AIP Applied Physics Letters

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Citation: Applied Physics Letters **91**, 081505 (2007); doi: 10.1063/1.2773943

View online: http://dx.doi.org/10.1063/1.2773943

View Table of Contents: http://scitation.aip.org/content/aip/journal/apl/91/8?ver=pdfcov

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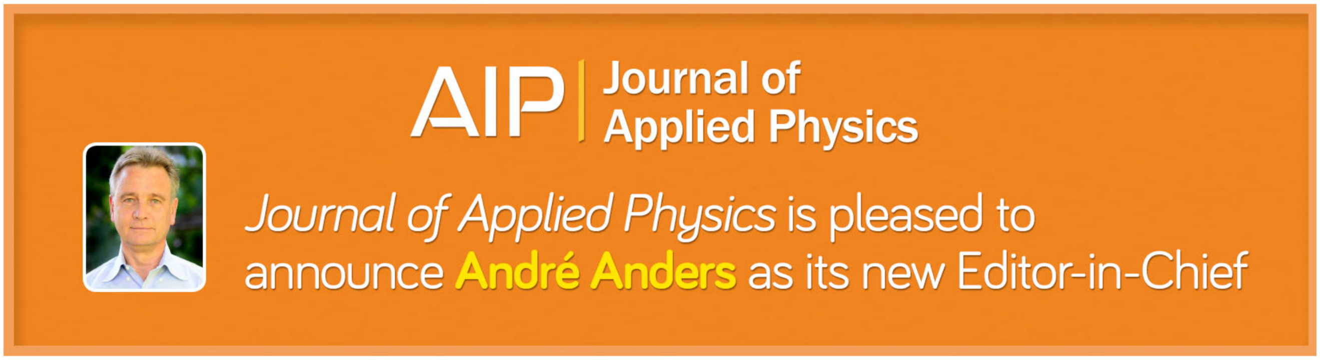

APPLIED PHYSICS LETTERS **91**, 081505 (2007)

Additional focusing of a high-intensity laser beam in a plasma with a density ramp and a magnetic field

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(Received 30 May 2007; accepted 31 July 2007; published online 23 August 2007)

Propagation of a high power Gaussian laser beam through a plasma with a density ramp where a magnetic field is present has been investigated. The spot size of the laser beam decreases as the beam penetrates into the plasma due to the role of a plasma density ramp. The studies show that the combined effect of a plasma density ramp and a magnetic field enhances the self-focusing property of the laser beam. Both factors not only reduce the spot size of the laser beam but also maintain it with only a mild ripple over several Rayleight lengths. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2773943]

Relativistic interaction of a laser beam with a plasma has been studied experimentally and theoretically by many authors in the past several decades.[1–4] Numerous applications were suggested and they include the laser-driven acceleration, x-ray laser, fast ignition for inertial confinement fusion,[5–7] etc. In these applications, one needs the laser beam to propagate over several Rayleigh lengths while preserving an efficient interaction with the plasmas. Therefore, self-focusing of a high-intensity laser beam in a plasma is a very important research issue nowadays.[8–14] The experimental results on relativistic self-focusing in hydrogen gas have been reported by Fedosejevs *et al.*[15] Hafizi *et al.*[16] studied the propagation of an intense laser beam in a plasma, including the relativistic and ponderomotive effects. Their results show that the laser beam can acquire a minimum spot size due to the relativistic self-focusing in a plasma. Beyond the focus, the nonlinear refraction starts weakening and the spot size of the laser increases, showing oscillatory behavior with the distance of propagation. To overcome the diffraction and the successive high amplitude oscillation of the spot size, Gupta *et al.*[17] have proposed to introduce a slowly increasing plasma density gradient and found that the plasma density gradient plays a crucial role in the laser self-focusing.

Pukhov and Meyer-ter-Vehn[18] have observed the strong magnetic field generated during laser-plasma interaction that can influence the laser beam propagation. Singh[19] applied this kind of a magnetic field for electron acceleration by laser-plasma interaction, where high energy gain was obtained. The magnetic field changes the dispersion relation of the laser beam and modifies the plasma electron density. It is very important in the context of laser self-focusing. In this letter, we report an additional focusing of an intense laser beam in a plasma. The additional focusing enhancement is caused by the magnetic field applied in the transverse direction of the laser propagation. Our study shows that the combined role of a plasma density ramp and a magnetic field can reduce the radius of the laser beam efficiently and can maintain it longer.

We consider the propagation of a Gaussian laser beam through a magnetized plasma of electron density n with a density ramp along the z direction. The fields of the laser beam can be written as

$$\vec{E}=\hat{x}A(r,z,t)\exp[-i(\omega_0 t-k_0 z)], \tag{1}$$

$$\vec{B}=c\vec{k}_0\times\vec{E}/\omega_0, \tag{2}$$

where $A^2|_{z=0}=A_0^2 g(t)\exp(-r^2/r_0^2)$, $k_0(z)=(\omega_0/c)\varepsilon_0^{1/2}$, $\omega_p(z)=[4\pi n(z)e^2/m]^{1/2}$ is the electron plasma frequency, ε_0 is the plasma dielectric constant, $g(t)$ characterizes the temporal shape of the pulse, $g(t)=1$ for $t>0$ and $g(t)=0$ for $t<0$, c is the speed of light in vacuum, and $-e$ and m are the electron's charge and mass, respectively. For $z>0$, we may write $A^2=(A_0^2/\sigma^2)\exp(-r^2/r_0^2\sigma^2)$, where $\sigma(z)$ is known as the beam-width parameter, A_0 is the axial amplitude of the laser, r_0 is the half width radius of the laser beam, and r is the radial coordinate of the cylindrical coordinate system. The upward plasma density profile can be modeled by a simple expression $n(\xi)=n_0\tan(\xi/d)$, where n_0 is the equilibrium plasma electron density, $\xi=z/R_{d0}$ is the normalized propagation distance, and d is an adjustable constant. We take this constant factor $d=2.5$ so that the electron density reaches twofold equilibrium plasma density during the ramp length of $3R_{d0}$, where $R_{d0}=\omega_0 r_0^2/2c$ and ω_0 is the laser frequency. This kind of density profile can be achieved in an experiment by using transient laser drilling on a gas jet. Here, the transverse magnetic field $(\vec{B}_0)$ in y direction is present. To use this magnetic field, we follow Singh,[20] where the transverse magnetic field was applied for laser electron acceleration. The wave equation governing the laser propagation can be written as

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}=\frac{4\pi}{c}\frac{\partial\vec{J}}{\partial t}, \tag{3}$$

where $\vec{J}=-ne\vec{v}$ is the plasma current density and $\vec{v}$ is the plasma electron velocity.

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0003-6951/2007/91(8)/081505/3/$23.00

Using the standard perturbation technique from the equation of electron motion and the continuity equation, the perturbed velocity and density components in the presence of a transverse magnetic field can be calculated. As a result, the total transverse current density can be calculated as

$$\vec{J}_{\perp} = -\hat{x}en(z)ca[\alpha_0 - a^2\{\alpha_0 c^2 k_0^2(\alpha_1 - \alpha_2) + \alpha_0\alpha_3\}]\sin(k_0 z - \omega_0 t), \tag{4}$$

where $a = eA/m\omega_0 c$, $\omega_c = eB_0/mc$, $\alpha_0 = \omega_0^2/\omega_0^2 - \omega_c^2$, $\alpha_1 = \alpha_0^4\omega_c^2(5\omega_0^4 - 11\omega_0^2\omega_c^2 - 6\omega_c^4)/4\omega_0^6(4\omega_0^2 - \omega_c^2)$, $\alpha_2 = \alpha_0^3(5\omega_0^4 - 11\omega_0^2\omega_c^2 - 6\omega_c^4)/4\omega_0^4(4\omega_0^2 - \omega_c^2)$, and $\alpha_3 = \alpha_0^4(3\omega_0^4 - 2\omega_0^2\omega_c^2 + 3\omega_c^4)/8\omega_0^4$. Here, the first term of Eq. (4) shows the linear current density and other terms are due to the nonlinear current density. The terms related to the nonlinear current density were modified due to the transverse magnetic field. This modified nonlinear current density due to the transverse magnetic field affects the nonlinear dielectric constant of the plasma, which can show the profound effect on laser self-focusing during propagation through an upward plasma density ramp. From the expression of the current density, the nonlinear dielectric constant can be written as $\Phi = a^2\omega_p^2(z) \times [\alpha_0(c^2k_0^2/\omega_0^2)(\alpha_1 - \alpha_2) + \alpha_0\alpha_3]$.

To find out the equation governing the laser spot size, we use the following mathematical steps: (a) we substitute the current density in the wave equation and obtain the envelope amplitude equation in the Wentzel-Kramers-Brillouin approximation; (b) second, following the approach given by Akhmanov,[21] we introduce an eikonal, $A = A_{00}(z,r)\exp[ik_0 S(z,r)]$, where $A_{00}(z,r)$ and $S(z,r)$ are the real function of space; and (c) we substitute the value of A and separate the real and imaginary parts of the resulting equation. The coupled set of equations for A_{00}^2 and S is solved in the paraxial approximation by expanding S as $S = S_0 + (r^2/2)(1/\sigma)(d\sigma/dz)$, where $S_0(z)$ is the axial phase.

By following the above steps, we obtain $A_{00}^2 = (A_0^2/\sigma^2)\exp(-r^2/r_0^2\sigma^2)$ and the resultant equation governing the spot size of the laser is as follows:

$$\frac{\partial^2\sigma}{\partial\xi^2} = \frac{1}{\sigma^3} - \frac{1}{2}\left(\frac{\partial\varepsilon_0}{\partial\xi}\right)\frac{\partial\sigma}{\partial\xi} - \frac{R_{d0}^2 a_0^2}{2r_0^2\sigma^3}\frac{\Phi}{\Phi_0}, \tag{5}$$

where $\Phi_0 = 1 - \omega_p^2(z)/(\omega_0^2 - \omega_c^2)$, $a_0 = eA_0/m\omega_0 c$, and $\xi = z/R_{d0}$. Here, Φ is the change in the dielectric constant of the plasma due to the nonlinear effects introduced by the magnetic field and the plasma density ramp.

In Eq. (5), the first term is due to the diffraction effect, the second term is due to the plasma inhomogeneity, and the last is the nonlinear term with the effect of a magnetic field and plasma density ramp which is responsible for the additional self-focusing of the laser beam. Equation (5) is a second order differential equation and we can solve it by choosing suitable laser and plasma parameters. In this way, one can find the suitable condition for self-focusing of the laser during propagation through the density ramp in a magnetized plasma. In general, we use the boundary conditions, $\sigma = 1$ and $\partial\sigma/\partial\xi = 0$.

For the experimental parameters of the equilibrium plasma electron density $n_0 = 1.1 \times 10^{19}$ cm^{-3}, maximum laser intensity $I_0 = 5 \times 10^{18}$ W cm^{-2}, laser wavelength $\lambda_0 = 1$ μm, and laser spot size $r_0 = 10$ μm, we solve Eq. (5) to find out the variation of the beamwidth parameter with the distance of propagation of the laser beam. The used magnetic field is about 30 MG. Figure 1 displays the variation of the beamwidth parameter against the normalized propagation distance of the laser beam with a magnetic field (black curve) and without a magnetic field (red curve). This figure shows that the combined effect of a plasma density ramp and a magnetic field enhances the laser focusing. In the absence of a magnetic field (where only the plasma density ramp is present), we find that the beamwidth parameter decreases with the distance of propagation to a value of $\sigma = 0.4$ at $z = 2R_{d0}$. As the equilibrium electron density is an increasing function of the distance of propagation of the laser beam, the plasma dielectric constant decreases rapidly as the beam penetrates deeper and deeper in the plasma. Consequently, the self-focusing effect is enhanced and the laser beam is focused more in the plasma density ramp. If the magnetic field is applied, the beamwidth parameter is observed to decrease with the propagation distance to a value of $\sigma = 0.2$ at $z = R_{d0}$. Because of the magnetization of the plasma, the laser beam spot size not only decreases to its minimum value but also maintains it with only a mild ripple over several Rayleight lengths. If we compare both cases (with and without the magnetic field), it is found that the magnetic field enhances the self-focusing property of the laser beam. Figure 2 shows the effect of plasma magnetization on laser focusing for different values of the magnetic fields. It is observed that the increasing magnetic field leads to a significant decrease in beamwidth parameter of the laser beam.

In conclusion, the combined role of the plasma density ramp and the transverse magnetic field is found to enhance the laser beam focusing in a plasma. The spot size of the laser beam shrinks as the beam penetrates into the plasma

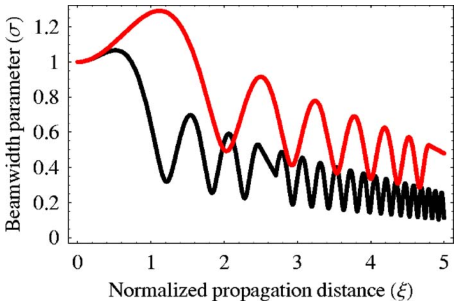

FIG. 1. (Color online) Variation of the beamwidth parameter (σ) with the normalized propagation distance ($\xi = z/R_{d0}$) with a magnetic field (black curve) and without a magnetic field (red line, only plasma density ramp is present). The used parameters are $n_0 = 1.1 \times 10^{19}$ cm^{-3}, $I_0 = 5 \times 10^{18}$ W cm^{-2}, $\lambda_0 = 1$ μm, $r_0 = 10$ μm, and $B_0 = 30$ MG.

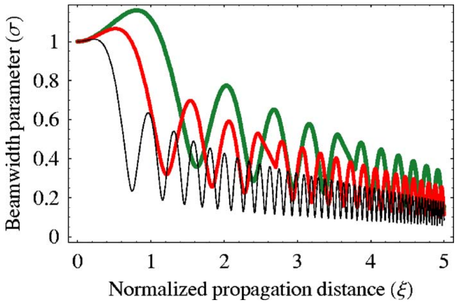

FIG. 2. (Color online) Variation of the beamwidth parameter (σ) with the normalized propagation distance ($\xi = z/R_{d0}$) for different values of the magnetic field. The green, red, and black curves are for the magnetic field values of 20, 30, and 45 MG, respectively. Other parameters are the same as those of Fig. 1.

due to the role of a plasma density ramp. The magnetic field acts as a catalyst for self-focusing of a laser beam during propagation in a plasma density ramp. By using this scheme, the laser not only becomes focused but it can also propagate over a long distance without divergence, which is a basic requirement for many laser-driven applications.

This work was supported by the Korean Ministry of Science and Technology through the Creative Research Initiative Program/KOSEF.

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