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One-dimensional broadband phononic crystal filter with unit cells made of two non-uniform impedance-mirrored elements

Il Kyu Lee, Yoon Jae Kim, Joo Hwan Oh, and Yoon Young Kim[a]
WCU Multiscale Design Division, School of Mechanical and Aerospace Engineering, Seoul National University, 599 Gwanak-ro, Gwanak-gu, Seoul 151-744, Korea

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A one-dimensional finite-sized phononic crystal(PC) made of a specially-configured unit cell is proposed to realize broad bandpass, high-performance filtering. The unit cell is specially-configured with two elements having mirrored impedance distributions of each other. One element has a non-uniform impedance distribution that is so engineered as to maximize wave transmission in the pass band and to minimize transmission in the adjacent stop band while the other, exactly the mirrored distribution. The mirroring approach naturally yields the overall impedance contrast within the resulting unit cell, necessary to form stop bands in a PC of the unit cells. More importantly, the good transmission performance of the orginally-engineered element can be preserved by the approach because no additional impedance mismatch is introduced along the interface of the two impedance-mirrored elements. Extraordinary performance of the PC filter made of the proposed unit cell, such as high transmission, large bandwidth and sharp roll-off, is demonstrated by using one-dimensional longitudinal elastic wave problems. [http://dx.doi.org/10.1063/1.4790638]

Because phononic crystals (PC's) of an infinite length form ideal stop and pass bands,[1–4] they may be also used to make bandpass filters. However, it is difficult to preserve the high-contrast stop/pass band characteristics of PC's when they become finite-sized. When typical PC's having unit cells made of two distinct high- and low- impedance elements become finite-sized, they cannot exhibit sharp roll-off characteristics or have a ripple-less flat bandpass region.[5–7] Instead of using PC's, one could use an impedance matching concept, as recently done for plasmonic waveguide design (see Xu *et. al.*[8] and references therein), but it is still difficult to realize the quasi-ideal bandpass filters processing the above-mentioned characteristics. PC's using cavity[9–12] or defect[13,14] modes may be also considered but we aim to engineer a filter having a bandwidth much wider than those realized by these PC's.

In order to device a quasi-ideal finite-sized bandpass PC filter (having high transmission, large bandwidth and sharp roll-off), we propose a specially-engineered unit cell. The unit cell consists of two impedance-mirrored elements each of which has an optimal non-uniform impedance distribution that maximizes wave transmission in the pass band and minimizes transmission in the adjacent stop bands. The optimal impedance variation in the element is represented by discrete values assigned to a number of sub-elements discretizing the element. Note that unlike earlier works mentioned above, the proposed method to engineer the unit cell also works even when the impedance of an input port is far different from that of an output port.

[a]Author to whom correspondence should be addressed, Electronic mail: yykim@snu.ac.kr. Phone: +82-2-880-7154. Fax: +82-2-872-1513

Once the impedance distribution of the element is determined, an element having the mirrored impedance distribution is placed next to the original element to complete the formation of a unit cell of a PC. Because the element placed next to the original element has the mirrored impedance distribution, there will be no apparent discontinuity in the interface of the two elements. Thus, the bandpass filtering characteristics of the original element can be virtually maintained. On the other hand, the mirroring approach guarantees a contrast in impedance over the length of the unit cell, a necessary property that a unit cell must possess in forming a PC with it. When a PC is formed with a period equal to the unit cell length, it will undoubtedly form stop bands. Definitely, the stop bands enhance the transmission suppression of the engineered element even if the PC becomes finite-sized. To make a use of the band gap phenomenon[15] of the resulting PC, therefore, the element size, equivalently, the unit cell size is determined by considering the Bragg frequencies of the selected unit cell. To explain the proposed impedance-optimizing and mirroring approach, we will mainly use one-dimensional longitudinal elastic waves. However, the developed method should be equally applicable to other types of wave problems.

Before presenting the proposed approach in some details, it will be useful to give an idea of the problem considered and also to demonstrate the performance of the proposed method. Note that all problems considered in this work deal with one-dimensional elastic waves propagating as depicted in Fig. 1(a). Let us begin with Fig. 1(a) that schematically shows that a bandpass filter is to be inserted between an input port and an output port. The two ports are allowed to have different media or impedances. As Case 1, we deal with a bandpass filter connecting input and output ports made of same impedance and as Case 2, we deal with another filter connecting input and output ports made of different impedances. Figure 1(b) shows the transmittance curves obtained by different approaches where the range of the pass band is targeted to lie between 50 kHz and 55 kHz. The curve marked with "Proposed" is obtained by using the proposed filter (to be explained later) while the curve marked with "Cavity-mode" is obtained by using a cavity-mode based PC filter.[9] With the cavity- or defect-mode approach,[9,13] one may tune the center frequency of the pass band at a desired frequency. However, the bandwidth realized by this approach is very narrow in compared with the curve obtained by the proposed method. Existing attempts to broaden the bandwidth of a pass band filter typically result in severe fluctuations in the transmission curve[6] unlike the curve obtained by the proposed approach. Furthermore, the value of transmittance tends to drop significantly when a cavity-mode PC filter is used to connect input and output ports of different impedances.

In addition to the curve by a cavity-mode approach, the classical result by the well-known impedance matched Salmon filter,[16] marked with "Impedance matching," is also plotted in Fig. 1(b). The reason to present this classical result is to suggest a possibility of incorporating the impedance matching concept in engineering a PC-based bandpass filter. Not that the concept of the matching appears to produce excellent transmission in the pass band although it is difficult to use if the transmission in the nearby stop bands is to be minimized below a certain value. To engineer a finite PC bandpass filter having broad bandwidth, ripple-less high transmission in the pass band, sharp roll-off, it may be therefore possible to utilize the good transmission capability of the matching concept and also the band gap phenomenon of a PC. With this background in mind, we will now present how to engineer a high-performance broad bandpass filter

If the longitudinal displacement u in the waveguide depicted in Fig. 1(a) is assumed to be a function of t (time) and x (axial coordinate), the wave equation is simply written as $\partial^2 u/\partial t^2 = c^2 \partial^2 u/\partial x^2$($c$: wave speed). To simplify the subsequent analysis, only the cross-sectional area S is assumed to vary along the x axis while intrinsic material properties, such as density ρ and wave speed c, are assumed to be uniform everywhere. Therefore, different media will be characterized primarily by the mechanical impedance Z (relating force and velocity) that depends on the cross-sectional area S because $Z = \rho c S$. For all problems considered here, we used $\rho = 2700$ kg/m^3 and $c = 5230$ m/s that correspond to an aluminum medium. Also, Z_A(A: input port medium) is assumed to be 1109 kg/s. The symbol f_L and f_U will be used to denote the lower and upper frequencies of the desired pass band. After an overview of the proposed method to engineer a bandpass filter made of a finite-sized PC is given, the detailed procedure will be explained with examples. In presenting the overview, the related physics will be also carefully investigated.

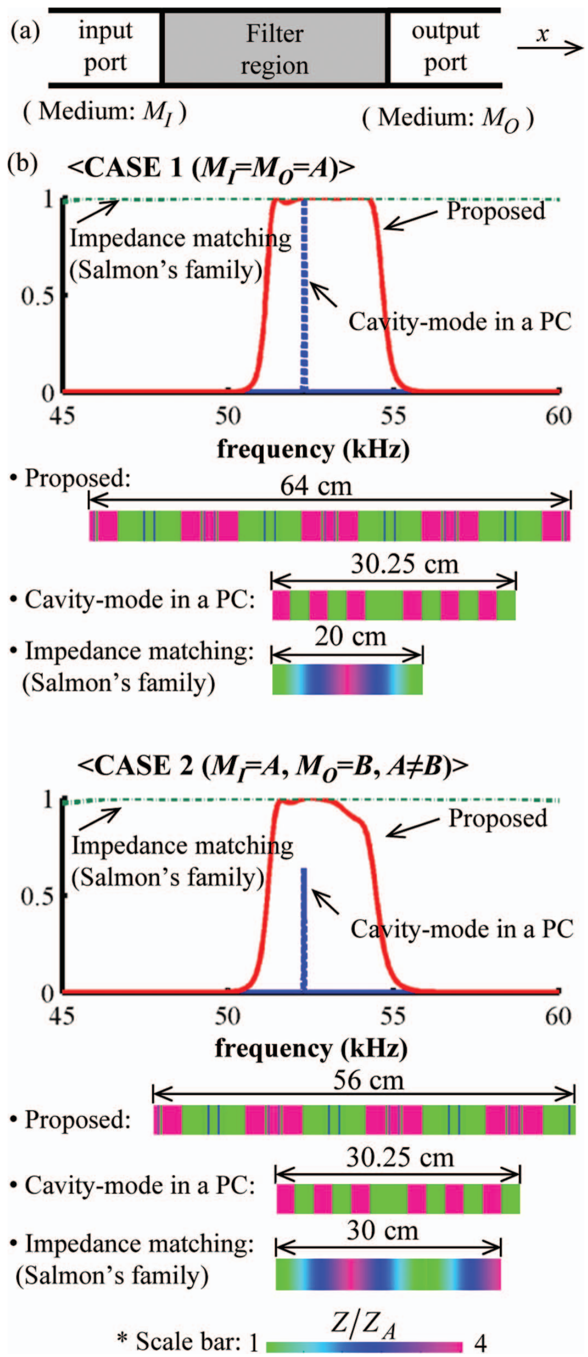

FIG. 1. (a) A bandpass filter to be inserted between an input port (medium M_I) and an output port (medium M_O), (b) Illustration of the transmittance curves by filters realized by different approaches for Case 1 (same medium for input and output ports) and Case 2 (different media for input and output ports).

Since a PC structure will be used, the length L of the unit cell of a PC needs to be determined first. This procedure is a routine one but we try to choose the minimum value of L through it in order to make the resulting PC filter as compact as possible. The value of L can be simply chosen to ensure that the corresponding infinitely-long PC has two consecutive Bragg frequencies[17] near the two frequencies (f_L^S,f_U^S) of the stop bands lying adjacent to the target pass band. Typically, $f_L^S = f_L - \Delta f_L^S/2$ and $f_U^S = f_U + \Delta f_U^S/2$ where Δf's can be viewed as the bandwidths of the stop

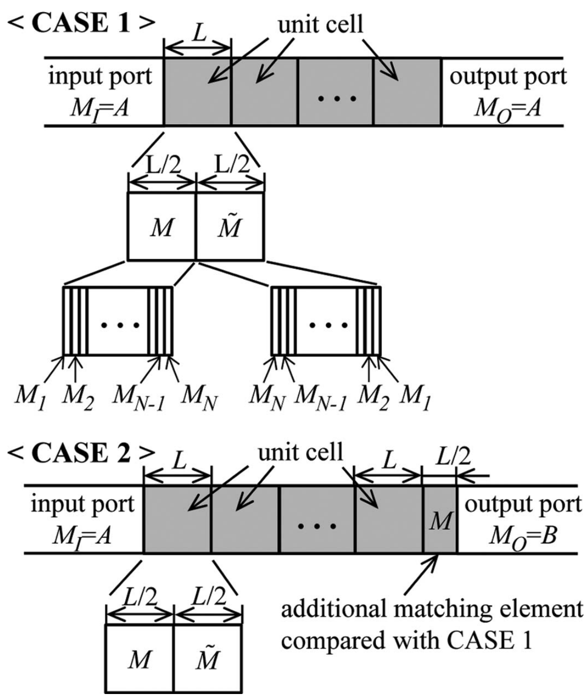

FIG. 2. The proposed layouts of the PC-based bandpass filter for Case 1 and Case 2. The unit cell of length L consists of two elements M and $\tilde{M}$ where the impedance distribution $\tilde{M}$ is the mirrored version of that of M. The use of an additional end element of M is needed for the Case 2 problems.

bands. Since the Bragg frequencies are given by $nc/2L$ (n: integers),[17] one should select the value of L for which there exists an integer n^* such that $n^*c/2L \leq f_L^S$ and $(n^*+1)c/2L \geq f_U^S$. As explained above, it is preferred to select the smallest L value allowing $n^*c/2L \approx f_L^S$ and $(n^*+1)c/2L \approx f_U^S$.

Once the value of the unit length L is selected, the key step in the proposed approach is to engineer a unit cell of a PC, specifically, to determine its desired impedance distribution. As pointed out earlier, constructing a unit cell with two $L/2$-long elements each of which have different but uniform impedance value causes undesirable ripples in the transmission curve when the resulting PC is finite-sized. Nevertheless, impedance contrast must be retained to make the resulting PC exhibit band gaps. With this in mind, we propose a unit cell composed of two elements, an element M of length $L/2$ and the other element $\tilde{M}$ of length $L/2$ as illustrated in Fig. 2. Because $\tilde{M}$ is supposed to have the mirrored impedance distribution of M (the reason behind this choice will be explained later), as schematically illustrated in Fig. 2, the impedance distribution of M will be optimized to have high transmission in the target pass band and low transmission in the adjacent stop bands. To facilitate the optimization, element M will be discretized into a set of sub-elements denoted by M_i ($i = 1, 2, \cdots, N_e$). Then, the impedance value Z_i of the ith element will be determined by and optimization scheme. For all problems considered here, the value N_e between 40 and 60 is used.

Let us now present the specific procedure to determine Z_i's. We will use Case 1 ($M_I = M_O = A$) to explain the procedure. The proposed strategy is to find the impedance distribution of element M that maximizes the transmittance in the target pass band frequencies between medium A (which represents the medium of both the input and output ports) and an artificial medium (say, C) having an impedance different from that of medium A. The main reason to introduce an artificial medium (say, C) is to ensure a non-uniform impedance distribution in element M; if the distribution were uniform, the resulting PC structure could not form stop bands. Note that the impedance medium C can be arbitrarily chosen as long as $Z_C \neq Z_A$. However, larger impedance contrast between C and A would more efficiently form appreciable stop bands[7] with a smaller number of unit cells. In this work, $Z_C/Z_A = 4$ was selected. (It is noted that the use of other Z_C/Z_A values yielded similar filter performance.) to find the desired impedance distribution $\mathbf{Z} = \{Z_1, Z_2, \cdots, Z_{N_e}\}^T$, the following maximization problem is solved numerically:

$$\max_{\{\mathbf{Z}\}} \left\{ \sum_{f=f_L}^{f_U} T(\mathbf{Z}; f) - \sum_{f=f_L-\Delta f_L^S}^{f_L} T(\mathbf{Z}; f) - \sum_{f=f_U}^{f_U+\Delta f_U^S} T(\mathbf{Z}; f) \right\}. \tag{1}$$

The first term in Eq. (1) represents the transmittance in the pass band while the second and third terms, the transmittances in the lower and upper stop bands. Here, the standard transfer matrix approach[18,19] is used in order to calculate the transmittance T for longitudinal waveguide made of N sub-elements. The side constraint $Z_A \leq Z_i \leq Z_C$ is imposed on Z_i. Once the formulation (1) is set up, the solution to Eq. (1), i.e., the distribution of impedance $\{Z_1, Z_2, \cdots, Z_{N_e}\}^T$ can be found by using any optimization algorithm. Thus, the details will be skipped here.

When element M of length $L/2$ is determined from Eq. (1), $\tilde{M}$ the mirror layout of M, is then placed next to M to form a complete unit cell of length L; see Fig. 2. Let us now explain the main reason why we propose the $M\tilde{M}$ arrangement, not just element M of length L alone in forming a unit cell. If element M of length L is repeatedly stacked to form a finite PC, there will be also an impedance mismatch along the interface between two adjacent unit cells. (This may be better understood if actual impedance distributions shown in Fig. 3(a) are examined.) On the other hand, putting $\tilde{M}$ of length $L/2$ next to M of length $L/2$ will not produce impedance mismatch either between the resulting two adjacent unit cells or between M and $\tilde{M}$. This way, the optimized transmission performance achieved by Eq. (1) can be well maintained even when the resulting unit cell composed by M and $\tilde{M}$ is periodically rearranged to form a PC. Note that the $M\tilde{M}$ arrangement, as a whole, clearly exhibits an impedance contrast within a unit cell, necessary in making a PC. Thereby, one can preserve the good transmittance in the pass band by the property inherited from the optimized impedance distribution of M and reduce the transmittance in the stop band mainly by the property inherited from the periodic PC arrangement.

When input and output port media are different ($Z_A \neq Z_B$), as also sketched by Case 2, an additional matching element M of length $L/2$ needs to attached to the last unit cell of the finite PC. The added element must be used to avoid unwanted impedance mismatch between the unit cell and the output port. It is also important in Case 2 to ensure that the artificial element C has the same impedance as that of B. Otherwise, the resulting impedance mismatch would alter significantly the transmission property inherited from element M.

As the first example to use the proposed method, let us consider Case 1 ($M_I = M_O = A$) with the target pass band dictated by $f_L = 50$ kHz and $f_U = 55$ kHz. The adjacent stop bands are assumed to be described by $\Delta f_L^S = \Delta f_U^S = 4$ kHz. First of all, one can easily find the smallest L value as 16 cm from the Bragg frequency calculation explained earlier. Specifically, the selected two consecutive Bragg frequencies are 49 kHz and 65 kHz when $n^* = 3$ is chosen. Figure 3(a) shows the impedance distribution of the unit cell obtained by solving Eq. (1) while the first plot in Fig. 3(b), the transmittance by the finite-sized PC consisting of 4 stacks of the unit cell found in Fig. 3(a). Note that the impedance variation in Fig. 3(a) is plotted in color while its actual implementation may be realized by the variation of the cross-sectional area of a one-dimensional elastic bar. It is apparent from Fig. 3(b) that the transmittance curve by the finite PC consisting of the proposed unit cell exhibits high transmission, large bandwidth and sharp roll-off which appear to be nearly impossible to realize with any existing approach. Note that the symbol N_c in Fig. 3(b) and elsewhere denotes the number of the unit cells used. The behavior of the transmittance curve shown in Fig. 3(b) may be examined further by investigating the dispersion curve for the engineered PC of an infinite length. The result is plotted in Fig. 3(c). The comparison of the plots in Figs. 3(b) and 3(c) shows that the stop bands of the PC filter with $N_c = 4$ match well with the band gaps of the corresponding PC of an infinite length. In choosing the value of N_c, one may actually examine its effects on the transmittance curve; see Fig. 3(d). Note that the use of a larger value of N_c makes the roll-off of the resulting bandpass filter sharper. However, it causes more ripples to appear in the pass band because of the well-known resonance phenomenon observed in finite-sized PC's.[6,7] Numerical tests have indicated that satisfactory results were obtained with $N_c \approx 4$ when the present approach is used to engineer unit cells.

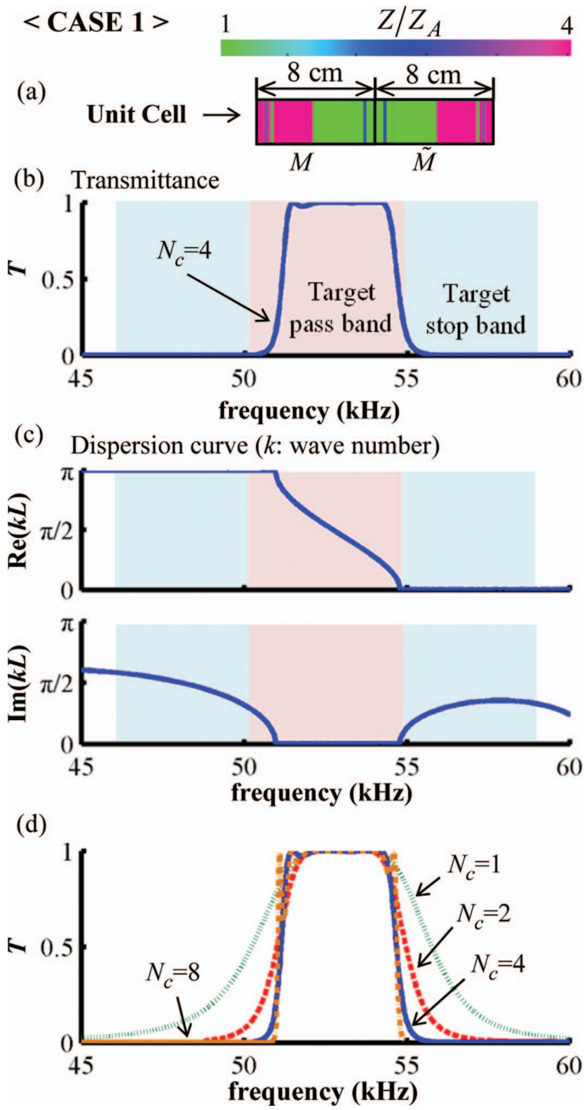

FIG. 3. (a) The impedance distribution in the unit cell engineered for Case 1 with the prescribed band frequencies of $f_L = 50$ kHz, $f_U = 55$ kHz, and $\Delta f_L^S = \Delta f_U^S = 4$kHz. (b) The transmittance curve of the proposed finite-sized PC with $N_c = 4$. (c) The dispersion curve obtained for the proposed PC of an infinite length. (d) The effects of N_c values on the transmittance curve.

Let us consider one more example for Case 1 that deals with a broader pass band: $f_L = 50$ kHz, $f_H = 70$ kHz, $\Delta f_L^S = 5$ kHz and $\Delta f_U^S = 5$ kHz. Following the same procedure employed in the previous example, the unit cell length of $L = 10.2$ cm is determined with $n^* = 2$. Figure 4 shows the impedance distribution in the unit cell and the transmittance of the engineered finite PC by the proposed method. The observations made with Fig. 3 equally apply to the results shown in Fig. 4. Therefore, extensive discussions with Fig. 4 may be skipped.

Finally, we consider Case 2, the case where the input and output ports have different impedance ($M_I \neq M_O$, i.e. $Z_A \neq Z_B$). The selected stop and pass bands are given by $f_L = 50$ kHz, $f_U = 55$ kHz and $\Delta f_L^S = \Delta f_U^S = 4$ kHz (as used in the first problem in Case 1). The problem with any value of Z_B can be solved by the proposed approach but the value of $Z_B = 4Z_A$ is selected in order to compare the results of this problem with those of the previous problem for $Z_B = Z_A$. Recall that $Z_C = 4Z_A$ (Z_C: the impedance of an artificial medium) was used in the previous problem for $Z_A = Z_B$. Because Z_B for this problem is selected to be the same as Z_C in the earlier problem,

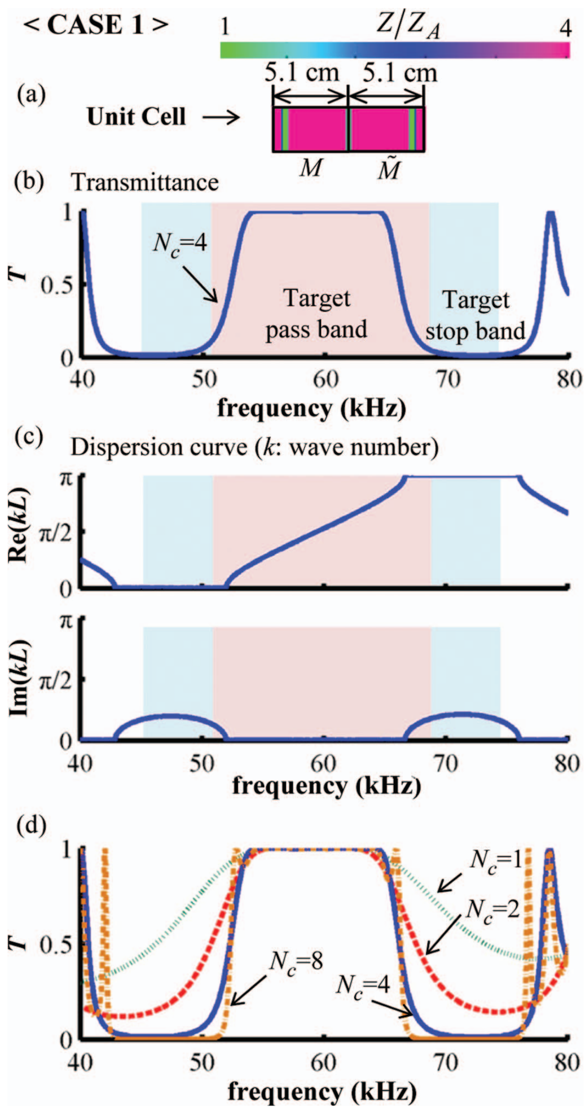

FIG. 4. (a) The impedance distribution in the unit cell engineered for Case 1 with prescribed band frequencies of $f_L = 50$ kHz, $f_U = 70$ kHz, and $\Delta f_L^S = \Delta f_U^S = 5$kHz. (b) The transmittance curve of the proposed finite-sized PC with $N_c = 4$. (c) The dispersion curve obtained for the proposed PC of an infinite length. (d) The effects of N_c values on the transmittance curve.

one can find the same impedance distribution and unit cell length as those obtained for the previous problem. This means that the distribution in Fig. 3(a) will be used for this problem. The main difference between the current PC layout and the previous PC layout is that an additional matching element M of length $L/2$ should be attached to the last unit cell interfacing M in the present case. The transmittance curves obtained with different N_C values for $Z_B = 4Z_A$ are shown in Fig. 5(a). The results for $Z_B = 4Z_A$ with different stop and pass bands of $f_L = 50$ kHz, $f_U = 70$ kHz and $\Delta f_L^S = \Delta f_U^S = 5$ kHz are shown in Fig. 5(b). To obtain the results in Fig. 5(b), the impedance distribution obtained in Fig. 4(a) is used because of the same reason explained in obtaining the results in Fig. 5(a). From the plots in Fig. 5, one can confirm that the engineered filters by the proposed approach even for the case of $Z_B \neq Z_A$ result in high transmission, large bandwidth and sharp roll-off. As expected, the filters for $Z_B = Z_A$ perform somewhat better than the filters for $Z_B \neq Z_A$.

In this study, a novel unit cell layout of a one-dimensional PC was proposed for realizing a bandpass filter characterized by wide bandwidth, high transmission in the pass band and sharp

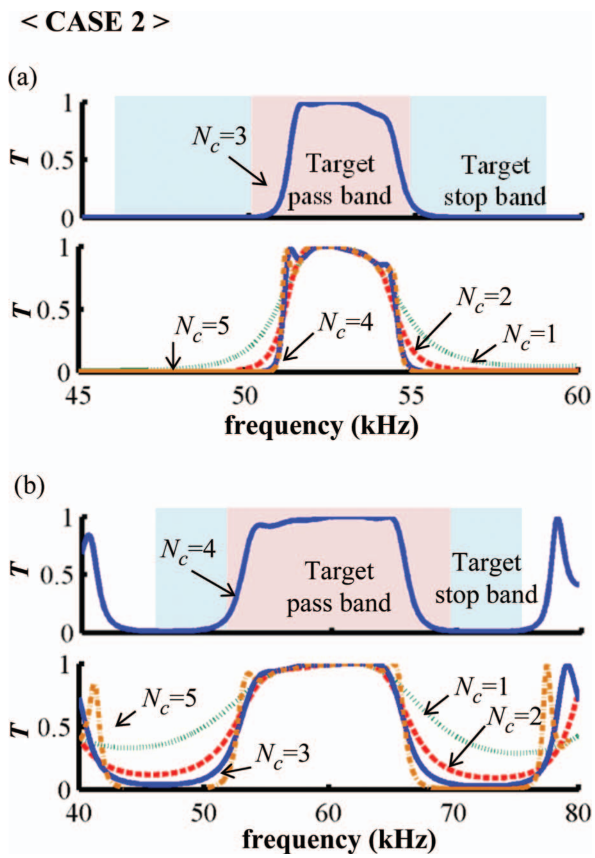

FIG. 5. The transmittance curves of the proposed finite-sized PC for Case 2 ($Z_B = 4Z_A$) with the prescribed band frequencies of (a) $f_L = 50$ kHz, $f_U = 55$ kHz, and $\Delta f_L^S = \Delta f_U^S = 4$ kHz and (b) $f_L = 50$ kHz, $f_U = 70$ kHz, and $\Delta f_L^S = \Delta f_U^S = 5$ kHz.

roll-off. The proposed unit cell is composed of two elements of non-uniform impedances. The discretized impedance distribution in each element is optimally engineered to maximize transmission in the pass band and minimize transmission in the stop bands. Furthermore, one of the two elements is made to the mirrored impedance distribution of the other element. This mirroring method was critical for avoiding any adverse effect due to the impedance mismatch between the two elements that would result in poor transmission performance. Obviously, the mirroring also ensures an overall impedance contrast within the proposed unit cell, necessary in forming stop bands with a PC made from periodically-arrange unit cells. Another advantage of the proposed approach is that unlike existing approaches, the proposed method also works for engineering a bandpass filter connecting input and output ports made of different impedances. Numerical examples confirmed the effectiveness of the proposed method. Extensions of the proposed method in other types of waves or higher-dimensional problems are expected to produce more promising results.

ACKNOWLEDGMENTS

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