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Nonlinear Systems with Parity-Time Symmetry

Yuri Kivshar[1*]

Abstract
We review our recent progress in the study of nonlinear systems with balanced gain and loss described by the models with parity-time (PT) symmetry. We discuss the nonlinear dynamics in optical waveguide couplers and show that for intensities below a threshold, the field oscillates between the waveguides, whereas above the threshold the symmetry breaking is observed. This nonlinearity-induced effect can be generalized to other systems, as well as can be observed experimentally. We then move to the systems with continuous variables and study the soliton dynamics and soliton scattering in the systems with PT-symmetric potentials. In particular, we demonstrate that both single and multiple soliton scattering can exhibit almost perfect unidirectional flows associated with an energy exchange between the soliton center of mass and its internal mode.

Keywords
Parity-time symmetry, nonlinear photonics, symmetry-breaking instability, soliton, nonlinear coupler

[1]Nonlinear Physics Center, Research School of Physics and Engineering, Australian National University, Canberra ACT 2615, Australia
* **Corresponding author:** ysk@internode.on.net

Introduction

Recent concepts of plasmonics and metamaterials [1] have opened technologically important capabilities for a design of novel types of photonic devices with the subwavelength confinement of light, however strong dissipative losses are the main obstacle for practical implementations of any of such systems. Exploration of various ideas of gain is very useful for lasing and light amplification, and considerable research efforts have been invested in eliminating or mitigating undesirable absorption by employing the gain mechanisms. However, it is generally impossible to achieve complete loss compensation for all wavelengths [2]. Another, rather nontrivial approach for the loss manipulation is to employ balanced amplification and absorption regions and create a new class of synthetic materials, the so-called *parity-time (or PT) symmetric* materials which can exhibit intriguing properties, including power oscillations and non-reciprocal light propagation.

Parity (P) and time-reversal (T) symmetries are fundamental notions in physics. There has already been much interest in systems that do not obey P and T symmetries separately but which respect the combined PT symmetry. Such systems are described by a Hamiltonian that commutes with the combined PT operator. Despite the fact that PT-Hamiltonians can, in general, be non-Hermitian, their spectra can be entirely real whereas the generated dynamics is (pseudo) unitary. This observation led Bender [3] to propose an extension of quantum mechanics based on non-Hermitian but PT-symmetric operators. Such an approach is distinct from the traditional formulation of Dirac and von Neumann, which requires that all observables (and thus also the Hamiltonian) be represented by Hermitian operators. The property of PT symmetry implies a necessary condition on the quantum potential,

$$V(-r) = V^*(r),$$

with r denoting a vector position and a star being a complex conjugation. The above condition is in general not sufficient, and it can be shown that there exists a critical non-Hermitian threshold beyond which an abrupt phase transition occurs due to the spontaneous PT-symmetry breaking, and the spectrum of eigenstates becomes (partially or entirely) complex [3].

The notion of PT-symmetry has been introduced in optics as a new paradigm to mold the flow of light [4]. This idea has led to new strategies in achieving an interplay between optical gain and loss.

Recently, it was proposed that optics can provide a fertile ground where PT-related concepts can be realized and experimentally tested (see details in Ref. [4]). This is due to the fact that there is close similarity between the effective wave propagation equation and the Schrodinger equation from quantum mechanics where the role of potential in the Schrodinger equation is played by the refractive index in optics. Motivated by this connection, optical systems which exhibit PT symmetry have been formulated [4]. Given that the complex refractive index of an optical one-dimensional system can be written as $n(x) = n_R(x) + i\, n_I(x)$, where $n_R(x)$ is the real part and $n_I(x)$ is the imaginary part of the refractive index, respectively, and x is the transverse coordinate, a PT-symmetric effective optical potential n(x) can be realized by a suitable design for the distribution of gain and loss in the medium, $n_I(x)$. The PT symmetry condition will, in this case, be satisfied with the real part of the refractive index profile of the medium, $n_R(x)$, being even and the gain/loss profile, $n_I(x)$, being an odd function of coordinate x. Recently, light transport in large-scale temporal PT-symmetric lattices has been reported (see Ref. [4] and references therein).

The concept of PT-symmetric potentials can be extended to nonlinear models, and a new class of localized modes was found to exist below the PT symmetry breaking transition. Subsequently, nonlinear modes were also studied in different types of complex PT symmetric potentials. Existence of periodically oscillating solitons in PT symmetric nonlinear couplers with gain or loss were also predicted (see references in Ref. [4]).

This talk aims to give a brief review of our recent progress in the study of nonlinear photonic systems with balanced gain and loss described by the models with the PT symmetry, including the nonlinear dynamics in optical couplers and soliton scattering exhibiting almost perfect unidirectional flow.

Nonlinear modes and oscillating solitons

The net balance of loss and gain can be achieved in a simple geometry of an optical coupler, that was recently realized experimentally [4] using two coupled PT-symmetric waveguides fabricated from iron-doped $LiNbO_3$. Each of the waveguides supports one propagating mode, one providing gain for the guided light and the other experiencing an equal amount of loss.

As was demonstrated in the original study of directional couplers with gain and loss, such structures can offer benefits for all-optical switching in the nonlinear regime, lowering the switching power and attaining sharper switching transition. Recently, Sukhorukov et al. [5] demonstrated that the origin of nonlinear switching in the PT-symmetric directional couplers is related to the effect of suppression of time-reversals. This conclusion was based on the symmetry analysis which is applicable to a broad class of nonlinear local responses, including in particular the cases of cubic or saturable responses. This is an important results in view of experimental realizations of such couplers in different material systems with various nonlinear.

Thus, this study demonstrates that PT-symmetry breaking in directional waveguide coupler with balanced gain and loss occurs through nonlinearly-induced switching, when the input intensity exceeds a threshold value. The switching mechanism is based on the suppression of time-reversals, and a transition of dynamics from periodic oscillations to light concentration and amplification in a single waveguide.

Reflection and transmission of solitons through scattering potentials represents an interesting problem, being of the fundamental interest in condensed matter physics and nonlinear optics. For instance, it was shown that a quantum reflection from barriers and wells has a resonant character, where the internal modes of solitons and local impurity modes are interacting. The case when defects exhibit equal gain/loss coefficients with specific type of PT symmetry is of great interest. Recently, scattering of a broad soliton in a nonlinear lattice with a PT-symmetric defect has been considered [4], where amplification of the soliton during the process of scattering was studied numerically. More recently, unidirectional reflectionless flow was demonstrated experimentally near the spontaneous parity-time symmetry phase transition point. Here we show that under special conditions we can have non-reciprocity in the soliton transmission or a unidirectional soliton flow of solitons giving rise to a diode-type transmission effect.

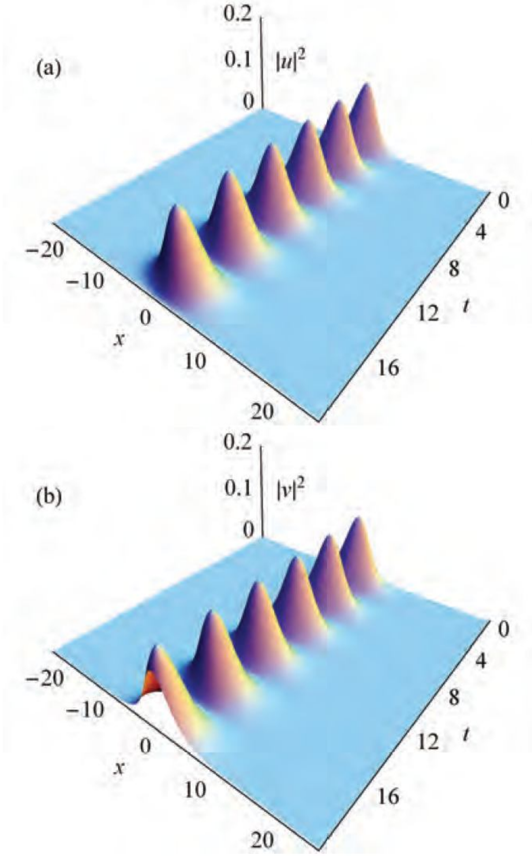

Figure 1. Numerical evolution of the localized initial condition.
Shown are (a) first and (b) second field components of the breather [7].

Finally, the unidirectional flow was also demonstrated for a train of six solitons where the distance between center-of-masses of adjacent solitons and low initial speed. Below this value of separation, the scattered solitons start to interact with incident solitons in a manner that destroys the unidirectional flow. In particular, the reflected dispersive wave excited from the scattering of one soliton will perturb the profile of adjacent incident soliton and its center-of-mass speed leading to a different outcome than for the unperturbed soliton. For the used values of V0 and W0, a minimum separation of 15 guarantees that such perturbations will not affect the unidirectional flow. The potential strength in both cases was fixed with V0=4 and W0=2.5. It is noticed that while the amplitude of the transmitted string of solitons is almost equal to that of the incident one, the amplitude of the reflected solitons is almost doubled. This asymmetry is again a consequence of the fact that while in one case the soliton experience gain followed by loss, in the other case the case it experiences loss followed by gain.

Optical solitons are formed when nonlinear effects compensate the diffractive broadening of light beams (spatial solitons) or dispersive spreading of optical pulses (temporal solitons). Although these localized structures arise both in conservative settings and in the systems with active and loss elements, the properties of dissipative optical solitons show significant differences from those of their conservative counterparts. In particular, in conservative systems the soliton amplitudes vary over continuous ranges of parameters whereas the generic dissipative systems can only support solitons at special amplitudes determined by the balance between gain and loss.

Recently, Alexeeva at al [6] studied the optical solitons and their stability in the PT-symmetric couplers with an extra spatial or temporal degree of freedom. They considered two cases when stationary light beams propagate in a system two coupled planar waveguides [i.e. waveguides extended in the transverse direction], as shown in Fig. 1(a), and the propagation of optical pulses in coupled one-dimensional waveguides [see Fig. 1(b)]. The configuration shown in Fig. 1(a) can also be seen as the continuous limit of the array of coupled waveguides recently discussed in Ref. [5]. We analyze spatial and temporal solitons in such couplers, examine their stability properties, and simulate their nonlinear evolution in the cases of instability.

The PT-symmetric solitons are usually represented as stationary self-localized modes. These arise due to the exact compensation of the gain and loss at each moment of time. A more general type of localized structures are either unstable solitons or spatially-localized temporally-periodic states [7]. These structures look similar to the breathers in conservative systems (such as the φ4 and sine-Gordon equation); hence they were referred to simply as breathers [7].

Recently, Barashenkov et al. [7] presented a comprehensive analytical and numerical study of breathers in the planar PT-symmetric nonlinear optical couplers. They derived the amplitude equations for the oscillatory solutions in this system (equations for the envelopes of the oscillatory wavepackets). These amplitude equations were then be used to show that the (zero-velocity) PT-symmetric breathers form two-parameter families with variable amplitude, localization width, and contrast of power density oscillations. Barashenkov et al. employed these equations to establish the stability of the breathers with small amplitude. Finally, numerical simulations demonstrated that the breathers are generic objects which are commonly formed as a result of soliton collisions.

Barashenkov et al. [7] demonstrated that the breather solution can be constructed as the asymptotic expansion, and they have proved that all small-amplitude breathers are stable on timescale t ~ ε-2, the small-amplitude breather was shown to be a simple oscillation --- it cannot have any incommensurate frequencies in its spectrum. Thus, breathers were shown to be common occurrences in the PT-symmetric chains of dimers. In particular, breathers are born in collisions of the low- and high-frequency solitons.

Importantly, the PT-symmetric breathers are different from their conservative counterparts in that their associated physical observables (e.g. energy and momentum) are not stationary but oscillate in time. From this point of view, the PT-breathers are similar to the time-periodic solitons in dissipative systems. However there is an important distinction between the latter two categories too. Namely, the dissipative solitons are limit cycles (in an infinite-dimensional phase space); their amplitudes and periods are determined uniquely by the parameters of the system. On the contrary, the PT-breathers arise as members of two-parameter families, similar to periodic trajectories in Hamiltonian systems.

Conclusion and outlook

In this talk, we present a brief review of our recent results on the study of nonlinear photonic systems with balanced gain and loss described by the models with parity-time symmetry. First, we have discussed the nonlinear dynamics in optical waveguide couplers and showed that for intensities below a threshold level, the amplitudes oscillate between the waveguides, whereas above a threshold level we observe the symmetry breaking. This major effect can be generalized to other systems, as well as observed experimentally. Then we have analyzed the systems with continuous variables and studied the soliton dynamics and soliton scattering in the systems with PT-symmetric potentials. In particular, we have demonstrated that, for a certain range of parameters, both single and multiple soliton scattering exhibit almost perfect unidirectional flow associated with nontrivial energy exchange between the soliton center of mass and its internal mode.

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