Master's Thesis

PAPR Reduction in GFDM Systems Using an SLM Technique

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Abstract

In the fifth generation (5G) cellular network system, user capacity should be improved compare with the current 4G network system. To this end, higher resource efficiency is an essential. Orthogonal frequency division multiplexing (OFDM) and orthogonal frequency division multiple access (OFDMA), which has high spectral efficiency resorting to orthogoanlity between subcarriers, is the most commonly used modulation technique in the current 4G network system. To maintain orthogonality, several types of frame structures are used for synchronized signal transmission and reception in Long Term Evolution (LTE). However, these fixed frame structures result in a fundamental limit for reducing latency. Thus an asynchronous communication scheme has been emerged as one of the solutions to reduce latency. On the contrary, without synchronization, OFDM signals generate interference to each other. Recently, generalized frequency division multiplexing (GFDM) has been proposed for the asynchronous multiple access. Many studies have evaluated that GFDM has higher sum-rate than OFDM for the asynchronous systems owing to the higher spectral efficiency and lower out-of-band emission (OOB). Despite the many advantages, GFDM also has disadvantages such as a high peak-to-average power ratio (PAPR). If the numbers of GFDM and OFDM subcarriers are equal, GFDM will get higher PAPR than OFDM due to multiple subsymbols. To reduce the PAPR, various PAPR reduction techniques have been studied on OFDM such as clipping, selective mapping (SLM), partial transmit sequence (PTS), Tone reservation (TR), and single-carrier frequency division multiple access (SC-FDMA) for LTE uplink. In GFDM, precoded GFDM and generalized frequency division multiple access (GFDMA) have been proposed as PAPR reduction techniques. Among PAPR reduction techniques, SLM is one of applicable techniques to the GFDM without signal distortions. In this paper, GFDM SLM is proposed as a PAPR reduction technique. In addtion, the performance analysis is compared in terms of the PAPR, OOB, and spectral efficiency among SC-FDMA, OFDMA, GFDMA, precoded GFDM, and GFDM SLM.

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Nomenclature

3GPP	3rd Generation Partnership Project
BW	bandwidth
CP	cyclic prefix
DFT	Discrete Fourier transform
FDMA	frequency-division multiple access
FFT	Fast Fourier transform
GFDM	Generalized Frequency Division Multiplexing
GFDMA	generalized FDMA
ICI	intercarrier interference
IFDMA	interleaved SC-FDMA
IFFT	Inverse Fast Fourier transform
ISI	Intersymbol Interference
LFDMA	localized SC-FDMA
LTE	long term evolution
LTE-A	LTE-Advanced
MIMO	Multiple-input and multiple-output
OFDM	Orthogonal frequency-division multiplexing
OFDMA	Orthogonal frequency-division multiple access
OOB	Out-of-band emission

PAPR	**Peak-to-Average Power Ratio**
PSK	**phase shift keying**
QAM	**quadrature amplitude modulation**
SC-FDMA	**Single-carrier FDMA**
SLM	**Selective mapping**
ZF	**zero-forcing**

1 INTRODUCTION

In the 5G network systems, there are several topics to achieve, such as massive MIMO, machine-to-machine communication, millimeter wave, ultra-dense network and ultra-low latency communication. The success and failure of topics are definitely related the developments on the user capacity, data rates, delay performance, and applications. Therefore, compared with the current 4G system, the importance of lower latency and resource efficiency are much emphasized in the upcoming 5G mobile network system [5]. The guideline of 5G proposed by some research groups. Among them, there are two main project groups. One is the METIS 2020 project. The METIS has proposed applications such as device-to-device communication, ultra-dense network, and ultra-reliable communications [32]. An other research project is 5GNow. Internet of things, gigabit wireless connectivity, and tactile internet are target applications of the 5GNow . Among the reseach projects, 5GNow has focused on non-orthogonal, asynchronous communication for an unifed frame structure concept [43, 44].

OFDM is a muticarrier modulation scheme using orthogonal subcarriers. It has attractive advantages such as high frequency efficiency, low interference of subcarriers to each other, elimination of ISI by the CP, and narrow band experiences [9]. For this reason, OFDM has been used on various standards such as digital video broadcasting (DTV), digital audio broadcasting (DAB), worldwide interoperability for microwave access (WiMAX), Wi-Fi, LTE, and LTE-A [14,16]. However, to keep the orthogonality, OFDM needs to be synchronized between users and an enhanced Node B(eNB) in both time and frequency domain [21,37]. In the current 4G network system, several frame structures are used for the synchronized transmission. Either time division duplex (TDD) or frequency division duplex (FDD), needs to wait available time slot to transmit a signal [2]. According to 3GPP ITU-R document, the latency is about 10ms assuming connected transmission in the 4G LTE system. The delay can be enlarged as increasing user density due to the limited time-frequency resource and waiting time on resource allocation [39]. It means an asynchronous communication system could be one of the solutions for decreasing the latency of current LTE system. This is the reason why 5GNow emphasized the necessity of asynchronous systems [43].

Another topic is new wavform designs for high spectral efficiency. In asynchronous environments, loss of orthogonality is a reason of high ICI because of large OOB of OFDM. To solve this problem, several analysis and solutions have been represented [4, 18, 23]. Among these, 5GNow has focused on waveform designs, introduced following four waveform designs, filtered OFDM, filter band based multicarrier, universal filtered multi-carrier, and GFDM with performance analysis [7, 24, 25, 46]. GFDM is one of a new wave form design as an alternative to the asynchronous and non-orthogonal environments which is proposed by Professor G. Fettwis. GFDM is possible to use CP like OFDM and as parameter setting, completely compatible to current synchronized OFDM systems. A sub-symbol concept of GFDM makes possible to reduce OOB, thus more suitable condition for an asynchronous system than OFDM [12, 27].

On the other hand, the power is one of considerable problems. High PAPR is a performance factor related with increasing of circuit build cost and a distortion of signals owing to non-linearity. It is caused by transmitted, received power and modulation. Basically, OFDM has high PAPR because of it's superposition of subcarriers. Therefore, the methods of reducing PAPR have been proposed up to now such as clipping [22], SLM and partial transmit sequence (PTS) [10, 17, 28],, and tone reservation [19]. In the current 4G LTE, OFDMA [36] and SC-FDMA [29] are universal modulations for uplink and downlink. OFMDA is based on OFDM, but a difference is consideration about other user's subcarriers at once. In the same side, SC-FDMA is considered for multiuser environments. The main purpose of SC-FDMA is focused on uplink scenarios with low PAPR signal transitions [2]. Therefore, those two modulation techniques are standard of performance analysis of PAPR and spectral efficiency.

The main theme of thesis is the PAPR reduction of GFDM using a SLM technique. SLM does not change subsymbol carrier waveform designs such as raised cosine (RC) or root raised cosine (RRC). Many OFDM SLM techniques have been researched, but GFDM. In addtion, because of the similarity between OFDM SLM and GFDM SLM, SLM is suitable for applying to GFDM. In this thesis, theoretical expectation for GFDM SLM will be estimated according to the estimation of OFDM SLM. From simulations of GFDM SLM, proper compensation constants are also represented. Finally, PAPR and spectral efficiency are compared among the current major modulations and GFDM SLM.

The composition of the paper is as follows. In Section II, detail backgrounds are describes basic concepts which definition of PAPR, basic concept and the system models of OFDM, GFDM, and SC-FDMA. Some of recently proposed GFDM PAPR reduction will be discussed in Section III. A concept of proposed GFDM SLM is described in Section IV. Section V shows performance analysis between modulations. Section VI is end of main contents, conclusion.

2 BACKGROUND

2.1 OFDM System Model

OFDM is a major modulation in current communication systems like cellular network and Wi-Fi. OFDM has significant number of advantages to use such as high spectral efficiency, frequency selective channel and CP. Those advantages come from two key concepts. One is multicarrier, another one is orthogonality. Compare with single carrier modulation, several subcarriers are modulated at once in OFDM. OFDM subcarriers are orthogonaly distributed, for higher frequency band efficiency. Also, it makes less interference between ICI than the single carrier modulation. The narrow band signal of OFDM is a same context. For understanding about a system model of OFDM , first, an OFDM system uses K subcarriers is considered. Each subcarrier is matched to the $k = 0$ to $K - 1$ data symbol array which is same as an index of the subcarriers. The array of data symbol $\mathbf{d}(k)$, is consisted with QAM or PSK (e.g. 16QAM or QPSK). In base band, OFDM can be represented by

Fourier transform

$$x(t) = \sum_{k=0}^{K-1} \mathbf{d}(k)\, e^{j2\pi k \triangle f\, t} \tag{1}$$

where $\triangle f$ is difference between subcarriers, T is one OFDM symbol duration, $0 \leq t \leq T$. k-th subcarrier can be noted by $f_k = k\triangle f$. Then, the subcarriers of OFDM are orthogonal to other subcarriers like (2).

$$\int_{t=0}^{t=T} (e^{j2\pi k \triangle f\, t})^* e^{j2\pi l \triangle f\, t} = \begin{cases} T & l = k \\ 0 & l \neq k \end{cases} \tag{2}$$

Due to the orthogonality, OFDM can be represented by inverse DFT (IDFT) or IFFT. The time domain sampling can be assumed by $\triangle\ f = 1/T$. Thus, (1) is equal to (3) for N, assumed $n = 0$ to $N - 1$.

$$x[n] = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} \mathbf{d}(k)\, e^{j2\pi k\, n/N} \tag{3}$$

$\sqrt{K}$ is normalize factor, and $N = K$. Then (3) is same as a multiplication of IFFT matrix and the data symbol vector, (3) can be written by

$$\mathbf{x} = \mathbf{F}_N^{\mathrm{H}} \mathbf{d} \tag{4}$$

where ${\mathbf{F}_N}^{\mathrm{H}}$is a $N \times N$ IFFT matrix. H means Hermitian [21, 37].

When a receiver gets the transmitted signal $\mathbf{x}$, the signal is changed by a channel. If an LTI system and a multipath channel is assumed, the received signal $\mathbf{y}$ is extended by a multipath propagation. The transmitted $\mathbf{x}$ will be ISI to another OFDM symbols which are just after transmitted. The impulse response of multipath propagation is represented by

$$h(t, \tau) = \sum_{i=1}^{N_{tap}} c_i(t)\, \delta(\tau - \tau_i) \tag{5}$$

where τ_i is a delay of i-th multipath, $c_i(t)$ means a complex amplitude of each impulse response. To solve this problem, a guard time is placed between each transmission. However, the guard time is inefficient to use time resources. The CP is a solution for the ISI and time inefficiency of the guard time. In OFDM system, (5) can be assumed by

$$\mathbf{h} = [h_0, h_1, h_2, , h_{N_{tap}-1}]^{\mathrm{T}} \tag{6}$$

where $h_i \in \mathbb{C}$. Then the received signal is noted by $\mathbf{y} = \mathbf{h} * \mathbf{x}$, where$*$ is convolution. Then(6) is written by a matrix $\mathbf{H}$ and $\mathbf{y} = \mathbf{h} * \mathbf{x}$ is equal to (7).

$$\mathbf{y} = \mathbf{H}\mathbf{x} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ \\ y_{N+N_{cp}-2} \end{bmatrix} = \begin{bmatrix} h_0 & 0 & & & 0 \\ h_1 & h_0 & & & \vdots \\ \vdots & \vdots & \ddots & & \\ h_{N_{tap}-1} & h_{N_{tap}-2} & & & h_0 \\ 0 & h_{N_{tap}-1} & & & \vdots \\ \vdots & & & & h_{N_{tap}-2} \\ 0 & 0 & \cdots & 0 & h_{N_{tap}-1} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ \\ x_{N-1} \end{bmatrix} \tag{7}$$

From (7), a CP is added which is a last part of $\mathbf{x}$ to the before $\mathbf{x}$. After removing the additional added parts, $\mathbf{H}$ is change to a circulant matrix $\mathbf{H}_{cp}$and $\mathbf{y}$ is represented by (8).

$$\mathbf{y} = \mathbf{H_{cp}}\mathbf{x} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_0 & 0 & \cdots & h_{N_{tap}-1} & h_{N_{tap}-2} & \cdots & h_1 \\ h_1 & h_0 & \cdots & 0 & h_{N_{tap}-1} & \cdots & h_2 \\ \vdots & \vdots & \ddots & & & & \vdots \\ h_{N_{tap}-1} & h_{N_{tap}-2} & & & & & 0 \\ 0 & h_{N_{tap}-1} & \cdots & & & & 0 \\ \vdots & & & & & & \vdots \\ 0 & 0 & & & & \cdots & h_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ \\ x_{N-1} \end{bmatrix} \tag{8}$$

By multiplying an FFT matrix, the circular matrix $\mathbf{H}_{cp}$ is decomposed by singular value decomposition (SVD). Then the channel frequency response is digonalized [11]. The channel frequency response can be detected by pilot, based on detected channel response, the data is able to estimate.

The basic concept of OFDMA is distribute the BW to multiple users using OFDM. The users are synchronized by LTE frame structures. In downlink, each user knows the assigned own frequency BW and time slot from resource block. Therefore, they can get their own data from the received signal. In uplink, the synchronized users are transmit data to eNB. For more elaborate transmission, some techniques are used like time advanced [3]. In (8), a difference between OFDMA and OFDM is that among total data symbol array, only assigned successive data symbols have data. Other data symbols are occupied by zeros for the user.

2.2 PAPR

2.2.1 Definition and general effect of PAPR in OFDM systems

As the name implies, PAPR means the ratio of maximum The definition of PAPR means the ratio of peak power to average power. Then, the definition of PAPR is written by (9).

$$\mathrm{PAPR}(x(t)) = \frac{\max\limits_{0 \le t \le T}(|x(t)|^2)}{E[|x(t)|^2]} \tag{9}$$

The main disadvantage of high PAPR is shown in Fig.1. The high power amplifier (HPA) needs power backoff for input signals due to the limitation of the input-output linearity region. Low PAPR signals get the high efficiency from small backoff with a high gain of circuit. On the over hand, high PAPR signal needs high input power backoff, it's efficiency is decreased in OFDM transceivers [42]. Another problem is a signal distortion. If the input signal is over the tolerance of circuit, the circuit cuts the overwhelming peaks.

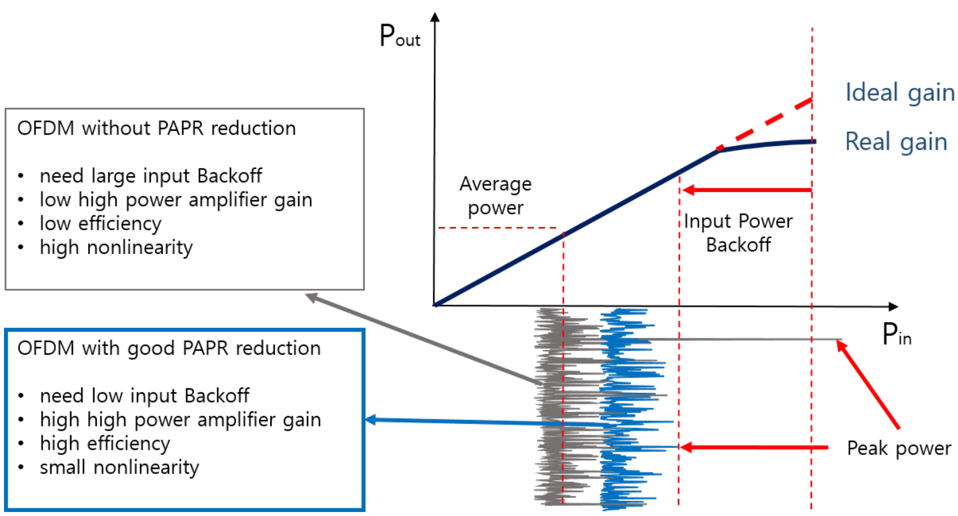

Figure 1: The major benefits of PAPR reduction for the power amplifier of OFDM transmitters.

The definition of PAPR is defined in continuous time domain. It needs to be transferred from an analog signal to the digital for the analysis of PAPR. OFDM is easily adaptable the oversampling using zero-padded IFFT. Therefore, the PAPR is assumed as oversampled discrete signals like (10) in this paper.

$$\mathrm{PAPR}(x[n]) = \frac{\max\limits_{0 \le n \le N}(|x[n]|^2)}{E[|x[n]|^2]} \tag{10}$$

Table 1: PAPR of OFDM comparison for different modulations (at $\gamma = 0.01$)

PAPR (dB)	4-QAM	16-QAM	64-QAM
K=64	9.3dB	9.3dB	9.3dB
K=128	9.7dB	9.7dB	9.7dB
K=256	10.0dB	10.0dB	10.0dB
K=512	10.3dB	10.3dB	10.3dB
K=1024	10.6dB	10.6dB	10.6dB

2.2.2 Theoretical estimation of OFDM PAPR

OFDM has high PAPR because of it's superposition of subacrriers. It is hard to consider about all cases of possible signals. For example, if K subcarrier OFDM system is assumed with 16-QAM, there are 16^K possible cases of data. Fortunately, there are only few of high PAPR signals. Thus, PAPR is usually expressed by the statistical distribution, complementary cumulative distribution function (CCDF). Physical meaning of CCDF is the probability of the given PAPR value over an given threshold. OFDM symbols are constructed by real and imaginary values with i.i.d. Gaussian random variables. It means the distribution of power of OFDM symbol follows Chi-squared distribution. Then, CCDF of OFDM PAPR can be expressed by

$$\text{CCDF}(\gamma) = P(\text{PAPR} > \gamma) = 1 - (1 - e^{-\gamma})^{\beta K} \tag{11}$$

where K is the number of subcarriers [17]. It is a reasonable trend considering that PAPR of OFDM is increased by increasing number of subcarriers. Of cause, PAPR is change by modulations, however it is much smaller than influence of changing the number of subcarriers [42]. In Table. 1, PAPR values are represented as different modulations at CCDF ($\gamma = 0.01$). It is known that (11) is not fit on the large K [34]. To solve this problem, β is given as a compensation coefficient for more accurate prediction. [30]. In the four times oversampled OFDM, β is known as 2.3 [15]. The detail derivation is explained in Appendix.

2.3 OFDM PAPR Reduction Schemes

2.3.1 Clipping

Clipping is the simplest PAPR reduction scheme. The maximum power of signal $|\mathbf{A}_{\max}|^2$is predetermined. If the signal is over a limited value of maximum power, the signal value is limited by $|\mathbf{A}_{\max}|^2$ like

$$|x(t)|^2 = \begin{cases} |\mathbf{A}_{\max}|^2 & |x(t)|^2 > |\mathbf{A}_{\max}|^2 \\ |x(t)|^2 & |x(t)|^2 \le |\mathbf{A}_{\max}|^2 \end{cases} \tag{12}$$

in the time domain. However, because of changes of the signal, there is possibility to make a signal distortion. In OFDM, those distorted peaks are related with unexpecatable subcarriers. To solve this problem, iterative clipping and filtering method was proposed in [6]. The main concept is prevent the ICI caused by removing not allowed high peaks using sharp filters to each subcarrier. As the name is represented, the technique repeats clipping and filtering until all overed peaks are removed. In Fig.2, the block diagram of the system is described.

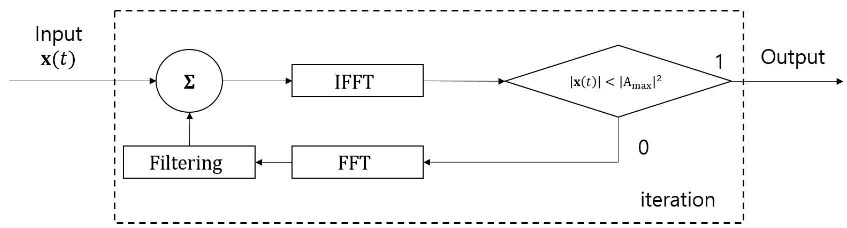

Figure 2: Block diagram of iterative clipping and filtering

2.3.2 Selective mapping and partial transmit sequence

The main concept of SLM or PTS is multiplying phase-shifting set to the part of subcarriers. As shown in the Fig.3, the phase shift can be represented by multiplication between complex exponential values and subcarriers in the OFDM system. Therefore, SLM is easy to adapt phase-shifting sets to each subcarrier [10]. The phase-shifting sets are assumed to be independent to each other. Then, from (11), the PAPR of OFDM SLM is represented by

$$\mathrm{CCDF}(\gamma) = P(\mathrm{PAPR} > \gamma) = (1 - (1 - e^{-\gamma})^{\beta K})^{U} \tag{13}$$

where U is the number of independant phase-shifting sets.

Partial transmit sequence is little bit different with SLM. PTS separates subcarriers with smaller set and IFFT those subcarrier sets. These IFFTed subcarriers are multiplied by optimal weighting factors [28]. It is well represented by Fig.4.

Both techniques transmit the information about selected phase-shifting set or weighting factors. It is called side information. A lot of SLM or PTS are not use data subcarriers as side information transmission carrier. However, it is clear that additional resource is needed such as complexity or pilot carriers. The reason why the SLM is focused in this paper is, SLM do not change data rate and OOB.

2.3.3 Tone reservation

Tone reservation(TR) use redundant data symbols to reduce PAPR. Therefore, this method doesn't need to care about the signal distortion. TR uses predetermined tones as reducing PAPR tones. If the PAPR is not over a threshold, those reserved tones are used for data transmission. In other words, this reservation system losses it's data rate caused by reserved tone which can't be used

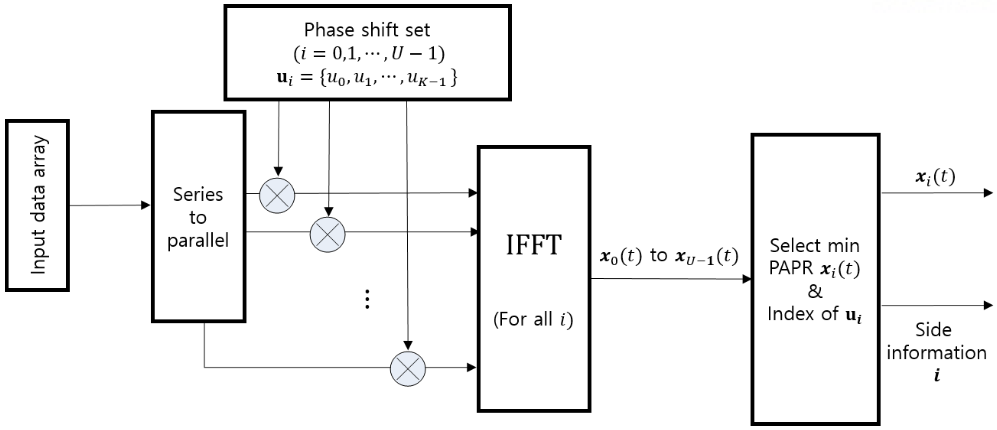

Figure 3: Block diagram of OFDM SLM

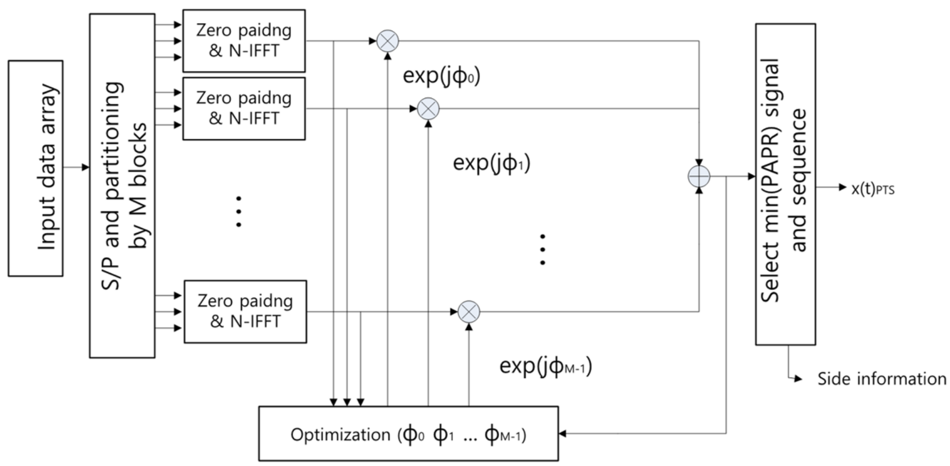

Figure 4: Block diagram of OFDM PTS

for data transmission and PAPR reduction at the same time. To minimize the data loss, about selecting optimal PAPR reduction tone rules or algorithms is researched on [8, 20].

2.4 SC-FDMA System Model

SC-FDMA has same advantages as OFDMA. It is suitable for a multiuser system, the high data rate and high spectral efficiency. It is proposed for low-PAPR in uplink system. SC-FDMA is one of precoded OFDMA. Therefore, subcarriers are orthogonal to each other, users need to be synchronized. The total possible subcarriers are devided to each user. If there are N subcarriers, the number of available subcarriers is $K_{user} = K/Q$, where Q is a number of users. There are two major distributing subcarrier approaches. One is localized FDMA. The LFDMA is almost same as OFDMA except a precoding matrix for each user. Another one is interleaved FDMA. IFDMA has even-spaced subcarriers with a Q intervals, other users subcarrier are located between them [29]. The examples of two subcarrier distribution are shown in the Fig. 5.

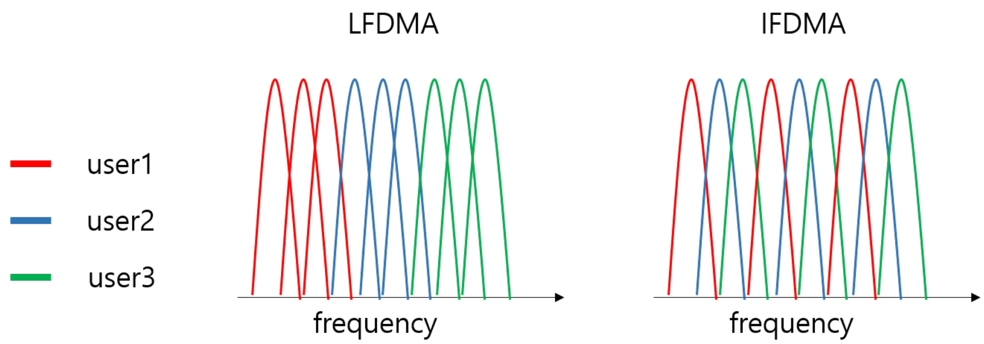

Figure 5: Subcarrier distribution for each user in LFDMA and IFDMA (K=9, K_{user}=3)

Assuming that among the Q users, a user makes a transmit uplink signal using SC-FDMA for . Then the formulation will be

$$\mathbf{x} = \mathbf{F}_K^H \mathbf{P} \mathbf{F}_{K_{user}} \mathbf{d}_{K_{user}} \tag{14}$$

where $\mathbf{P}$ **a** is permutation matrix which depends on LFDMA or IFDMA, $\mathbf{F}_{K_{user}}$ is a $K_{user} \times K_{user}$ FFT matrix.$\mathbf{d}_{K_{user}}$is a data symbol vector K_{user} by one. The main idea of SC-FDMA is, the FFT precoding matrix makes signals which have same as symbols at sampling point.

For example, LFDMA has each$\mathbf{d}_{K_{user}}$value in time domain at the each Q sampling point. The other points have unexpectable complex exponential value.

IFDMA has higher PAPR reduction than LFDMA by using different roll-off factor filters like Fig.6. Even if change roll-off factor 0 to 1, the difference of PAPR of LFDMA is smaller than 1dB. Used number of subcarrier is 64, and used filter is RC filter.

However, IFDMA needs to be strongly synchronized with other users. 7 is show spectrums of LFDMA and IFDMA. In asynchronous environment, it is hard to use IFDMA.

According to [1], considerable OFDMA/SC-FDMA parameters are represented in Table. 3.

Table 2: Parameters for DL OFDMA and UL SC-FDMA transmission

	OFDMA (FDD)	SC-FDMA
Spectrum Allocation (MHz)	20 / 15 / 10 / 5 / 2.5 / 1.25	20 / 15 / 10 / 5 / 2.5 / 1.25
# of occupied subcarriers	1201 / 901 / 601 / 301 / 151 / 96	600 / 450 / 300 / 150 / 75 / 38
resource block BW (frequency)	375kHz	
subframe duration	0.5 ms	0.5 ms
subcarrier spacing	15kHz	
FFT size	2048 / 1536 / 1024 / 512 / 256 / 128	1024 / 768 / 512 / 256 / 128 / 64

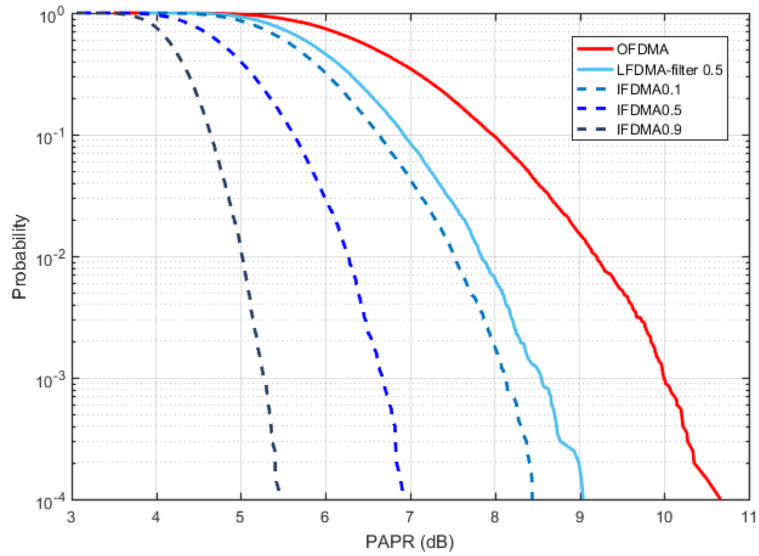

Figure 6: PAPR of LFDMA and IFDMA

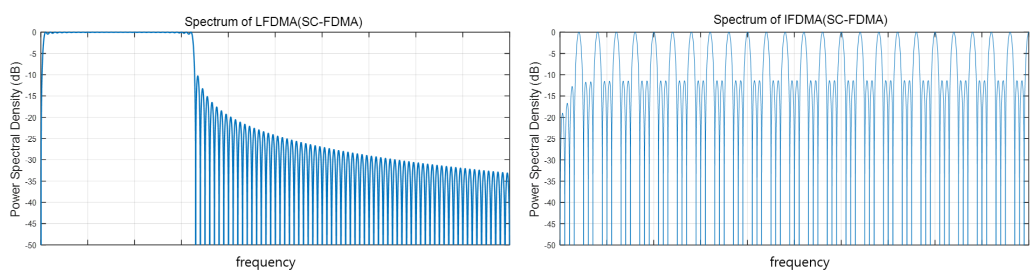

Figure 7: Spectrums of LFDMA and IFDMA

2.5 GFDM

GFDM is proposed by G.Fettweis at 2009 [12]. GFDM, which has K subcarriers, transmits KM data symbols at once. The M subsymbols are represented in time domain as amplitidues multiplied with circdcular shifted filters. The cyclic shift filter can be denoted by $g(t-mT)e^{j2\pi f_k t}$ where $k=0$ to $K-1$, $m=0$ to $M-1$, T is a time duration of a OFDM symbol. If then, total symbol duration of GFDM will be MT. f_k does not have to orthogonal to each other. If data symbol array of GFDM is assumed by $\mathbf{d}$, then those KM symbols are multiplied to KM subsymbol carriers by (15).

$$x(t) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{k,m} g(t-mT) e^{j2\pi f_k t} \tag{15}$$

For the L times oversampled LKM points in time domain, the GFDM signal is represented by

$$x[n] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{k,m} g(n - mKL) e^{j2\pi k\, n/LK} \tag{16}$$

. Like (4), (16) is noted by a matrix form

$$\mathbf{x} = \mathbf{A}\mathbf{d} \tag{17}$$

where $\mathbf{A}$ is a LKM by KM matrix. According to [13], (17) is same as

$$\mathbf{x} = \mathbf{F}_N^{\mathrm{H}} \sum_{k=0}^{K-1} \mathbf{P}_k' \boldsymbol{\Gamma} \mathbf{R} \mathbf{F}_M \mathbf{d}_k \tag{18}$$

$\mathbf{R}$ is a repetition matrix with $2M$ by M, $\boldsymbol{\Gamma}$ is a diagonalized waveform filter which is transformed to frequency domain and $\mathbf{P}_k'$is a mapping and an upsampling matrix. $\mathbf{d}_k$is M symbols in k-th subcarrier. It can be derived by characteristic of DFT. The block diagrams are represented in Fig.8 and Fig.9.

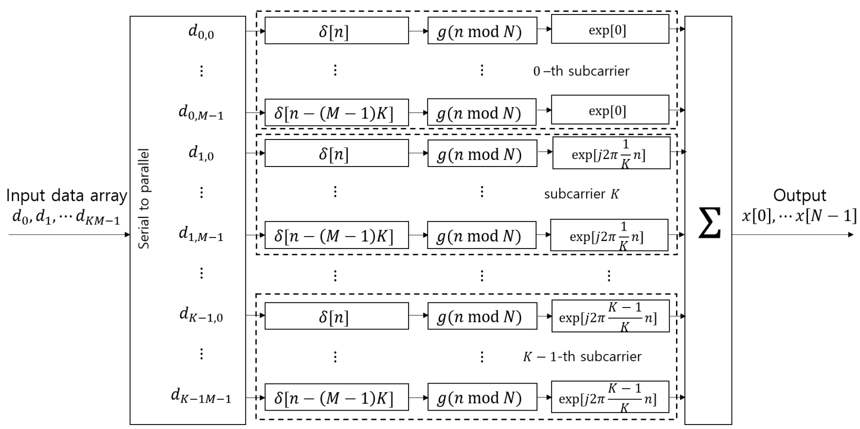

Figure 8: Block diagram of GFDM from [12]

A CP is applicable to the GFDM transmit signal $\mathbf{x}$. As like (8), it is able to represent GFDM with a CP channel matrix by

$$\mathbf{y} = \mathbf{H}_{cp}\mathbf{A}\mathbf{d} \tag{19}$$

If a GFDM modulation matrix $\mathbf{A}$ has orthogonally distributed subsymbol carriers in all available frequency band, $\mathbf{H}_{cp}$ is diagonalized by an FFT matrix. The channel coefficient is matched to the subsymbol carriers. However, in many cases, GFDM is assumed non-orthogonal environments and asynchronized multiusers. There are three major demodulation techniques for GFDM. One is ZF which means multiply an $\mathbf{A}^{-1}$ matrix after removing channel effects. Matched filter is also possible, but higher interference to adjacent subcarriers, the usually the performance is not enough. The last one is MMSE which has the best performance in those techniques. It is come from higher

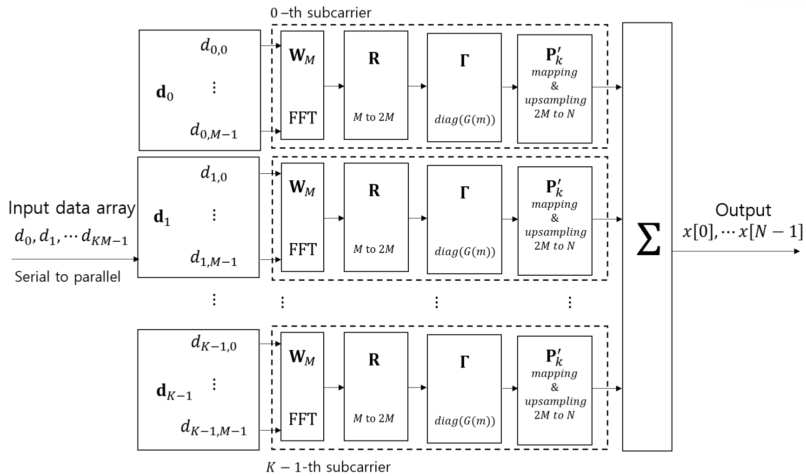

Figure 9: Block diagram of GFDM from [13]

computation cost. In [13], proposed iterative cancellation using matched filter, which has a high performance.

GFDM has high degree of freedom in parameter setting. Assuming time duration of an OFDM symbol is T, time duration of GFDM is MT duration. To transmit KM symbols, K subcarrier OFDM needs M time duration and CP duration. Due to the GFDM transmit KM symbols at once, GFDM needs only one CP for transmit KM symbols. Therefore, GFDM can have higher time efficiency than OFDM in some environments. Also it means $\triangle f$ between subsymbol carrier is closer than OFDM. In same word, long MT rectangular window makes $1/MT$ zero-crossing sharp power spectral density for each subsymbol carrier. For this reason, GFDM has a lower OOB than OFDM. If duration of OFDM and GFDM are same, GFDM will have higher data rate. However, it means GFDM loses it's lower OOB and time efficiency. The lower OOB especially important in asynchronous multiuser environments. In [33], spectral efficiency is shown by Fig. 10, simulated under an asynchronous environment in Rayleigh fading channels. In [45] , sum rate is analyzed comparing between GFDMA and SC-FDMA under asynchronous AWGN channels by Fig. 11. The result clearly shows that GFDM has higher spectral efficiency than OFDM and SC-FDMA.

3 RELATED WORK

3.1 GFDMA

The exact definition of GFDMA is distributing a few sets of subsymbol carriers in same subcarrier to each users. It means, if GFDM is used with some subcarrier sets such as empty other subcarriers for different users, it is satisfies the condition of GFDMA. In [26], a low PAPR GFDMA is proposed comparing with LFDMA and IFDMA. To get the low PAPR, a special parameter setting is used. The number of subsymbol carriers is matched to K_{user} of SC-FDMA, number of subcarriers is

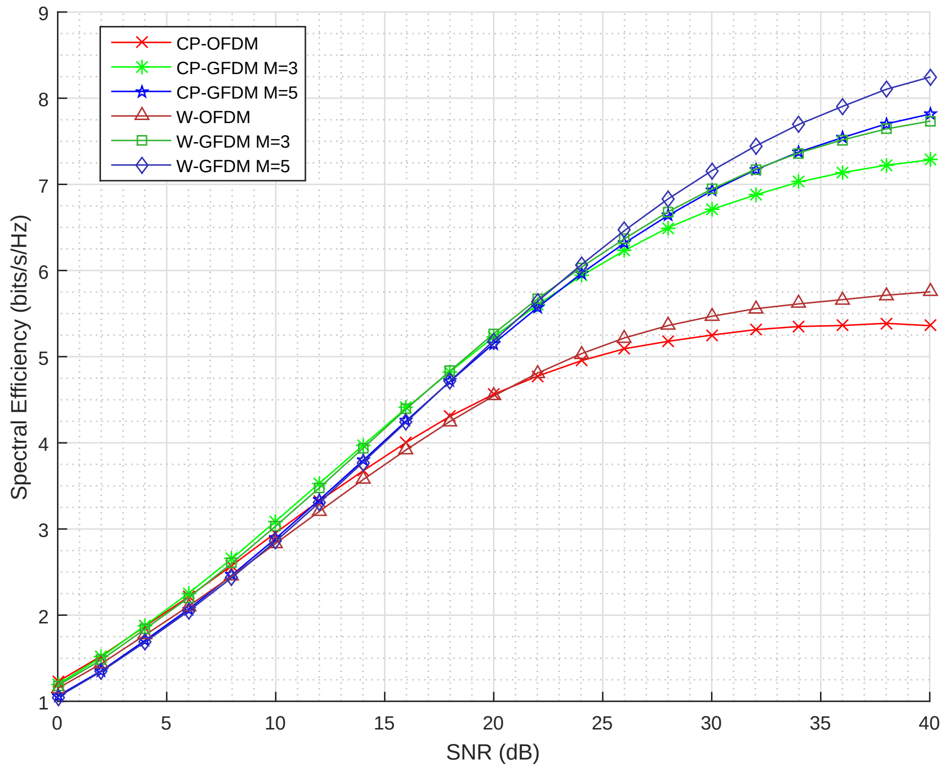

Figure 10: spectral efficiency of GFDM vs OFDM in the asynchronous system

matched to Q of SC-FDMA. In this paper, GFDMA means the low PAPR GFDMA. Fig.12 shows that GFDMA has similar PAPR performance with IFDMA and similar OOB with LFDMA.

However, if it is assumed that $\triangle f$ of between subsymbol carriers as same as SC-FDMA, GFDMA loses the advantages of GFDM such as low OOB and time efficiency from CP. Fig.13 shows the spectrum of difference modulations. For fair comparison, assumed same number of subcarriers K=32, M=5 for GFDM. K=32, Q=4 for OFDMA and SC-FDMA without filter. Because of parameter set of GFDMA, assumed K=4, M=32. As well shown as Fig.13, GFDMA has a trade off between loss of OOB and low PAPR performance compare with other techniques.

3.2 Precoded GFDM

In [40], several precoded GFDM techniques are proposed. The basic concpets of main precoded GFDM techniques, such as Block IDFT GFDM(BIDFT GFDM) and DFT precoded GFDM, are start form (16). Those proposed schemes are assumed to use CP. BIDFT GFDM is use characteristic of block diagonalization of $(\mathbf{HA})^{\mathrm{H}}\mathbf{HA}$. $\mathbf{HA}$ means $\mathbf{H_{cp}A}$ of the (19). This phenomenon is caused by IDFT precoding. In [40], the size of IDFT is assumed by N, it called by BIDFT-N GFDM. DFT precoded GFDM is same concept of the precoding matrix with IFDMA or LFDMA. Therefore, those two method called by LFDMA-GFDM and IFDMA-GFDM. For example, LFDMA-GFDM can be represented by

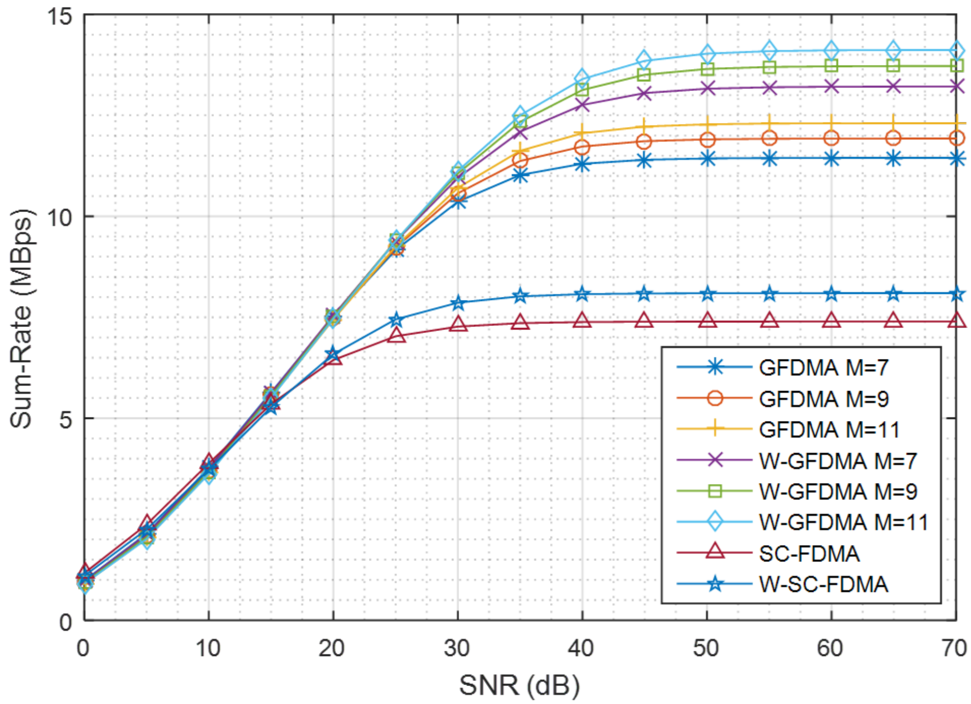

Figure 11: Sum rate of GFDMA vs SC-FDMA in the asynchronous system

$$\mathbf{x}_{LFDMA-GFDM} = \mathbf{A}\mathbf{P}_{KM}(\mathbf{F}_{N_{DFT}} \otimes \mathbf{I}_{KM/N_{DFT}})\mathbf{d} \tag{20}$$

where $\otimes$ means Kronecker product, $\mathbf{P}_{KM}$ is a permutation matrix. In LFDMA-GFDM, $\mathbf{P}_{KM}$ is an identity matrix. The block diagram of DFT precoded GFDM is represented in Fig.14.

Linear precoding with iterative algorithm GFDMA also proposed by [38]. It is improved version of BIDFT GFDM iteratively using a precoding matrix to data symbols.

Clipping is also available for GFDM. [35] proposed a concept with iterative receiver for a clipped GFDM signal. Basically, a block diagram of clipped GFDM is same as Fig.2 without that GFDM modulation and demodulation are used in the processing. In the Fig. 2 of [35] shows around 4 dB PAPR at CCDF(0.1%). However, it is clear that the clipping method has disadvantages on the exact signal transmission, it is shown as lower performance of bit error rate. The self-interference of GFDM makes negative synergy with clipping. Thus, the clipping method doesn't be considered in this paper.

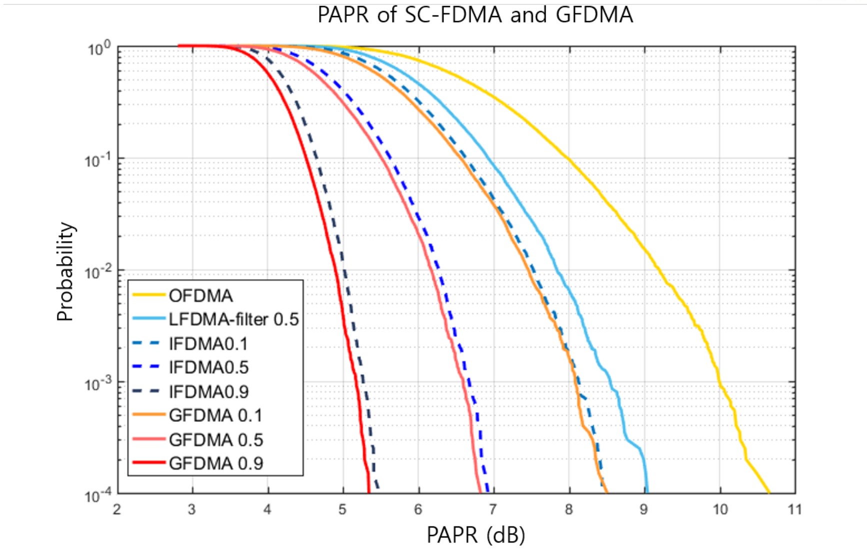

Figure 12: PAPR of SC-FDMA and GFDMA

4 CONCEPT OF GFDM SLM

Compare with OFDM, GFDM has more superpositions of M subsymbol carriers. If OFDM and GFDM are assumed to use same number of subcarriers, as mentioned in Section 2.2.1, PAPR of GFDM is higher than OFDM. The basic structure of GFDM is similar with OFDM. It means traditional OFDM PAPR reduction, introduced in Section 2.3, would be adaptable to GFDM system. In this paper, SLM is considered for GFDM PAPR reduction among several OFDM PAPR reduction techniques. An SLM technique doesn't make changes of basic data symbols and waveform of subsymbol carriers on GFDM. Also, there are no clipping distortions and rate losses by redundant symbols like TR.

The simplest way to apply the GFDM SLM is to apply KM phase-shifting arrays to the matched subsymbol carriers. However, since the GFDM modulation is performed on the time axis, it is not easy to directly apply SLM in frequency domain like OFDM SLM. Therefore, the block diagram of GFDM SLM can be represented by Fig.15 [31].

The important note is, GFDM SLM should be avoided to overlapping subsymbol waveforms which have same subcarrier index. For this problem, the simple overlapping avoidance algorithm is applied in the simulation. The random phase-shifting set has too much possible cases, and it is not realistic applicable. As same as OFDM SLM limits it's specific phase-shifting set, the simplified phase-shifting set is proposed [41].

Assumed phase-shifting set $\{\pm 1, \pm i\}$ has two major advantages. First, the structure is able to simplified. As phase-shifting set has determined inverse or not. The phase shift can be possible to multiply to the data symbols before GFDM modulation. Therefore, Fig.15 is replaced to Fig.16. [31] Another advantage is it does not need to add the overlapping avoidance algorithm. The proposed

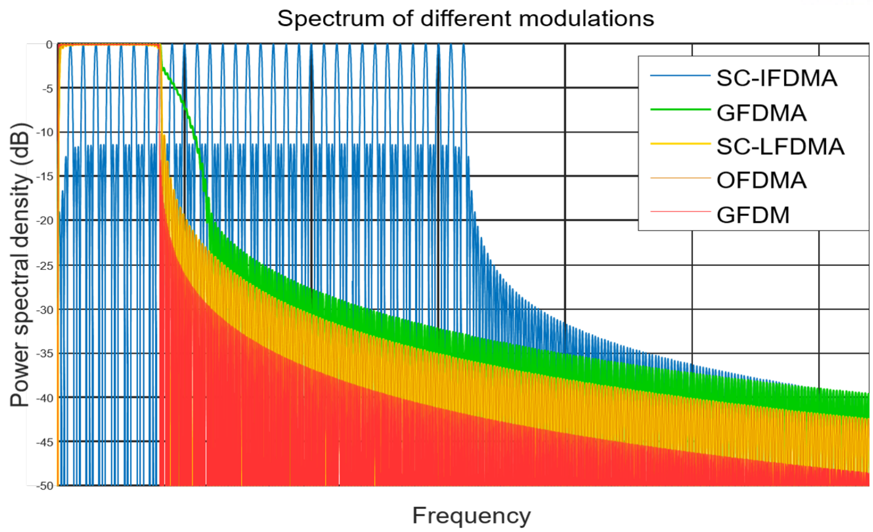

Figure 13: The spectrum of OFDMA, SC-FDMA, GFDM, and GFDMA

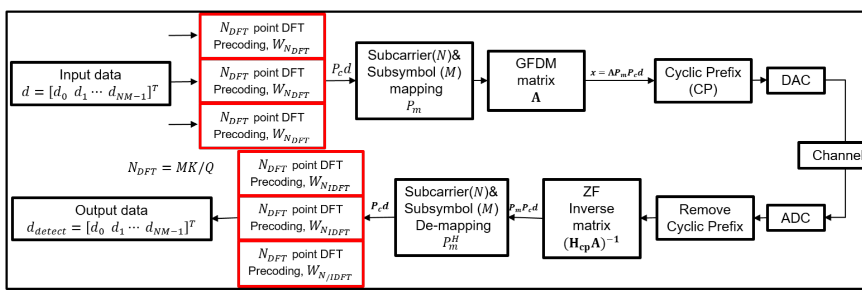

Figure 14: Block diagram of DFT precoded GFDM

phase-shifting set doesn't affect to circular shift of subsymbol carrier waveform. Therefore, the subsymbol carriers are never overlap to each other. It can be also available for low-complexity GFDM [13].

In GFDM, subsymbol carriers with the same frequency can be assumed to be subcarriers multiplied by a cyclic shift filter and a data symbol on the time domain. Therefore, (11) can be expressed by

$$\mathrm{CCDF}(\gamma) = P(\mathrm{PAPR} > \gamma) = (1 - (1 - e^{-\gamma})^{\alpha KM})^{U} \tag{21}$$

where compensation coefficient α is different with β of OFDM, U is the number of independant phase-shifting set.

Determines the OOB because it is the period MT of the GFDM signal, since it does not affect the period or the frequency band of the signal. The received signal is distinguished at the receiving end by a square filter, where the sinc spectrum is multiplied by the square frequency filter for each frequency component. In OFDM, the interference between data symbols is 0 due to orthogonality,

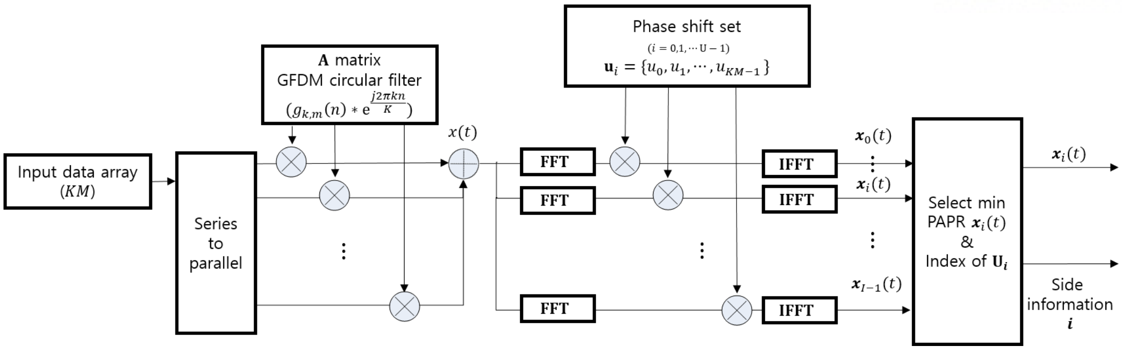

Figure 15: Block diagram of basic GFDM SLM

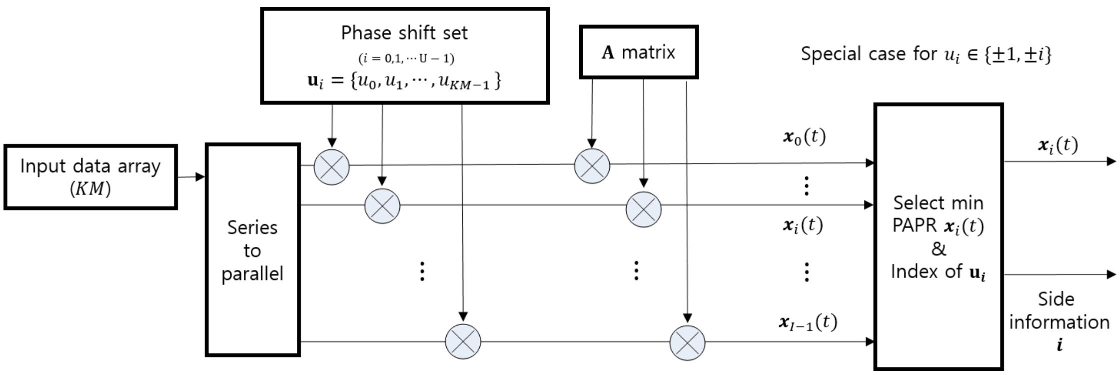

Figure 16: Block diagram of simplified GFDM SLM

but in the asynchronous situation, interference is greatly increased by adjacent OFDM signals. On the other hand, since the GFDM signal having a long period has a lower OOB as the period of the square filter increases, the influence of interference is less than that of OFDM in an asynchronous situation. Even if SLM is applied, OOB is not changed for same parameters.

5 PERFORMANCE ANALYSIS

5.1 Performance Evaluation of PAPR

Consider about Table. 2, the parameter setting of PAPR for OFDM SLM, GFDM SLM and SC-FDMA are defined as Table. 3. $U = 0$ means the original signal without SLM.

Fig.17 is shown difference between simulation results and theoretical estimations. From [15], PAPR of OFDM SLM is well fit on (11) with $\beta = 2.3$ and $K = 128$. For the(21),α is approximately 2.8 from simulation for given conditions. The average error of GFDM SLM is 0.1dB. In same sense, Fig. (18) plots the simulations and estimations for different parameters. As same as OFDM SLM,α is changed by as changing the parameters in GFDM SLM. This result means that the estimation is closely matched to simulations. With feasible coefficients, the estimation is possible to replace the

Table 3: OFDM SLM, GFDM SLM and SC-FDMA system parameters

Parameters	Value
Subcarrier spacing, $\triangle f_k$	15KHz
Number of subcarrier, K	16, 64, 128, 256
Number of subsymbols, M	5
Number of oversampling, L	4
Number of trial	50000
Number of phase-shifting set, U	0 (w/o SLM), 2, 4, 8, 16, 32, 64, 128
phase-shifting set	$\{\pm 1, \pm i\}$
Filter	Raised cosine (roll off factor, a = 0.1, 0.5, 0.9)
Number of users, Q	4
Block size of BIDFT-N	64
DFT precoding size, N_{DFT}	4
Data symbol modulation	16-QAM

simulations for significant parameters.

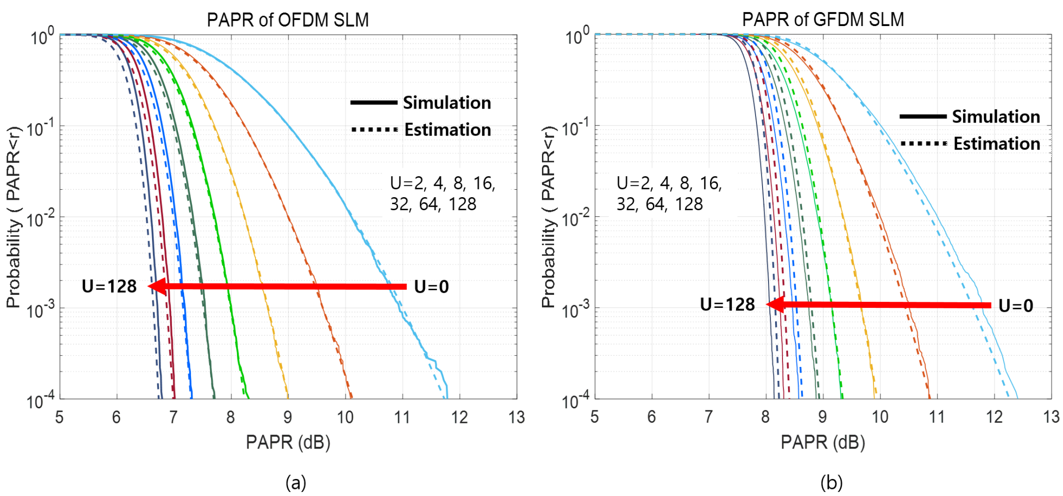

Figure 17: PAPR of simulations and theoretical estimations. (a) OFDM SLM $K = 128$, $\beta = 2.3$, (b) GFDM SLM $K = 128$, $M = 5$, $\alpha = 2.8$ and a = 0.5

Fig.19 represents PAPR of GFDM SLM and other PAPR reduction techniques. The parameters are $K = 64$, $M = 5$ and $Q = 4$ for OFDMA, GFDMA and SC-FDMA. GFDM SLM makes significant PAPR reduction compare with original GFDM. Maximum PAPR reduction is 3.8dB, the PAPR difference with OFDM SLM is around 1dB. Reducing the lowest PAPR is hard to achievable. Because of large number of subsymbol carriers, the minimum PAPR of GFDM SLM is larger than SC-FDMA around 2.5dB. At CCDF(0.1%), U=8 or above condition is required to approach to the LFDMA PAPR. U=4 is enough for higher PAPR performance than OFDMA system.

The PAPR of precoded matrices have higher PAPR than GFDMA. Because, the parameters are same as GFDM SLM, there are physical limitation to reduce PAPR. The main key of reducing PAPR

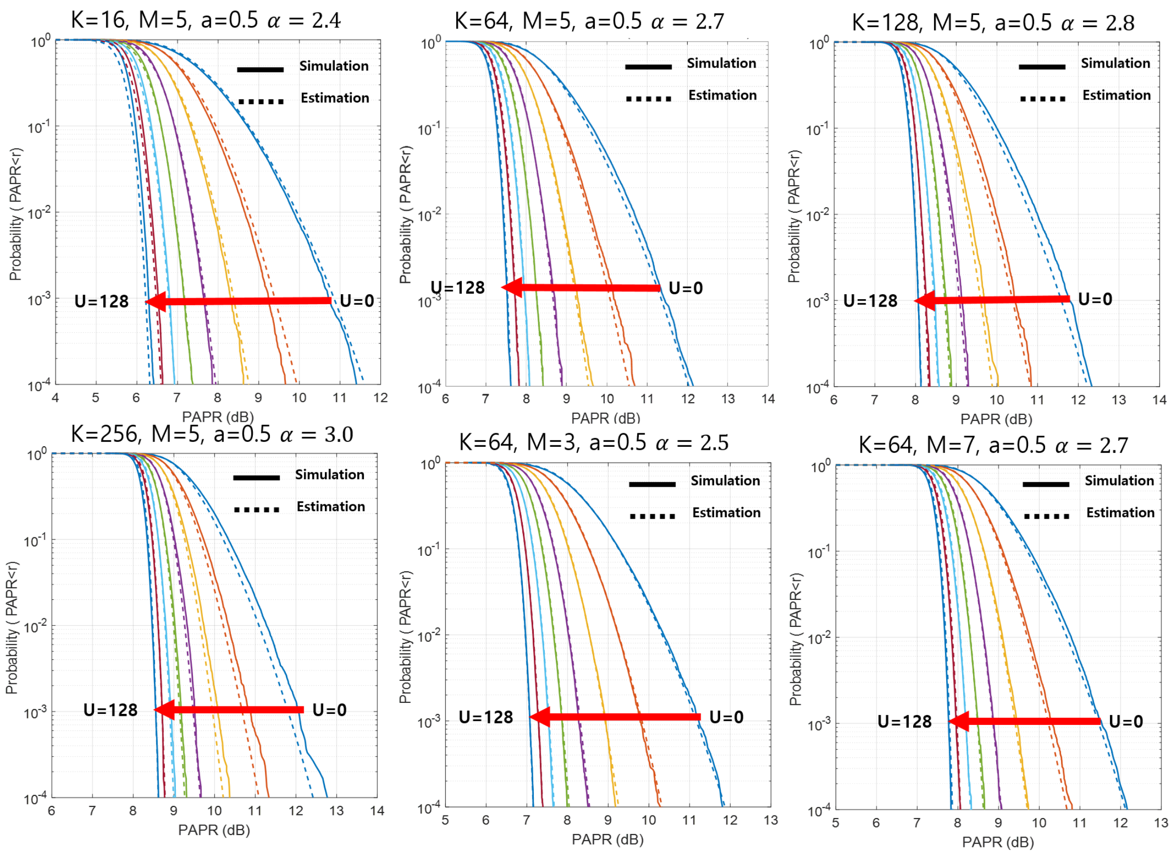

Figure 18: PAPR of simulations and theoretical estimations with different parameters

is IDFT spreading. If assume GFDM has KM sampling points in time domain, each mK sample point has same amplitude with data symbols. Without filter, interleaved M GFDM subsymbols become same as IFDMA modulation. In time domain, those M subsymbols are repeated and it's amplitude is spreaded by $\frac{1}{\sqrt{K}}$. Therefore, BIDFT-N has lower PAPR than general GFDM. However IFDMA-GFDM and LFDMA-GFDM used a partial FFT matrix. Even if used a permutation matrix, it doesn't make large changes. Because of DFT spreading effect, the PAPR is slightly reduced than original GFDM. Compare with GFDM SLM, U=2 is enough to get same performance with DFT precoded GFDM and U=4 or 8 is enough to get better PAPR performance than BIDFT-N GFDM at CCDF(0.1%).

5.2 Spectrum Analysis

The introduced GFDM PAPR reduction techniques have a common similarity. Data symbol array of GFDM, $\mathbf{d}$ is consisted by i.i.d. QAM. The expectation of each symbol power is 1 and the PSD will not changed by DFT or IDFT precoding. It means the precoded GFDM has a same spectrum with GFDM when the common parameters are same. In the GFDM SLM case, the power of a phase

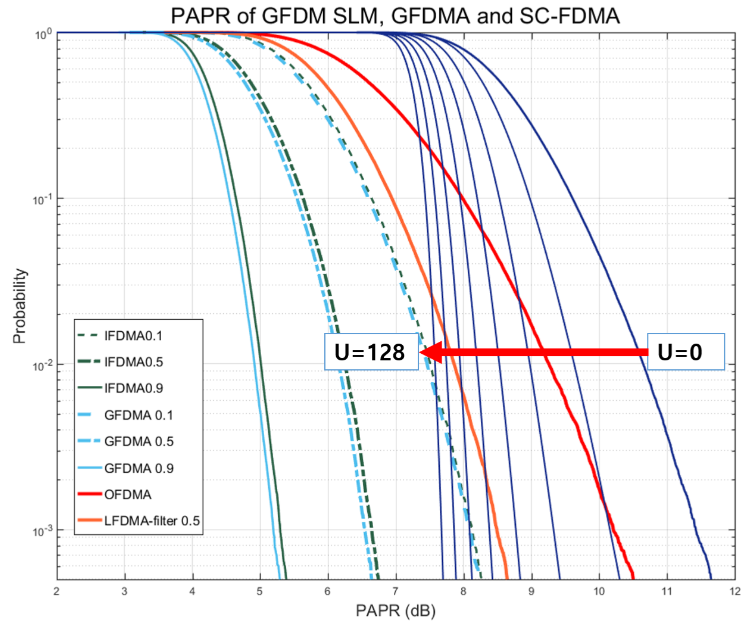

Figure 19: PAPR comparison of GFDM SLM, GFDMA and SC-FDMA.

shift is 1. Therefore, the spectrum of GFDM SLM is same as GFDM. GFDMA is assumed the specific parameters for the lowest PAPR GFDMA, the spectrum of GFDMA is higher than general GFDM and LFDMA. It is shown in the Fig. 21. Table. 4 shows detail interference to adjacent user considering some guard subcarrier by different modulations from Fig. 21.

Table 4: Interference to adjacent user as guard subcarriers

$K = 64$, $M = 5$, $Q = 4$	0 guard subcarrier	1 guard subcarrier	3 guard subcarrier
OFDMA, LFDMA	-10 dB	-14 dB	-16 dB
GFDM, GFDM SLM, precoded GFDM	-17 dB	-20 dB	-23 dB
GFDMA	-7 dB	-8.5 dB	-10 dB

5.3 Spectral Efficiency

Spectral efficiency is one of useful tool for performance analysis which is unit of $(bit/s)/Hz$. Sometimes spectral efficiency called by sum rate. According to parameters from 5, the spectral efficiency is simulated for noted PAPR reduction techniques. In this simulation,only Rayleigh fading is assumed, not AWGN channel. One of important notice for the simulation setting is GFDM and GFDMA is devided by $2\triangle f_k$. In fact, GFDM use less than $2\triangle f_k$ BW. Due to the frequency domain RC filter, around edge subsymbol carriers have zero values. Which means, effective BW of GFDM is smaller than $2M$. Especially in GFDMA, it use 24 subsymbol carriers with $\triangle f_M$. The difference between subsymbol carriers are increased to $\triangle f_k$, GFDMA has much spreaded frequency

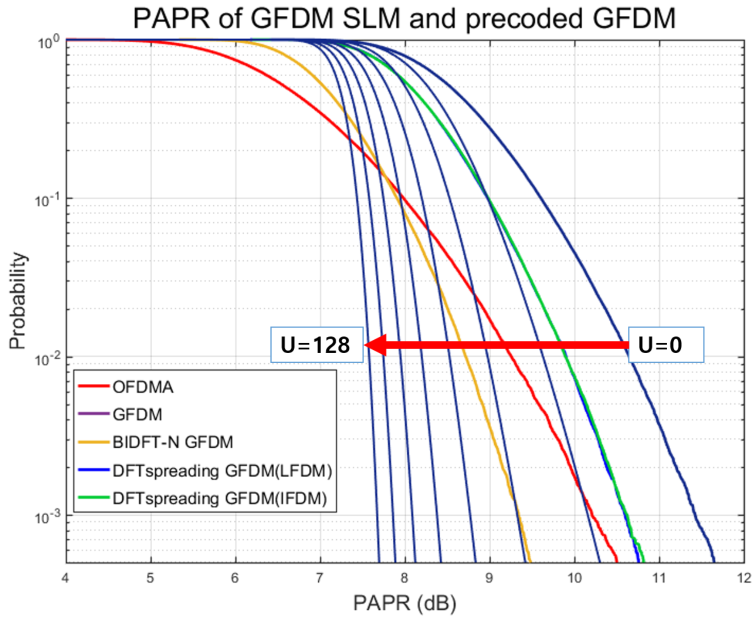

Figure 20: PAPR comparison of GFDM SLM and precoded GFDM.

domain RC filter than original GFDM. From Shannon's theorem $C = B * log_2(1 + \frac{P_s}{P_N})$, the spectral efficiency, S is represented by

$$S = log_2(1 + \frac{P_s}{P_N})/B \tag{22}$$

where B is BW, P_s and P_N are average signal power and average interference power. The effect of channel removed by ZF channel equalizer, it enhance the noise power if the channel is an ill-conditioned matrix. Considering noise enhancement and effective BW of all method, the simulation result of spectral efficiency is able to represent by the Fig. 22. GFDM SLM has better performance than precoded GFDM or GFDMA with 5.

6 CONCLUSION AND FUTURE WORK

In this paper, the GFDM SLM is proposed for reducing PAPR of GFDM without signal distortions. From a traditional OFDM SLM technique, the limited phase-shifting set is applied for a simplifying structure. Also the PAPR estimation equation is compared with simulation for approximated compensated coefficients by different parameters. For comparing performance, other PAPR reduction schemes are considered such as SC-FDMA, low PAPR GFDMA and precoded GFDM. GFDM SLM technique can be achieved a significant PAPR reduction compare with original GFDM, as increasing number of phase-shifting sets. Compare with other GFDM PAPR reduction techniques, the proposed GFDM SLM technique has lower PAPR than precoded GFDM by a few number of phase-shifting set. Due to the same spectrum characteristics with GFDM, GFDM SLM has low

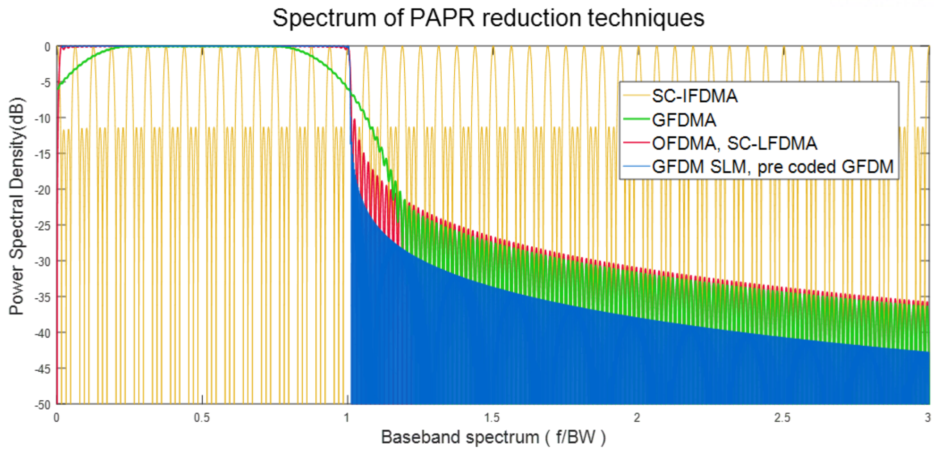

Figure 21: Spectrum of PAPR reduction techniques.

Table 5: Parameters for spectral efficiency of PAPR reduction techniques in ZF-receiver

Parameters	Value
Number of trial	100000
Subcarrier spacing,$\triangle f_k$	15KHz
Number of effective subcarriers, K	12
Number of subsymbols, M	5
Filter and roll off factor	Raised cosine, 0.5
Number of users, Q	4
Number of oversampling, L	4
Block size of BIDFT-N	K
DFT precoding size, N_{DFT}	4
Data symbol modulation	16-QAM
Rayleigh fading with 6 taps	

OOB and enough spectral efficiency. The cases of SC-FDMA and low PAPR GFDMA have the lowest boundary of PAPR. However SC-FDMA is not suitable for asynchronous environments as shown as Fig. 10 and Fig. 11. As shown as Fig. 21 and Fig. 22, GFDMA have loss in OOB and spectral efficiency.

The proposed SLM technique is focused on PAPR and efficiency. Even if the complexity is reduced by limited phase-shifting set, it has higher complexity than precoded GFDM or SC-FDMA. In OFDM SLM, there are some trials to pick optimal phase-shifting set for low complexity and optimal PAPR. Those schemes are one of considerable way to develop proposed idea. Another developable topic is the side information. SLM technique use side information for transmit information about used phase-shifting set. Even if the side information is assumed that excluded in the data symbols, it makes loss in other transmission processing. This is the second challangable problem as a future works.

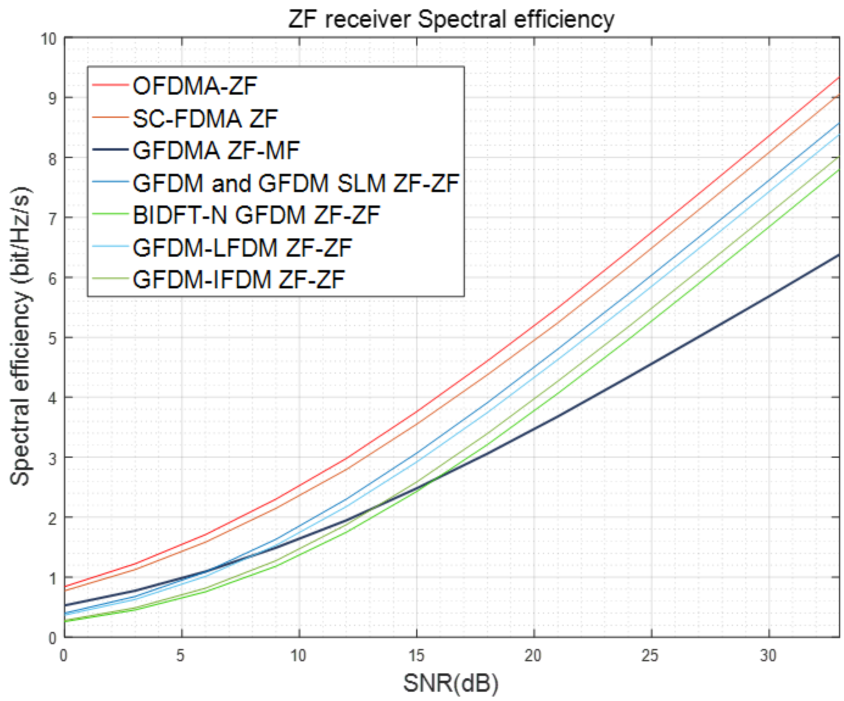

Figure 22: Spectral efficiency of PAPR reduction methods

APPENDIX. A

From (3), $\mathbf{x}[n]$ is consisted by uncorrelated in-phase and quadrature-phase like

$$\mathbf{x}[n] = \mathbf{x}_I[n] + \mathbf{x}_Q[n] \tag{23}$$

for $0 \leq n \leq N-1$ in . $\mathbf{d}$ is array of OFDM symbols that has QAM constellation. Every elements of $\mathbf{d}$ is i.i.d.. For large N, by the central limit theorem, it will follows Gaussian distribution with $CN \sim (0, \frac{\sigma^2}{2})$. The probability density function of amplitude is noted by

$$p(\mathbf{x}_I) = p(\mathbf{x}_Q) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \tag{24}$$

which same as Rayleigh distribution. PAPR is equal to (10), $|\mathbf{x}[n]|^2 = (x_I[n])^2 + (x_Q[n])^2$, the power of $|\mathbf{x}[n]|^2$ has Chi-square distribution,

$$P(|\mathbf{x}[n]|^2 = \gamma) = 2\gamma\, e^{-\gamma^2} \tag{25}$$

From probability density function, CDF can be noted by $1 - e^{-\gamma}$. Because, DFT of i.i.d. is also i.i.d, time domain value $\mathbf{x}[n]$ are i.i.d. For the N symbols, the cumulative density function is $(1 - e^{-\gamma})^N$. Therefore

$$CCDF(PAPR > \gamma) = 1 - (1 - e^{-\gamma})^N \tag{26}$$

APPENDIX. B

In LFDMA, let's assume $d_k$is k-th complex data symbol, $d_k{}'$ is DFT of d_k of user Q.

$\mathbf{X}_q$ is a frequency domain data array of user q where ($0 \leq q \leq Q-1$) for total subcarrier K, K=$K_{user}\,Q$.

Then $\mathbf{X}_q$ is same as

$$\mathbf{X}_q[l] = \begin{cases} d'_l & l = k + \frac{qK}{Q}\ (0 \leq k \leq K_{user} - 1) \\ \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

The Tx signal $\mathbf{x}_q$ is noted by

$$\mathbf{x}_q[r] = \frac{1}{K} \sum_{l=0}^{K-1} \mathbf{X}_q[l]\, e^{\frac{j\,2\pi\,l\,r}{K}} = \frac{1}{K} \sum_{l=0}^{K_{user}-1} \mathbf{X}_q[l]\, e^{\frac{j\,2\pi\,l\,r}{K}} \tag{28}$$

At the Qk points in time domain,

$$\mathbf{x}_q[Qk] = \frac{1}{K}\left(\frac{K}{Q} \sum_{l=0}^{K-1} \mathbf{X}_q[l]\, e^{\frac{j\,2\pi\,k\,r}{K/Q}}\right) = \frac{1}{Q}[d_0\,, d_1\,, \cdots, d_{K_{user}-1}\,] \tag{29}$$

Therefore

$$\mathbf{x}_q = \frac{1}{Q}[d_0\,, *\,, *\,\cdots d_1\,, *, *\,\cdots\,, d_{K_{user}-1}, *, *,\,\cdots] \tag{30}$$

In IFDMA, Then $\mathbf{X}_q$ is same as

$$\mathbf{X}_q[l] = \begin{cases} d'_l & l = Qk + q\ (0 \leq k \leq K_{user} - 1) \\ \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

The Tx signal $\mathbf{x}_q$ is noted by

$$\mathbf{x}_q[r] = \frac{1}{K} \sum_{l=0}^{K-1} \mathbf{X}_q[l]\, e^{\frac{j\,2\pi\,l\,r}{K}} \tag{32}$$

For 0 to $K_{user} - 1$ points in time domain,

$$\mathbf{x}_q[r] = \frac{1}{K}\left(\frac{K}{Q}\sum_{l=0}^{K-1}\mathbf{X}_q[l]\, e^{\frac{j\,2\pi\,k\,r}{K_{user}}}\right) = \frac{1}{Q}[d_0\,,d_1\,,\cdots\,,d_{K_{user}-1}\,] \tag{33}$$

$r = K_{user}$, then $e^{\frac{j\,2\pi\,k\,K_{user}}{K_{user}}} = 1$.
Therefore,$\mathbf{x}_q[K_{user}] = \frac{d_0}{Q}$ is repeated.

$$\therefore \mathbf{x}_q = \frac{1}{Q}[d_0\,,d_1\,,\cdots\,,d_{K_{user}-1},d_0\,,d_1\,,\cdots\,,d_{K_{user}-1},\cdots\,,\,d_{K_{user}-1}] \tag{34}$$

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