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Breather Modes Induced by Localized RF Radiation: Analytical and Numerical Approaches

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Abstract
Numerical computations and collective variables approach are applied to analytical and numerical study of spatially localized excitations of one-dimensional magnetic system in external high-frequency magnetic field It is demonstrated the hysteresis character of dependence for amplitude of local soliton-like states on external field magnitude. The system shows a variety of interesting nonlinear phenomena such as periodicity doubling and chaos.

Keywords
AC field, damping, soliton, quasi-soliton, collective variables, hysteresis, bifurcation, chaos.

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Introduction

The nonlinear dynamics of many quasi-one-dimensional physical systems (surface Tamm states, long Josephson junctions [1], optical fibers and magnetic transmission lines [2]) show a rich variety of interesting nonlinear phenomena, which have been observed experimentally and explained theoretically. In particular, the study of the influence of a locally applied high-frequency signal on the features of excited quasi-soliton (breather type) states is of fundamental as well as practical interest.

1. Quasi-soliton excitations in the system under the localized pumping

In the cases of quasi-one-dimensional magnetic and optical systems (thin magnetic ribbon and optical fiber transmissions) this problem may be investigated in the framework of 1D nonlinear Schrodinger equation (NLSE)

$$i\psi_t + \psi_{xx} - \omega_0\psi + |\psi|^2\psi + i\gamma\psi = h\delta(x)\exp(-i\omega t) \qquad (1)$$

for a deviation of magnetization from the easy axis ground state or an envelope of the electrical field in an optical pulse. In this equation ω_0 is frequency of the ferromagnetic resonance, γ is the damping coefficient, ω and h are the frequency and amplitude of external circular magnetic field.

In homogeneous medium without pumping and damping the localized soliton excitations (magnon droplets or breathers) exist due to the nonlinearity of a system and have the form:

$$\psi_s(x,t) = \sqrt{2}\,\varepsilon_s \exp(-i\omega_s t)\operatorname{sech}\left(\varepsilon_s(x - x_s)\right) \qquad (2)$$

with $\varepsilon_s = \sqrt{\omega_0 - \omega_s}$ and arbitrary position x_s in space (the curve S for $x_s = 0$ in Fig. 1a). Under the action of the external AC field at point $x = 0$ in the absence of a damping there exist different types of quasi-soliton stationary states localized near the pumping point (the lines A, B, C, D in Fig. 1a):

$$\psi_i(x,t) = u_i(x)\exp(-i\omega t) = \sqrt{2}\,\varepsilon\exp(-i\omega_s t)\operatorname{sech}(\varepsilon(x-x_{0i})), \tag{3}$$

where $\varepsilon = \sqrt{\omega_0 - \omega}$ and $sh(\varepsilon x_{0i}(h)) = \left(\sigma_i s + \mu_i\sqrt{s^2-1}\right)$ with $s(\varepsilon,h) = \sqrt{2}\,\varepsilon^2/h$, $\sigma_{A,B} = -1$, $\sigma_{C,D} = 1$, $\mu_{A,C} = -1$, $\mu_{B,D} = 1$. Such the localized excitations does exist only in the fields with amplitudes smaller then the critical one: $h_c = \sqrt{2}\varepsilon^2$.

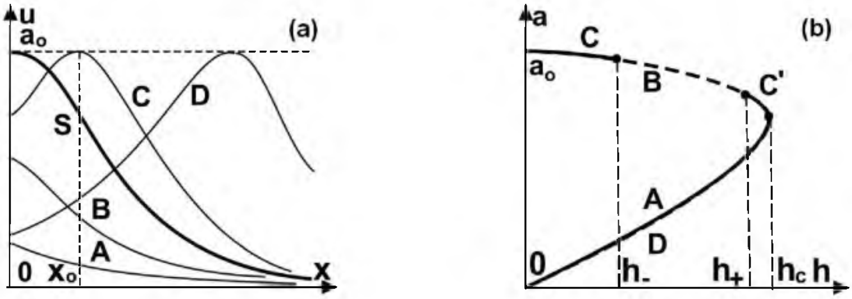

Figure 1. The profiles of quasi-soliton states (a) and the field dependence of soliton amplitude (b)

In experiments the frequency ω of AC field is fixed and only its amplitude h changes. The corresponding dependences of the magnetization in the launched point on the field amplitude

$$a_i(h) = |u_i(x=0)| = \sqrt{2}\varepsilon/\sqrt{1+\left(\sigma_i s + \mu_i\sqrt{s^2-1}\right)^2} \tag{4}$$

are shown in Fig. 1b, where the soliton amplitude $u_s(0)$ is $a_0 = \sqrt{2}\varepsilon$. The $A-$ and $D-$dependences are the same as well as for $B-$ and $C-$ states, but the correspondent solutions differ essentially.

2. Quasi-soliton states in the dissipative medium. Numerical simulation

In the presence of a damping the structure of quasi-soliton excitations transforms considerably. The equation (1) with $\gamma \neq 0$ for $\psi(x,t) = u(x,t)\exp(-i\omega t)$ was solved numerically in discrete approach $(u(x,t) \to u_n(t))$ for the magnetic chain with N spins ($N = 20 \div 80$). The results for the parameters values $\varepsilon = 0.4$ and $\gamma = 0.1$ are represented in Fig. 2a.

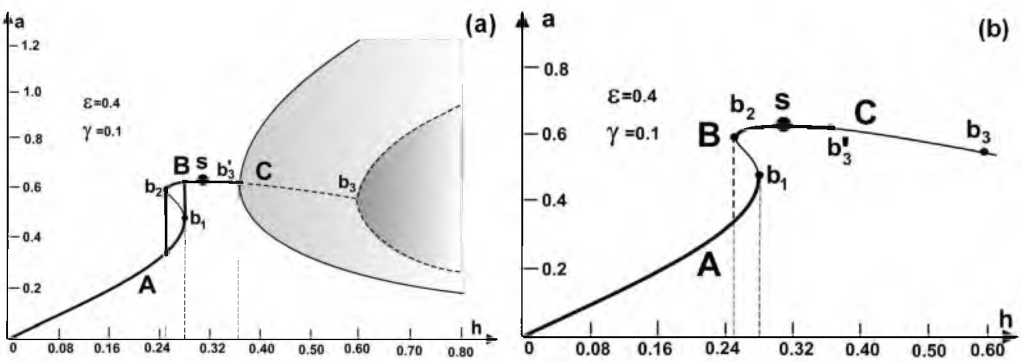

Figure 2. The results of numerical calculation (a) and ones for collective variables approach (b)

The numerical results demonstrate the set of new features of nonlinear soliton dynamics in the presence of the damping (compare Fig. 1b and Fig. 2a): the existence of hysteresis phenomenon, quasi-periodicity and chaotic behavior of the system. The hysteresis of the magnetization takes place in the fields near the critical one $h = h_c$. For the stationary states the quasi-magnon branch A of the dependence $a = a(h)$ is stable till the first bifurcation point b_1. The soliton-like branch of $B-$type is stable in the domain of field values between the bifurcation point b_2 and critical point s, where the solution is similar to the soliton excitation in undisturbed system with the phase shift $\delta = \pi/2$ between the phases of external field and rotation of magnetization. Soliton-like solution of $C-$type is stable in the field interval between points s and b_3 in which the stationary state becomes unstable. After this last bifurcation in the fields $h > h_{b3}$ the nutation of the magnetic vector with the frequency Ω additional to the precession motion with the frequency ω arises. The amplitude of the additional magnetization motion grows up with the increasing of the external field (see dark hatched domain in Fig. 2a). With the following growth of the amplitude the doubling of the period with the frequency 2Ω takes place and finally the dynamics become chaotic. For the taking values of parameters $\varepsilon = 0.4$ and $\gamma = 0.1$ the critical fields are the following: $h_{b1} \approx 0.28$, $h_{b2} \approx 0.25$ and $h_{b3} \approx 0.6$.

Unfortunately the numerical approach has some defects because of the finite size of the system under the consideration and its discreteness. Our main goal is to explain analytically the results of these numerical simulations. Therefore the collective coordinate approach in certain sense similar to this used in [3] is applied to analyze the phenomena.

3. Collective variables approach to nonlinear soliton dynamics

The main idea of the method consists in taking the solution of Eq. (1) in the form of the soliton in undisturbed system, but with slowly varying in time arbitrary parameters (amplitude B, phase φ, wave number k and position $x_0 = z/B = \exp(w)/B$), which play the role of collective variables:

$$u(x,t) = \sqrt{2}B(t)\exp(-ik(t)x - i\varphi(t))\operatorname{sech}\left(B(t)x - z(t)\right). \tag{5}$$

Following [3] let us use the Lagrangian variational principle and represent Eq. (1) as the dynamical equation for the Lagrangian density

$$l(u,\bar{u}) = \left(i(\dot{u}\bar{u} - \dot{\bar{u}}u)/2 - |u'|^2 + \varepsilon^2|u|^2 + |u|^4/2 - h(u+\bar{u})\delta(x)\right)\exp(2\gamma t). \tag{6}$$

After substitution of the Ansatz (5) into (6) and integrating it over the infinite space we obtain the effective Lagrangian for the collective variables as the dynamical generalized coordinates

$$L = \left(B(1 + th\,z)(\dot{\varphi} - \varepsilon^2 + B^2 - k^2) + (z + \ln(2ch\,z))\dot{k} - \frac{2B^3}{3}(1 + th^3 z) - h\frac{B\cos\varphi}{\sqrt{2}\,ch\,z}\right)\exp(2\gamma t)$$

and the corresponding system of differential equations for them:

$$\dot{w} = -2kBw - \gamma(1 + w^2)\ln(1 + w^2)/w^2, \tag{7}$$

$$\dot{B} = 4kB^2/(1 + w^2) + hB\sin\varphi/\sqrt{2}w + 2\gamma B(\ln(1 + w^2)/w^2 - 1), \tag{8}$$

$$\dot{\varphi} = (k^2 + \varepsilon^2 - B^2) + 4B^2(1 - w^2)/(1 + w^2)^2 + h\cos\varphi/\sqrt{2}w, \tag{9}$$

$$\dot{k} = -4B^3(1 - w^2)/(1 + w^2)^2 - hB\cos\varphi/\sqrt{2}w. \tag{10}$$

In nondissipative medium ($\gamma = 0$) after the linearization of Eqs. (7-10) over the stationary states (with w_0 from the equation $4s(\varepsilon,h)w_0(w_0^2 - 1)/(w_0^2 + 1)^2 = \cos\varphi_0$, $B_0 = \varepsilon$, $\varphi_0^{A,B} = \pi$, $\varphi_0^{C,D} = 0$ and

$k_0 = 0$), for the small deviations $\delta w, \delta B, \delta\varphi, \delta k \sim \exp(\lambda t)$ it is easy to obtain the characteristic equation $\lambda^4 + F(\varepsilon, h)\lambda^2 + G(\varepsilon, h) = 0$ with the known functions F and G ($F > 0$). Then the stability criterion for different localized states reduces to the inequality $F^2 > G > 0$. Its analyses demonstrates the instability of $B-$ and $D-$types of quasi-solitons, stability of quasi-magnon-type $A-$solitons and stability of $C-$type solitons in the fields domains $0 < h < h_- \approx 0.397 h_c$ and $h_+ \approx 0.987 h_c < h < h_c$ (see solid lines A, C and C' in Fig. 1b).

The dissipation leads to the complication for stationary states parameters B_0, k_0, φ_0 and w_0 dependences on field $h = h_c H$ and attenuation $\gamma = \Gamma\varepsilon^2/2$ in the following implicit form

$$(B_0, k_0) = \frac{\varepsilon}{\sqrt{2}}\sqrt{f \pm 1}, \quad \varphi_0 = \arcsin\left(\frac{w_0\Gamma}{H}\right), \quad \left(\frac{H}{w_0\Gamma}\right)^2 = 1 + \left(\frac{2(1-w_0^3)(1+f)}{\Gamma(1+w_0^2)^2}\right)^2, \tag{11}$$

where $f(w_0) = \sqrt{1 + \left(\Gamma(1+w_0^2)\ln(1+w_0^2)/2w_0^2\right)^2}$. From these algebraic equations and the definition of magnetization amplitude in the pumping point $a = a_0 2 w_0 B_0 / \varepsilon(1 + w_0^2)$ it is possible to identify the dependence $a = a(h)$, which is illustrated in Fig. 2b for the parameters values $\varepsilon = 0.4$ and $\gamma = 0.1$.

This dependence is practically the same as obtained from the initial PDE (1) for the stationary quasi-soliton states and is represented in Fig. 2a as a solid line. For the fields in the interval $0 < h < h_{b1}$ the profile of the soliton is the same as for $A-$soliton in Fig. 1b. In segment $(b_1 - b_2 - s)$ it is similar to such for $B-$soliton, and in the fields $h > h_s$ is similar to $C-$soliton. Quasi-solitons of $D-$type do not exist in the system with a dissipation.

4. Stability of the quasi-solitons and the nature of nonstationary states

To clear up the question about the stability of different branches of soliton excitations with $\gamma \neq 0$ let us linearize the system (7-10) with respect to stationary solutions with parameters $B_0(\varepsilon, h)$, $k_0(\varepsilon, h)$, $\varphi_0(\varepsilon, h)$ and $w_0(\varepsilon, h)$ from (11). The solution of Eqs. (7-11) for the small deviations $\delta w, \delta B, \delta\varphi, \delta k \sim \exp(\lambda t)$ gives the characteristic equation which was solved numerically. The stability analysis demonstrates the following results: quasi-magnon $A-$states are stable in the fields $0 < h < h_{b1} \approx 0.28$, $B/C-$quasi-solitons are stable in the interval $h_{b2} \approx 0.25 < h < h_{b3'} \approx 0.37$. Thus the hysteresis picture of nonlinear soliton dynamics are the same in the framework of PDE and in collective variables approach not only qualitatively but quantitatively as well. Though the boundaries of the existence for non-stationary soliton dynamics in these two approaches are different: $h = h_{b3} \approx 0.6$ from PDE solution and $h = h_{b3'} \approx 0.37$ in the collective variables method. But the last one gives the possibility for the analysis of the problem on the physical level.

In the high amplitude fields $h > h_{b3'}$ the instability of stationary solitons has the oscillation nature with the frequency $\Omega \approx 0.28$ and gain increment $\mathrm{Re}\,\lambda \sim \sqrt{h - h_{b3'}}$. The additional to the external field frequency ω the nutation frequency Ω depends on the field amplitude and the magnitude of the attenuation. In the field $h = h_{b3'}$ the above-mentioned stationary point $(w_0, B_0, \varphi_0, k_0)$ of the dynamical system in the four-dimensional phase space (w, B, φ, k) loses its stability and transform into unstable focal point. At the same time the stable limit cycle appears.

In Fig. 3a the two-dimensional cross-section (B, z) of the phase space through the unstable stationary point (1 in Fig. 3a) is displayed for $h = 0.38 > h_{b3'} = 0.37$. The motion of the represented point along the phase trajectory corresponds to the dynamics of the magnetization in quasi-soliton state, which is pointed in Fig. 3b. From the figure we notes that this state corresponds to the pair of solitons, which are bounded in the pumping point and oscillate ("breath") with the frequency Ω. The soliton envelope 2 in Fig. 3b corresponds to the point 2 in Fig. 3a and line 3 in Fig. 3b – to the point 3 in Fig. 3a. The dashed line 1 corresponds to the unstable stationary state.

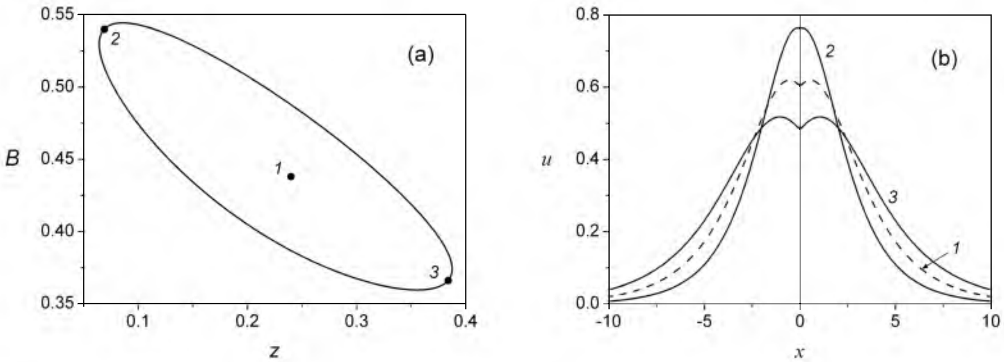

Figure 3. The limit circle (a) and the "breathing" (b) of non-stationary quasi-soliton state for $h = 0.38$

Qualitatively the soliton dynamics varies only slightly with the further grows of the field h till its value of order of $h \approx 0.7$. (Though the amplitude of the oscillations steadily rises, as it is visible from Fig. 2a). The typical phase portrait of the system in this interval of the fields is shown in Fig. 4a. In Fig. 4b the results of the Fourier analysis of the soliton oscillations i.e. the function $g(\omega) = (1/T)\int_0^T dt\, u(t)\exp(i\omega t)$ are represented. (Here the limit of integration T denotes the interval of the numerical calculation of the oscillations).

The spectrum of non-stationary excitations consists of the peaks with the multiple frequencies $n\Omega$, which are connected with the nonlinear character of limit cycle. The width of the peaks $\delta\omega \sim 1/T$ tends to zero with $T \to \infty$.

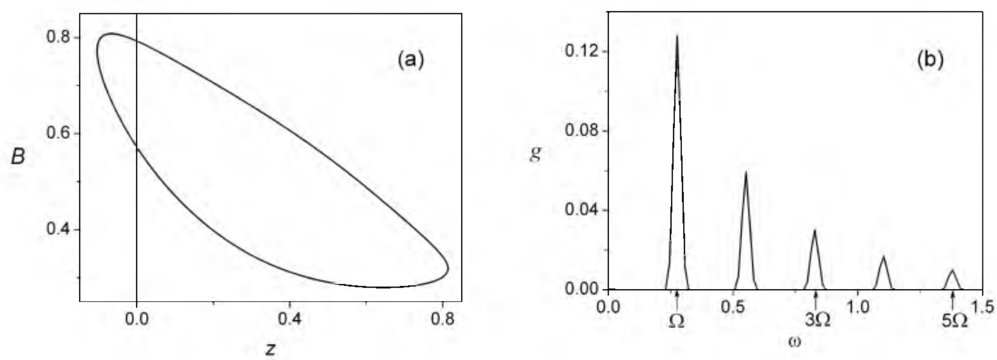

Figure 4. The limit circle (a) and Fourier transformation (b) of quasi-soliton dynamics for $h = 0.5$

In the next bifurcation point at $h \approx 0.7$ the period doubling of the limit cycle takes place (See Fig. 5a for the phase portrait of the system in the field $h = 0.7$). From Fig. 5b we observe that the additional peaks with the half-frequencies $n\Omega/2$ appear in the Fourier spectrum.

At last at higher amplitudes of the pumping the dynamics becomes chaotic. It is illustrated in Fig. 6 for the phase portrait and Fourier spectrum of the system for the field amplitude $h = 10$.

From these figures it is evident the conservation of the following prominent feature of the soliton dynamics in this limit: it corresponds to the oscillations of two bounded in pair solitons with the average frequency Ω. The amplitude of these oscillations weakly changes in time and these deviations are chaotic.

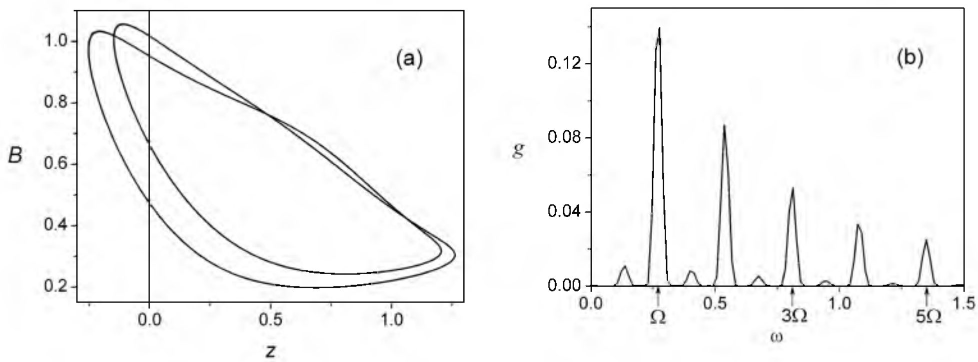

Figure 5. The period doubling of the limit circle (a) and Fourier spectrum (b) for $h = 0.7$

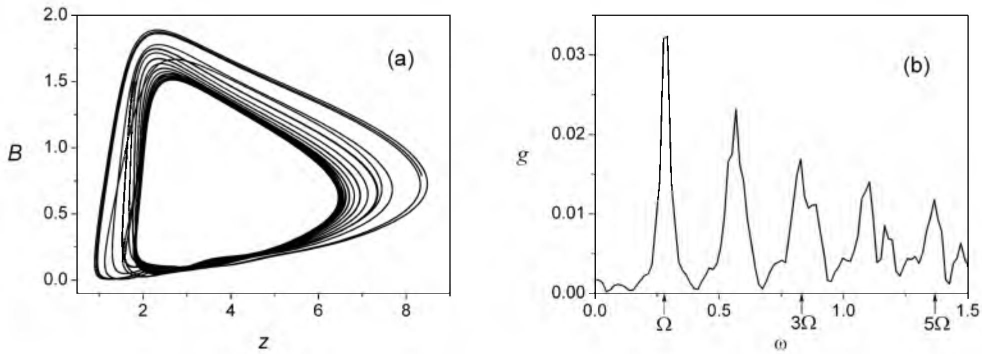

Figure 6. The chaotic behavior of quasi-soliton in phase space (a) and Fourier spectrum (b) for $h = 10$

Conclusions

Thus under the external localized high frequency pumping in nonlinear dissipative media the specific soliton-like excitations may exist. In the range of the small field amplitude these states have the stationary form with the hysteresis of the field dependence. In the larger fields quasi-soliton states transform into the non-stationary breather-like oscillating excitation with the chaotic component in the limit of very strong fields. The results obtained by numerical computations and in the framework of the collective variables approach are qualitatively the same.

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