritz Pasch and the start of geometry in Germany and eventually spread of geometry to Italy.

Websites:
http://math.rice.edu/~lanius/Geom/his.html
Lanius, C. (2008). *History of Geometry*. Retrieved from http://math.rice.edu/~lanius/Geom/his.html
A Brief Synopsis of Geometry throughout history. Describes the contributions of Egyptians, Babylonians, and Greeks. Includes links to biographies of major contributors to geometry.

http://softsurfer.com/history.htm
Sunday, D. (2006). *A Short History of Geometry*. Retrieved from http://softsurfer.com/history.htm
This is a short outline of geometry's history, exemplified by major geometers responsible for its

evolution. Has the option to click on a person's picture or name for an expanded biography. Chronology starts with 30,000BC and ends in 2000AD.

<u>**Algebra**</u>

Web-sites:

Snell, M. In *The History of Algebra*. Retrieved Oct. 12, 2009, from http://historymedren.about.com/od/aentries/a/11_algebra_2.htm

This article is originally from 1911 Encyclopedia. This article states that the invention of Algebra dates back to 1700 B.C., if not earlier. The earliest Indian mathematician of whom we have certain knowledge is Aryabhata, who flourished about the beginning of the 6th century of our era. A period of mathematical stagnation then appears to have possessed the Indian mind for an interval of several centuries. The question as to whether the Greeks borrowed their algebra from the Hindus or vice versa has been the subject of much discussion. There is no doubt that there was a constant traffic between Greece and India, and a transfer of ideas is highly probable.

Lawrence, S. In *Al-Khwarizmi*. Retrieved Oct. 12, 2009, from http://www.smallsweetboffin.com/people/alkhwarizmi.htm

Al-Khwarizmi was one of the learned men who worked in the House of Wisdom, an academy in Baghdad. His interests lied in the fields of algebra, geometry, astronomy and geography. His now most famous work is that from which we got the name for algebra itself - *Hisab al-jabr w'al-muqabala* . The period in which Al-Khwarizmi lived and the House of Wisdom in which he worked, preserved for us most of the Greek and Byzantine mathematics and science that eventually led to the revival of learning in Europe.

Books:

Kleiner, I. (2007). *A History of Abstract Algebra*. Boston, Birkhauser

Prior to the nineteenth century, algebra meant the study of the solution of polynomial equations. By the twentieth century algebra came to encompass the study of abstract, axiomatic systems such as groups, rings, and fields. This presentation provides an account of the intellectual lineage behind many of the basic concepts, results, and theories of abstract algebra. The development of abstract algebra was propelled by the need for new tools to address certain classical problems that appeared unsolvable by classical means. A major theme of the approach in this book is to show how abstract algebra has arisen in attempts to solve some of these classical problems, providing context from which the reader may gain a deeper appreciation of the mathematics involved.

Brezina, C. (2006). *Al-Khwarizmi: The Inventor of Algebra*. New York, New York: Rosen Publishing Group.

Al-Khwarizmi is arguably the most important mathematician of the Middle Ages. He developed two distinct branches of mathematics, both of which owe their name to him: algebra and algorithms. This carefully crafted biography shines a long-overdue light on these achievements, documents Khwarizmi's contributions to geography and astronomy, and paints a picture of life in the ninth-century Muslim Empire. Supports history-social science context standards mandating exploration of intellectual exchanges and contributions of Muslim scholars, and their influence on the science, geography, mathematics, philosophy, and medicine of later civilizations.

Articles:

Katz, V.. (2007, Oct). Stages in the History of Algebra with Implications for Teaching. *Educational Studies in Mathematics*. 66(2), 185-201. Retrieved Oct 12, 2009, from EBSCOhost

This article takes a rapid journey through the *history of algebra*, noting the important

developments and reflecting on the importance *of* this *history* in the teaching *of algebra* in secondary school or university. Frequently, *algebra* is considered to have three stages in its historical development: the rhetorical stage, the syncopated stage, and the symbolic stage. But besides these three stages *of* expressing algebraic ideas, there are four more conceptual stages which have happened alongside *of* these changes in expressions.

Bowen, J. P. (1994, Jan. 11). In *A Brief History of Algebra and Computing*. Retrieved Oct. 12, 2009, from http://74.125.47.132/search?q=cache:F3coC4XAZowJ:utopia.duth.gr/~ap4225/ebooks/Maths/A%2520Brief%2520History%2520Of%2520Algebra%2520And%2520Computing.pdf+a+brief+history+of+algebra+and+computing&cd=2&hl=en&ct=clnk&gl=us

*Sorry this address is so long- I wasn't sure how to convert a text document to a PDF :/

This document journeys through English, Boolean, and recent developments in algebra programs. Cuthbert Tonstall (1474-1559) and Robert Recorde (1510-1558) were two of the foremost English mathematicians. They were the first mathematicians at the University of Cambridge. Recorde is considered the more important of the two because he introduced the equality sign (=).

Data Analysis and Probability

Websites:

Electronic Journal for History of Probability and Statistics. Retrieved Oct. 13, 2009, from http://www.emis.de/journals/JEHPS/index-2.html

The Electronic Journal for History of Probability and Statistics has a double vocation. It publishes original papers on history of both domains and also older documents of exceptional interest, and makes them available as downloadable files. They can be downloaded freely by anyone interested without previous registration or subscription.

Aldrich, J. (2009, May). In *Figures from the History of Probability and Statistics*. from http://www.economics.soton.ac.uk/staff/aldrich/Figures.htm

This website lists in chronological order over 200 mathematicians who contributed to the development of probability and statistics. The timeline starts in 1650 all the way to 1980+. The origins of probability and statistics were discovered through the mathematical games of chance and in the studies of mortality data. During the 1650-1700, the age of Scientific Revolution, some of the biggest contributors were Galileo and Newton. The period from 1708-1718 is often referred to as the "great leap forward" because of the numerous important contributions to the subject. Each time period is clearly mapped out on the website, I could write about the specific times/dates for pages!

Books:

Hald, A. (2003). *A history of probability and statistics and their applications before 1750*. New York, New York: Wiley-Interscience.

This book contains an exposition of the history of probability theory and statistics and their application before 1750 together with some background material. It covers three aspects of history: problems, methods, and persons. "The driving force behind the development of probability theory and statistics was pressure from society to obtain solutions to important problems for practical use, as well as competition among mathematicians." "Probability theory before 1750 was inspired mainly by games of chance. Dicing, card games, lotteries, public and private, were important social and economic then as today." Curiosity and economic interests led to mathematical investigations.

Todhunter, I. (2007). *A History of the Mathematical Theory of Probability: From the Time of Pascal to That of Laplace*. Cambridge, London: Macmillan and Co..

This book focuses on the mathematicians behind the theory of probability. Some mathematicians discussed include: Carden, Kepler, Galileo, Chevalier de Mere, and Pascal. Because many mathematicians never published their work, or never completed their research, many of their theories are listed without completion. However, Laplace was a mathematician whose full accounts of writing are given and explained.

Articles:

Doob, J. L. (1996, Aug). The Development of Rigor in Mathematical Probability. *The American Mathematical Monthly*. 103(7), 586-595. Retrieved Oct 13, 2009, from JSTOR

This paper is a brief informal outline of the history of the introduction of rigor into mathematical probability in the first half of this century. Specific results are mentioned only in so far as they are important in the history of the logical development of mathematical probability.

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