IDENTIFICATION ALGORITHMS FOR THERMAL CONDUCTIVITY COEFFICIENT OF WOOD USING THE INVERSE PROBLEM

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ABSTRACT

A realization for the challenges of thermal conductivity coefficient identification of wood by using the inverse heat conduction problem is proposed. An overview of mathematical tools involved in the construction of a mathematical model of the experiment is given. The object of research is the thermal conductivity coefficient of wood raw material. The results of this work are functions that showing dependences between thermal conductivity coefficient and certain temperatures specified using polar coordinate system.

Key words: approximating, heat transfer, process, thermal conductivity

INTRODUCTION

Posing of the problem and formulation of the goals

During the processing any material, it should be a clear understanding of all of its technological characteristics. For the correct carrying wood pyrolysis processes the key parameter is a thermal conductivity of this material. On the basis of experimental data, identification of wood thermal conductivity coefficient through the use of mathematical apparatus is provided.

Processing of the results based on the inverse problems solution methodology for heat conduction. In its general form, such task assumes multiple modeling of temperature field in the investigated sample with simultaneous selection of the desired dependences between the thermal conductivity coefficients and given temperatures [1, 2].

Inverse heat conduction problem requires specifying an analytic justification and in choosing the correct mathematical techniques. The essence of the proposed algorithm consecutively is shown below in the work.

Determination of the analytical framework

For the creating an identification algorithm of the thermal conductivity coefficient for wood the following inputs are required.

1. A mathematical model based on the differential equation

$$\frac{\partial t}{\partial \tau} = \frac{\lambda}{c \cdot \rho} \cdot \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) + \frac{q_v}{c \cdot \rho}, \qquad (1)$$

$\frac{\partial t}{\partial \tau}$ – change of the internal energy over the time. In our case, considered each stationary state, means $\frac{\partial t}{\partial \tau} = 0$;

λ – coefficient of thermal conductivity, W/(m·K);
c – specific thermal capacity, J/(kg·K);
ρ – density, kg/m^3;
$\frac{\partial^2 t}{\partial x^2}, \frac{\partial^2 t}{\partial y^2}, \frac{\partial^2 t}{\partial z^2}$ – projection of the heat flux density vector on the coordinate axes x, y and z, respectively;
q_v – specific heat evolution density, W/m^3 [3].

2) The geometrical characteristics of the wood sample that detail its parameters.
3) Border conditions (BC):
3a) BC of the second kind.They assume the preset of the heat flux for each point of the body surface and any moment in time. These boundary conditions are specified on the external surface of the cylinder, which is adjacent to the heater. Analytically these BC can be represented as follows:

$$q_{fl} = f(x, y, z, \tau), \tag{2}$$

q_{fl} – the heat flux density on the body surface;
x, y, z – body surface coordinates defined in therectangular Cartesian coordinates[3].

3b) BC of the third kind. They assume the preset of the ambient temperature and the law of heat exchange between the surface of the body and the environment. For a correct description the heat transfer process between the medium and the surface of the body is usually applied Newton-Richman's law.

$$q = \alpha \cdot (t_b - t_{am}), \tag{3}$$

α – heat convection coefficient, W/m^2·K;
t_b – body temperature, K;
t_{am} – ambient temperature, K[2].

After a series of transformations, the boundary condition of the third kind can be written as follows:

$$\left(\frac{\partial}{\partial \tau}\right)_b = -\frac{\alpha}{\lambda} \cdot (t_b - t_{am}). \tag{4}$$

Expression (4) is essentially a special case of the law of energy conservation for the body surface area [3].

The heat transfer coefficient depends on many factors. However, in many cases, the heat transfer coefficient can be considered constant, so in the future solving problems of thermal conductivity the value of α will be adopted constant [4].

In ourstudy, these BC will be applied with respect to copper tube located in the center of the heat chamber, and the water that passes through it during the experiment.

3c) BC of the fourth kind characterize the conditions of the heat exchange for system of bodies or body with the environment by thermal conductivity laws. Here we must start from the idea that the contact between the bodies is perfect (ie, the temperatures of the contacting surfaces are the same) [4].

In such conditions we have the equality of heat flows through the contact surface, that is

$$\lambda_1 \cdot \left(\frac{\partial t}{\partial \tau}\right)_b = \lambda_2 \cdot \left(\frac{\partial t}{\partial \tau}\right)_b . \tag{5}$$

In the described experiment BC of the fourth kind will take place between the outer wall of the metal tube and the inner surface of the hole in the middle of the cylinder wood sample.

Differential equation (1) in combination with the terms of uniqueness, mentioned above, gives a complete mathematical formulation for the problem of determining of the specific thermal conductivity [4].

4) Physical dependences for auxiliary elements of the experiment: water and copper – thermal conductivity coefficient $\lambda(T)$ (for both) and kinematic viscosity coefficient υ (just for water).

5) Definition of a number of supporting characteristics for the future mathematical model:

5a) determination of the target functions type;

5b) decision-making concerning whether the physical quantity of thermal conductivity is a function or constant (in our case it's a function of the temperature);

5c) the question about the isotropy of heat transfer properties of the sample (in our case it's an anisotropy);

5d) selection of the coordinate system of anisotropy (decided: axis of abscissa and applicate are directed along the fibers and sample height, and the ordinate axis directed crosswise the fibers). Nevertheless, this issue needs to be further developed, because we must correctly take into account the angle of rotation φ after repeated switching between the cylindrical and Cartesian coordinates during the calculations.

6) A set of reference points for the measurement of temperatures.

MATERIALS AND METHODS

The methodological basis of the model of process

Having a full set of base line data, you can proceed to the consideration of the mathematical formalism of their treatment.

The numerical model of the process, which is a direct problem of heat conductivity, is compiled by using the finite element method [5].

The essence of the method lies in the fact that the area, in which the search for solving differential equationsis carried out, is divided into a finite number of elements. Then it should be selected the form of the approximating function for each element. The values of the functions at the boundaries of the elements are known in advance.

The coefficients of the approximating functions are usually looking for, based on the condition about equality of the values for neighboring functions on the boundaries between the elements (the nodes).Then these coefficients are expressed through values of the functions at the nodes of the elements. Then system of linear algebraic equations is compiled. The number of equations is equal to the number of unknown values of the nodes.

Having a numerical model, we perform the identification with extreme methods using standard root-mean-square deviation of the solutions for the direct problem from the experimental data.

Then we find the target values, using Nelder-and-Meade' method of the deformed polyhedron [6].

In the method of Nelder-and-Meade the function of n independent variables is minimized with the use of $n+1$ vertices of the deformed polyhedron. Each vertex can be identified by the vector x.

The vertex (point), in which the value of $f(x)$ is the highest, projected through the centroid (center of gravity) of other vertexes.

Improved (lower) value of the objective function can be found by serial replacing the points with the maximum value of $f(x)$ to the "best" points until wefinds a minimum of $f(x)$.

In more detail, this algorithm can be described as follows:

let

$$x_i^{(k)} = \left[x_{i1}^{(k)},\dots,x_{ij}^{(k)},\dots,x_{in}^{(k)}\right]^T, \; i=1,\dots,n+1, \tag{6}$$

is the i-th vertex (point) on the k-th stage of the search,

$k=0,1,\dots$, and let the value of the objective function in $x_i^{(k)}$ be $f\left(x_i^{(k)}\right)$. In addition, we determine those vectors of the polyhedron, which give the maximum and minimum value of the function $f(x)$.

Define

$$f\left(x_i^{(k)}\right) = \max\left\{f\left(x_j^{(k)}\right),\dots, f\left(x_{n+1}^{(k)}\right)\right\}, \tag{7}$$

where $x_n^{(k)} = x_i^{(k)}$, and

$$f\left(x_i^{(k)}\right) = \min\left\{f\left(x_j^{(k)}\right),\dots, f\left(x_{n+1}^{(k)}\right)\right\}, \tag{8}$$

where $x_n^{(k)} = x_i^{(k)}$.

Since the polyhedron consists of $(n+1)$ vertices $x_1,\dots,x_{n+1}$, let x_{n+2} be the center of gravity for all vertices except x_h.

Then the coordinates of the center are determined by formula

$$x_{n+2,j}^{(k)} = \frac{1}{n}\left[\left(\sum_{i=1}^{n+1} x_{i1}^{(k)}\right) - x_{h,j}^{(k)}\right] \; j=1,\dots,n, \tag{9}$$

where the index j indicates the coordinate direction [5].

After couple of transformations we find desired value of thermal conductivity coefficient as a function between thermal conductivity and temperature for each of the coordinate axes, chosen at the beginning: $\lambda_x(T)$, $\lambda_y(T)$, $\lambda_z(T)$.

RESULTS AND DISCUSSION

The experiment was conducted for two samples of wood, representatives of different species and even different breed types - coniferous and deciduous - pine and oak. Briefly consider the results on the example of pine.

The data collected using afore mentioned equipment is summarized in Table 1.

Table 1 - Experimental data. Wood sample – pine

# of thermocouple	Stage 1 (0 – 150 ºC)			Stage 2 (150 – 315 ºC)				Stage 3 (315 – 500 ºC)		
	up to 100ºC	up to 120ºC	up to 150ºC	up to 170ºC	up to 220ºC	up to 275ºC	up to 315ºC	up to 350ºC	up to 425ºC	up to 500ºC
1	49	55	62	71	87	95	102	109	177	273
2	72	85	95	114	147	173	196	221	303	399
3	98	117	142	165	217	270	310	347	428	513
4	68	81	90	104	136	164	187	204	224	307
5	87	103	118	143	190	237	271	300	340	402
6	101	122	145	174	223	280	318	355	421	488
7	69	82	96	115	146	187	217	246	286	331
8	27	28	29	30	33	35	38	39	42	45
Power, W	15.63	23.74	27.34	39.06	46.88	58.59	70.31	78.13	117.19	140.63
Water inlet 19 ºC					Water flow rate 30 ml/min					

After the treatment of this information using computer-mathematical methods, the function dependencies for the thermal conductivity (defined in polar coordinates) have been obtained:

$$\lambda_r = 2.858615 - 0.023120 \cdot T; \tag{10}$$

$$\lambda_\varphi = 1.507020 + 0.019104 \cdot T; \tag{11}$$

$$\lambda_z = 0.036863 + 0.144163 \cdot T \,. \qquad (12)$$

Polar coordinates are arranged as shown in Fig. 1.

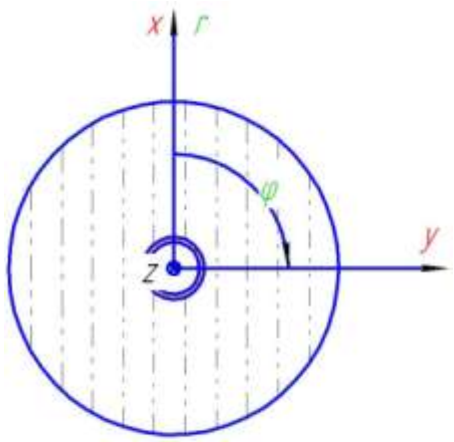

Fig. 1. Arrangement of coordinates. Replacing of rectangular to the polar

As you can see from the dependences, anisotropy of thermal conductive properties for all coordinate directions in the sample is observed. This issue should be considered in more detail on the subject of a full alignment of the calculated data during the switching to the usual rectangular coordinates.

CONCLUSION

The use of the inverse problem in the proposed algorithm permits reducing the problem of thermal conductivity coefficient identification for wood to the form of its functional dependencies along the coordinate axes. The anisotropy of the heat-conducting abilities of the wood raw material is confirmed.The necessity of the return to Cartesian coordinates during the thermal conductivity measurement in order to avoid the distortions introduced by the radial direction coordinates φ is proved.

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