2016

The Physics of Bicycling

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The Physics of Bicycling (https://rampages.us/bishop/)

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The Great VCU Bike Race Book

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(//rampages.us/bishop/wp-content/uploads/sites/6872/2015/12/AuthorCollage1.jpg)

The left-hand picture was taken on Broad Street approaching the finish line at the end of the Team Time Trial Training. From left to right are James Lukas, Michael Barber, Andreea Popescu, and James Wingfield. The right-hand picture shows the faculty for the "Physics of Bicycling". From left-to right are Drs. Tom McMullen, Marilyn F. Bishop, and Muruges Duraisamy.

The "Physics of Bicycling" course gave the students an opportunity to learn some basic principles of physics in the context of an exciting world championship bicycle race, the UCI Worlds Championships of 2015. After studying the lecture notes elucidating the basic principles of physics as related to bicycles, the students met with the instructors, expecting to be told exactly what they needed to do during the race. Instead, they were asked to think about how to measure quantities such as velocity and acceleration, using common tools such as tape measures, smart phones, and video camera. There were questions raised about the accuracy of GPS data from sources such as Google Maps and Mathematica's Wolfram Alpha data bases. It was decided to meet first on the afternoon of Saturday, September 19, 2015 at the bottom of the Governor Street climb to decide what we needed to measure. We used a tape measure and took video and still pictures to determine length scales and the percent grade of the ascending and descending paths. We met again on the afternoon of Wednesday, September 23, for the Men's Elite Time Trials, on the afternoon of Friday, September 25, for the Men's Under 23 Road Race, on

Saturday afternoon, September 26, for the Women's Elite Road Race, and on Sunday, September 27, for the Men's Elite Road Race. Some of the students independently took photos and videos at other races. We thought our systematic methods of doing measurements would go unnoticed by those around us. But we were wrong.

Danger! Danger! Physicists at Work!

The time was 2 pm on Friday, September 25, 2015, for the Men's Under 23 Road Race. Dr. Bishop and I (Tamesh Surujpaul) were standing on the sidewalk directly in front of the Capitol Police building. Dr. Bishop and I got out a measuring tape and bent over to begin doing length measurements. Along the race route were barriers used to blockade the roads to prevent onlookers from interfering with the actual contestants. Fastened to these barriers were signs with words, like "SHIMANO" or "MAPEI". We thought we could use the length of these words as a scale in our photographs and movies. We were measuring the distance from one "SHIMANO" to the next, the length from an "S" to the beginning of the sign, the length of an "O" to the end of the sign, and the lengths of the barriers themselves. We had only finished our first measurements when a large man wearing a uniform emerged and walked over to us. Soon he stood towering over us, and we saw a patch over his heart entitled, "Capitol Police." We quickly stood up, only to realize that off in the distance to each side were other officers approaching rapidly. I began to grow nervous of what crime we could have possibly committed. The officer finally said, "What outfit are you with?" Fearlessly, Dr. Bishop replied, "We are with VCU, and we are teaching short courses associated with the race. This course is called "The Physics of Bicycling". Then she asked, "Is there a problem?" He replied that he had gotten a call. She explained, "If we know the distance between one SHIMANO and the next, and we know the time between frames, we can determine the velocity and acceleration of a bicycle." His eyes glazed over halfway through the explanation, as if someone had told him to build a spaceship with a toothpick and some string. He immediately apologized from making an assumption that we were up to no good. He signaled through his radio to the other officers, "They're ok; they're doing physics."

Carrying out the measurements

In the Physics of Bicycling course, we wanted to analyze different portions of the bike race to help determine how quickly the bikes were moving, how they might be accelerating or decelerating, what the angle the bikes were at going around a turn, as well as what the inclines of the hills for descending and ascending. In order to do this, we used some tools that almost everyone has. These tools included a tape measure, a smartphone, and a video camera. The tape measure allowed us to measure distances that we then could use as a scale, in conjunction with a video camera, to measure at what velocity the bikes were traveling at a point in time. The Smartphone with its different applications could be used as a video camera, as well as a way to find height using a barometer that is built into some devices. With these tools, we could hopefully collect data that would be as accurate as possible.

In order to analyze the video data, we used a computer program call Pasco Capstone. This program would allow us to load videos into Pasco and determine the speeds of the bikes. In this program, one first sets the scale using a virtual caliper that can be stretched across some object in the first frame of the video. The time scale is set by telling the program the number of frames per second. For instance, the usual video frame rate is 29.97 frames per second. Then one can go frame by frame in the video, clicking on the same part of the bicycle in each frame. The program then collects all the data either in a chart or a graph and calculates the velocity and acceleration.

The original length measurements were done on Saturday, September 19 with a cloth tape measure that Dr. Bishop happened to have in her purse. We measured from one "I" in Mapei to the next "I" a distance of 154.5 centimeters. We also measured from the "O" in Shimano to the next "O" to be 198.5 centimeters. Later, on Friday, September 35, we used a metal tape measure, which was easier to keep straight. Then we measured from an "S" to and "S" in Shimano to be 196 inches, from the front edge of "S" in Shimano to the edge of the banner to be 111/4 inches, and from the end of the "O" in Shimano to be 25 1/4inches. For the Mapei banner, we measured from an "I" to an "I" in Mapei to be 153 1/4 inches, and from the end of the banner to the top of the right-hand end of "I" to be 27.5 inches.

Pasco has a measurement tool which we originally used with the measurement of the barricade that was put up along the street and was found to be 220 cm in length. From this we felt we could then determine how fast the bikes were going based off of how long it might take for them to pass the barricade sections. However, we later found that this would be an inaccurate way to measure the bikes speed. The reason why is because the camera could capture the image of the bike passing by the barricade. However, the barricade was farther away so it would appear smaller than the bike which was closer to us. So when you use Pasco's ruler, the barricade would have the number of 220cm but the bikes wheel would have an incorrect measurement since it was visually closer which causes closer things to look bigger than things farther away. So from determining this we decided to measure a road bike wheel which should be standard for the bikes running in the race and found it to be 70 cm in diameter. From this we could then use Pasco to run frame by frame to determine how long it takes to travel a particular distance traveled and calculate a speed.

Some thoughts on the Richmond 2015 UCI bike race (James Lukas)

The Richmond 2015 UCI bike race for me was a very exciting and enjoyable time. From seeing the signs saying the race is coming September nineteenth through twenty-seventh to seeing it wrap up and end, it really was a good time. I was able to be at a number of different locations throughout many of the races over the course of the week due to having a bike. Another plus about going by bicycle for me was that I could get from one end of the race to the next without too much trouble since many roads were blocked off. The crowds at places did make it slightly harder to see the race with having the bike in hand but for the most part it was not a problem.

There were several places that I decided to watch from. One of which was over by Monument and Davis which was a turnaround point for many of the races and was a good spot to watch if you wanted to be very close to the racers without as much of a crowd. Plus there was a median you could get on and have the bikes passing by on both sides which allowed me to get some good pictures and video. I spent a fair bit of time at Governor St. which was the last big hill before the end of the race. It was very crowded at the top and bottom of the hill but not so much in the middle. The hill made for a good place to watch for the bicyclist to compete with each other to make it the top the fastest. Plus you could see how much they might or might not be struggling physically since the races were rather long. Another place that I spent a bit of time on Dock Street near the Canal. From there it was a long flat straight away that there were very few people around which while not as exciting to watch you could see them coming and going for a good distance. And finally there was Libby Hill. Libby Hill was as far as I am concerned a very challenging segment of the course due to there being turns going up the hill plus having a road surface made of cobblestones. Here you would see the racers struggling to get up the hill and stay ahead of their opponent. This made this part of the course the most interesting of all of them since they could gain or lose considerable time and position. Besides the finish this segment drew the biggest crowed especially on the last day of the race on Sunday which meant it was not so easy to get a good spot to view with a bike. Other things to mention about Libby hill was that a number of vendors were set up there, plus a screen to watch the race happen from other areas of the race.

Other Things that surprised me was the fact that there were so many cars and motorcycles involved with the race. The cars being made up of all Lexus's as support vehicles carried the teammate backup bikes in the event of a mechanical issue. The cars at times appeared to have trouble keeping up with the bike which made for some interesting scenes of which the cars would be squealing tires going around the turns to catch or keep up. Also I saw at least once where there was a close call with a vehicle almost hitting a bike or another car which kept it interesting.

Needless to say the UCI bike race while disruptive for some, was a great event. The volunteers and police and all the other participants really did a great job with running the show. I hope to see another big bike race in the future, hopefully in Richmond sooner rather than later. Because it was nice to see all the unity with all the people from different countries come and have a good time seeing a really great sport play out in front of them.

How we could have improved the measurements

Other stuff we could have measured would be the G-force of the bikes going around a corner including their acceleration and deceleration.

If we had access to the .GPX files for the individual racers we than could have considerably more data to analyze as well as much more data from different parts of the course that we were not at. This was due to not being able to be at more than one location at a time.

Another way we could have possibly measure the speed of the bikes would be by determining the gear ratio and which gear they were in with the revolutions per minute of the crank. You would need to know the wheel size as well but they seem to be standard across the race bikes. Or the other way would be to find the gear ratio and see how fast they were pedaling.

From the data we collected we determined we had too much data collected to analyze with the time constraints that are in place for the course.

Appendix A: Maps, Descriptions, and Times of the Race Courses Used in the UCI Bicycle Races

The following kml file can be opened in Google Maps or Google Earth in order to displace all the race courses used in the bicycle races. There are also the paths for the alternate VCU bus routes. Click on this link to download the file.

Bike Race and Bus Routes (//rampages.us/bishop/wp-content/uploads/sites/6872/2015/09/Bike-Race-and-Bus-Routes.kml)

Race Courses for the 2015 Union Cycliste Internationale (UCI) Road World Championships Richmond, Virginia, USA September 19-27, 2015

The description of the course was taken from the Richmond 2015 web site richmond2015.com. This description will help put the analyses in context.

Team Time Trial

The first set of races were the team time trials, with training taking place on Saturday, September 19, 2015.

ABOUT THIS COURSE

The Team Time Trial was reintroduced to the Road World Championships in 2012 for Elite Men and Women and remains the only discipline contested by "trade teams" (all other events at Road Worlds are contested by national teams). This course was 38.8 km long for both the women's and men's team trial training.

Teams rolled off from Henrico County at beautiful Lewis Ginter Botanical Garden, originally the Lakeside Wheel Club, founded in 1895 as a gathering spot for turn-of-the-century cyclists. The opening kilometers race through Richmond's historic Northside neighborhoods leading into downtown.

The course continued east of Richmond down rural Route 5, which parallels the 50-mile Virginia Capital Trail. The first few kilometers are scenic, flat, open roads that eventually narrow and wind through Richmond National Battlefield Park, a historic Civil War site.

The race re-entered the city through Shockoe Bottom, eventually making a hard right turn on Governor Street to ascend 300 meters. At the top, teams took a sharp left turn onto the false-flat finishing straight, 680 meters to the finish.

COURSE SCHEDULE

Saturday, 9.19.2015 | Team Time Trial Training | 9:00 a.m. to 12:00 p.m.

Sunday, 9.20.2015 | Women's Team Time Trial | 11:30 a.m. to 12:55 p.m.
(38.8 km)

Sunday, 9.20.2015 | Men's Team Time Trial | 1:30 p.m. to 3:35 p.m.
(38.8 km)

Time Trial Circuit

The time trial circuit was used for all the individual time trials, with training on Saturday afternoon, September 19. The time trials are individual competitions. Different groups did different numbers of laps.

ABOUT THIS COURSE

In 2015, Elite Women, U23 Men and Junior Men & Women will competed for Individual Time Trial championships on a technical course that winds through the city of Richmond.

Racers headed west from downtown to Monument Avenue, a paver-lined, historic boulevard that has been named one of the "10 Great Streets in America." From there, the course maked a 180-degree turn at N. Davis Avenue and continued in the opposite direction. The race cut through the Uptown district before coming back through Virginia Commonwealth University and then crossing the James River.

After a technical turnaround, the race comes back across the James and worked its way through downtown Richmond, eventually heading up the 300-meter-long climb on Governor Street. At the top, riders faced a false flat 680 meters to the finish.

Elite Women, U23 and Junior Men will each completed two laps of the circuit and Junior Women completed one lap. One lap of the circuit was 15 km.

COURSE SCHEDULE

Saturday, 9.19.2015 | Time Trial Training | 1:00 p.m. to 2:30 p.m.

Monday, 9.21.2015 | Women's Junior Time Trial | 10:00 a.m. to 11:10 a.m.
(1 lap, for a total of 15 km)

Monday, 9.21.2015 | Men's Under 23 Time Trial | 11:30 a.m. to 3:50 p.m.
(2 laps, for a total of 30 km)

Tuesday, 9.22.2015 | Men's Junior Time Trial | 9:30 a.m. to 1:05 p.m.
(2 laps, for a total of 30 km)

Tuesday, 9.22.2015 | Women's Elite Time Trial | 1:30 p.m. to 4:45 p.m.
(2 laps, for a total of 30 km)

Men's Elite Individual Time Trial

Men's Elite Individual Time Trial

This was the path of the Men's Elite Individual Time Trial. The path is 53 km (32.9 mi) long and has an elevation change of 245 m (803 ft).

ABOUT THIS COURSE

The Elite Men will began their "race of truth" 20 miles north of Richmond at Kings Dominion, Virginia's premier amusement park in Hanover County. Racers then sped past Meadow Event Park, home to the State Fair of Virginia and birthplace of thoroughbred racing legend Secretariat.

Racers headed south on long, open straights past the Hanover County Courthouse, the third oldest courthouse still in use in the U.S. and dating back to about 1740.

Long hills on Brook and Wilkinson roads brought the racers back into the city through Virginia Union University before turning into downtown. Nearly half the turns of the entire route fall within the closing kilometers, the second to last of which was onto the 300-meter-long climb up Governor Street. At the top, riders turned left and finally faced a false flat 680 meters to the finish.

COURSE SCHEDULE

Wednesday, 9.23.2015 | Men's Elite Individual Time Trial | 1:00 p.m. to 3:35 p.m.
(53 km)

Road Circuit

This is the circuit for the road races. Different groups did a different number of laps. The Men's Elite Road Race had an extra section starting at the University of Richmond. Training was done on Thursday, September 24, 2015. One lap around the circuit is 16.2 km

ABOUT THIS COURSE

All road races will take place on a challenging, technical and inner-city road circuit.

The peloton heads west from Downtown Richmond, working their way onto Monument Avenue, a paver-lined, historic boulevard that's been named one of the "10 Great Streets in America." Racers will take a 180-degree turn at the Jefferson Davis monument and then maneuver through the Uptown district and Virginia Commonwealth University.

Halfway through the circuit, the race heads down into Shockoe Bottom before following the canal and passing Great Shiplock Park, the start of the Virginia Capital Trail. A sharp, off-camber turn at Rocketts Landing brings the riders to the narrow, twisty, cobbled 200-meter climb up to Libby Hill Park in the historic Church Hill neighborhood.

A quick descent, followed by three hard turns leads to a 100-meter-long climb up 23rd Street. Once atop this steep cobbled hill, riders descend into Shockoe Bottom. This leads them to the final 300-meter-long climb up Governor Street. At the top, riders face a 680-meter false flat to the finish.

COURSE SCHEDULE

Thursday, 9.24.2015 | Road Circuit Training | 10:00 a.m. to 12:00 p.m.

Thursday, 9.24.2015 | Conquer the Cobbles Run | 7:00 p.m. to 9:00 p.m.

Friday, 9.25.2015 | Women's Junior Road Circuit | 10:00 a.m. to 11:50 a.m.
(4 laps, for a total of 64.9 km)

Friday, 9.25.2015 | Conquer the Cobbles Ride | 7:00 p.m. to 9:00 p.m.

Friday, 9.25.2015 | Men's Under 23 Road Circuit | 12:45 p.m. to 4:50 p.m.
(10 laps, for a total of 162.2 km)

Saturday, 9.26.2015 | Men's Junior Road Circuit | 9:00 a.m. to 12:15 p.m.
(8 laps, for a total of 129.6 km)

Saturday, 9.26.2015 | Women's Elite Road Circuit | 1:00 p.m. to 4:25 p.m.
(8 laps, for a total of 129.6 km)

Sunday, 9.27.2015 | Men's Elite Road Circuit | 9:00 a.m. to 3:40 p.m.
(16 laps, plus the 4.9 km stretch from the University of Richmond, for a total of 264 km)

Strava Data From Men's Elite Road Race

Strava is a website that contains data from athletes for various sports. Several of those who participated in the Men's Elite Road Race posted results containing quantities like average speed and maximum speed for various segments of the race course. Extracting information is a painful process. You have to click on a segment and wait for it to open up. It will show a map of the segment, with the approximate distances from the beginning of the race. From this information, you can determine which lap goes with that data. For the Men's Elite Road Race, there were 16 laps, and so many of the segments appear 16 times, one for each lap. Sometimes the segment for a lap gets dropped, perhaps because the device used lost it, or it didn't get uploaded to the site. I gathered some of the information in a Mathematica notebook (a .nb file that needs Mathematica to open it). It's in a rough stage right now, but make use of it if you like. Here is the link (Oct. 12, 2015):

Strava Segments (//rampages.us/bishop/wp-content/uploads/sites/6872/2015/10/Strava-Segments.nb)

If you want to investigate further, you need to sign up (for free) with Strava and login. When you click any of the links below, it will ask you to sign up or login, unless you're already logged in. Here are the links to various athletes' data for the Men's Elite Road Race, with their names, and their rank in the race:

Kwiatowski, Michal #8
//www.strava.com/activities/404609309 (//www.strava.com/activities/404609309)

Mezgec, Luka, Slovenia, #48
//www.strava.com/activities/401942575 (//www.strava.com/activities/401942575)

King, Benjamin, USA, #53
//www.strava.com/activities/405026810 (//www.strava.com/activities/405026810)

Yates, Adam, Great Britain #57
//www.strava.com/activities/401789248 (//www.strava.com/activities/401789248)

Duchesne, Antoine, Canada #61
//www.strava.com/activities/403098000 (//www.strava.com/activities/403098000)

Quintero, Carlos Julian, Colombia #68
//www.strava.com/activities/403544539 (//www.strava.com/activities/403544539)

Bole, Grega, Slovenia #86
//www.strava.com/activities/403386415 (//www.strava.com/activities/403386415)

Gesink, Robert, Netherlands #91
//www.strava.com/activities/401682744 (//www.strava.com/activities/401682744)

Woods, Michael, Canada #94
//www.strava.com/activities/403137270 (//www.strava.com/activities/403137270)

Thwaites, Scott, Great Britain #101
//www.strava.com/activities/402618689 (//www.strava.com/activities/402618689)

Voss, Paul, Germany #103
//www.strava.com/activities/401689003#9628713381 (//www.strava.com/activities/401689003#9628713381)

Martens, Paul, Germany, #104
//www.strava.com/activities/402618689 (//www.strava.com/activities/401705600)

Greipel, Andre, Germany #105
//www.strava.com/activities/404311946 (//www.strava.com/activities/404311946)

Rogina, Radoslav, Croatia #108
//www.strava.com/activities/401679092 (//www.strava.com/activities/401679092)

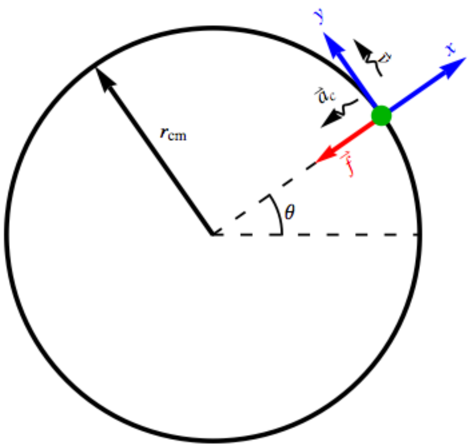
Here the link to a video of the Men Under 23 Road Race going around the corner.
//drive.google.com/file/d/0B7LcSVuGGFSnV3llak5XVTZ1dUk/view? (//drive.google.com/file/d/0B7LcSVuGGFSnV3llak5XVTZ1dUk/view?usp=sharing)

Here is the video embedded.

Appendix B: Bicycle motion going around a corner

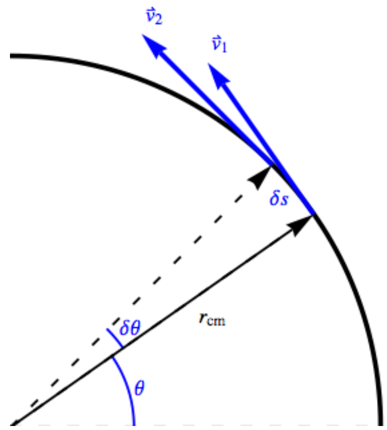
B.1 Center of Mass of Bicycle Going Around a Circle

Newton's first law tells us that a bicycle will keep going in a straight line at constant speed unless there is a force making it speed up or slow down, or making it go in a circle. Moving around a circle is an acceleration, since the velocity is changing direction. The force that enables a bicycle to turn is a frictional force perpendicular to the wheel. Suppose for the moment that we represent the bicycle by a green dot and draw the motion of the center of mass around the circle.



(//rampages.us/bishop/wp-content/uploads/sites/6872/2015/12/figB.1-1.png)

The direction of the velocity $\vec{v}$ is in the same direction that the center of mass of the bicycle plus rider is going. The acceleration $\vec{a}_c$, known as the centripetal acceleration is perpendicular to the velocity and points toward the center of the circle. To see how the acceleration comes from a change in velocity, consider two velocities along the circle that are separated by a short time δt, as shown in the picture below:



(//rampages.us/bishop/wp-content/uploads/sites/6872/2015/12/figB.1-2.png)

In this picture, the distance traveled is δs, which is the same as the arc length of the circle. The distance traveled is simply the radius r_{cm} times the angle $\delta\theta$,

$$\delta s = r_{cm}\delta\theta \quad , \tag{1}$$

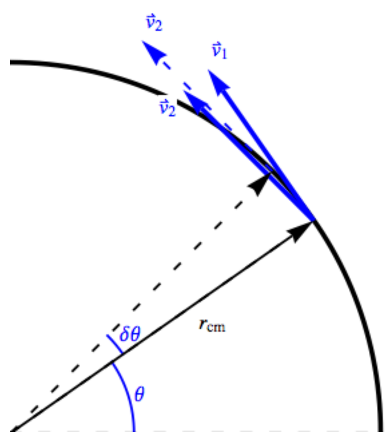
If we divide both sides by δt, we get the speed v, and in the limit that the time interval approaches zero, we have

$$v = \frac{ds}{dt} = \lim_{\delta t 0} \frac{\delta s}{\delta t} = \lim_{\delta t 0} r_{cm} \frac{\delta\theta}{\delta t} = r_{cm}\frac{d\theta}{dt} = r_{cm}\omega \quad , \tag{2}$$

where $\omega = \frac{d\theta}{dt}$ is magnitude of the angular velocity. To summarize,

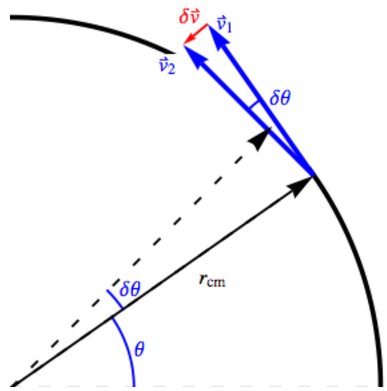
$$v = r_{cm}\omega \quad . \tag{3}$$

Now to subtract the two vectors $\vec{v}_1$ and $\vec{v}_2$, we must slide $\vec{v}_2$ over so that its tail is at the tail of $\vec{v}_1$, without changing its slope. Here is the picture with the two tails together.



(//rampages.us/bishop/wp-content/uploads/sites/6872/2015/12/figB.1-3.png)

The vectors $\vec{v}_1$ and $\vec{v}_2$ are separated by angle $\delta\theta$. To see difference of these two vecors, we will remove the original (dashed) $\vec{v}_2$ to show the difference in red as $\delta\vec{v}$. If we consider that the bicycle goes around the circle at constant speed, the magnitudes of $\vec{v}_1$ and $\vec{v}_2$ are the same and are both equal to v, the speed.



(//rampages.us/bishop/wp-content/uploads/sites/6872/2015/12/figB.1-4.png)

We now see that $\delta\vec{v}$ points toward the center of the circle. Its magnitude is given approximately by the length of the arc of a circle going from the tip of $\vec{v}_1$ to the tip of $\vec{v}_2$. Since the arc length of this circle is just the angle $\delta\theta$ times the radius v, we have

$$|\delta\vec{v}| = v\delta\theta \ \ . \tag{4}$$

The rate of change of the velocity, the magnitude of the centripetal acceleration, is this quantity divided by the time $\delta]t$, and so we have

$$a_c = \frac{|\delta\vec{v}|}{\delta t} = v\frac{\delta\theta}{\delta t} \ \ . \tag{5}$$

In the limit that the time interval becomes infinitesimal, this becomes

$$a_c = \frac{|d\vec{v}|}{dt} = v\frac{d\theta}{dt} = v\omega \ \ , \tag{6}$$

where $\omega = \frac{d\theta}{dt}$ is the magnitude of the the angular velocity of motion about the center of mass. We found earlier that $v = r_{cm}\omega$, which is the same as $\omega = vr_{cm}$, and so we can write the magnitude of the centripetal acceleration as

$$a_c = v\omega = \omega^2 r_{cm} \ \ . \tag{7}$$

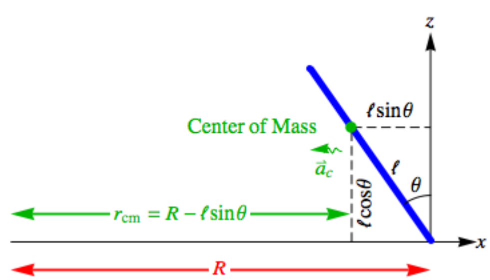
Since the centripetal acceleration is toward the center of the circle, using the x and y coordinates of the beginning of this section, we can write its vector form as

$$\vec{a}_c = -\frac{v^2}{r_{cm}} \ \ . \tag{8}$$

This form will be useful in the following section.

B.2 Stick Model of a Bicycle Going Around a Circle

To begin our study of the stability of a bicycle going a around a corner, we will first model the bicycle and its rider as a stick, with the center of mass of the stick at a distance ℓ along the stick above the ground. When the bicycle and rider go around a corner, they tilt toward the center of the circle at an angle θ relative to the vertical, as in the picture below. In this picture, we have represented the rider and bicycle as a thin blue stick with a green dot at the center of mass.

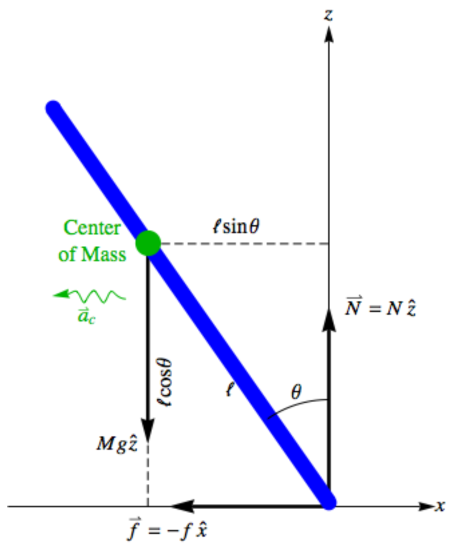


(//rampages.us/bishop/wp-content/uploads/sites/6872/2015/12/figB.2-1.png)

Notice that the wheel touches the ground at a distance R from the center of the circle describing the turn, but the center of mass is closer by a distance $\ell\sin\theta$. Therefore, the centripetal acceleration is

$$\vec{a}_c = -\frac{v^2}{r_{cm}}\hat{x} = -\frac{v^2}{R - \ell\sin\theta}\hat{x} \ . \tag{9}$$

Now let's look in detail at the forces on this model of the bicycle and rider, where we zoom in on the position of the bicycle.



(//rampages.us/bishop/wp-content/uploads/sites/6872/2015/12/figB.2-2.png)

The forces are shown on this diagram, with the weight Mg acting at the center of mass, and the ground pushing straight up on the wheel with the normal force N and pushing to the left along the ground with a frictional force f. The sum of the forces along the z direction must add to zero, since there is no acceleration in the vertical direction. This is

$$\sum F_z = N - Mg = 0 \ . \tag{10}$$

Therefore, we find that the normal force is given by

$$N = Mg \ . \tag{11}$$

In the horizontal direction, along the x-axis, there is a frictional force f to the left, and there is a centripetal acceleration $a_c = \frac{v^2}{R}$, also to the left. Therefore, the sum of the forces along x must be equal to the mass M times that acceleration.

$$\sum F_x = -f = -Ma_c = -\frac{Mv^2}{R - \ell\sin\theta} \ . \tag{12}$$

Thus, the frictional force is

$$f = \frac{Mv^2}{R - \ell\sin\theta} = \frac{Mv^2}{1 - \frac{\ell}{R}\sin\theta} \ . \tag{13}$$

where the factor $(1 - \frac{\ell}{R}\sin\theta)$ is approximately equal to one if the distance ℓ is much less than the radius of the turn R. The vertical and horizontal forces obeying Newton's second law do not give us enough conditions to ensure stability. We also have to make sure that there is no tendency for the bicycle to start doing somersaults. Therefore, the sum of the torques about the center of mass must be zero. The first torque is that due to the normal force $\vec{N}$, and its magnitude is given by the moment arm $\ell\sin\theta$ times the magnitude N of the normal force. The direction is out of the page, gotten by using the right-hand rule, which is the negative y direction. The other torque is that due to the frictional force $\vec{f}$, and its magnitude is given by the moment arem $\ell\cos\theta$ times the magnitude f of the frictional force. From the right-hand rule, this is into the page, which is the positive y direction. We can therefore write

$$_{aboutcm}\vec{\tau} = -N\ell\sin\theta\hat{y} + f\ell\cos\theta\hat{y} = 0 \ . \tag{14}$$

We can now substitute our expressions for the magnitudes of the normal force and frictional forces, N and f, to obtain

$$-Mg\ell\sin\theta\hat{y} + \frac{Mv^2}{R}\left(\frac{1}{1 - \frac{\ell}{R}\sin\theta}\right)\ell\cos\theta\hat{y} = 0 \ . \tag{15}$$

Dividing through by $-Mg\ell\cos\theta$, we have

$$\left[\frac{\sin\theta}{\cos\theta} - \frac{v^2}{Rg}\left(\frac{1}{1 - \frac{\ell}{R}\sin\theta}\right)\right]\hat{y} = 0 \ . \tag{16}$$

Therefore, we can get an expression for $\tan\theta = \frac{\sin\theta}{\cos\theta}$, which is

$$\tan\theta = \frac{v^2}{Rg}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right) \quad . \tag{17}$$

Therefore, the angle will be greater for higher speeds, and it will be smaller for large turning radius R. For a large turning radius, the factor $\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)$ has a negligible effect on the result. Otherwise, it would tend to make the angle slightly larger.

B.3 Unicycle Model of a Bicycle Going Around a Curve

In the previous section, we treated the bicycle and the rider as a stick going around a curve and obtained an expression for the angle of tilt required for stability. In that simple model, we ignored the fact that the bicycle wheels rotate. The significance of this is that a rotating wheel has an angular momentum along its axle, and if one turns the wheel, one changes the direction of the angular momentum. A change in angular momentum requires a torque. Including this effect changes the torque equation we wrote down in the previous section, and would presumably change the angle. Because a bicycle is rather complicated when going around a curve, since the front and back wheels point in different directions, it is desirable to find a simple model that will give an idea of the importance of this effect. If the effect is small, it doesn't make sense to do a detailed calculation to include it. Therefore, in this section, we will approximate the bicyle by a unicyle that has a wheel radius r_w. We will retain all the other dimensions in the problem. Therefore, we first need to find the angular momentum of a wheel going around its axle. Because the wheel rolls without slipping (or we hope it does), the distance along the circumference of the wheel that touches the ground is the same as the distance traveled along the ground. For that reason, the speed of the bicycle is given by

$$v = r_w\omega_w \quad . \tag{18}$$

where ω_w is the angular velocity of the wheel around its axle. Note that this is different than the angular velocity $omega$ of the center of mass around the center of the turn. The next step is to find the angular momentum of the wheel $\vec{L}_w$. The magnitude of the angular momentum of the wheel about its axis is given by

$$L_w = I_w\omega_w \quad , \tag{19}$$

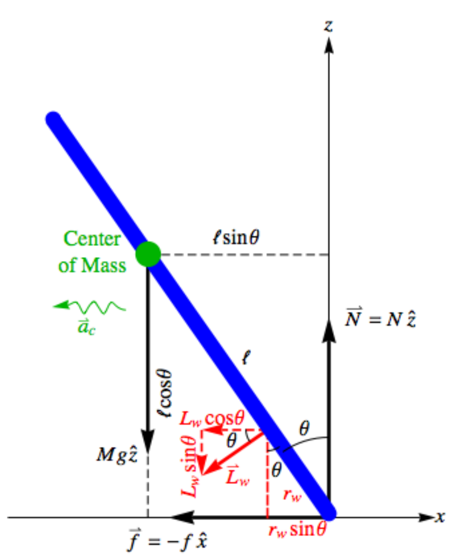
where I_w is the momentum of inertia of the wheel about its axle. In the simplest approximation, we could assume that all the mass of the wheel is in its tire, and then we would maximize this effect. In that case, if the mass of the wheel is m, the moment of inertia would be

$$I_w = mr_w^2 \quad , \tag{20}$$

Rather than assuming this form, we will work through the derivation until the end and insert the moment of inertia at the last step. Using the rolling without slipping condition, we can write the magnitude of the angular momentum of the wheel in terms of the speed of the bicycle as

$$L_w = I_w\frac{v}{r_w} \quad . \tag{21}$$

and this way, we will have expressions the radius r_w of the wheel and the speed of the bicycle. The direction of the angular momentum of the wheel is found by the right-had rule, wrapping the fingers of the right hand in the direction of rotation of the wheel, with the extended thumb giving the direction of the angular momentum. The next step is to add the radius of the wheel and the angular momentum to our picture.



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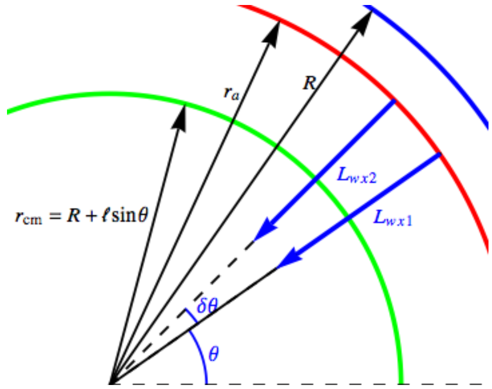
The angular momentum of the wheel can be written in vector form as

$$\vec{L}_w = -L_{wx}\hat{x} - L_{wz}\hat{z} = -L_w \cos\theta \hat{x} - L_w \sin\theta \hat{z} \ \ , \tag{22}$$

In going around a corner, the vertical z component of the angular momentum L_{wz} does not change, since the bicycle will keep the same tilt as long it is on the circular path. However, the horizontal x component L_{wx} will change. Let's go back to the picture of the bicycle on the circular path to see what this change is. We define the distance from the center of the turn to the axle of the wheel to be

$$r_a = R - r_w \sin\theta \ \ , \tag{23}$$



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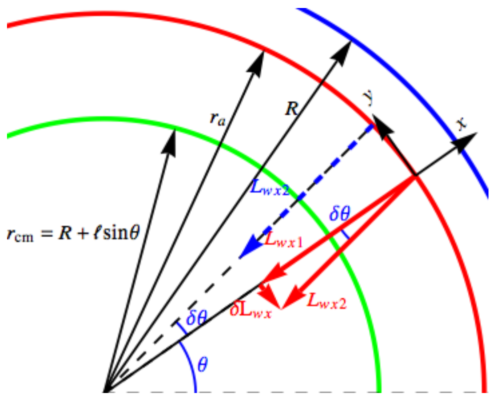
Again, we need to put the tails of the vectors together in order to subtract them, and so we have the following picture.



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We see that the change in angular momentum is in the $-\hat{y}$ direction, and the magnitude is given by L_{wx} times the angle $\delta\theta$,

$$\delta\vec{L}_w = -L_{wx}\delta\theta\hat{y} = -L_x \cos\theta\delta\theta\hat{y} \ \ . \tag{24}$$

If we divide both sides by δt, we obtain the rate of change of angular momentum,

$$\frac{d\vec{L}_w}{dt} = -\lim_{\delta t\to\infty} \frac{\delta\vec{L}_w}{\delta t} = \lim_{\delta t\to\infty} L_{wx}\frac{\delta\theta}{\delta t}\hat{y} = -L_w \cos\theta \frac{d\theta}{dt}\hat{y} \ \ . \tag{25}$$

Substituting $\omega = \frac{d\theta}{dt} = \frac{v}{r_{cm}}$, we have

$$\frac{d\vec{L}_w}{dt} = -L_w \frac{v}{r_{cm}} \cos\theta\hat{y} \ \ . \tag{26}$$

Now we can substitute $L_w = I_w \frac{v}{r_{cm}}$ to obtain

$$\frac{d\vec{L}_w}{dt} = -I_w \frac{v^2}{r_{cm}^2} \cos\theta\hat{y} \ \ . \tag{27}$$

where, as before, $r_{cm} = R - \ell\sin\theta = R(1 - \frac{\ell}{R}\sin\theta)$. The change in angular momentum about the center of mass is the same as the change in angular momentum about the axle of the wheel, if we ignore rotation of the bicycle and wheel about their centers of mass during the turn. Therefore, we have

$$\frac{d\vec{L}_{cm}}{dt} = \frac{d\vec{L}_w}{dt} = -I_w \frac{v^2}{r_{cm}^2} \cos\theta\hat{y} \ \ . \tag{28}$$

Now we need to set the torque about the center of mass equal to the change in angular momentum about the center of mass, and we have from the previous section,

$$\sum_{aboutcm} \vec{\tau} = -N\ell \sin\theta \hat{y} + f\ell \cos\theta \hat{y} = \frac{d\vec{L}_{cm}}{dt} = -I_w \frac{v^2}{r_{cm}^2} \cos\theta \hat{y} \quad . \tag{29}$$

We can now substitute our expressions for the magnitudes of the normal force and frictional forces, N and f, to obtain

$$-Mg\ell \sin\theta \hat{y} + \frac{Mv^2}{R}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\ell \cos\theta \hat{y} = -I_w \frac{v^2}{r_{cm}^2} \cos\theta \hat{y} \quad . \tag{30}$$

Equating the coefficients of $\hat{y}$, changing the sign of both sides, and substituting for r_{cm} we have

$$Mg\ell \sin\theta - \frac{Mv^2}{R}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\ell \cos\theta = I_w \frac{v^2}{r_{cm}^2} \cos\theta \quad . \tag{31}$$

Dividing both sides by $Mg\ell \cos\theta$, we obtain

$$\tan\theta - \frac{v^2}{gR}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\ell \cos\theta = \frac{v^2}{gR}\frac{I_w}{M\ell R\left(1-\frac{\ell}{R}\sin\theta\right)^2} \quad . \tag{32}$$

Isolating $\tan\theta$, we have

$$\tan\theta = \frac{v^2}{gR}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\left[1+\frac{I_w}{M\ell R\left(1-\frac{\ell}{R}\sin\theta\right)^2}\right] \quad . \tag{33}$$

Now, using the approximation for the moment of inertia of the bicycle wheel, $I_w = mr_w^2$, we have

$$\tan\theta = \frac{v^2}{gR}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\left[1+\frac{mr_w^2}{M\ell R\left(1-\frac{\ell}{R}\sin\theta\right)^2}\right] \quad . \tag{34}$$

Writing the second term in square brackets to emphasize ratios of like quantities that are small compared with one, we have

$$\tan\theta = \frac{v^2}{gR}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\left[1+\left(\frac{m}{M}\right)\left(\frac{r_w}{\ell}\right)^2\left(\frac{\ell}{R}\right)\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\right] \quad . \tag{35}$$

We see that there are many factors less than one. The first is the ratio of the mass of the wheel to the mass of the bicycle plus rider, $\left(\frac{m}{M}\right)$. The second is the ratio of the radius of the wheel to the distance of the center of mass to the bottom of the wheel, $\left(\frac{r_w}{\ell}\right)$, and this factor is squared. The third is the ratio of the distance of the center of mass to the bottom of the wheel to the radius of the turn $\left(\frac{\ell}{R}\right)$. This last factor also makes the factor $\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)$ nearly unity. Therefore, we see that the angle is fairly accurately determined by the stick model of the bicycle plus rider.

To see the size of the angle, we can start with

$$\tan\theta \approx \frac{v^2}{gR} \quad . \tag{36}$$

We have not mentioned the limitation imposed by the coefficient of friction between the bicycle tire and the road surface. We know that the frictional force is

$$f = \frac{Mv^2}{R-\ell\sin\theta} = \frac{Mv^2}{R}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right) \quad . \tag{37}$$

The maximum frictional force is given by

$$f_{max} = \mu_s N = \mu_s Mg \quad , \tag{38}$$

where μ_s is the coefficient of static friction between the bicycle tire and the road. Substituting this into the expression for the friction, this will limit the speed,

$$\mu_s Mg = \frac{Mv_{max}^2}{R}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right) \quad , \tag{39}$$

Dividing both sides by M and multiplying both sides by $R\left(1-\frac{\ell}{R}\sin\theta\right)$, we obtain

$$v_{max}^2 = \mu_s Rg\left(1-\frac{\ell}{R}\sin\theta\right) \quad . \tag{40}$$

This assumes, of course, that the factor $\left(1-\frac{\ell}{R}\sin\theta\right)$ does not have much effect on determining the maximum speed. Now we can put this maximum speed into the expression for $\tan\theta$ to get an expression for the maximum possible angle.

$$\tan\theta_{max} \approx \frac{\mu_s Rg\left(1-\frac{\ell}{R}\sin\theta_{max}\right)}{gR}\left(\frac{1}{1-\frac{\ell}{R}\sin\theta_{max}}\right)\times \times \left[1+\left(\frac{m}{M}\right)\left(\frac{r_w}{\ell}\right)^2\left(\frac{\ell}{R}\right)\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\right] . \quad (41)$$

Simplifying, this expression becomes

$$\tan\theta_{max} \approx \mu_s\left[1+\left(\frac{m}{M}\right)\left(\frac{r_w}{\ell}\right)^2\left(\frac{\ell}{R}\right)\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\right] . \quad (42)$$

Therefore, we need to know some coefficients of static friction to see what the maximum angle must be, and this will also tell us what the maximum speed must be. From the Engineer's Handbook (//www.engineershandbook.com/Tables/frictioncoefficients.htm), we have:

MATERIAL#1	MATERIAL #2	μ_s STATIC??	μ_k KINETIC
Rubber	Asphalt (Dry)	1	0.5-0.8
Rubber	Asphalt (Wet)	1	0.25-0.75
Rubber	Concrete (Dry)	1	0.6-0.85
Rubber	Concrete (Wet)	1	0.45-0.78

To determine the fastest possible angle, we can make estimates of the speed v and turning radius R. Suppose fastest speed around a corner is about 30 miles per hour. This is probably an overestimate.

ArcTan[1.18]/\°
49.7201

ArcTan[1.]/\°
45.

Let's try some numbers in our expression for the angle:

$$\tan\theta_{max} \approx \mu_s\left[1+\left(\frac{m}{M}\right)\left(\frac{r_w}{\ell}\right)^2\left(\frac{\ell}{R}\right)\left(\frac{1}{1-\frac{\ell}{R}\sin\theta}\right)\right] . \quad (43)$$

We want to try an extreme case. Suppose

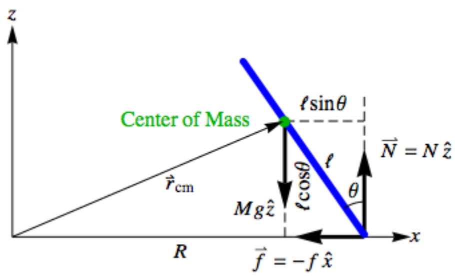
$$\begin{aligned} v &= 35\frac{km}{h} \\ g &= 9.81\frac{m}{s^2} \\ r_w &= 27inches = 0.6858m \\ \ell &= 2r_w \\ \ell &= 0.1R \\ R &= 10\ell = 20r_w = 20(0.6858m) = 13.716m \end{aligned} \quad (44)$$

Let's try substituting and plotting the results.

******A Manipulate Plot should go here!********

B.4 Including the rotation of the bicycle about the center of mass.

This problem can also be done by considering the rate of change of angular momentum about a fixed point at the center of the turn, which must be equal to the total torque about that same point. We will still need the fact that the magnitude of the normal force is equal to the weight, $N = Mg$, but we do not need to know the frictional force. This approach will make it fairly easy to include both the spinning of the bicycle wheel (still in the unicycle model), but also the rotation of the bicycle about its center of mass. This latter rotation occurs because the bicycle always has the same side facing the center of the turn, and so if it goes in a complete circle, it makes a complete rotation about its center of mass.



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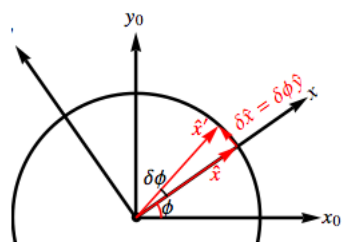
If we consider the bicycle to be a point mass located at the center of mass, ignoring rotations about the center of mass, the angular momentum of the bicycle plus the rider about the origin is equal to

$$\vec{L} = \vec{r} \times M\vec{v} = [(R - \ell\sin\theta)\hat{x} + \ell\cos\theta\hat{z}] \times M(v\hat{y}) \quad . \tag{45}$$

Performing the cross product, we have

$$\vec{L} = Mv[-\ell\cos\theta\hat{x} + (R - \ell\cos\theta)\hat{z}] \quad . \tag{46}$$

Now the rate of change of angular momentum only depends on the rates of change of the unit vectors. The unit vector $\hat{z}$ does not change as the bicycle goes around the curve, but the unit vector $\hat{x}$ does change.



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For a change in angle $\delta\phi$, the change in the $\hat{x}$ unit vector is

$$\delta\hat{x} = \delta\phi\hat{y} \quad . \tag{47}$$

Dividing both sides by δt and taking the limit as δt approaches zero, we have

$$\frac{d\hat{x}}{dt} = \frac{d\phi}{dt}\hat{y} = \omega\hat{y} \quad . \tag{48}$$

Therefore, the rate of change of angular momentum, which uses this change in unit vector, is

$$\frac{d\vec{L}}{dt} = -Mv\ell\cos\theta\frac{d\hat{x}}{dt} = -Mv\omega\ell\cos\theta\hat{y} \quad . \tag{49}$$

Since the radius of the center of mass is at $r_{cm} = (R - \ell\sin\theta)$, the magnitude of the velocity of the center of mass is

$$v = (R - \ell\sin\theta)\omega = R(1 - \frac{\ell}{R}\sin\theta)\omega \quad . \tag{50}$$

Therefore, the rate of change of angular momentum with ω replaced by its expression in terms of v, is

$$\frac{d\vec{L}}{dt} = -M\frac{v^2}{R(1 - \frac{\ell}{R}\sin\theta)}\ell\cos\theta\hat{y} \quad . \tag{51}$$

This must be equal to the net torque about the origin. From the picture, that torque is

$$\vec{\tau} = -RN\hat{y} + (R - \ell\sin\theta)Mg\hat{y} \quad . \tag{52}$$

Substituting $N = Mg$, we have

$$\vec{\tau} = -RMg\hat{y} + (R - \ell\sin\theta)Mg\hat{y} \quad . \tag{53}$$

We see that the first term cancels the first part of the second term to give

$$\vec{\tau} = -Mg\ell\sin\theta\hat{y} \quad . \tag{54}$$

We can now equate this to our expression for the rate of change of angular momentum to get

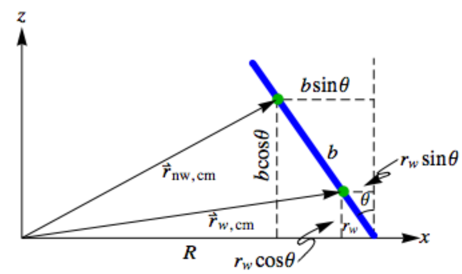
$$\begin{aligned} \frac{d\vec{L}}{dt} &= \vec{\tau} \\ -M\frac{v^2}{R(1-\frac{\ell}{R}\sin\theta)}\ell\cos\theta\hat{y} &= -Mg\ell\sin\theta\hat{y} \quad . \end{aligned} \tag{55}$$

Equating components of $\hat{y}$ and dividing both sides by $-Mg\ell\cos\theta$, we have

$$\tan\theta = \frac{v^2}{Rg}\frac{1}{\left(1-\frac{\ell}{R}\sin\theta\right)} \quad . \tag{56}$$

This is the same expression we got for the simplest model earlier.

The next step is to considering the bicycle without the wheel and the wheel as two separate objects, and to show that we can get the same results as the simplest model above, we will begin by ignoring the rotation of the bicycle wheel about its axis and the rotation of the bicycle about its center of mass. We will use the subscripts nw and w for the bicycle without the wheel and the wheel alone. Here b the length along the bicycle from the ground to the center of mass of the bicycle without the wheel, and r_w is the corresponding distance to the axle of the wheel. Here is the picture.



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From the picture, the coordinates of the center of masses of the bicycle without the wheel and of the wheel are

$$\begin{aligned} \vec{r}_{nw,cm} &= (R-b\sin\theta)\hat{x} + b\cos\theta\hat{z} \\ \vec{r}_{w,cm} &= (R-r_w\sin\theta)\hat{x} + r_w\cos\theta\hat{z} \quad . \end{aligned} \tag{57}$$

Since $\hat{z}$ is constant, the velocities of the two centers of mass are given by

$$\begin{aligned} \vec{v}_{nw,cm} &= (R-b\sin\theta)\frac{d\hat{x}}{dt} = \omega(R-b\sin\theta)\hat{y} \\ \vec{v}_{w,cm} &= (R-r_w\sin\theta)\frac{d\hat{x}}{dt} = \omega(R-r_w\sin\theta)\hat{y} \quad . \end{aligned} \tag{58}$$

Using the notation,

$$\begin{aligned} \vec{v}_{nw,cm} &= v_{nw,cm}\hat{y} \\ \vec{v}_{w,cm} &= v_{w,cm}\hat{y} \quad , \end{aligned} \tag{59}$$

we have

$$\begin{aligned} v_{nw,cm} &= \omega(R-b\sin\theta) \\ v_{w,cm} &= \omega(R-r_w\sin\theta) \quad . \end{aligned} \tag{60}$$

Then the angular momentum about the origin of these two point masses is

$$\begin{aligned} \vec{L} &= \vec{r}_{nw,cm}\times(M-m)\vec{v}_{nw,cm} + \vec{r}_{w,cm}\times m\vec{v}_{w,cm} \\ &= [(R-b\sin\theta)\hat{x} + b\cos\theta\hat{z}]\times(M-m)(v_{nw}\hat{y}) + \\ &+ [(R-r_w\sin\theta)\hat{x} + r_w\cos\theta\hat{z}]\times(mv_w\hat{y}) \quad . \end{aligned} \tag{61}$$

Performing the cross products, we have

$$\begin{aligned} \vec{L} &= (M-m)v_{nw}[-b\cos\theta\hat{x} + (R-b\sin\theta)]\hat{z} + \\ &+ mv_w[-r_w\cos\theta\hat{x} + (R-r_w\sin\theta)\hat{z}] \quad . \end{aligned} \tag{62}$$

Grouping by components, we have

$$\begin{aligned}\vec{L} &= [-(M-m)v_{nw}b\cos\theta\hat{x} + (M-m)v_{nw}(R-b\sin\theta)]\hat{z} + \\ &+ [-mv_w r_w\cos\theta\hat{x} + mv_w(R-r_w\sin\theta)\hat{z}] \\ &= -[(M-m)v_{nw}b + mv_w r_w]\cos\theta\hat{x} + \\ &+ [(M-m)v_{nw}(R-b\sin\theta) + mv_w(R-r_w\sin\theta)]\hat{z} \ .\end{aligned} \tag{63}$$

Substituting v_{nw} and v_w, we have

$$\begin{aligned}\vec{L} &= -[M\left(1-\frac{m}{M}\right)\omega(R-b\sin\theta)b + m\omega(R-r_w\sin\theta)r_w]\cos\theta\hat{x} + \\ &+ [M\left(1-\frac{m}{M}\right)\omega(R-b\sin\theta)^2 + m\omega(R-r_w\sin\theta)^2]\hat{z} \ .\end{aligned} \tag{64}$$

Factoring out $MR^2\omega$ in each component, we have

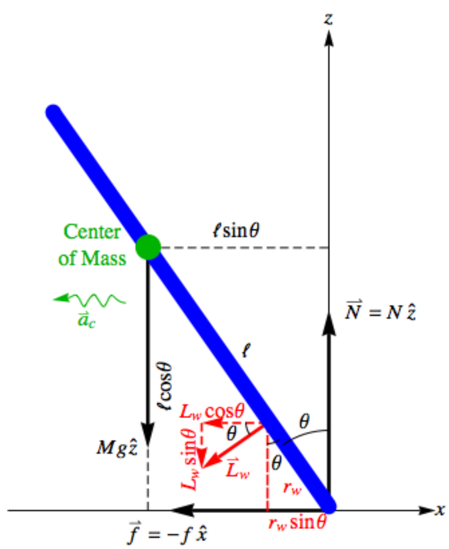
$$\begin{aligned}\vec{L} &= -MR^2\omega\left[\left(1-\frac{m}{M}\right)(1-\frac{b}{R}\sin\theta)\frac{b}{R} + \right. \\ &+ \left.\frac{m}{M}\omega\left(1-\frac{r_w}{R}\sin\theta\right)\frac{r_w}{R}\right]\cos\theta\hat{x} + \\ &+ MR^2\omega\left[\left(1-\frac{m}{M}\right)(1-\frac{b}{R}\sin\theta)^2 + \right. \\ &+ \frac{m}{M}\left(1-\frac{r_w}{R}\sin\theta\right)^2\hat{z} \qquad .\end{aligned} \tag{65}$$

(2)

Along the rod from the bottom, the center of mass is at

$$\tan\theta = \frac{v^2}{Rg}\frac{1}{(1-\frac{\ell}{R}\sin\theta)} \ . \tag{66}$$

xxx



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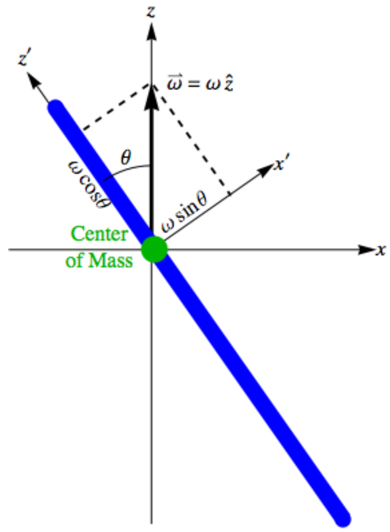

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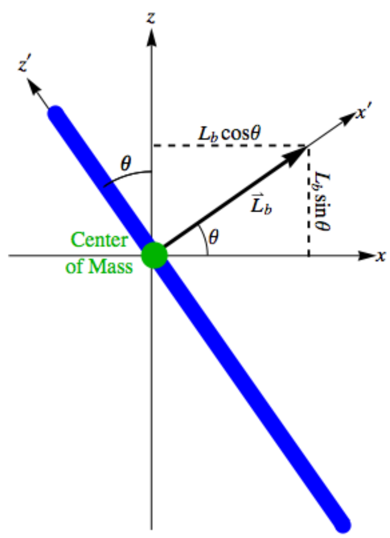

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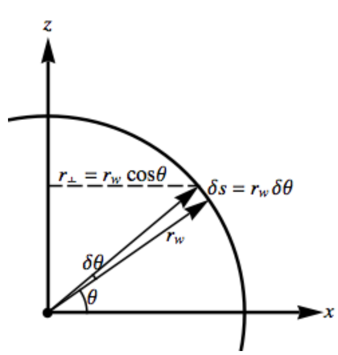

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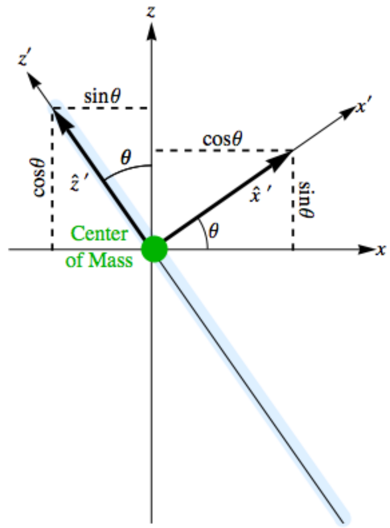

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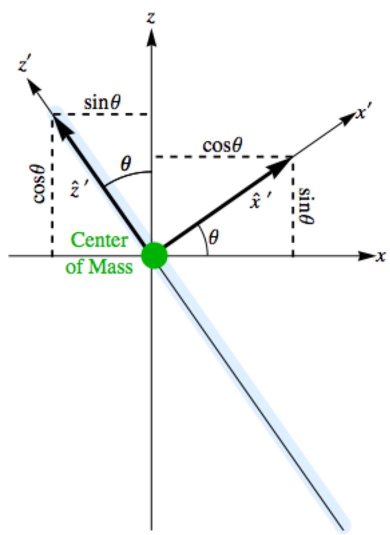

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Appendix C: Message from the Vice Provost

Welcome! This course is part of a larger project called The Great VCU Bike Race Book. Your work in this course counts not only for the course, but also for the entire university—and the world. In fact, The Great VCU Bike Race Book demonstrates to the world what happens when our students and faculty think together about a single event, question, or challenge. You are doing work connected to historic event, and you are helping to create history yourself. Eventually, the best student work will be preserved in the final edition of the "book," archived in Scholars Compass at Cabell Library for future generations to enjoy and learn from.

We hope you will find your course a perfect blend of fun, learning, and creativity. Bring your best work to this opportunity! The world will be following along.

Dr. Gardner Campbell

Vice-Provost for Learning Innovation and Student Success

Dean, University College

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Pages

Header Photo

The third and final day of the USA Cycling Collegiate National Road Championships, held in Richmond May 2-4, 2014, included road course races. [Photos by Phil Riggan from Richmond Times-Dispatch]