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A universal characterization of nonlinear self-oscillation and chaos in various particle-wave-wall interactions

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The comprehensive parameter space of self-oscillation and its period-doubling route to chaos are shown for bounded beam-plasma systems. In this parametrization, it is helpful to use a potentially universal parameter in close analogy with free-electron-laser chaos. A common parameter, which is related to the velocity slippage and the ratio of bounce to oscillation frequencies, is shown to have similar significance for different physical systems. This single parameter replaces the dependences on many input parameters, thus suitable for a simplifying and diagnostic measure of nonlinear dynamical and chaotic phenomena for various systems of particle-wave interactions. The results of independent kinetic simulations verify those of nonlinear fluid simulations. © *1998 American Institute of Physics.* [S0003-6951(98)01012-2]

The self-oscillation in an undriven physical system and its standard routes to chaos are the subject of intense studies for plasma systems[1] and for free-electron-laser (FEL) systems.[2–4] The previous plasma studies were mostly experimental,[5,6] reporting the observation of various nonlinear dynamical and chaotic oscillations in such plasma systems. The overall parameter window including all such oscillations is particularly needed for designing the parameters of future studies, experimental or theoretical.

The self-oscillations in undriven[5] (without external ac driver) and driven[6] plasma systems are observed in many parallel-plate thermionic (with an electron beam injected from the cathode) discharges, electron-cyclotron-resonance plasma discharge,[7] and rf plasmas.[8] Once excited, these self-oscillations follow one of the standard routes to chaos via period-doubling, intermittency, or quasiperiodic oscillation. Some of these are shown in the present letter for the parallel-plate beam-plasma discharges. Despite the differences in the plasma-formation mechanism of these various discharges, most discharges have some forms of interactions between energetic beam and collective wave, which become nonlinearly important at large wave amplitudes. The proposed diagnostic parameter μ contains the nonlinear wave-amplitude resulting from the particle-wave interaction and the velocity slippage between particle and wave. The chaos of driven systems[6] can also be classified using μ in a similar fashion to the undriven plasma system in the present study.

The physical model for the simulation is that of the extended Pierce diode system,[9,10] where an electron beam of velocity u_0, density n_0, and plasma frequency ω_p is injected to a collisionless plasma-diode system of length L with the applied dc voltage V_a and background ion density n_i. The past simulations showed the significance of the Pierce parameter[10] $\alpha=\omega_p L/u_0$, the ion-electron ratio[9,11] $\eta=n_i/n_0$, and the applied voltage[12] $\phi=eV_a/mu_0^2$.

For the numerical simulations, we have employed two different methods, nonlinear fluid and kinetic simulations, which showed similar results. Our fluid simulations employ the equations of continuity and motion along with Poisson's equation in one dimension.[9–11] The kinetic simulation has been carried out by using PDP-1[13] with a change to give the initial perturbation.

The overall parameter window in terms of control input variables for various nonlinear dynamical phenomena leading to chaos is useful and illuminating for the comparison with other experiments and simulations. These parameter regimes, shown in Fig. 1, are the results of our nonlinear fluid simulations extended to three-parameter space involving α, ϕ, and η. The regions for stable focus (thus no-oscillation), self-oscillation, period-doubling, chaos, and unstable region[10] are marked SF, $1P$, $2P$, C, and U, in Fig. 1, respectively. The time series, power spectra, and phase spaces of each case are shown in Fig. 2. The new results noted in Fig. 1 are that the increase in α at a constant ϕ can also

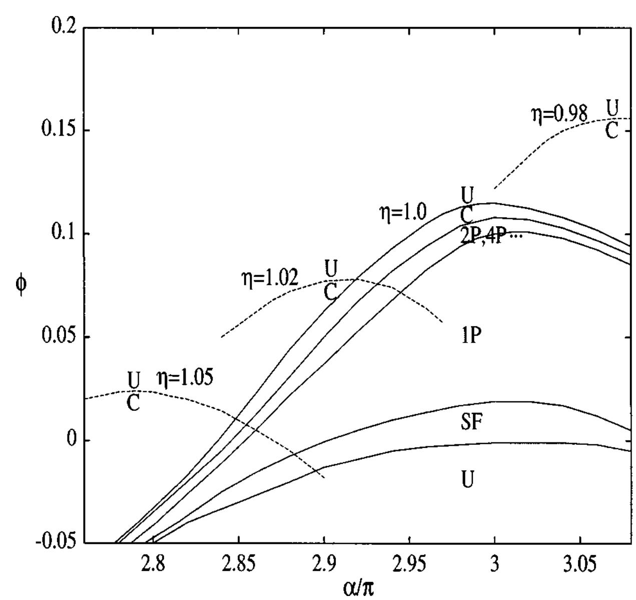

FIG. 1. Phase diagram of the extended Pierce system with the fluid model. The regions for stable focus, self-oscillation, period-doubling, chaos, and unstable region are marked SF, $1P$, $2P$, C, and U.

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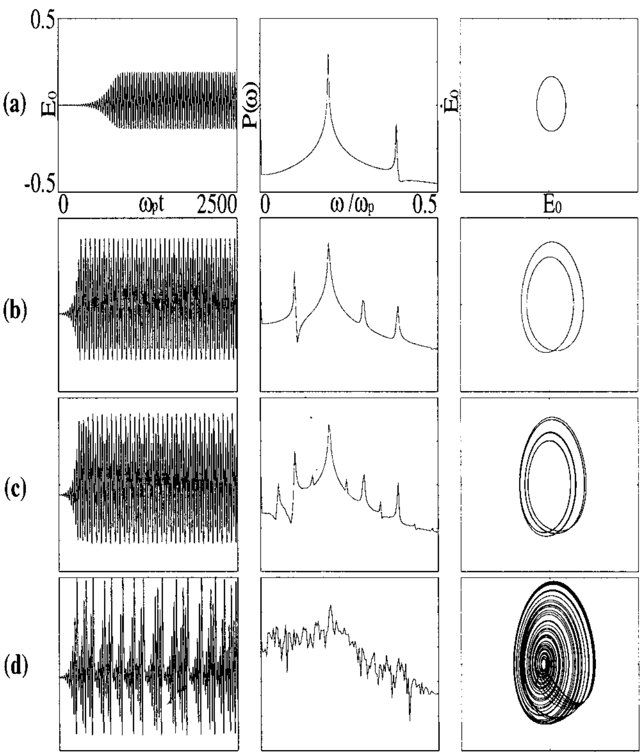

FIG. 2. Time series, power spectra, and phase spaces of the normalized electric fields on the cathode $E_0 = E/\omega_p u_0$ for various ϕ's with $\eta=1$, $\alpha = 2.88\pi$. (a) $\phi=0$, Self-oscillation, (b) $\phi=0.025$, period-doubled, (c) $\phi = 0.0285$, period-quadrupled, and (d) $\phi=0.04$, chaotic.

reveal the period-doubling route to chaos shown in Fig. 2 at larger values of α and that the increase in η moves the peak of the parabola-like curve to the small α and ϕ regime.

The phase diagram for various nonlinear dynamical phenomena can be classified in terms of the parameter μ similar to one used in describing the free-electron-laser (FEL) chaos.[2–4] In studying the FEL chaos, an empirical parameter μ was introduced as in[2,3]

$$\mu \equiv \frac{L_s}{L_{\rm syn}} = N_w \lambda \left[\frac{2\pi c(c-u_0)}{\omega_b u_0} \right]^{-1}, \quad (1)$$

where $L_s(\equiv N_w \lambda)$ is slippage length (the effective interaction length between electrons and wave) with the number of wiggler periods N_w and the radiation wavelength λ, $L_{\rm syn}$ is synchrotron slippage distance, c is the speed of light, u_0 is the average axial velocity of electron beam, and $\omega_b = \sqrt{ekE/m}$ is the synchrotron bounce frequency with wave number k and electric field intensity E. For the beam-plasma case,

$$\mu \equiv \frac{L_{\rm int}}{L_{\rm syn}} = \frac{\pi}{k} \left[\frac{2\pi v_\phi (u_0 - v_\phi)}{\omega_b u_0} \right]^{-1} = \frac{\omega_b/\omega_r}{2(1-v_\phi/u_0)}, \quad (2)$$

where $L_{\rm int}(\equiv \pi u_0/\omega_p = \pi/k)$ is the effective interaction length and $v_\phi = \omega_r/k$ is the phase velocity of the wave with oscillation frequency ω_r. The numerator of μ is related to the amount of the trapped particles in the wave, which is proportional to the electric field intensity of the excited wave. The denominator of μ, the velocity slippage between the beam and wave, is associated with resonance and energy delivery between beam and wave. Bonifacio[14] pointed out the similarity of the bounded FEL and the bounded plasma

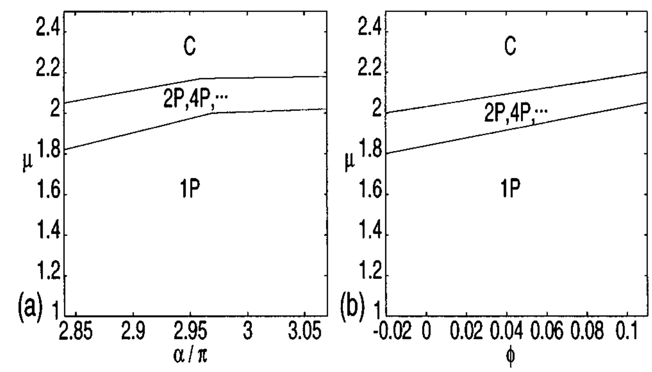

FIG. 3. Phase diagrams by fluid simulation in terms of the deterministic parameter μ vs. (a) α and (b) ϕ for $\eta=1$.

equations along with the importance of the velocity slippage. The significance of the slippage in determining the nonlinear amplitude has also been recognized for a homogeneous beam-plasma instability[15] and for an FEL system.[16]

For a homogeneous beam-plasma instability,[15] Eq. (2) can be expressed as

$$\mu = \frac{\sqrt{K}}{W} \left[\frac{R}{4(1-W/K)^3} \right]^{1/4}, \quad (3)$$

where $W = \omega_r/\omega_{pp}$, $K = kv_b/\omega_{pp}$, and $R = n_b/n_p$ with the plasma density n_p, beam density n_b, plasma frequency of the bulk plasma ω_{pp}, and cold beam velocity v_b. In this system, the oscillation of saturated field energy of large μ case is more chaotic than that of small μ case because of small velocity slippage despite low saturation level. The significance of the diagnostic quantity μ is that the degree of the nonlinearity can be determined not only by the amplitude of the electric field but also by the difference between the beam and wave velocities. This parameter is in a fashion similar to the other useful diagnostic quantities such as Lyapunov exponent and Kolmogorov entropy.

The regimes where various oscillations such as $1P$, $2P$, $4P$, and chaos appear are indicated in Fig. 3 with specific μ's. The large values of μ exceeding approximately 2 are associated with the chaotic oscillations. The transitions of the FEL chaos[3] from $1P$, to $2P$, then to C occur at $\mu \sim 1.7$ and 2.0, which compare with the plasma case at $\mu \sim 1.8$ and 2.0. The dependence of these transition μ-values on the input parameters such as α, ϕ, and η is weak as seen from the weak horizontal slopes in Fig. 3. The usefulness of a single parameter μ as a diagnostic measure of various types of nonlinear phenomena leading to chaos is pronounced especially when the physical systems have complex dependences on many input parameters.

These nonlinear dynamical and chaotic phenomena for the beam-plasma system are analogous to the case of FEL. The phase diagram of our FEL work [Fig. 12 of Ref. 2] in terms of the input parameters resembles the modest α-regimes for the plasma of Fig. 1. The μ-classification for the FEL chaos (Fig. 1 of Ref. 3), is qualitatively similar to that for the plasma chaos in Fig. 3. The intermittent chaos and the quasiperiodic oscillations are also observed in our plasma systems.

The Feigenbaum constants δ[17] for these period-doubling sequences up to $16P$ for several typical cases of our fluid simulations are calculated. These are 4.8308 and 4.6429 for $\phi=0$, $\eta=1$; 4.7586 and 4.8333 for $\phi=0.05$, $\eta=1$; 4.9714 and 4.6667 for $\phi=0$, $\eta=1.05$ which are close to the universal value 4.6692.

The results of nonlinear fluid simulations as shown in Figs. 1 and 3 are verified by our independent kinetic results using the PDP-1 code.[13] The results from two entirely different origins agree very well despite the slight differences in the simulation conditions, most notably in the initial equilibria.

The preliminary results from our simulations for driven cases and for collisional cases indicate the similar conclusion. For the driven cases, the dominant route to chaos is via quasiperiodic oscillations unless the driven ac voltage-amplitude becomes comparable to the dc amplitude. For the latter case, the period-doubling route to chaos becomes dominant.

In summary, the major results are shown in Fig. 1 for the phase diagrams in terms of the control input parameters for the period-doubling route to chaos of the beam-plasma system as well as in Fig. 3 in terms of the potentially universal diagnostic parameter μ in close analogy with the FEL chaos. The parameter μ, reflecting the ratio of the interaction to the synchrotron slippage lengths, has similar significance for different physical systems of FEL and plasma. The dependence on this single parameter represents the multiple dependences on many input parameters, the latter of which vary from system to system. It can thus be used as a simplifying, quantitative, and diagnostic measure of various nonlinear dynamical and chaotic phenomena for many different physical systems. The results of our independent kinetic-simulations verify closely those of the nonlinear fluid simulations.

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