1990

National Assessment of Educational Progress in Mathematics: Analysis and Interpretive Remarks of Teh State of Mathematics Achievement

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NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS IN MATHEMATICS

ANALYSIS AND INTERPETIVE REMARKS

of

THE STATE OF MATHEMATICS ACHIEVEMENT

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NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS IN MATHEMATICS

...[We] can't pretend to not know what is known...

Joyce Carol Oates (1989)

The National Assessment of Educational Progress's Mathematics assessment (hereafter the Report) provides the nation, educational policy makers, and practitioners the opportunity to stop pretending not to know. The Report plainly confronts our own professional knowledge regarding mathematics education. In many instances it confirms much of what research has all ready described and what we have feared regarding mathematics education.

In some instances the Report agrees with Carl Glickman's view that "professionals have gone about the business of teaching and operating schools in ways they privately admit are not in the best interests of students." From this view one of the Report's most important contributions is that practitioners of mathematics education no longer can pretend to know and policy makers can no longer pretend not to know.

The purpose of this Report is to provide interpretative comments of the important findings in the 1990 National Assessment of Educational Progress in Mathematics.

Four questions guided our analysis: 1) what important findings were confirmed?, 2) what assumptions were challenged?, 3) how effective has the reform agenda been?, and 4) what new directions do the findings point toward? We framed our thoughts through the use of three concepts: Rhetoric, Reality and Remarks. Rhetoric refers to policy statements and research findings regarding mathematics education. Reality refers to findings of the NAEP 1990 math results. And, Remarks refers to our interpretive comments and conclusions regarding the structure, emphasis, delivery and broader impact of mathematics education. While we found the data fascinating and worthy of lengthy analysis, we chose to confine our remarks to three major categories and twelve conclusions.

A. The Report confirms the portrayal of a nation at-risk.

1. The Report confirms that our mathematics educational system is unable to educate the nation's youth to a level of performance capable of maintaining a leadership position among the world's developed nations.
2. The Report confirms that one third of the nation's youth are resistant to educational opportunities and are not achieving at levels that strengthen our national fabric.
3. The Report confirms that the reform agenda of the 80's has made modest improvements in producing the results the nation needs.

4. The Report confirms that family background is a major influence and perhaps the decisive one regarding student achievement.

B. The Report describes an antiquated core technology of teaching and learning in math education and supports recent efforts to restructuring the teaching and learning of mathematics.

5. The Report confirms and points to new directions needed in curriculum structure and emphasis.

6. The Report confirms and points to new directions needed in teacher education and training.

7. The Report confirms and points to new directions needed in instructional approaches.

C. The Report challenges assumptions that have been broadly accepted ...

8. The Report challenges that assumptions that expectations and perceptions are related to higher performance.

9. The Report challenges the assumption that resources do not relate to improved performance.

10. The Report challenges the assumption that private school students perform better than public school students.

11. The Report challenges the assumption that tracking is necessarily bad for students.

12. The Report challenges the assumption that teachers should be empowered to make instructional decisions and points toward the need for further study of the empowerment movement.

INTERPRETIVE ANALYSIS AND REMARKS

1. The Report confirms that we are a nation at risk.

Rhetoric. The Nation's goal is that by the year 2000 American students will be first in math and science achievement. The challenge is to develop an educational system second to none in the world, so all Americans are as well educated and as highly skilled as our competitors. The common goals are:

- o Performance of our highest achievers should be boosted to levels equal or above the performance of the best students everywhere.
- o Performance of our lowest achievers should be substantially increased to higher levels than current levels.
- o Average students should achieve what our best students achieve now.

Reality.

- o Most 4th graders should have reached level 200 (the primary curriculum). Yet, only 77% attain the level. *

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* The expectations for the four proficiency levels; 200, 250, 300 and 350 are described in appendix A.

- o Most 4th graders should have reached the 250 level (the upper elementary curriculum). Yet, only 11% attain the level. No 4th graders attained levels 300 or 350.
- o Many 8th graders should reach the 250 level. Yet, only 66% did reach the level. No 8th graders achieved the 350 level.
- o Most seniors should have reached the 300 level (the middle school curriculum). Yet, only 41% attain the level. Many seniors should have reached level 350 (the high school curriculum), yet only 5% attain the level. However, 91% of the seniors did attain level 250.

Remarks. The reality portrays an educational system at-risk; and when taken as a whole a nation at-risk. By broad agreement, the principal problem in mathematics education is the inefficient and narrow curriculum provided the great majority of students. The Report reinforces the notion that even under the best of current conditions it remains difficult to raise the achievement levels of our most proficient children. Considering the energy and effort devoted to improving educational opportunities over the past 25 years, the performance between the top and bottom was substantive and discouraging.However, even in our top performing schools, achievement is less than exceptional. While disadvantaged children are worse off; other students are not doing well either.

2. The Report confirms that 1/3 of the nation's youth are at-risk and not achieving at levels to strengthen our national fabric.

Rhetoric. The National Council of Teachers of Mathematics believe that the comprehensive education of every child is the most compelling goal of mathematics education ... its essential that schools and communities accept the goal of mathematics education for every child.

The goal recognizes that every child can learn regardless of background or ability. The Council believes that success requires:

- o A significant number of students from all races and ethnic groups should be found among our top performers.
- o Schools must be able to effectively educate children when they arrive at the school house door, regardless of variation in student interest, capacities or learning styles.
- o Efforts to restructure education must work toward guaranteeing that all students, regardless of background or disability, acquire the knowledge and skills necessary to succeed in a changing economy.

Reality.

- o There are an insignificant number of Blacks and Hispanics among our top performers at any grade level.
- o At 4th grade, 28% the of students did not attain level 200 (the primary curriculum) and are considered at-risk. 89% who didn't attain level 250 (the upper elementary curriculum) should be considered at-risk.
- o At 8th grade, 33% of the students did not attain level 250 indicating they are still struggling with elementary curriculum. 86% did not attain the 300 level (the middle school curriculum) which means that they may not make the transition from arithmetic driven curriculum to algebraic curriculum in middle schools.
- o At the 12th grade, 10% of the students were below level 251. Fifty-four percent were below level 300 indicating a lack of readiness for advanced math.
- o Thirty-three percent of the student population scoring at the lower proficiency levels contains 66% of the Black, 50% of Hispanic population and 66% of the students attending schools in disadvantaged urban communities. In 16 states, at least 56% of the Asians attended top performing schools. In the states a sizable and sometimes the majority of all students in advantaged urban centers where found among the top 1/3 of performing schools. In 24 states, no disadvantaged urban public schools were in the top performing schools.

- o Higher performing students take more math classes.
- o Asian students exhibit higher levels of performance at all levels followed by Whites, Hispanics and Blacks.
- o There are large performance increases between grades 4 and 8 by Asians. Twenty-three percent (23%) of Asians achieve level 250 at grade 4; 33% achieve level 300 at grade 8; and 31% achieve level 350 at grade 12.
- o Student performance in the bottom 33% of the student population is dramatically lower. In fact, 8th graders in top 33% had better performance than seniors in bottom 33% of the student population.
- o The lower performing states appeared to be concentrated in the southeast which parallels NAEP findings for the nations. Yet, students in the southeast in the pre-algebra or algebraic track in the 8th grade score at levels comparable to students in those tracks in the rest of the nation.
- o The gender differences favoring males are more pervasive in some states than the national results suggested.
- o In 27 states, the majority of Black public school 8th graders were attending lower performing schools.

Remarks. Children represented in the bottom one-third of our school population travel a predictable route in school; retention in grade, enrollment in remedial or special programs, placement in a bottom track, and dropping out in high school.

Research supports several findings in this Report. For example, schools enrolling low socio-economic status (SES) students emphasize more computation and less focus on applications and concepts than do other schools (Porter, 1988). Schools serving disadvantaged students tend to have less capable teachers and inadequate resources for mathematics education (Corcoran, Walker & White, 1988). Schools tend to have low expectations of disadvantaged student's ability to learn mathematics (Oakes, 1985; Good & Biddle, 1988). Disadvantaged student's families, and communities, are typically less able to provide concrete assistance to students for mathematics learning (Committee on Research in Mathematics, Science and Technology Education, 1985).

The education of at-risk disadvantaged children represented by these data are an economic drain and the biggest failure of American public education. Weigh the fact that the available labor force is contracting. Our total graduates were 2.8 million in '80, 2.5 million in '89, and projections are 2.2 million in '94. This contraction is exacerbated in two ways. First, 48% of the education pool will be composed of minority students, with Hispanics being the largest minority by the year 2000. This fact is important because it is well documented that some minorities have not benefitted from public education at a rate equal to the majority population.

Secondly, despite gains in graduation rates, one fourth of the students enrolling in our public schools still fail to graduate thirteen years later. Further, consider that these dropouts are a large part of the bottom third of our educational pools projected to fill 80 percent of the jobs available in the year 2000. It is from this bottom third; presently weakest in basic and employability skills; composed of minorities, women and immigrants, to whom we must look to meet our labor needs. The problem is dramatically brought home when we hear the Japanese extol the fact that they have the best educated bottom third of the labor pool of any country in the world.

The reality is that we have been skimming the cream off the top of the educational pool, serving the easiest to serve. Today, when students are scarce, it is imperative that we educate all of them. In tomorrow's world we will not be able to afford losing the abilities of a single student. The reality is stark; it is challenging; but not insurmountable. The task facing us is not to lower the educational expectations for at-risk children, but to provide the time and supportive services that will enable them to meet the standards. It means we will have to examine our fundamental expectations about what children can learn and how well they can perform academically. And, we will have to strive to create learning environments in which raised expectations for children can be met.

The growing low achievement for students in 33% of our schools documents growing feelings about poverty, ignorance, and despair in the heart of large cities. We suggest quickly moving on initiatives that address the educational needs of those who are most resistant to education. Some would argue that without a commitment to increase support services we will continue to fail students whose educational problems spring from poverty; they lack appropriate role models, have poor language development, suffer from cultural deprivation, or lack proper parental guidance.

3. REFORM AGENDA. The Report confirms that the rhetoric of the reform agenda of the 80's has made modest improvements in producing the results the nation needs.

Rhetoric. The 80's have seen the greatest surge of educational reform in the nation's history. There have been three distinct strategies pursued: reform, restructure and replacement.

The first wave was an effort to "make" schools better through new controls that targeted every aspect of the schools - curriculum, discipline, personnel, textbooks, instructional methods and more. They championed more rigorous academic curriculum through stricter graduation requirements, raising teacher quality, and holding schools accountable for requiring new formal tests of student performance.

Reformers in the late 80's believed that the first wave of reform did not get at the underlying causes of the problem. They believed reform efforts wouldn't increase student performance if the basic structure was left in place. These reformers thought schools had to be restructured by granting more autonomy and teachers had to be professionalized and empowered. Two key features distinguish restructuring from earlier reform efforts: 1) it is driven by a focus on student performance, based on the premise that all student must learn at higher levels; and 2) it is a long term commitment to fundamental systematic change.

Reformers of the 90's believe that all schools are shaped in pervasive and subtle ways by their institutional settings. The kind of organization and how effectively it performs are largely reflections on the institutional context in which it operates (Chubb & Moe, 1990). Therefore, they argue one must not only change the assumptions governing teaching and learning but also those controlling the school. Since this has proven to be very difficult and almost impossible from third wave view, they advocate replacing the current schools system with choice systems utilizing a strategy reminiscent of the development of the national interstate highway system. As Governor Lamar Alexander, when Chancellor of the University of Tennessee asserted, "We just need to start from square one and create new schools, not change the old ones."

Reality. The results of the math assessment indicate that early reform efforts have had little significant impact on how much students learn.

- o Some states, school systems, and schools have guaranteed that students take what appears to be a more rigorous curriculum and increased course work. But few students take advantage of the increased opportunities.
- o States have required higher teacher credentials particularly in the breadth of mathematics course work with modest gains at high cost.
- o Some states have increased mathematics time allocation but they have received little performance gain in return.

Remarks. It is our expectation that proponents of the reforming, restructuring, and replacing strategies will be vocal participants in the discussion the Report generates. Therefore, we proved some interpretive remarks to place their responses in context.

Reform. Reformers could assume, and the data will support, that the current mathematics education programs and performance could improve to some extent if appropriate resources were made available; teachers were better trained and properly used instructional strategies for different student populations; students reduced their levels of television watching, did their homework, had parents who graduated from high school and college and lived together in advantaged communities. However, these first wave reform efforts apparently have little effect on teaching and learning in the classroom. Measured by changes in what is taught and how, and student performance, the rewards for first wave reforms are few and the performance improvements they offer may have topped out. For example, students in higher performing schools must take more of every subject than students in lower performing schools. But, the actual difference is a little more than a year. It appears that requirements such as these are less important than first wave reformers suggest.

Restructure. Restructuring advocates can assume, and the data will support, that the curriculum should be restructured and teachers retrained in strategies to deliver the new curriculum. The restructuring proponents will argue that no matter how many resources are applied and how instructional strategies are changed, the current mathematics education system - its core technology - teaching and learning must be restructured. Restructuring goes far beyond traditional curricular improvements through attempts to change the underlying assumptions of mathematics education and the environment is which it is delivered. This strategy has the potential to address the most obvious weaknesses in mathematics education pointed out by the NAEP 1990 Mathematics Assessment.

Replacement. Replacement advocates could assume, and the data would support, that the current system cannot reform or restructure mathematics education dramatically enough to improve proficiency. These reformers would argue that no matter how many resources were made available the educational system cannot reform the mathematics program itself and/or efforts to restructure will be exceedingly slow. Therefore, the current public educational system should be replaced with an alternative public educational structure.

One could become overwhelmed when considering the economic imperative of present mathematics performance in light of a complex educational governance system. Historically, education is perceived as a national interest, a state responsibility and a local operation. Consider for a moment that there are over 15,000 local school boards, many elected, each with its own Chief Executive Officer and staff. Given the diverse, pluralistic nature of our society and the governance of education, many observers believe the educational system is difficult to change. What works one place may not work in another place. Add to the mix that there is continual stress between uniformity and pluralism in this nation resulting in uneven societal development. We are also ambivalent, as a nation, about setting a single set of educational goals. And, isn't it true that we are the only nation that believes pluralism and ethnic diversity are a competitive edge? The data reported tend to indicate that homogeneity in culture and values relates to increased performance.

In summary, there is a great deal of hope that our current romance with educational reform will be successful even though the early reforms have been inadequate to make the gains we seek as a nation in mathematics performance. There is an unlimited supply of new ideas and no shortage of political and social pressure to put these ideas on the agenda. However, this continual thesis, antithesis, synthesis cycle either confuses or freezes action at the classroom level. For example, its apparent that the goals for mathematics learning have not been constant been over time. Years ago it was basic skills and direct instruction that spurred policies (Purken & Smith, 1983). Today basic skills are concerns of teachers and parents but policy makers have shifted to higher order skills and problem solving. This constant shifting sends the wrong message to teachers. While the answers must be informed by those closest to the student; there must be a consistent policy framework that recognizes the realities and allows those within the system to work toward the solutions.

4. **FAMILY BACKGROUND/HOME ENVIRONMENT.** The report confirms that family background is a major influence and perhaps the decisive one in a student's math proficiency.

Rhetoric. American homes must be places of learning. Studies consistently show lower achievement for less advantaged, some minorities, females, and students of single parents. And, lower performance appears to be related to demography and home characteristics.

Reality.

- o There is a strong positive relationships between well educated parents and higher math proficiency in all areas. The impact of parents education was noticeable in every state.
- o Students with at least one parent graduating from high school out performed those where neither parent graduated from high school.
- o Lower performance is related to disadvantaged areas, poorly educated parents, fewer than two parents in house, watching excessive TV, unlikely to do homework or read for school. For example, at the 8th grade, North Dakota, Montana and other higher performing states had higher percentages of two-parent families, more newspapers and books in their homes, and low television viewership.
- o At all levels students who had access to greater numbers of reading and resource materials at home had higher performance.

Remarks. The results create a discouraging picture. The influence of family background is overwhelming. Most students seem unable to break the cycles of their parents. In fact some people believe that when we fail to educate the disadvantaged student we are in fact rejecting their parents. The strongest and most consistent finding in research on student achievement is that family background is a major influence perhaps a decisive one, mainly because income and education better equip a student for learning.

While individual socio-economic status (SES) is a known predictor of mathematics achievement, the school's SES is an even stronger predictor. Therefore, if we want to get to the root of the problem of school performance we must move out of the school into the school environment where many of the forces shaping school processes are found. For example, higher performing states tended to have fewer students in large city schools; fewer students in free lunch programs; fewer disadvantaged students; and less population density. Also, the educational experience of disadvantaged youth attending schools with higher percentage of students from low income families is known to be substantially different from similar students serving predominantly middle and upper income families (Knapp & Shields, 1990).

Teachers also report that the background of many children is more than an "excuse." As one Virginia teacher recently remarked, "In my previous assignment I had many students with college educated parents and homes in which there were books, travel and many enriching social and education advantages. These are advantages many students, where I now teach, do not have."

Further consider that in the national data higher performance is attributed to Asians and Whites. But, the state data shows that West Virginia with a White student population above 95% ranks among the low performing states. However, it is also a state which is characterized by low literacy levels in the adult population; low per capita income; low college going rates; and extreme ruralness.

It seems reasonable to conclude that family SES, not race, is a strong influence on educational performance. It is also likely that years of effort will be needed to lessen the impact of generations of poverty, accompanied by social and educational deprivation.

C. The report describes an antiquated core technology of teaching and learning that has little to do with today's reality. The current technology has resulted in ... teachers doing little better than their own mathematics teacher have done.

The environment in which teachers teach is as important to their success as the environment in which students learn is to them. But in the final analysis the focus for real and lasting change lies with the classroom teacher.

5. The report points to the need for new curricular structure and emphasis.

Rhetoric. Three years ago The National Council of Teachers of Mathematics released a report calling for sweeping changes in the mathematics curriculum. The Report envisioned the development of mathematical power for all students and that knowledge should emerge from experience with problems before they have learned the arithmetic operation which adults would use to solve such problems. Their opinion was that to reach the goal will require the creation of a curriculum and a teaching and learning environment that is very different from much of current practice.

The National Council of Teachers of Mathematics (NCTM) called for sweeping changes in the ways in which we inspire our children's knowledge of mathematics. NCTM assumes that teachers are key figures in changing the way mathematics is taught and learned. The curriculum goals envisioned require an environment in which teaching and learning are to occur that is very different from much of current practice. Their standards propose five shifts in the environment needed to improve mathematics teaching: 1) individuals to communities, 2) teachers as sole authority to logic and evidence as authority, 3) memorizing to reasoning, 4) mechanistic answer finding toward inventing and problem solving, and 5) isolated concepts and procedure to connecting mathematics, its ideals and application.

Reality.

- Performance is constant with what is taught.
- The substance of elementary and middle school mathematics content may be more problematic than allocation of instructional time. The arithmetic driven middle school curriculum reflects the elementary curriculum rather than the algebraic driven high school curriculum.

- o Percentage of students enrolled in advanced math courses is about the same as other countries.
- o At the 4th grade 86% of students receive a heavy emphasis on whole numbers, moderate emphasis on common factors and low emphasis on decimals and fractions. Sixty-six percent receive heavy emphasis on measurement. No differences were reported across ability levels.
- o At the 8th grade, low ability students received a heavy emphasis on number operations and measurement.
- o Course taking, especially in algebraic functions, is a positive indicator of achievement.

Remarks. The American curriculum is not a world class curriculum. It is greatly different than the curriculum provided in other economically developed countries. And, it is well behind that envisioned by the NCTM and needed by society. It does not emphasize the application of mathematics concepts central to a technological society. Still, performance lags the curriculum currently being taught in American classrooms let alone the curriculum of other developed countries.

The content of instruction is a key determinant of what children learn. Although mathematics associations have taken the lead in public statements, guides and frameworks, such as NCTM in 1980 and 1989, these statements have not led to a consensus on mathematics curriculum or instructional strategies. It is true that teachers, curriculum specialists, and state departments of education influence what is taught through state and locally developed curriculum guides and currently adopted texts which to a great degree provide the structure. However, even with a recommended curriculum structure, in virtually all American elementary and middle schools, it is the teacher who ultimately decides what students will study and how they will be taught. For example, a recent study indicates that teachers in national systems are more likely to teach the same things than those in locally controlled systems. In nationally controlled curriculum systems the amount of content taught depends less on teacher or student characteristics. In locally controlled curriculum systems, teachers were more sensitive to student level of mastery and abilities and number and length of math sessions. These findings lead us to believe that student centered teachers have lowered their expectations for some students (Stevenson & Baker, 1991).

We conclude that most American students receive the "plain vanilla" curriculum content. The "higher order thinking skills" are a national concern that is reemphasized by policy and research reports. But, as the data indicate, they are not taken seriously in our nations classrooms. For example, improving student reasoning ability is a universally accepted goal for mathematics education. However, the majority of students are not receiving a heavy emphasis in this area. Computational facility and learning rote skills continues to dominate the grade K-8 curriculum in spite of the many recommendations over the past 15 years to broaden what is viewed as basic mathematics skills.

One goal for our schools could be to maximize the probability that worthwhile content is being delivered to all students. The Report strongly suggests that students do less well on problem solving and higher order thinking skills primarily because these are not emphasized in the curriculum or instructional approaches currently used. The study of geometry, measurement, estimation, and problem solving while an integral part of the existing grade K-8 curriculum is all too often delayed for many students until they have mastered basic computational skills. In many instances this emphasis on procedural knowledge continues in the secondary curriculum where the teaching of algebra and geometry is dominated by more computational "grinding" of symbols and the memorization of isolated facts.

The rhetoric makes no distinctions among types of students. However, the Report indicates that these distinctions are being made in practice. Disadvantaged children, minorities and girls receive more instruction on mastery of basic skills and less on developing conceptual understanding and application (Porter, et at., 1988).

The National Governors Association states that the present system contains too many teachers who focus "largely on the mastering of discrete, low level skills and isolated facts." By doing so, the system denies opportunities for students to master subject matter in depth, learn more complex problem solving skills, or apply the skills they learn (Henry, 1990). This conclusion supports the assertion that teachers with limited mathematics backgrounds may restrict student learning which is not compensated for in later school years.

We feel that no matter how many resources are applied, instructional strategies are improved; the current curriculum will not lead us to higher numbers of students at the advanced proficiency levels. Students are consistently hashing the same information over and over again. In fact, a study of textbooks indicates a steady decrease in the amount of new material being introduced up to grade 8 where less than one third of material used is new (Flanders, 1987).

In the elementary and middle grades this structure is characterized by repetition and review whereby topics are introduced in a particular grade and reviewed in succeeding grades. This spiral approach provides students with multiple chances to learn the material. Yet, it only offers a small amount of new content to be introduced each year with little expectation of content mastery when first presented.

6. The report points to new directions in teacher eduction and training.

Rhetoric. The National Governors Association (NGA) indicates that 1) the number of teachers with a substantive background in mathematics must increase by 5%, 2) there must be an all out effort to recruit and prepare excellent math teachers, 3) there must be a major teacher retraining effort, and 4) the on-going training program for elementary teachers must be up-graded to learn of research developments, train to overcome ethnic and gender stereotypes, and develop new instructional strategies.

Reality.

- o Most 4th graders are taught by education majors; only 15% of the 4th grade students are taught by teachers with math certificates. Those taught by math majors had higher proficiency levels. Elementary math may be arithmetically bound because teacher don't have training to go beyond.
- o There was no relationship between methods courses taken by teachers and proficiency levels of students.
- o Hispanic and Black students are being taught by teachers with 10 years experience or less.
- o Forty percent of Black students are being taught by Black teachers at grade 4, 27% at grade 8.

- o Only modest evidence relates the amount of inservice training and student achievement. In the majority of the states, 8th graders having teachers with more inservice training performed better than those with less.
- o At 8th grade, greater teacher's course breadth and inservice is positively related to achievement. Students who were taught by teachers who majored in mathematics had the highest proficiency levels.

<u>Remarks</u>. We must build the capacity of people charged with educating our youth. The success of the educational system to meet the needs of its clients very much depends on the cooperation and attitudes of those who do the work.

Yet, the Report does not give clear guidance to policy makers. For many teacher variables, the relationship between proficiency and teacher background, suggest no consistent pattern. However, there is a tendency for better performing students to have teachers with stronger course work in mathematics and more inservice.

Still only a fraction more of the teachers in the high performing schools than in the low performing schools have worked in current institutions for at least ten years. Given the personnel rules which reward seniority rather than performance, it is quite plausible, as Chubb and Moe have noted, that teacher's experience and student achievement are unrelated (Chubb & Moe, 1990).

Rather than credentials, the answer may lie in the answer to the question, "Is low achievement a student or teacher problem?" In successful schools teachers don't work from prescriptive lists; they work from professional judgments. Teachers in higher performing schools have more efficacy. They believe that success is within their control (Chubb & Moe, 1990). The most important factor seems to be the teacher's willingness to take responsibility for student achievement. Effective teachers see student difficulty as something to be corrected. Less effective teachers see the difficulty as something over which they have little control (Brophy & Rohrkemper, 1981).

7. The report points to new directions in instructional approaches.

Rhetoric. Students can no longer be merely passive receptors of knowledge. They must have an active part in the constructing of their mathematics knowledge. This can only happen when students engage in activities which foster interaction, encourage intuition, and build upon conceptual knowledge rather than procedural knowledge. Instructional time can be better spent. Instead of teachers watching students complete paper and pencil worksheets, they should be catalysts who help students think mathematically.

Research supports use of concrete objects and hands on activities. The standards and reforms suggest changes in instructional approaches including use of technology, small group work, using manipulatives, and problem solving in the context of projects to improve proficiency.

Reality.

- o 8th graders spend an average of 3 1/2 hours per week on math instruction; 4th graders about 4 hrs. After 4 hours of instruction there is no increase in proficiency.
- o Thirty-three percent of students across all grade levels reported never working in small groups. Performance of students that work in groups once a week is about the same at every grade level. Performance of students who work in groups less than once a week is higher. Working in groups once a week related to higher scores for high ability groups. Working in groups less than once a week or never is related to higher scores for low and mixed ability groups.
- o Thirty-three percent of students at all grade levels report never working with manipulatives. There was no difference between high and moderate use of manipulatives. At 8th grade using manipulatives less than once a week related to higher scores for high ability groups and lower scores for low ability groups. There was no significant difference at 4th grade.

- o Confounding the data, the less emphasis on reports and projects the higher the performance at all grade levels. Using reports/projects once a week related to higher scores for high ability groups and lower scores for low ability groups.
- o Students who use calculators perform better. Using calculators more than once a week or less than once a week rather than never related to higher scores for high ability groups. And, using calculators less than once a week related to higher scores for low ability groups at grade 8. The proficiency levels associated with unrestricted use suggest that teachers of proficient students are more likely to use calculators in instruction. Sixty-two percent of the 4th graders and 39% of the 8th graders reported never using a calculator in their mathematics class. Although almost all students indicated they had access to a calculator at home, only about half had access to school-owned ones. Only 57% of the 4th graders, 44% of the 8th graders, and 30% of the 12th graders were categorized as having strong knowledge in how and when to use a calculator.
- o More proficient students are given more opportunity to use calculators. At grade 12 there is a positive relationship between calculator use and proficiency. However, fewer than one third of students were permitted unrestricted use of calculators. Teachers indicate that only 4% of 4th graders and 19% of 8th graders have unrestricted use in the

mathematics classroom. In every state there was a clear relationship between facility with calculators and performance on the assessment. Calculator usage was more prevalent in high performing states.

- o Working problems on worksheets related to lower scores in high ability students at grade 8 and higher scores at 4th grade.
- o Homework and higher scores are related to a point. For example, 15 minutes improves scores at grade 4, 45 minutes at grade 8, and 30 minutes at grade 12. There is no significant increase if students go beyond these times.
- o Twenty-five percent of 4th graders and 58% of 8th graders never use a computer. At grade 8, low and mixed ability students use computers more often than high ability. At grade 12 students with no access to computers score higher than those with access. Teachers appear reluctant to embrace the concept of computers. The computer is used for drill and review; but, its potential is in spread sheets and graphs. Computers are found more at the elementary than other levels.

Remarks. The findings confirm much of what is known about teaching. For example, in a century of public education, little structural change has occurred in classroom teaching (Cuban, 1984). Math instruction is still characterized by teacher explanation and individual work on paper/pencil assignment and textbooks. And, worksheets still comprise the primary instructional tools. The majority of classroom time is spent on teachers lecturing and students listening; students reading textbooks or filling-out work sheets. To observe classrooms now is to observe them fifty years ago (Goodlad, 1984).

In the face of this gloomy picture we also know that real projects with primary sources, real problems to solve, and real discussions show dramatic and significant gains in student achievements and motivation (Slavin & Madden, 1988). However, the Report indicates that these strategies are reserved for high ability students. And, the Report provides little evidence that would indicate NCTM recommended practices are more effective than other practices except for high ability students.

On the other hand, we know that effective teaching is not a set of generic practices, but is a set of context driven decisions about teaching. Effective teaching is a set of decisions about the use of a variety of classroom materials and methods used to achieve certain learning goals. According to the Report, practicioners have chosen to stay with familiar practices even though the performance results of students is confined to a narrow range.

A reliance on textbooks and worksheets emphasizing paper and pencil procedures is all too common. As a result students view mathematics as something that is routine and solitary. Textbooks, worksheets, manipulative materials, reports and projects, calculators and computers, as well as large and small group instruction have a place in the mathematics classroom. How each is utilized can be the difference between a dull, routine, passive learning experience and an inviting, energetic, active environment where students are engaged in problem solving and critical thinking.

In essence mathematics requires the manipulation of symbols or numbers. In the mathematics driven curriculum we manipulate numbers. In the algebraic driven curriculum we manipulate symbols. Instead of "grinding it out", calculators can change how we teach and how fast children are able to make the transition from numbers to symbols. In the upper elementary grades and certainly by the 8th grade, the question should not be whether or not calculators should be permitted, but how best can they be used in exploring mathematical ideas and problem solving. This is just another indication that low performing students may be hindered from learning new mathematics because computational facility with paper and pencil is still perceived as a prerequisite to using technology.

Knowing when to use a calculator is just as, if not more, important than knowing how to use it. What is encouraging is that students who used calculators properly had higher performance levels than those that didn't. The use of calculators must become more widespread in order to close the gap between those who "can" and those who "can't". Their use appears to provide more instruction time and effort can be spent on developing concepts, understanding processes, solving problems and applying mathematical ideas in the real world. We can no longer afford to not allow full use of calculators in mathematics classes. However, as the state data indicates, if the algebraic curriculum is not being delivered the effect of calculators on performance is negligible.

In summary, a high quality mathematics experience is not determined simply by the presence of computers, or calculators, or the use of small groups, manipulatives, or student discussions. The nature of the mathematical task posed and what is expected of students and the particular students needs, ability and achievement levels are critical aspects against which to judge the effectiveness of the instructional strategy utilized.

The Report data indicate that the desired strategies projected by reformers are being used differently and with different effects by teachers. Much more needs to be known before we adopt wholesale the latest "silver bullet" from the mathematics reformers, but their ideas are important to place the data in context.

Finally, at all levels textbooks and worksheets are the primary source of instruction. This must change and the textbooks must change. Texts and worksheets alike should be sources of information, applications and problems, not what currently exists, which are pages of drill and practice.

D. The Report challenges the assumptions that a) perceptions are related to performance, b) tracking is necessarily bad for students, c) that resources do not relate to improved performance, d) private school students perform better than public school students, and e) teacher empowerment is an appropriate reform strategy.

8. The Report challenges the assumption that perceptions are related to performance and points toward new directions worthy of study.

Rhetoric. Students, especially those who are disadvantaged, are low in self-esteem. This problem must be attached before one can expect students to learn.

Reality.

- o In general, the results support links between positive perception and learning. In most states there was a direct relationship between the degree of positive student perception and mathematics proficiency. Those with higher positive perception also have higher mathematics achievement. However, positive perception toward math changes from elementary to middle to high school.
- o The relationship did not hold because more students in lower performing states and fewer students in some of the higher performing states also reported positive attitudes. In fact, fewer students in higher

performing states reported stronger confidence. For example, in grades 8 and 12 more Blacks reported positive attitudes but do not take more advanced courses. Also, in grades 8 and 12 a higher percent of Blacks believe they are good in math.

Remarks. American expectations of what students can learn is relatively low. American parents more than Chinese and Japanese parents seem to believe that ability is "in born" and therefore hard work is less important (Stigler & Perry, 1988). Schools are no longer leading students - or teachers - to do their best. Schools have become undemanding. Teachers may hold strict or lenient standards for student achievement. They may teach to expose children to content or demand mastery (Bernstein, 1985).

The informal manifestation of what schools expect students to accomplish may be more important. Successful schools rank academic excellence higher than lower performing schools, 30% of all higher performing schools rank academic excellence as top priority and 12% of low performance schools.

These findings are potentially quite important because observers of effective schools repeatedly stress the great motivator that high expectations can have. A school will be more likely to find academic success if it makes academic excellence its major goal. Low performing schools focusing on basic literacy skills and good work habits are taking a less rigorous path. Naturally schools with resources find it easier to focus on academic excellence.

It also appears that the self-esteem proponents need to rethink their position. Research has shown that speeding the pace of mathematics does more good than harm. In controlled studies, students who didn't take an accelerated course show higher self-esteem rating and lower overall mathematics proficiency. The accelerated course takers had a lower self-esteem rating but higher test scores. Evidentially the course takers met stiff competition in the program giving them a realistic perception of their own capabilities.

The findings also contradict a National Science Foundation report of 1983 indicating that dislike for mathematics is more prevalent among minorities at the end of junior high. The Report portrays just the opposite, and also contradicts the findings that positive perceptions and performance are related. For example, as a result of the repeating nature of the curriculum, many students perceive mathematics as irrelevant and boring. Others, develop a false sense of accomplishment thinking they have learned mathematics when in actuality they are inadequately prepared to study higher mathematics.

Finally, on the one hand, it is difficult to improve performance without increasing enrollment. Yet, a large number of high school students opt out. For example, Black students report more often than any group that they are good at math. But these proficiency levels on the whole are the lowest. While the better students pursue more math classes the rest pursue less challenging work. Evidentally, we are not presenting a challenging curriculum to some minority students.

On the other hand, others believe they can't do mathematics when in fact what they can't to is compute primarily due to years of failure in performing computations. The challenge to introduce more rigorous course work has gone unheeded. In fact, many practitioners appear to believe that students cannot be taught to think mathematically without having fully mastered facts and procedures. These students are being denied the opportunity to develop skills needed to improve achievement.

9. The Report challenges the assumptions that tracking and ability grouping are bad for students and points toward new directions worthy of study.

Rhetoric. It is highly likely that rapid acceleration in high school mathematics may be influenced by tracking policies implemented in the early grades. Tracking of students into ability groups does not help students - the evidence shows no benefits are gained by tracking students into ability groups (Oakes, 1985; Slavin, 1987 and 1990; George, 1987). Higher achieving students do not do better when together, and lower achieving students do much worse when together. The SIM Study finds that American 8th graders are the most ability tracked group of 13 year olds in the world.

Almost all observers agree that the use of ability grouping can create vast differences in a students exposure or opportunity to learn numbers.

Reality.

- o The system of different math study for various students in our country begins early and is well established by middle schools despite concerns of tracking and balanced curriculum. Without exception, students receiving heavy instructional emphasis in algebra and functions had higher average proficiency than those receiving less instructional emphasis.
- o The curriculum begins to differentiate in middle schools. Typically 8th graders take one of three different courses: a) 8th grade math, b) pre-Algebra; or c) Algebra.
- o The less able are consistently in curricula consistent with elementary school levels. Hispanic, Blacks, and disadvantaged urban area youths are over represented in the sample that had not studied algebra and under represented in groups completing Algebra II.
- o Only 26% of the student population is grouped by policy. But, 60% are in classes of equal ability; 26% are grouped by ability at elementary; and 66% in middle school.
- o Higher ability students perform higher when grouped together. There were no performance differences for mixed and average ability groups.

Remarks. Many people believe a student's high school mathematics study is determined as soon as the initial ability group placement is made in the elementary or early middle grades. There also seems to be uniform agreement among researchers that curriculum tracking and focused, long term ability grouping are undesirable (Slavin, 1989). These results are not fully supported by the findings of the NAEP Report when a distinction is made between ability tracking and curriculum tracking.

Tracking students into ability groups is a common practice in American schools. The Report clearly depicts the influence of the practice on student performance in mathematics. Normally the extent to which students are grouped varies by school and grade level. In many instances entire classes are designated as high, average, or low ability. In others, students in regular classrooms are divided into small groups based on ability. It is not unusual for these different groups to pursue different curricula. Those students identified as having high ability often pursue an accelerated and/or enriched curriculum. Students placed in low or remedial groups, however, spend more time attempting to master a given concept or skill than students in average or high ability groups, thereby falling behind in a given year in the amount of content actually covered. It appears that in many cases this process continues throughout the elementary years, offering little opportunity for these designated low ability students to go beyond the computational based curriculum that now exists.

In effect, many students are being denied the opportunity to develop skills needed to improve achievement. For example, should thinking only be stressed after the mastery of facts and procedures? The Report portrays vast curriculum differences for high and low ability students. For example, at grade 4, 64% of high ability students receive a heavy emphasis on reasoning compared to 40% of low ability students. At grade 8, 69% of high ability students receive heavy emphasis on reasoning compared to 28% of low ability students.

On one hand it is easy to agree with the Rhetoric. At its worse, it clearly discriminates and perpetuates inequalities among students; higher track students tend to be White, wealthy and from highly educated families and lower track tend to be poor, Black and Hispanic, and from poorly educated families. Our worst fears are realized when both ability and curriculum tracking often result in resegregation of students by socio-economic status or race.

On the other hand, tracking into <u>curricular tracks</u> is another matter. Among policies and practices, academic course work displayed the largest difference between high and low performing schools. Apparently academic program participation has strong independent effect on achievement gains. Since academic programs promote academic achievement, the percentage of students enrolled in an academic track could be considered an indicator of the programmatic orientation of school.

Effectively organized schools seem more likely to place the typical student in an academic program. Chubb & Moe (1990) indicate that curricular tracking practices may account for as much as 30% of the total influence of school organization on achievement. Chubb and Moe go on to say that 1) aggressive tracking into academic programs distinguishes high performing schools from low performing schools, and 2) tracking is a more important determinant of performance than SES or student ability. Apparently high performing schools are able to track aggressively because the culture supports and encourages academic performance. However, if low performing schools tried to track they would probably meet resistance. This paradox poses quite a dilemma for policy makers and practitioners. Yet, one cannot dismiss the impact of curricular tracking on performance.

Further consider, that high performing countries take tracking one step further. Perhaps it's time to acknowledge the European standard of worker preparation that is school and industry based. In Germany and Denmark, leaders in this field, students pursue an education after primary school that leads either to University studies or technical education. Official apprenticeships are written with the employer, and students then go to school one or two days and work four days. They must pass theoretical and competency tests of craft unions in order to be certified as journeymen. The government funds education, the employer pays a salary, and the result has been low dropouts, a highly skilled work force and competitive products in the international market. This may be the carrot and stick for the one-third at-risk population needed to allow us to rise to international prominence.

10. The Report challenges the assumption that resources do not relate to improved performance and points toward new directions worthy of study.

Rhetoric. Many current observers say that the one thing mainstream educational reformers can always agree on is that more money needs to be spent. The latter day reformers view the performance problems of the public schools as having little or nothing to do with inadequate funding, and their problems cannot be corrected by digging deeper into the public purse. They are supported by numerous studies which concluded that economic resources of various kinds are unrelated to school or student performance.

Reality.

- Adequate resources are related to student achievement. At grade 4, students score higher whose teachers get needed resources, all or most of the time. At grade 8 resources are not related to scores in advantaged areas; positively related in rural areas; and conversely related in disadvantaged areas.

- o Advantaged areas get significantly more instructional resources. They get all or most of what they request. Disadvantaged or rural areas get most or some.
- o Thirty-three percent of the national student population attend classes with serious resource problems.
- o Most students in disadvantaged areas are educated in classrooms where teachers report receiving some or none of the resources needed.
- o In no state were more than 33% of the students in fully equipped classrooms. Again reflective of the national data, students in classrooms with more resources performed better.

Remarks. It makes sense that schools ought to operate more successfully when they have more resources they have to work with. And, the Report, supports this assumption - resources matter for school performance.

For example, schools serving higher percentages of disadvan- taged students often have fewer resources than schools serving high percentages of students from middle-income and high-income families (Porter, et. al., 1988). Schools in the top percentile of student achievement gains spend about 20% more per pupil than schools in bottom percentile. The funds identified in the Report are not used to pay higher salaries; but for more teaching resources. This differs from Chubb and Moe's finding that the main difference money buys is more teachers. Therefore, higher performing schools have lower numbers of students to teach.

In interpreting this finding it is wise to consider that the money in the Report is tied to instructional resources, not drained off by the school system for personnel or overhead needs. It is true that resources are limited but clearly disadvantaged schools are in most need, and that is where new resources should be invested.

11. The Report challenges the assumption that private education is better than public education and points toward new directions worthy of study.

Rhetoric. Coleman et al., concluded that private schools are academically more effective than public schools. Chubb and Moe (1990) also say private schools out perform public schools on the average, and also tend to spend less than public schools do in educating their students. They get better schools for the money.

Reality.

- At grades 4 and 8 private schools out performed the public schools.
- By grade 12 there were little differences in student performance. Students in private schools take more course work but proficiency is

not significantly higher than public schools. Fewer public school students take geometry but they score higher.

- o At grades 8 and 12 those students in pre-algebra and algebra tracks in public schools scored equally well to private schools.

Remarks. Some public schools are able to achieve comparable levels of proficiency to private schools, and this is most note-worthy. What seems to occur is that the public school population becomes more like the private school population after the 25% that drop out exit. Private schools appear to look more effective than they really are. In effect private education is in its simplest conception a form of tracking mainly accomplished through self selection.

12. The report challenges the assumption that teachers should be empowered to make curriculum and instruction decisions and points toward the need for further study of the empowerment movement.

Rhetoric. Studies show that formal qualities such as credentials, years of service, scores on competency tests, or teacher pay does not seem to make significant difference. What does seem to matter is a set of informal characteristics such as teachers who are organized as a community of professionals with autonomy that encourages and supports effective teaching (Chubb & Moe, 1990.)

For example, we know that outstanding teachers do not teach for external incentives but for the pleasure of seeing the effects of their decision on students. We also know the motivating factors for excellent teachers have to do with discretion and control over resources, time, instructional material and teaching strategies. Others state that "the nature of the curriculum, the choice of learning materials, and the means of testing students all work against the best interest of learning if they are imposed 'top down'" (MSEB, 1991). Therefore, teachers must be empowered to make the changes needed to improve mathematics education.

On the other hand, conclusions reached in the fifteen thousand hours of study indicate that in the less successful schools, teachers were often left completely alone to plan what to teach, with little guidance or supervision from their ... colleagues and little coordination with other teachers to ensure a coherent course from year to year (Rutter et al., 1979).

The conclusion drawn from these lines of reasoning is that we must decentralize decision making to the school and classroom levels if we expect to improve our educational system. Advocates of the teacher empowerment strategy contend that most of the reform proposals are never enacted because reformers fail to take into account two fundamental realities about schools: 1) teachers are professionals who are predisposed to do what is best for their clients, and 2) teachers have made an investment of time, energy, and personal and professional pride in their current practices. Because they have so much to lose, they will not change merely for the sake of change (Evans, 1991).

Reality.

- At grade 4, twenty-four percent of students are ability grouped through decisions of practitioners as opposed to mandated by school policy. In grade 8, less than one-half of the students were in mixed ability classes.
- Content emphasis is on computation rather than algebraic function.
- Teachers place limited reliance on instructional use of strategies designed to foster higher order skill development such as calculators, computers, group projects and reports.

Remarks. Examining the decisions made by practitioners as depicted in the Report causes us to question the expansion of the teacher empowerment strategy. We agree that the move towards professionalism is theoretically desirable; and ultimately correct. However, in reviewing the Report one can get uneasy with our "current love" affair with empowering those closest to the child to make the curriculum and instruction decisions that will lead to higher mathematics performance until a consensus is reached and accepted on curriculum content and emphasis for all children. The Report's findings suggest that it may take light years to make the educational improvements needed through this strategy.

Secondly, the Report demonstrates that the decisions being made are not in the best interest of our nation. Academics through the reports of learned societies talk about the importance of higher order skills, the use of groups, projects, calculators and computers. At the level of the crucible one gets a different view. Practitioners place limited reliance on the newer approachers and tend to remain with the tried and true instructional strategies of textbooks and worksheets. It is easy to blame regulations, curriculum guides and textbooks if one will dismiss the fact that almost always teachers make up the committees that develop these constructs.

Thirdly, comparing the Report's findings with the Rhetoric, one could assume that teachers have a misguided view of the best interest of their clients even though student performance does not reach the advanced level. They continue to use ability grouping, emphasize computation and employ outdated instructional approaches. We also have to consider the research which concluded that teachers in locally controlled systems modifying the curriculum focus on student ability and mastery (Stephenson & Baker, 1991).

Simply put, the restructuring reformers are projecting a new curriculum structure and emphasis, touting new instructional approaches. However, there is little evidence that teachers are changing either curriculum or instruction. In essence, the practitioners are telling the reformers that their strategies are only good for a small number of students. As Evans indicates, teachers have made an investment and take pride in current practice and are reluctant to change. After reviewing these findings they can no longer pretend not to know the effects of their decisions.

If the above perceptions are correct, it may make more sense to adopt combinations of top down and bottom up strategies and replacement strategies. For example, develop a consensus on content structure and emphasis and move quickly to get the information found in the Report in the hands of teachers in the hope that practitioners will change their practice. And, at the same time develop new schools with practitioners knowledgeable and committed to delivering the algebraic curriculum through appropriate instructional approaches to all children.

APPENDIX A

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Figure 1.1 Description of Mathematics Proficiency at Four Anchor Levels on the NAEP Scale

Level 200--Simple Additive Reasoning and Problem-Solving with Whole Numbers

Students at this level have some degree of understanding of simple quantitative relationships involving whole numbers. They can solve simple addition and subtraction problems with and without regrouping. Using a calculator, they can extend these abilities to multiplication and division problems. These students can identify solutions to one-step word problems and select the greatest four-digit number from a list.

In measurement, these students can read a ruler as well as common weight and graduated scales. They also can make volume comparisons based on visualization and determine the value of coins. In geometry, these students can recognize simple figures. In data analysis, they are able to read simple bar graphs. In the algebra dimension, these students can recognize translations of word problems to numerical sentences and extend simple pattern sequences.

Level 250--Simple Multiplicative Reasoning and Two-Step Problem-Solving

Students at this level have extended their understanding of quantitative reasoning with whole numbers from additive to multiplicative settings. They can solve routine one-step multiplication and division problems involving remainders and two-step addition and subtraction problems involving money. Using a calculator, they can identify solutions to other elementary two-step word problems. In these basic problem-solving situations, they can identify missing or extraneous information and have some knowledge of when to use computational estimation. They have a rudimentary understanding of such concepts as whole number place value, "even," factor," and "multiple."

In measurement, these students can use a ruler to measure objects, convert units within a system when the conversions require multiplication, and recognize a numerical expression solving a measurement word problem. In geometry, they demonstrate an initial understanding of basic terms and properties, such as parallelism and symmetry. In data analysis, they can complete a bar graph, sketch a circle graph, and use information from graphs to solve simple problems. They are beginning to understand the relationship between proportion and probability. In algebra, they are beginning to deal informally with a variable through numerical substitution in the evaluation of simple expressions.

Level 300--Reasoning and Problem-Solving Involving Fractions, Decimals, Percents, Elementary Geometric Properties, and Simple Algebraic Manipulations

Students at this level are able to represent, interpret, and perform simple operations with fractions and decimal numbers. They are able to locate fractions and decimals on number lines, simplify fractions, and recognize the equivalence between common fractions and decimals, including pictorial representations. They can interpret the meaning of percents less than and greater than 100 and apply the concepts of percentages to solve simple problems. These students demonstrate some evidence of using mathematical notation to interpret expressions, including those with exponents and negative integers.

In measurement, these students can find the perimeters and areas of rectangles, recognize relationships among common units of measure, and use proportional relationships to solve routine problems involving similar triangles and scale drawings. In geometry, they have some mastery of the definitions and properties of geometric figures and solids.

In data analysis, these students can calculate averages, select and interpret data from tabular displays, pictographs, and line graphs, compute relative frequency distributions, and have a beginning understanding of sample bias. In algebra, they can graph points in the Cartesian plane and perform simple algebraic manipulations such as simplifying an expression by collecting like terms, identifying the solution to open linear sentences and inequalities by substitution, and checking and graphing an interval representing a compound inequality when it is described in words. They can determine and apply a rule for simple functional relations and extend a numerical pattern.

Level 350--Reasoning and Problem-Solving Involving Geometric Relationships, Algebraic Equations, and Beginning Statistics and Probability

Students at this level have extended their knowledge of number and algebraic understanding to include some properties of exponents. They can recognize scientific notation on a calculator and make the transition between scientific notation and decimal notation. In measurement, they can apply their knowledge of area and perimeter of rectangles and triangles to solve problems. They can find the circumferences of circles and the surface areas of solid figures. In geometry, they can apply the Pythagorean theorem to solve problems involving indirect measurement. These students also can apply their knowledge of the properties of geometric figures to solve problems, such as determining the slope of a line.

In data analysis, these students can compute means from frequency tables, and determine the probability of a simple event. In algebra, they can identify an equation describing a linear relation provided in a table and solve literal equations and a system of two linear equations. They are developing an understanding of linear functions and their graphs, as well as functional notation, including the composition of functions. They can determine the nth term of a sequence and give counter examples to disprove an algebraic generalization.

Draft Descriptions Prepared Independently

by the Two Groups of Panelists

Group A

DRAFT DESCRIPTION

LEVEL 200

Students at this level have a beginning intuitive understanding of quantitative relationships among whole numbers, particularly in the area of additive reasoning. They can read and interpret basic mathematical symbols, add and subtract whole numbers without a calculator, perform straightforward multi-operations problems with a calculator, and compare four digit whole numbers. They can identify models that represent concepts, including region models of fractions. They can use addition and subtraction to solve one-step story problems and find the solutions to simple number sentences. They can read weight and volume scales, determine the value of coins, and read a ruler. They have a beginning knowledge of symmetry and can extend simple geometric patterns. They can read bar graphs and locate the coordinates on a grid.

LEVEL 250

Students at this level are developing their understanding of the quantitative relationships among whole numbers, to include multiplicative reasoning. They can select from among the four basic operations to solve one-step word problems, including some division problems requiring interpretation of remainders. They can use addition and subtraction to solve two-step word problems, some of which deal with money and apply their understanding of whole number place value. They can convert units of measure, use their understanding of multiplication to solve simple number sentences, and analyze simple problem-situations to determine extraneous or missing information. They can measure with a ruler and have a beginning understanding of basic geometric terms. They can complete bar graphs and pie charts, as well as use the information from graphs and scales to solve problems. They have an initial understanding of basic probability concepts and can evaluate simple algebraic expressions.

LEVEL 300

Students at this level demonstrate a beginning understanding of the relationships between fractions, decimals, and percents. For example they can locate fractions and decimals on number lines, reduce fractions, and recognize the equivalence between common fractions and decimals, including picture representations. They can interpret the meaning of percents less than and greater than 100 and apply the concepts of simple percentages to solve word problems. They show some indications of proportional reasoning and an extended ability to read mathematical symbols, including negative numbers and exponents. They can find the perimeter and area of rectangles in simple situations, recognize relationships among common units of measure, and use proportions to solve problems, including scale drawings and similar triangles. They understand the definitions and properties of geometric figures and can use visionalization skills with two- and three-dimensional figures. When given a set of data, they can compute the mean. They also can identify the probability of a simple event and have a beginning understanding of bias in sampling. They have an expanded facility in reading a variety of tables and graphs, including line graphs and pictographs. Students can identify a solution or solution sets and graph the solutions of simple linear inequalities. They can collect like terms in a simple algebraic expression and evaluate multiplicative algebraic expressions that include integers. They can find and apply the rule for functional relations and extend a numerical pattern. They can identify coordinates of a point and plot the point on a coordinate grid. They have some familiarity with algebraic identities.

LEVEL 350

Students at this level can recognize scientific notation on a calculator and transfer from scientific to regular notation. They can apply their knowledge of area and perimeter or rectangles (including squares) and triangles to solve problems. They can find the surface areas of solid figures, and apply their knowledge of area and circumference of circles to solve problems. They are familiar with the concept of precision in measurement. They can apply the pythagorean theorem to solve problems. They can also apply their knowledge of the properties of geometric figures to solve problems, such as determining the slope of a line, identifying the line of symmetry in a rotated figure, and identifying perpendicular line segments embedded in two-dimensional figures. They can compute weighted means from frequency tables, use a sample space to determine the probability of an event, and construct a sample space for a simple event. Students can identify an equation to describe a linear relation given in a table. They can solve a literal equation and a system of linear equations. They can simplify expressions involving powers of ten. They are developing an understanding of functions and their graphs, as well as functional notation, including composition of functions. They can determine the nth term of a sequence and give counterexamples to disprove a generalization.

Group B

DRAFT DESCRIPTION

LEVEL 200

Learners at this level can solve simple addition and subtraction problems with and without regrouping. Using a calculator, their problem-solving abilities extend to simple multiplication and division settings. They are able to solve one-step word problems involving translation from verbal to numerical form as well as interpret place value to order whole numbers. Using models, they are able to recognize fractions.

Students are able to identify common symmetrical figures. In measurement they can read a variety of scales, including the direct reading of a ruler. These learners also have some sense of gross measurement based on visualization. In data interpretation, they are able to read data from a bar graph. Given a visual shape pattern, they are able to recognize and extend the patterns. They are also capable of solving open sentences with missing addends.

LEVEL 250

Learners at this level can solve one-step multiplication and division whole number translation problems without calculators and most forms of one- and two-step whole number translation problems involving any operation with a calculator. They are able to handle decimal problems involving using money and apply place value concepts to decimal settings. The number concepts of factor, multiple, even, and odd are familiar, and whole number estimation skills are developing.

Students' measurement skills include the ability to use a ruler to measure objects, convert simple unit measures within a system, and translate verbal measurement descriptions to numerical representations in application problems. In geometry, students can draw a line of symmetry for common figures and demonstrate basic understanding of two- and three-dimensional shapes by relating vocabulary and elementary properties of shapes and solids in real-world contexts.

In data representation, they can sketch and interpret bar graphs and circle graphs. They also have an elementary understanding of the relationship of proportion and chance. In algebraic settings, these learners can solve open sentences involving subtraction. They are beginning to be able to deal informally with the concept of variable through substitution in the evaluation of expressions.

LEVEL 300

Learners at this level are able to interpret, represent, and operate with fractions and decimal numbers. Their knowledge of percent includes both percents greater than and less than 100% and they are able to perform multi-step problems involving simple calculations with percent. There is evidence of the beginning of proportional reasoning at this level.

These learners have use of exponential notation and are capable of performing simple algebraic manipulations such as simplifying an expression by collecting like terms, solving open linear sentences and inequalities by substitution, and checking and graphing an interval representing a compound inequality when it is described in words.

Students at this level also have the ability to operate with integers and graph points on the Cartesian plane. There is the emergence of students' ability to identify, establish, and apply simple functional relationships.

Learners at level 300 are also able to both calculate an average and use an average value to discuss a population total. They are capable of selecting and interpreting data from a tabular display, pictographs, and two-group comparison graphs. Their understanding of probability includes the calculation of relative frequency probabilities and relating such information to models. Some simple understanding of sample bias is also present.

LEVEL 350

Learners at this level have extended their knowledge of number and algebraic understanding to include exponential representations, including properties of exponents, both on paper and with calculators. They have command of percent in all forms, including markup and discount problems. These learners can also generate required terms to extend or describe patterns in linear sequences or establish a general formula. Other evidence suggests they have considerable understanding of functional notation and the ability to represent and interpret situations involving the graphs of linear functions. Their manipulation skills include the ability to solve a system of linear equations.

Students at level 350 are able to calculate group averages from a grouped frequency table as well as create the sample space for and calculate the probability of events involving more than one object.

Chapter Three

Mathematics Content Area Proficiency for the Nation and Subpopulations

Background and Description of the Mathematics Content Areas

In contrast to the previous chapters, which contain results on overall mathematics achievement for the nation and subpopulations, this chapter presents results separately for each of the content areas. In accordance with the mathematics framework underlying the assessment, results are presented for the following five content area scales: *Numbers and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; and Algebra and Functions.* In addition, as a result of the special paced-audiotape portion of the assessment conducted for the nation at grades 4, 8, and 12, results are presented for a sixth content area scale: *Estimation.*[18] The estimation questions included a broad array of situations, ranging from measurement, monetary value, and time estimates to the results of various numerical operations. The pacing format made any direct calculations of answers difficult, and thus, the information from the estimation study is intended to supplement that obtained from the numbers and operations as well as the measurement questions administered using the more traditional paper and pencil approaches. Brief descriptions of the six content areas are presented in FIGURE 3.1.

[18] To create each of the six content areas scales, the distribution for the total population was set to have a mean of 250.5 with a standard deviation of 50.

FIGURE 3.1
Description of Content Areas

Numbers and Operations

This content area focuses on students' understanding of numbers (whole numbers, fractions, decimals, integers) and their application to real-world situations, as well as computational and estimation situations. Understanding numerical relationships as expressed in ratios, proportions, and percents is emphasized. Students' abilities in estimation, mental computation, use of calculators, generalization of numerical patterns, and verification of results are also included.

Estimation

Estimation involving whole numbers, fractions, and decimals pervades most of the content areas in mathematics. Presented using a paced-tape procedure, questions assess students' abilities to make estimates appropriate to a given situation. Estimates take into consideration such factors as knowing when to estimate and whether to overestimate or underestimate in a particular problem.

Measurement

This content area focuses on students' ability to describe real-world objects using numbers. Students are asked to identify attributes, select appropriate units, apply measurement concepts, and communicate measurement-related ideas to others. Questions are included that require an ability to read instruments using metric, customary, or nonstandard units, with emphasis on precision and accuracy. Questions requiring estimation, measurements, and applications of measurements of length, time, money, temperature, mass/weight, area, volume capacity, and angles are also included under this content area.

Geometry

This content area focuses on students' knowledge of geometric figures and relationships and on their skills in working with this knowledge. These skills are important at all levels of schooling as well as in practical applications. Students need to be able to model and visualize geometric figures in one, two, and three dimensions and to communicate geometric ideas. In addition, students should be able to use informal reasoning to establish geometric relationships.

Data Analysis, Statistics, and Probability

This content area focuses on data representation and analysis across all disciplines, and reflects the important and prevalence of these activities in our society. Statistical knowledge and the ability to interpret data are necessary skills in the contemporary world. Questions emphasize appropriate methods for gathering data, the visual exploration of data, and the development and evaluation of arguments based on data analysis.

Algebra and Functions

This content area is broad in scope, covering algebraic and functional concepts in more informal, exploratory ways for the eighth- grade Trial State Assessment. Proficiency in this concept area requires both manipulative facility and conceptual understanding; it involves the ability to use algebra as a means of representation and algebraic processing as a problem-solving tool. Functions are viewed not only in terms of algebraic formulas, but also in terms of verbal descriptions, tables of values, and graphs.

Although results are not reported separately for the mathematical abilities dimension of the matrix comprising the framework underlying the assessment, each content area included *Conceptual Understanding, Procedural Knowledge, and Problem Solving*. These are briefly described in FIGURE 3.2.[19]

FIGURE 3.2
Description of Mathematical Abilities

The following three categories of mathematical abilities are not to be construed as hierarchical. For example, problem solving involves interactions between conceptual knowledge and procedural skills, but what is considered complex problem solving at one grade level may be considered conceptual understanding or procedural knowledge at another.

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples and counterexamples of concepts; can use and interrelate models, diagrams, and varied representations of concepts; can identify and apply principles; know and can apply facts and definitions; can compare, contrast, and integrate related concepts and principles; can recognize, interpret, and apply the signs, symbols, and terms used to represent concepts; and can interpret the assumptions and relations involving concepts in mathematical settings. Such understandings are essential to performing procedures ina meaningful way and applying them in problem-solving situations.

Procedural Knowledge

Students demonstrate procedural knowledge in mathematics when they provide evidence of their ability to select and apply appropriate procedures correctly, verify and justify the correctness of a procedure using concrete models or symbolic methods, and extend or modify procedures to deal with factors inherent in problem settings. Procedural Knowledge includes the various numerical algorithms in mathematics that have been created as tools to meet specific needs in an efficient manner. It also encompasses the abilities to read and produce graphs and tables, execute geometric constructions, and perform noncomputational skills such as rounding and ordering.

Problem Solving

In problem solving, students are required to use their reasoning and analytic abilities when they encounter new situations. Problem solving includes the ability to recognized and formulate problems; determine the sufficiency and consistency of data; use strategies, data, models and relevant mathematics; generate, extend, and modify procedures; use reasoning (i.e. spatial, inductive, deductive, statistical, and proportional); and judge the reasonableness and correctness of solutions.

[19] See Procedural Appendix for weighting of items in the content area by mathematical ability framework.