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Structural Modeling of Elastoplastic Deformation Processes of the Bodies of Non-classical Shape

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Abstract
An approach based on the theory of small elastoplastic deformations is proposed to study the aims of the stress-strain state determining of finned cylindrical and conical bodies of finite sizes.
We consider small elastoplastic deformations described by the nonlinear equations system, for linearization of which the variable elastic parameters method is applied
Approximate solution of the linearized elasticity problem at each k-th iteration is made with the use of the R-functions theory in the form of a single analytical expression.
Determination of the stress-strain state, the plasticity areas and analysis of the results obtained has been performed with the POLE software package.

Keywords
Elasticity, small elastoplastic deformations, R-functions

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Relevance

At the investigation of practical important technical issues, sometimes we can see questions, which connected with choosing of investigation direction. It will be both practical testing and usage of classical and numerous methods.

Conducting of practical experiments leads to usage of expansive materials, time, and it will be unacceptable in those cases, than physical sizes of components small enough, setting up of sensitive devices and sensors inside it – it will be very difficult issue and in some other cases. Looking on these reason is very actual of usage of determination of physical fields of elements and apparats by analytical and numerous methods.

Analytical methods provide solutions for limited classes of problems. In the cases of investigation of difficult physical processes in random object we need to appeal to numerous-analytical and analytical methods.

In modern technique we can see increased requirement to accuracy of determination of fields of investigated elements and nodes, therefore understandable desire of creation of universal methods and high-accuracy algorithms for such problems.

Now we can see popularity of systems and software complexes, which developed on the basis of finite elements methods for solving of problems for physics-mechanical fields [1-4].

One of the competitive methods for finite element method – it's R-function theory and software complex Pole (Field), which was created on the basis of this method [5-7].

Usage of R-function theory for investigation of physical tasks in random areas provide investigation of fields of non-classical bodies without limitation on forms of areas and types of mechanical, wave and other influences on investigated object.

Analytical solutions – structures (GSS) or structure formulas has view as functional ratios, which consists from elementary functions or super-position of elementary and special functions.

When is taken into account on analytical level border conditions and geometry of field of investigated problem, furthermore, possibility of taking into consideration apriori information about exact solution (if it's exists) and possibility to approach him in the metrics of corresponding functional space.

Theory of R-function brought methods of mathematical physics and constructive methods of logics algebra. This option, in this case, allowed to formulate concept of structure solution (GSS) (i.e. form, for which we can find solution) as functional ration

$$U=B(\Phi),$$

where U – problem solution, B– known operator, which taking into account border conditions of boundary value problem, which can be determine on multiplicity M, and element Φ is selected, to well satisfy of initial equation (in one way or another).

Main idea of building of structure formulas, giving into account of border solutions, consist in decomposition in range of desired solution on exponent function ω or ωi, where ω, ωi – left parts of normalized equations of border of area S ($\partial\Omega$) or their sites.

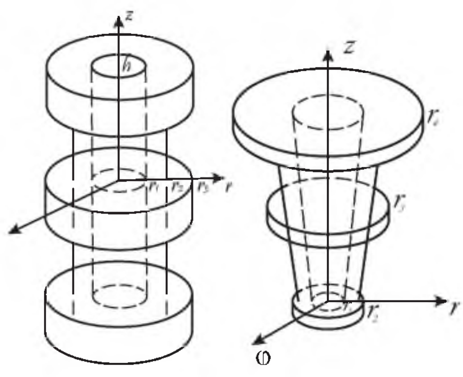
Next stage – it's satisfying of initial differential equation, which described physical process. In most cases this process at numerous solutions can be realized by grid or variable methods. In variable methods record of information about area Ω, it's border $\partial\Omega$ and boundary conditions carried during process of building of coordinate (basis) function, which has needed properties of completeness and linear independence. It's give a possibility during building of solutions structure (GSS) for taking into account for geometrical information of boundary value problem on analytical level, without any approximation, i.e. for structure models.

This approach proposed for investigation of physical processes in elements of tooling, which used for production of pipes of different diameter [8].

The elements are thick-walled cylindrical or conical shell, reinforced by external ring stiffening ribs (Fig. 1), and loaded with internal pressure. Internal loads greater than the elasticity limits are permissible, but which do not result in the elastoplastic deformation and shell's destruction. Selecting of the design geometric ribs parameters under different load values will allow the use of thick-walled shells with ribs in the necessary processes and predict the modes of their optimal operation.

The most common are shell configurations with single external central reinforcing rib, with the two ribs at the ends, and in particularly difficult cases of load—with two end ribs and a central rib on a cylindrical or conical surface.

Relatively well developed classical methods of the elasticity theory provide solutions for smooth thick-walled cylindrical bodies, loaded with internal pressure [9,10]. The presence of the ring ribs on the outer surface of the cylinder substantially changes the stress distribution pattern inside the body.



a) b)

Figure 1. Finned Bodies General View

Thus, the problem of the stress-strain state study and permissible loads determining due to the geometric parameters optimization of the finned thick-walled shells is still relevant.

In practice of investigation of finned bodies, as usual, using special methods [11,12] of registration of influence of edges for approximated solutions on inner and outer surfaces of elements.

In this work we consider new approach for investigation of physical fields in finned bodies, which based on theory of R-function, and which can provide exact accounting of geometry of finned surface and also can get analytical solutions for free configurations of elements of tooling.

Problem Formulation

It is known that solids under loading can be considered elastic only up to certain limits, above which the bodies do not subject to the elastic properties. The linear relationships between stress and strain break down due to the appearance of plastic strains. In solving the problem of the body's stress state determining, loaded beyond the elastic limits, the equations of the materials' plastic state are considered [9–10].

The mathematical model of the elastoplastic strains process study is described by a system of differential equations [13-14] as shown below (1)

$$\frac{1}{3}\frac{\sigma_i}{\varepsilon_i}\nabla^2 u_z + \left(\frac{E}{3(1-2\nu)} + \frac{1}{9}\frac{\sigma_i}{\varepsilon_i}\right)\frac{\partial e}{\partial z} + \\ + \frac{1}{3}\frac{\partial}{\partial r}\left(\frac{\sigma_i}{\varepsilon_i}\right)\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) - \frac{2}{9}e\frac{\partial}{\partial z}\left(\frac{\sigma_i}{\varepsilon_i}\right) = 0, \qquad (1)$$

where the relation σ_i / ε_i characterizes the material's deformation diagram; E –modulus of elasticity; ν – Poisson's ratio,

$$e = \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} + \frac{u_r}{r}; \quad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{\partial}{\partial z^2} + \frac{1}{r}\frac{\partial^2}{\partial r^2},$$

u_r, u_z – displacement tensor components.

The boundary conditions are written in the following relations

$$\sigma_n = \left(\frac{E}{3(1-2\nu)} + \frac{4}{9}\frac{\sigma_i}{\varepsilon_i}\right)\left(\frac{\partial u_r}{\partial n}l_1 + \frac{\partial u_z}{\partial n}l_2\right) + \\ + \left(\frac{E}{3(1-2\nu)} - \frac{2}{9}\frac{\sigma_i}{\varepsilon_i}\right)\left(\frac{\partial u_z}{\partial \tau}l_1 - \frac{\partial u_r}{\partial \tau}l_2 + \frac{u_r}{r}\right) = f_1, \\ \tau_n = \frac{1}{3}\frac{\sigma_i}{\varepsilon_i}\left(\frac{\partial u_r}{\partial n}l_2 - \frac{\partial u_z}{\partial n}l_1 + \frac{\partial u_r}{\partial \tau}l_1 + \frac{\partial u_z}{\partial \tau}l_2\right) = f_2, \qquad (2)$$

where l_1, l_2 – are the direction cosines of the outward normal n; *f1*, *f2* – load intensity, respectively at the inner and outer boundary surfaces of the elements.

Solution Method

Relations (1)–(2) are non-linear, for their linearization the variable elasticity parameters method has been used [4–5], according to which the given elastic modulus E* and Poisson's ratio ν* depend on the stress-strain state of the material and are determined in this way

$$E^* = \frac{\dfrac{\sigma_i}{\varepsilon_i}}{1 + \dfrac{1-2\nu}{3E}\dfrac{\sigma_i}{\varepsilon_i}}, \quad \nu^* = \frac{\dfrac{1}{2} - \dfrac{1-2\nu}{3E}\dfrac{\sigma_i}{\varepsilon_i}}{1 + \dfrac{1-2\nu}{3E}\dfrac{\sigma_i}{\varepsilon_i}}. \qquad (3)$$

Parameters λ^*, μ^* are functions of the coordinates and determined by solving of the problem of the previous $(k-1)$ approximation, based on received deformation diagrams

$$\lambda^* = \lambda^{(k)} = \frac{E}{3(1-2\nu)} - \frac{2}{9}\frac{\sigma_i^{(k-1)}}{\varepsilon_i^{(k-1)}}; \qquad \mu^* = \mu^{(k)} = \frac{1}{3}\frac{\sigma_i^{(k-1)}}{\varepsilon_i^{(k-1)}}. \tag{4}$$

As the initial (zero) approximation the parameters $E^*=E$, $\nu^*=\nu$ are taken and at the first iteration the problem of elasticity theory for an isotropic body is solved.

At the next iterations the linear problems for inhomogeneous elastic bodies with parameters λ^*, μ^*conditioned by relations are considered (4). Stabilization of solutions and parameters λ^*, μ^*at iterations can serve as the ground for the iterative process completion.

An approximate solution of the linear elastic problem at the k-th iteration is shown in accordance with the theory of R-functions [15] by relations

$$u_r^{(k)} = u_{r0}^{(k)} + \sum_{i=1}^{n} C_i^{(k)} u_{ri}^{(k)},$$
$$u_z^{(k)} = u_{z0}^{(k)} + \sum_{i=1}^{n} F_i^{(k)} u_{zi}^{(k)}, \tag{5}$$

where u_r, u_z – are the displacement vector components;

$$u_{r0}^{(k)} = \frac{1}{\lambda^{(k)} + 2\mu^{(k)}}\omega\frac{\partial\omega}{\partial r}f_1 + \frac{1}{\mu^{(k)}}\omega\frac{\partial\omega}{\partial z}f_2;$$

$$u_{ri}^{(k)} = \Phi_1 - \omega D_1\Phi_1 - \frac{\lambda^{(k)}}{\lambda^{(k)} + 2\mu^{(k)}}\omega\frac{\partial\omega}{\partial r}\frac{\Phi_1}{r} + 2\frac{\lambda^{(k)} + \mu^{(k)}}{\lambda^{(k)} + 2\mu^{(k)}}\omega\frac{\partial\omega}{\partial r}\frac{\partial\omega}{\partial z}T_1\Phi_1 +$$
$$+\omega\left[\left(\frac{\partial\omega}{\partial z}\right)^2 - \frac{\lambda^{(k)}}{\lambda^{(k)} + 2\mu^{(k)}}\left(\frac{\partial\omega}{\partial r}\right)^2\right]T_1\Phi_2;$$

$$u_{z0}^{(k)} = \frac{1}{\lambda^{(k)} + 2\mu^{(k)}}\omega\frac{\partial\omega}{\partial z}f_1 - \frac{1}{\mu^{(k)}}\omega\frac{\partial\omega}{\partial r}f_2;$$

$$u_{zi}^{(k)} = \Phi_2 - \omega D_1\Phi_2 - \frac{\lambda^{(k)}}{\lambda^{(k)} + 2\mu^{(k)}}\omega\frac{\partial\omega}{\partial z}\frac{\Phi_1}{r} - 2\frac{\lambda^{(k)} + \mu^{(k)}}{\lambda^{(k)} + 2\mu^{(k)}}\omega\frac{\partial\omega}{\partial r}\frac{\partial\omega}{\partial z}T_1\Phi_2 +$$
$$+\omega\left[-\left(\frac{\partial\omega}{\partial z}\right)^2 - \frac{\lambda^{(k)}}{\lambda^{(k)} + 2\mu^{(k)}}\left(\frac{\partial\omega}{\partial r}\right)^2\right]T_2\Phi_2. \tag{6}$$

Here ω – is the equation of the research area boundary; operators $T = \frac{\partial\omega}{\partial r}\frac{\partial}{\partial z} - \frac{\partial\omega}{\partial z}\frac{\partial}{\partial r}$, $D = \frac{\partial\omega}{\partial r}\frac{\partial}{\partial r} - \frac{\partial\omega}{\partial z}\frac{\partial}{\partial z}$, F_1, F_2 – undefined components of structural formulas (GSS) (6), which are shown by expansions $\Phi_1 = \sum_{i,j}^{n} C_{ij}^{(1)}\varphi_i(r)\varphi_j(z)$, $\Phi_2 = \sum_{i,j}^{n} C_{ij}^{(2)}\psi_i(r)\psi_j(z)$, $\varphi_i(r)$, $\varphi_j(z)$, $\psi_i(r)$, $\psi_j(z)$ – basis functions or functions with local carriers.

The coefficients of expansions $C_{ij}^{(1)}$, $C_{ij}^{(2)}$ are determined with the appropriate condition minimum for the boundary value problem (1)–(2) functional

$$I(u_r^{(k)},u_z^{(k)})=\frac{1}{2}\iint_{\Omega}\left(\lambda^{(k)}\left[\frac{\partial u_r^{(k)}}{\partial r}+\frac{\partial u_z^{(k)}}{\partial z}+\frac{u_r^{(k)}}{r}\right]^2+2\mu^{(k)}\left(\left(\frac{\partial u_r^{(k)}}{\partial r}\right)^2+\left(\frac{\partial u_z^{(k)}}{\partial r}\right)^2+\left(\frac{u_r^{(k)}}{r}\right)^2\right)+\right.$$
$$\left.+\mu^{(k)}\left(\frac{\partial u_r^{(k)}}{\partial r}+\frac{\partial u_z^{(k)}}{\partial z}\right)^2\right)rdrdz-\int_{\partial\Omega}\left(f_1u_r^{(k)}-f_2u_z^{(k)}\right)rdr. \quad (7)$$

It should be noted that the relation (7) of the structural model of elastoplastic deformation of the bodies of non-classical shape has a general view, allows due to the function ω selection to describe the area boundary and study the stress-strain state of the various structural elements of arbitrary geometry.

Here are the functional relationships for the function ω for the element case, reinforced by three ribs on the outer surface

$$\omega=(((f_1\,\&_0\,(r-a))\,\&_0\,f_6)\,\&_0\,((f_3!_0\,f_4)!_0\,(f_{5!}!_0\,f_4));$$
$$\omega_1=r-a;\quad \omega_2=(f_1\,\&_0\,f_6)\,\&_0\,(f_3!_0\,f_4)\,\&_0\,(f_5!\,f_4),$$

the equation of the hollow truncated cone boundary with three reinforcing ribs is expressed by the formula

$$\omega=(((f_1\,\&_0\,(-f_7))\,\&_0\,f_{11})\,\&_0\,((f_9!_0\,f_8)!_0\,(f_{10!}!_0\,f_8)).$$

In these formulas

$$f_1=z^2-h^2,\quad f_2=z^2-h_3^2,\quad f_3=(z-b)(h_2-z),$$
$$f_4=a_0-r,\quad f_5=(z-b)(-h_2-z),\quad f_6=a_1-r,$$
$$f_7=h_1^2-z^2,\quad b=(h_1-h_2)/2,\quad f_8=\mathrm{a}_{10}\mathrm{z}-\mathrm{r}+\mathrm{b}_1,$$
$$f_{11}=a_2z\text{-}r+b_2,\quad f_{11}=h_{11}^2-z^2,\quad f_{10}=h_{10}^2-z^2,$$

$h,\ h_1,\ h_2,\ h_3,\ b,\ a_0,\ a_1,\ a_{10},\ b_1,\ a_2,\ b_2,\ h_{11},\ h_{10}$ – geometric parameters of the studied objects. Computer symbols &0 and !0 using for next R-operations: R- соиспользуются для следующих R-оиераций: R – conjunction and R – disjunction, which can be presented as next

$$a\,\&_0\,b=a+b-\sqrt{a^2+b^2},$$
$$a!_0b=a+b+\sqrt{a^2+b^2}.$$

Structural models built in certain functional spaces with the use of the R-functions theory [5,6] are common and serve as the basis for the research models development of a certain class of problems, and are used for the qualitative analysis of this class of problems solutions, i.e. to determine the general properties of the solutions. These models allow taking into account the necessary from a physical point of view data contained in the mathematical formulation of the initial problem, to analyze and optimize physical and mechanical systems under the predetermined optimality criterion.

The solution algorithm assumes such highlights of the problem study:

- entering of the physical characteristics and geometrical parameters of the studied problem;
- description of study area geometry and its boundaries;
- representation of analytical problem solution (1)–(2) in the form (6);
- determination of the unknown coefficients with the conditions of functional minimum (7) at each step of the iterative process;
- determination of the stress-strain state, plasticity areas and the analysis of the results obtained.

Analysis of the Results

On the basis of the structural model under POLE software system operation [7] there has been developed a computer model elastoplastic deformations study in the objects of complex geometric shape that are under the arbitrary nature of loading. The computer model has allowed carrying out a wide computational experiment and determining the plasticity area and optimum operation parameters of the thick-walled shells of finite length, reinforced with ribs.

Computational experiment has been carried out under the condition of equality of the masses of the cylindrical (conical) bodies with a different number of ribs at certain values of the geometric parameters h, h1, r_0, R1 and under the load P.

Due to this, the size R_p which defines an outer radius of the finned body has been changed.

The calculations were made for the steel material 40X in the annealed condition with such characteristics

yield strength σ_S = 400 MPa,

E = 2,05 $\cdot 10^7$ MPa,

ν= 0,3.

The plastic deformation zone estimation in the section of the workpiece has been carried out on the basis of inequation

$$\varepsilon_i - \varepsilon_T \geq 0 .$$

Stabilization of the numerical results for the stresses σ_θ with numerical implementation took place at 5–7 iterations.

The numerical results analysis showed that in the thick-walled cylindrical assembly the plastic deformation zone on the inner surface begins at the load value which is equal to P = 0,75σ_θ; with the load P growth this zone increases and covers in the central section the range of 0,36≤r/r1≤0,55.

The central rib on the cylindrical surface has such influence on plastic deformation: in the central section, they begin to appear under the load P = 0,25σ_θ and increase respectively the height of the cylinder.

With the same load the end ribs on the cylindrical surface cause plastic deformation increase in the section z = 0.

The end ribs in the section z = 0 under the load P = 0,25σ_T also cause plastic deformations. The zone of these deformations is considerably greater than in case of the central rib availability, but still plastic deformations are not observed at the ends. They begin to appear under the load P = 0,4σ_T.

Completely plastic deformation in the cylindrical body with two end and one central rib are observed in the section z=0 under P = 0,4σ_T and in the end z=h under P = 0,25σ_T (Fig. 2)

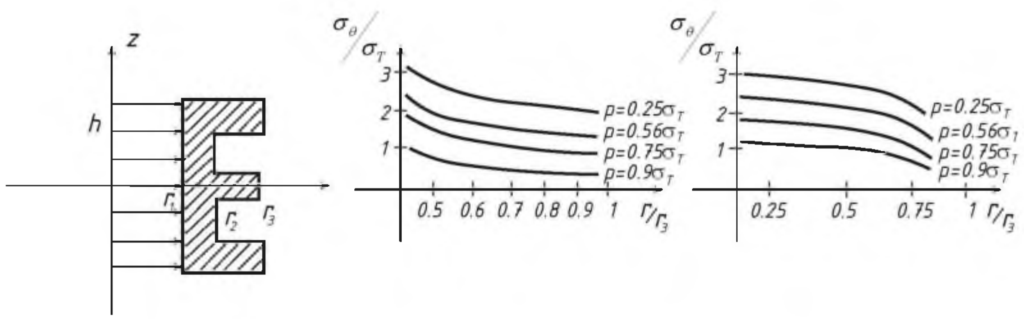


Figure 2. Distribution of the stresses σ_θ / σ_T in the cylindrical element with three reinforcing ribs

Counting experiment of investigation of stress-stained state of conical element with three stiffening edges led to the following conclusions (pic.3).

Results presented for planes z=0 (0,2≤r≤0,3), z=h/2 (0,4≤r≤0,7), z= h (0,8≤r≤1) for load values P, which changing from 0, 25σT to 0,9 σT..

In plane z=0 plastic deformations appearing on inner surface at the loading values, more than 0,5σT,.

On the central edge at z=h/2 plastic deformation also appear on inner surface and and it tends to decrease stress σ_θ along the edge radius r. On upper end surface z= h tension σ_θ along the edge radius r decreasing with increasing of loading P.

Results was taking by system Pole [7] with the using of basis function of Chebyshev polynomials at 21 (basis) coordinate function.

Reliability of results tested at their comparisons with limit cases for solid cylinder, changing of the number of coordinate functions, stabilization of calculation processes on 5-th iteration.

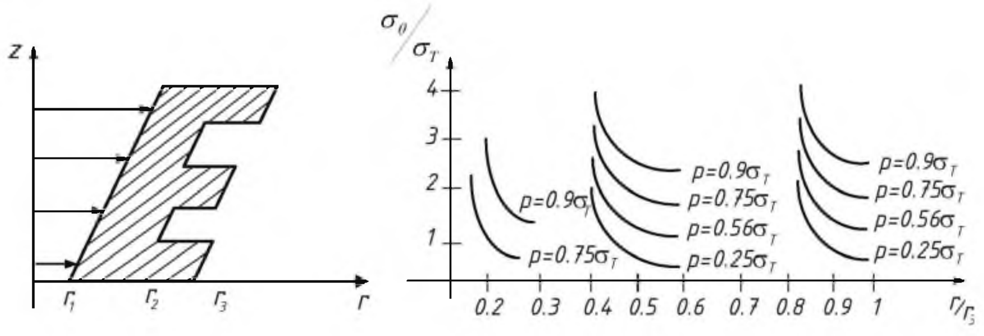


Figure 3. Stress distribution σ_θ / σ_T in the conical element with three reinforcing ribs

Conclusions

The study of the physical processes that occur in the operation of various structural elements of the production field, involves the model construction, and on its basis—the basic mathematical relations that allow to evaluate performance, reliability and operational stability of the components and devices at the stage of theoretical elaborations.

New approach for investigation of elastic-plastic state of finned elements of tooling, which accurately taking into account geometric parameters of edges and allows to obtain of analytical solutions without approximation of the outer and inner surfaces of the elements.

On the basis of the R-functions theory, structural and computer studying models of stress-strain state of shells, reinforced with ribs have been established. Computational experiment has been performed and the optimal operating parameters of the elements with ribs in technological equipment have been determined.

The numerical results analysis allows making conclusions on the plasticity zones in cylindrical and conical elements of the assembly depending on the availability of the central rib, end ribs, two end ribs and one central rib.

The geometric parameters of the ribs, heights of the cylindrical and conical elements, the optimum loads on the elements and characteristics of plasticity and deformation zones have been determined.

A similar approach can be used to study and optimize the structural elements characteristics of complex geometric shapes.

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